Quasi PDF as observables

Denn eigentlich unternehmen wir umsonst,
das Wesen eines Dinges auszudrücken.

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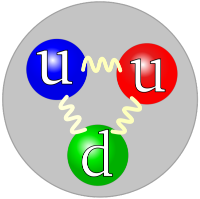
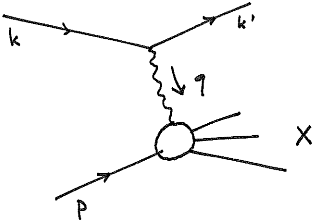
the parton model & deep inelastic scattering





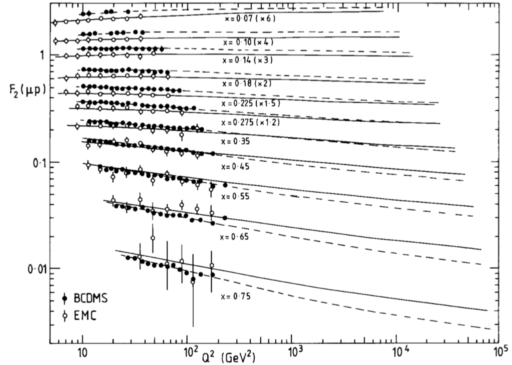
kinematics

$$k = k' + q\,, \quad Q^2 = -q^2\,, \quad \nu = q \cdot p$$

$$p^2 = M^2\,, \quad x = \frac{Q^2}{2\nu}\,, \quad y = \frac{q \cdot p}{k \cdot p}$$

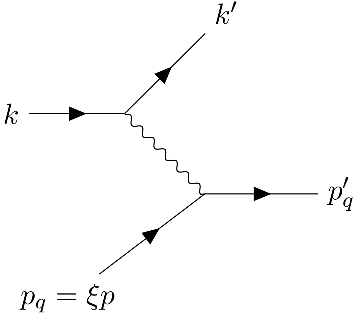
Bjorken scaling – pointlike constituents

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4}\Big\{ \left[1+(1-y)^2\right] F_1(x,Q^2) + \\ + \frac{1-y}{x}\left[F_2(x,Q^2) - 2xF_1(x,Q^2)\right] \Big\}$$



[MRS, 1988]

scattering off a parton



$$\frac{d\hat{\sigma}}{dQ^2} = \frac{2\pi\alpha^2 e_q^2}{Q^4} \left[1 + (1-y)^2\right]$$

$$p_q'^2 = -2p \cdot q \; (x - \xi) = 0$$

$$\boxed{\frac{d^2\hat{\sigma}}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4} \; \left[1 + (1-y)^2\right] \; \frac{1}{2} e_q^2 \delta(x - \xi)}$$

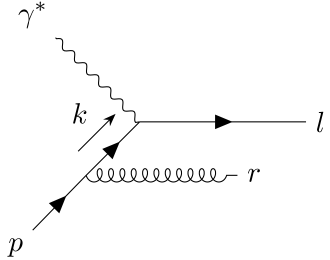
parton distribution functions

$$\hat{F}_2(x, Q^2) = x e_q^2 \delta(x - \xi) = x \hat{F}_1(x, Q^2)$$

- F_2 *measures* the fraction of momentum of the parton
- introduce a probability density $q_0(\xi)$

$$\begin{aligned} F_2(x) &= \sum_{q,\bar{q}} \int d\xi\, q_0(\xi)\, \hat{F}_2(x, Q^2) \\ &= \sum_{q,\bar{q}} \int d\xi\, q_0(\xi)\, x e_q^2 \delta(x - \xi) \\ &= \sum_{q,\bar{q}} e_q^2\, x q_0(x) \end{aligned}$$

QCD corrections



$$d\Phi_2 = \int \frac{d^4r}{(2\pi)^4}\frac{d^4l}{(2\pi)^4}\,(2\pi)\delta^+\left(r^2\right)\,(2\pi)\delta^+\left(l^2\right)\,(2\pi)^4\delta\left(p+q-r-l\right)$$
$$= \frac{1}{4\pi^2}\int d^4k\,\delta^+\left((p-k)^2\right)\delta^+\left((k+q)^2\right)$$

$$k^\mu = \eta p^\mu + \frac{k_T^2 - k^2}{2\eta}n^\mu + k_T^\mu$$

factorization

$$\hat{F}_2\left(x, Q^2\right) = e_q^2 x \left[\delta(1-x) + \frac{\alpha_s}{2\pi} \left(P(x)\, \log\frac{Q^2}{\kappa^2} + C(x)\right)\right]$$

convolution of the quark structure function (at fixed order) with PDF

$$F_2\left(x, Q^2\right) = x \sum_{q,\bar{q}} e_q^2 \left[q_0(x) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi}\, q_0(\xi) \times \right.$$
$$\left. \times \left(P\left(\frac{x}{\xi}\right) \log\frac{Q^2}{\kappa^2} + C\left(\frac{x}{\xi}\right)\right) + \ldots\right]$$

$$= x \sum_{q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi}\, q(\xi, \mu^2) \times$$
$$\times \left[\delta\left(1 - \frac{x}{\xi}\right) + \frac{\alpha_s}{2\pi} P\left(\frac{x}{\xi}\right) \log\frac{Q^2}{\mu^2} + \ldots\right]$$

DIS observables

- PDFs are defined in a given scheme and at a given order in PT

$$F_2(x, Q^2) = \sum_i C_i(x, Q^2/\mu^2) \otimes q_i(x, \mu^2) + \dots$$

- PDFs are nonperturbative quantities - not accessible in perturbation theory

- the scale dependence is determined DGLAP equations

$$\mu^2 \frac{\partial}{\partial \mu^2} q(x, \mu^2) = \frac{\alpha_s}{2\pi} P(x) \otimes q(x, \mu^2)$$

kinematic cuts

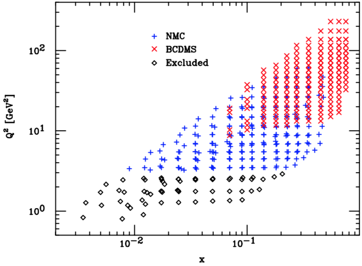
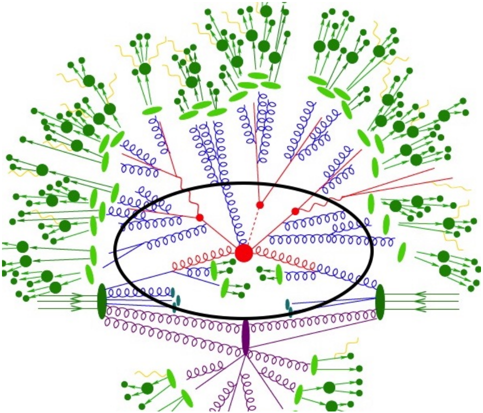


Figure 2: Kinematical coverage of the available experimental data on the nonsinglet structure function in the (x, Q^2) plane.

Experiment	x range	Q^2 range	$N_{\rm dat}$	$\langle\sigma_s\rangle$	$\langle\sigma_c\rangle$	$\left\langle\frac{\sigma_c}{\sigma_s}\right\rangle$	$\langle\sigma_N\rangle$	$\langle\sigma_{\rm tot}\rangle$	$\langle\rho\rangle$	$\langle{\rm cov}\rangle$
NMC	$9.0\ 10^{-3}$ – $4.7\ 10^{-1}$	$3.2 - 61$ GeV^2	229	103.2	85.6	0.75	72.5	157.5	0.038	0.199
BCDMS	$7.0\ 10^{-2}$ – $7.5\ 10^{-1}$	$8.7 - 230$ GeV^2	254	22.3	12.2	0.50	22.3	37.3	0.163	0.085

universality

$$\sigma(H_1 H_2 \to X) = \sum_{i,j} \int dx_1 dx_2\; q_i(x_1, \mu^2) q_j(x_2, \mu^2) \times \\ \times\, \hat{\sigma}_{ij\to X}(x_1 x_2 s, \mu^2, \mu_R^2)$$



NNPDF methodology

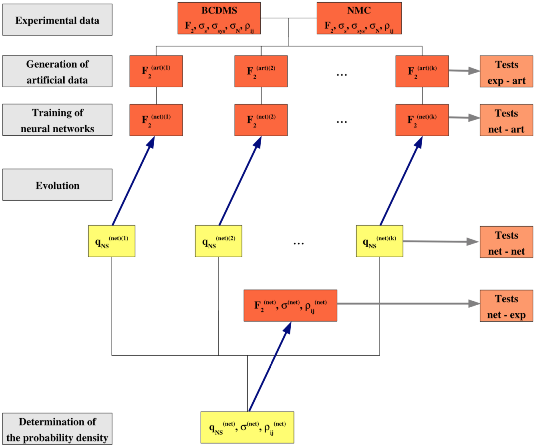
- parametrise the PDF at some initial scale Q_0^2
- compute the observables (efficiently)

$$F_2(x, Q^2) = C(x, Q^2) \otimes \Gamma(x, Q^2, Q_0^2) \otimes f(x, Q_0^2)$$

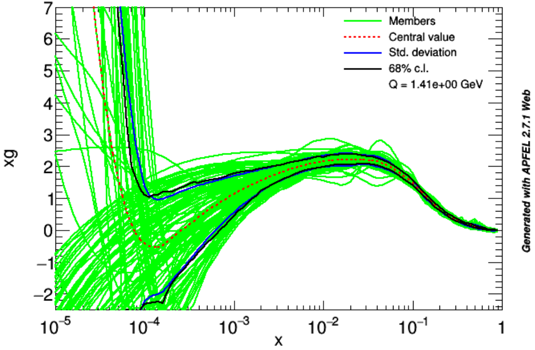
$$F_I = \sum_\alpha K_{I\alpha}\, f_\alpha \qquad \text{(FK tables)}$$

- sufficient accuracy / small bias $\hookrightarrow$ closure tests
- faithful error propagation
  $\hookrightarrow$ Monte Carlo sampling of exp data (replicas)

methodology

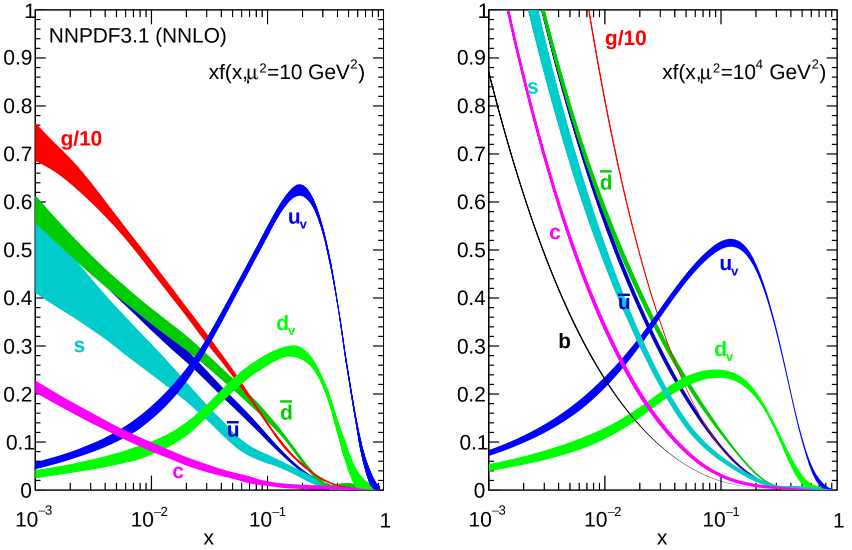


replicas



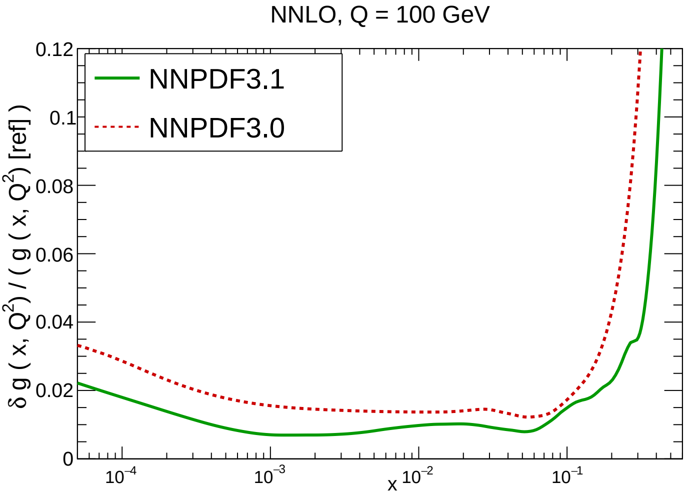
Members
Central value
Std. deviation
68% c.l.
Q = 1.41e+00 GeV
xg
x
Generated with APFEL 2.7.1 Web

NNPDF3.1

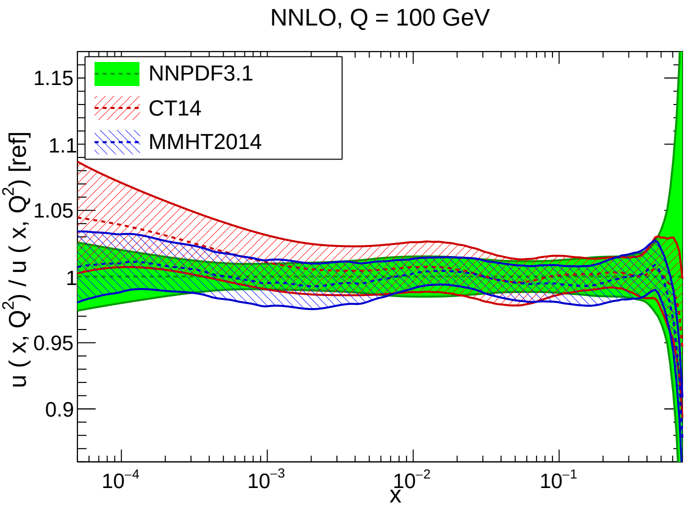


NNPDF3.1 [1706.00428]

gluon distribution

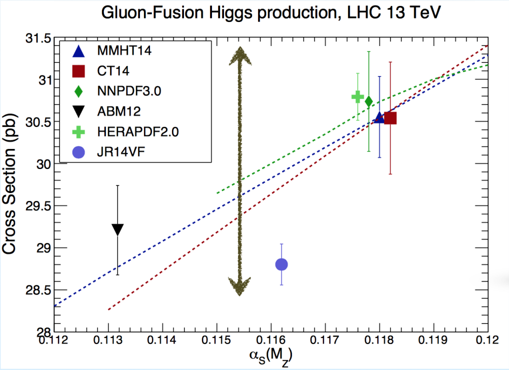


agreement between different determinations

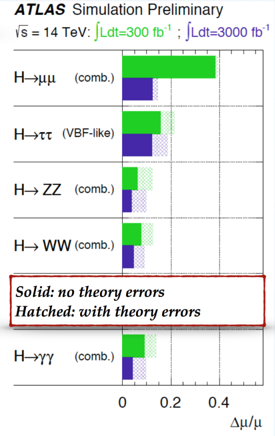


PDF uncertainties for Higgs physics

Uncertainties from Parton Distributions** are one of the limiting factors of theory predictions of Higgs production, **degrading the exploration of the Higgs sector

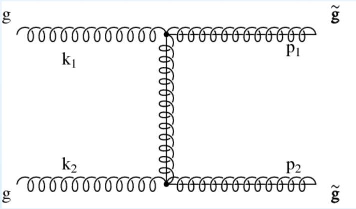


Higgs Cross-Section Working Group YR4 16

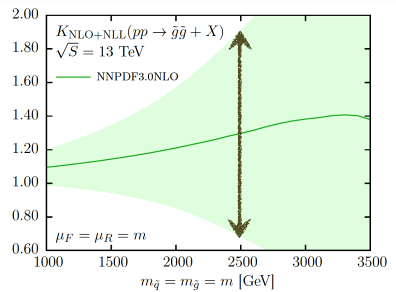


[plot courtesy of J Rojo]

PDF uncertainties for new physics



Gluino pair production at the LHC



Beenakker, Borchensky, Kramer, Kulesza, Laenen, Marzani, Rojo 15

Beenakker et al [1510.00375]

matrix elements

$$\boxed{\mathcal{M}_{\Gamma,A}(\zeta) = \bar{\psi}(\zeta)\Gamma\lambda_A \,\mathrm{P}\exp\left(-ig\int_0^\zeta d\eta\, A(\eta)\right)\psi(0)}$$

Ioffe time distributions

$$M_{\gamma^\mu,A}(\zeta,P) = \langle P|\mathcal{M}_{\gamma^\mu,A}(\zeta)|P\rangle$$

Lorentz covariance

$$M_{\gamma^\mu,A}(\zeta,P) = P^\mu h_{\gamma^\mu,A}(\zeta\cdot P, z^2) + \zeta^\mu h'_{\gamma^\mu,A}(\zeta\cdot P, z^2)$$

PDFs and quasi-PDFs

light-cone PDF – $P = (P^+, 0, \vec{0}_\perp), \quad \nu = \zeta \cdot P = P^+ \zeta^-$:

$$f_3(x,\mu) = \int \frac{d\zeta^-}{4\pi}\, e^{-i(xP^+)\zeta^-}\, P^+ h_{\gamma^+,3}(\nu, z^2)$$
$$= \int \frac{d\nu}{2\pi}\, e^{-ix\nu} h_{\gamma^+,3}(\nu, z^2)$$

quasi-PDF, time-independent quantity – $\zeta = (0,0,0,z)$:

$$q_3(x,\mu,P_z) = \int \frac{dz}{4\pi}\, e^{-i(xP_z)z}\, h_{\gamma^z,3}(\nu, z^2)$$

recent review [K Cichy & M Constantinou 18]

from quasi-PDFs to PDFs

Extracting PDFs from lattice simulations:

- renormalization of the lattice operator
  - ◇ RI/MOM prescription
  - ◇ matching to $\overline{\mathrm{MS}}$
  - ◇ trace operators and power divergencies

- Euclidean to Minkowski space

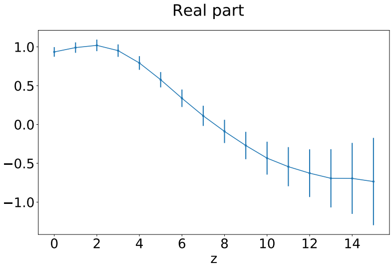
- factorization theorem for the renormalized quasi-PDF

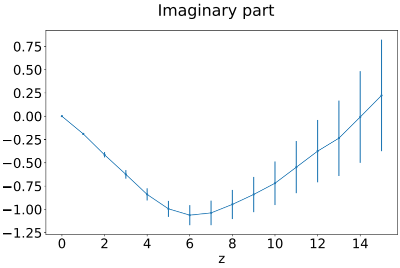
$$\boxed{q_3(x,\mu,P_z) = \int_{-1}^{+1} \frac{dy}{y}\, C_3\left(\frac{x}{y}, \frac{\mu}{|y|P_z}\right) f_3(y,\mu^2) + \ldots}$$

lattice data as observables

[work in collaboration with K Cichy and T Giani]

$$\mathcal{O}^{\text{Re}}_{\gamma^0}\left(zP_z, z^2\right) \equiv \text{Re}\left[\text{h}_{\gamma^0,3}\left(zP_z, z^2\right)\right] \qquad \mathcal{O}^{\text{Im}}_{\gamma^0}\left(zP_z, z^2\right) \equiv \text{Im}\left[\text{h}_{\gamma^0,3}\left(zP_z, z^2\right)\right]$$





[C Alexandrou et al 18]

lattice observables

evolution basis for PDFs

$$f_3\left(x,\mu^2\right)=\begin{cases} u\left(x,\mu^2\right)-d\left(x,\mu^2\right) & \text{if} \quad x>0 \\ -\bar{u}\left(-x,\mu^2\right)+\bar{d}\left(-x,\mu^2\right) & \text{if} \quad x<0 \end{cases}$$

inverse Fourier transform

$$\mathcal{O}^{\mathrm{Re}}_{\gamma^0}\left(zP_z,z^2\right)=\int_{-\infty}^{\infty}dx\,\cos\left(xP_zz\right)\int_{-1}^{+1}\frac{dy}{|y|}\,C_3\left(\frac{x}{y},\frac{\mu}{|y|P_z}\right)\,f_3\left(y,\mu^2\right)$$

$$\mathcal{O}^{\mathrm{Im}}_{\gamma^0}\left(zP_z,z^2\right)=\int_{-\infty}^{\infty}dx\,\sin\left(xP_zz\right)\int_{-1}^{+1}\frac{dy}{|y|}\,C_3\left(\frac{x}{y},\frac{\mu}{|y|P_z}\right)\,f_3\left(y,\mu^2\right)$$

factorization formula for ME

using the explicit expressions for C_3

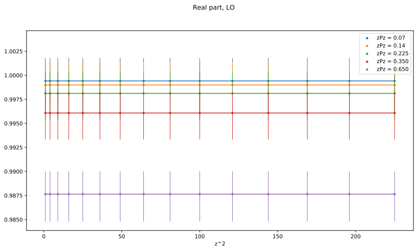
$$\mathcal{O}^{\mathsf{Re}}_{\gamma^0}(z,\mu) = \int_0^1 dx\; \mathcal{C}_3^{\mathsf{Re}}\left(x, z, \frac{\mu}{P_z}\right) V_3(x,\mu) = \mathcal{C}_3^{\mathsf{Re}}\left(z, \frac{\mu}{P_z}\right) \circledast V_3\left(\mu^2\right)$$
$$\mathcal{O}^{\mathsf{Im}}_{\gamma^0}(z,\mu) = \int_0^1 dx\; \mathcal{C}_3^{\mathsf{Im}}\left(x, z, \frac{\mu}{P_z}\right) T_3(x,\mu) = \mathcal{C}_3^{\mathsf{Im}}\left(z, \frac{\mu}{P_z}\right) \circledast T_3\left(\mu^2\right)$$

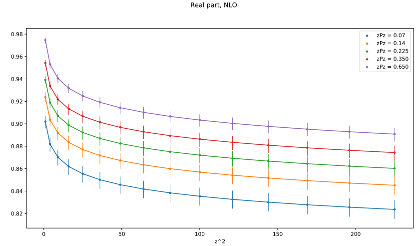
where V_3 and T_3 are the nonsinglet distributions defined by

$$V_3(x) = u(x) - \bar{u}(x) - \left[d(x) - \bar{d}(x)\right]$$
$$T_3(x) = u(x) + \bar{u}(x) - \left[d(x) + \bar{d}(x)\right]$$

$$\mathrm{LO}: \quad \mathcal{O}^{\mathsf{Re}}_{\gamma^0}\left(zP_z, z^2\right) = \int dx \cos(zP_z x) V_3(x,\mu^2)$$

Bjorken scaling of ME



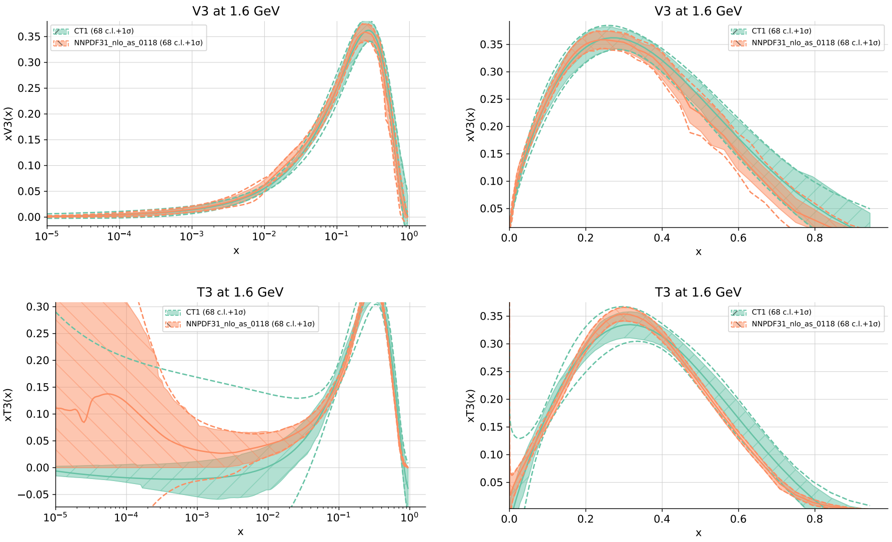


systematic errors

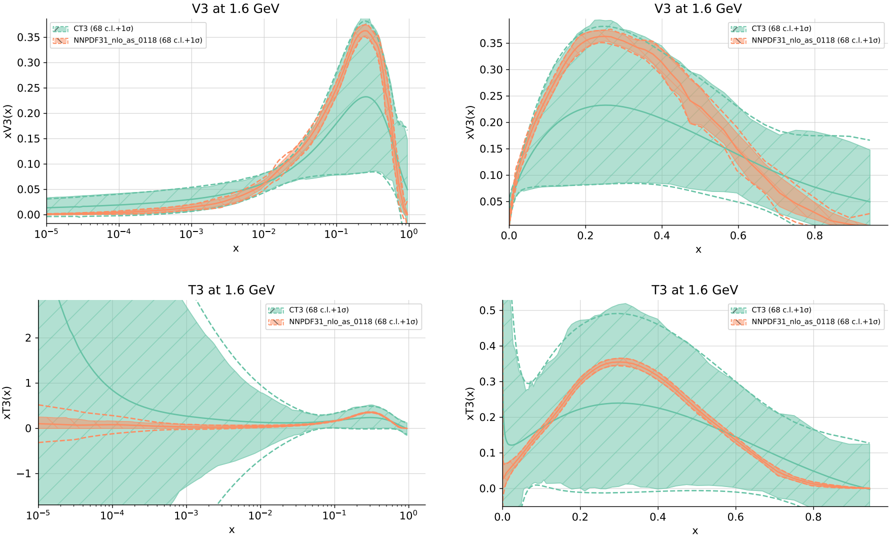
- cut-off effects
- finite volume effects
- excited states contamination
- truncation effects
- higher-twist terms
- isospin breaking

Scenario	Cut-off	FVE	Excited states	Truncation
S1	10%	2.5%	5%	10%
S2	20%	5%	10%	20%
S3	30%	$e^{-3+0.062z/a}\%$	15%	30%
S4	0.1	0.025	0.05	0.1
S5	0.2	0.05	0.1	0.2
S6	0.3	$e^{-3+0.062z/a}$	0.15	0.3

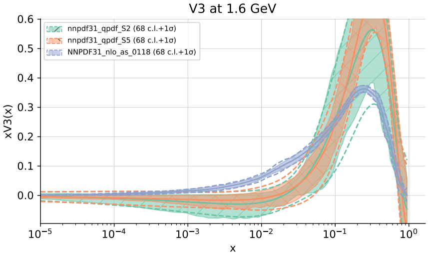
closure test – 1

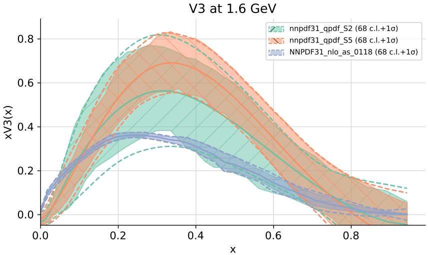


closure test – 2



fit results


V3 at 1.6 GeV
nnpdf31_qpdf_S2 (68 c.l.+1σ)
nnpdf31_qpdf_S5 (68 c.l.+1σ)
NNPDF31_nlo_as_0118 (68 c.l.+1σ)
xV3(x)
x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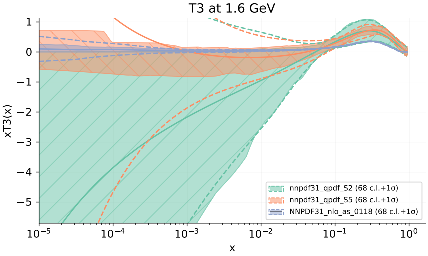
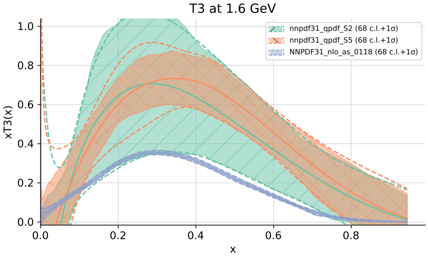

T3 at 1.6 GeV
nnpdf31_qpdf_S2 (68 c.l.+1σ)
nnpdf31_qpdf_S5 (68 c.l.+1σ)
NNPDF31_nlo_as_0118 (68 c.l.+1σ)
xT3(x)
x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NNPDF31_nlo_as_0118 (68 c.l.+1σ)
xT3(x)
x

outlook

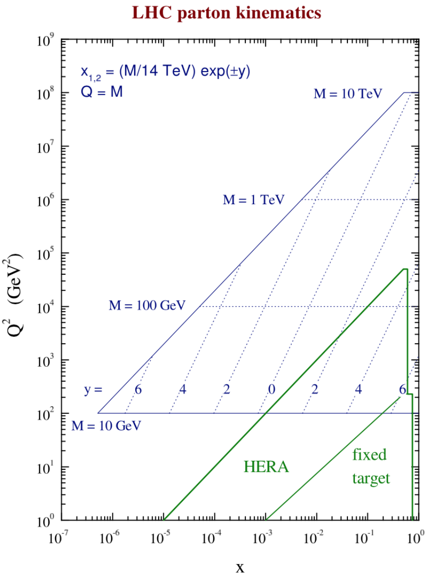
- light-cone PDFs + factorization describe the structure of the proton
- necessary input for the exploitation of LHC, HL-LHC
- current extraction from data is very precise + improving
- lattice data provide complementary information, can be included in global fits like any other data
- identify the areas where a significant phenomenological impact from lattice QCD is possible

global fits

Exp.	Obs.	Ref.	$N_{\mathbf{dat}}$	Kin$_1$	Kin$_2$ (GeV)	Theory
ATLAS	W, Z 2010	[49]	30 (30/30)	$0 \le \lvert\eta_l\rvert \le 3.2$	$Q = M_W, M_Z$	MCFM+FEWZ
	W, Z 2011 **(*)**	[72]	34 (34/34)	$0 \le \lvert\eta_l\rvert \le 2.3$	$Q = M_W, M_Z$	MCFM+FEWZ
	high-mass DY 2011	[50]	11 (5/5)	$0 \le \lvert\eta_l\rvert \le 2.1$	$116 \le M_{ll} \le 1500$	MCFM+FEWZ
	low-mass DY 2011 **(*)**	[77]	6 (4/6)	$0 \le \lvert\eta_l\rvert \le 2.1$	$14 \le M_{ll} \le 56$	MCFM+FEWZ
	[Z p_T 7 TeV (p_T^Z, y_Z)] **(*)**	[78]	64 (39/39)	$0 \le \lvert y_Z\rvert \le 2.5$	$30 \le p_T^Z \le 300$	MCFM+NNLO
	Z p_T 8 TeV (p_T^Z, M_{ll}) **(*)**	[71]	64 (44/44)	$12 \le M_{ll} \le 150$ GeV	$30 \le p_T^Z \le 900$	MCFM+NNLO
	Z p_T 8 TeV (p_T^Z, y_Z) **(*)**	[71]	120 (48/48)	$0.0 \le \lvert y_Z\rvert \le 2.4$	$30 \le p_T^Z \le 150$	MCFM+NNLO
	7 TeV jets 2010	[57]	90 (90/90)	$0 \le \lvert y^{\rm jet}\rvert \le 4.4$	$25 \le p_T^{\rm jet} \le 1350$	NLOjet++
	2.76 TeV jets	[58]	59 (59/59)	$0 \le \lvert y^{\rm jet}\rvert \le 4.4$	$20 \le p_T^{\rm jet} \le 200$	NLOjet++
	7 TeV jets 2011 **(*)**	[76]	140 (31/31)	$0 \le \lvert y^{\rm jet}\rvert \le 0.5$	$108 \le p_T^{\rm jet} \le 1760$	NLOjet++
	$\sigma_{\rm tot}(t\bar{t})$	[74, 75]	3 (3/3)	-	$Q = m_t$	top++
	$(1/\sigma_{t\bar{t}})d\sigma(t\bar{t})/y_t$ **(*)**	[73]	10 (10/10)	$0 < \lvert y_t\rvert < 2.5$	$Q = m_t$	Sherpa+NNLO
CMS	W electron asy	[52]	11 (11/11)	$0 \le \lvert\eta_e\rvert \le 2.4$	$Q = M_W$	MCFM+FEWZ
	W muon asy	[53]	11 (11/11)	$0 \le \lvert\eta_\mu\rvert \le 2.4$	$Q = M_W$	MCFM+FEWZ
	$W + c$ total	[60]	5 (5/0)	$0 \le \lvert\eta_l\rvert \le 2.1$	$Q = M_W$	MCFM
	$W + c$ ratio	[60]	5 (5/0)	$0 \le \lvert\eta_l\rvert \le 2.1$	$Q = M_W$	MCFM
	2D DY 2011 7 TeV	[54]	124 (88/110)	$0 \le \lvert\eta_{ll}\rvert \le 2.2$	$20 \le M_{ll} \le 200$	MCFM+FEWZ
	[2D DY 2012 8 TeV]	[84]	124 (108/108)	$0 \le \lvert\eta_{ll}\rvert \le 2.4$	$20 \le M_{ll} \le 1200$	MCFM+FEWZ
	$W^\pm$ rap 8 TeV **(*)**	[79]	22 (22/22)	$0 \le \lvert\eta_l\rvert \le 2.3$	$Q = M_W$	MCFM+FEWZ
	Z p_T 8 TeV **(*)**	[83]	50 (28/28)	$0.0 \le \lvert y_Z\rvert \le 1.6$	$30 \le p_T^Z \le 170$	MCFM+NNLO
	7 TeV jets 2011	[59]	133 (133/133)	$0 \le \lvert y^{\rm jet}\rvert \le 2.5$	$114 \le p_T^{\rm jet} \le 2116$	NLOjet++
	2.76 TeV jets **(*)**	[80]	81 (81/81)	$0 \le \lvert y_{\rm jet}\rvert \le 2.8$	$80 \le p_T^{\rm jet} \le 570$	NLOjet++
	$\sigma_{\rm tot}(t\bar{t})$	[82, 88]	3 (3/3)	-	$Q = m_t$	top++
	$(1/\sigma_{t\bar{t}})d\sigma(t\bar{t})/y_{t\bar{t}}$ **(*)**	[81]	10 (10/10)	$-2.1 < y_{t\bar{t}} < 2.1$	$Q = m_t$	Sherpa+NNLO
LHCb	Z rapidity 940 pb	[55]	9 (9/9)	$2.0 \le \eta_l \le 4.5$	$Q = M_Z$	MCFM+FEWZ
	$Z \to ee$ rapidity 2 fb	[56]	17 (17/17)	$2.0 \le \eta_l \le 4.5$	$Q = M_Z$	MCFM+FEWZ
	$W, Z \to \mu$ 7 TeV **(*)**	[85]	33 (33/29)	$2.0 \le \eta_l \le 4.5$	$Q = M_W, M_Z$	MCFM+FEWZ
	$W, Z \to \mu$ 8 TeV **(*)**	[86]	34 (34/30)	$2.0 \le \eta_l \le 4.5$	$Q = M_W, M_Z$	MCFM+FEWZ

NNPDF3.1 [1706.00428]

kinematic range



lattice perturbation theory

an instructive example [1705.11193]

- $O_i(z) = \bar{\psi}(z)\Gamma_i W(z,0)\psi(0)$
- renormalization pattern: $O^Y_{R,i}(z) = Z^{X,Y}_{ij}(z) O^X_j(z)$
- one-loop computations: $Z = 1 + g^2 C_F/(16\pi^2) Z^{(1)} + \dots$

$$Z^{\mathrm{LR},\overline{\mathrm{MS}}} = 1 + \frac{g^2 C_F}{16\pi^2}\left(\dots + e_2 \frac{|z|}{a} + \dots - 3\log(a^2\mu^2)\right)$$

$$Z^{\mathrm{LR},\overline{\mathrm{MS}}}_{12} = 0 + \frac{g^2 C_F}{16\pi^2}\left(\dots\right)$$

- one-loop matching:

$$\mathcal{C}^{\overline{\mathrm{MS}},\mathrm{RI}} = (Z^{\mathrm{LR},\overline{\mathrm{MS}}})^{-1} \cdot (Z^{\mathrm{LR},\mathrm{RI}}) = (Z^{\mathrm{DR},\overline{\mathrm{MS}}})^{-1} \cdot (Z^{\mathrm{DR},\mathrm{RI}})$$

RI/MOM prescription

renormalization condition:

$$
\begin{aligned}
&O_{R,i}(z) = Z_{ij}(z)O_j(z) \\
&\Lambda_i(p,z) = S(p)^{-1}\left\langle O_i(z)\psi(-p)\bar{\psi}(p)\right\rangle S(p)^{-1} \\
&Z_q^{-1}\frac{1}{12}\,\mathrm{Tr}\left[\Lambda_{R,i}(p,z)(\Lambda_j^{\mathrm{tree}}(p,z))^{-1}\right]\Big|_{p^2=\mu^2} = \delta_{ij}
\end{aligned}
$$

linear divergence from the Wilson line:

$$
Z(z) = \mathcal{Z}(z)e^{\delta m|z|/a - c|z|}
$$

automatically subtracted in this framework

ETMC [1706.00265], J-W Chen et al [1706.01295]

Euclidean/Minkowski

matrix element at $t = 0$ is agnostic about space-time signature

$$h_{\gamma^z,3}(z, P_z) = \langle P_z | O_z(z) | P_z \rangle$$

computed from different correlators in Euclidean and Minkowski

$$\langle N(\tau', P_z) O_z(z) N(\tau, P_z) \rangle = \mathcal{Z}\mathcal{Z}' \mathcal{M}_z(z, P_z) e^{-\omega_P(\tau'-\tau)} + \dots$$

$$\int d^D y \, d^D y' \, e^{i(P(y'-y))} \langle T N(y') O_z(z) N(y) \rangle \sim$$

$$\sim \frac{i\mathcal{Z}'}{P^2 - m^2} \mathcal{M}_z(z, P_z) \frac{i\mathcal{Z}}{P^2 - m^2}$$

Carlson et al [1702.05775], Briceno et al [1703.06072]