UNIVERSITY OF CAMBRIDGE

The b_1 resonance from lattice QCD

Antoni Woss

Advance in Lattice Gauge Theory 2019

CERN

based on work with the **Hadron Spectrum Collaboration**:
AW, Christopher Thomas, Jo Dudek,
Robert Edwards, David Wilson - **arXiv:1904.04136**

hadspec

hadspec.org

why the b_1?

'natural' spin-parities: $J^P = 0^+, 1^-, 2^+, ...$

e.g.: $\sigma(600),\ \rho(770),\ f_2(1270)$

seen in pseudoscalar-pseudoscalar scattering

'unnatural' spin-parities: $J^P = 0^-, 1^+, 2^-, ...$

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'unnatural' spin-parities: $J^P = 0^-, 1^+, 2^-, \ldots$

e.g.: $h_1(1170),\ b_1(1235),\ a_1(1260)$

cannot access quantum numbers in pseudoscalar-pseudoscalar
need something else...
e.g.: vector-pseudoscalar scattering

$b_1(1235)$

$I^G(J^{PC}) = 1^+(1^{+-})$

Mass $m = 1229.5 \pm 3.2$ MeV (S = 1.6)
Full width $\Gamma = 142 \pm 9$ MeV (S = 1.2)

$b_1(1235)$ DECAY MODES	Fraction (Γ_i/Γ)	Confidence level	p (MeV/c)
$\omega\pi$	seen		348
[D/S amplitude ratio $= 0.277 \pm 0.027$]			
$\pi^\pm\gamma$	$(1.6\pm0.4) \times 10^{-3}$		607
$\eta\rho$	seen		†
$\pi^+\pi^+\pi^-\pi^0$	< 50 %	84%	535
$K^*(892)^\pm K^\mp$	seen		†
$(K\overline{K})^\pm\pi^0$	< 8 %	90%	248
$K^0_S K^0_L \pi^\pm$	< 6 %	90%	235
$K^0_S K^0_S \pi^\pm$	< 2 %	90%	235
$\phi\pi$	< 1.5 %	84%	147

PDG

3 volumes: 2 - 3 fm

$L/a_s = 16, 20, 24 \quad T/a_t = 128 \quad m_\pi L \sim 4 - 6$

anisotropic action: $\xi = a_s/a_t \sim 3.5$

Symanzik-improved Wilson-Clover fermions

Distillation (Peardon *et al* 2009) to efficiently handle the many wick contractions

heavier-than-physical light quark masses
$m_\pi \sim 391$ MeV

used in many calculations to-date

earlier lattice studies:
Lang et al JHEP 04 162 (2014)
Michael & McNeile PRD 73 074506

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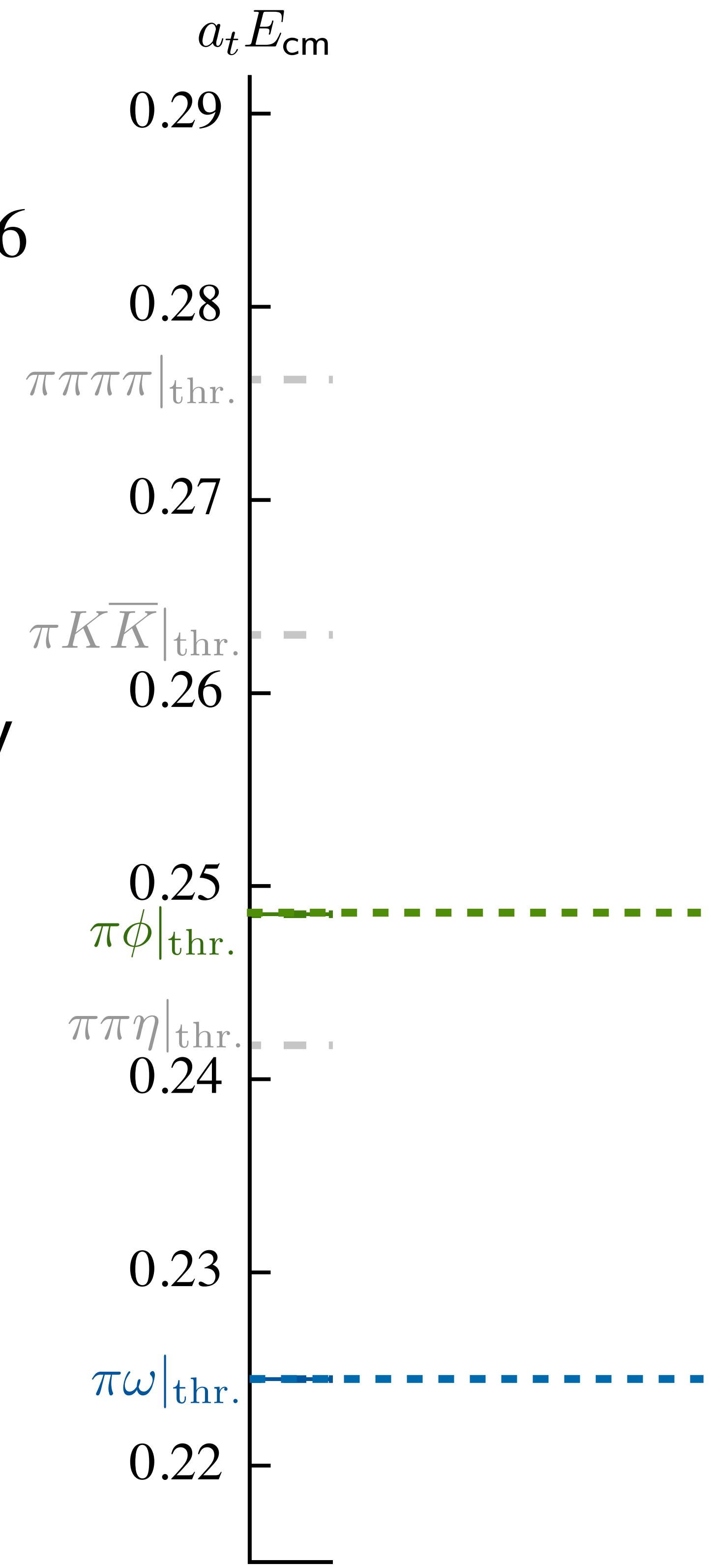

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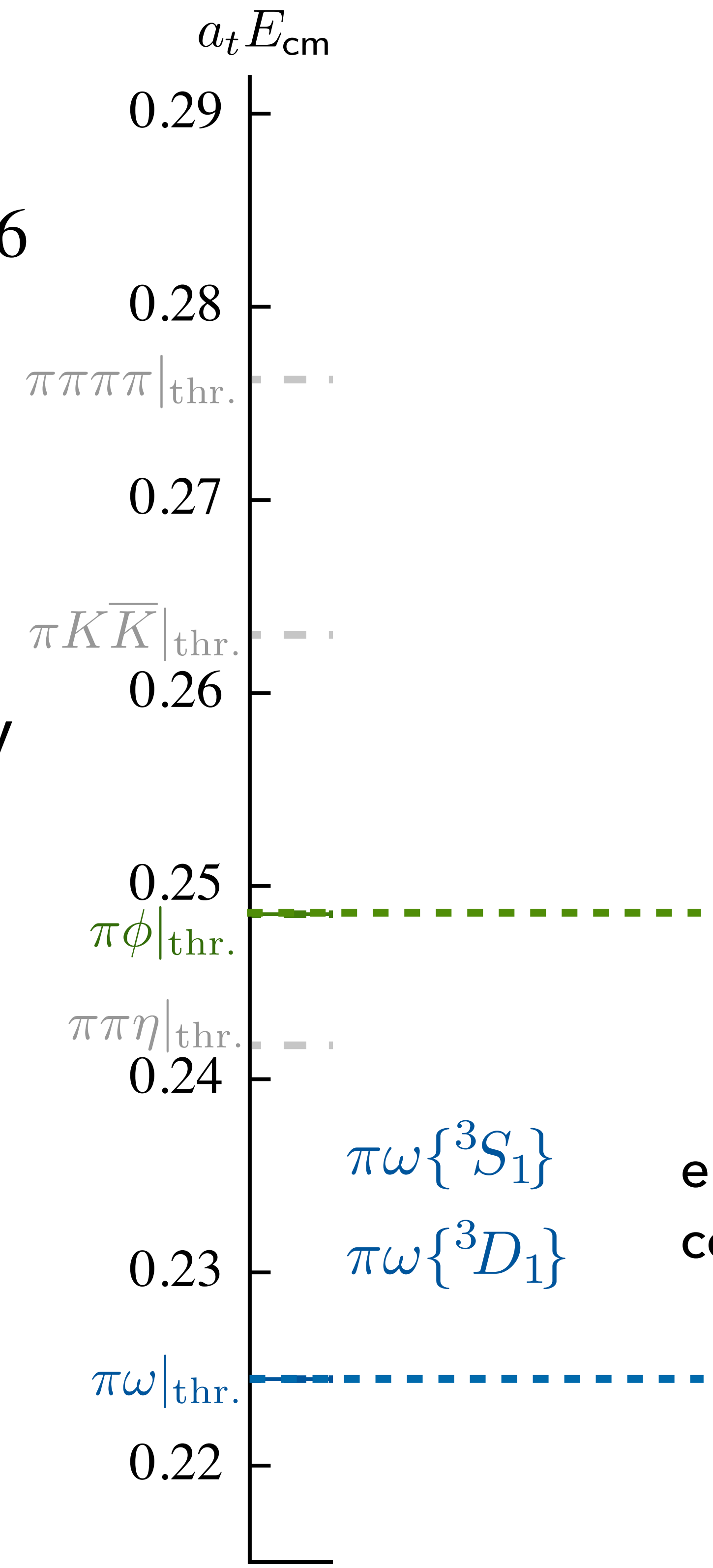

elastic hadron-hadron coupled partial-waves

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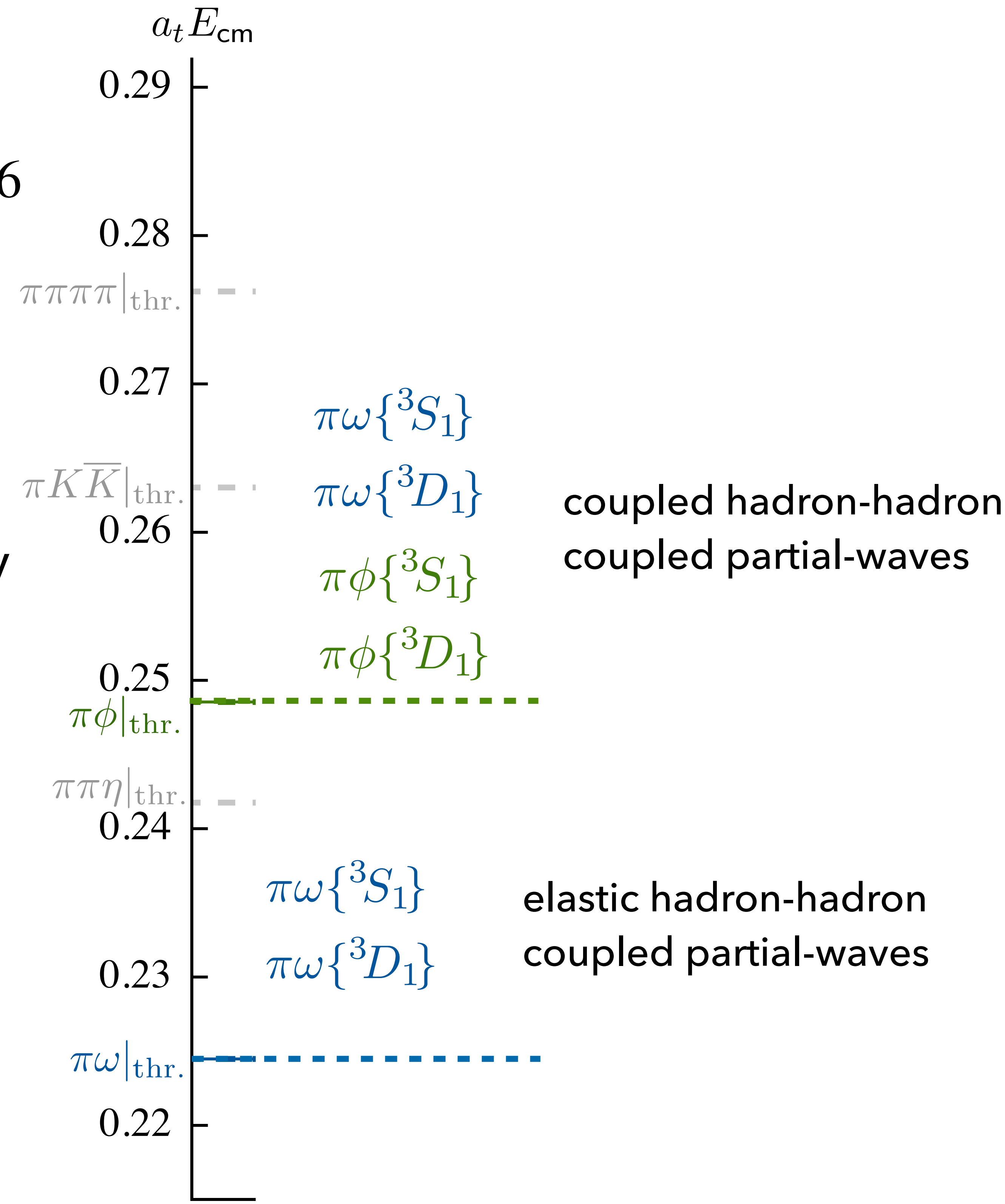
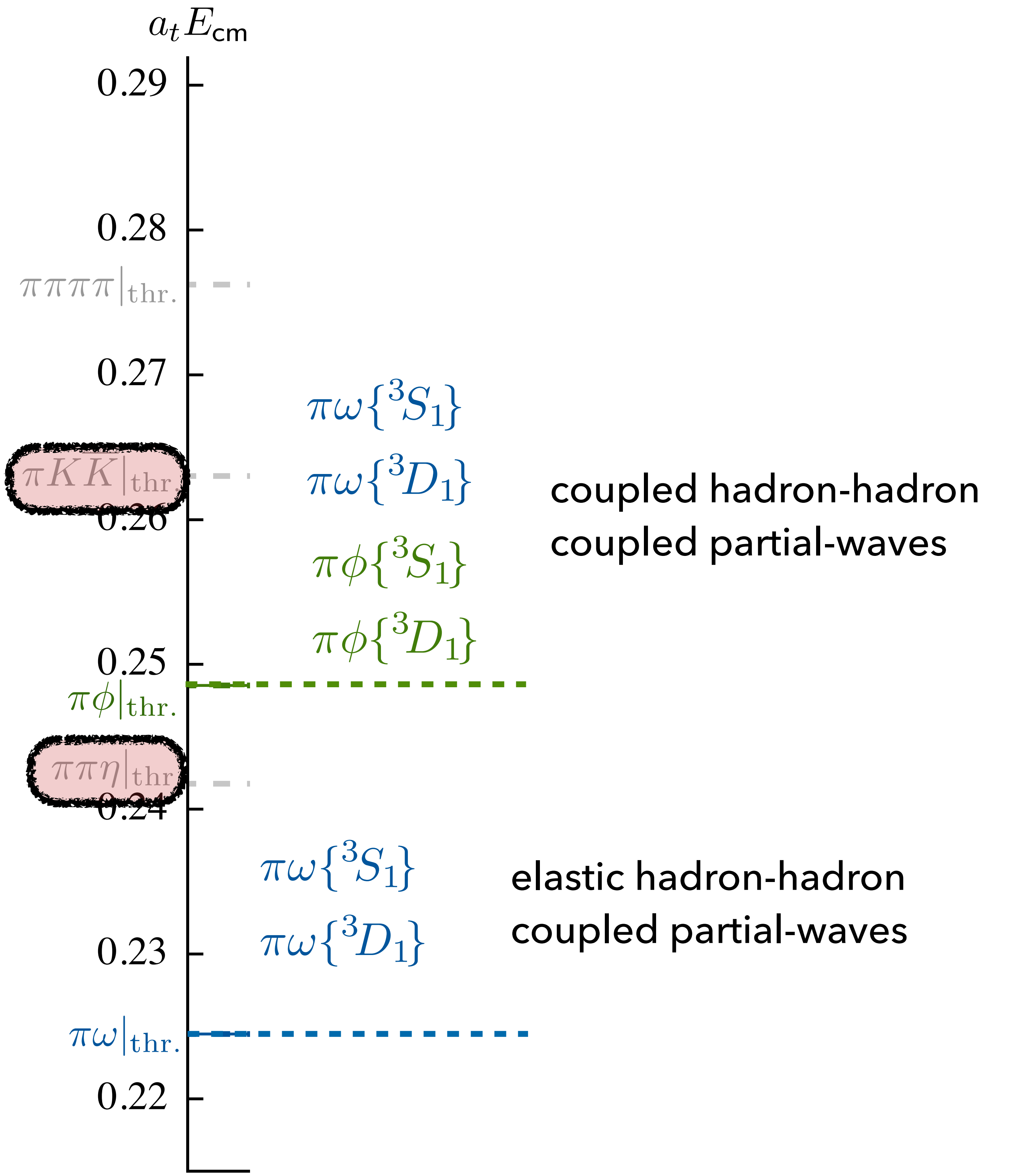


there are open three-body thresholds

we will return to these later...



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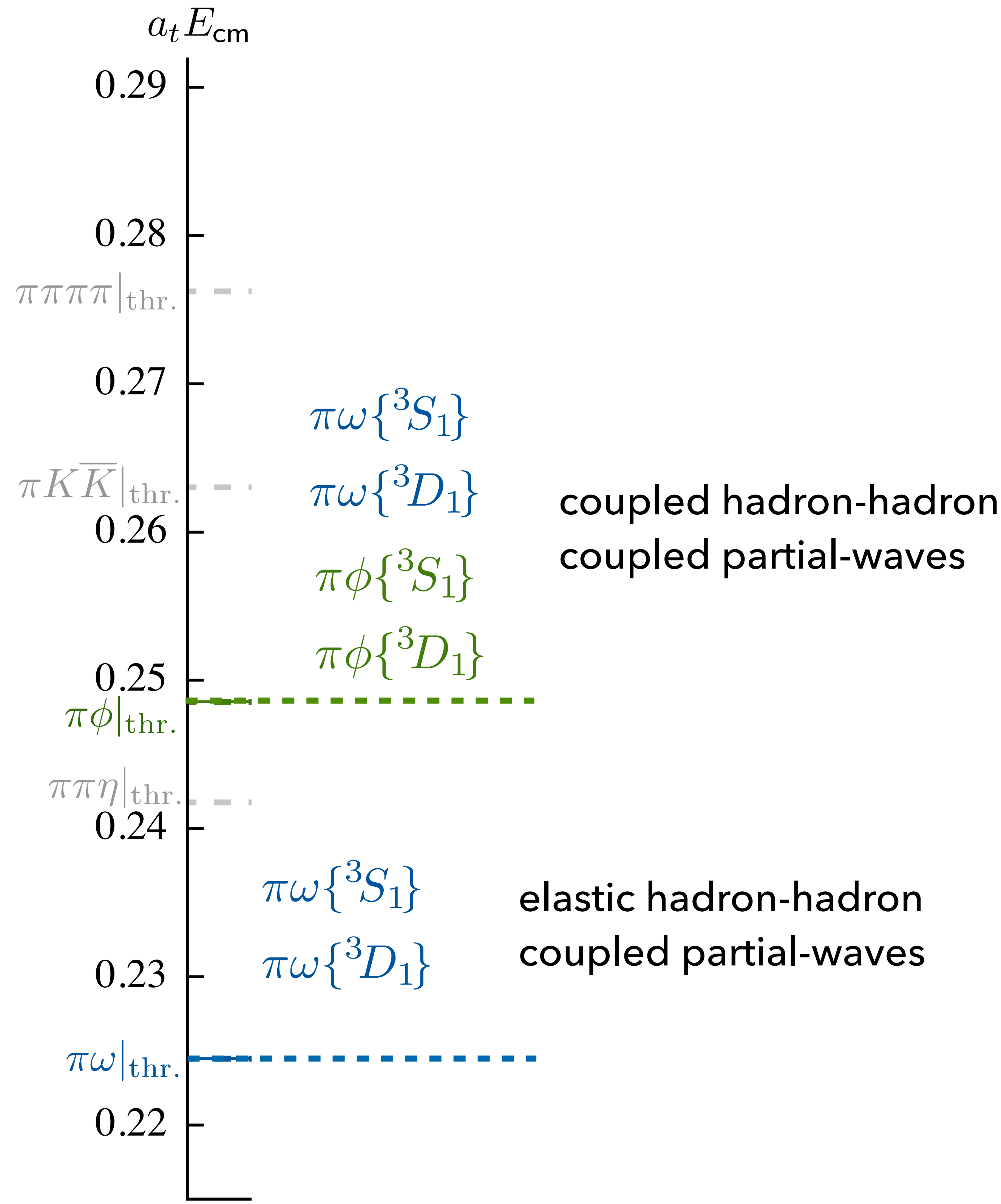

necessitates the inclusion of single-meson-like, two-meson-like and three-meson-like operators in the operator bases

[000] T_1^+ operator basis for each lattice volume

L/a_s	16	20	24
	$22 \times \bar{\psi}\mathbf{\Gamma}\psi$	$22 \times \bar{\psi}\mathbf{\Gamma}\psi$	$22 \times \bar{\psi}\mathbf{\Gamma}\psi$
	$\pi_{[000]}\omega_{[000]}$	$\pi_{[000]}\omega_{[000]}$	$\pi_{[000]}\omega_{[000]}$
	$\pi_{[000]}\phi_{[000]}$	$\pi_{[000]}\phi_{[000]}$	$\pi_{[000]}\phi_{[000]}$
	$\rho_{[000]}\eta_{[000]}$	$\rho_{[000]}\eta_{[000]}$	$\rho_{[000]}\eta_{[000]}$
	$K^*_{[000]}\overline{K}_{[000]}$	$K^*_{[000]}\overline{K}_{[000]}$	$K^*_{[000]}\overline{K}_{[000]}$
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	$K^*_{[000]}\overline{K}_{[000]}$	$K^*_{[000]}\overline{K}_{[000]}$	$K^*_{[000]}\overline{K}_{[000]}$
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optimised η – follows from variational analysis of a matrix of correlation functions of $u\bar{u} + d\bar{d}$ and $s\bar{s}$ quark bilinear operators in relevant irreps

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operator bases

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optimised ω and ϕ operators – follows from variational analysis of a matrix of correlation functions of $u\bar{u} + d\bar{d}$ and $s\bar{s}$ quark bilinear operators in relevant irreps

ω appears as ground state, ϕ appears as first excited state

$[000]\, T_1^+$ operator basis for each lattice volume

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	$\rho_{[000]}\eta_{[000]}$	$\rho_{[000]}\eta_{[000]}$	$\rho_{[000]}\eta_{[000]}$
	$K^*_{[000]}\overline{K}_{[000]}$	$K^*_{[000]}\overline{K}_{[000]}$	$K^*_{[000]}\overline{K}_{[000]}$
			$\{2\}\pi_{[001]}\omega_{[001]}$

optimised ρ and K^* operators – follow from variational analysis of a matrix of correlation functions of quark bilinear operators and two-meson operators

will return to these later…

partial-waves

$[000]\, T_1^+$	$[00n]\, A_2$	$[0nn]\, A_2$	$[nnn]\, A_2$
	$0^- \;\; ({}^3P_0)$	$0^- \;\; ({}^3P_0)$	$0^- \;\; ({}^3P_0)$
$1^+ \begin{pmatrix} {}^3S_1 \\ {}^3D_1 \end{pmatrix}$	$1^+ \begin{pmatrix} {}^3S_1 \\ {}^3D_1 \end{pmatrix}$	$1^+ \begin{pmatrix} {}^3S_1 \\ {}^3D_1 \end{pmatrix}$	$1^+ \begin{pmatrix} {}^3S_1 \\ {}^3D_1 \end{pmatrix}$
		$2^+ \;\; ({}^3D_2)$	
	$2^- \begin{pmatrix} {}^3P_2 \\ {}^3F_2 \end{pmatrix}$	$2^- \begin{pmatrix} {}^3P_2 \\ {}^3F_2 \end{pmatrix}_{[2]}$	$2^- \begin{pmatrix} {}^3P_2 \\ {}^3F_2 \end{pmatrix}$
$3^+ \begin{pmatrix} {}^3D_3 \\ {}^3G_3 \end{pmatrix}$	$3^+ \begin{pmatrix} {}^3D_3 \\ {}^3G_3 \end{pmatrix}$	$3^+ \begin{pmatrix} {}^3D_3 \\ {}^3G_3 \end{pmatrix}_{[2]}$	$3^+ \begin{pmatrix} {}^3D_3 \\ {}^3G_3 \end{pmatrix}_{[2]}$

restrict to $[\vec{P}]\, A_2$ irreps at non-zero momentum to circumvent the need to disentangle $J^P = 1^-$ contributions

$[000]\,T_1^+$	$[00n]\,A_2$	$[0nn]\,A_2$	$[nnn]\,A_2$
	$0^-\ ({}^3P_0)$	$0^-\ ({}^3P_0)$	$0^-\ ({}^3P_0)$
$1^+ \begin{pmatrix} {}^3S_1 \\ {}^3D_1 \end{pmatrix}$	$1^+ \begin{pmatrix} {}^3S_1 \\ {}^3D_1 \end{pmatrix}$	$1^+ \begin{pmatrix} {}^3S_1 \\ {}^3D_1 \end{pmatrix}$	$1^+ \begin{pmatrix} {}^3S_1 \\ {}^3D_1 \end{pmatrix}$
		$2^+\ ({}^3D_2)$	
	$2^- \begin{pmatrix} {}^3P_2 \\ {}^3F_2 \end{pmatrix}$	$2^- \begin{pmatrix} {}^3P_2 \\ {}^3F_2 \end{pmatrix}_{[2]}$	$2^- \begin{pmatrix} {}^3P_2 \\ {}^3F_2 \end{pmatrix}$
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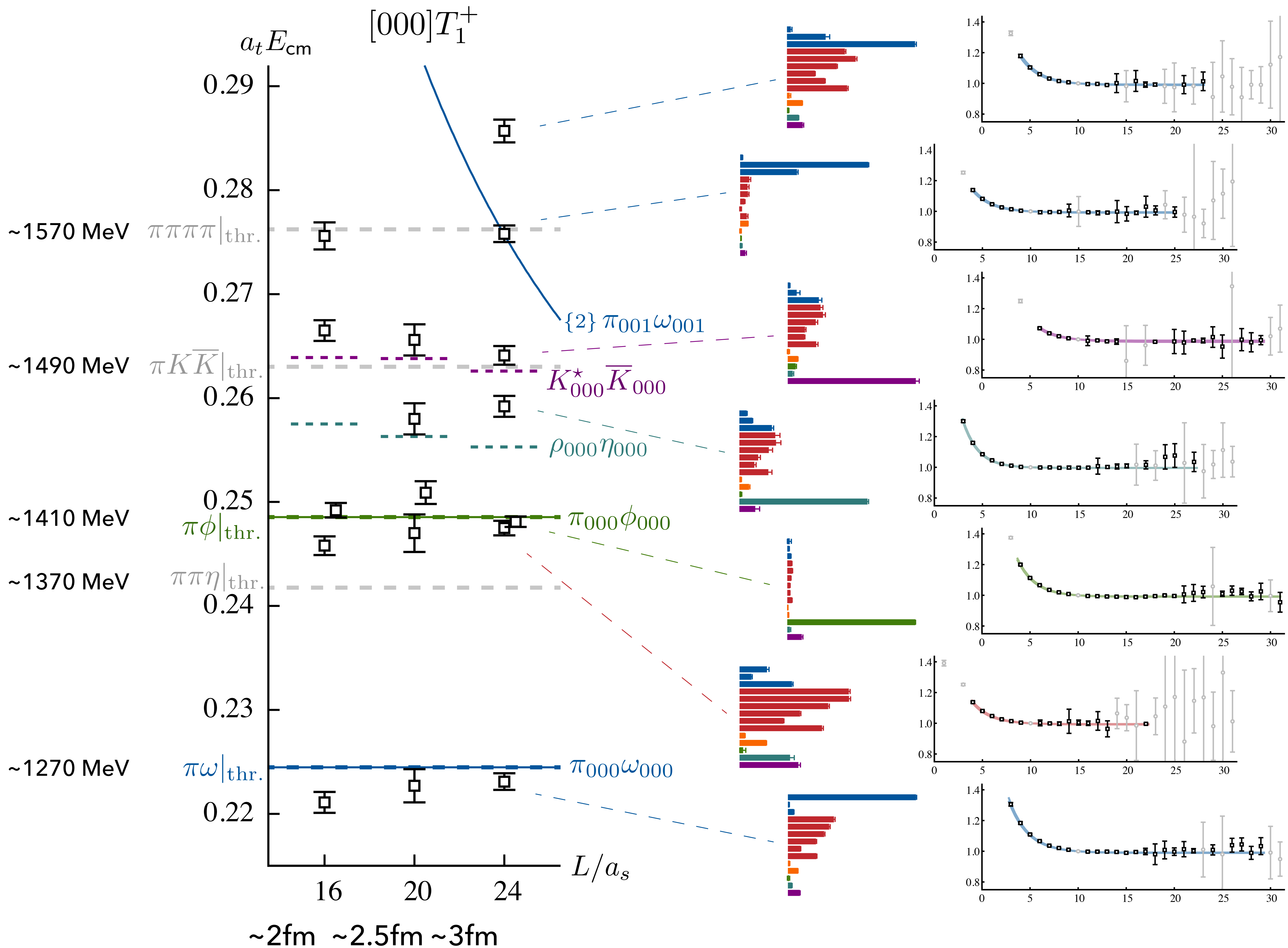
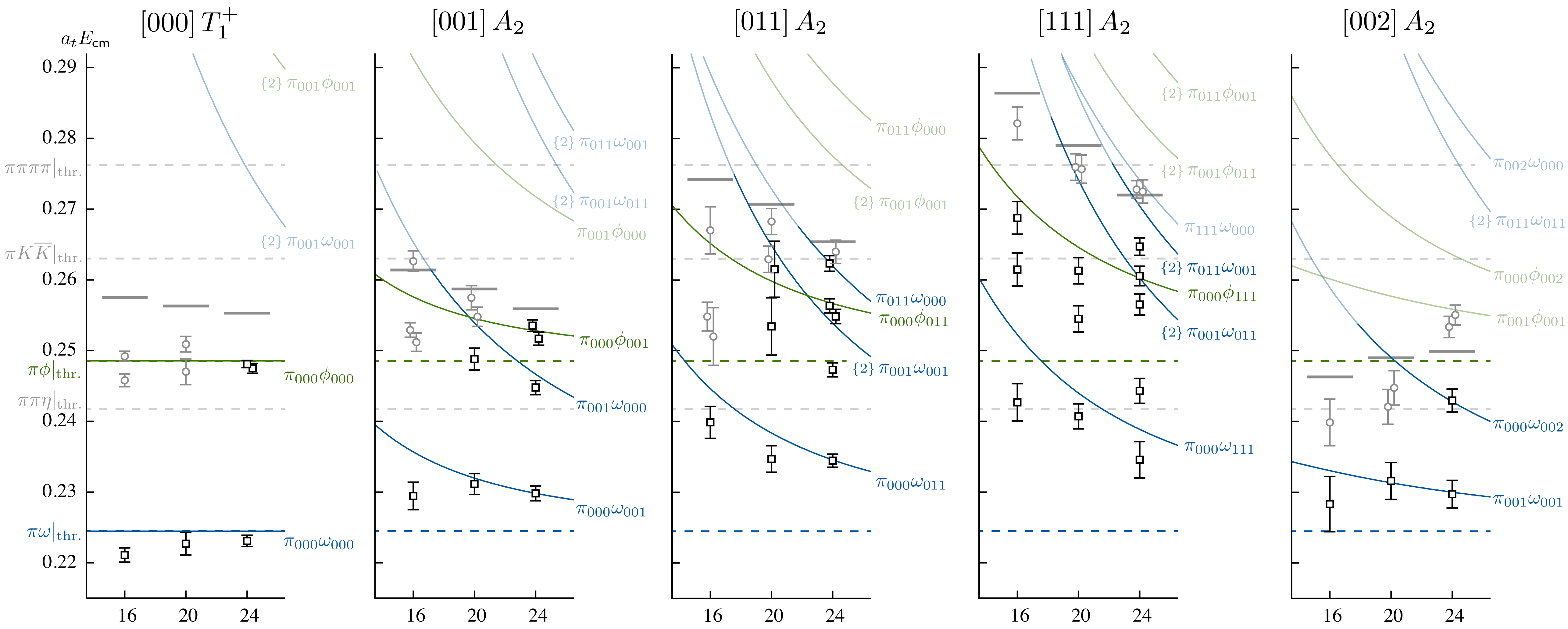
focus on the $J^P = 1^+$

spectrum



$[000]T_1^+$
$a_t E_{cm}$
0.29
0.28
0.27
0.26
0.25
0.24
0.23
0.22
~1570 MeV
$\pi\pi\pi\pi|_{thr.}$
~1490 MeV
$\pi K\overline{K}|_{thr.}$
~1410 MeV
$\pi\phi|_{thr.}$
~1370 MeV
$\pi\pi\eta|_{thr.}$
~1270 MeV
$\pi\omega|_{thr.}$
$\{2\}\ \pi_{001}\omega_{001}$
$K^\star_{000}\overline{K}_{000}$
$\rho_{000}\eta_{000}$
$\pi_{000}\phi_{000}$
$\pi_{000}\omega_{000}$
L/a_s
16
20
24
~2fm ~2.5fm ~3fm

36 energy levels used to constrain the scattering amplitudes



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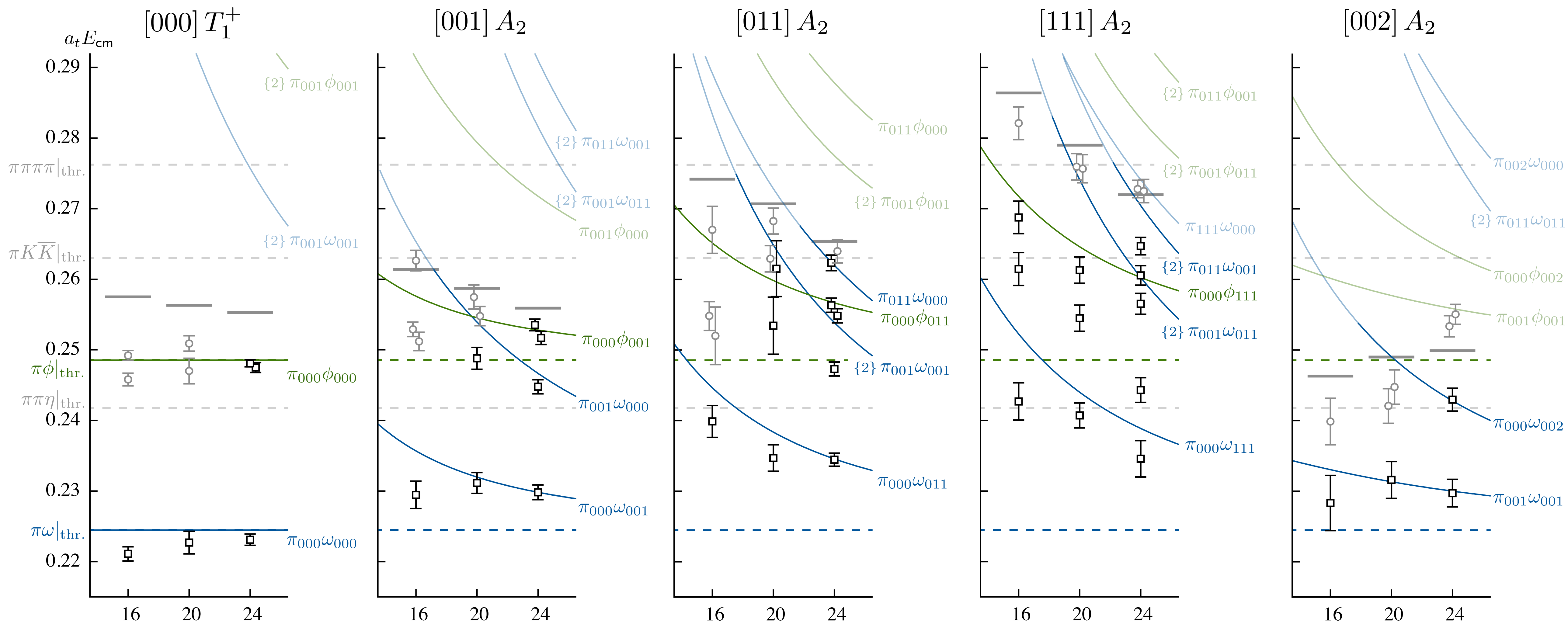

exclude any energies that appear to show sensitivity to three-body operators

General two-body quantisation condition

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\boldsymbol{t}(E)\cdot\left(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\right)\right] = 0$$

phase space

infinite volume scattering *t*-matrix

known finite-volume functions

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phase space

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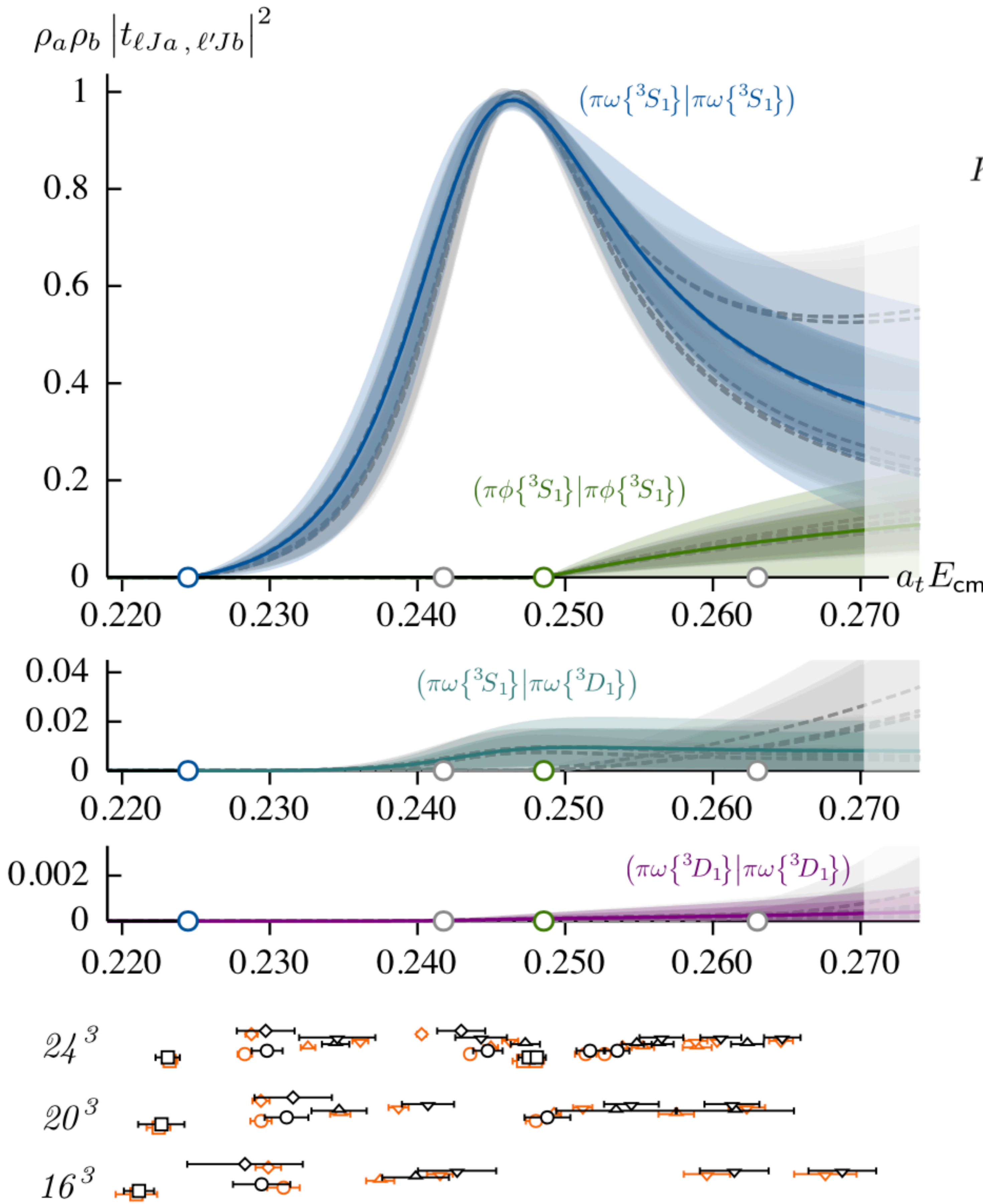
known finite-volume functions

$$\mathbf{t} = \begin{pmatrix} t(\pi\omega\{{}^3S_1\}|\pi\omega\{{}^3S_1\}) & t(\pi\omega\{{}^3S_1\}|\pi\omega\{{}^3D_1\}) & t(\pi\omega\{{}^3S_1\}|\pi\phi\{{}^3S_1\}) \\ & t(\pi\omega\{{}^3D_1\}|\pi\omega\{{}^3D_1\}) & t(\pi\omega\{{}^3D_1\}|\pi\phi\{{}^3S_1\}) \\ & & t(\pi\phi\{{}^3S_1\}|\pi\phi\{{}^3S_1\}) \end{pmatrix}$$

K-matrix approach: $\mathbf{t}^{-1} = \mathbf{K}^{-1} - i\boldsymbol{\rho}$ Simple phase space

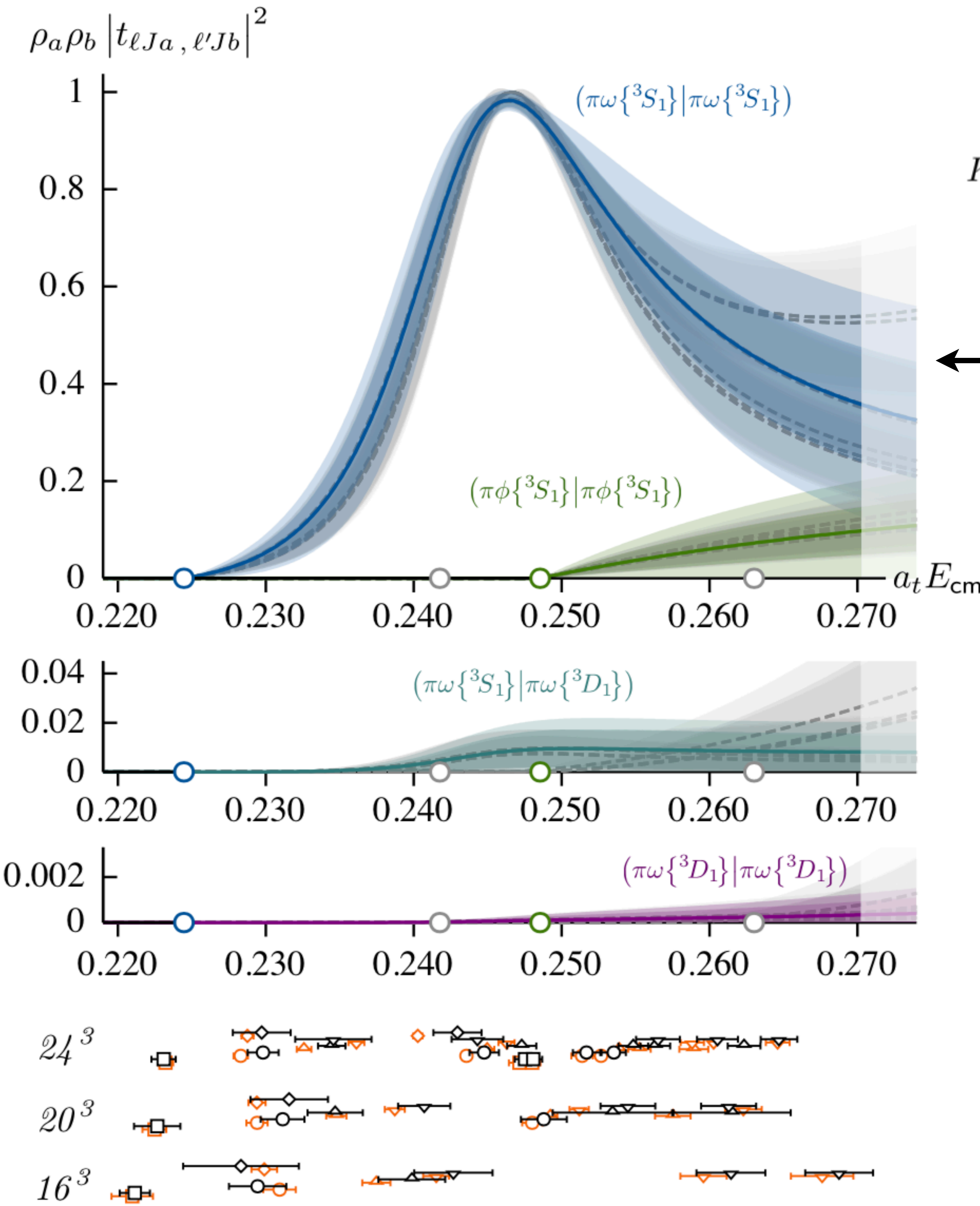
$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \boldsymbol{I}$$ Chew-Mandelstam phase space

three-channel amplitudes



many parameterisations

$$K_{\ell Ja,\ell' Jb}(s) = \frac{g_{\ell Ja}(s)\, g_{\ell' Jb}(s)}{m^2 - s} + \sum_{n=0}^{N} \gamma^{(n)}_{\ell Ja,\ell' Jb} \cdot s^n$$



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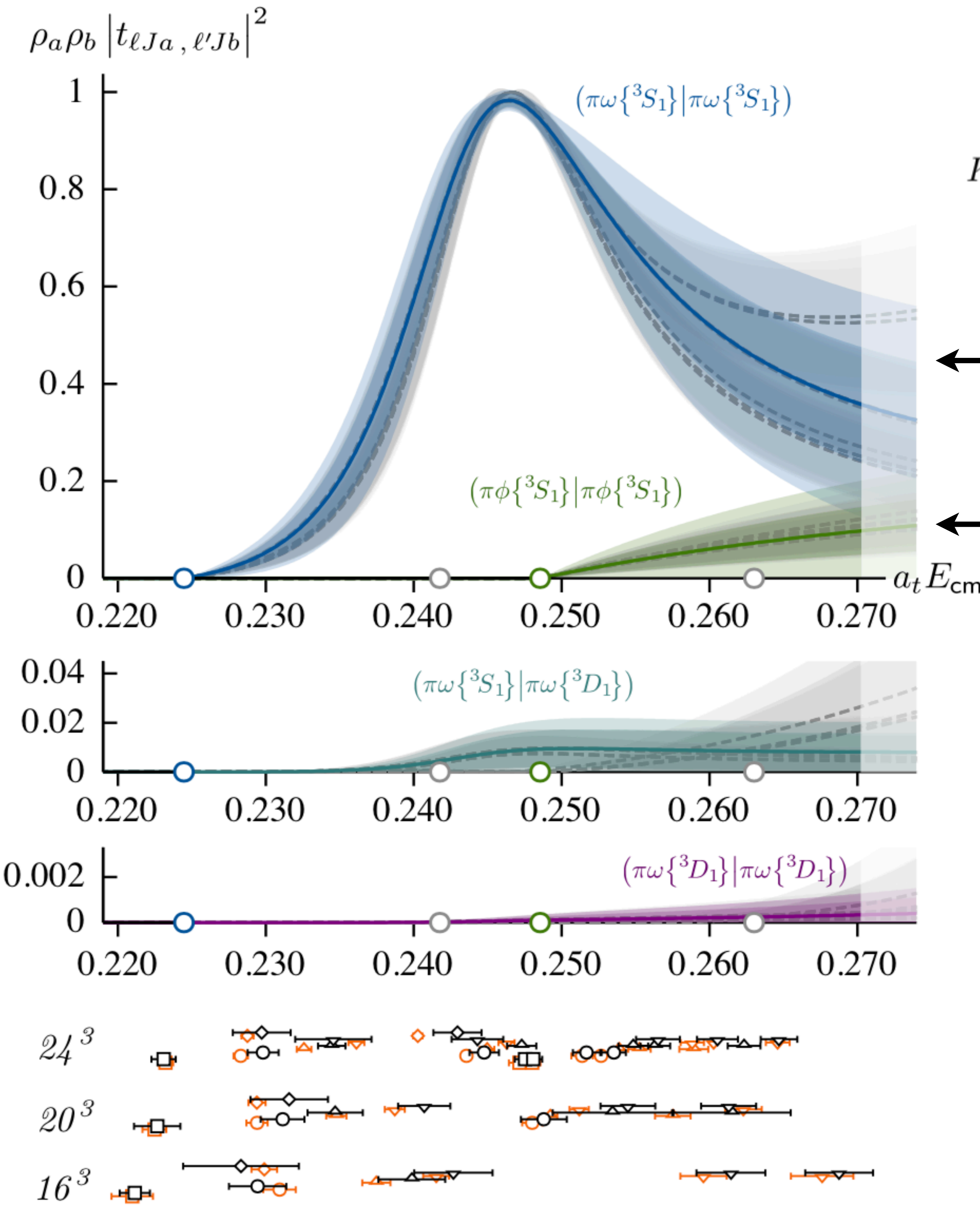
$$K_{\ell J a,\ell' J b}(s) = \frac{g_{\ell J a}(s)\, g_{\ell' J b}(s)}{m^2 - s} + \sum_{n=0}^{N} \gamma^{(n)}_{\ell J a,\ell' J b} \cdot s^n$$

large canonical \`bump' enhancement in $\pi\omega\{^3S_1\}$

three-channel amplitudes



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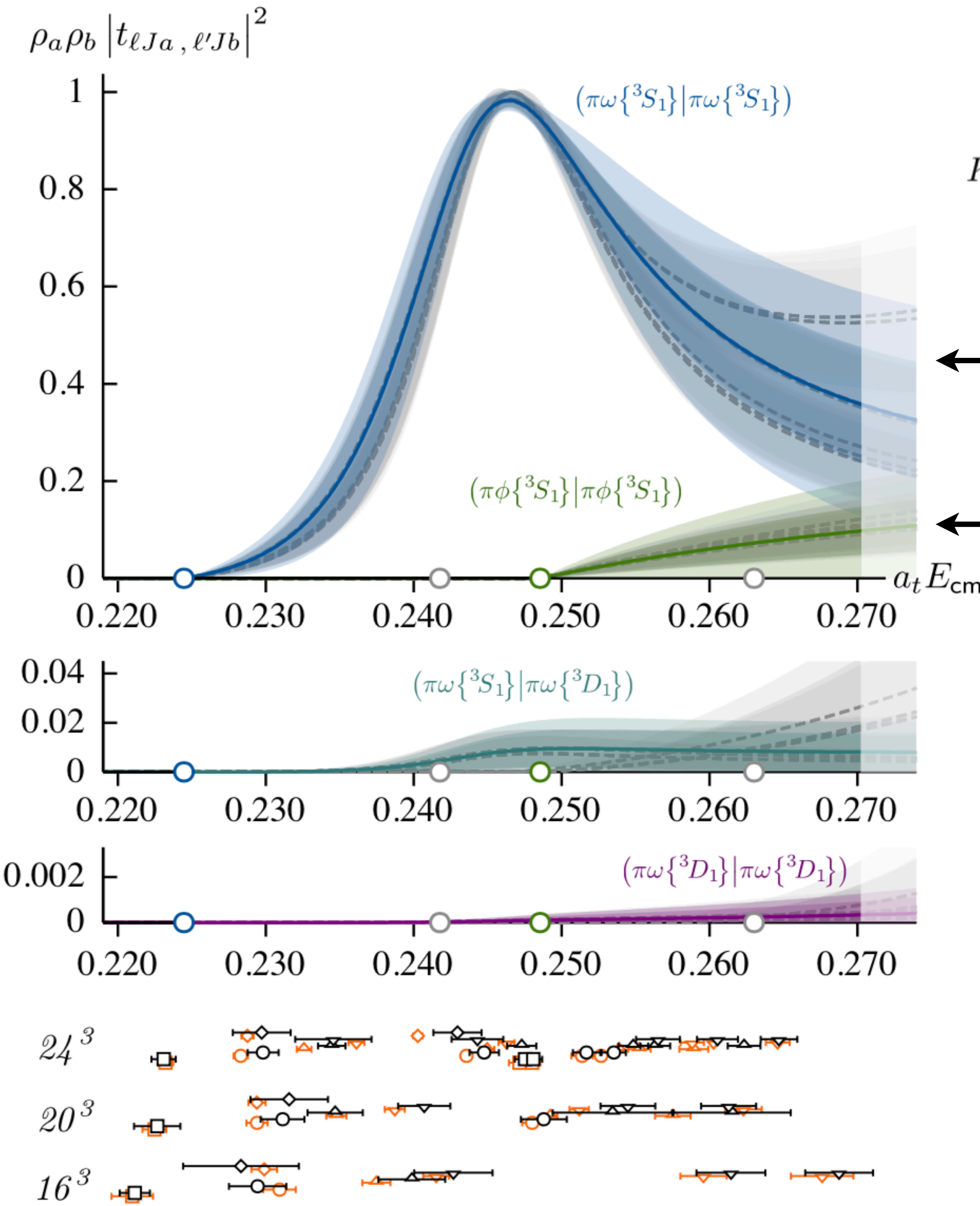

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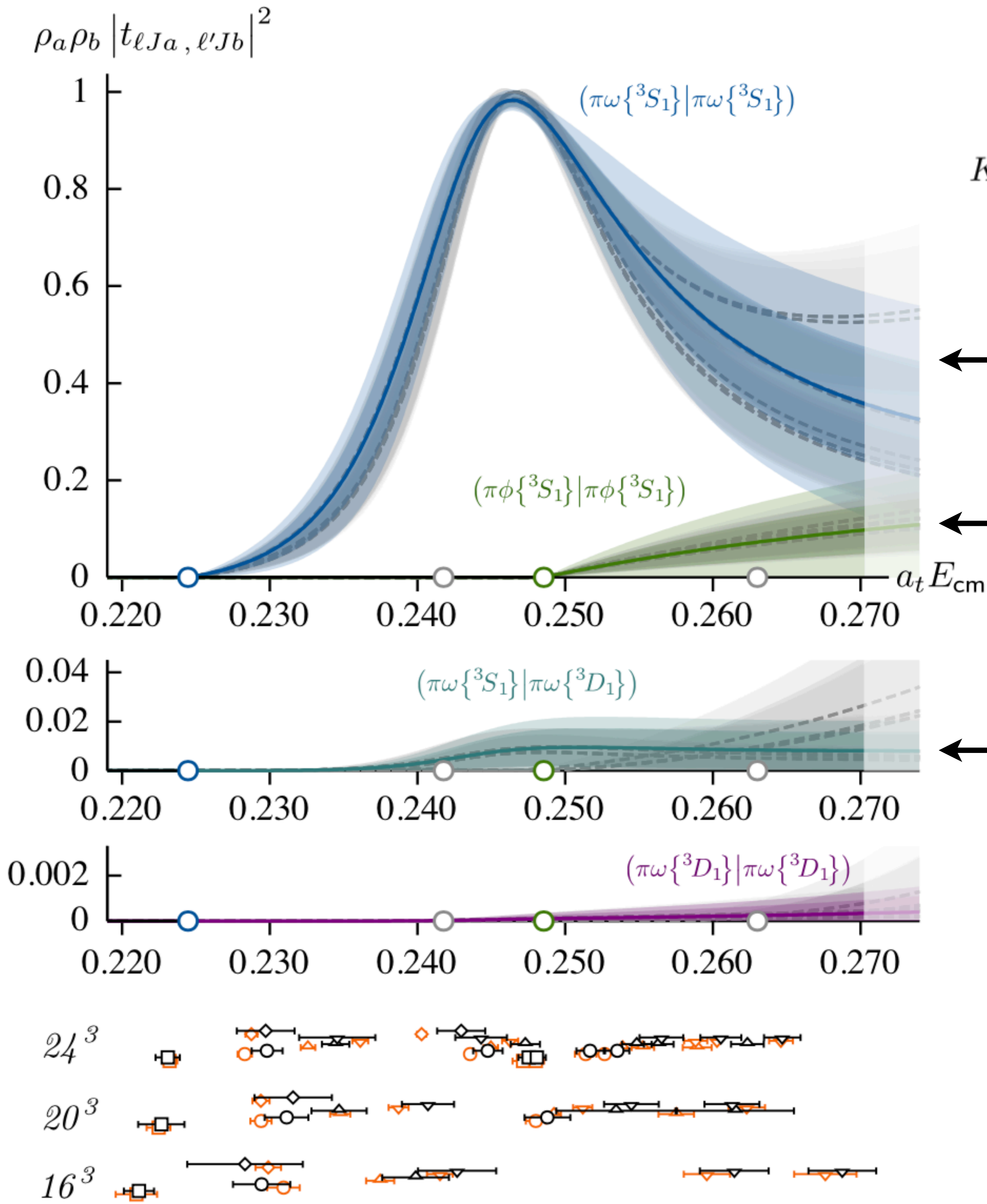

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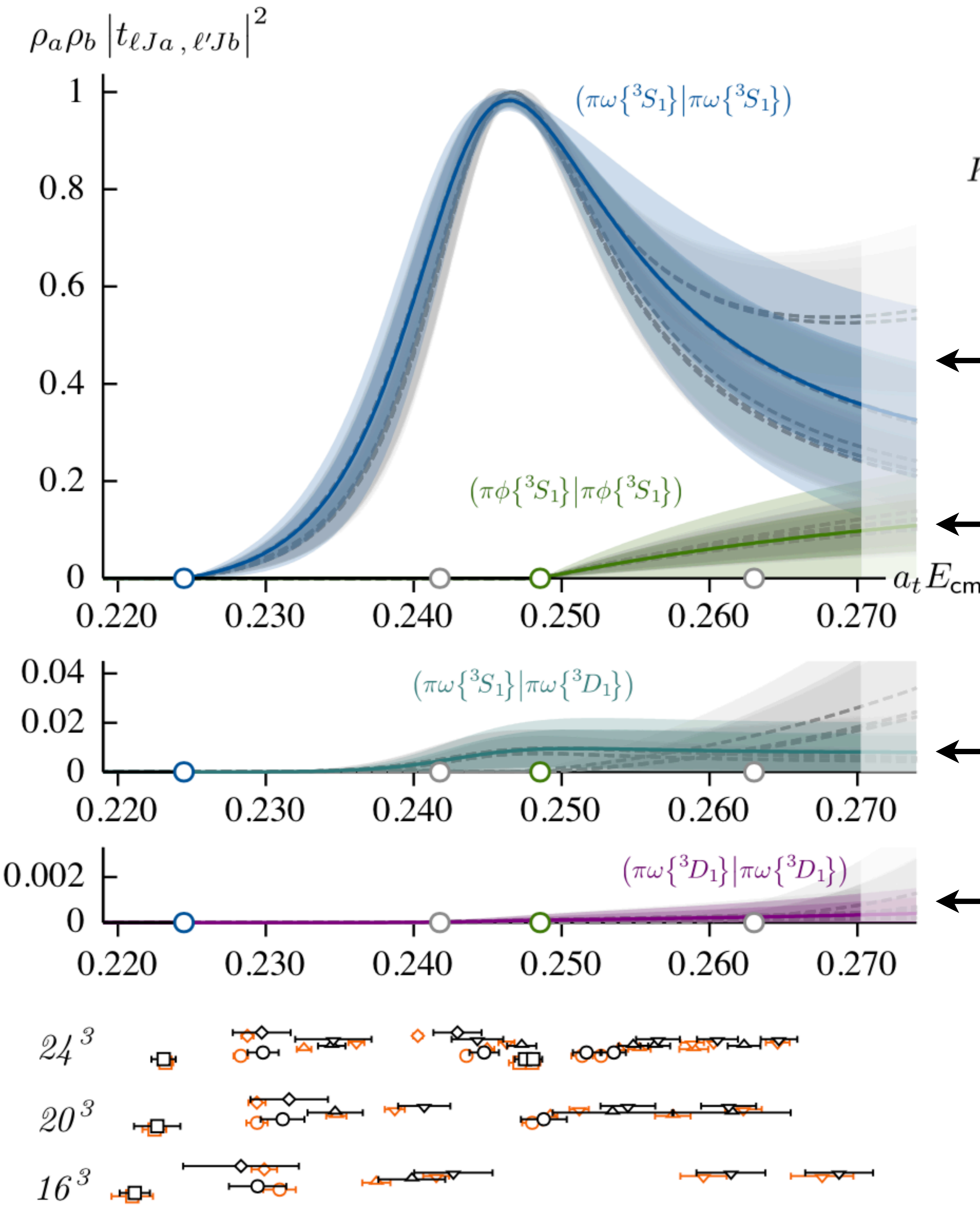

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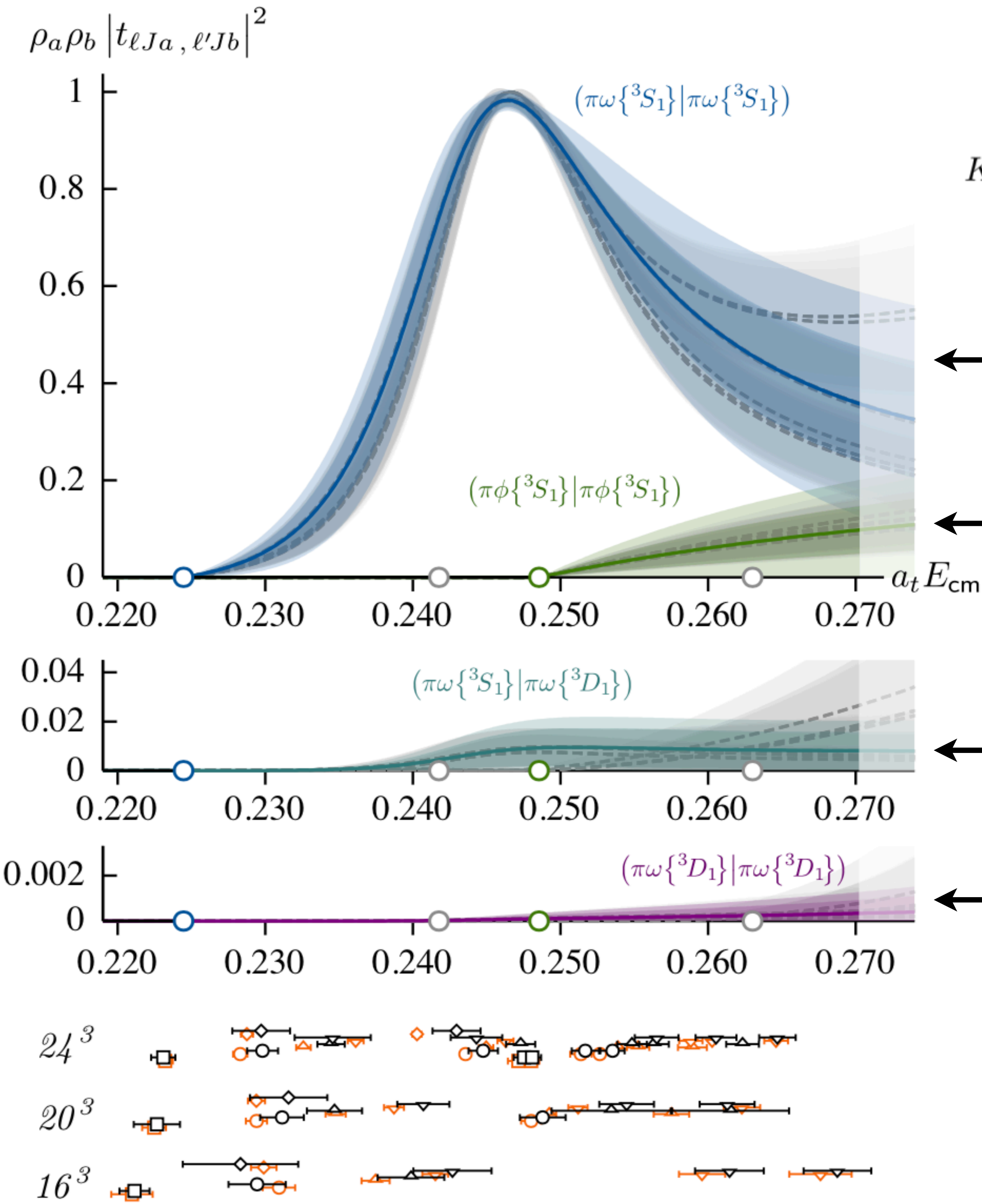

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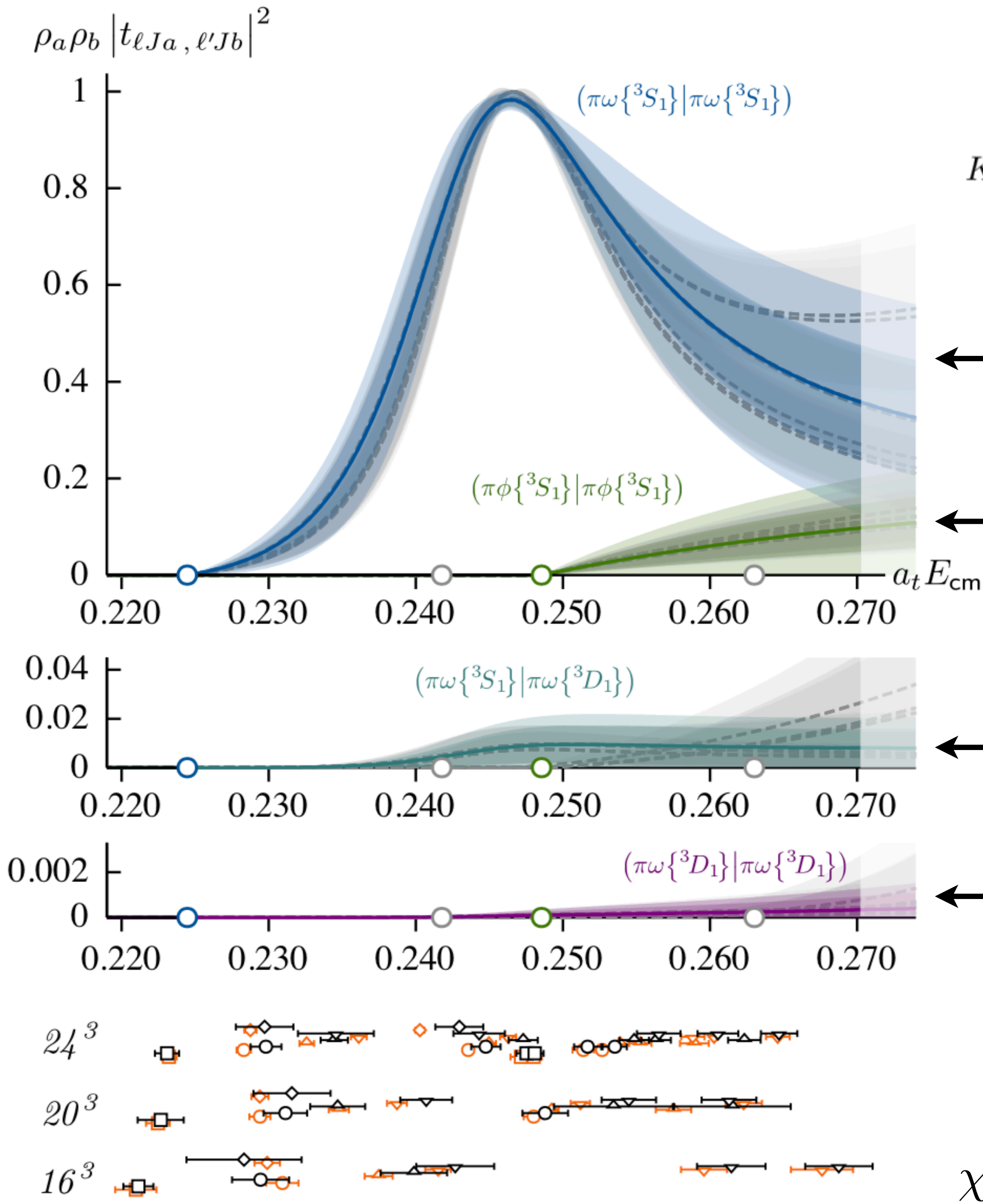
extremely small $\pi\omega \,|\, \pi\phi$ amplitudes consistent with zero



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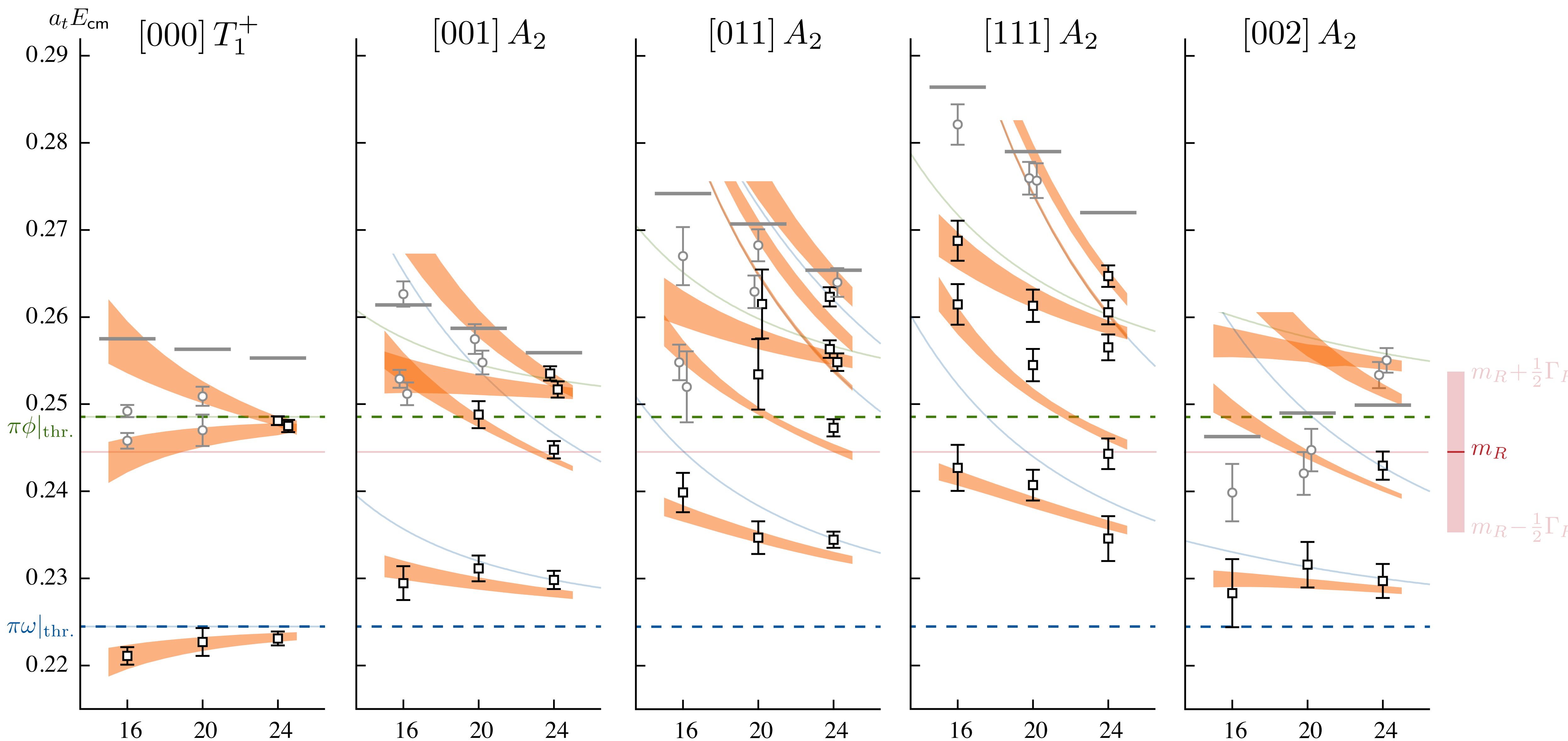
$$\chi^2/N_{\mathrm{dof}} = \frac{36.8}{36-5} = 1.19.$$

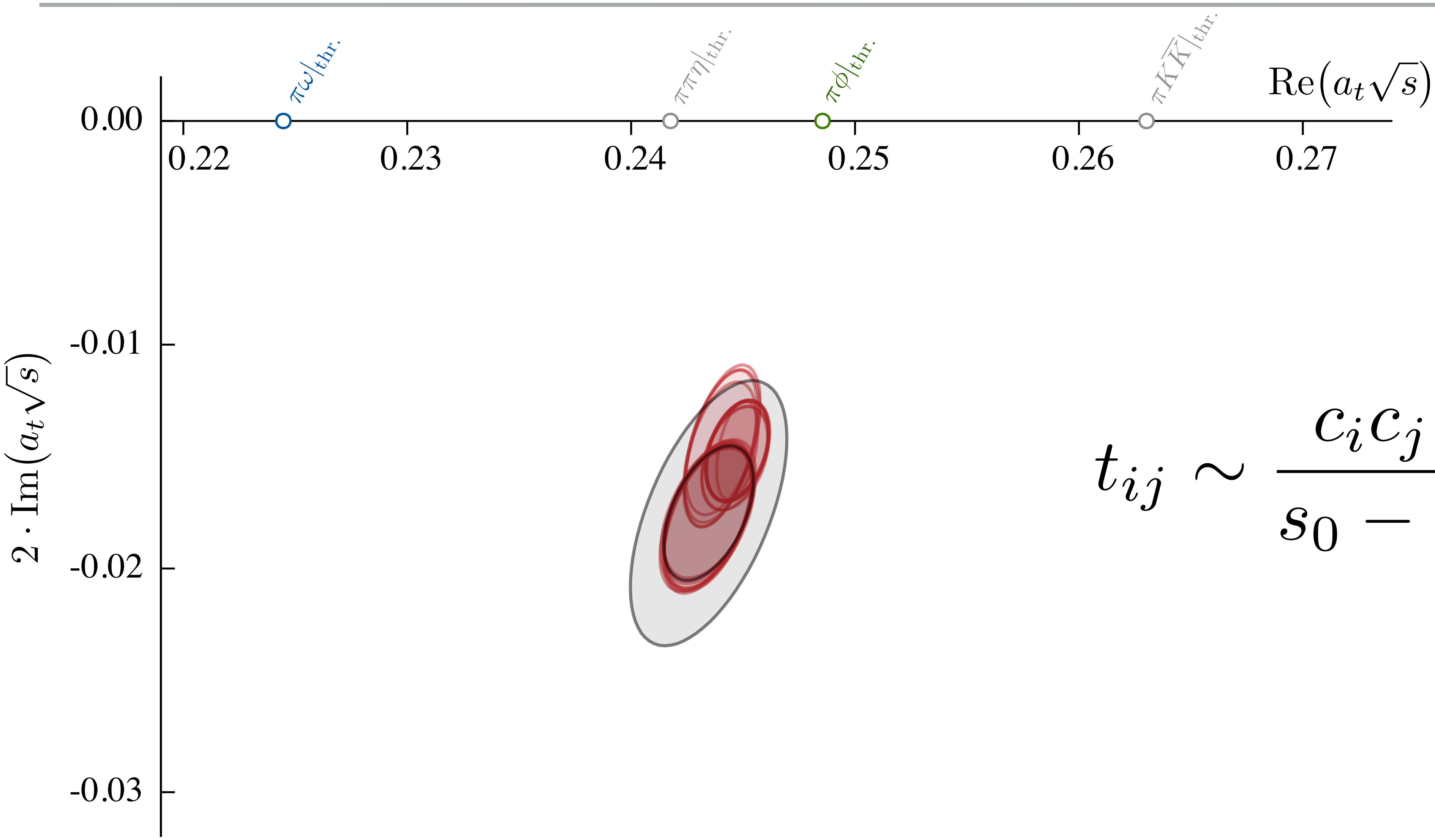
levels from the determinant

$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\boldsymbol{t}(E)\cdot\left(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\right)\right] = 0$

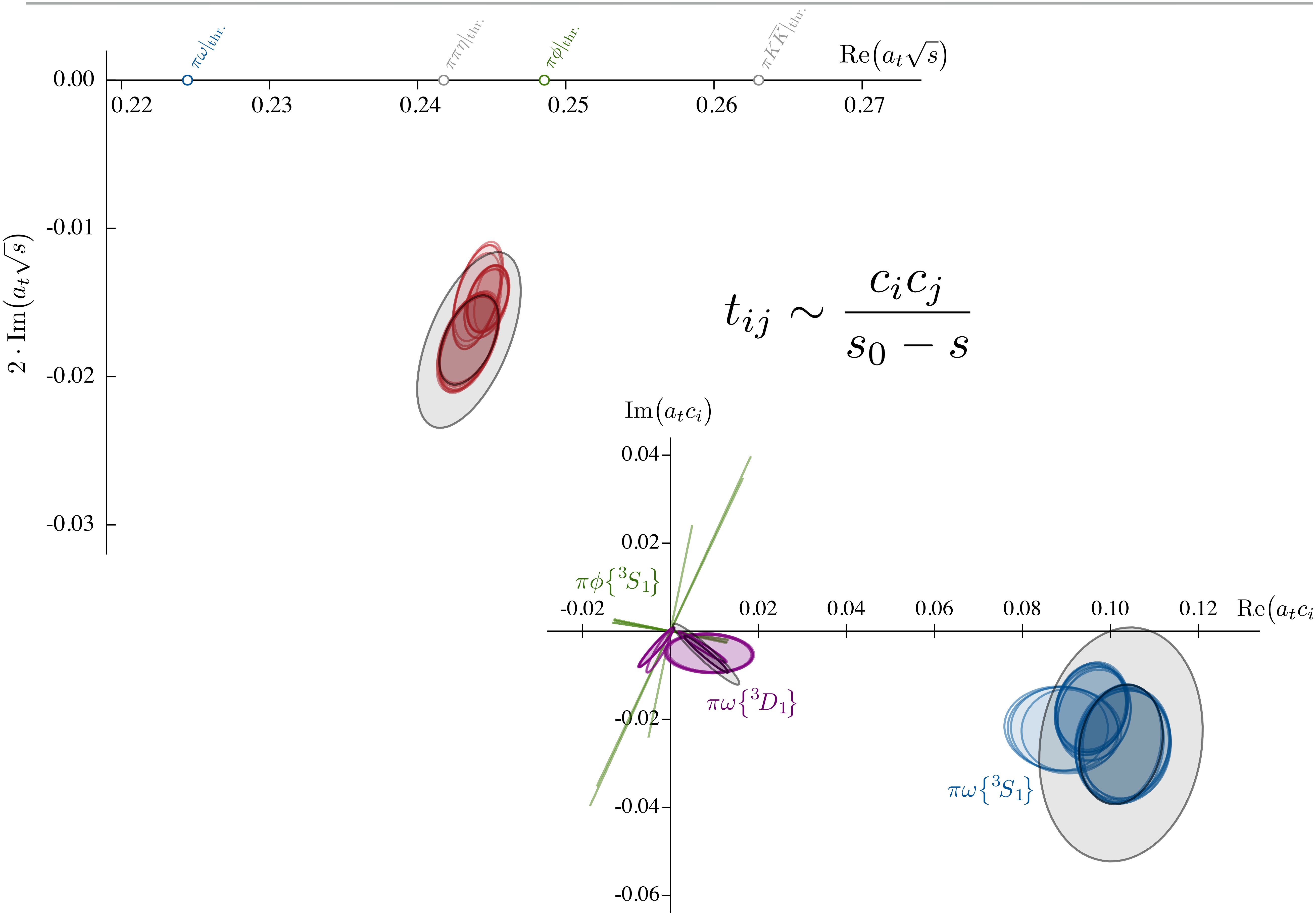


$$\chi^2/N_{\text{dof}} = \frac{36.8}{36-5} = 1.19.$$

poles & couplings



$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$



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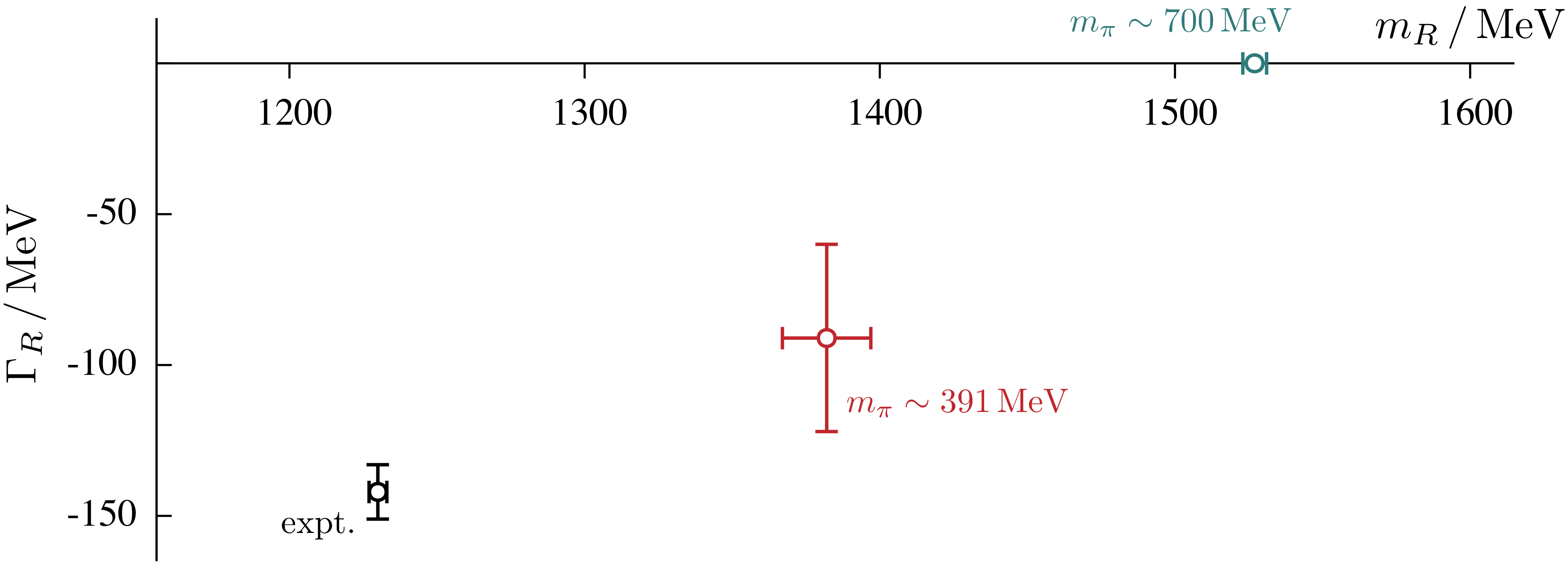
pole position at various pion masses



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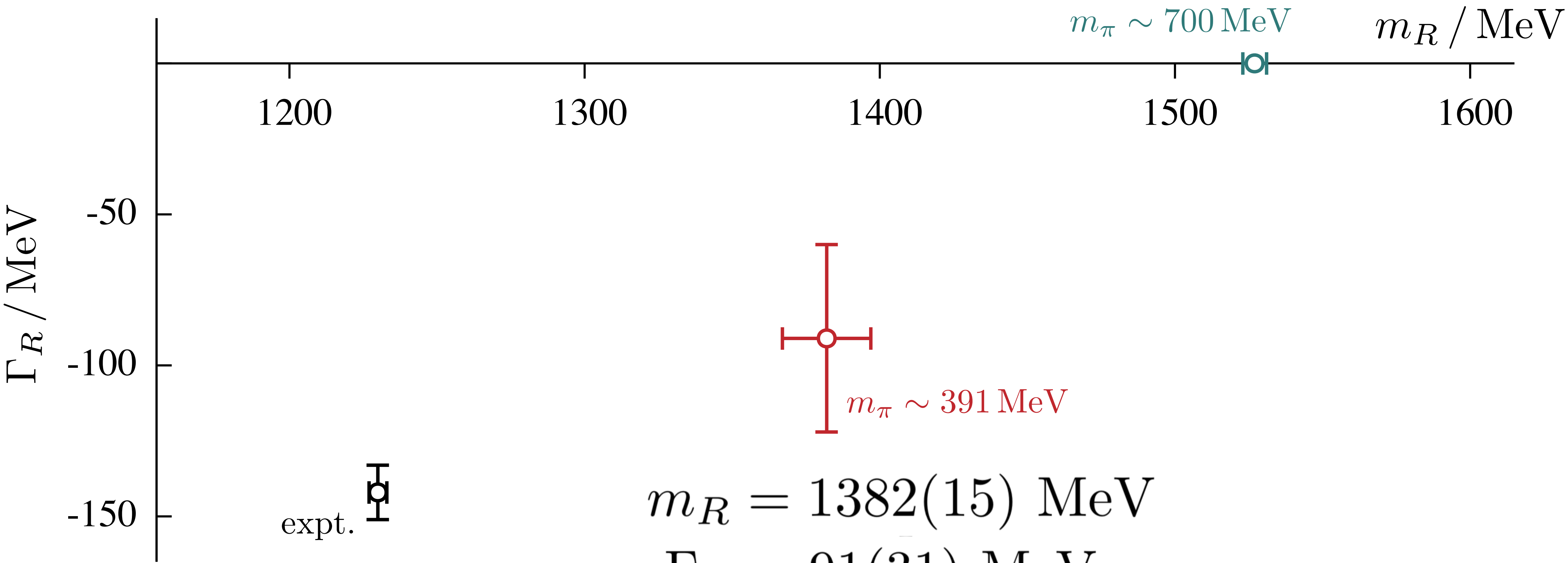
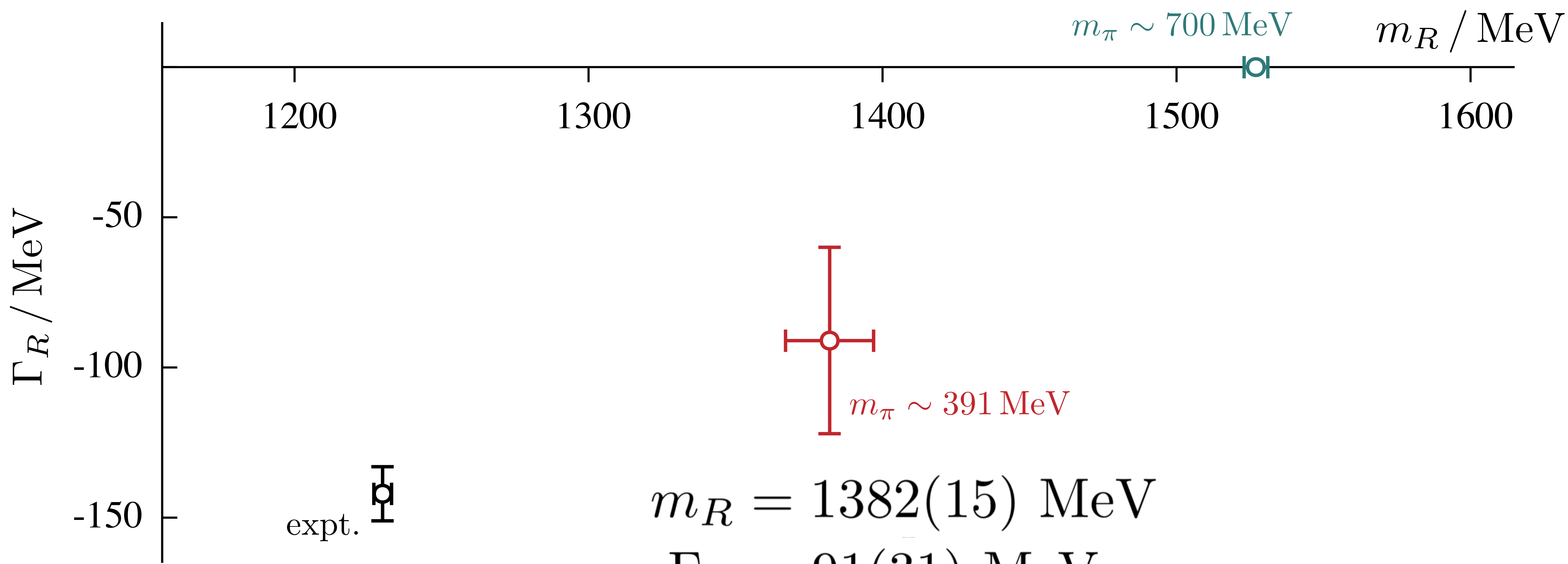


$$m_R = 1382(15)\ \text{MeV}$$
$$\Gamma_R = 91(31)\ \text{MeV}$$

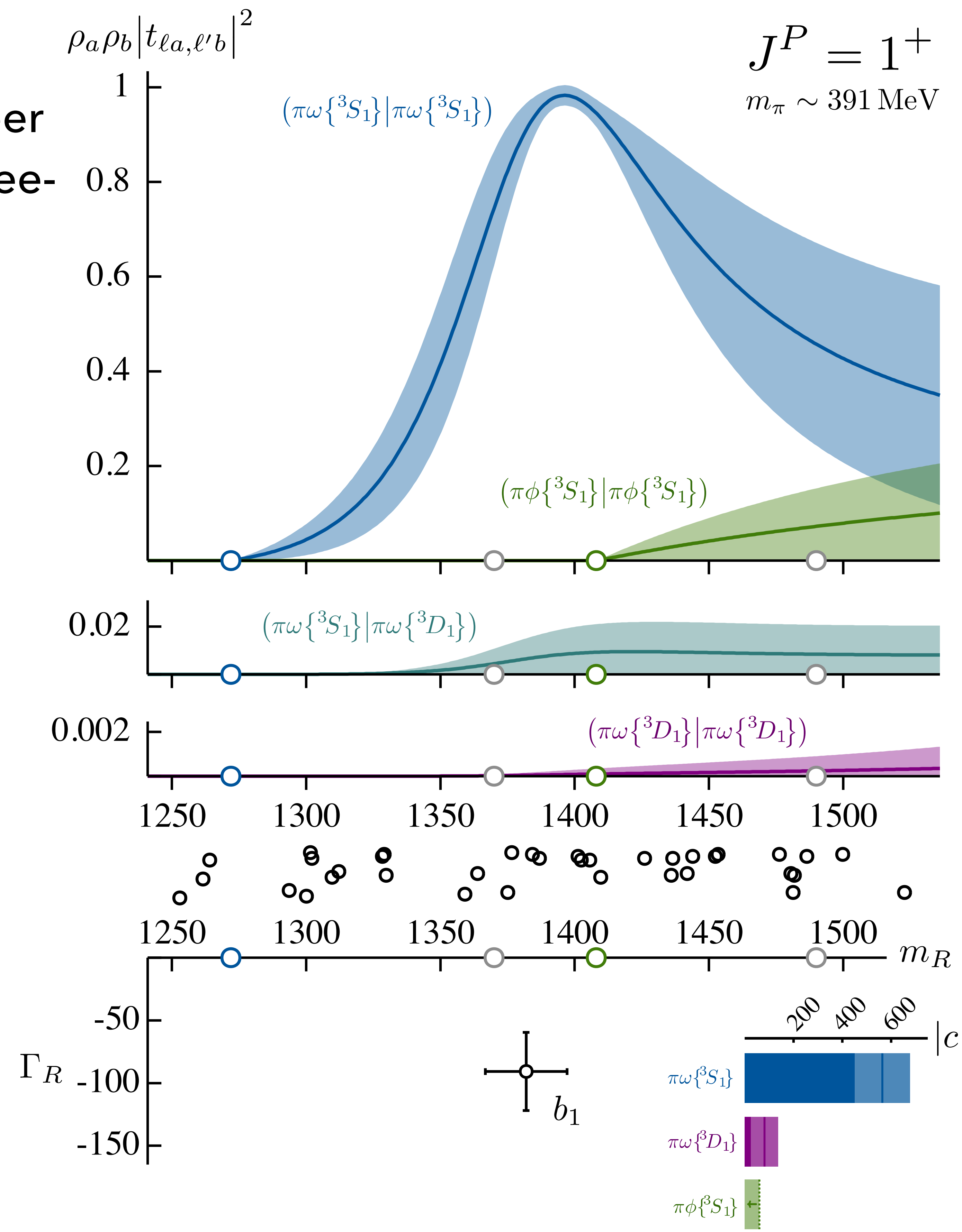
pole position at various pion masses



$$m_R = 1382(15)\ \text{MeV}$$
$$\Gamma_R = 91(31)\ \text{MeV}$$

$$\left|c_{\pi\omega\{^3S_1\}}\right| = 564(114)\ \text{MeV}$$
$$\left|c_{\pi\omega\{^3D_1\}}\right| = 81(56)\ \text{MeV},$$
$$\left|c_{\pi\phi\{^3S_1\}}\right| = 59(41)\ \text{MeV}.$$

calculated lattice spectra in a number of irreps using single-, two- and three-meson operators



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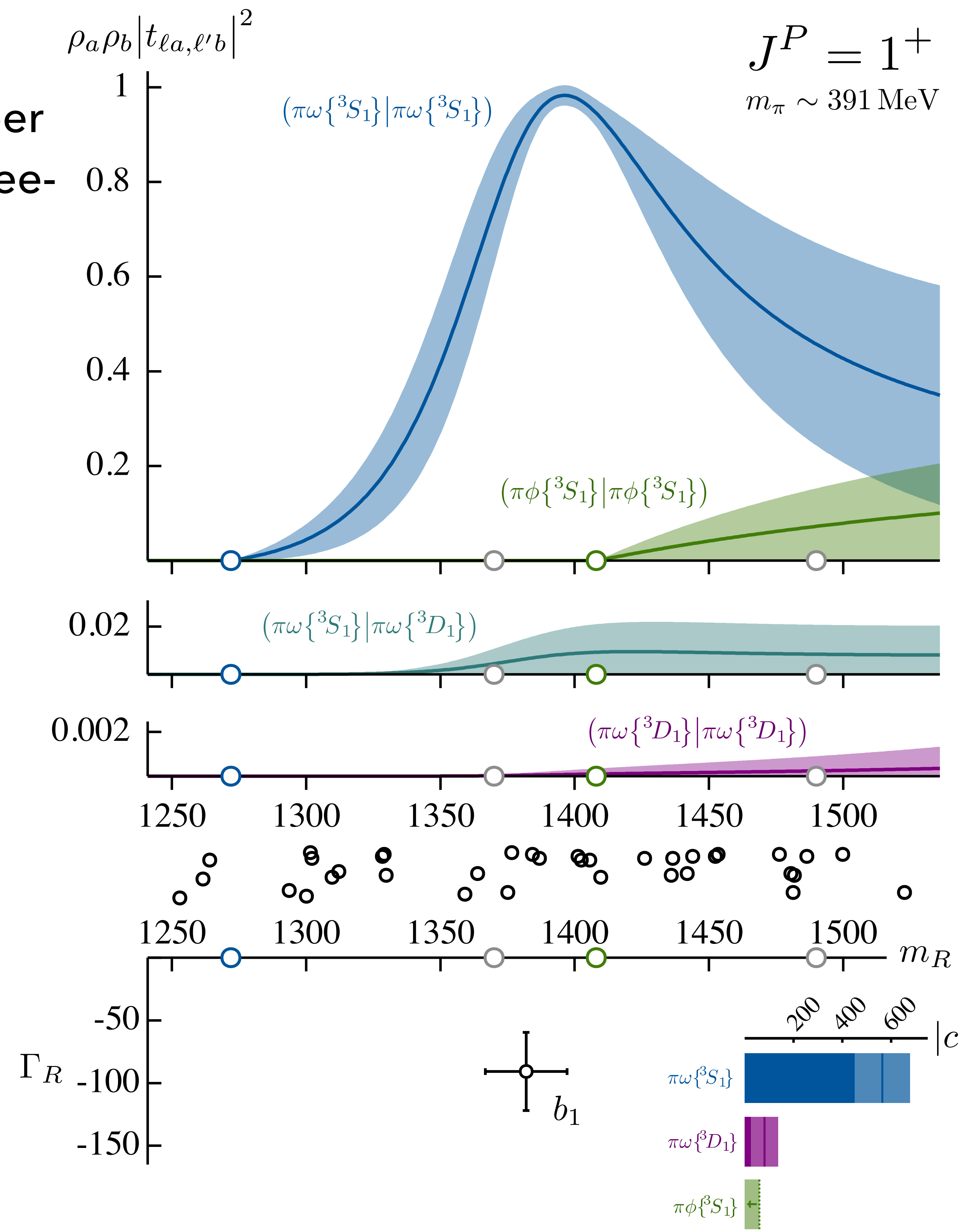

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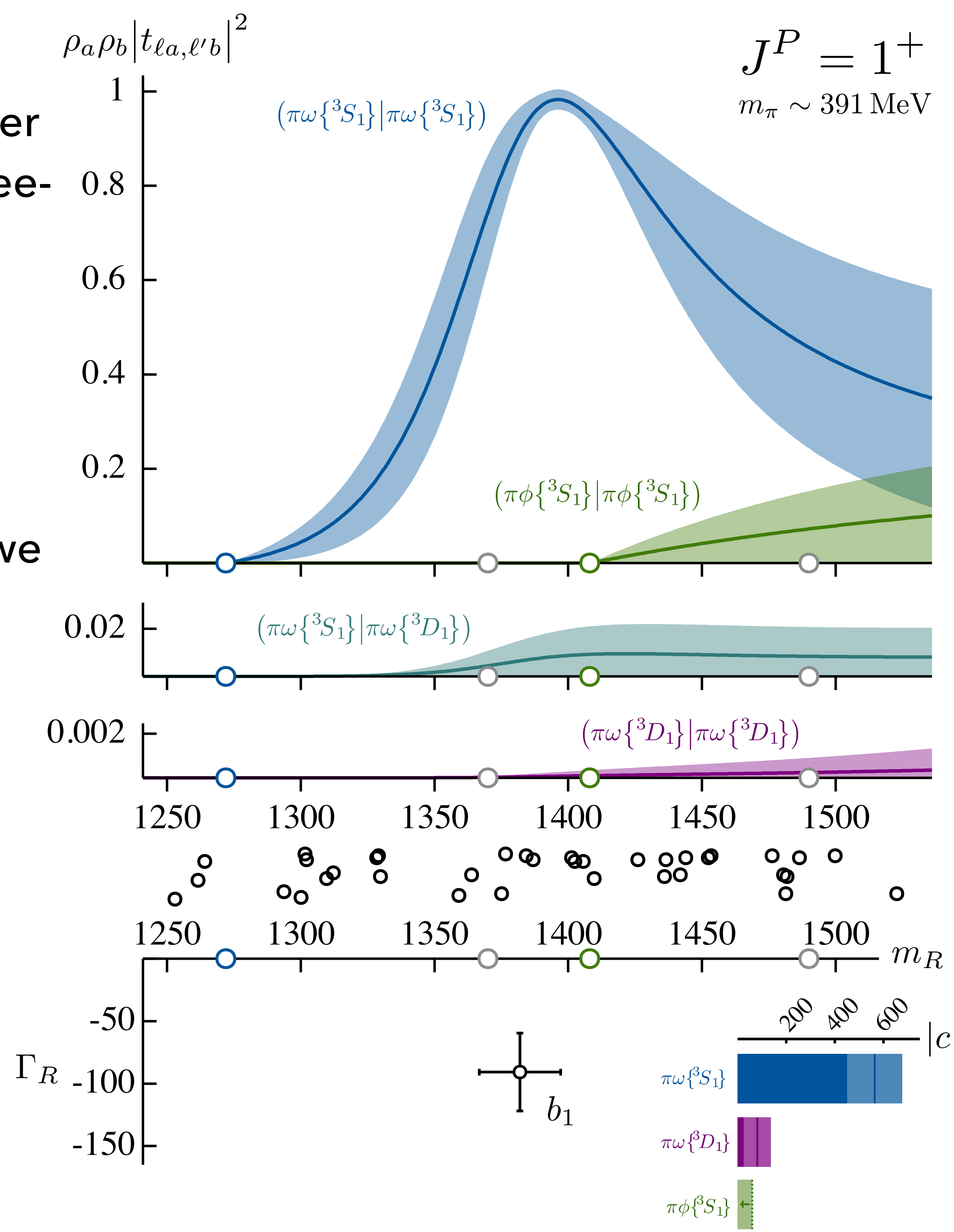

three-meson-like operators

region of study includes the opening of several two- and three-meson thresholds:

necessitates the inclusion of single-meson-like, two-meson-like and three-meson-like operators in the basis

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previous talks have discussed the construction of single-meson-like and two-meson-like operators region of study includes the opening of several two- and three-meson thresholds:

necessitates the inclusion of single-meson-like, two-meson-like and three-meson-like operators in the basis

previous talks have discussed the construction of single-meson-like and two-meson-like operators

what about three-meson-like?

one way is to iteratively apply the two-meson construction

$$\mathcal{O}^{\dagger\Lambda\mu}_{\mathbb{M}_1\mathbb{M}_2}(\vec{p}_{12}) = \sum_{\substack{\vec{p}_1,\vec{p}_2\\ \mu_1,\mu_2}} \mathcal{C}([\vec{p}_{12}]\Lambda,\mu;[\vec{p}_1]\Lambda_1,\mu_1;[\vec{p}_2]\Lambda_2,\mu_2)\,\Omega^{\dagger\Lambda_1\mu_1}_{\mathbb{M}_1}(\vec{p}_1)\,\Omega^{\dagger\Lambda_2\mu_2}_{\mathbb{M}_2}(\vec{p}_2),$$

optimised single-meson operators

lattice Clebsch–Gordans

efficient at interpolating energies near the corresponding two-meson non-interacting energies

$$E^{(2)}_{\text{n.i.}} = \sqrt{m_1^2 + |\vec{p}_1|^2} + \sqrt{m_2^2 + |\vec{p}_2|^2}$$

three-meson-like operators

three-meson construction

$$\mathcal{O}^{\dagger\Lambda\mu}_{\mathbb{M}_1\mathbb{M}_2\mathbb{M}_3}(\vec{p}_{123}) = \sum_{\substack{\vec{p}_{12},\vec{p}_3 \\ \mu_{12},\mu_3}} \mathcal{C}([\vec{p}_{123}]\Lambda,\mu;[\vec{p}_{12}]\Lambda_{12},\mu_{12};[\vec{p}_3]\Lambda_3,\mu_3)\mathcal{O}^{\dagger\Lambda_{12}\mu_{12}}_{\mathbb{M}_1\mathbb{M}_2}(\vec{p}_{12})\,\Omega^{\dagger\Lambda_3\mu_3}_{\mathbb{M}_3}(\vec{p}_3)$$

lattice Clebsch–Gordans

two-meson operator

optimised single-meson operator

efficient at interpolating energies near the corresponding three-meson non-interacting energies

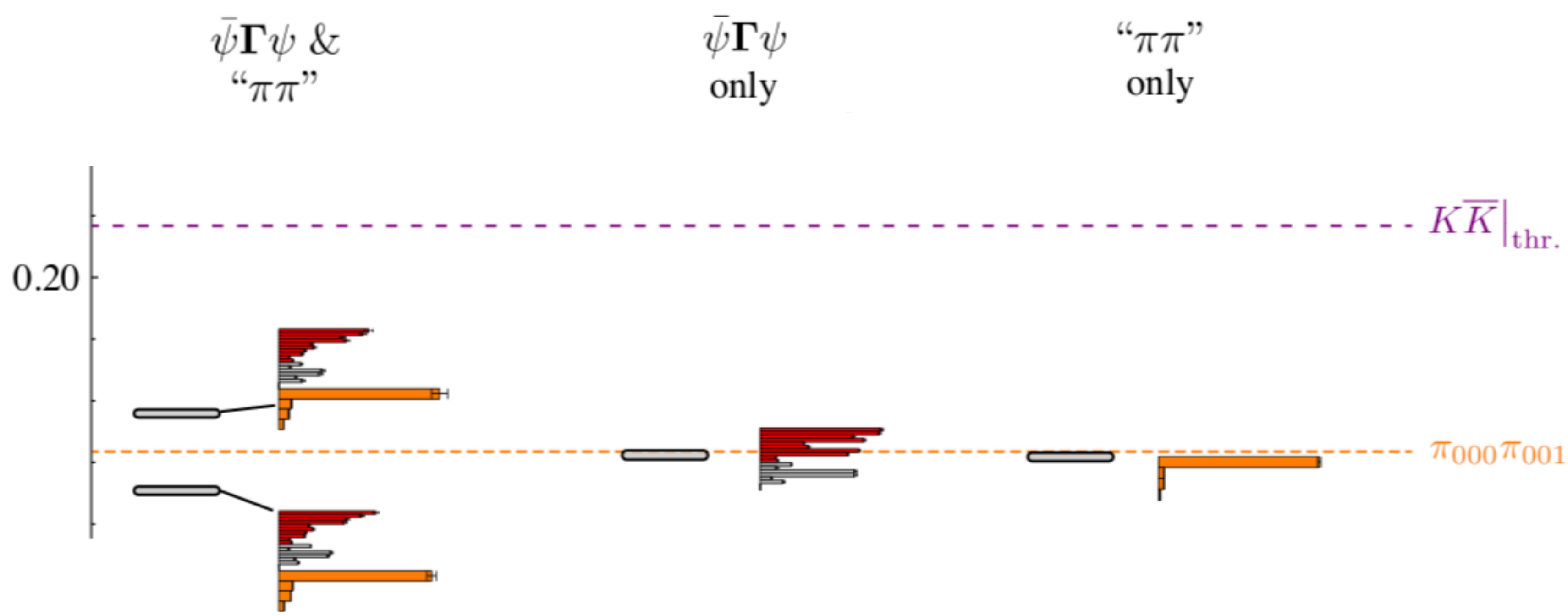
$$E^{(3)}_{\text{n.i.}} = \sqrt{m_1^2 + |\vec{p}_1|^2} + \sqrt{m_2^2 + |\vec{p}_2|^2} + \sqrt{m_3^2 + |\vec{p}_3|^2}$$

however, these operators pay no attention to interactions in the two-meson subsystems...

as an example consider isospin-2 $\pi\pi\pi$

previous construction would attempt to describe isospin-1 $\pi\pi$ using only $\pi\pi$ -like operators…

we know this is bad…



PhysRevD.87.034505

three-meson-like operators

rather build three-meson-like operators that incorporate subsystem interactions

use optimised two-meson operators in the three-meson operator construction

$$\mathcal{O}^{\dagger\Lambda\mu}_{\mathbb{R}_{12}\mathbb{M}_3}(\vec{p}_{123}) = \sum_{\substack{\vec{p}_{12},\vec{p}_3\\ \mu_{12},\mu_3}} \mathcal{C}([\vec{p}_{123}]\Lambda,\mu;[\vec{p}_{12}]\Lambda_{12},\mu_{12};[\vec{p}_3]\Lambda_3,\mu_3)\,\Omega^{\dagger\Lambda_{12}\mu_{12}}_{\mathbb{R}_{12}}(\vec{p}_{12})\,\Omega^{\dagger\Lambda_3\mu_3}_{\mathbb{M}_3}(\vec{p}_3)$$

lattice Clebsch–Gordans

optimised two-meson operator

optimised single-meson operator

by design we anticipate these operators to efficiently interpolate energies near

$$E^{(2+1)}_{\mathrm{n.i.}} = E^{\Lambda_{12}}_{\mathbb{R}^{\mathfrak{n}}_{12}}(\vec{p}_{12}) + \sqrt{m_3^2 + |\vec{p}_3|^2},$$

finite-volume energies in the two-meson subsystem in irrep $[\vec{p}_{12}]\Lambda_{12}$

three-meson-like operators example

as an example consider $\pi\pi\eta$ transforming in $[000]T_1^+$

$\vec{p}_1 = [000], \vec{p}_2 = [000], \vec{p}_3 = [000]$

$$\underbrace{\overbrace{[000]A_1^-}^{(I^G=1^-)}}_{\pi} \otimes \underbrace{\overbrace{[000]A_1^-}^{(I^G=1^-)}}_{\pi} \otimes \underbrace{\overbrace{[000]A_1^-}^{(I^G=0^+)}}_{\eta} \rightarrow [000]A_1^-$$

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$\vec{p}_1 = [001], \vec{p}_2 = [001], \vec{p}_3 = [000]$

$$\overbrace{\underbrace{[001]A_2}_{\pi}}^{(I^G=1^-)} \otimes \overbrace{\underbrace{[001]A_2}_{\pi}}^{(I^G=1^-)} \otimes \overbrace{\underbrace{[000]A_1^-}_{\eta}}^{(I^G=0^+)} \to \overbrace{[000]T_1^+}^{(I^G=1^+)} \oplus \ldots$$

$$E^{(3)}_{\text{n.i.}} = 2\sqrt{m_\pi^2 + \left(\tfrac{2\pi}{L}\right)^2} + m_\eta$$

two distinct two-meson subsystems:

three-meson-like operators example

First:

$$\vec{p}_1 = [001], \vec{p}_2 = [001], \vec{p}_3 = [000]$$

$$\Big(\underbrace{\overbrace{[001]A_2}^{(I^G=1^-)}}_{\pi} \otimes \underbrace{\overbrace{[001]A_2}^{(I^G=1^-)}}_{\pi} \Big) \otimes \underbrace{\overbrace{[000]A_1^-}^{(I^G=0^+)}}_{\eta} \rightarrow \overbrace{[000]T_1^+}^{(I^G=1^+)}$$

$$\underbrace{\overbrace{[000]T_1^-}^{(I^G=1^+)}}_{\rho} \otimes \underbrace{\overbrace{[000]A_1^-}^{(I^G=0^+)}}_{\eta} \rightarrow \overbrace{[000]T_1^+}^{(I^G=1^+)}\,.$$

$$E_{\text{n.i.}}^{(2+1)} = E_{\rho^{\mathfrak{n}}}^{T_1^-}([000]) + m_\eta$$

Second:

$\vec{p}_1 = [001], \vec{p}_2 = [001], \vec{p}_3 = [000]$

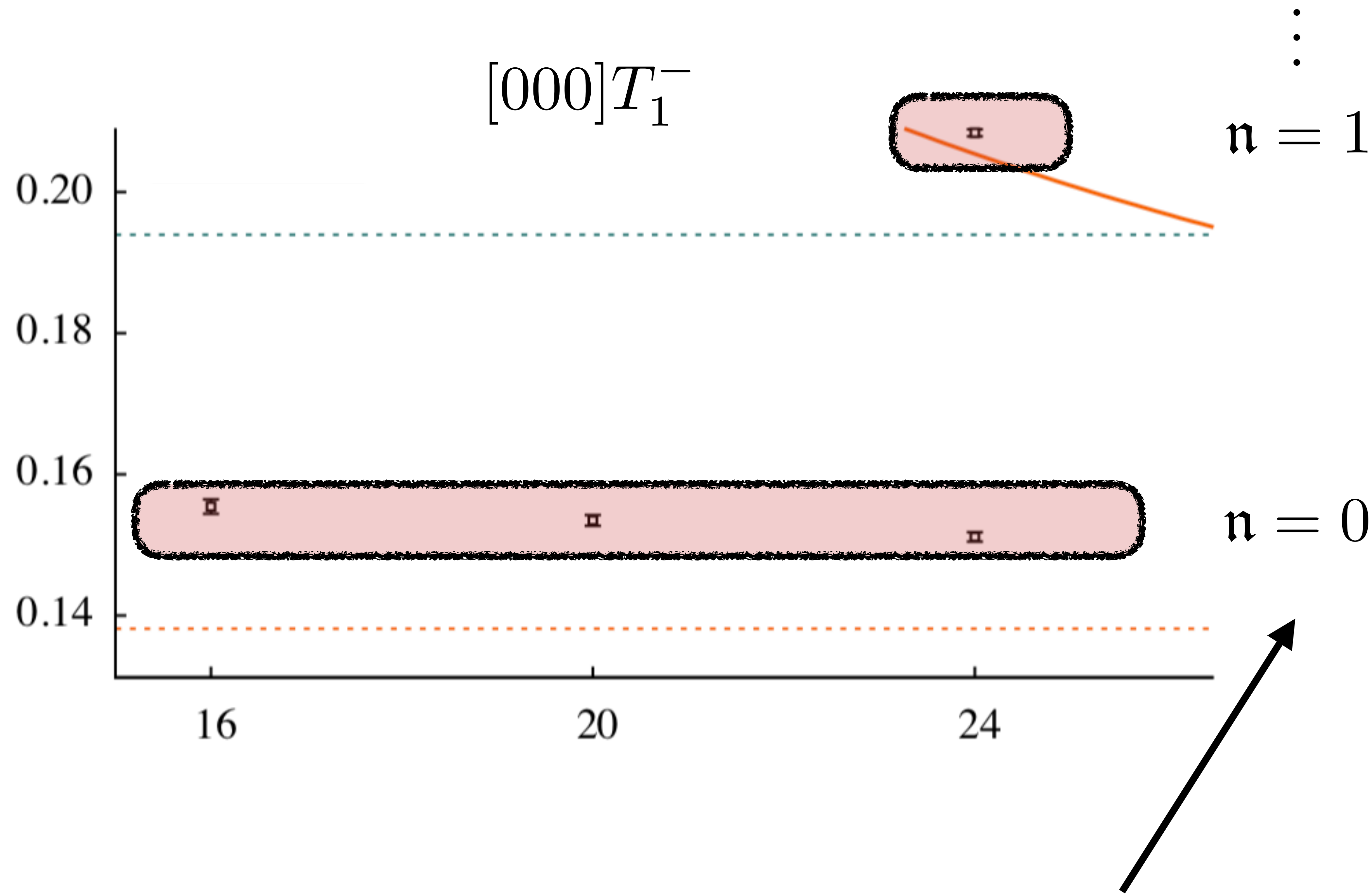
$$\Big(\overbrace{\underbrace{[001]A_2}_{\pi}}^{(I^G=1^-)} \otimes \overbrace{\underbrace{[000]A_1^-}_{\eta}}^{(I^G=0^+)} \Big) \otimes \overbrace{\underbrace{[001]A_2}_{\pi}}^{(I^G=1^-)} \rightarrow \overbrace{[000]T_1^+}^{(I^G=1^+)}$$

$$\overbrace{\underbrace{[001]A_1}_{a_0}}^{(I^G=1^-)} \otimes \overbrace{\underbrace{[001]A_2}_{\pi}}^{(I^G=1^-)} \rightarrow \overbrace{[000]T_1^+}^{(I^G=1^+)}$$

$$E_{\text{n.i.}}^{(2+1)} = E_{a_0}^{A_1}{}_{\mathfrak{n}}([001]) + \sqrt{m_\pi^2 + \left(\frac{2\pi}{L}\right)^2}$$

three-meson-like operators example

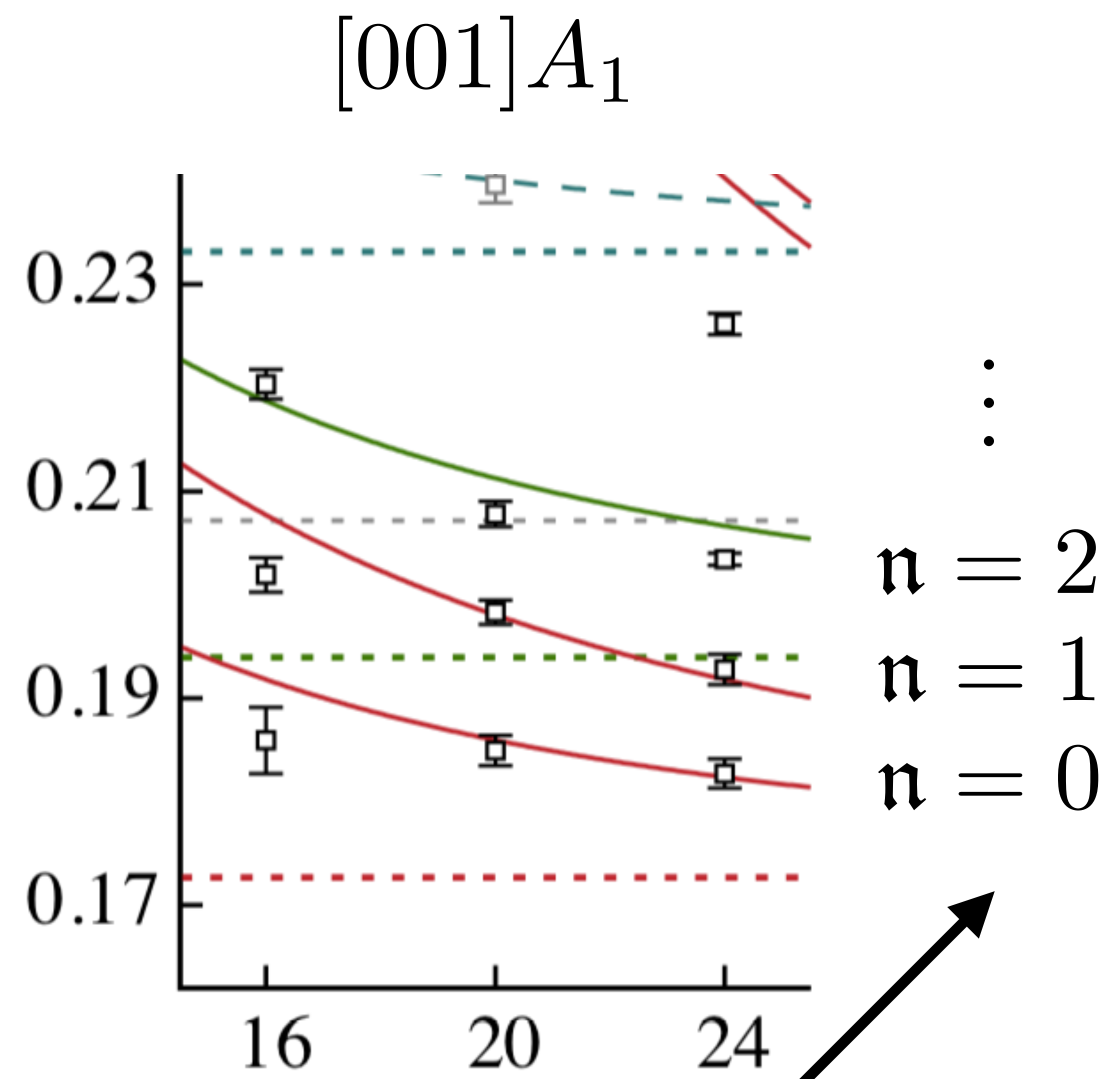
First:



$$E_{\text{n.i.}}^{(2+1)} = E_{\rho^{\mathfrak{n}}}^{T_1^-}([000]) + m_\eta$$

PhysRevD.87.034505

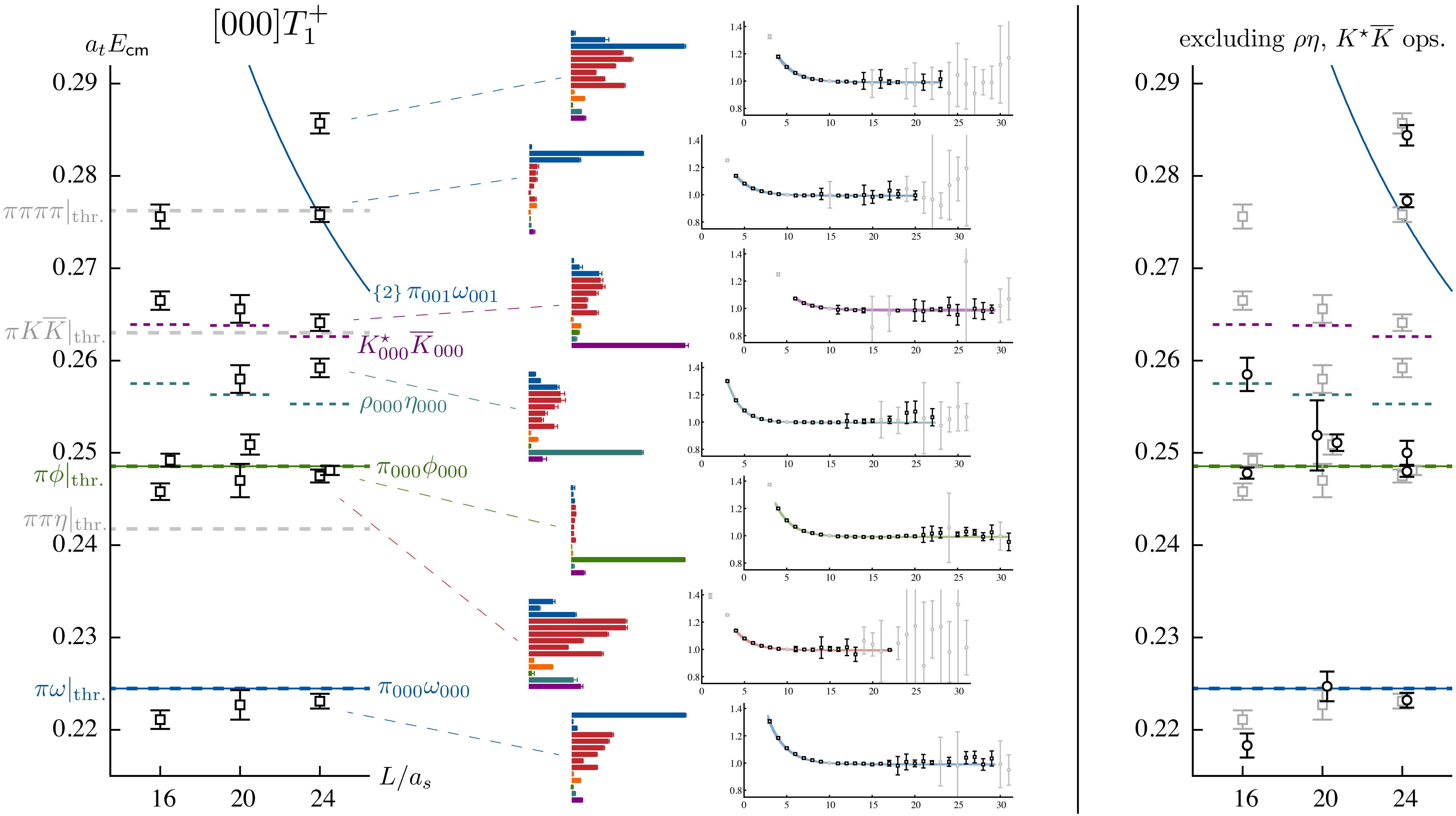
Second:



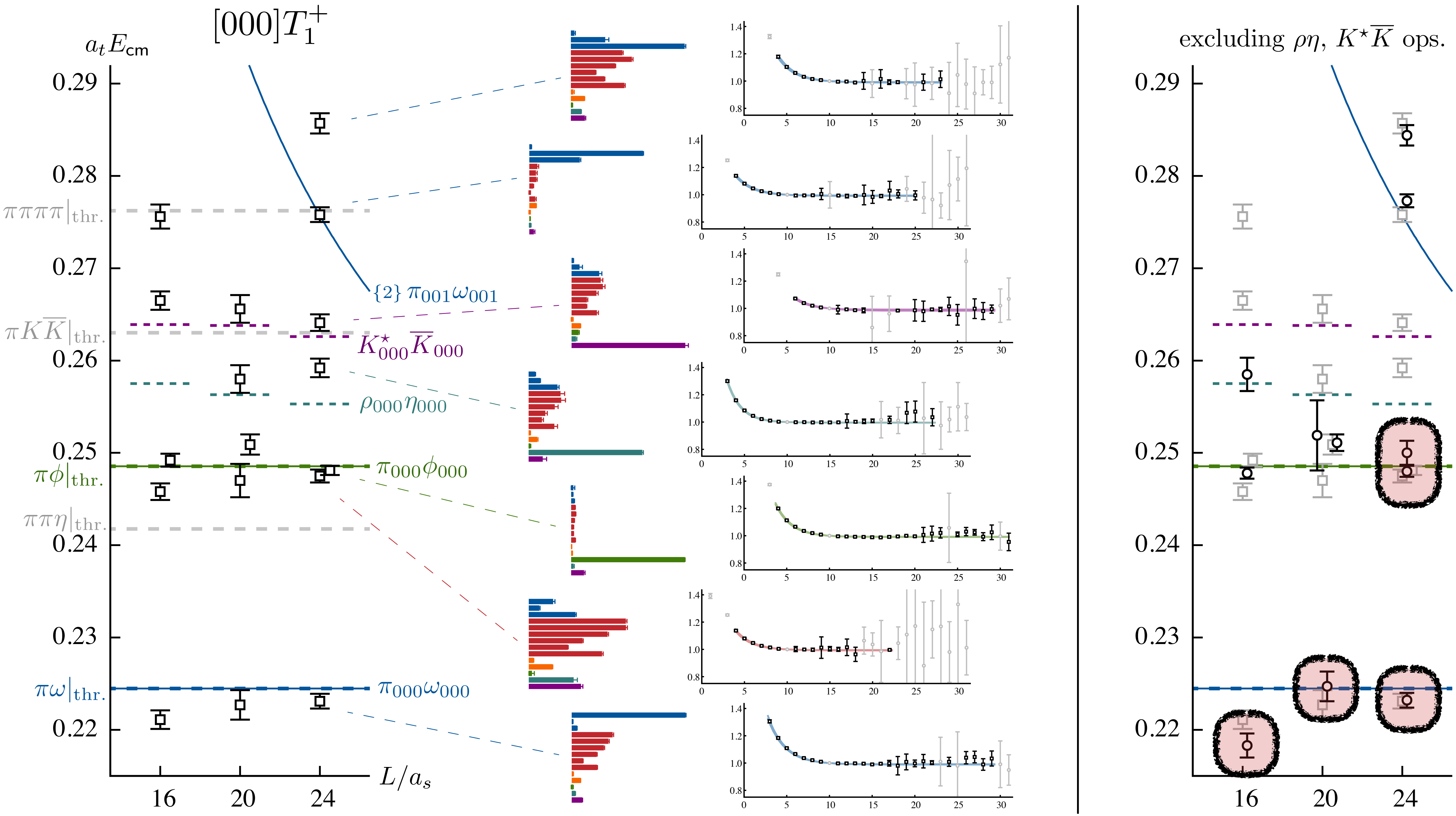
$$E_{\text{n.i.}}^{(2+1)} = E_{a_0{}^{\mathfrak{n}}}^{A_1}([001]) + \sqrt{m_\pi^2 + \left(\frac{2\pi}{L}\right)^2}$$

PhysRevD.93.094506

spectrum with/without three-meson operators



spectrum with/without three-meson operators



lots of excellent work on a general quantisation condition for three-particle scattering

not quite ready for use in systems such as $\pi\pi\eta$ or $\pi\overline{K}K$ in isospin-1

systematic tests – `three-body' amplitudes

as a crude test of how three-body amplitudes may influence the spectra and our determination of the two-body amplitudes, approximate ρ and K^* as `*stable*' as a crude test of how three-body amplitudes may influence the spectra and our determination of the two-body amplitudes, approximate $\rho$ and $K^*$ as `*stable*'

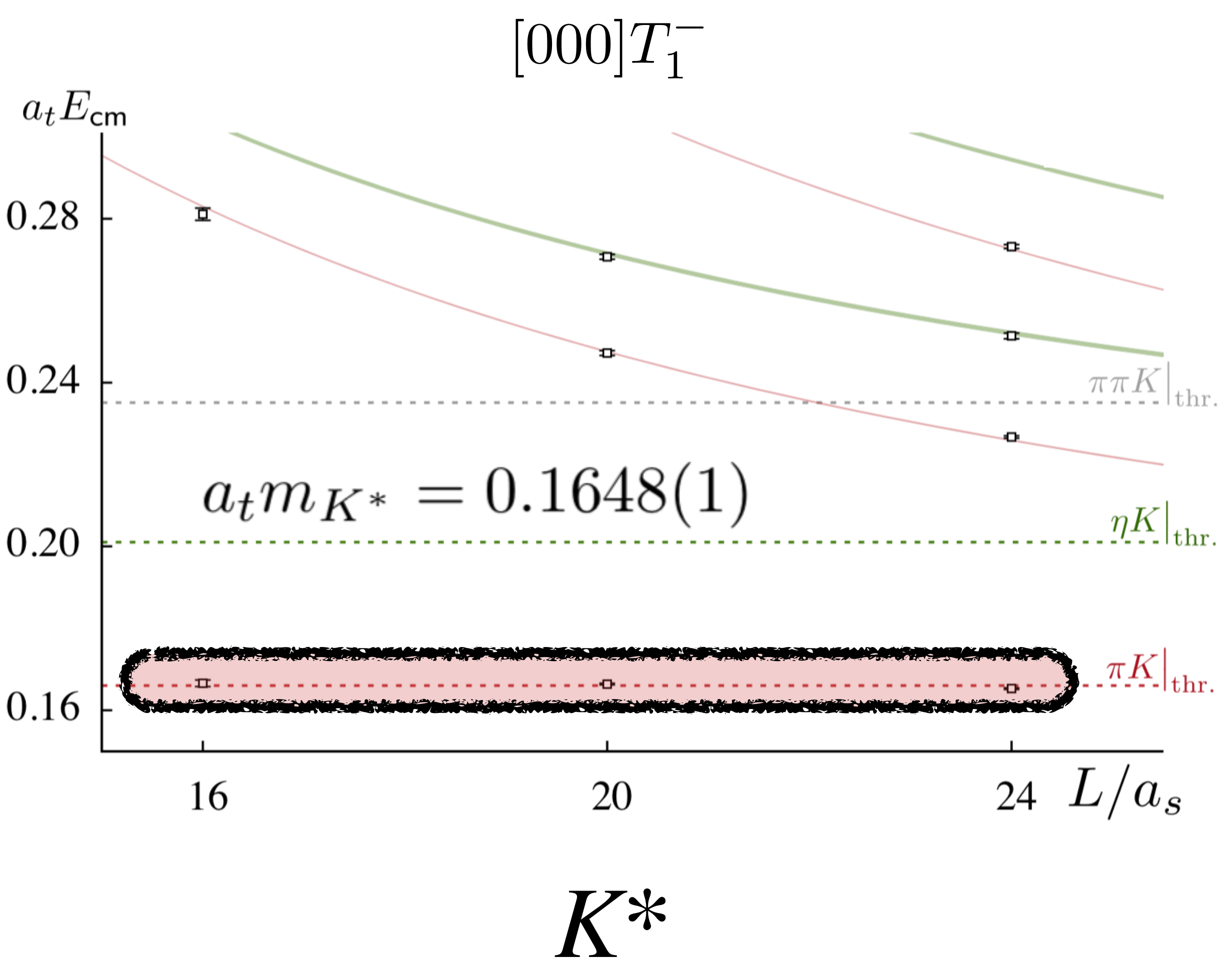
take `*stable*' masses to be the pole masses determined in previous scattering calculations

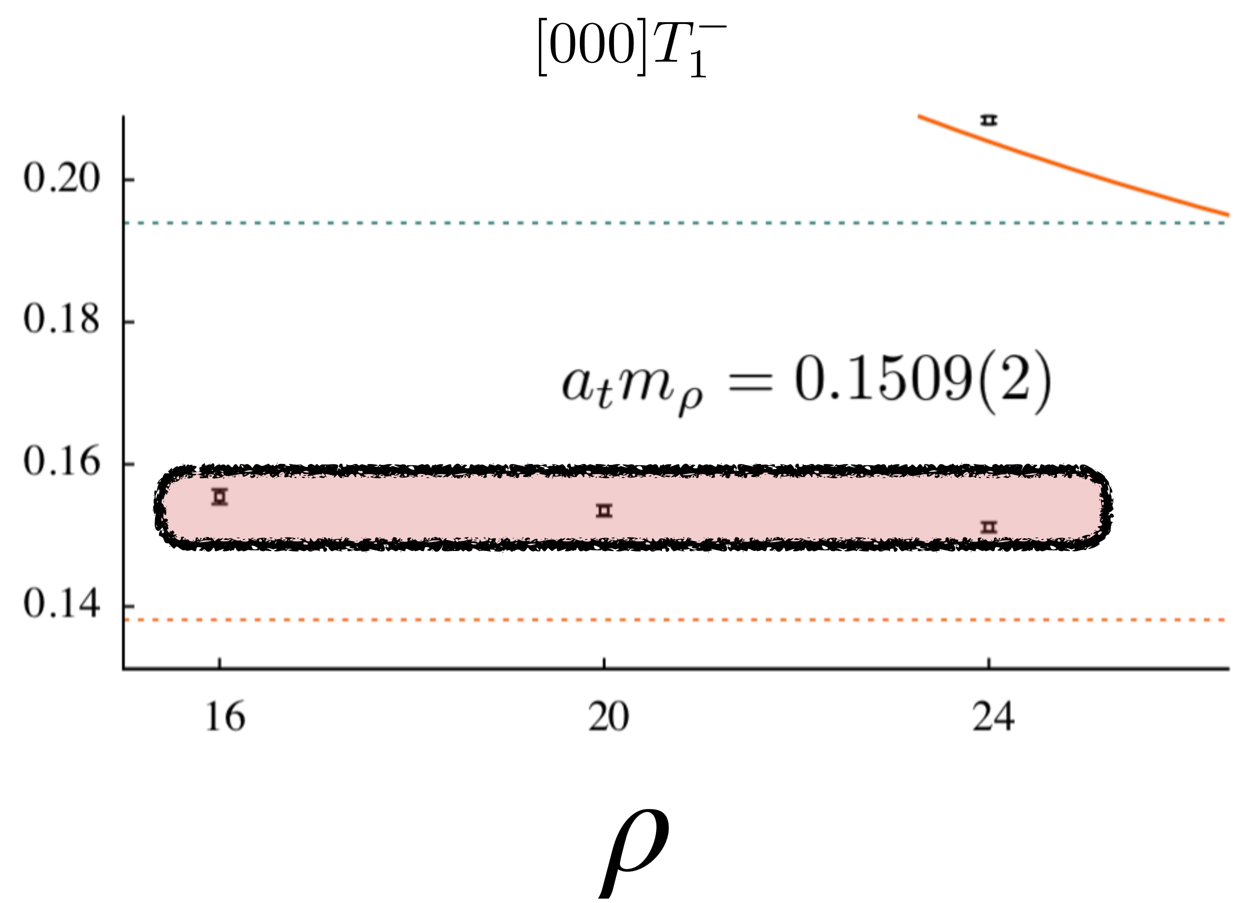
systematic tests – `three-body' amplitudes

as a crude test of how three-body amplitudes may influence the spectra and our determination of the two-body amplitudes, approximate ρ and K^* as `*stable*'

take `*stable*' masses to be the pole masses determined in previous scattering calculations

this appears to be reasonable when the ρ and K^* are at zero momentum:
at these volumes ground-state appears dominated by `$q\bar{q}$'-like operators





PhysRevD.91.054008

PhysRevD.87.034505

augment t-matrix to accommodate additional two-channels

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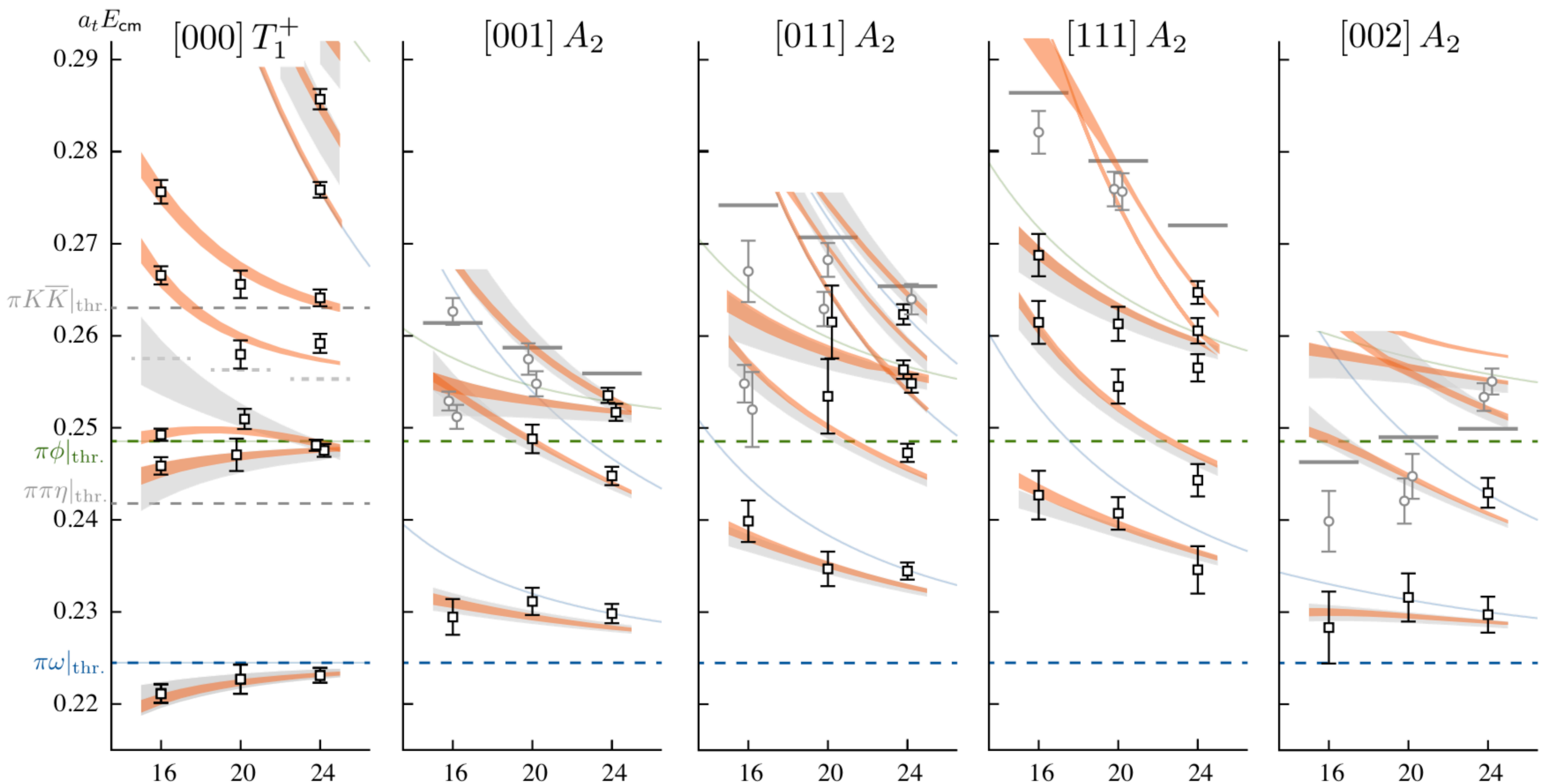
fit the 5 coupled-channels to the lattice spectra: 36 levels + 12 additional levels



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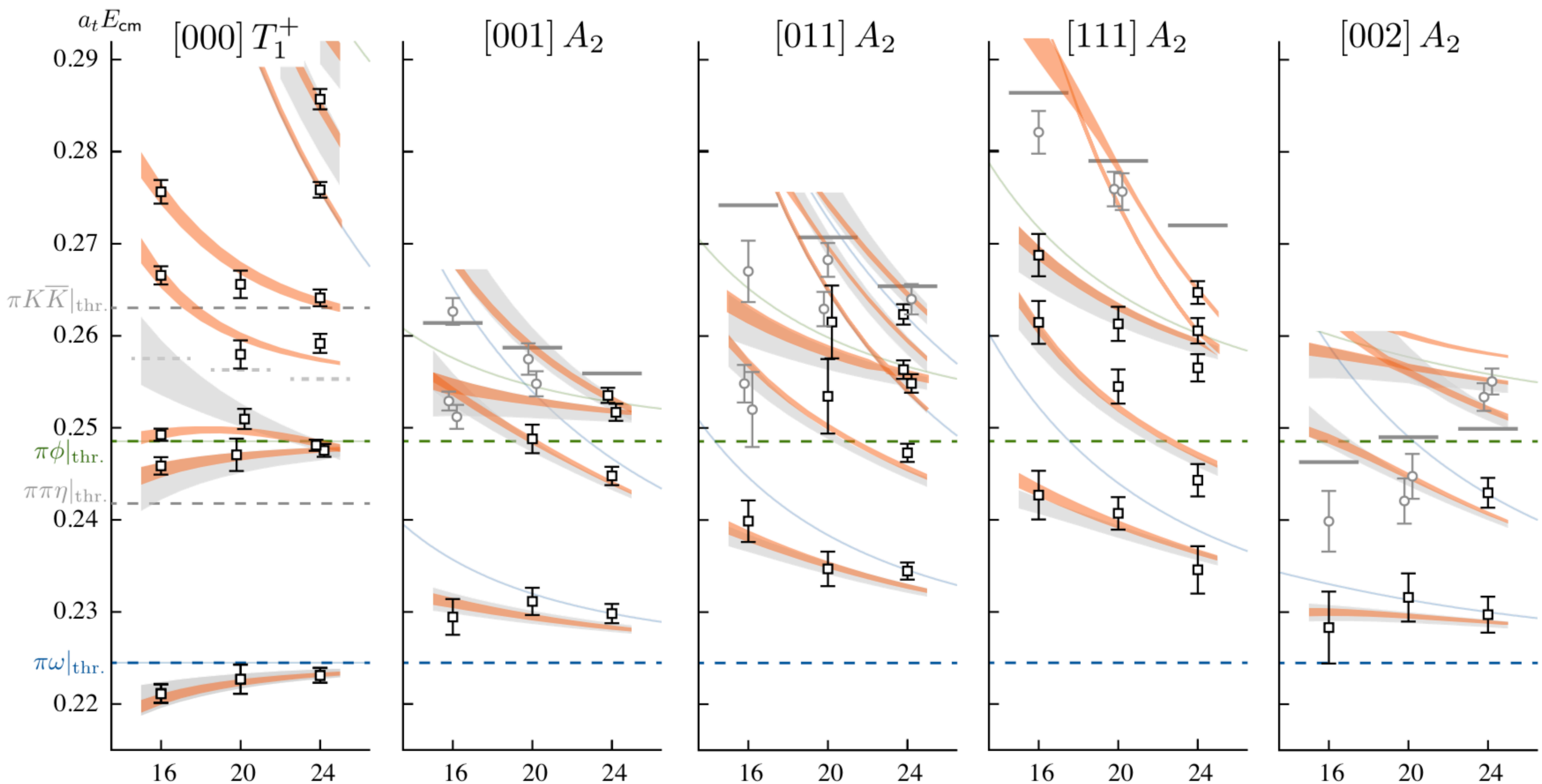

$$m_R = 1387(7)\ \text{MeV}$$
$$\Gamma_R = 122(12)\ \text{MeV}$$

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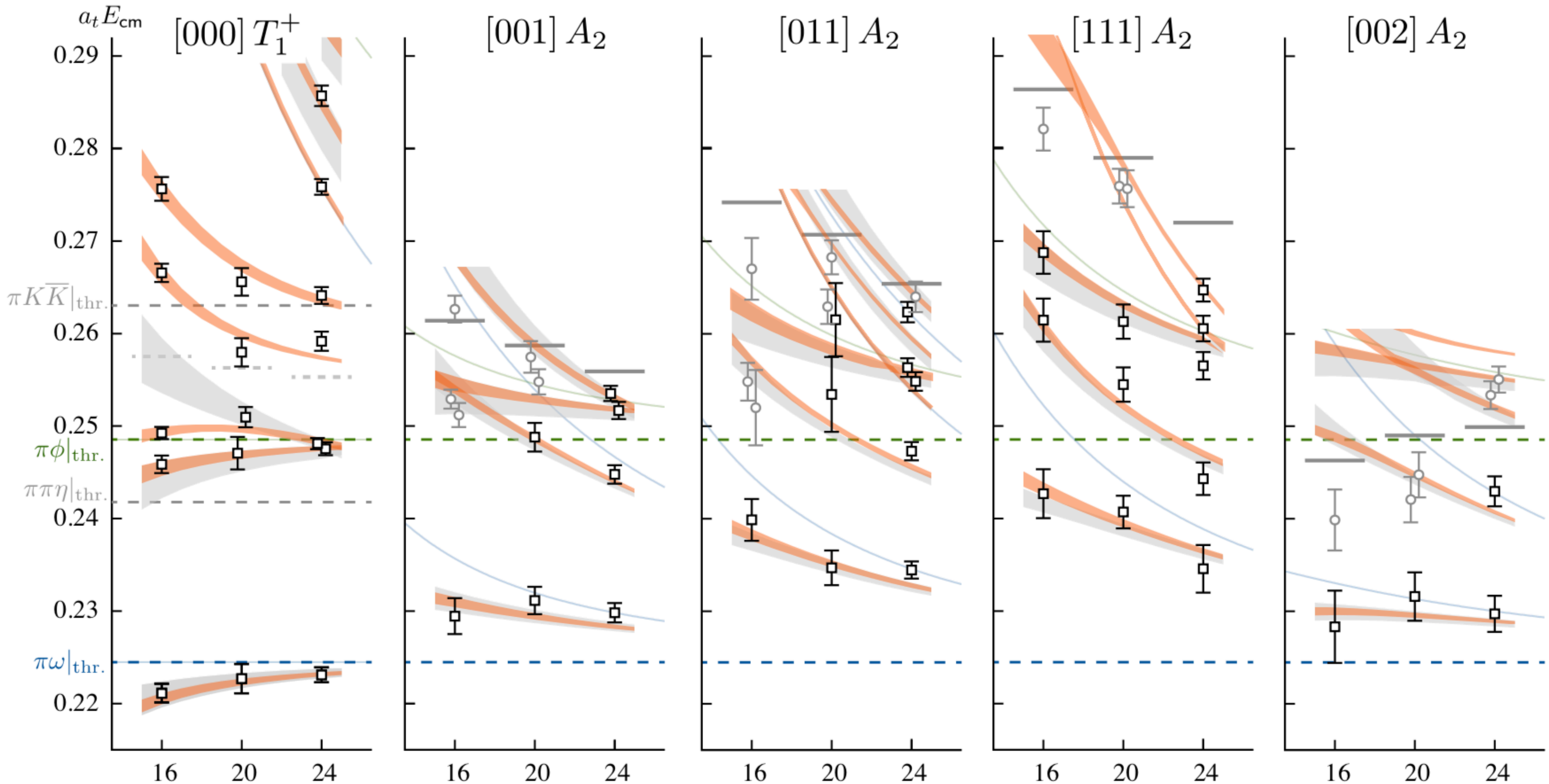


$$m_R = 1387(7) \text{ MeV}$$
$$\Gamma_R = 122(12) \text{ MeV}$$

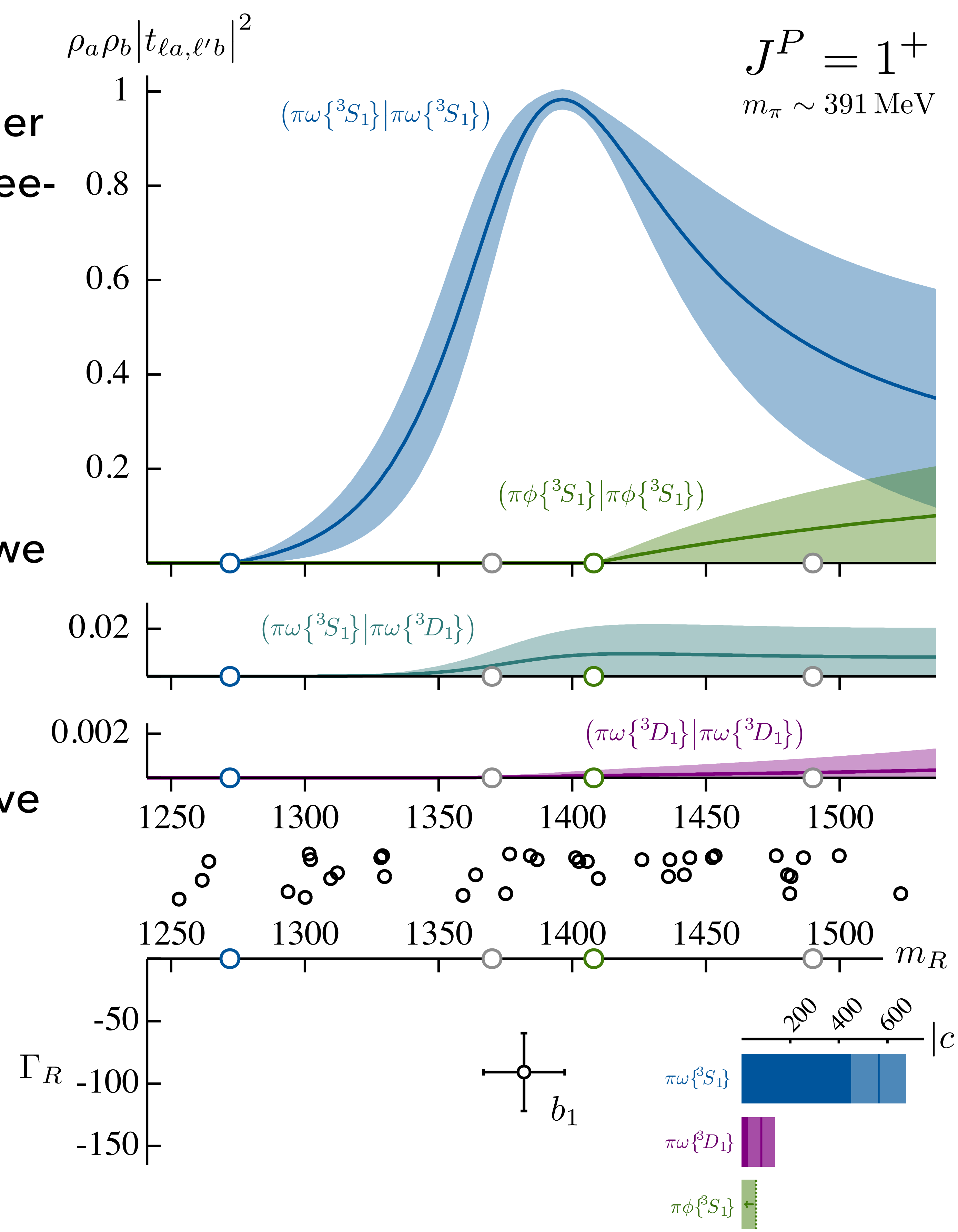
recall:
$$m_R = 1382(15) \text{ MeV}$$
$$\Gamma_R = 91(31) \text{ MeV}$$

calculated lattice spectra in a number of irreps using single-, two- and three-meson operators

calculated three-coupled vector-pseudoscalar amplitudes

determined a resonance pole that we interpret as the b_1 resonance

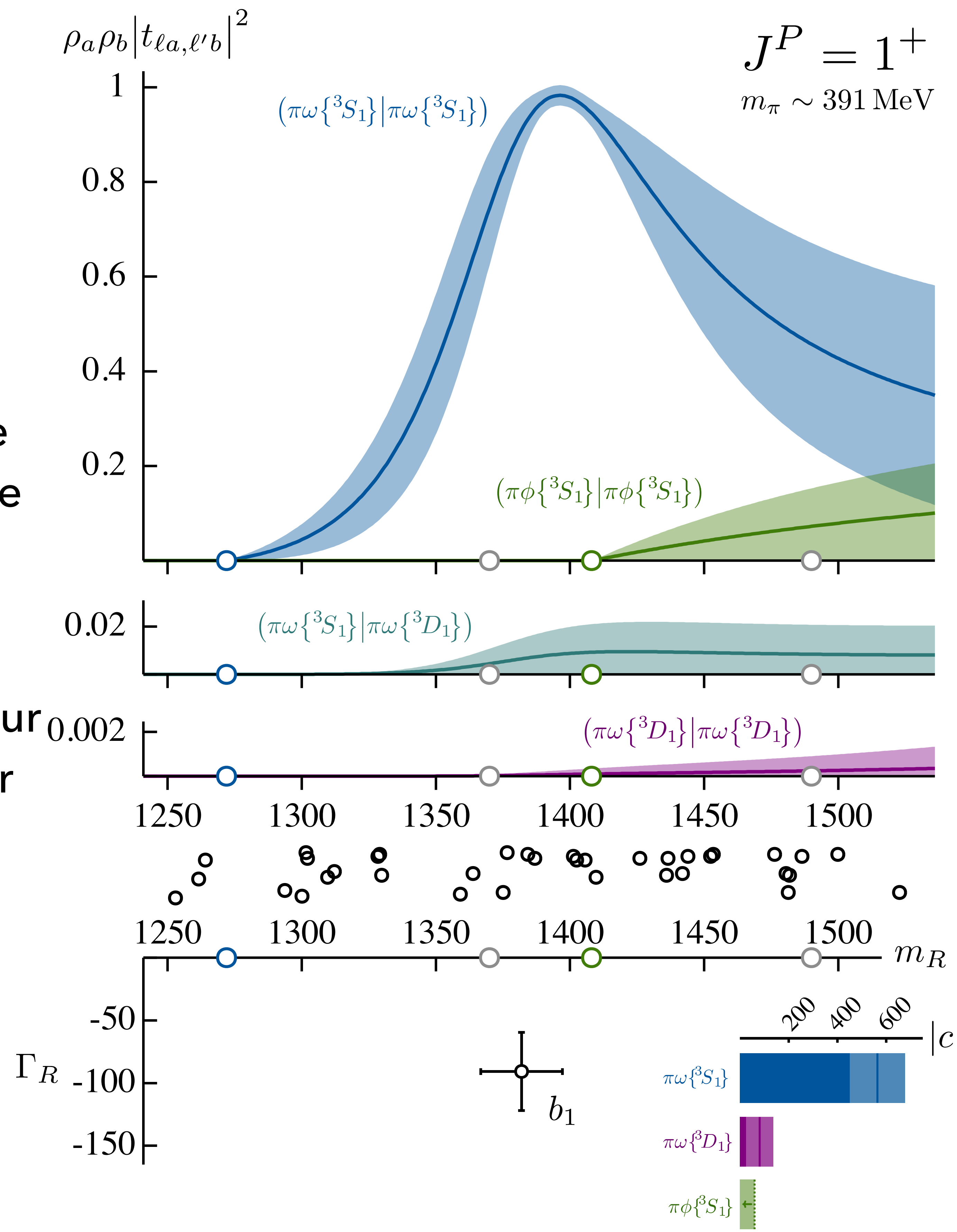
explored the role of three-body channels – found evidence they have negligible effect in this case



a step in understanding the role of non-zero intrinsic spin in scattering: essential to calculate, e.g. a_1, π_1, Z_c

this calculation provides a good testing ground for extensions to the three-body formalism to incorporate isospin and coupled channels

desirable to determine the behaviour of the pole and couplings at a lower pion mass



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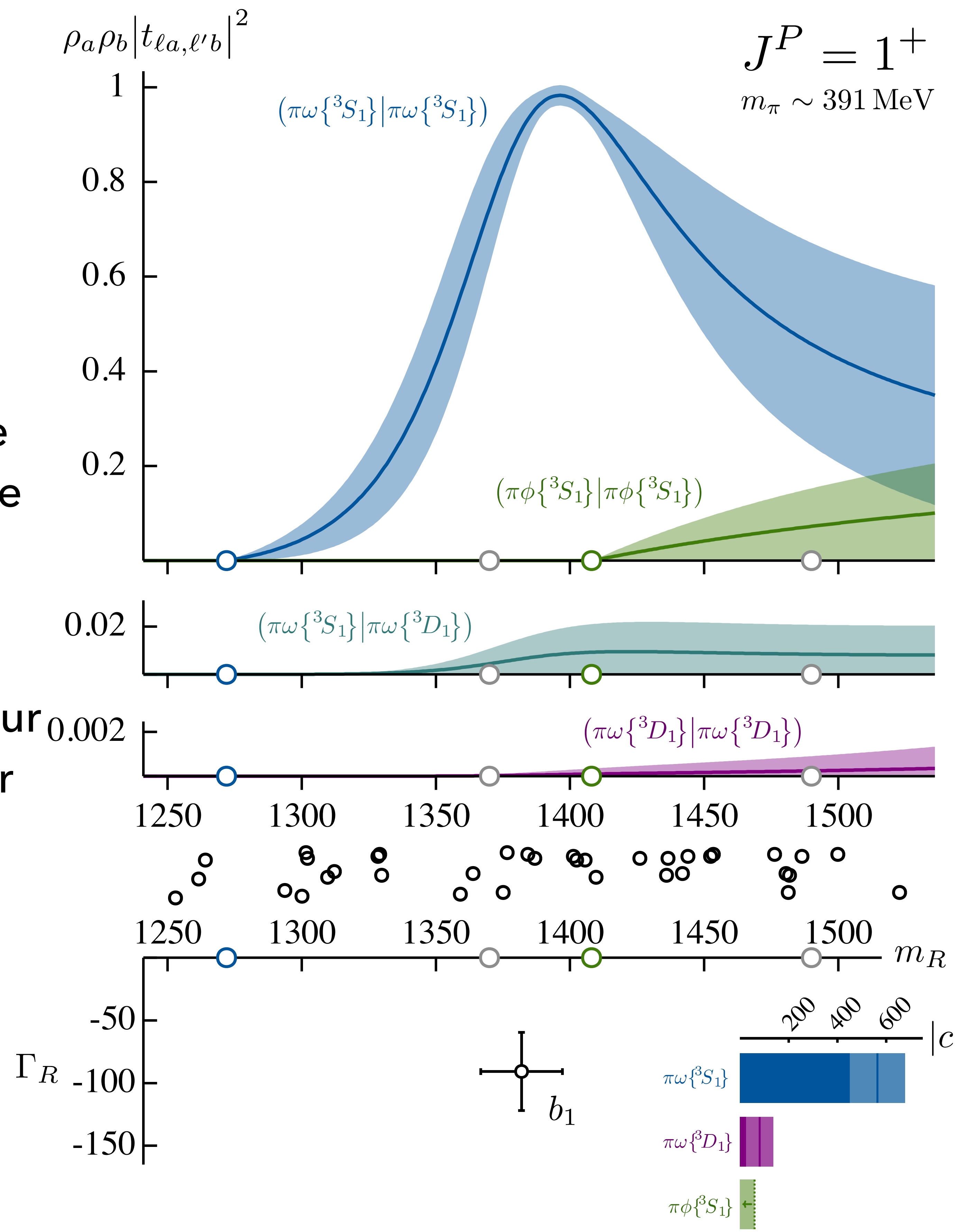
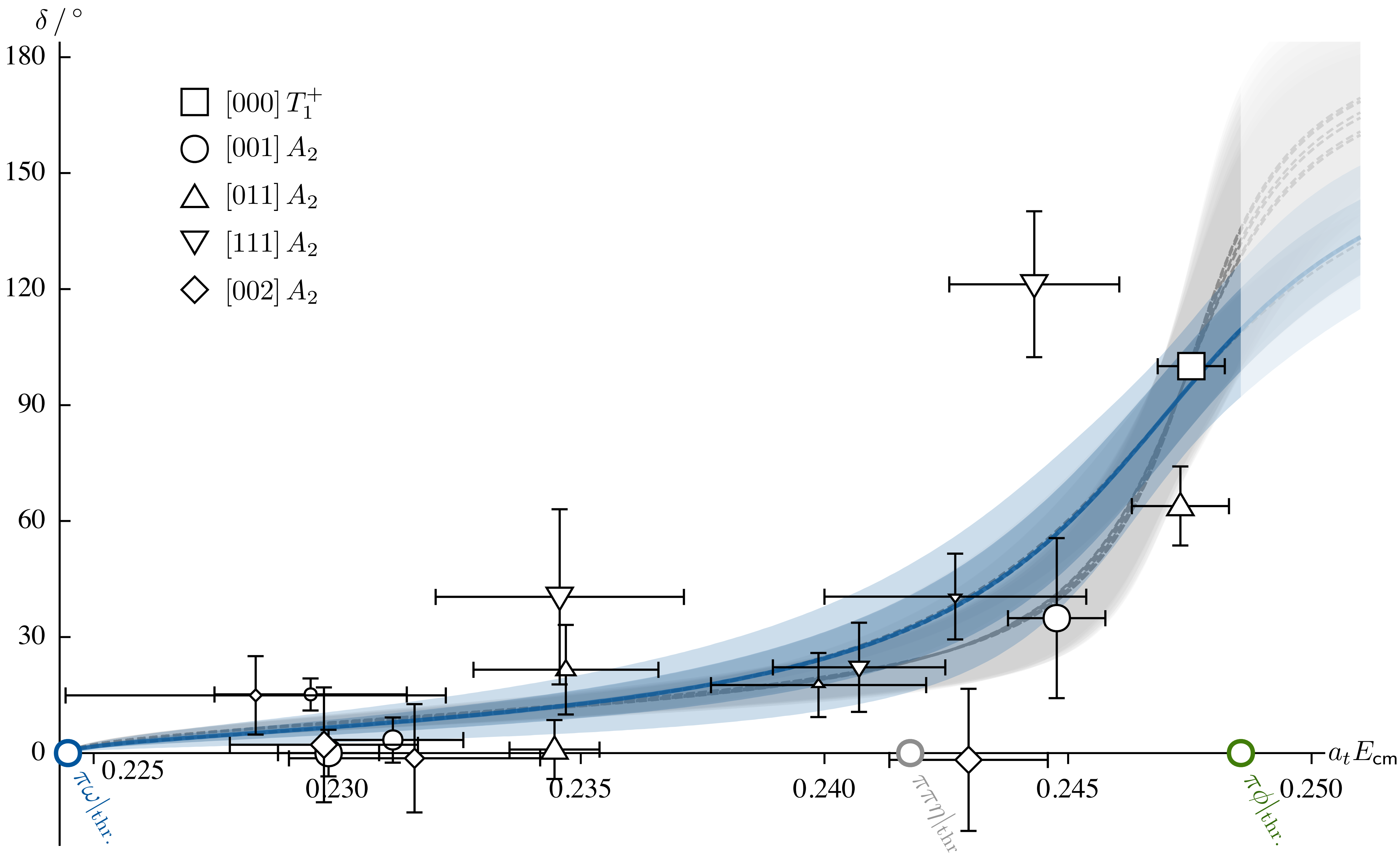
thank you for listening!



backup

backup – `single-channel' phase-shift

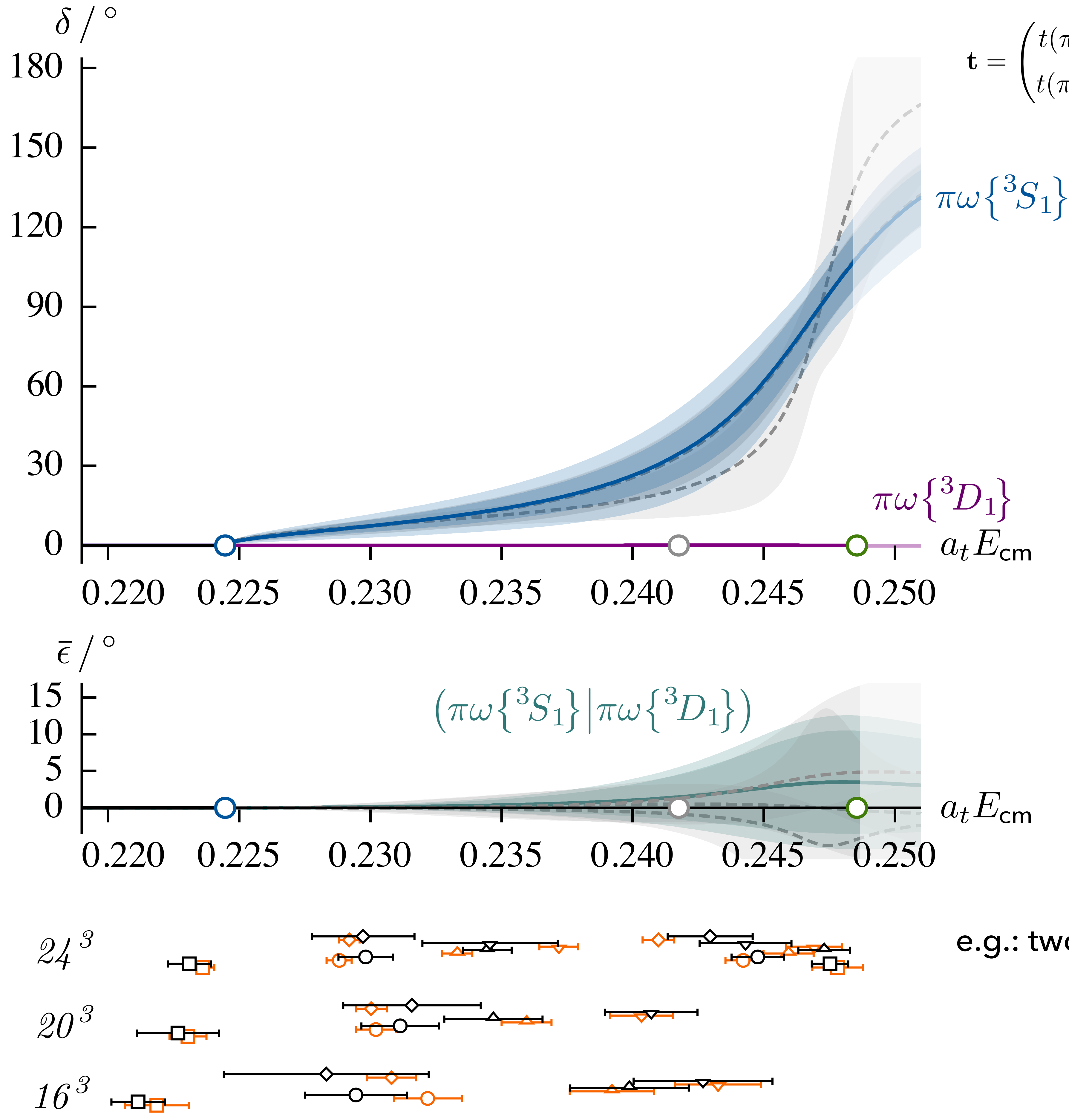
$t(\pi\omega\{^3S_1\}|\pi\omega\{^3S_1\})$



e.g.: single-channel K-matrix with one pole: $\chi^2/N_{\mathrm{dof}} = \frac{15.1}{20-2} = 0.84$

backup – two-channel phase-shifts + mixing-angles

$$\mathbf{t} = \begin{pmatrix} t(\pi\omega\{^3S_1\}|\pi\omega\{^3S_1\}) & t(\pi\omega\{^3S_1\}|\pi\omega\{^3D_1\}) \\ t(\pi\omega\{^3S_1\}|\pi\omega\{^3D_1\}) & t(\pi\omega\{^3D_1\}|\pi\omega\{^3D_1\}) \end{pmatrix},$$

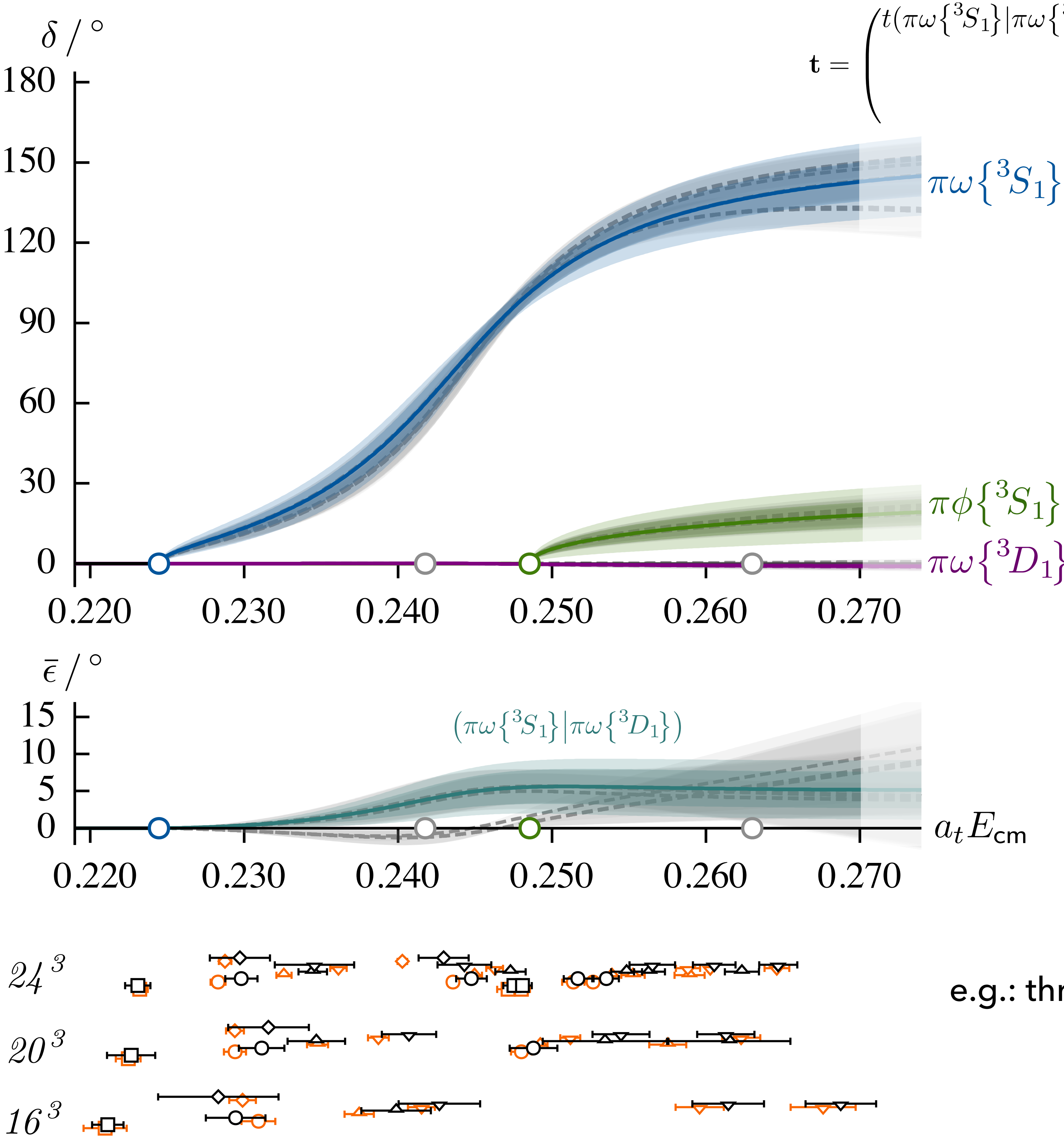


e.g.: two-channel K-matrix with one pole:

$$\chi^2/N_{\mathrm{dof}} = \frac{14.9}{20-3} = 0.87$$

backup – three-channel phase-shifts + mixing-angles

$$\mathbf{t} = \begin{pmatrix} t(\pi\omega\{^3S_1\}|\pi\omega\{^3S_1\}) & t(\pi\omega\{^3S_1\}|\pi\omega\{^3D_1\}) & t(\pi\omega\{^3S_1\}|\pi\phi\{^3S_1\}) \\ & t(\pi\omega\{^3D_1\}|\pi\omega\{^3D_1\}) & t(\pi\omega\{^3D_1\}|\pi\phi\{^3S_1\}) \\ & & t(\pi\phi\{^3S_1\}|\pi\phi\{^3S_1\}) \end{pmatrix}$$



e.g.: three-channel K-matrix with one pole:

$$\chi^2/N_{\mathrm{dof}} = \frac{36.8}{36-5} = 1.19.$$

backup – operator tables

L/a_s	16	20	24
$\rho_{[000],\,T_1^-}$	$26 \times \bar{\psi}\mathbf{\Gamma}\psi$ $3 \times \pi\pi$	$26 \times \bar{\psi}\mathbf{\Gamma}\psi$ $2 \times \pi\pi$	$12 \times \bar{\psi}\mathbf{\Gamma}\psi$
$\rho_{[001],\,A_1}$	$8 \times \bar{\psi}\mathbf{\Gamma}\psi$ $4 \times \pi\pi$	$18 \times \bar{\psi}\mathbf{\Gamma}\psi$ $4 \times \pi\pi$	$18 \times \bar{\psi}\mathbf{\Gamma}\psi$ $4 \times \pi\pi$
$\rho_{[011],\,A_1}$	$27 \times \bar{\psi}\mathbf{\Gamma}\psi$ $3 \times \pi\pi$	$27 \times \bar{\psi}\mathbf{\Gamma}\psi$ $3 \times \pi\pi$	$27 \times \bar{\psi}\mathbf{\Gamma}\psi$ $3 \times \pi\pi$
$\rho_{[111],\,A_1}$	$8 \times \bar{\psi}\mathbf{\Gamma}\psi$ $3 \times \pi\pi$	$21 \times \bar{\psi}\mathbf{\Gamma}\psi$ $3 \times \pi\pi$	$21 \times \bar{\psi}\mathbf{\Gamma}\psi$ $3 \times \pi\pi$

L/a_s	16	20	24
$K^*_{[000],\,T_1^-}$	$6 \times \bar{\psi}\mathbf{\Gamma}\psi$	$16 \times \bar{\psi}\mathbf{\Gamma}\psi$	$9 \times \bar{\psi}\mathbf{\Gamma}\psi$
$K^*_{[001],\,A_1}$	$8 \times \bar{\psi}\mathbf{\Gamma}\psi$ $2 \times \pi K$	$16 \times \bar{\psi}\mathbf{\Gamma}\psi$ $6 \times \pi K$	$8 \times \bar{\psi}\mathbf{\Gamma}\psi$
$K^*_{[011],\,A_1}$	$8 \times \bar{\psi}\mathbf{\Gamma}\psi$ $3 \times \pi K$	$26 \times \bar{\psi}\mathbf{\Gamma}\psi$ $6 \times \pi K$	
$K^*_{[111],\,A_1}$	$8 \times \bar{\psi}\mathbf{\Gamma}\psi$ $4 \times \pi K$	$9 \times \bar{\psi}\mathbf{\Gamma}\psi$ $4 \times \pi K$	$9 \times \bar{\psi}\mathbf{\Gamma}\psi$

L/a_s	16	20
$a_{0\,[001],\,A_1}$	$14 \times \bar{\psi}\mathbf{\Gamma}\psi$ $4 \times \pi\eta$ $2 \times \bar{K}K$	$14 \times \bar{\psi}\mathbf{\Gamma}\psi$ $4 \times \pi\eta$ $2 \times \bar{K}K$
$a_{0\,[011],\,A_1}$	$18 \times \bar{\psi}\mathbf{\Gamma}\psi$ $4 \times \pi\eta$ $2 \times \bar{K}K$	$18 \times \bar{\psi}\mathbf{\Gamma}\psi$ $4 \times \pi\eta$ $2 \times \bar{K}K$
$a_{0\,[111],\,A_1}$		$15 \times \bar{\psi}\mathbf{\Gamma}\psi$ $4 \times \pi\eta$ $2 \times \bar{K}K$

backup – operator tables

L/a_s	16	20	24
	$12 \times \bar{\psi}\mathbf{\Gamma}\psi$	$12 \times \bar{\psi}\mathbf{\Gamma}\psi$	$12 \times \bar{\psi}\mathbf{\Gamma}\psi$
$[001]A_2$	$\pi_{[000]}\omega_{[001]}$	$\pi_{[000]}\omega_{[001]}$	$\pi_{[000]}\omega_{[001]}$
	$\pi_{[000]}\phi_{[001]}$	$\pi_{[001]}\omega_{[000]}$	$\pi_{[001]}\omega_{[000]}$
	$\rho_{[001]}\eta_{[000]}$	$\pi_{[000]}\phi_{[001]}$	$\pi_{[000]}\phi_{[001]}$
	$a_{0\,[001]}\pi_{[000]}$	$\rho_{[001]}\eta_{[000]}$	$\rho_{[001]}\eta_{[000]}$
	$\pi_{[001]}\omega_{[000]}$	$a_{0\,[001]}\pi_{[000]}$	$K^*_{[001]}\bar{K}_{[000]}$
	$K^*_{[001]}\bar{K}_{[000]}$	$K^*_{[001]}\bar{K}_{[000]}$	$\rho^1_{[001]}\eta_{[000]}$
	$\rho_{[000]}\eta_{[001]}$		$\rho_{[000]}\eta_{[001]}$
	$\pi_{[001]}\phi_{[000]}$		$\pi_{[001]}\phi_{[000]}$
			$K^*_{[000]}\bar{K}_{[001]}$
			$\{2\}\pi_{[001]}\omega_{[011]}$
			$\{2\}\pi_{[011]}\omega_{[001]}$

L/a_s	16	20	24
	$21 \times \bar{\psi}\mathbf{\Gamma}\psi$	$21 \times \bar{\psi}\mathbf{\Gamma}\psi$	$21 \times \bar{\psi}\mathbf{\Gamma}\psi$
$[011]A_2$	$\pi_{[000]}\omega_{[011]}$	$\pi_{[000]}\omega_{[011]}$	$\pi_{[000]}\omega_{[011]}$
	$\pi_{[000]}\phi_{[011]}$	$\pi_{[000]}\phi_{[011]}$	$\{2\}\pi_{[001]}\omega_{[001]}$
	$\rho_{[011]}\eta_{[000]}$	$\{2\}\pi_{[001]}\omega_{[001]}$	$\pi_{[000]}\phi_{[011]}$
	$K^*_{[011]}\bar{K}_{[000]}$	$\rho_{[011]}\eta_{[000]}$	$\pi_{[011]}\omega_{[000]}$
	$\{2\}\pi_{[001]}\omega_{[001]}$	$a_{0\,[011]}\pi_{[000]}$	$\rho_{[011]}\eta_{[000]}$
	$a_{0\,[011]}\pi_{[000]}$	$K^*_{[011]}\bar{K}_{[000]}$	
	$\pi_{[011]}\omega_{[000]}$		

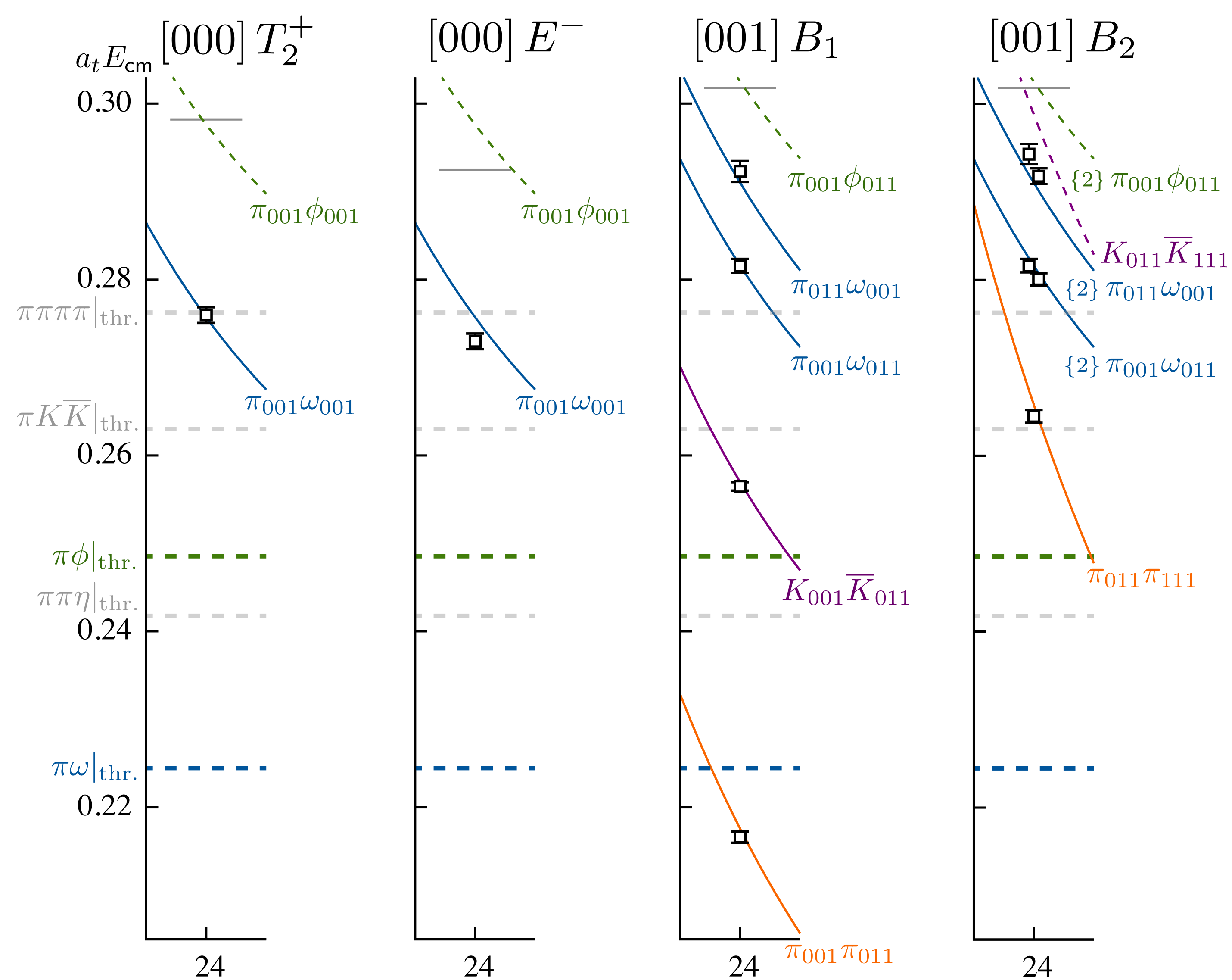
L/a_s	16	20	24
	$20 \times \bar{\psi}\mathbf{\Gamma}\psi$	$20 \times \bar{\psi}\mathbf{\Gamma}\psi$	$20 \times \bar{\psi}\mathbf{\Gamma}\psi$
$[002]A_2$	$\pi_{[001]}\omega_{[001]}$	$\pi_{[001]}\omega_{[001]}$	$\pi_{[001]}\omega_{[001]}$
	$\rho_{[001]}\eta_{[001]}$	$\rho_{[001]}\eta_{[001]}$	$\pi_{[000]}\omega_{[002]}$
	$K^*_{[001]}\bar{K}_{[001]}$	$\pi_{[000]}\omega_{[002]}$	$\rho_{[001]}\eta_{[001]}$
	$\pi_{[000]}\omega_{[002]}$	$\pi_{[001]}\phi_{[001]}$	$\pi_{[001]}\phi_{[001]}$
	$\pi_{[001]}\phi_{[001]}$	$K^*_{[001]}\bar{K}_{[001]}$	$K^*_{[001]}\bar{K}_{[001]}$
	$\rho^1_{[001]}\eta_{[001]}$	$a_{0\,[001]}\pi_{[001]}$	$\pi_{[000]}\phi_{[002]}$

L/a_s	16	20	24
	$15 \times \bar{\psi}\mathbf{\Gamma}\psi$	$15 \times \bar{\psi}\mathbf{\Gamma}\psi$	$15 \times \bar{\psi}\mathbf{\Gamma}\psi$
$[111]A_2$	$\pi_{[000]}\omega_{[111]}$	$\pi_{[000]}\omega_{[111]}$	$\pi_{[000]}\omega_{[111]}$
	$\pi_{[000]}\phi_{[111]}$	$\pi_{[000]}\phi_{[111]}$	$\pi_{[000]}\phi_{[111]}$
	$\rho_{[111]}\eta_{[000]}$	$\{2\}\pi_{[001]}\omega_{[011]}$	$\{2\}\pi_{[001]}\omega_{[011]}$
	$K^*_{[111]}\bar{K}_{[000]}$	$\rho_{[111]}\eta_{[000]}$	$\{2\}\pi_{[011]}\omega_{[001]}$
	$\{2\}\pi_{[001]}\omega_{[011]}$	$K^*_{[111]}\bar{K}_{[000]}$	$\rho_{[111]}\eta_{[000]}$
	$\pi_{[111]}\omega_{[000]}$	$a_{0\,[111]}\pi_{[000]}$	$\pi_{[111]}\omega_{[000]}$
	$\{2\}\pi_{[011]}\omega_{[001]}$		$K^*_{[111]}\bar{K}_{[000]}$

backup – higher partial-waves

[000] T_2^+	[000] E^-	[001] B_1	[001] B_2
2^+ $(^3D_2)$		2^+ $(^3D_2)$	2^+ $(^3D_2)$
	2^- $\begin{pmatrix} ^3P_2 \\ ^3F_2 \end{pmatrix}$	2^- $\begin{pmatrix} ^3P_2 \\ ^3F_2 \end{pmatrix}$	2^- $\begin{pmatrix} ^3P_2 \\ ^3F_2 \end{pmatrix}$
3^+ $\begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}$		3^+ $\begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}$	3^+ $\begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}$

$[000]T_2^+$	$[000]E^-$	$[001]B_1$	$[001]B_2$
$14 \times \bar{\psi}\mathbf{\Gamma}\psi$	$12 \times \bar{\psi}\mathbf{\Gamma}\psi$	$9 \times \bar{\psi}\mathbf{\Gamma}\psi$	$9 \times \bar{\psi}\mathbf{\Gamma}\psi$
$\pi_{[001]}\omega_{[001]}$	$\pi_{[001]}\omega_{[001]}$	$\pi_{[011]}\pi_{[001]}$	$\pi_{[111]}\pi_{[011]}$
		$\bar{K}_{[011]}K_{[001]}$	$\{2\}\pi_{[001]}\omega_{[011]}$
		$\pi_{[001]}\omega_{[011]}$	$\{2\}\pi_{[011]}\omega_{[001]}$
		$\pi_{[011]}\omega_{[001]}$	



backup – reference amplitude

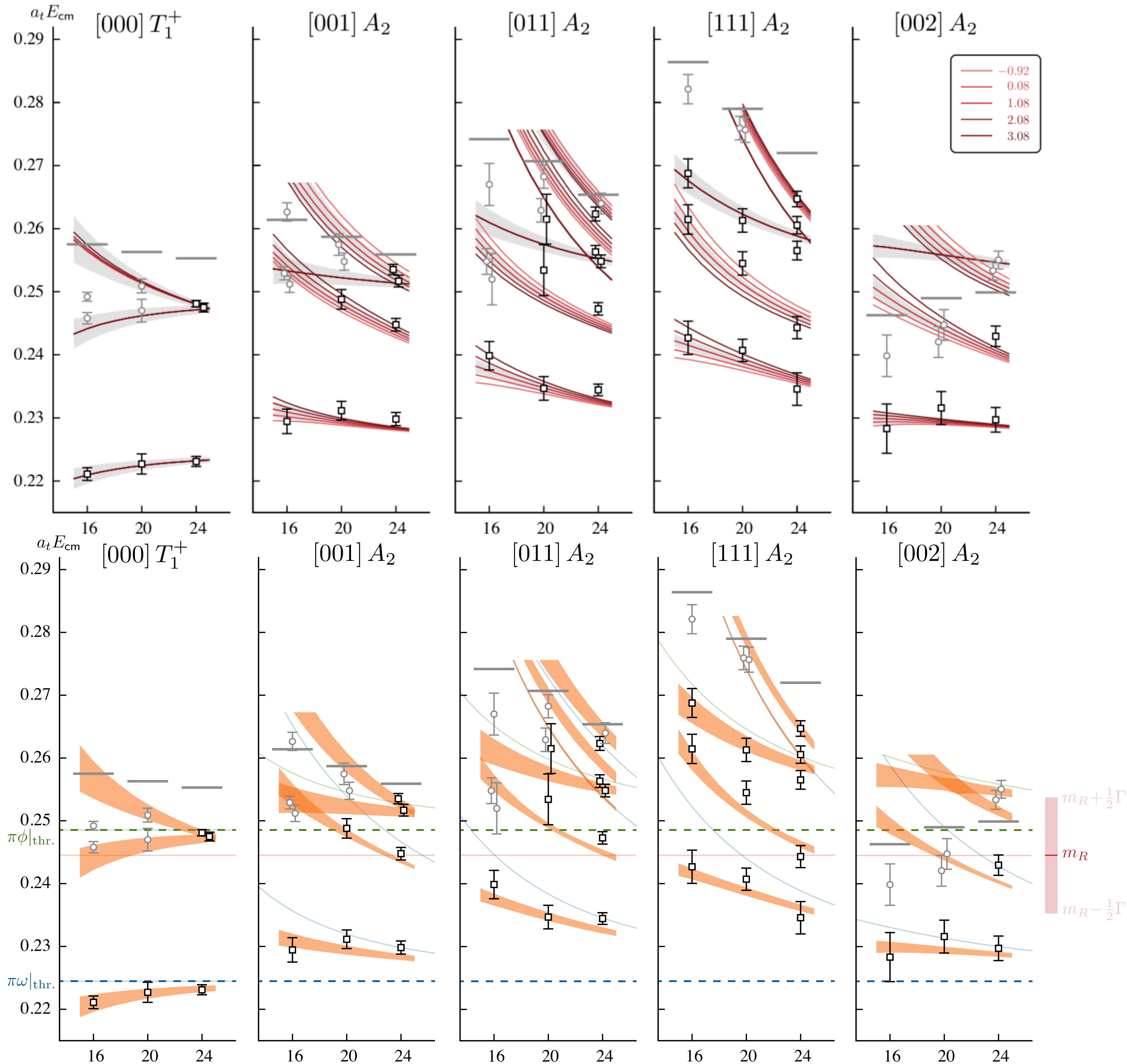
$$\boldsymbol{K}(s) = \frac{1}{m^2 - s}\begin{pmatrix} g^2_{\pi\omega\{^3S_1\}} & g_{\pi\omega\{^3S_1\}}\, g_{\pi\omega\{^3D_1\}} & 0 \\ g_{\pi\omega\{^3S_1\}}\, g_{\pi\omega\{^3D_1\}} & g^2_{\pi\omega\{^3D_1\}} & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} \gamma^{(0)}_{\pi\omega\{^3S_1\},\pi\omega\{^3S_1\}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \gamma^{(0)}_{\pi\phi\{^3S_1\},\pi\phi\{^3S_1\}} \end{pmatrix}$$

used with the Chew-Mandelstam prescription with $\mathrm{Re}\, I_a(s = m^2) = 0$

$$\begin{aligned} m &= (0.2465 \pm 0.0007 \pm 0.0001) \cdot a_t^{-1} \\ g_{\pi\omega\{^3S_1\}} &= (0.106 \pm 0.007 \pm 0.007) \cdot a_t^{-1} \\ g_{\pi\omega\{^3D_1\}} &= (1.08 \pm 0.47 \pm 0.28) \cdot a_t \\ \gamma^{(0)}_{\pi\omega\{^3S_1\},\pi\omega\{^3S_1\}} &= -0.35 \pm 0.19 \pm 0.18 \\ \gamma^{(0)}_{\pi\phi\{^3S_1\},\pi\phi\{^3S_1\}} &= 0.90 \pm 0.24 \pm 0.27 \end{aligned} \quad \begin{bmatrix} 1 & -0.05 & 0.05 & -0.01 & -0.23 \\ & 1 & 0.70 & -0.54 & -0.06 \\ & & 1 & -0.39 & -0.06 \\ & & & 1 & 0.22 \\ & & & & 1 \end{bmatrix}$$

$$\chi^2/N_{\mathrm{dof}} = \tfrac{36.8}{36-5} = 1.19.$$

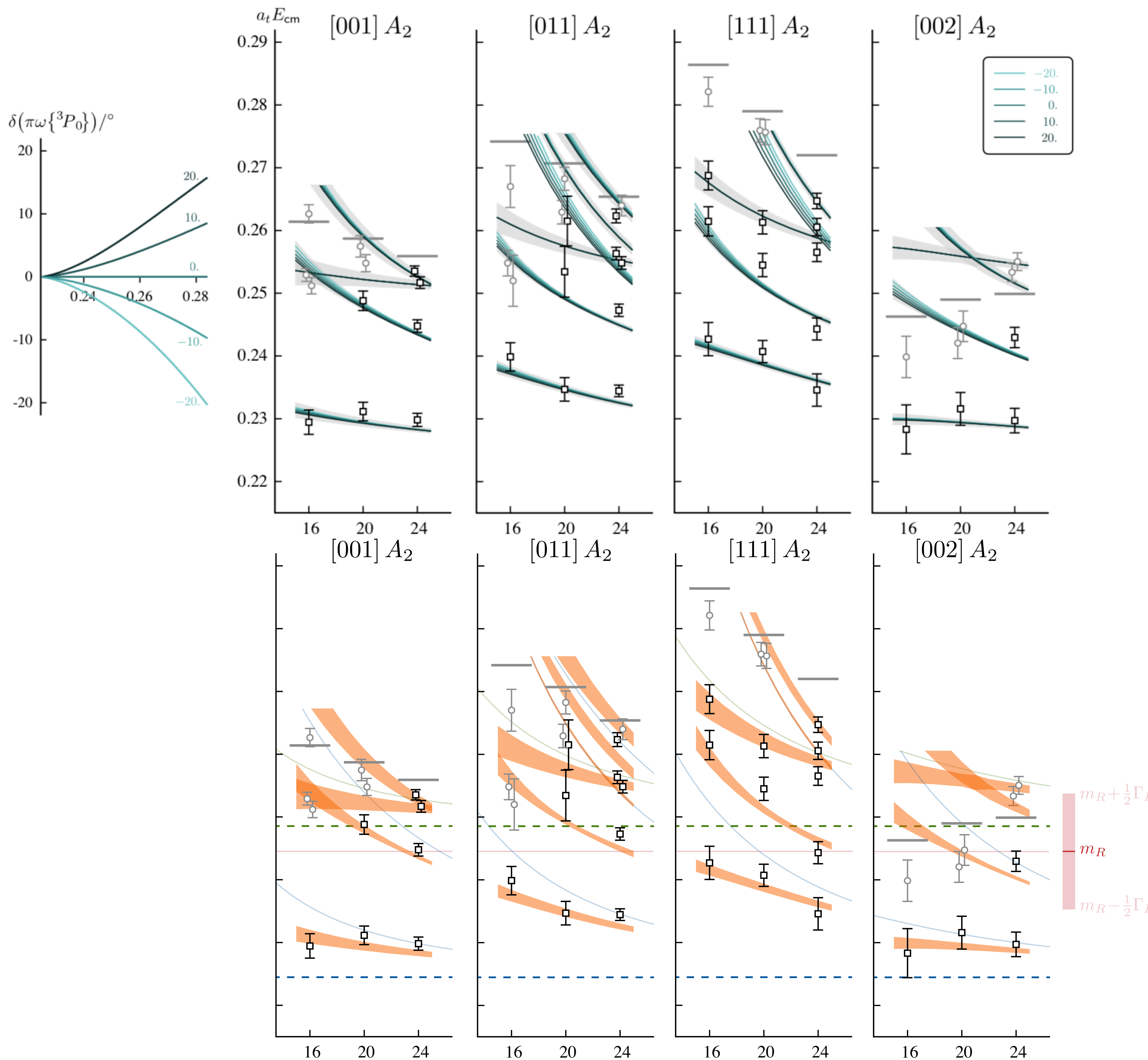
backup – systematic tests $g(^3D_1)$ variation



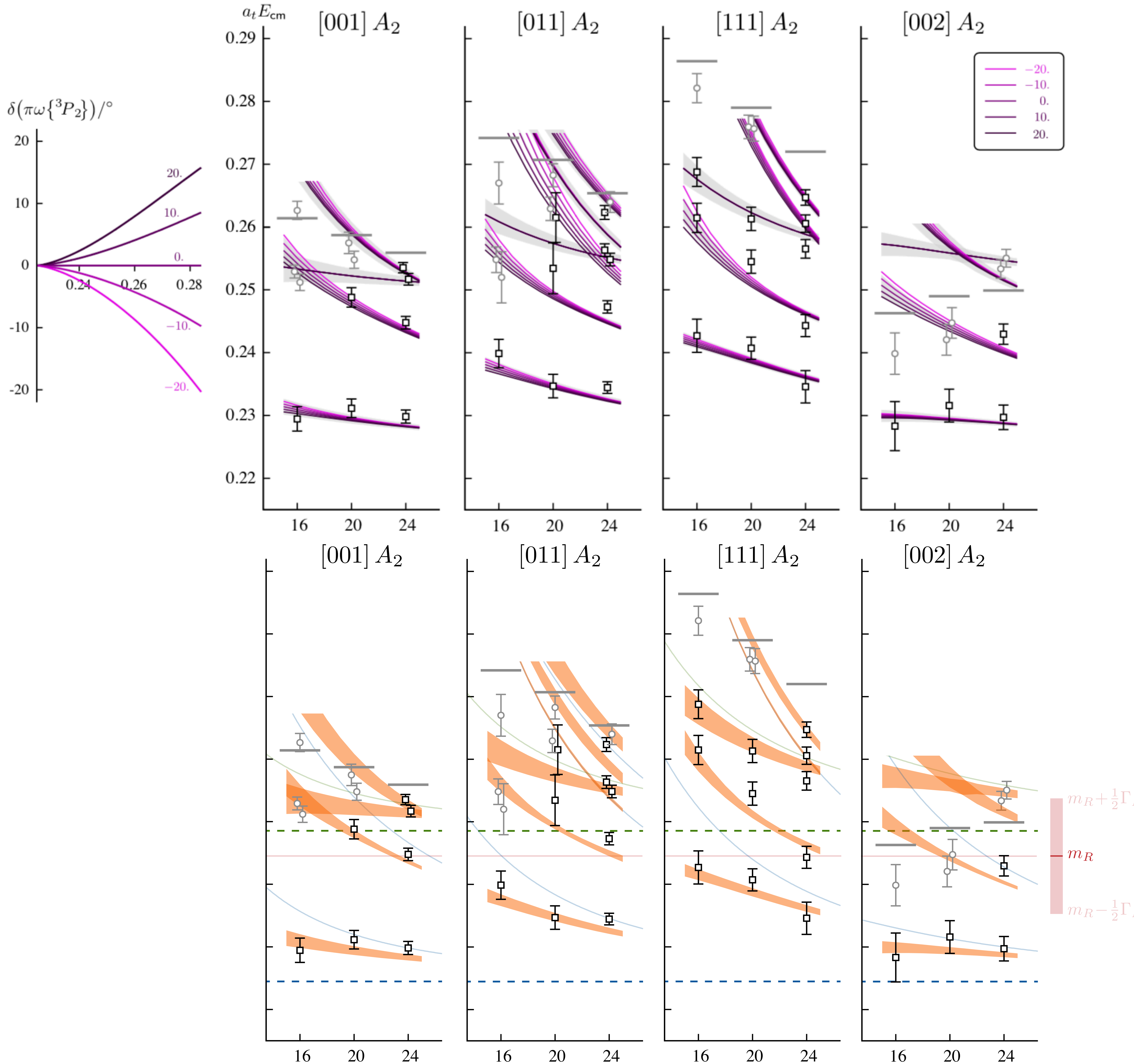
backup – mixed P-waves

$[000]\,T_1^+$	$[00n]\,A_2$	$[0nn]\,A_2$	$[nnn]\,A_2$
	$0^-\ ({}^3P_0)$	$0^-\ ({}^3P_0)$	$0^-\ ({}^3P_0)$
$1^+ \begin{pmatrix} {}^3S_1 \\ {}^3D_1 \end{pmatrix}$	$1^+ \begin{pmatrix} {}^3S_1 \\ {}^3D_1 \end{pmatrix}$	$1^+ \begin{pmatrix} {}^3S_1 \\ {}^3D_1 \end{pmatrix}$	$1^+ \begin{pmatrix} {}^3S_1 \\ {}^3D_1 \end{pmatrix}$
		$2^+\ ({}^3D_2)$	
	$2^- \begin{pmatrix} {}^3P_2 \\ {}^3F_2 \end{pmatrix}$	$2^- \begin{pmatrix} {}^3P_2 \\ {}^3F_2 \end{pmatrix}_{[2]}$	$2^- \begin{pmatrix} {}^3P_2 \\ {}^3F_2 \end{pmatrix}$
$3^+ \begin{pmatrix} {}^3D_3 \\ {}^3G_3 \end{pmatrix}$	$3^+ \begin{pmatrix} {}^3D_3 \\ {}^3G_3 \end{pmatrix}$	$3^+ \begin{pmatrix} {}^3D_3 \\ {}^3G_3 \end{pmatrix}_{[2]}$	$3^+ \begin{pmatrix} {}^3D_3 \\ {}^3G_3 \end{pmatrix}_{[2]}$

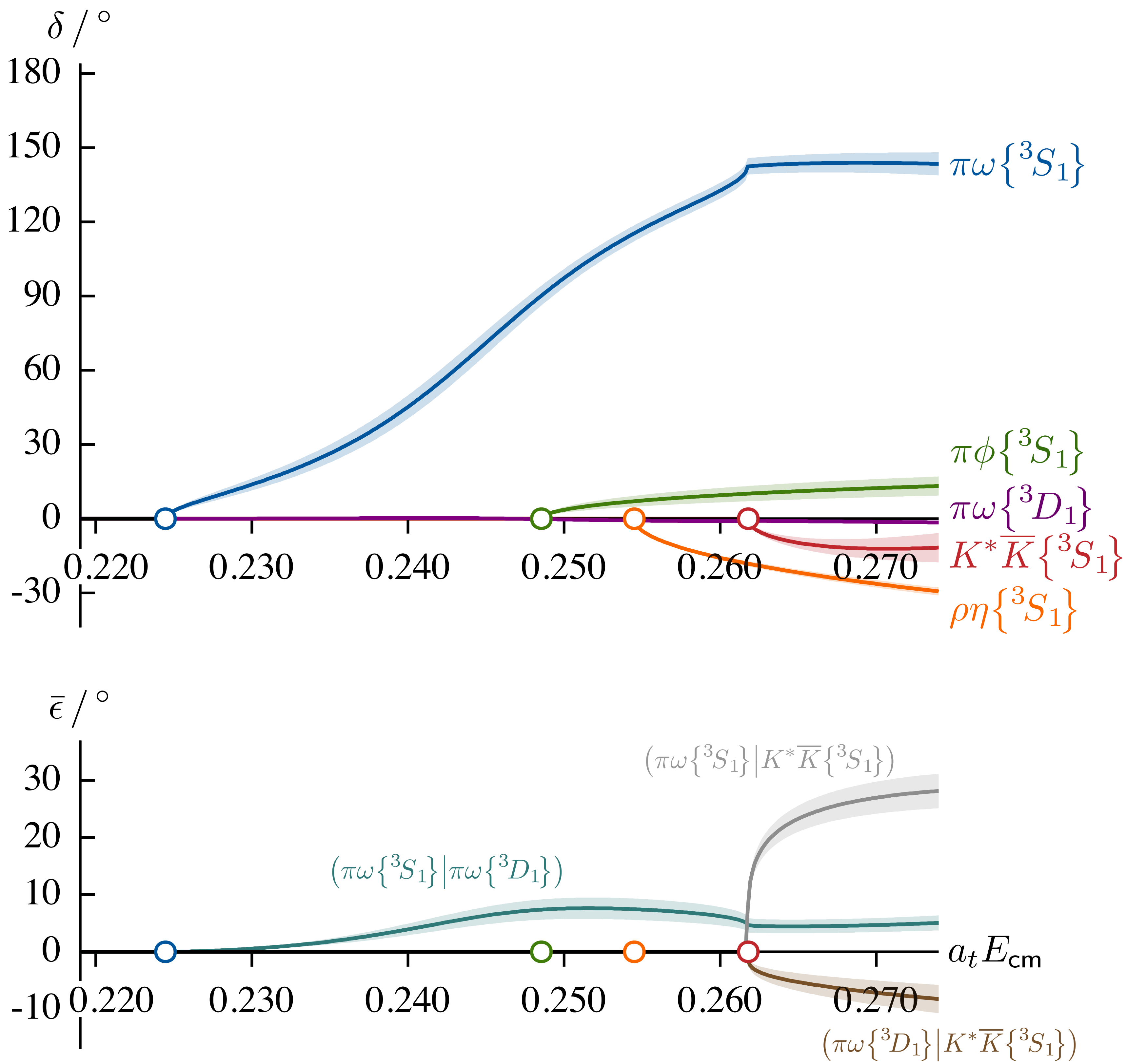
backup – mixed P-waves



backup – mixed P-waves



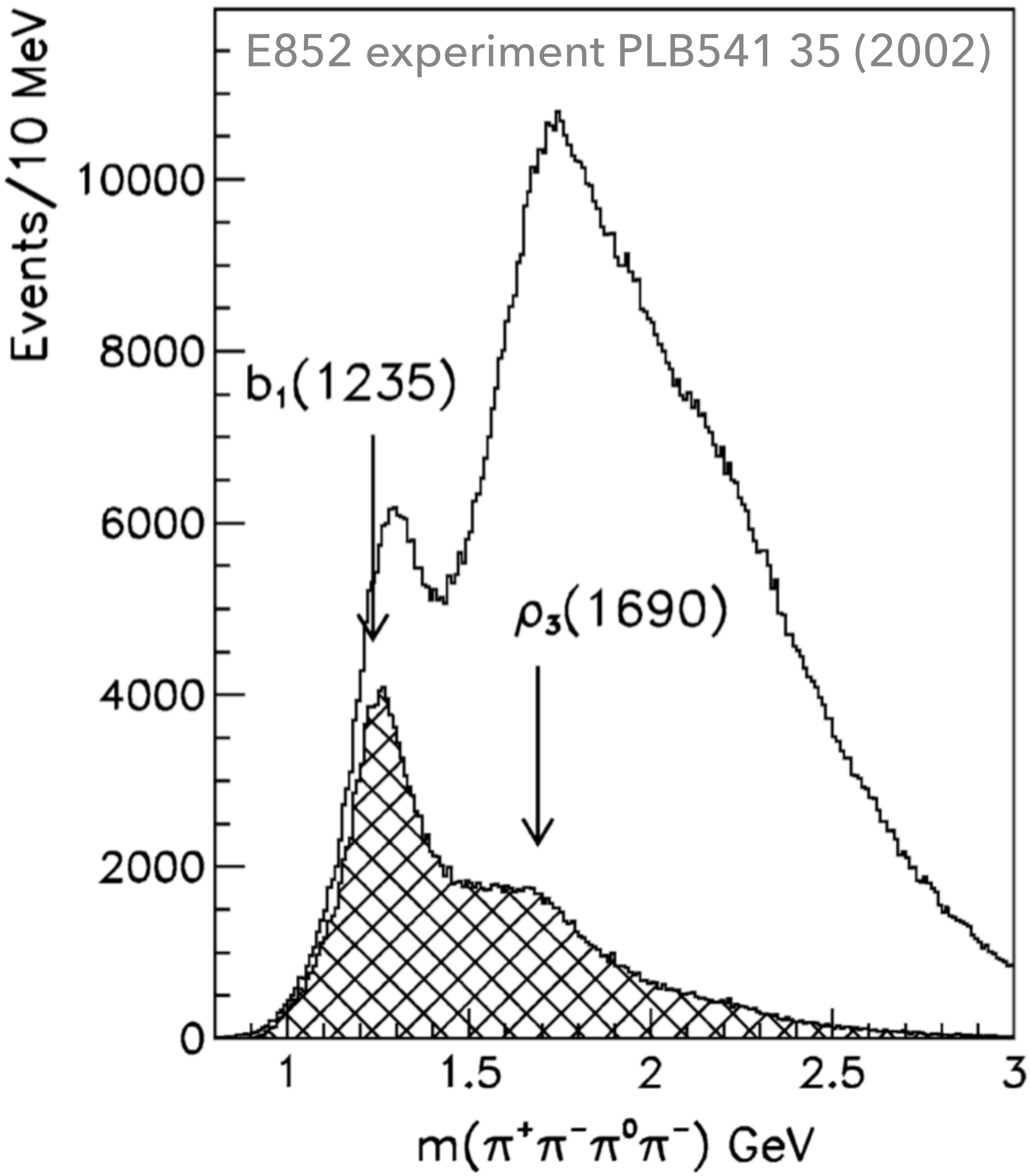
backup – five coupled-channel phase-shifts and mixing-angles



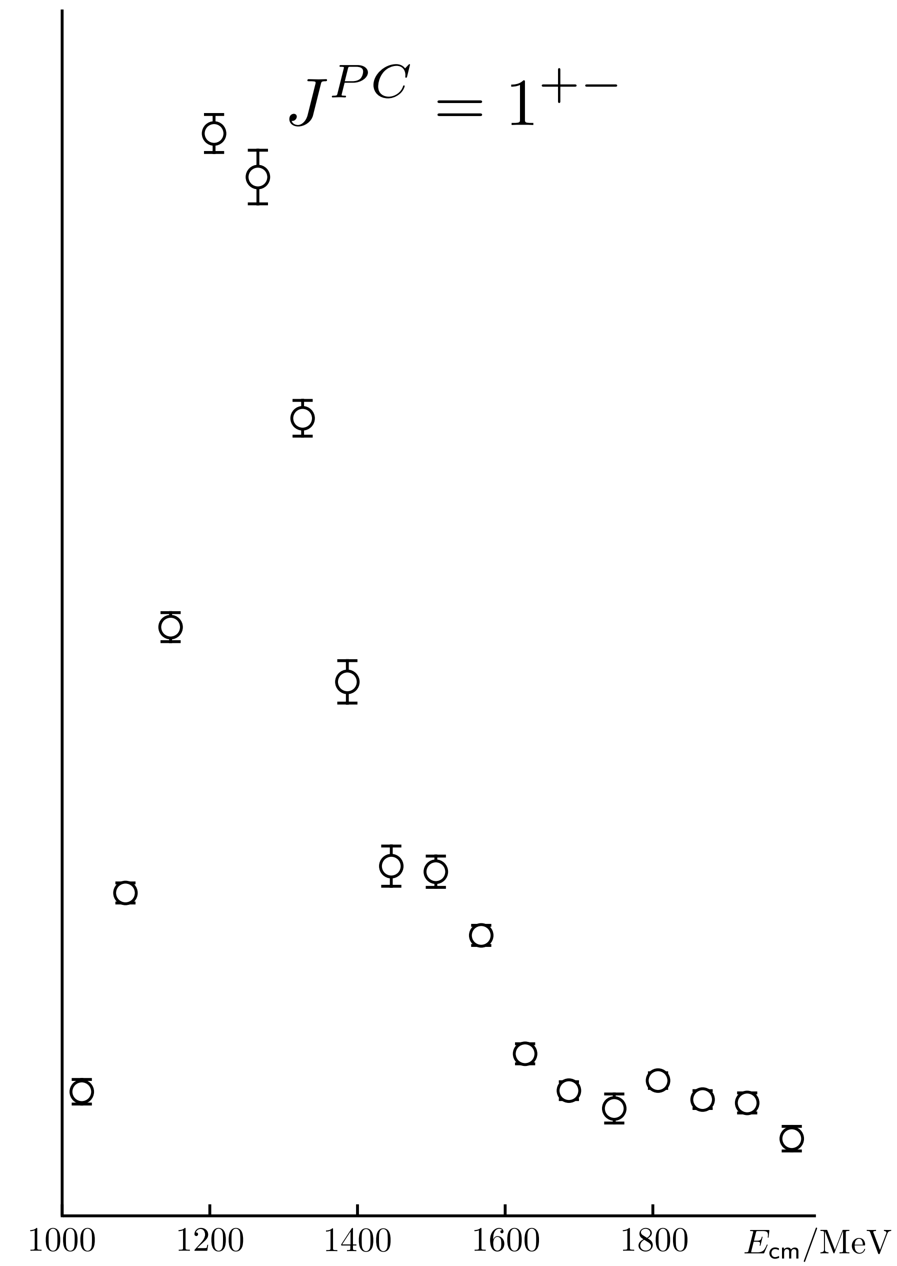
$$\chi^2/N_{\mathrm{dof}} = \frac{46.6}{48-9} = 1.19$$

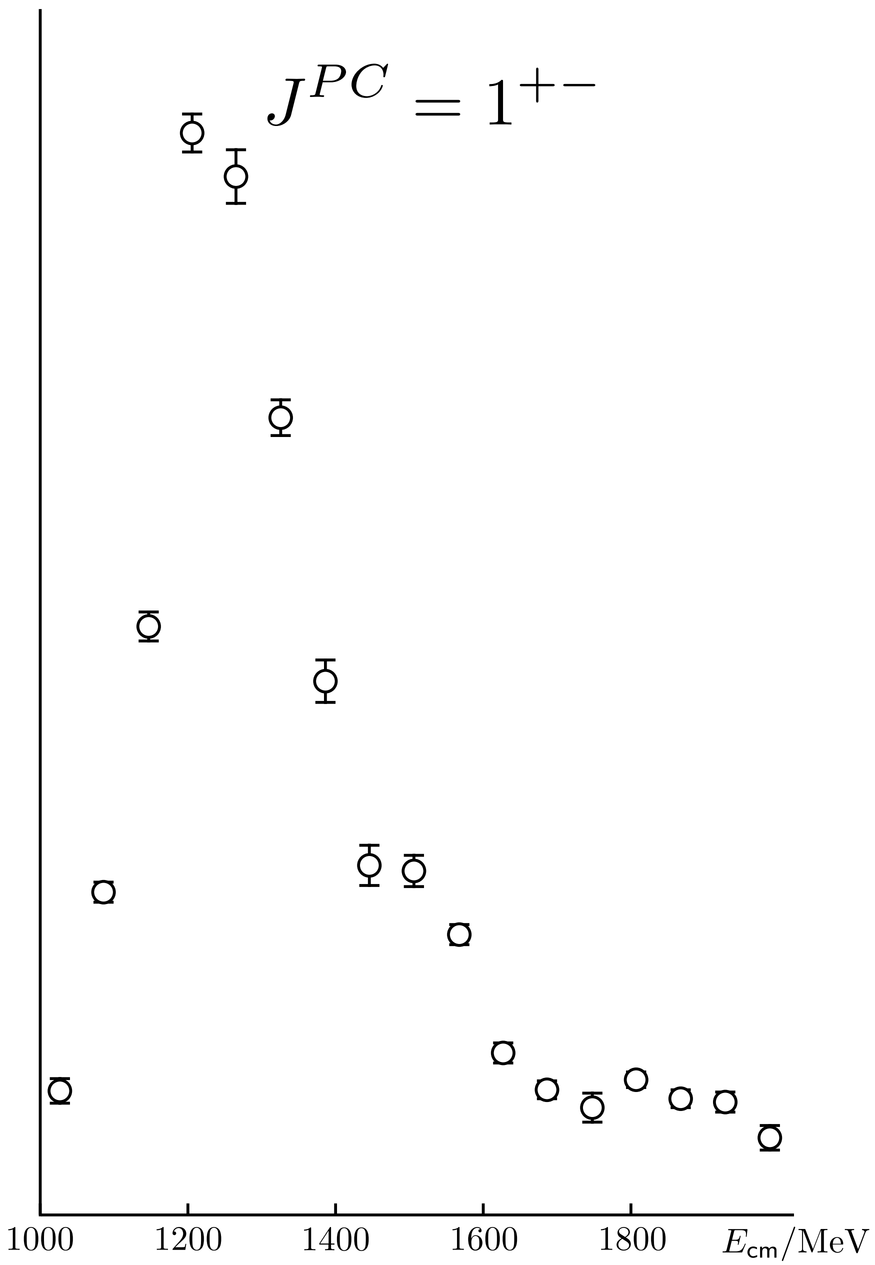
backup

$$(I^G)J^{PC} = (1^+)1^{+-} \quad \begin{aligned} &\to \pi\omega \to \pi(\pi\pi\pi) \\ &\to \ldots \end{aligned}$$

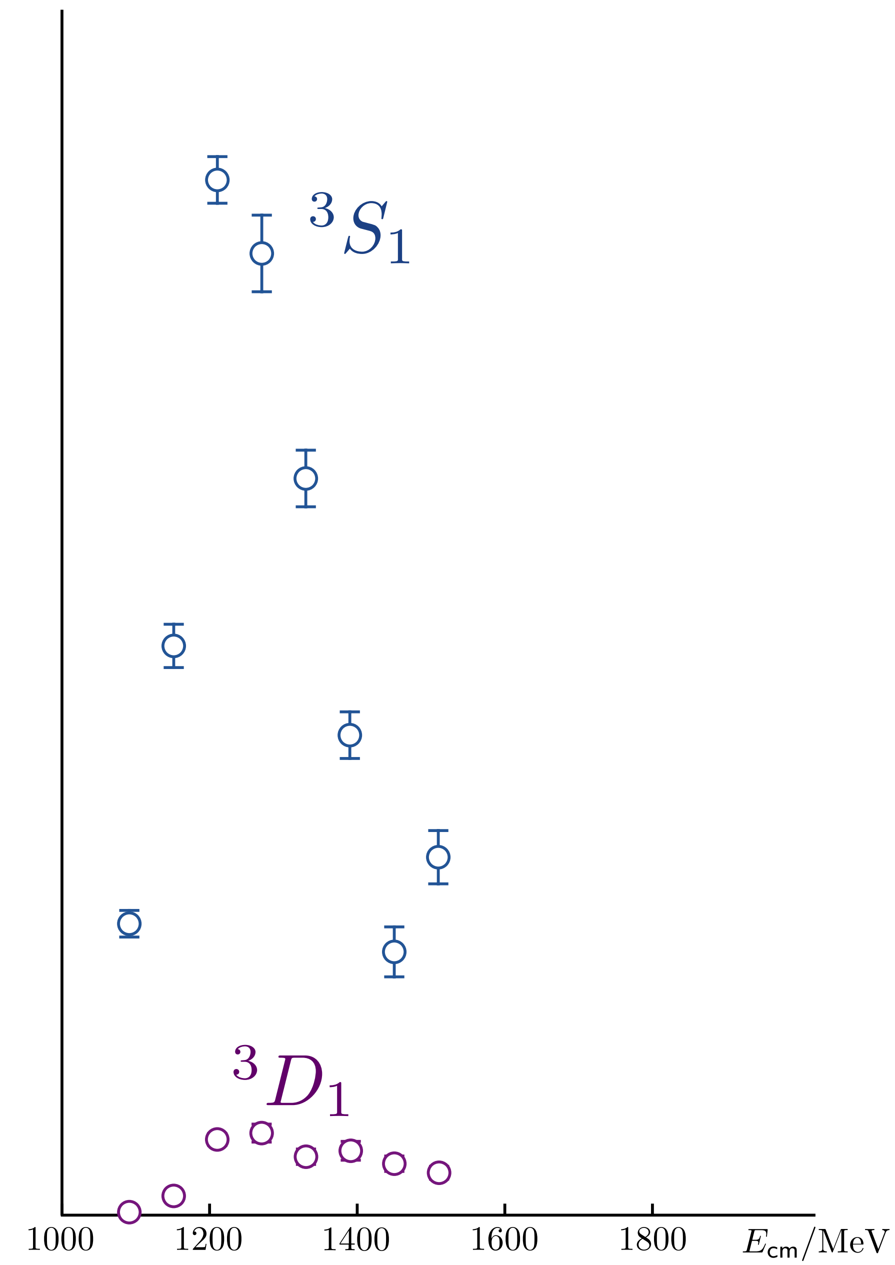


partial wave analysis →





$$
\begin{aligned}
b_1 &\to \pi\omega \quad {}^3S_1 \\
&\to \pi\omega \quad {}^3D_1 \\
&\to \ldots
\end{aligned}
$$



E852 experiment PLB541 35 (2002)