$B_c \to B_{s(d)}$ form factors with NRQCD and heavy-HISQ

Laurence Cooper

DAMTP, University of Cambridge

L.J.Cooper@damtp.cam.ac.uk

Christine Davies, Judd Harrison, Javad Komijani, Matthew Wingate

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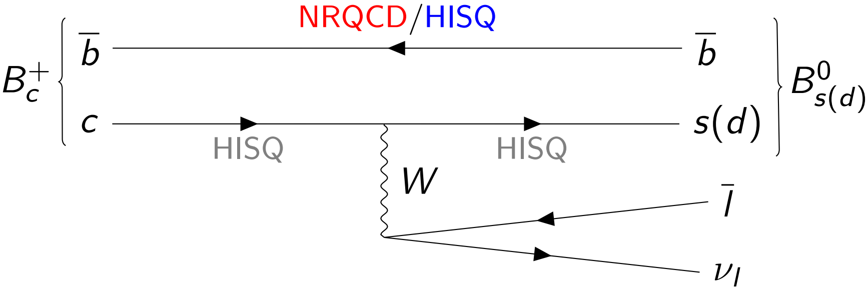
HPQCD



NRQCD: arXiv:hep-lat/9205007, Lepage et al
HISQ: arXiv:hep-lat/0610092, Follana et al

$B_c \longrightarrow B_s(d)$ form factors

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} = \frac{G_F^2|V|^2}{24\pi^3}\Delta(q^2)^3|f_+(q^2)|^2$$

in the limit of massless leptons, where f_+ is one of two form factors that parametrise the weak matrix element

$$\langle B_{s(d)}(\mathrm{p}_2)|V^\mu|B_c(\mathrm{p}_1)\rangle = f_0(q^2)\left[\frac{M_{B_c}^2 - M_{B_{s(d)}}^2}{q^2}q^\mu\right] + f_+(q^2)\left[p_2^\mu + p_1^\mu - \frac{M_{B_c}^2 - M_{B_{s(d)}}^2}{q^2}q^\mu\right]$$

and $q = p_1 - p_2$ with $0 < q^2 < t_- = (M_{B_c} - M_{B_{s(d)}})^2$.

Extracting f_0 with PCVC

Partially Conserved Vector Current (PCVC) Ward identity

$$\partial_\mu V^\mu = (m_c - m_{s(d)})S$$

relates the c to $s(d)$ conserved vector current V^μ and scalar density S.

Using only local lattice operators, need a single q^2 independent renormalisation factor Z_V satisfying

$$q_\mu \langle B_{s(d)} | V^\mu | B_c \rangle Z_V = (m_c - m_{s(d)}) \langle B_{s(d)} | S | B_c \rangle .$$

Hence, f_0 is solely determined by the scalar density matrix element through

$$f_0(q^2) = \langle B_{s(d)} | S | B_c \rangle \frac{m_c - m_{s(d)}}{M_{B_c}^2 - M_{B_{s(d)}}^2} .$$

Extracting f_+

Trivial rearrangement of form factor definition with $\mu = 0$ yields

$$f_+(q^2) = \frac{Z_V \mathcal{V}^0 - q^0 f_0(q^2) \frac{M^2_{B_c^+} - M^2_{B_s^0}}{q^2}}{p_2^0 + p_1^0 - q^0 \frac{M^2_{B_c^+} - M^2_{B_s^0}}{q^2}},$$

requiring $\mathcal{V}^0$ and $\mathcal{S}$ only.

Problem: numerator and denominator grow like 3-momentum away from zero-recoil

$\Longrightarrow$ large errors on f_+ near zero-recoil, data is uninformative

$\Longrightarrow$ cannot obtain f_+ data near zero-recoil

Extracting f_+ with V^i

With $\mu = i \neq 0$, require $\mathcal{V}^0$, $\mathcal{S}$ *and* $\mathcal{V}^i$ to extract f_+ through

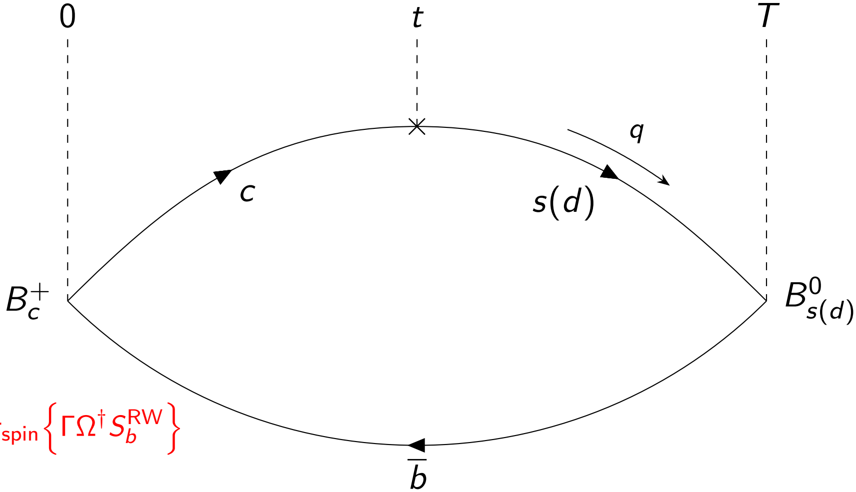
$$f_+(q^2) = \frac{-\frac{Z_V \mathcal{V}^i}{q^i} + f_0(q^2)\frac{M^2_{B_c^+} - M^2_{B_s^0}}{q^2}}{1 + \frac{M^2_{B_c^+} - M^2_{B_s^0}}{q^2}}$$

Possessing $\mathcal{V}^i$ allows scrutiny of the q^2 independence of Z_V through

$$q_\mu \langle B_{s(d)} | V^\mu | B_c \rangle Z_V = (m_c - m_{s(d)}) \langle B_{s(d)} | S | B_c \rangle.$$

Problem solved: Both the numerator and denominator are away from zero around zero-recoil

Complication: How expensive will it be to obtain $\mathcal{V}^i$ and well as $\mathcal{V}^0$?

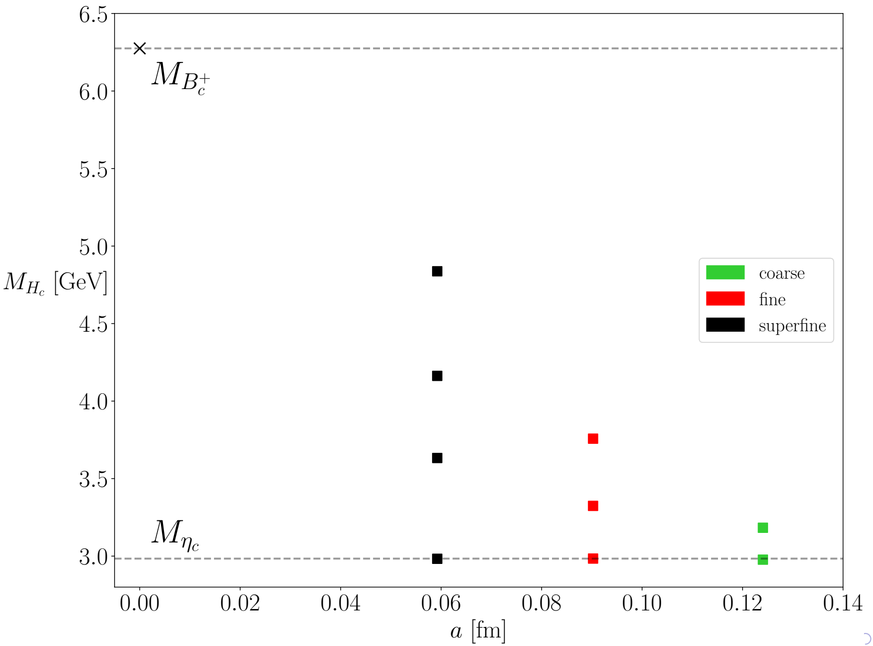


$$\mathrm{Tr}_{\mathrm{spin}}\left\{\Gamma\Omega^\dagger S_b^{\mathrm{RW}}\right\}$$

Lattice parameters

set	handle	a [fm]	$N_x^3 \times N_t$	n_{cfg}	am_l^{sea}	am_s^{sea}	am_c^{sea}
1	very-coarse	0.15	$16^3 \times 48$	1000	0.013	0.065	0.838
2	very-coarse-physical	0.15	$32^3 \times 48$	500	0.00235	0.00647	0.831
3	coarse	0.12	$24^3 \times 64$	1053	0.0102	0.0541	0.635
4	coarse-physical	0.12	$48^3 \times 64$	1000	0.00184	0.0507	0.628
5	fine	0.09	$32^3 \times 96$	504	0.0074	0.037	0.440
6	superfine	0.045	$48^3 \times 144$	250	0.0048	0.024	0.286

Lattice spacing determined from the Wilson flow parameter. Sets 1 to 5 are used by the NRQCD calculation. Heavy-HISQ data is obtained on sets 3, 5 and 6.



6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
M_{H_c} [GeV]
$M_{B_c^+}$
M_{η_c}
coarse
fine
superfine
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
a [fm]

Correlators

$$\langle \mathcal{O}_{B_c}(t)\mathcal{O}^\dagger_{B_c}(0)\rangle = \sum_i b[i]^2 e^{-E_b[i]t} - \sum_i b_o[i]^2(-1)^t e^{-E_{b_o}[i]t}$$

$$\langle \mathcal{O}_{B_{s(d)}}(t)\mathcal{O}^\dagger_{B_{s(d)}}(0)\rangle = \sum_i a[i]^2 e^{-E_a[i]t} - \sum_i a_o[i]^2(-1)^t e^{-E_{a_o}[i]t}$$

$$\begin{aligned}\langle \mathcal{O}_{B_{s(d)}}(T)J(t)\mathcal{O}^\dagger_{B_c}(0)\rangle = &\sum_{i,j} a[i]e^{-E_a[i]t}V_{nn}[i,j]b[j]e^{-E_b[j](T-t)} \\ &-\sum_{i,j}(-1)^{T-t}a[i]e^{-E_a[i]t}V_{no}[i,j]b_o[j]e^{-E_{b_o}[j](T-t)} \\ &-\sum_{i,j}(-1)^{t}a_o[i]e^{-E_{a_o}[i]t}V_{on}[i,j]b[j]e^{-E_b[j](T-t)} \\ &+\sum_{i,j}(-1)^{T}a_o[i]e^{-E_{a_o}[i]t}V_{oo}[i,j]b_o[j]e^{-E_{b_o}[j](T-t)}\end{aligned}$$

For fitting methodology, see:

G.P. Lepage et al, Nucl.Phys.Proc.Suppl. 106 (2002) 12-20

K. Hornbostel et al, Phys.Rev. D85 (2012) 031504

C. Bouchard et al, Phys.Rev. D90 (2014) 054506

Form factor fit functions: NRQCD and Heavy-HISQ

$$t_\pm = (M_{B_c} \pm M_{B_{s(d)}})^2, \quad z' = k\frac{\sqrt{t_+ - q^2} - \sqrt{t_+}}{\sqrt{t_+ - q^2} + \sqrt{t_+}}, \quad P(q^2)^{-1} = 1 - \frac{q^2}{M^2_{\text{res}}}$$

$$f(q^2) = P(q^2)\sum_{n=0}^{3} b^{(n)} z'^n$$

where $b^{(n)} = A^{(n)}\left\{1 + B^{(n)}(am_c/\pi)^2 + C^{(n)}(am_c/\pi)^4\right\}.$

$$f(q^2) = P(q^2)\sum_{n,i,j,k=0}^{3} A^{(n)}_{ijk} z'^n \left(\frac{am_c}{\pi}\right)^{2i}\left(\frac{am_h}{\pi}\right)^{2j}\Delta^{(k)}_M$$

where $\Delta^{(k)}_M = (\Lambda_{\text{QCD}}/M_{H_c})^k - (\Lambda_{\text{QCD}}/M_{B_c})^k.$

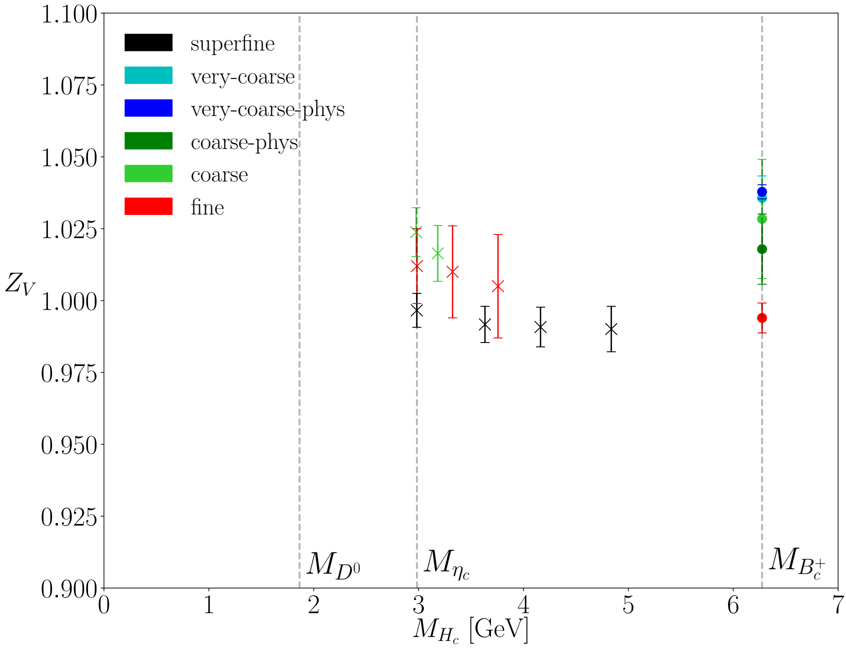
Vector current renormalisation Z_V

set	handle	Z_V
1	very-coarse	1.0357(77)
2	very-coarse-physical	1.0378(25)
3	coarse	1.028(21)
4	coarse-physical	1.018(12)
5	fine	0.9940(52)

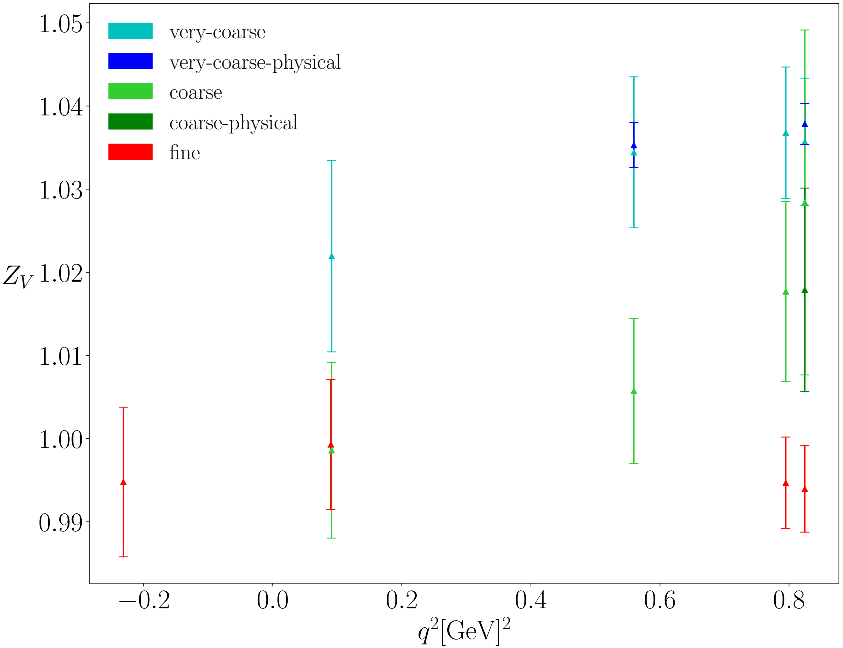
Table: Z_V for $c \to s$ from NRQCD

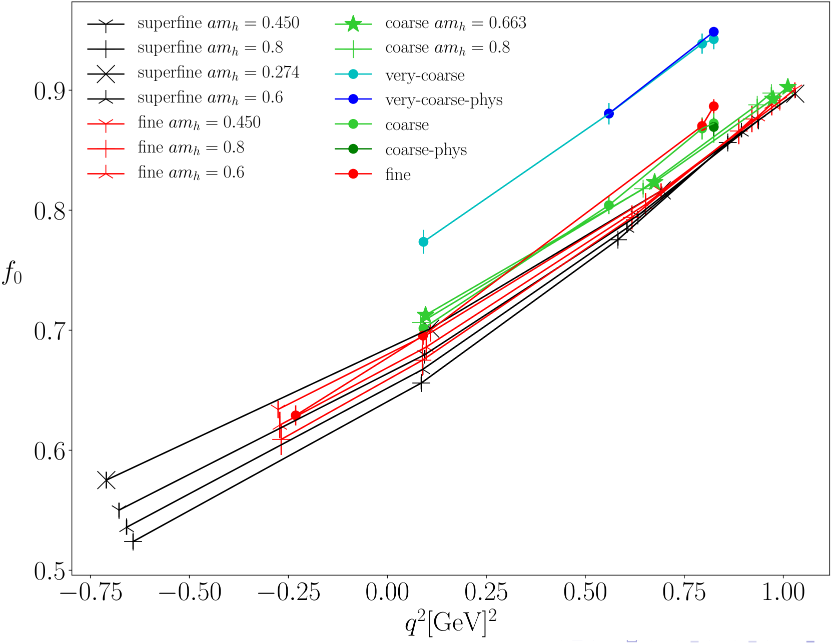
set $\backslash am_h$	0.274	0.450	0.6	0.663	0.8
3	-	-	-	1.0165(99)	1.017(11)
5	-	1.0057(96)	1.004(10)	-	1.003(11)
6	0.9963(91)	0.994(10)	0.993(11)	-	0.992(12)

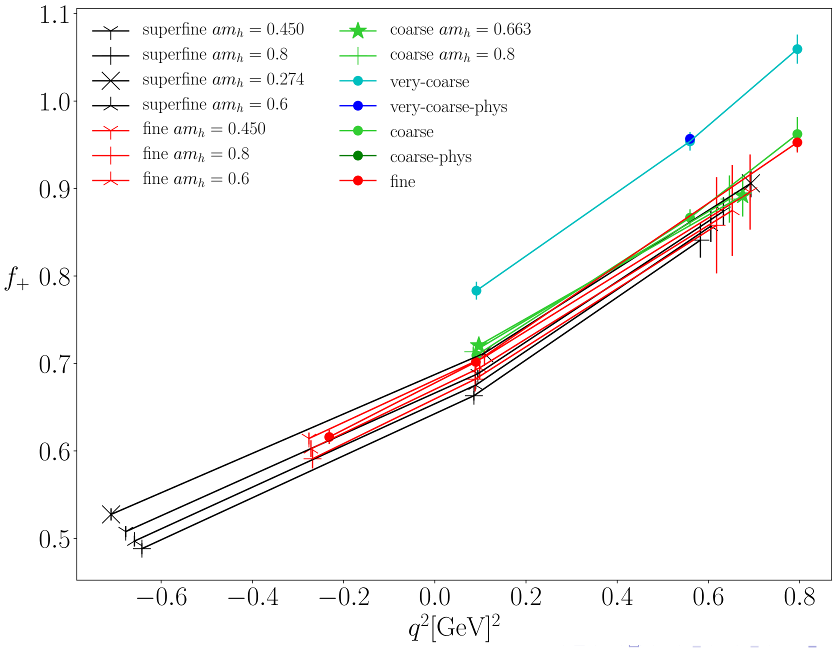
Table: Z_V for $c \to s$ from heavy-HISQ

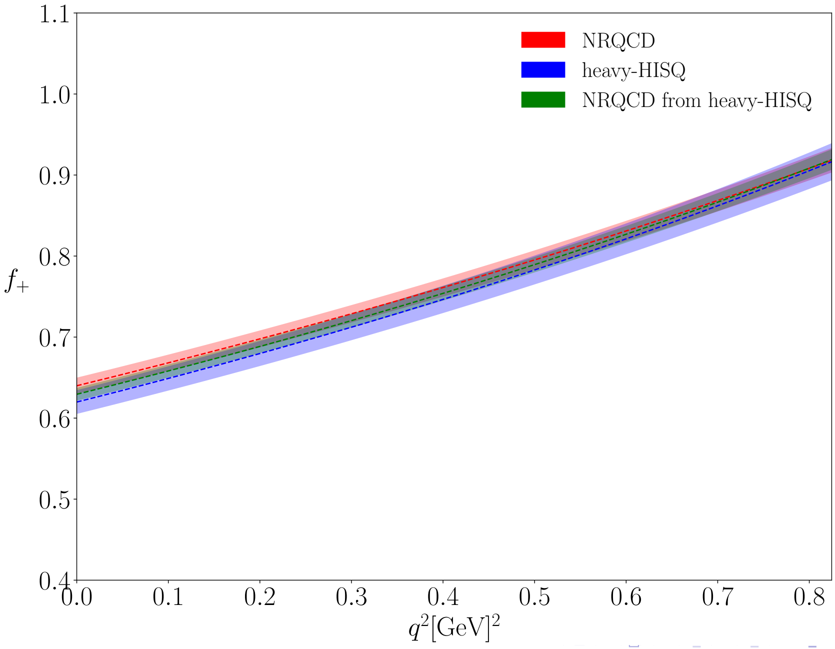


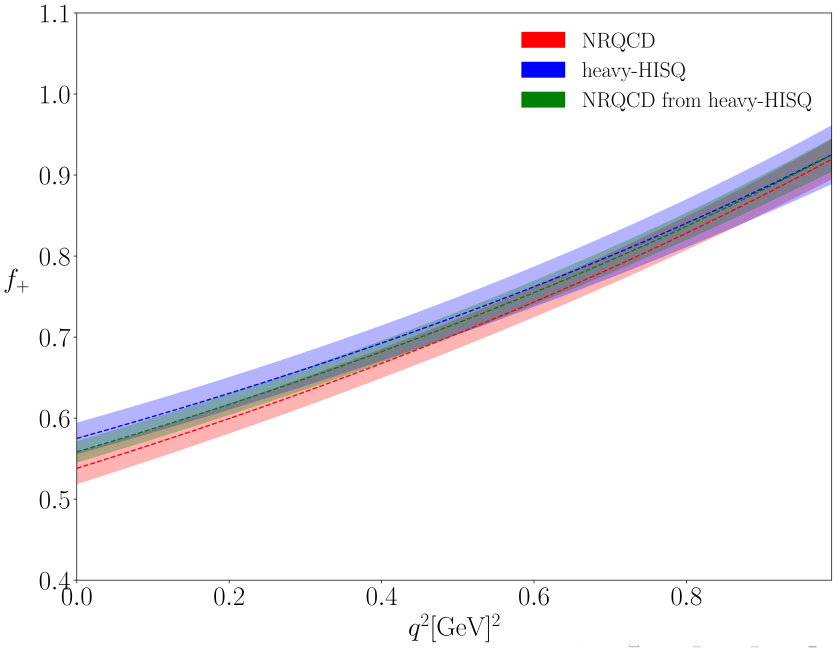
superfine
very-coarse
very-coarse-phys
coarse-phys
coarse
fine
Z_V
1.100
1.075
1.050
1.025
1.000
0.975
0.950
0.925
0.900
0
1
2
3
4
5
6
7
M_{D^0}
M_{η_c}
$M_{B_c^+}$
M_{H_c} [GeV]


very-coarse
very-coarse-physical
coarse
coarse-physical
fine
Z_V
1.05
1.04
1.03
1.02
1.01
1.00
0.99
−0.2
0.0
0.2
0.4
0.6
0.8
$q^2[\mathrm{GeV}]^2$

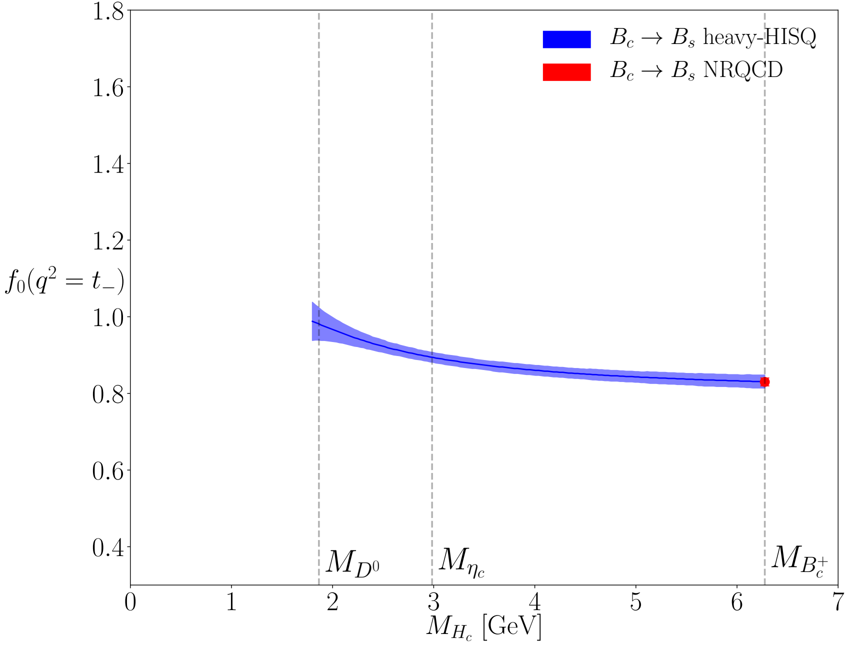

superfine $am_h = 0.450$
superfine $am_h = 0.8$
superfine $am_h = 0.274$
superfine $am_h = 0.6$
fine $am_h = 0.450$
fine $am_h = 0.8$
fine $am_h = 0.6$
coarse $am_h = 0.663$
coarse $am_h = 0.8$
very-coarse
very-coarse-phys
coarse
coarse-phys
fine
f_0
$q^2[\mathrm{GeV}]^2$
0.5
0.6
0.7
0.8
0.9
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


superfine $am_h = 0.450$
superfine $am_h = 0.8$
superfine $am_h = 0.274$
superfine $am_h = 0.6$
fine $am_h = 0.450$
fine $am_h = 0.8$
fine $am_h = 0.6$
coarse $am_h = 0.663$
coarse $am_h = 0.8$
very-coarse
very-coarse-phys
coarse
coarse-phys
fine
f_+
1.1
1.0
0.9
0.8
0.7
0.6
0.5
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
$q^2[\text{GeV}]^2$

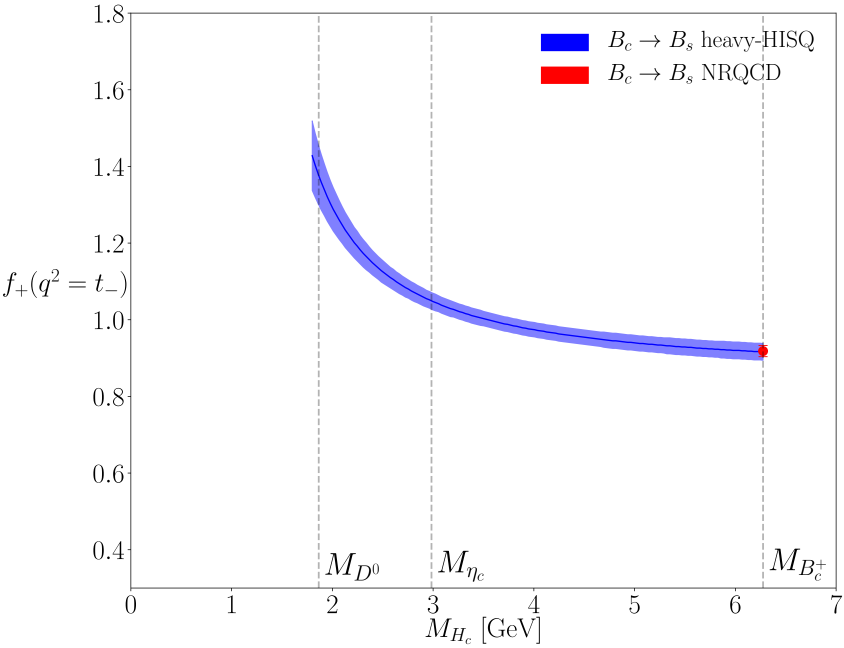

NRQCD
heavy-HISQ
NRQCD from heavy-HISQ
f_+
$q^2[\mathrm{GeV}]^2$
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8



1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
f_+
0.0
0.2
0.4
0.6
0.8
$q^2[\mathrm{GeV}]^2$
NRQCD
heavy-HISQ
NRQCD from heavy-HISQ



$B_c \to B_s$ heavy-HISQ
$B_c \to B_s$ NRQCD
$f_0(q^2 = t_-)$
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
M_{D^0}
M_{η_c}
$M_{B_c^+}$
0
1
2
3
4
5
6
7
M_{H_c} [GeV]



$B_c \to B_s$ heavy-HISQ
$B_c \to B_s$ NRQCD
$f_+(q^2 = t_-)$
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
M_{D^0}
M_{η_c}
$M_{B_c^+}$
0
1
2
3
4
5
6
7
M_{H_c} [GeV]

Comparing NRQCD and heavy-HISQ

NRQCD

- cheap propagators
- calculate at m_b
- extend to V^i
  - Z_V away from zero-recoil
  - f_+ near zero-recoil

heavy-HISQ

- B_c^+ not very NR: $v^2 \approx 0.5$
- smaller discretisation effects
- clean extraction of matrix elts
- probe spectator mass effects
- higher precision with ultrafine and more stats on superfine
- (non-pert renormalisation in all-HISQ heavy-light calculations enables high precision, e.g. 1904.02046)