Boosting hotQCD

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Introduction

The goal

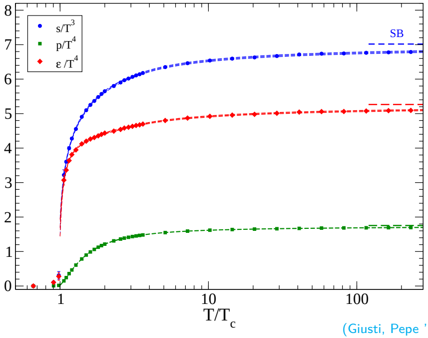
QCD equation of state (EoS)

$$s(T),\ p(T),\ \varepsilon(T)$$

Thermodynamics

$$s(T) = \frac{\partial p(T)}{\partial T}$$

$$Ts(T) = p(T) + \varepsilon(T)$$



(Giusti, Pepe '17)

SU(3) YM – $T_c \approx 300\,\mathrm{MeV}$

Why is this interesting?

- Fundamental property of QCD
- Heavy-ion collisions
- Cosmology
- ...

What do we know?

- EoS of $N_\mathrm{f} = 2+1$ QCD for $T \lesssim 500\,\mathrm{MeV}$ (Bazavov *et al.* '14; Borsanyi *et al.* '14; Bali *et al.* '14; ...)
- First exploratory results up to $T \approx 1 - 2\,\mathrm{GeV}$ (Borsanyi *et al.* '16; Bazavov, Petreczky, Weber '18)
- Most results use variants of staggered fermions
  Wilson quarks are catching up ... (tmfT Collab. '16; WHOT-QCD Collab. '18; MDB, Giusti, Pepe '18; ...)

Introduction

A non-perturbative problem

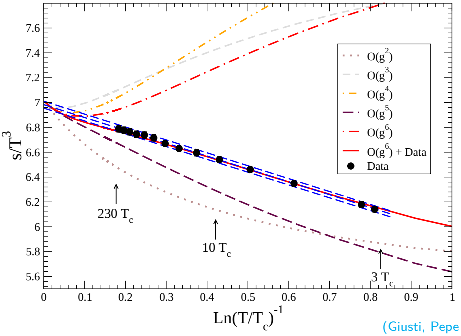
Asymptotic freedom

$$\alpha_s(\mu \approx T) \overset{T\to\infty}{\longrightarrow} 0$$

$\Rightarrow$ PT should work at large T

Free quarks & gluons gas

$$\frac{s_{SB}(T)}{T^3} = \frac{\pi^2}{45}\left(32 + 21 \times N_f\right) \underset{N_f=0}{\approx} 7.02$$



(Giusti, Pepe '17)

SU(3) YM – $T_c \approx 300\,\mathrm{MeV}$

Problems

- PT at finite T shows very **poor convergence**
  - Works only up to a **finite** order: no matter how **small** α_s is! (Lindé '80)
  - Here "O(g^6) + Data" at $T \approx 68\,\mathrm{GeV}$ is $\approx 50\%$ of the correction to free gas
- Resummation techniques seem to improve convergence but cf. (Andersen *et al.* '16)
  - Uncertainties are **hard** to quantify reliably within PT
  - Lindé issue is **not** solved

Introduction

A difficult non-perturbative problem

Free energy

$$f = -p = -\frac{T}{V}\ln\mathcal{Z}$$

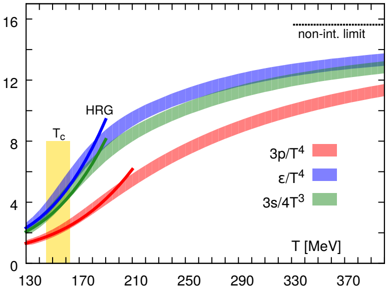
Trace anomaly

(Boyd *et al.* '96; Umeda *et al.* '09; ...)

$$\frac{I(T)}{T^4} \equiv \frac{\varepsilon - 3p}{T^4} = T\frac{\mathrm{d}}{\mathrm{d}T}\left(\frac{p}{T^4}\right)$$

Pressure

$$\frac{p(T)}{T^4} = \frac{p(T_0)}{T_0^4} + \int_{T_0}^{T}\mathrm{d}T'\,\frac{I(T')}{T'^5}$$



(HotQCD Bazavov *et al.* '14)

QCD with $N_\mathrm{f} = 2+1$ quarks

Lattice obs. ($\widehat{A} \equiv$ **lattice**)

$$\widehat{I}(T) = -\frac{T}{V}\frac{\mathrm{d}\ln\widehat{\mathcal{Z}}}{\mathrm{d}\ln a} = \frac{T}{V}\left(a\frac{\mathrm{d}\vec{b}}{\mathrm{d}a}\right)\left\langle\frac{\partial\widehat{S}_{\mathrm{QCD}}}{\partial\vec{b}}\right\rangle_T \qquad \vec{b} = \{g_0(a), m_{0,f}(a), \ldots\} \Leftarrow \mathrm{LCP}$$

Renormalization

$$I(T) = \lim_{a\to 0}\widehat{I}_R(T) = \lim_{a\to 0}\left[\widehat{I}(T) - \widehat{I}(0)\right]\Big|_{\vec{b}}$$

Problem

The renormalization **unnaturally** ties together two **separate** physical scales

$$L^{-1} \ll T \ll a^{-1} \text{ AND } L^{-1} \sim m_\pi \;\Rightarrow\; L/a = \mathrm{O}(100) \text{ for } T = \mathrm{O}(1\,\mathrm{GeV})$$

The energy-momentum tensor

Back to basics

EMT (continuum)

(Callan, Coleman, Jackiw '71; . . .)

$$\mathcal{T}^R_{\mu\nu} = \mathcal{T}_{\mu\nu} = \mathcal{T}^F_{\mu\nu} + \mathcal{T}^G_{\mu\nu}$$

$$\mathcal{T}^F_{\mu\nu} = \frac{1}{4}\left\{\overline{\psi}\gamma_\mu \overleftrightarrow{D}_\nu \psi + \overline{\psi}\gamma_\nu \overleftrightarrow{D}_\mu \psi\right\} - \delta_{\mu\nu}\mathcal{L}^F \qquad \mathcal{T}^G_{\mu\nu} = \frac{1}{g_0^2} F^a_{\mu\alpha} F^a_{\nu\alpha} - \delta_{\mu\nu}\mathcal{L}^G$$

Entropy density

$$Ts(T) = p(T) + \varepsilon(T) \qquad \varepsilon = \langle \mathcal{T}_{00}\rangle_T \qquad p = -\langle \mathcal{T}_{kk}\rangle_T$$

EMT (lattice)

(Caracciolo *et al.* '90 '91 '92)

$$\mathcal{T}^R_{\mu\nu} = \mathcal{T}^{R,[6]}_{\mu\nu} + \mathcal{T}^{R,[3]}_{\mu\nu} + \mathcal{T}^{R,[1]}_{\mu\nu} \qquad \mathcal{T}_{\mu\neq\nu} \in [6]; \quad \mathcal{T}_{00} - \mathcal{T}_{kk} \in [3]; \quad \mathcal{T}_{\mu\mu} \in [1]$$

- $\mathcal{T}^{R,[6,3]}_{\mu\nu} = Z^{[6,3]}_F(g_0)\mathcal{T}^{F,[6,3]}_{\mu\nu} + Z^{[6,3]}_G(g_0)\mathcal{T}^{G,[6,3]}_{\mu\nu}$
- $\langle \mathcal{T}_{\mu\mu}\rangle_T / T^4 \overset{a\to 0}{\propto} (aT)^{-4}$

Novel approaches

- Thermal QFT in a moving frame (Giusti, Meyer '13)
- Ward identities with flowed probes (Patella *et al.* '13)
- $\mathcal{T}^R_{\mu\nu}$ from small flow-time expansion (s. next talk!) (Suzuki '13)

Thermodynamics in a moving frame

The relativistic fluid **(Minkowski space)**

Rest frame

$$\mathcal{T}_{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

Moving frame

$$\mathcal{T}_{0k} = \gamma^2 (p + \varepsilon) v_k \qquad \boldsymbol{v} \equiv \text{velocity}, \;\; \gamma = \frac{1}{\sqrt{1 - \boldsymbol{v}^2}}$$

$$\mathcal{T}_{00} = \gamma^2 (p + \varepsilon) - p \qquad \mathcal{T}_{jk} = \gamma^2 (p + \varepsilon) v_j v_k + p\, \delta_{jk}$$

Entropy density (using $Ts = p + \varepsilon$)

$$Ts = \frac{\mathcal{T}_{0k}}{\gamma^2 v_k} \qquad [\, T \equiv \text{temp. rest frame} \,]$$

Kinematic relations

$$\mathcal{T}_{0k} = \frac{v_k}{1 + v_k^2} \left(\mathcal{T}_{00} + \mathcal{T}_{kk} \right) \qquad [\, v_k \neq 0 \,]$$

Thermodynamics in a moving frame

Shifted boundary conditions (continuum)

Thermal QCD path integral $[\,L=\infty\,]$

$$\mathcal{Z}(L_0',\theta) = \int [DA][D\overline{\psi}][D\psi]\, e^{-S_{\rm QCD}[A,\overline{\psi},\psi]} \qquad \begin{aligned} A_\mu(L_0',\boldsymbol{x}) &= A_\mu(0,\boldsymbol{x}) \\ \psi(L_0',\boldsymbol{x}) &= -e^{i\theta}\psi(0,\boldsymbol{x}) \end{aligned}$$

Euclidean boost / $\mathrm{SO}(4)$ **rotation** $\left[\,\boldsymbol{\xi}=-i\boldsymbol{v};\ \gamma=(1+\boldsymbol{\xi}^2)^{-1/2}\,\right]$

$$\begin{aligned} &\mathcal{L}_{\rm QCD} \\ A_\mu(L_0',\boldsymbol{x}) &= A_\mu(0,\boldsymbol{x}) \\ \psi(L_0',\boldsymbol{x}) &= -e^{i\theta}\psi(0,\boldsymbol{x}) \end{aligned} \quad \xrightarrow{\text{SO(4)}} \quad \begin{aligned} &\mathcal{L}_{\rm QCD} \\ A_\mu(L_0,\boldsymbol{x}) &= A_\mu(0,\boldsymbol{x}-\boldsymbol{\xi}L_0) \\ \psi(L_0,\boldsymbol{x}) &= -e^{i\theta}\psi(0,\boldsymbol{x}-\boldsymbol{\xi}L_0) \end{aligned} \qquad [\,L_0'=L_0/\gamma\,]$$

Partition function

$$\mathcal{Z}(L_0,\boldsymbol{\xi},\theta) = \mathrm{Tr}\left\{e^{-L_0(\hat{H}-i\boldsymbol{\xi}\cdot\hat{\boldsymbol{P}}_\theta)}\right\}$$

Free energy

$$f(L_0,\boldsymbol{\xi},\theta) = -\frac{1}{L_0 V}\ln\mathcal{Z}(L_0,\boldsymbol{\xi},\theta) \qquad f(L_0,\boldsymbol{\xi},\theta) \overset{V\to\infty}{=} f(L_0/\gamma,0,\theta)$$

Entropy density

$$Ts(T) = -\frac{\langle T_{0k}\rangle_{\boldsymbol{\xi},\theta=0}}{\gamma^2\xi_k} \qquad T=\frac{\gamma}{L_0}$$

Thermodynamics in a moving frame

Shifted boundary conditions (lattice)

Thermal QCD path integral $[L=\infty]$

$$\widehat{\mathcal{Z}}(L_0',\theta) = \int [DA][D\overline{\psi}][D\psi]\, e^{-S_{\rm LQCD}[U,\overline{\psi},\psi]} \qquad \begin{array}{c} U_\mu(L_0',\boldsymbol{x}) = U_\mu(0,\boldsymbol{x}) \\ \psi(L_0',\boldsymbol{x}) = -e^{i\theta}\psi(0,\boldsymbol{x}) \end{array}$$

Euclidean boost / $\mathrm{SO}(4)$ **rotation** $\left[\boldsymbol{\xi}=-i\boldsymbol{v};\ \gamma=(1+\boldsymbol{\xi}^2)^{-1/2}\right]$

$$\begin{array}{c} \mathcal{L}_{\rm LQCD} \\ U_\mu(L_0',\boldsymbol{x}) = U_\mu(0,\boldsymbol{x}) \\ \psi(L_0',\boldsymbol{x}) = -e^{i\theta}\psi(0,\boldsymbol{x}) \end{array} \quad \not\xrightarrow{\mathrm{SO}(4)} \quad \begin{array}{c} \mathcal{L}_{\rm LQCD} \\ U_\mu(L_0,\boldsymbol{x}) = U_\mu(0,\boldsymbol{x}-\boldsymbol{\xi}L_0) \\ \psi(L_0,\boldsymbol{x}) = -e^{i\theta}\psi(0,\boldsymbol{x}-\boldsymbol{\xi}L_0) \end{array} \quad [L_0' = L_0/\gamma]$$

Partition function

$$\widehat{\mathcal{Z}}(L_0,\boldsymbol{\xi},\theta) = \mathrm{Tr}\left\{e^{-L_0(\hat{H}-i\boldsymbol{\xi}\cdot\hat{\boldsymbol{P}}_\theta)}\right\}$$

Free energy

$$\widehat{f}(L_0,\boldsymbol{\xi},\theta) = -\frac{1}{L_0 V}\ln\widehat{\mathcal{Z}}(L_0,\boldsymbol{\xi},\theta) \qquad \widehat{f}(L_0,\boldsymbol{\xi},\theta) \;\not\overset{V\to\infty}{=}\; \widehat{f}(L_0/\gamma,0,\theta)$$

Entropy density

$$Ts(T) = \lim_{a\to 0} -\frac{\langle T^R_{0k}\rangle_{\boldsymbol{\xi},\theta=0}}{\gamma^2\xi_k} \qquad T = \frac{\gamma}{L_0}$$

Renormalization of the EMT

Ward identities (continuum)

Momentum identities

$$\langle \mathcal{T}_{0k} \rangle_{\boldsymbol{\xi},\theta} = -\frac{\partial}{\partial \xi_k} f(L_0, \boldsymbol{\xi}, \theta)$$

$$L_0 \langle \overline{\mathcal{T}}_{0k}(x_0) \mathcal{O} \rangle_{\boldsymbol{\xi},\theta,c} = \frac{\partial}{\partial \xi_k} \langle \mathcal{O} \rangle_{\boldsymbol{\xi},\theta} \qquad \overline{\mathcal{T}}_{0k}(x_0) = \int_V \mathrm{d}\boldsymbol{x}\, \mathcal{T}_{0k}(x_0, \boldsymbol{x})$$

Baryon number identities

$$i \langle V_0 \rangle_{\boldsymbol{\xi},\theta} = L_0 \frac{\partial}{\partial \theta} f(L_0, \boldsymbol{\xi}, \theta)$$

$$\langle \overline{V}_0(x_0) \mathcal{O} \rangle_{\boldsymbol{\xi},\theta,c} = i \frac{\partial}{\partial \theta} \langle \mathcal{O} \rangle_{\boldsymbol{\xi},\theta} \qquad V_0(x) = \overline{\psi}(x) \gamma_0 \psi(x)$$

Other identities

$$\frac{\partial}{\partial \xi_k} \frac{\partial}{\partial \theta} f = \frac{\partial}{\partial \theta} \frac{\partial}{\partial \xi_k} f \quad \Rightarrow \quad -i \frac{\partial}{\partial \xi_k} \langle V_0 \rangle_{\boldsymbol{\xi},\theta} = L_0 \frac{\partial}{\partial \theta} \langle \mathcal{T}_{0k} \rangle_{\boldsymbol{\xi},\theta}$$

$$\langle \mathcal{T}_{0k} \rangle_{\boldsymbol{\xi},\theta} = \frac{\xi_k}{1 - \xi_k^2} \left(\langle \mathcal{T}_{00} \rangle_{\boldsymbol{\xi},\theta} - \langle \mathcal{T}_{kk} \rangle_{\boldsymbol{\xi},\theta} \right) + \text{FV}$$

Renormalization of the EMT

Ward identities (lattice)

Momentum identities

$$\langle \mathcal{T}^R_{0k} \rangle_{\boldsymbol{\xi},\theta} = -\frac{\partial}{\partial \xi_k} \widehat{f}(L_0, \boldsymbol{\xi}, \theta) + \mathrm{O}(a) \qquad \frac{\partial}{\partial \xi_k} \widehat{f}(\boldsymbol{\xi}) = \frac{L_0}{2a} \left[\widehat{f}(\boldsymbol{\xi} + \tfrac{a\hat{k}}{L_0}) - \widehat{f}(\boldsymbol{\xi} - \tfrac{a\hat{k}}{L_0}) \right]$$

$$L_0 \langle \overline{\mathcal{T}}^R_{0k}(x_0) \mathcal{O} \rangle_{\boldsymbol{\xi},\theta,c} = \frac{\partial}{\partial \xi_k} \langle \mathcal{O} \rangle_{\boldsymbol{\xi},\theta} + \mathrm{O}(a)$$

Baryon number identities

$$i \langle \widetilde{V}_0 \rangle_{\boldsymbol{\xi},\theta} = L_0 \frac{\partial}{\partial \theta} \widehat{f}(L_0, \boldsymbol{\xi}, \theta)$$

$$\langle \overline{\widetilde{V}}_0(x_0) \mathcal{O} \rangle_{\boldsymbol{\xi},\theta,c} = i \frac{\partial}{\partial \theta} \langle \mathcal{O} \rangle_{\boldsymbol{\xi},\theta} \qquad \widetilde{V}_0(x) = \frac{1}{2} \left[\overline{\psi}(x)(\gamma_0 - 1) e^{i\theta} U_0(x) \psi(x + \hat{0}) + \text{c.c.} \right]$$

Other identities

$$-i \frac{\partial}{\partial \xi_k} \langle \widetilde{V}_0 \rangle_{\boldsymbol{\xi},\theta} = L_0 \frac{\partial}{\partial \theta} \langle \mathcal{T}^R_{0k} \rangle_{\boldsymbol{\xi},\theta} + \mathrm{O}(a)$$

$$\langle \mathcal{T}^R_{0k} \rangle_{\boldsymbol{\xi},\theta} = \frac{\xi_k}{1 - \xi_k^2} \left(\langle \mathcal{T}^R_{00} \rangle_{\boldsymbol{\xi},\theta} - \langle \mathcal{T}^R_{kk} \rangle_{\boldsymbol{\xi},\theta} \right) + \mathrm{O}(a) + \text{FV}$$

O(a)-improvement of the EMT

Massless case

Lattice action

O(a)-improved Wilson fermions

Lattice EMT

(Caracciolo *et al.* '90 '91 '92)

$$\mathcal{T}^G_{\mu\nu} = \frac{1}{g_0^2}\widehat{F}^a_{\mu\rho}\widehat{F}^a_{\nu\rho} - \delta_{\mu\nu}\widehat{\mathcal{L}}^G$$

$$\mathcal{T}^F_{\mu\nu} = \tfrac{1}{8}\left\{\overline{\psi}\gamma_\mu\left[\overleftrightarrow{\nabla}^*_\nu + \overleftrightarrow{\nabla}_\nu\right]\psi + \overline{\psi}\gamma_\nu\left[\overleftrightarrow{\nabla}^*_\mu + \overleftrightarrow{\nabla}_\mu\right]\psi\right\} - \delta_{\mu\nu}\widehat{\mathcal{L}}^F$$

O(a)-counterterms ($m_{\mathrm{q},R} = 0;\ i \in [6,3]$)

similar problem to (Capitani *et al.* '00)

$$\mathcal{O}_{1,\mu\nu} = \overline{\psi}\sigma_{\mu\rho}F_{\nu\rho}\psi\,;\quad \mathcal{O}_{2,\mu\nu} = \overline{\psi}\left\{\overleftrightarrow{D}_\mu, \overleftrightarrow{D}_\nu\right\}\psi\,;\quad \mathcal{O}_{3,\mu\nu} = \partial_\rho\left(\overline{\psi}\sigma_{\mu\rho}\overleftrightarrow{D}_\nu\psi\right)$$

$$\mathcal{T}^{R,[i]}_{\mu\nu,\mathrm{I}} = Z^{[i]}_G(g_0)\mathcal{T}^{G,[i]}_{\mu\nu} + Z^{[i]}_F(g_0)\left\{\mathcal{T}^{F,[i]}_{\mu\nu} + a\delta\mathcal{T}^{F,[i]}_{\mu\nu}\right\} \quad \delta\mathcal{T}^{F,[i]}_{\mu\nu} = \sum_{k=1,2,3} c^{[i]}_k(g_0)\widehat{\mathcal{O}}^{[i]}_{k,\mu\nu}$$

Remarks

- $c^{[i]}_k(g_0) = \mathrm{O}(g_0^2)$
- $\mathcal{O}_2$ and $\mathcal{O}_3$ can be neglected when considering $\langle\mathcal{T}^{R,[i]}_{\mu\nu,\mathrm{I}}\rangle_{\boldsymbol{\xi},\theta}$
- In finite volume **NO** spontaneous chiral symmetry breaking $\Rightarrow \langle\delta\mathcal{T}^{F,[i]}_{\mu\nu}\rangle_{\boldsymbol{\xi},\theta} = \mathrm{O}(a)$
  Note that this is also true in **infinite volume** if $T > T_c$!

O(a)-improvement of the EMT

Mass-degenerate case

O(a)-improved parameters

(Lüscher *et al.* '92)

$$\tilde{g}_0^2 = g_0^2\big(1 + b_{\rm g}(g_0)am_{\rm q}\big) \qquad m_{\rm q} = m_0 - m_{\rm crit}$$

$$\widetilde{m}_{\rm q} = m_{\rm q}\big(1 + b_{\rm m}(g_0)am_{\rm q}\big)$$

O(am)-counterterms ($m_{{\rm q},R} = m;\ \ i \in [6,3]$)

$$\mathcal{O}_{4,\mu\nu} = m\,\mathcal{T}^{G}_{\mu\nu}\,; \quad \mathcal{O}_{5,\mu\nu} = m\,\mathcal{T}^{F}_{\mu\nu}$$

O(a)-improved EMT

$$\mathcal{T}^{R,[i]}_{\mu\nu,{\rm I}} = Z^{[i]}_G(\tilde{g}_0)\mathcal{T}^{G,[i]}_{\mu\nu,{\rm I}} + Z^{[i]}_F(\tilde{g}_0)\mathcal{T}^{F,[i]}_{\mu\nu,{\rm I}}$$

with

$$\mathcal{T}^{G,[i]}_{\mu\nu,{\rm I}} = \big(1 + b^{G,[i]}_T(g_0)am_{\rm q}\big)\mathcal{T}^{G,[i]}_{\mu\nu} \qquad b^{G,[i]}_T = {\rm O}(g_0^2)$$

$$\mathcal{T}^{F,[i]}_{\mu\nu,{\rm I}} = \big(1 + b^{F,[i]}_T(g_0)am_{\rm q}\big)\big\{\mathcal{T}^{F,[i]}_{\mu\nu} + a\delta\mathcal{T}^{F,[i]}_{\mu\nu}\big\} \qquad b^{F,[i]}_T = 1 + {\rm O}(g_0^2)$$

Remarks

- For non-degenerate quarks we have one additional b coefficient of O(g_0^4)
- Assuming $aT \ll 1$, O(am)-effects are expected to be **small** ($\lesssim 1\%$) for the light quarks once $T \gtrsim 1\,{\rm GeV}$
- Perturbative estimates for the improvement coefficients are likely **sufficient** at these high temperatures

A test in perturbation theory

Renormalization constants and improvement coefficients at one-loop order

Ward identities

$$\langle \mathcal{T}^{R,[6]}_{0k,\mathrm{I}} \rangle_{\boldsymbol{\xi},\theta} = -\frac{\partial}{\partial \xi_k} \widehat{f}(L_0, \boldsymbol{\xi}, \theta) \quad \langle \mathcal{T}^{R,[6]}_{0k,\mathrm{I}} \rangle_{\boldsymbol{\xi},\theta} = \frac{\xi_k}{1-\xi_k^2} \left(\langle \mathcal{T}^{R,[3]}_{00,\mathrm{I}} \rangle_{\boldsymbol{\xi},\theta} - \langle \mathcal{T}^{R,[3]}_{kk,\mathrm{I}} \rangle_{\boldsymbol{\xi},\theta} \right)$$

Renormalization constants

$$Z_G(g_0) = 1 + g_0^2 \left[N Z_G^{(1),N} + \frac{1}{N} Z_G^{(1),\frac{1}{N}} + N_\mathrm{f} Z_G^{(1),F} \right] + \mathrm{O}(g_0^4)$$

$$Z_F(g_0) = 1 + g_0^2 C_F Z_F^{(1)} + \mathrm{O}(g_0^4)$$

Results: Unimproved and O(a)-improved Wilson for both $[6,3]$

Perfect agreement with the literature (when available) (Caracciolo *et al.* '92; Capitani, Rossi '95)

O(a)-improvement coefficients

$$c_k(g_0) = \mathrm{O}(g_0^2) \quad b_T^F(g_0) = 1 + g_0^2 b_T^{F,(1)} + \mathrm{O}(g_0^4) \quad b_T^G(g_0) = g_0^2 b_T^{G,(1)} + \mathrm{O}(g_0^4)$$

Results: O(a)-improved Wilson for both $[6,3]$

Perfect agreement with the literature (when available) (Capitani *et al.* '00)

For geeks

- $L_0/a = 4-32$, $R = L/L_0 = 5-15$, several $\boldsymbol{\xi}$ and θ values
- Gauge zero-mode removal $\Rightarrow$ $R \to \infty$ extrapolations
- Coordinate space calculation based on FFT

Towards the EoS at high temperature

General strategy

Master equation

$$\frac{s(T)}{T^3} = \lim_{a\to 0} -\frac{L_0^4 \langle \mathcal{T}^R_{0k,\mathrm{I}} \rangle_{\boldsymbol{\xi},\theta=0}}{\gamma^6 \xi_k} \qquad T = \frac{\gamma}{L_0}$$

Lattice set-up

- $N_\mathrm{f} = 3$ O(a)-improved Wilson quarks with shifted bc., $\boldsymbol{\xi} = (1,0,0)$

Lines of constant physics

(MDB *et al.* '16, '18; Campos *et al.* '18)

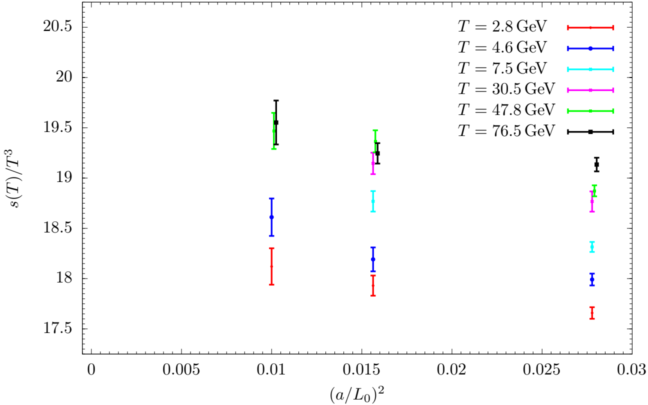
- 8 values of $T \approx 2.8 - 80\,\mathrm{GeV}$ fixed by $\bar{g}^2_\mathrm{SF}(\mu = T/\gamma)$
- $L_0/a = 6, 8, 10(, 12)$ and $L/a = 288 \Rightarrow TL \approx 34 - 17$
- $m_{\mathrm{q},R} = \mathrm{O}(a^2)$, i.e., massless quarks
- PT values improvement coefficients

Systematic effects

- **Mass effects:** $s(T)|_m = s(T)|_{m=0} + \mathrm{O}(m^2/T^2)$
  Expected to be quite small for light quarks for $T \gtrsim 2.8\,\mathrm{GeV}$
  Actual size needs to be estimated at the smaller T's
- **Finite size effects:** $s(T)|_L = s(T)|_{L=\infty} + \mathrm{O}(e^{-mL})$ w/ $m = \mathrm{O}(T)$ (Giusti, Meyer '13)
  $L/a = 96$ simulations and measure of $m(T)$ to estimate actual size
- $\mathrm{O}(a)$**-effects**: Monitor size of $\mathrm{O}(a)$-counterterms

Towards the EoS at high temperature

Some preliminary results. Perturbative Z's have been used for illustration!
Vertical scale should not be taken at face value!



Remarks

- $N_{\rm ms} = 100, 250, 450$ for $L_0/a = 6, 8, 10$
- $\mathrm{Var}(s(T))/s(T)^2 \propto (L_0/a)^8$; $\tau_{\rm int} \lesssim 2\,\mathrm{MDUs}$
- About 1% error for $L_0/a = 10$
- Small discretization errors (?)

Conclusions & Outlook

Conclusions

- QCD in a moving frame is a **powerful** framework for thermodynamics studies
- Offers alternative ways for computing thermodynamics quantities
- Provides many Ward identities for the non-perturbative **renormalization** of the EMT
- The framework passes with flying colours an analysis to 1-loop order in PT
- Preliminary results on the bare entropy are **encouraging**

Outlook

- The non-perturbative renormalization of the EMT is on its way
- Accurate determination of the EoS of $N_{\rm f} = 3$ QCD in a totally **unexplored** temperature range
- How accurate is PT in this regime?
- Heavy-quark effects? PT or non-PT?


Thanks