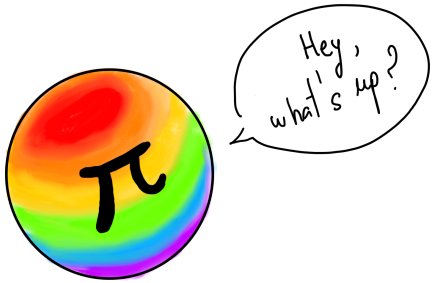
Meson interactions at Large N_c from Lattice QCD



Fernando Romero-López

In collaboration with: A. Donini, P. Hernández & C. Pena

arXiv:1607.03262, arXiv:1810.06285, arXiv:1907.11511 and on-going work

University of Valencia, IFIC

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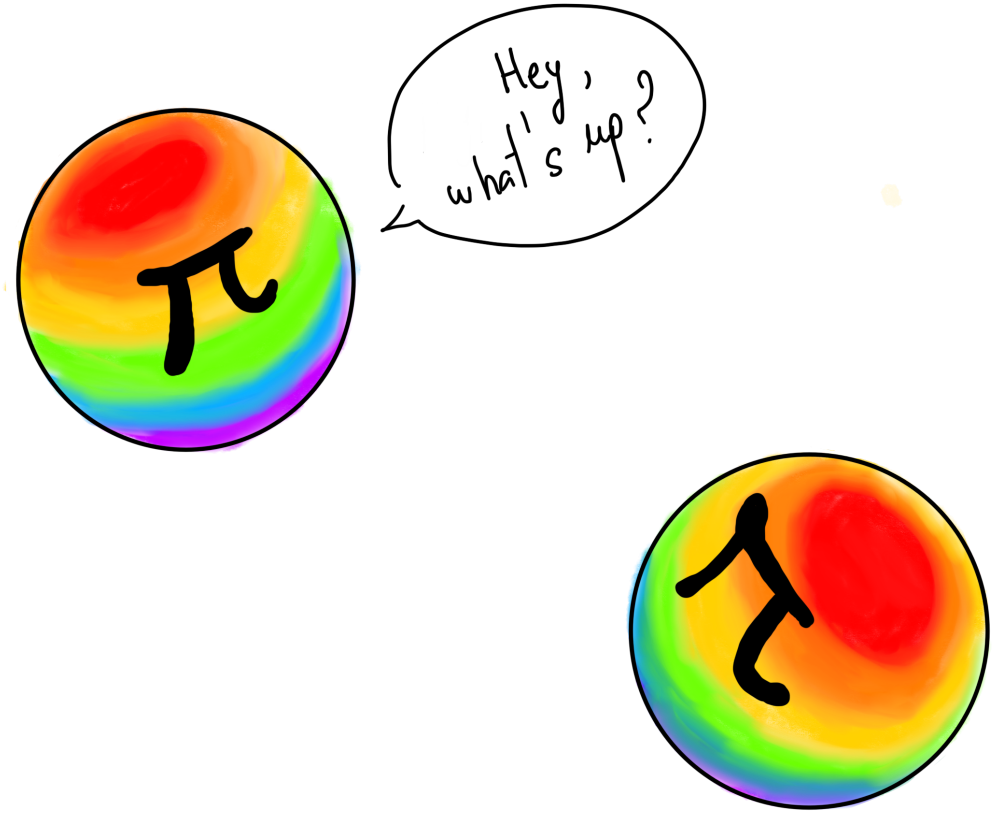
CERN, August 2nd, 2019

Overview of meson interactions at Large N_c



Motivation

The Large N_c limit ('t Hooft limit)

- Let QCD be a $SU(N_c)$ gauge theory
  - ★ $N_c \to \infty$
  - ★ $\alpha_s N_c =$ constant

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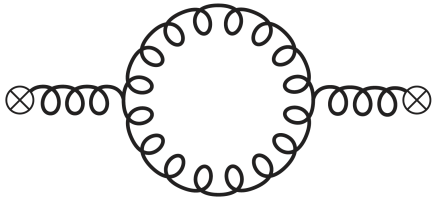
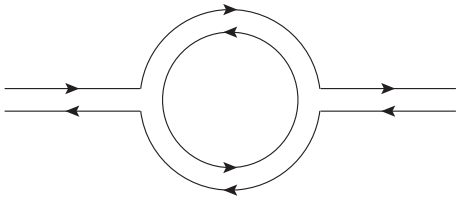
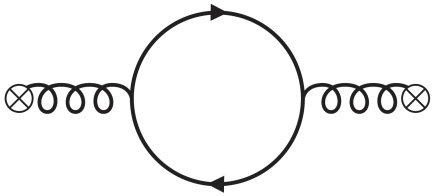
Example of N_c counting:

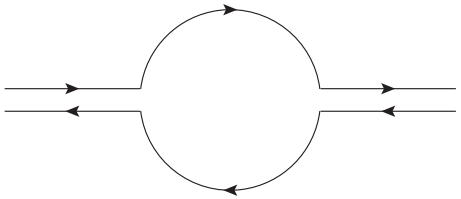
1. Gluon loops $\sim N_c^2$





2. Quark loops $\sim N_c$





Chiral Perturbation Theory

ChPT is an EFT for QCD at low energies in terms of mesons.

$$\phi = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}, \qquad U = e^{i\phi/F_0},$$

U transforms as:

$$U \to L^\dagger U R, \text{ with } L, R \in SU(N_f)$$

The lowest order ($p^2 \sim M$) Lagrangian with the QCD symmetries:

$$\mathcal{L}_2 = \frac{F_0^2}{4}\text{tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) + \frac{F_0^2 B}{2}\text{tr}\left(MU + M^\dagger U^\dagger\right),$$

with $M = \text{diag}\,(m_u, m_d, m_s)$.

Chiral Perturbation Theory at NLO and Large N_c

1. The LO Lagrangian ($\mathcal{L}_2$) of ChPT is very predictive with few parameters: F_0, B and quark masses.

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$$\mathcal{L}_4 = \sum_{i=0}^{10} L_i \mathcal{O}_i.$$

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- LECs encode the high energy physics information
- They have different Large N_c behaviour

L_i	Value	Order
$2L_1 - L_2$	-0.4 ± 0.2	1
L_4	-0.0 ± 0.3	1
L_6	0.0 ± 0.4	1
L_7	-0.3 ± 0.2	1
L_2	1.6 ± 0.2	N_c
L_3	-3.8 ± 0.3	N_c
L_5	1.2 ± 0.1	N_c
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Bijnens & Ecker, 2014

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- Large N_c allows for important simplifications!

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The Large N_c limit in Phenomenology

"Large N_c-inspired" approximations are usual in phenomenology.

Updated Standard Model Prediction for ε'/ε *

Hector Gisbert[a,**], Antonio Pich[a]

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Apt. Correus 22085, E-46071 València, Spain

6. The SM prediction for ε'/ε

Taking into account all computed corrections in Eq. (7), our SM prediction for ε'/ε is

$$\mathrm{Re}\,(\varepsilon'/\varepsilon) = \left(15 \pm 2_\mu \pm 2_{m_s} \pm 2_{\Omega_{\rm eff}} \pm 6_{1/N_C}\right) \times 10^{-4}$$

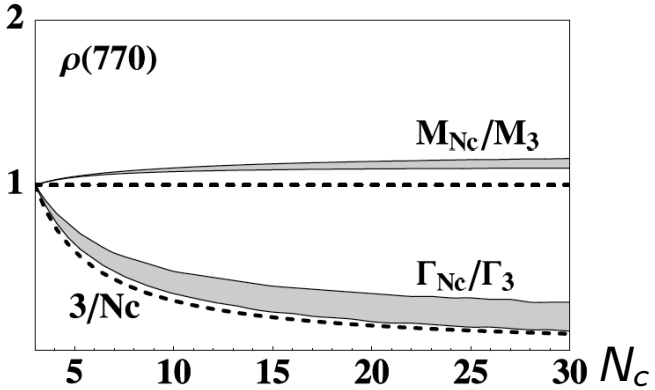
- Uncertainties from Large N_c are hard to estimate.
- Can Lattice QCD improve this?

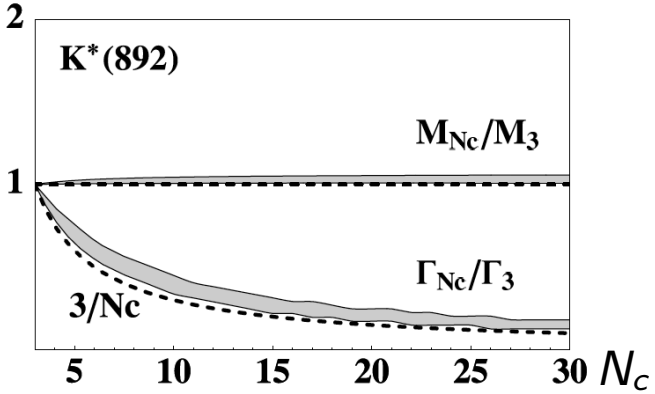
The Large N_c limit in Phenomenology

Properties of light resonances from unitarized Chiral perturbation theory: Nc behavior and quark mass dependence

J. R. Peláez[a],*) J. Nebreda[a], G. Ríos[a]

[a] *Dept. de Física Teórica II. Universidad Complutense. 28040 Madrid. Spain*





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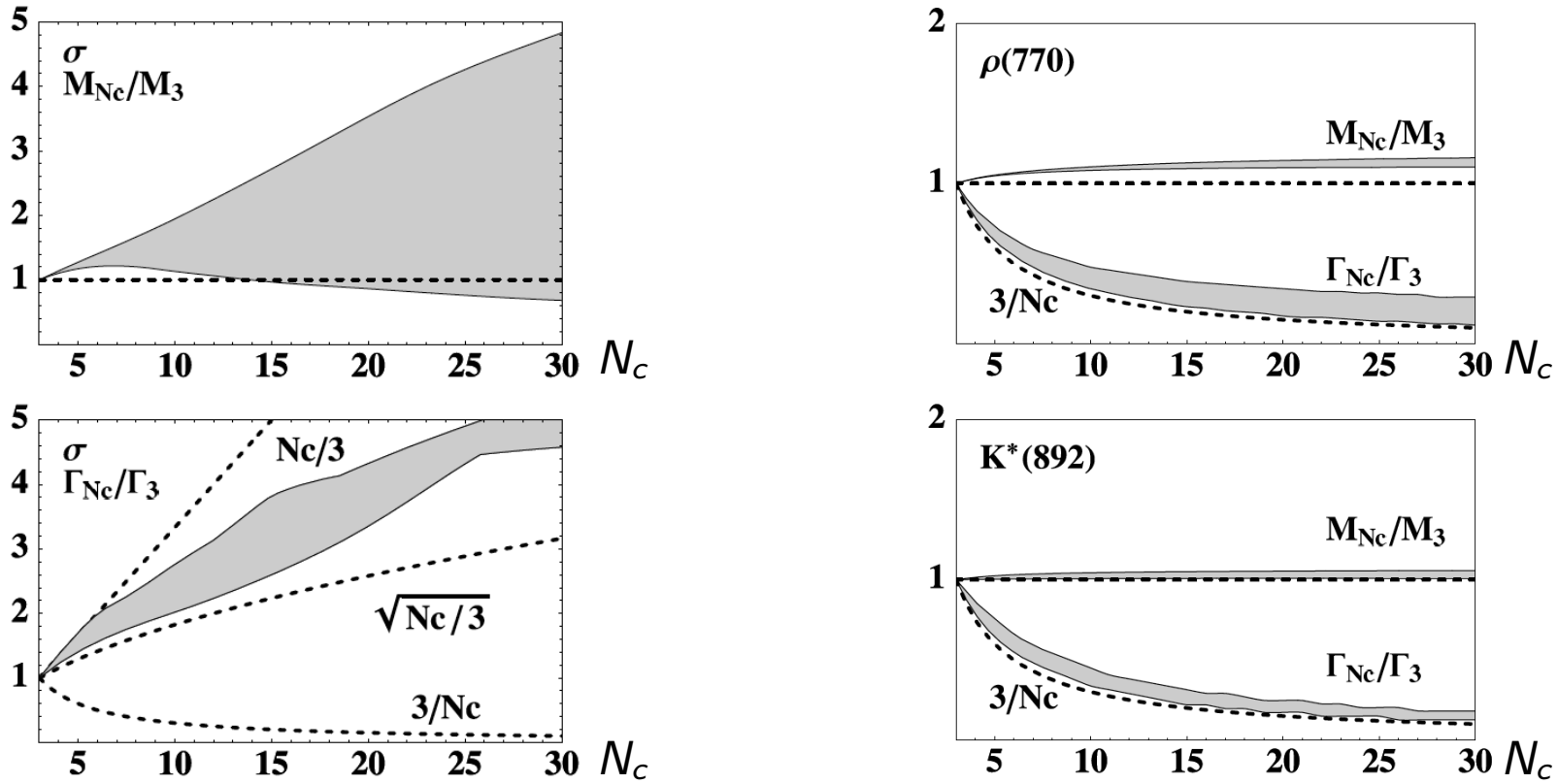

Lattice QCD at Large N_c

$\to$ The Large N_c is often very useful. However:

1. Systematic errors are hard to estimate.
2. It fails for few observables: $K \to \pi\pi$ ($\to$later!)

Lattice QCD can help!

Topics that we aim to address at Large N_c from lattice simulations:

- Meson masses and decay constants
- Meson scattering (isospin-2, isospin-1/ρ, isospin-0)
- $K \to \pi$ and $K \to \pi\pi$
- η' meson
- Tetraquarks

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1. Finished
2. Started
3. Planned

Ensembles at Large N_c

Our Large N_c Ensembles with $N_f = 4$

- Iwasaki gauge action and $O(a)$ improved Wilson fermions.

Ensemble	N_c	$L \times T$	β	m_0	aM	M (MeV)
A301	3	20×36	1.778	-0.4040	0.2191(36)	570
A302		24×48		-0.4060	0.1831(17)	480
A303		24×48		-0.4070	0.1612(24)	420
A304		32×60		-0.4080	0.1384(15)	360
A401	4	20×36	3.570	-0.3725	0.2035(14)	530
A402		24×48		-0.3752	0.1804(7)	470
A403		24×48		-0.3760	0.1714(8)	440
A404		32×60		-0.3780	0.1397(8)	360
A501	5	20×36	5.969	-0.3458	0.2128(9)	560
A502		24×48		-0.3490	0.1802(6)	470
A503		24×48		-0.3500	0.1712(6)	450
A504		32×60		-0.3530	0.1328(8)	350
A601	6	20×36	8.974	-0.3260	0.2150(7)	570
A602		24×48		-0.3300	0.1801(5)	470
A603		24×48		-0.3311	0.1690(7)	450
A604		32×60		-0.3340	0.1354(7)	360

P. Hernandez, C. Pena and FRL *et al.*, *arXiv:1810.06285, arXiv:1907.11511*
Generated with HiRep, M. Hansen *arXiv:1705.11010*

Scale Setting at Large N_c

Use observables from Gradient Flow (Lüscher):

$$\langle t^2 E(t) \rangle = \frac{3}{128\pi^2} \frac{N_c^2 - 1}{N_c} \lambda_{GF}(\mu)$$

with $\mu = 1/\sqrt{8t}$ and $\lambda_{GF} = N_c g_{GF}^2$ ('t Hooft coupling).

For QCD , t_0 is defined through the implicit equation:

$$\langle t^2 E(t) \rangle \Big|_{t=t_0} = 0.3.$$

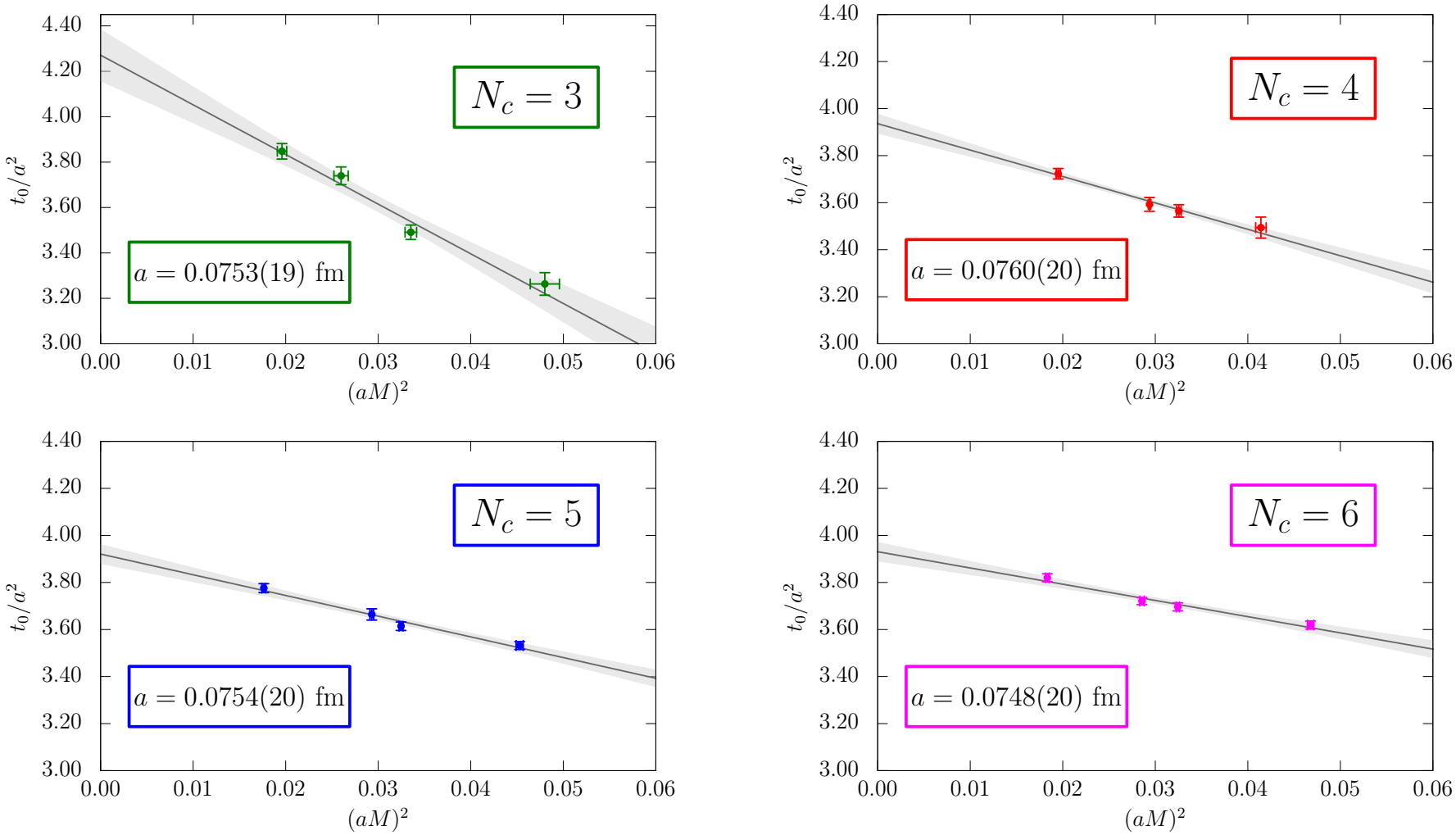
Input $\longrightarrow$ $\sqrt{t_0}$ from other lattice simulations

Generalization for arbitrary N_c:

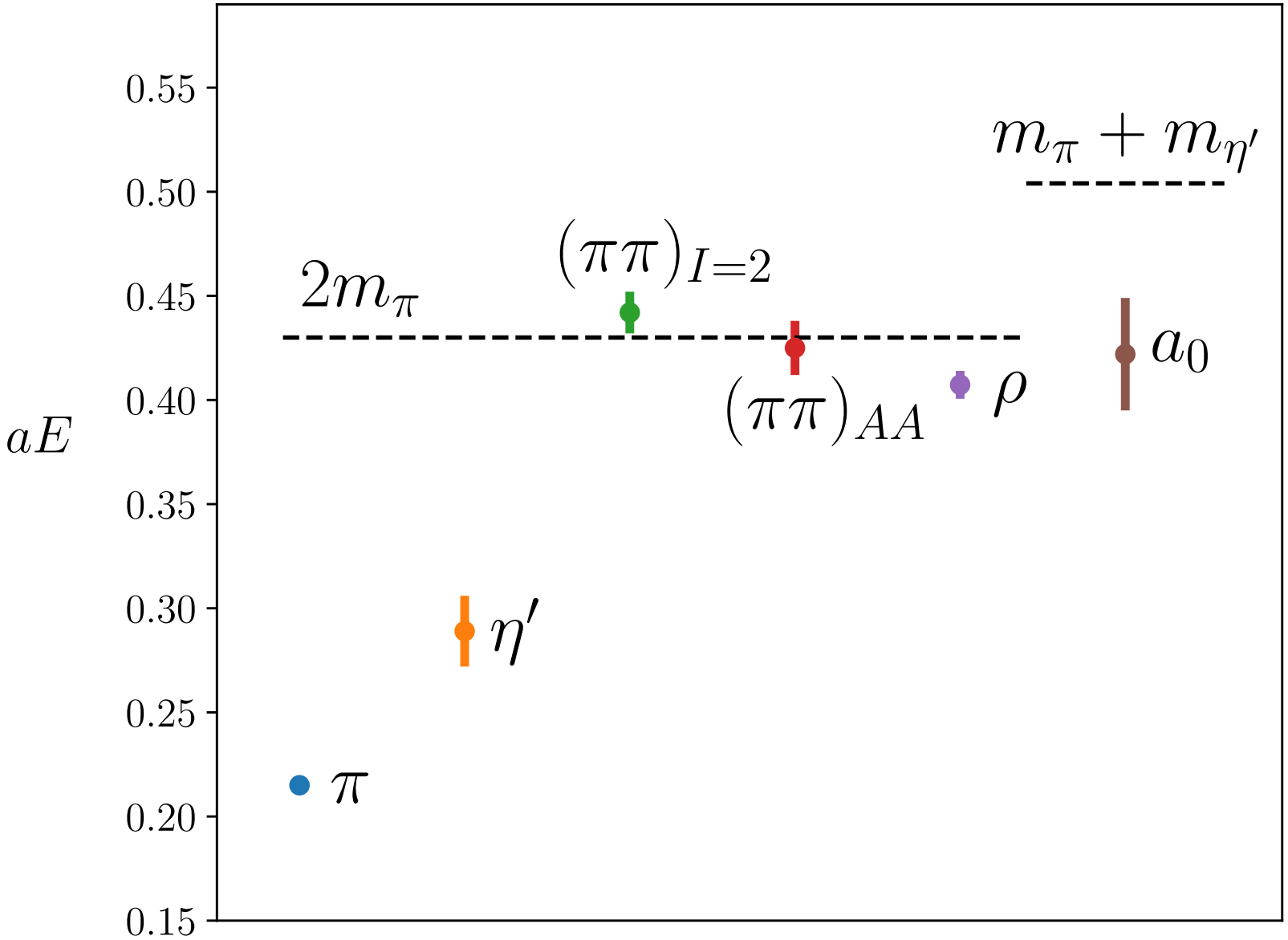
$$\langle t^2 E(t) \rangle \Big|_{t=t_0} = 0.1125 \frac{N_c^2 - 1}{N_c}, \quad (M\sqrt{t_0})\Big|_{M=420\text{ MeV}} = 0.3090(83)$$

Mass dependence of t_0

$$t_0(M^2) = t_0^{\text{chiral}}\left(1 + kM^2\right) + O(M^4) \xrightarrow{\text{Large } N_c} t_0^{\text{chiral}}$$



Spectrum: $N_c = 6$, $M_\pi = 560$ MeV



M_π and F_π at Large N_c

Large N_c scaling of meson masses and decay constants

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[1] *IFIC (CSIC-UVEG), Edificio Institutos Investigación, Apt. 22085, E-46071 Valencia, Spain*
[2] *Departamento de Física Teórica and Instituto de Física Teórica UAM-CSIC, Universidad Autónoma de Madrid, E-28049 Madrid, Spain*

(Dated: July 29, 2019)

We perform an *ab initio* calculation of the N_c scaling of the low-energy couplings of the chiral Lagrangian of low-energy strong interactions, extracted from the mass dependence of meson masses and decay constants. We compute these observables on the lattice with four degenerate fermions, $N_f = 4$, and varying number of colours, $N_c = 3 - 6$, at a lattice spacing of $a \simeq 0.075$ fm. We find good agreement with the expected N_c scaling and measure the coefficients of the leading and subleading terms in the large N_c expansion. From the subleading N_c corrections, we can also infer the N_f dependence, that we use to extract the value of the low-energy couplings for different values of N_f. We find agreement with previous determinations at $N_c = 3$ and $N_f = 2, 3$ and also, our results support a strong paramagnetic suppression of the chiral condensate in moving from $N_f = 2$ to $N_f = 3$.

Meson decay constant

- The decay constant parametrizes the hadronic part of $\pi^+ \to \ell^+ + \nu_\ell$.
- It is defined as:

$$\langle 0|A_\mu(0)|\pi^+(q)\rangle = -iq_\mu\sqrt{2}F_\pi$$

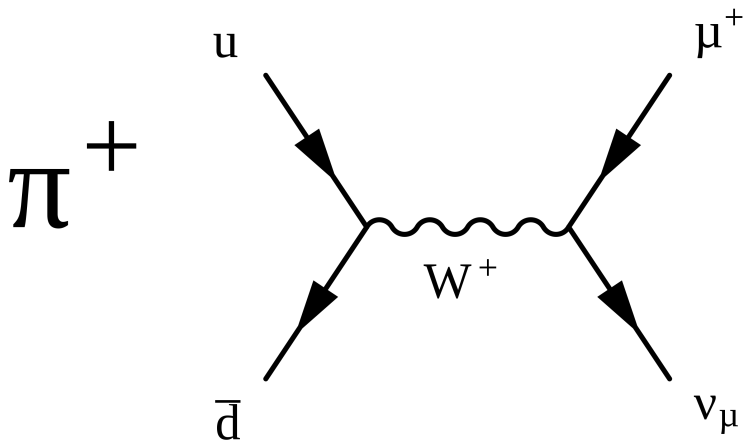
with $A_0 = \bar{q}\gamma_0\gamma_5 q$.

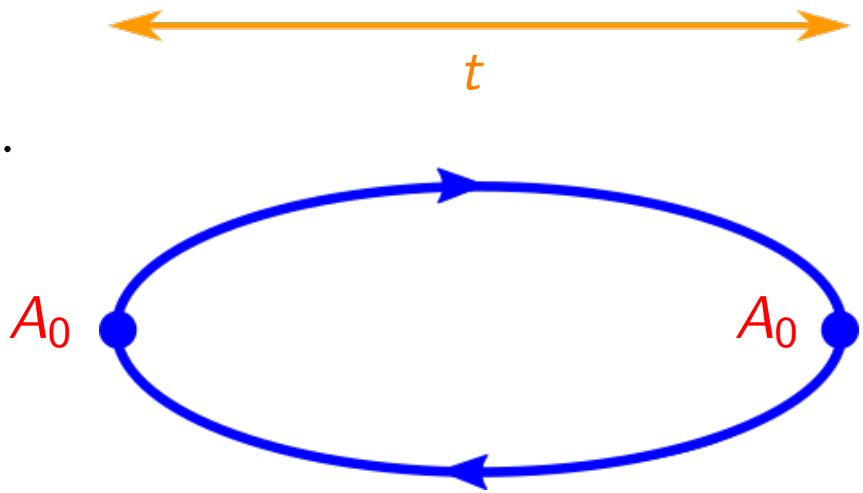
- On the lattice:

$$C_A(t) = \langle A_0(0)A_0(t)\rangle \propto F_\pi^2 e^{-M_\pi t}.$$

- By simple counting of traces,

$$F_\pi^2 = O(N_c)$$

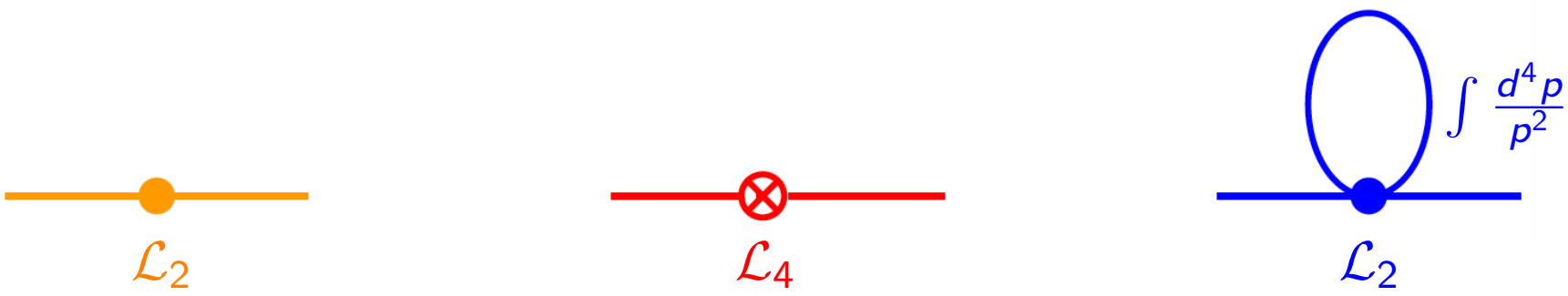




Meson decay constant at Large N_c

In Chiral Perturbation Theory:

$$F_\pi = F\left[1 + \frac{M_\pi^2}{F_\pi^2}\Big(4L_5(\mu) + 4N_f L_4(\mu)\Big) + \frac{N_f}{2}\frac{M_\pi^2}{(4\pi F_\pi)^2}\log\frac{M_\pi^2}{\mu^2}\right]$$



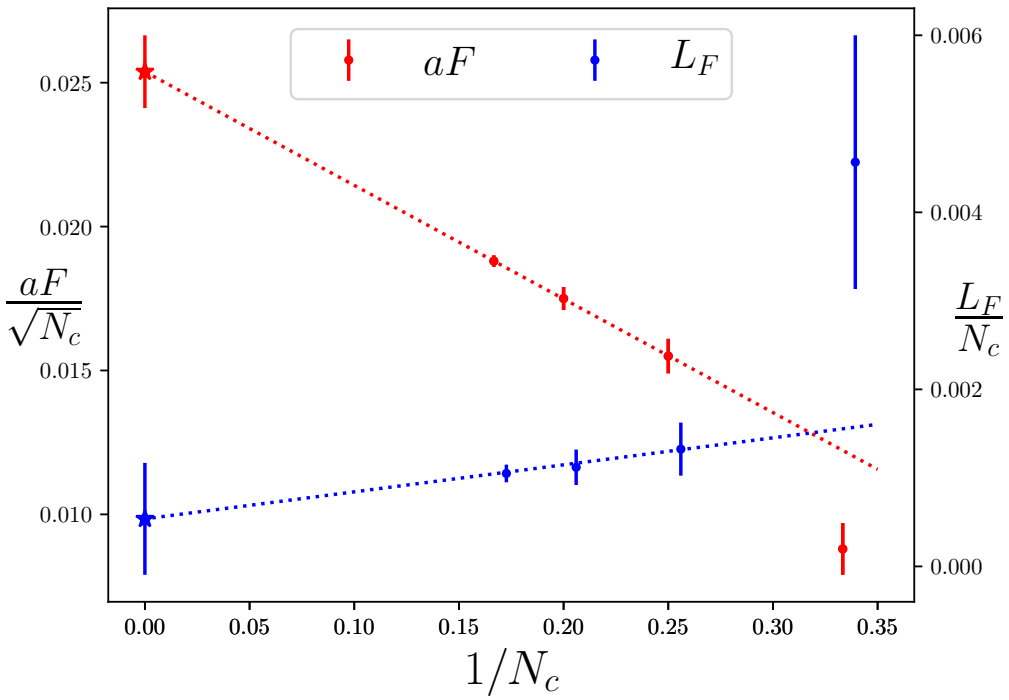
1. $F_\pi^2 = O(N_c)$
2. $L_5 = O(N_c)$
3. $L_4 = O(1)$

$$\xrightarrow{\text{Large } N_c} \quad F_\pi = F\left[1 + 4\frac{M_\pi^2}{F_\pi^2}L_5\right]$$

Large N_c Scaling of Low Energy Constants for F_π

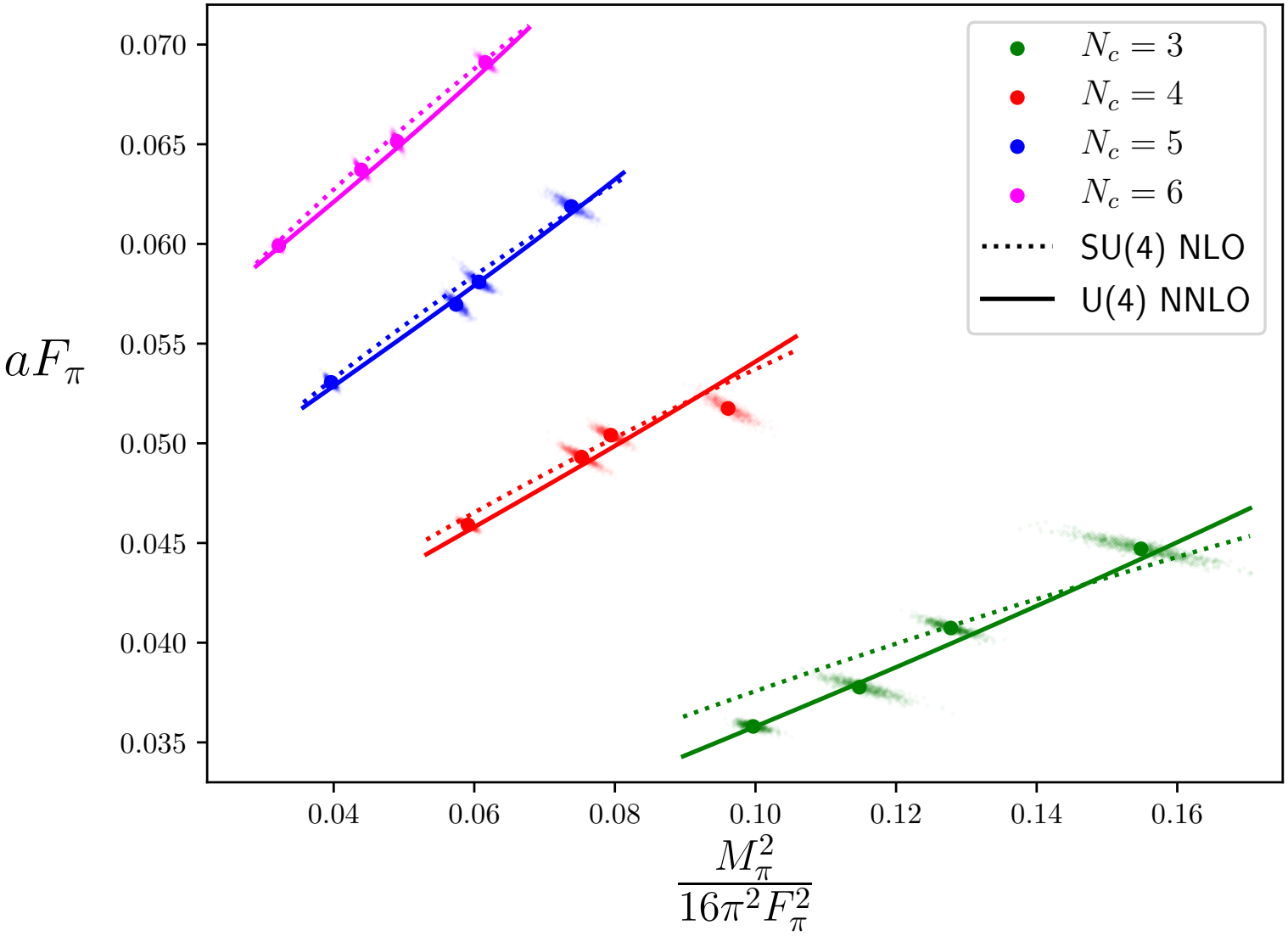
$$F_\pi = F\left[1 + \frac{M_\pi^2}{F_\pi^2}\left(4L_F\right) + \frac{N_f}{2}\frac{M_\pi^2}{(4\pi F_\pi)^2}\log\frac{M_\pi^2}{\mu^2}\right]$$

$$F = \sqrt{N_c}\left(F_0 + \frac{F_1}{N_c}\right) \qquad L_F = N_c L_F^{(0)} + L_F^{(1)}$$



F_π at Large N_c

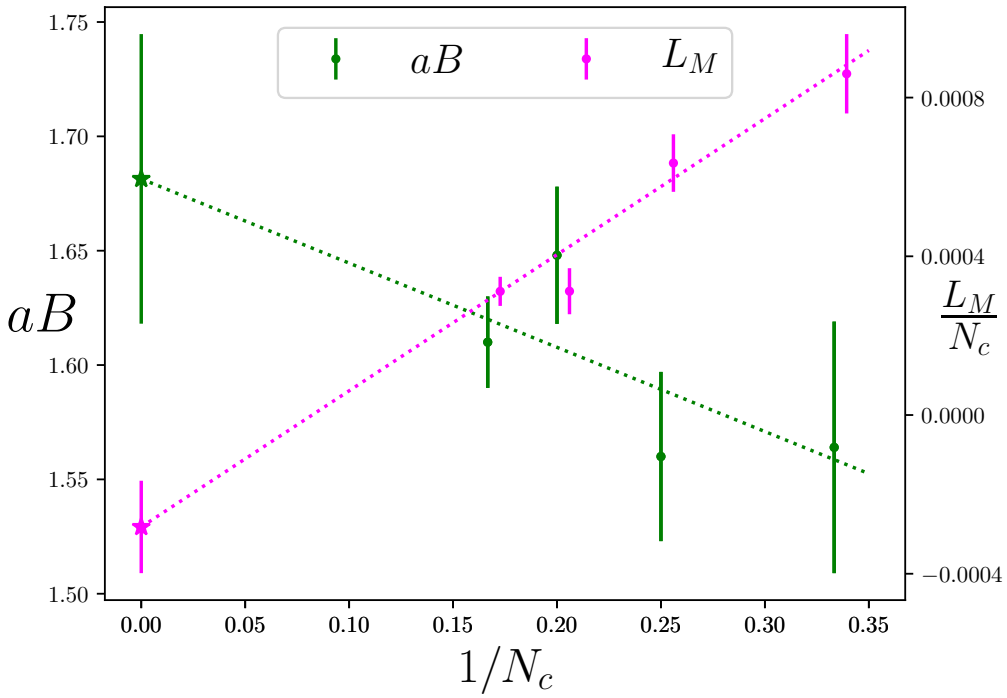
Simultaneous chiral and N_c fit.



Large N_c scaling of Low Energy Constants for M_π

$$M_\pi^2 = 2Bm\left[1 + \frac{M_\pi^2}{F_\pi^2}\left(8L_M\right) + \frac{1}{N_f}\frac{M_\pi^2}{(4\pi F_\pi)^2}\log\frac{M_\pi^2}{\mu^2}\right]$$

$$B = B_0 + \frac{B_1}{N_c} \qquad L_M = N_c L_M^{(0)} + L_M^{(1)}$$



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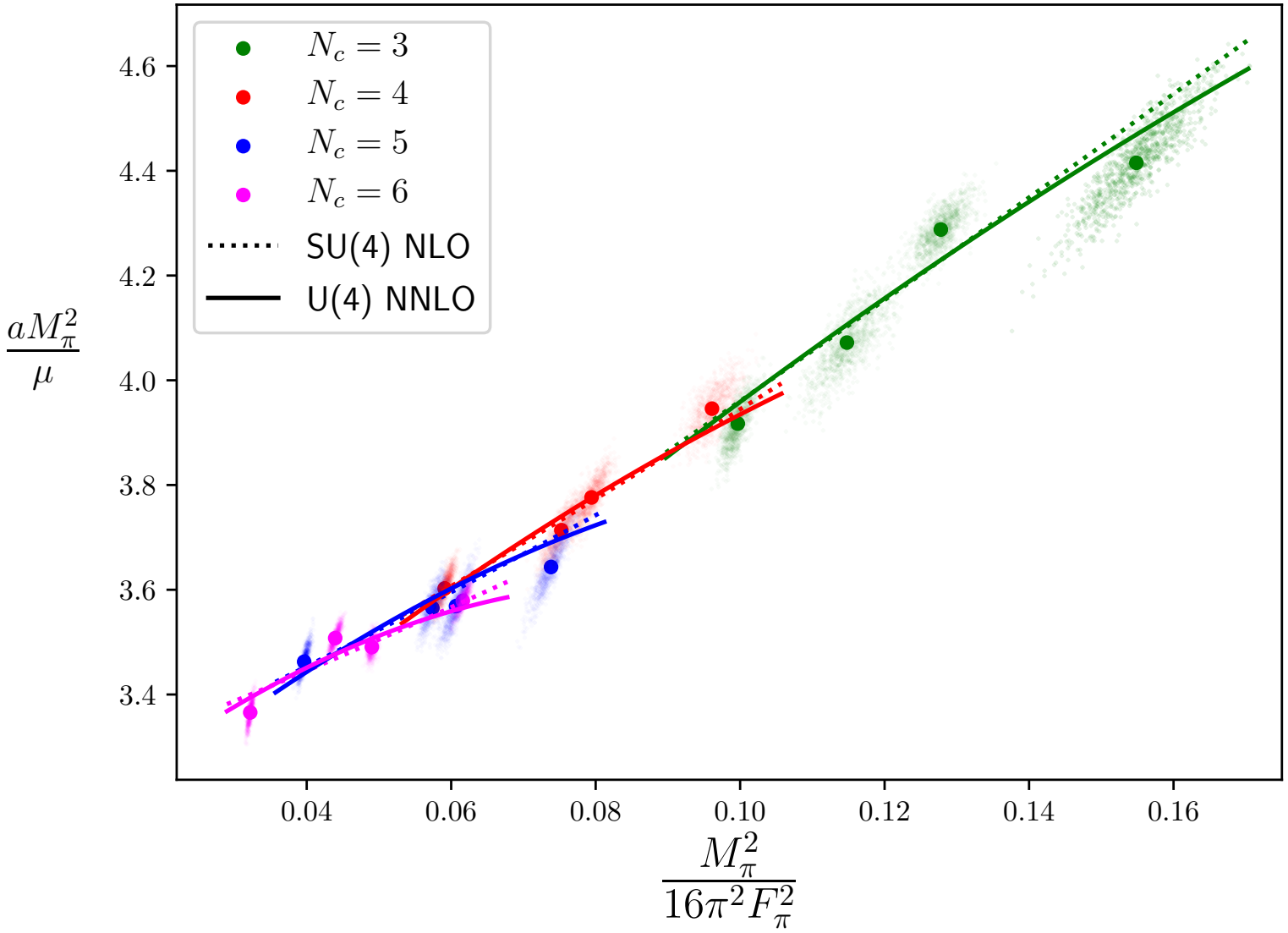


Selected Results

From the fit, we can infer the N_c and N_f dependence:

$$\frac{F}{\sqrt{N_c}} = \left[67(3) - 26(4)\frac{N_f}{N_c}\right] \quad \text{MeV}$$

$N_f = 2 \to F = 86(3)$ MeV

$N_f = 3 \to F = 71(3)$ MeV

ETM 15A	[125]	86.3(2.8)
Engel 14	[88]	85.8(0.7)(2.0)
Brandt 13	[147]	84(8)(2)
QCDSF 13	[161]	86(1)
TWQCD 11	[150]	83.39(35)(38)
ETM 09C	[111]	$85.91(07)\left(^{+78}_{-07}\right)$
ETM 08	[17]	86.6(7)(7)
Hasenfratz 08	[153]	90(4)
JLQCD/TWQCD 08A	[108]	$79.0(2.5)(0.7)\left(^{+4.2}_{-0.0}\right)$
JLQCD/TWQCD 07	[154]	87.3(5.6)
Colangelo 03	[162]	86.2(5)

Collaboration	Ref.	N_f	F_0
JLQCD/TWQCD 10A	[138]	3	71(3)(8)
MILC 10	[114]	2+1	80.3(2.5)(5.4)
MILC 09A	[112]	2+1	78.3(1.4)(2.9)
MILC 09	[110]	2+1	
PACS-CS 08	[145]	2+1	83.8(6.4)
RBC/UKQCD 08	[146]	2+1	66.1(5.2)

FLAG 2019

Selected Results

- Predict the decay constant LECs at $N_f = 2$

$$\bar{\ell}_4 = \mathbf{5.1(3)}, \text{ vs. FLAG 2019 } \bar{\ell}_4 = \mathbf{4.40(28)}$$

- Predict the chiral condensates at $N_f = 3$

$$\Sigma_0^{1/3}(N_f = 3) = \mathbf{223(9)} \text{ MeV vs. [Fukaya } et\ al.\text{] } \mathbf{214(6)(24)} \text{ MeV}$$

- Predict the ratio of chiral condensates at at $N_f = 2, 3$

$$\frac{\Sigma(N_f = 2)}{\Sigma(N_f = 3)} = \mathbf{1.49(10)} \text{ vs. [Bernard } et\ al.\text{] } \mathbf{1.51(11)}$$

- We find that the subleading parts to the LECs are sizeble. For the meson mass LEC we obtain:

$$\frac{L_M^{N_f=4}}{N_c} \cdot 10^3 = -\mathbf{0.2(2)} + \frac{\mathbf{2.9(6)}}{N_c} + O(N_c^{-2}).$$

Scattering at Large N_c

Scattering in infinite volume

- Define asymptotic states: $|\phi_{\text{IN}}\rangle\,, \quad |\phi_{\text{OUT}}\rangle$
- The Scattering Matrix relates these states:

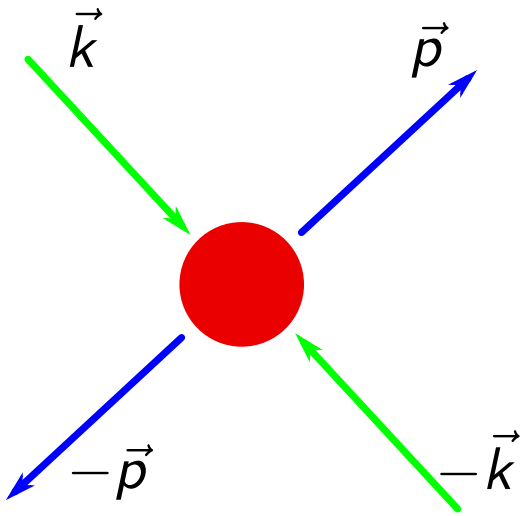
$$|\phi_{\text{OUT}}\rangle = \hat{S}\,|\phi_{\text{IN}}\rangle$$

- The Phase Shifts parametrize the S Matrix

$$\langle \vec{k}\ell m|\hat{S}|\vec{p}\ell m\rangle = S_\ell = e^{2\delta_\ell(k)}\delta\left(|\vec{k}| - |\vec{p}|\right)$$

- Effective range expansion (s-wave):

$$k\cot\delta_0 = -\frac{1}{a_0} + \frac{1}{2}rk^2 + O(k^4)$$



Scattering in finite volume

Lüscher method

Make use of finite volume artefacts to study interactions

$$\det\left[\cot\delta + \mathcal{M}\right] = 0$$

S Matrix

kinematical quantity

$$k\cot\delta_0 = -\frac{1}{a_0} + \frac{1}{2}rk^2 + O(k^4)$$

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scattering length

For the ground state,a simpler formula is available:

(Huang & Yang, Lüscher, Hansen & Sharpe)

$$E - 2m = \frac{4\pi a_0}{mL^3}\left(1 + c_1\frac{a_0}{L} + c_2\left(\frac{a_0}{L}\right)^2 + c_3\left(\frac{a_0}{L}\right)^3 + \frac{2\pi r(a_0)^2}{L^3} - \frac{\pi a_0}{m^2L^3}\right)$$

$\Rightarrow$ At order L^{-5}, the ground state is solely explained by a_0

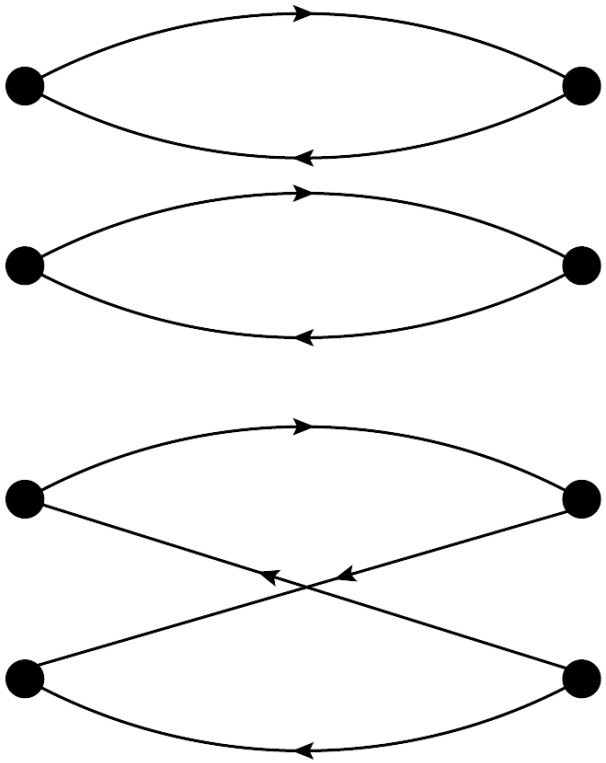
Isospin 2 $\pi\pi$ scattering

$\pi^+\pi^+ \to \pi^+\pi^+$ is the simplest application to QCD

Phenomenological value (Ynduráin, 2002)

$$M_\pi a_0^{I=2} = 0.0422(22)$$

- Weakly coupled
- Less noisy
- Extract LECs of ChPT



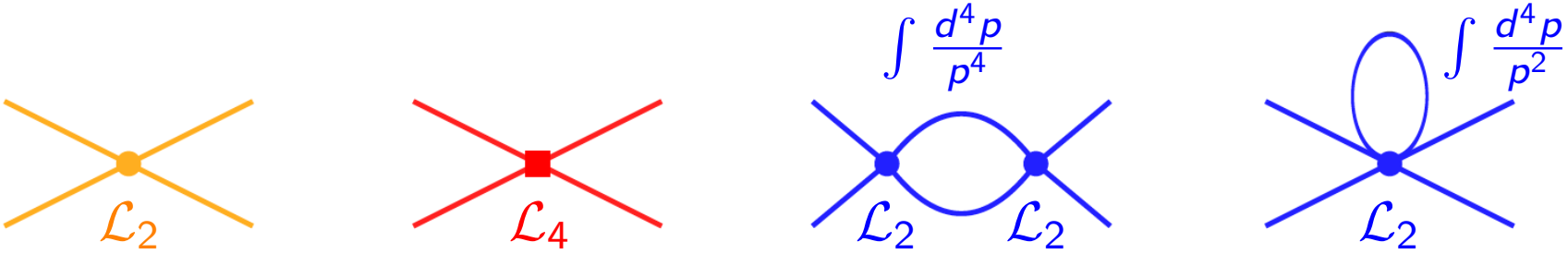
$$C_{\pi\pi} = \langle \pi\pi(0)\pi^\dagger\pi^\dagger(t)\rangle$$

I=2 $\pi\pi$ scattering at Large N_c

In Chiral Perturbation Theory with $N_f = 4$ (Bijnens *et al.*)

$$M_\pi a_0^{I=2} = -\frac{M_\pi^2}{16\pi F_\pi^2}\left[1 - \frac{16M_\pi^2}{F_\pi^2}L_{\pi\pi}(\mu) - \frac{M_\pi^2}{32\pi^2 F_\pi^2}\left(\frac{13}{4}\log\frac{M_\pi^2}{\mu^2} - \frac{3}{4}\right)\right]$$

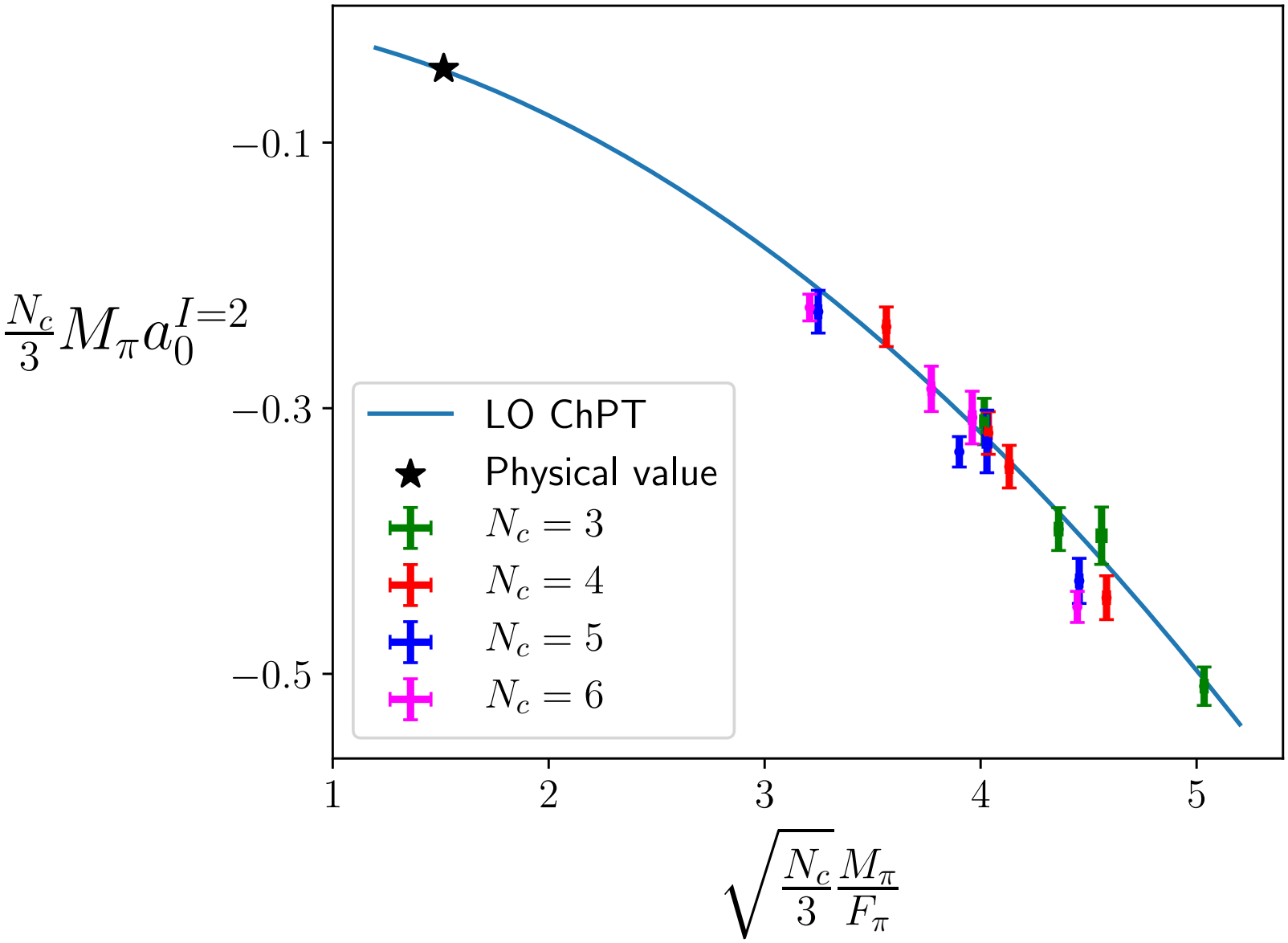
with $L_{\pi\pi} = L_0 + 2L_1 + 2L_2 + L_3 - 2L_4 - L_5 + 2L_6 + L_8$



1. $F_\pi^2 = O(N_c)$
2. $L_0, L_5, L_3, L_8 = O(N_c)$
3. $L_1, L_2, L_4, L_6 = O(1)$

$$\xrightarrow{\text{Large } N_c} \quad a_0^{I=2} \propto \frac{1}{N_c}(1 + \text{LECs})$$

Preliminary results for Isospin 2 $\pi\pi$ scattering



Other scattering channels

In a general theory $N_f > 3$, there are more scattering channels (Bijnens *et al.*)

$$a_0^{I,\text{tree}} = \frac{M_\pi^2}{16\pi F_\pi^2}\left(2N - \frac{1}{N}\right)$$

$$a_0^{S,\text{tree}} = \frac{M_\pi^2}{16\pi F_\pi^2}\left(N - \frac{2}{N}\right)$$

$\pi\pi$ isospin-0

$$a_1^{A,\text{tree}} = \frac{M_\pi^2}{48\pi F_\pi^2} N$$

$$a_1^{SA,\text{tree}} = a_1^{AS,\text{tree}} = 0$$

$\pi\pi$ isospin-1

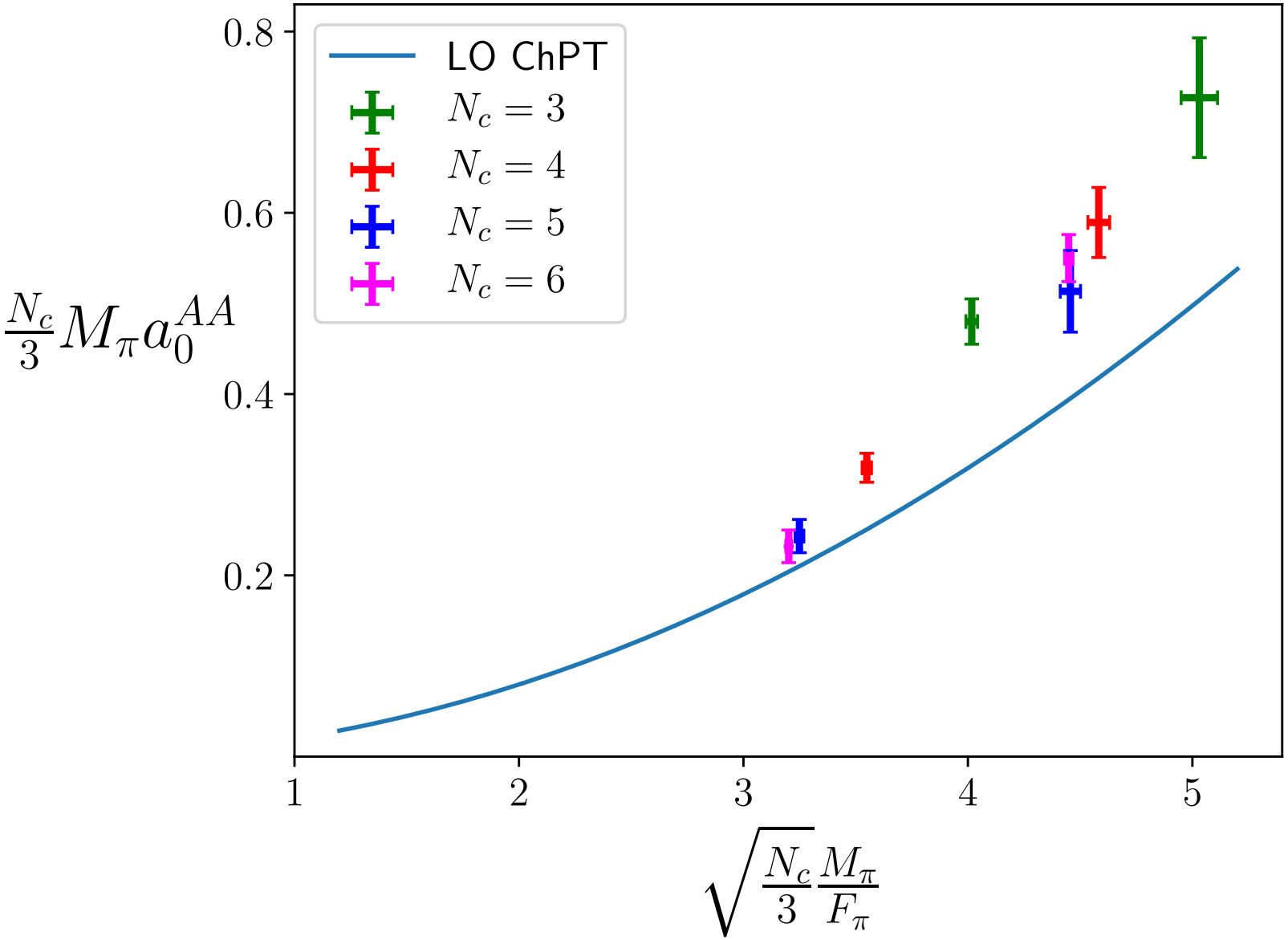
$$a_0^{SS,\text{tree}} = -\frac{M_\pi^2}{16\pi F_\pi^2}$$

$\pi\pi$ isospin-2

$$a_0^{AA,\text{tree}} = \frac{M_\pi^2}{16\pi F_\pi^2}\,.$$

$\pi D_s - KD$

Preliminary results for $\pi D_s - KD$ scattering



$K \to \pi\pi$ at Large N_c

Large N_c limit for $K \to \pi\pi$

Weak decay with two isospin final states, $I = 0, 2 \to A_0, A_2$

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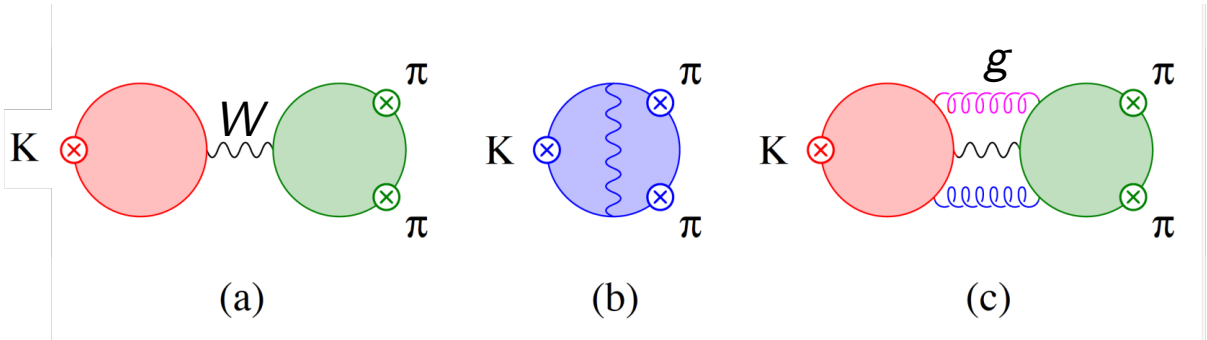
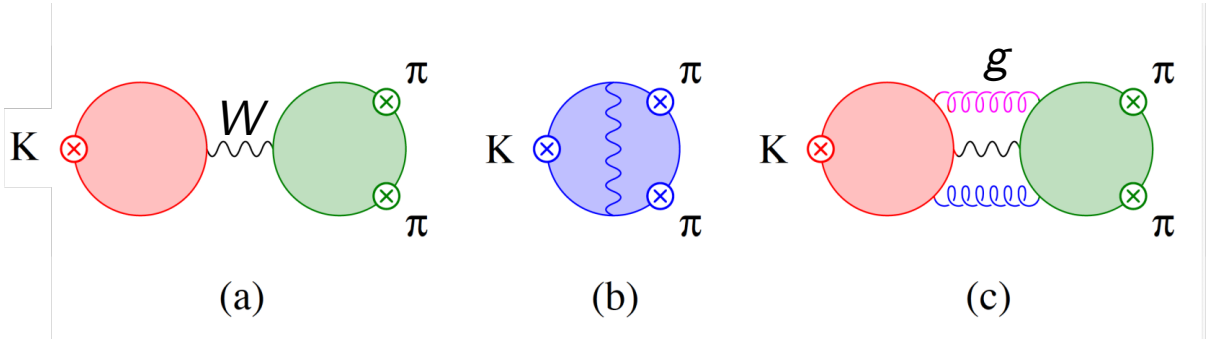


Large N_c limit for $K \to \pi\pi$

Weak decay with two isospin final states, $I = 0, 2 \to A_0, A_2$

- Diag.(a) has 2 quark loops $\to O(N_c^2)$
- Diag.(b) has 1 quark loop $\to O(N_c)$
- Diag.(c) has 2 quark loops and 4 vertices $\to O(\alpha_s^2 N_c^2) = O(1)$



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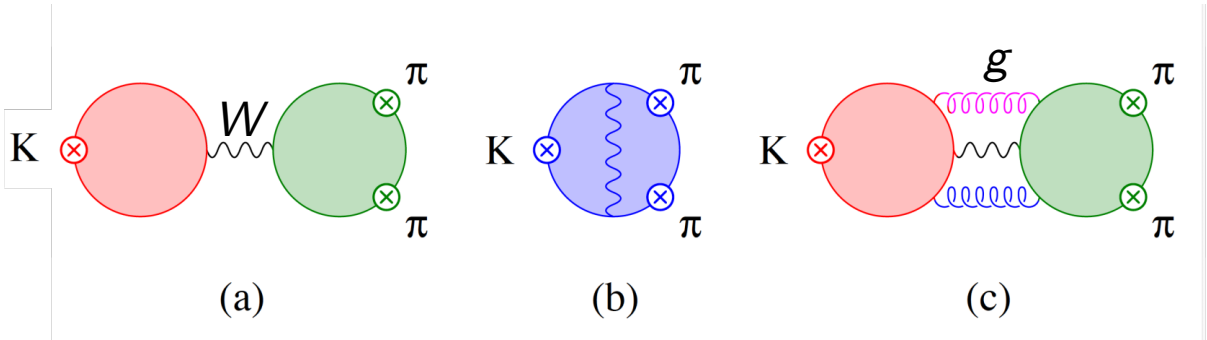

$\Rightarrow$For neutral particles, Diag.(a) is not present:

$$A(K^0 \to \pi^0\pi^0) = A_0 - \sqrt{2}A_2 = 0 \text{ at Large } N_c$$

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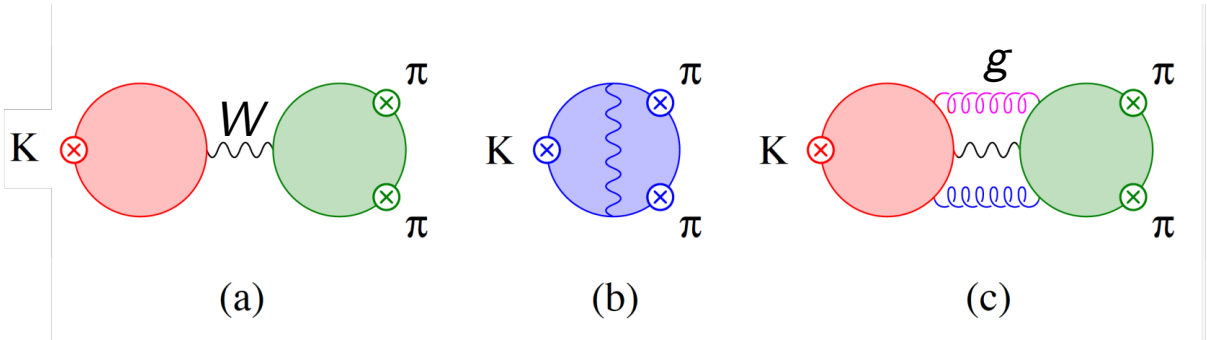

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$$\frac{\text{Re } A_0}{\text{Re } A_2} = \sqrt{2}$$

Large N_c limit for $K \to \pi\pi$

- The Large N_c prediction for $K \to \pi\pi$ is (*Manohar, Large N QCD*):

$$\frac{\text{Re } A_0}{\text{Re } A_2} = \sqrt{2}$$

- Experimental values for $K \to \pi\pi$ are very well measured in two isospin channels, $I = 0, 2$ and Large N_c fails.

$$\frac{\text{Re } A_0}{\text{Re } A_2} \simeq 22 \gg \sqrt{2}\Big|_{\text{Large } N_c},$$

- State of the art result by *RBC-UKQCD, 2015*:

$$\frac{\text{Re } A_0}{\text{Re } A_2} = 31(11),$$

- Why does Large N_c fail? Very large $1/N_c$ corrections? Can Lattice QCD help? .

Relating $K \to \pi$ to A_2 and A_0

$$\mathcal{H}^{SM}(W^\mu, u, d...) \longrightarrow \mathcal{H}_W^{N_f=4}(u, d, c, s) \longrightarrow \mathcal{H}_W^{ChPT}(\pi, K, D, \eta)$$

$$\mathcal{H}_W^{ChPT} \propto g^+\mathcal{O}^+ + g^-\mathcal{O}^-$$

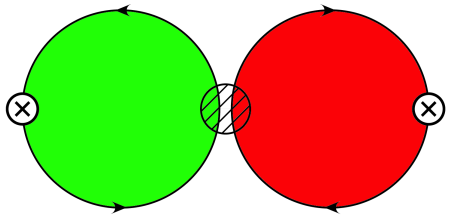
The tree level result in ChPT for the ratio is:

$$\frac{A_0}{A_2} = \frac{1}{2\sqrt{2}}\left(1 + 3\frac{g^-}{g^+}\right) \xrightarrow[g^+=g^-]{\text{Large } N_c} \sqrt{2}$$

Determine $g^\pm$ from Lattice QCD:

$$A^\pm = \langle K|\mathcal{O}^\pm|\pi\rangle \xrightarrow{M_\pi \to 0} g^\pm$$

$g^\pm \propto$



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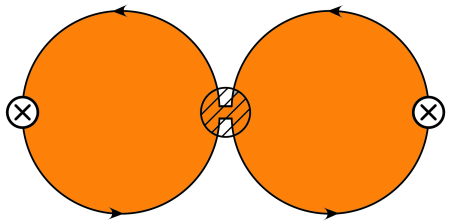
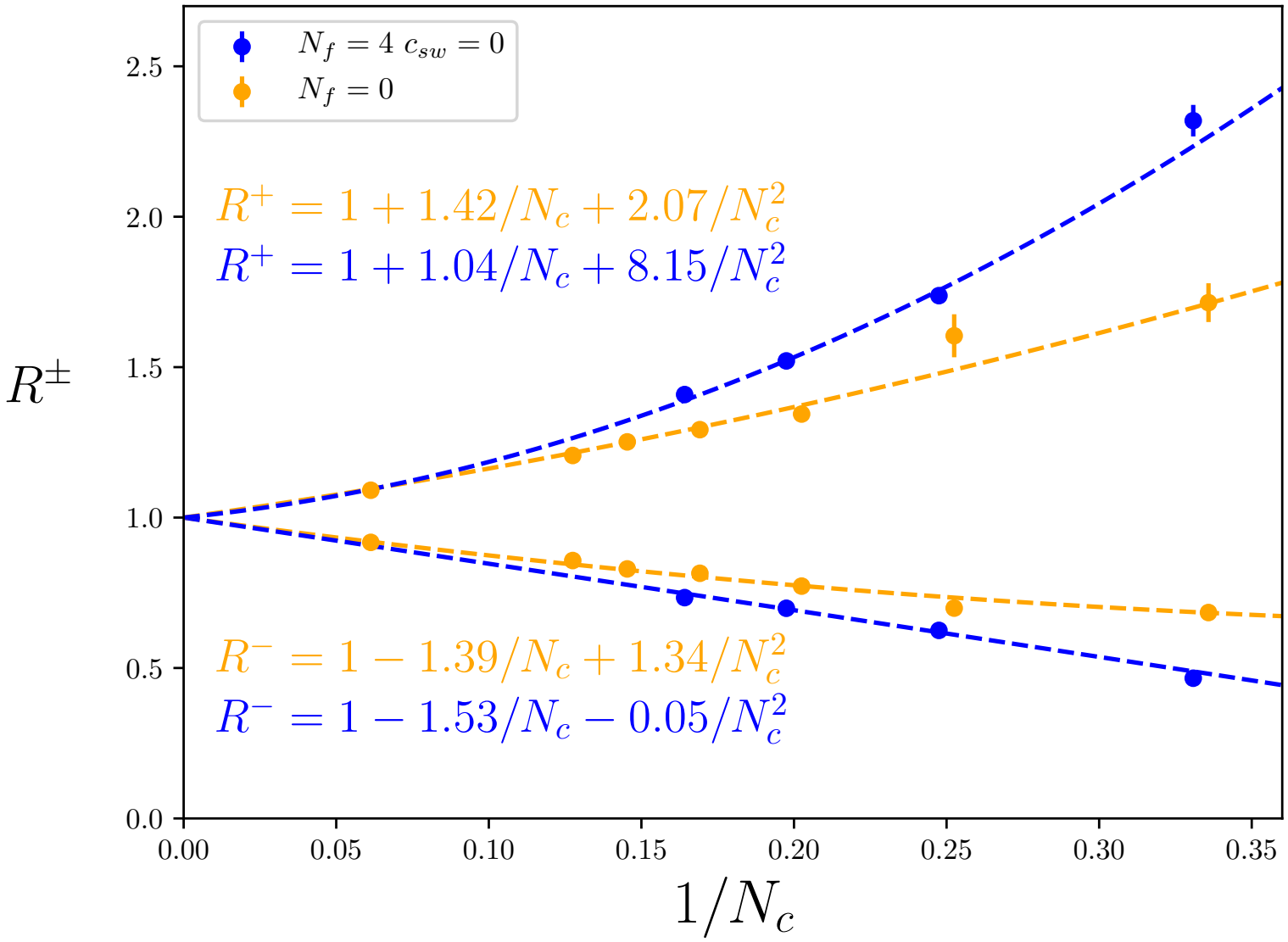


Color-disconnected $O(N_c^2)$ Color-connected $O(N_c)$

Preliminary results for $K \to \pi$ at Large N_c

Donini, Hernández, FRL, Pena, arXiv:1607.03262, arXiv:1810.06285 and on-going work



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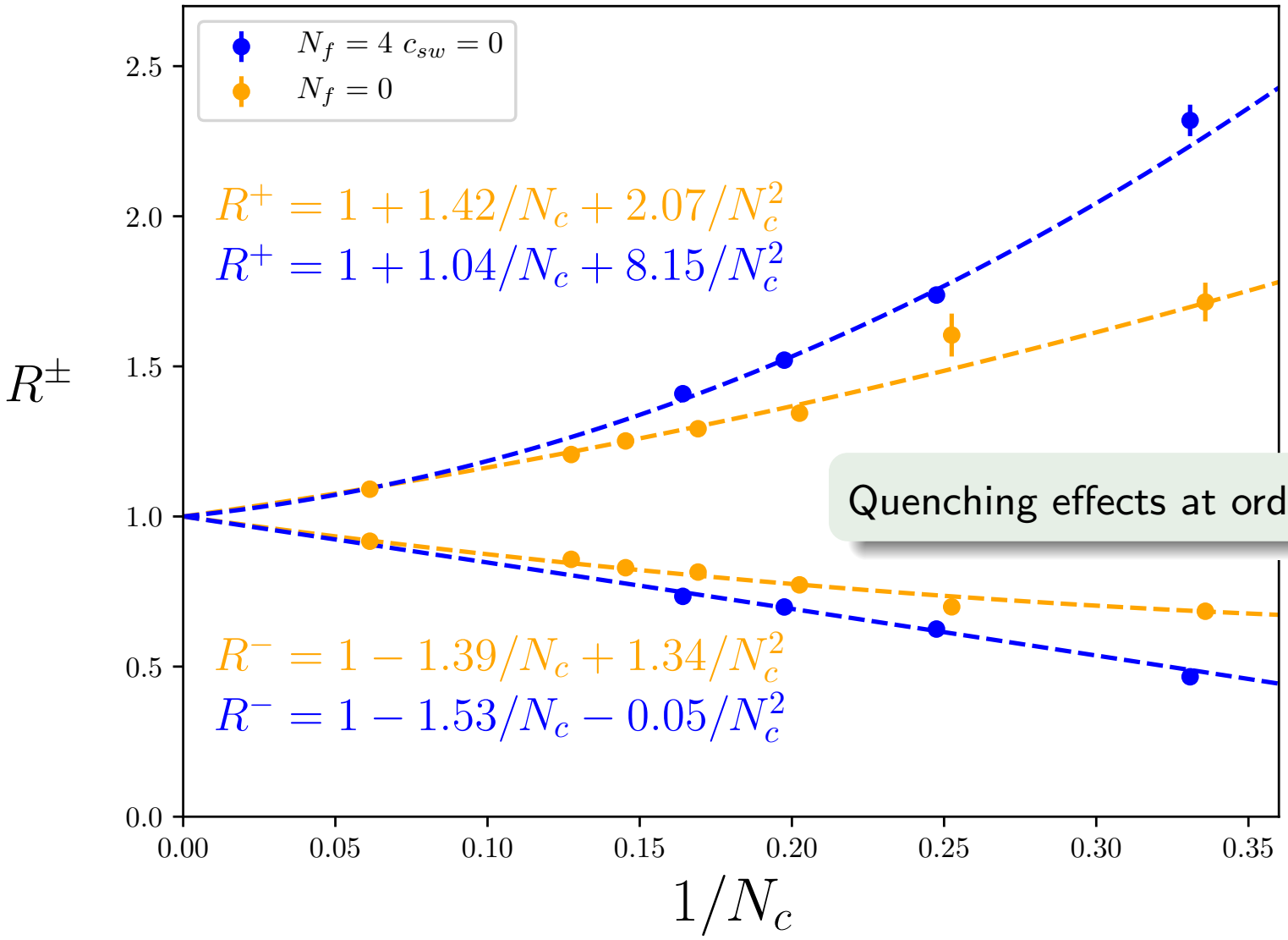

Quenching effects at order $1/N_c^2$

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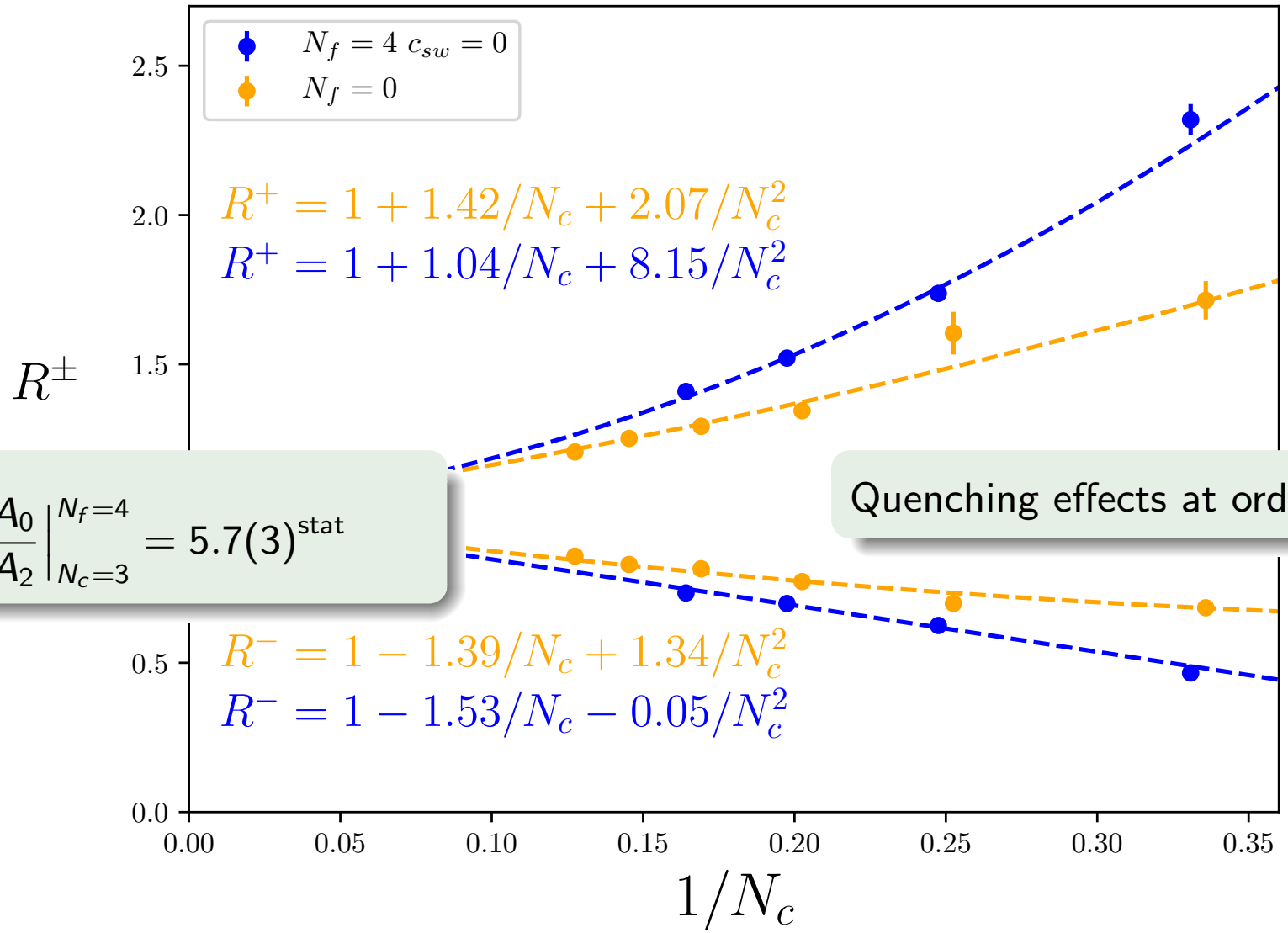

$$\frac{A_0}{A_2}\Big|_{N_c=3}^{N_f=4} = 5.7(3)^{\text{stat}}$$

Quenching effects at order $1/N_c^2$

Summary

Outlook

- Meson masses and decay constants
- Meson scattering (isospin-2, isospin-1/ρ, isospin-0)
- $K \to \pi$ and $K \to \pi\pi$
- η' meson
- Tetraquarks

1. Finished
2. Started
3. Planned

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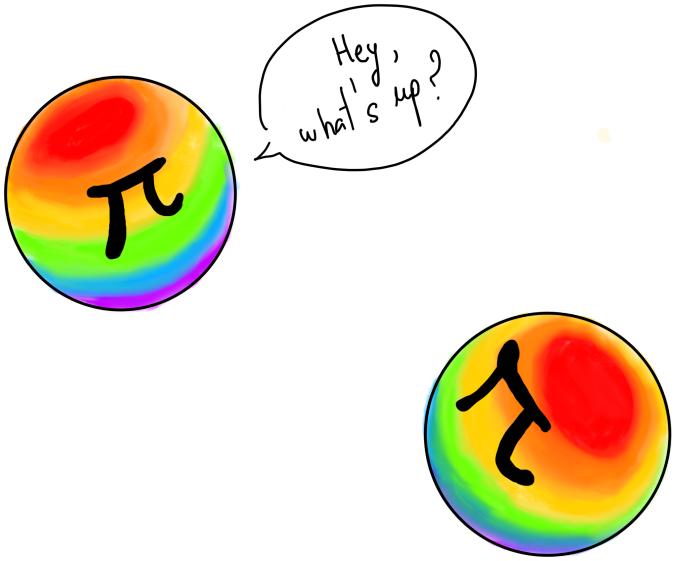
Thanks for your attention!



This project has received funding through the European Union's Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie grant agreement No. 713673 and "La Caixa" Foundation

Framework for $K \to \pi\pi$: weak interactions in ChPT

1. Take the lowest order Lagrangian is written as:

$$\mathcal{L}_2 = \frac{F^2}{4}\text{tr}\ \left(\partial_\mu U^\dagger \partial^\mu U\right) + \frac{F^2 B}{2}\text{tr}\ \left(MU + M^\dagger U^\dagger\right),$$

2. Use covariant derivative with an external left-handed source:

$$\partial_\mu U \to D_\mu U = \partial_\mu U + T^a A^a_\mu U,$$

3. Define the left current as:

$$\mathcal{J}^a_\mu = \frac{\delta \mathcal{L}}{\delta A^a_\mu} = F^2\ \text{Tr}\ \left(T_a U \partial_\mu U^\dagger\right) \leftrightarrow \bar{q} T^a \gamma^L_\mu q$$

4. Build operators with the right irreducible representation of the $SU(4)$ flavour group $(\mathcal{O}_{\Gamma=20}, \mathcal{O}_{\Gamma=84})$. One needs 4 left indices:

$$O_\Gamma = t_{ijkl}(U\partial_\mu U^\dagger)_{ij}(U\partial_\mu U^\dagger)_{kl}$$

The electroweak Hamiltonian in ChPT is:

$$\mathcal{H}_W = g^+ \mathcal{O}^+_{\Gamma=84} + g^- \mathcal{O}^-_{\Gamma=20}$$

U(N_f) Chiral Perturbation Theory

At Large N_c , the flavour singlet becomes a Goldstone boson.
A consistent power counting is:

$$\mathcal{O}(\delta) \sim \mathcal{O}(p^2) \sim \mathcal{O}(m_q) \sim \mathcal{O}(m_\pi^2) \sim \mathcal{O}(N_c^{-1}).$$

At NNLO for the mass:

$$M_\pi^2 = 2m\left(B_0 + \frac{B_1}{N_c} + \frac{B_2}{N_c^2}\right)\left[1 + \frac{1}{N_f}\frac{M_\pi^2}{(4\pi F_\pi)^2}\log\frac{M_\pi^2}{\mu^2}\right.$$
$$\left. - \frac{1}{N_f}\frac{M_{\eta'}^2}{(4\pi F_\pi)^2}\log\frac{M_{\eta'}^2}{\mu^2} + 8\frac{M_\pi^2}{F_\pi^2}\left(N_c L_M^{(0)} + L_M^{(1)}\right) + N_c^2 K_M^{(0)}\left(\frac{M_\pi^2}{F_\pi^2}\right)^2\right]$$

Matching of SU(N_f) and U(N_f):

$$[B]_{SU(N_f)} = [B]_{U(N_f)}\left(1 - \frac{1}{N_f}\frac{M_0^2}{(4\pi F_\pi)^2}\log\frac{M_0^2}{\mu^2}\right),$$

$$\left[L_M^{(1)}\right]_{SU(N_f)} = \left[L_M^{(1)}\right]_{U(N_f)} - \frac{1}{8N_f(4\pi)^2}\left(\log\frac{M_0^2}{\mu^2} + 1\right),$$