Inclusive B decay structure functions and their decomposition into final states

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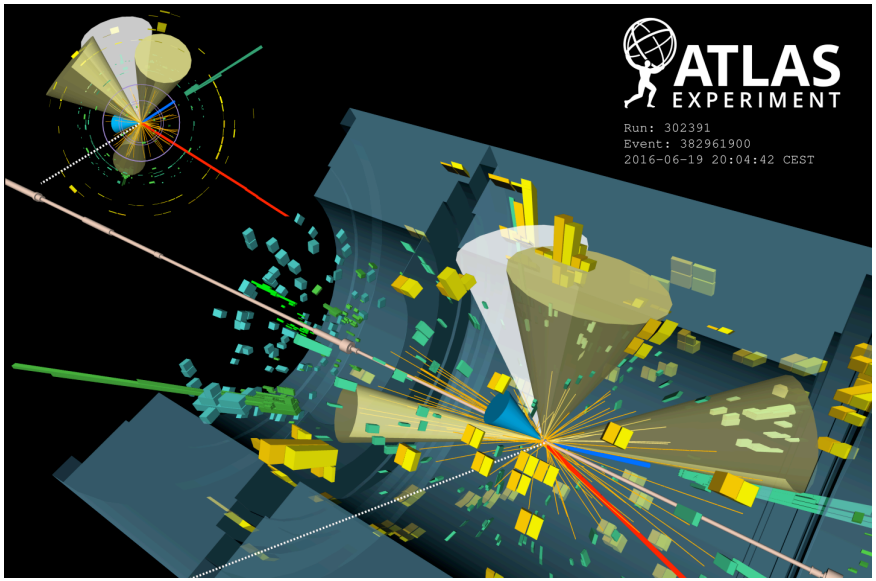
SOKENDAI

CERN = the biggest factory of QCD processes



LQCD is of little use to analyze them, because…

PQCD (plus some models/fits) are employed, instead.

Quark-hadron duality

= basic assumption behind the use of PQCD

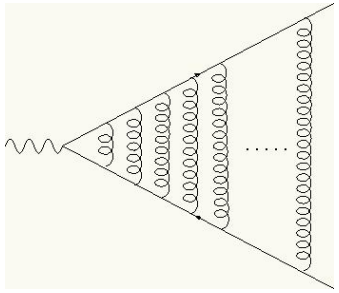
TH: calculations are done for quarks/gluons
EX: what one detects in experiments are hadrons

Duality = they coincide under certain circumstances.

- What are the conditions?
- How much precision can one achieve?
- How can we test the assumption?
- Any proof??

Duality is badly violated…

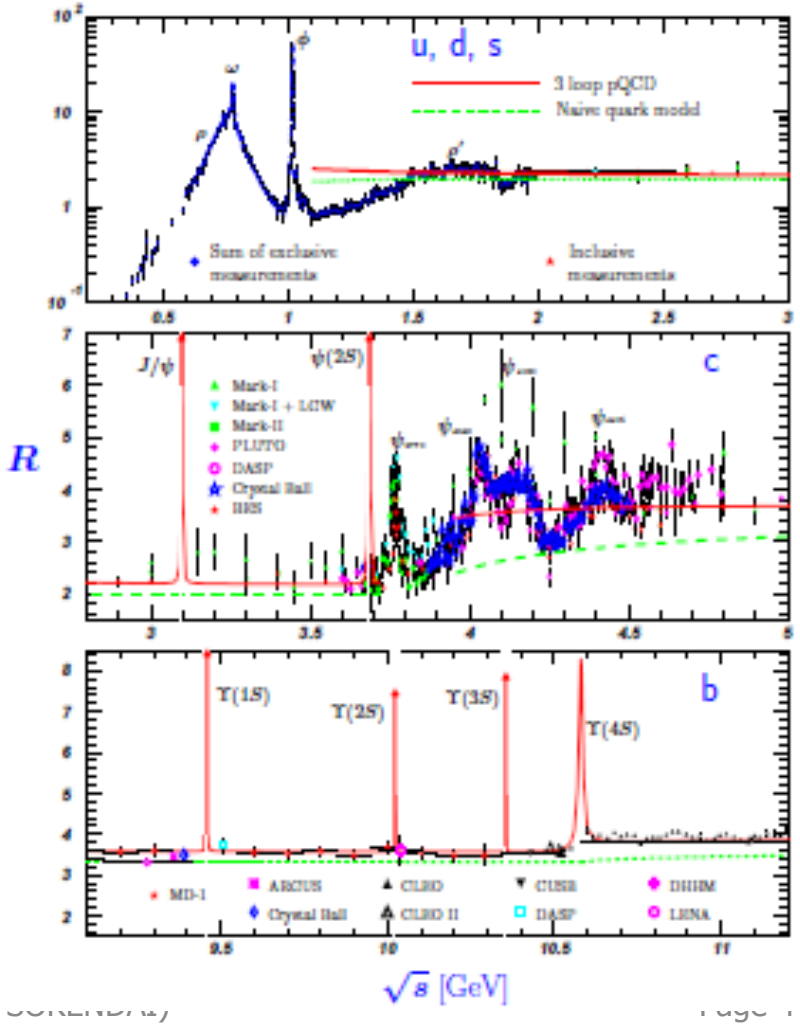
- A lot of resonances found in the e^+e^- collisions
  - Highly non-perturbative even for quarkonium:



$$\sim \left(\frac{\alpha_s}{v}\right)^n$$

(need to resum)

  - Truly non-perturbative for light sectors



Duality at work

Poggio, Quinn, Weinberg, PRD13, 1958 (1976)

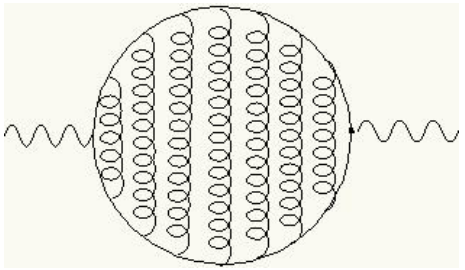
- Smearing
  - Consider a quantity smeared over some range.

$$\bar{R}(s,\Delta) \equiv \frac{\Delta}{\pi}\int_0^\infty ds' \frac{R(s')}{(s-s')^2+\Delta^2}$$

$$= \frac{1}{2\pi i}\int_0^\infty ds'\, R(s')\left(\frac{1}{s-s'+i\Delta} - \frac{1}{s-s'-i\Delta}\right)$$

$$= \frac{1}{2i}\left[\Pi(s+i\Delta) - \Pi(s-i\Delta)\right]$$

$$\mathrm{Im}\Pi(s) \propto R(s) = \frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



  - One can avoid the threshold singularity.
  - Δ must be larger than Λ_{QCD} to avoid non-perturbative physics, but how much?

Still qualitative…

Inclusive = smearing

TH: calculations are done for quarks/gluons
EX: what one detects in experiments are hadrons

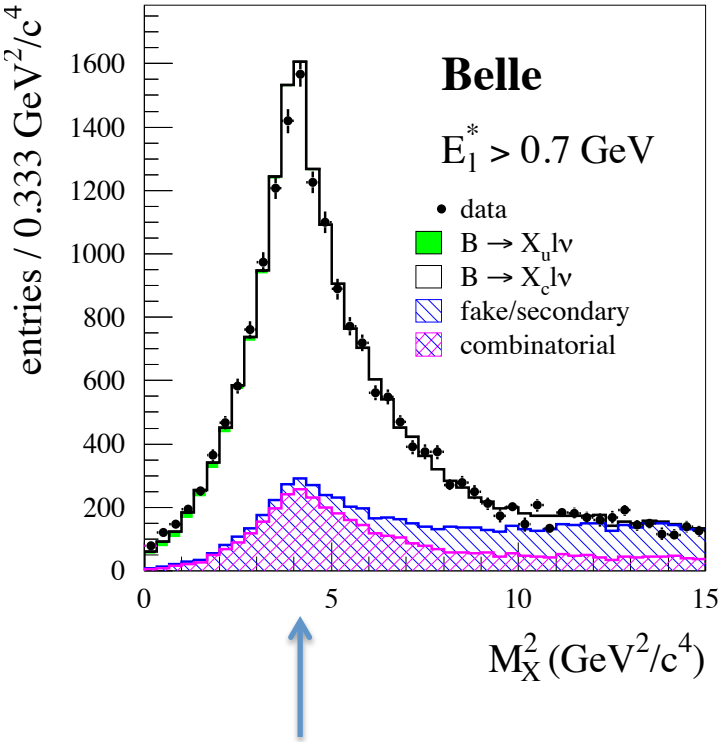
Duality = they coincide under certain circumstances.

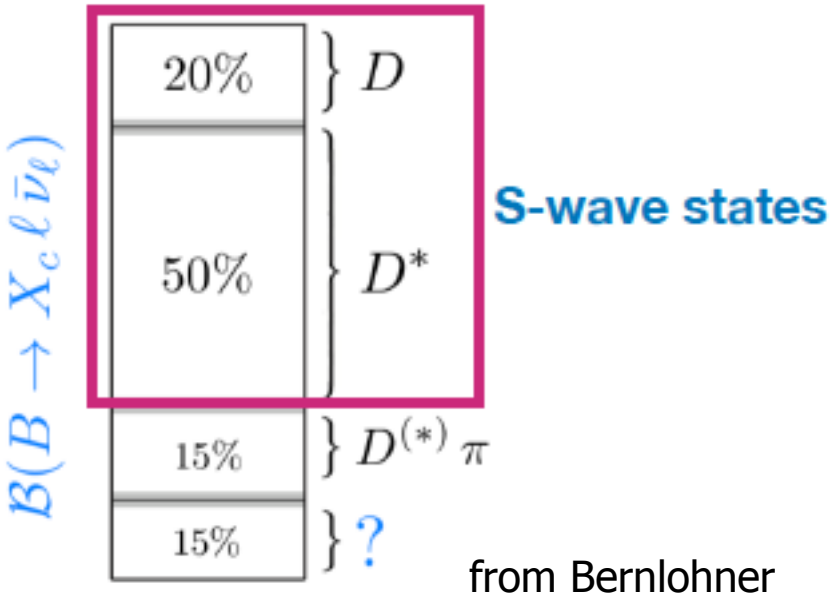
if sufficiently smeared

- How much smearing needed?
- Is the existing analysis okay?

Semileptonic B decay ($B \to X_c / \nu$):

M_X spectrum





from Bernlohner @ CKM2014

Dominated by D and D* by about 70%.

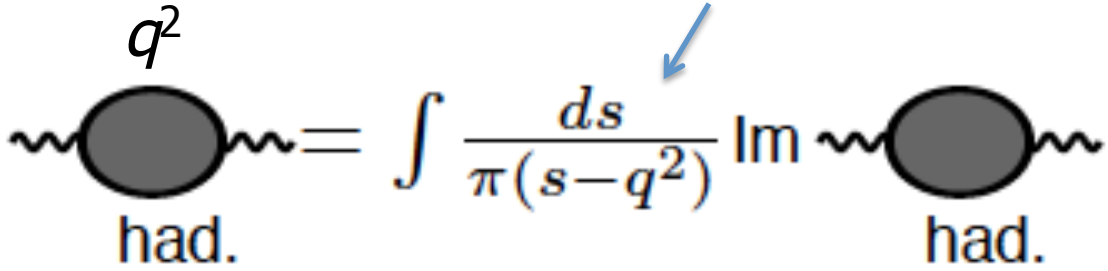
Duality? What if 90% ??

Theoretically-clean analysis

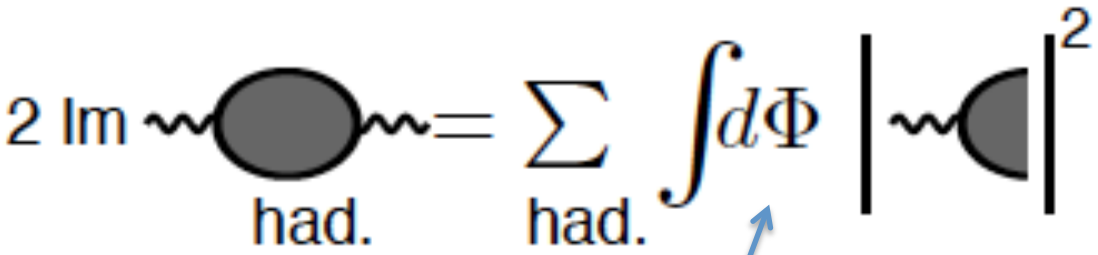
= smear according to Cauchy…

Weighted integral over momentum transfer

Dispersion relation:



Optical theorem:



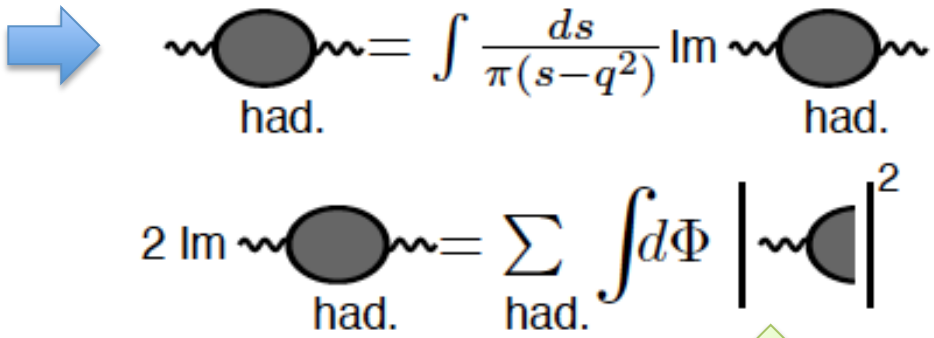
all possible final states

“QCD sum rule”

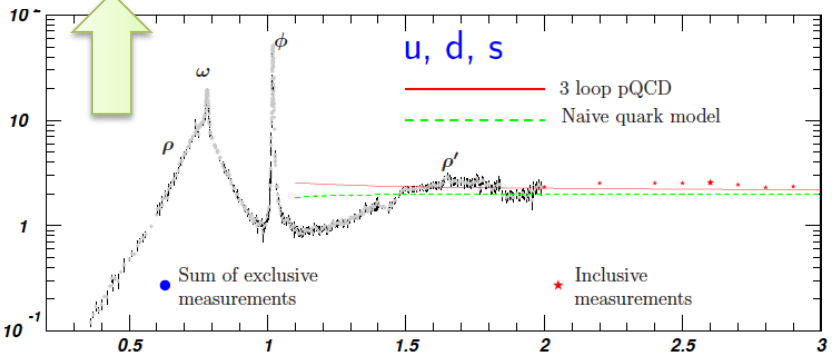
Shifman, Vainshtein, Zakharov, NPB147 385, 448 (1979)

$\Pi(Q^2)$: calculable by pQCD and OPE (+ Borel sum, etc)

space-like region: $Q^2 = -q^2 > 0$



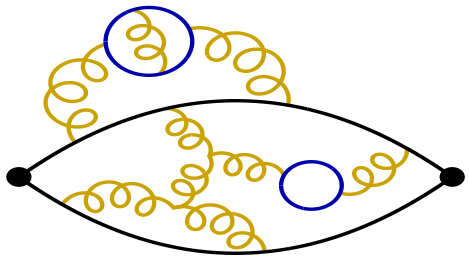
time-like region: $s=q^2 > 4m_\pi^2$



$\Pi(Q^2)$: Why not lattice?

Surely!

$$\Pi_{\mu\nu}(x) = \langle 0|T\{J_\mu(x)J_\nu(0)\}|0\rangle$$

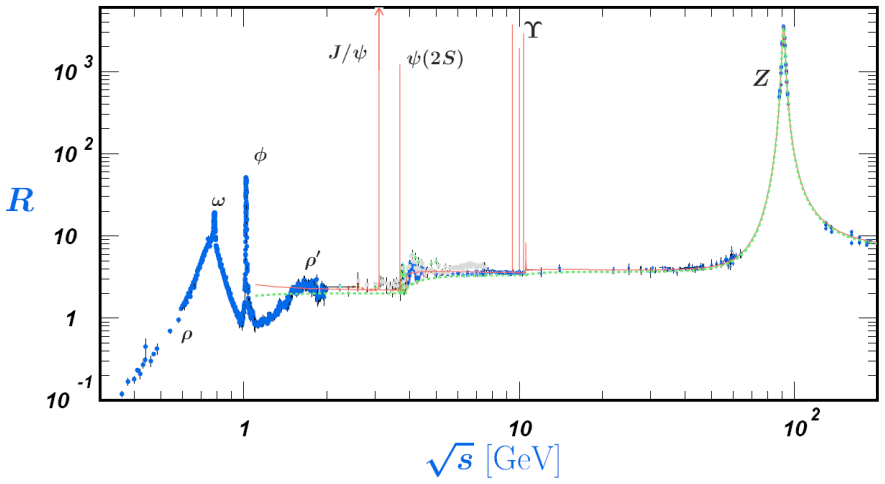


x and 0 are separated space-like

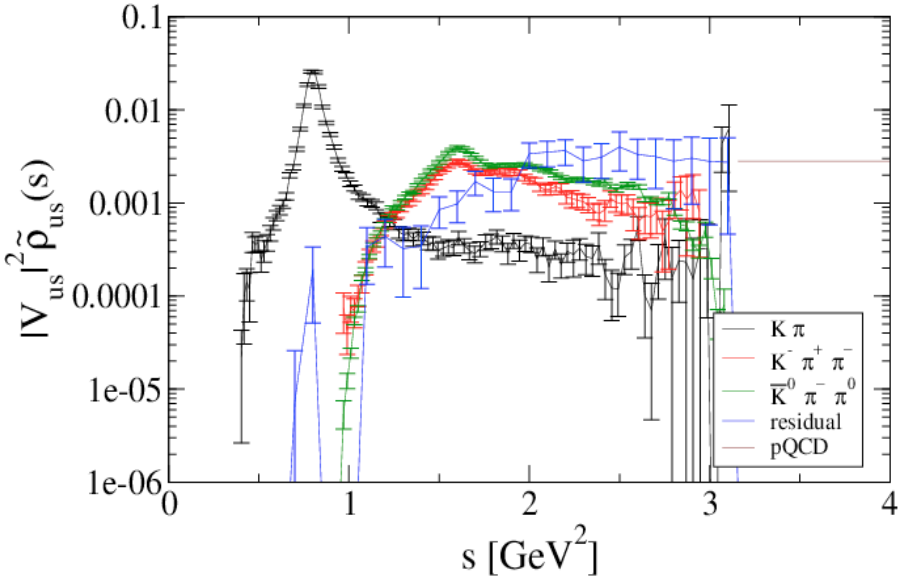
- Commonly used as an input for hadronic vacuum polarization (HVP) contribution to muon g-2.
- And, more…
  (see, e.g. the analysis of hadronic tau decay by RBC/UKQCD)

Inclusive processes of interest:

$e^+e^- \to q\bar{q}$



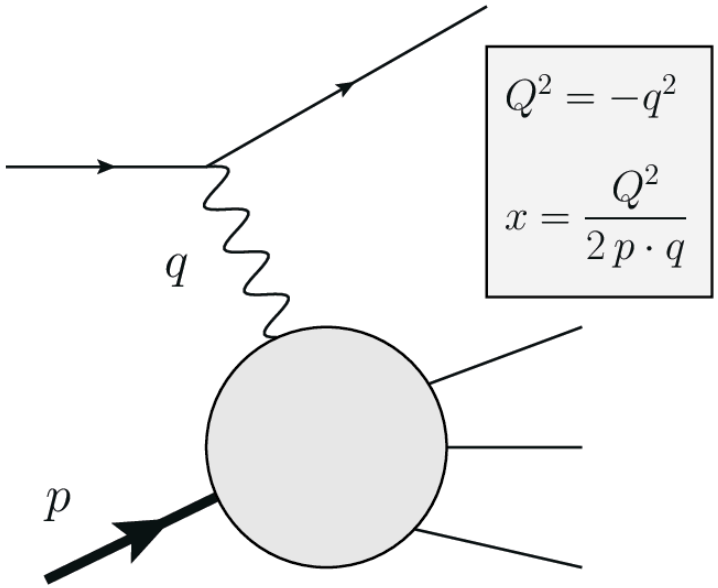
$\tau \to \nu_\tau q\bar{q}$



Boyle et al. [RBC/UKQCD], PRL121 202003 (2018).

(Deep) (In)elastic scattering

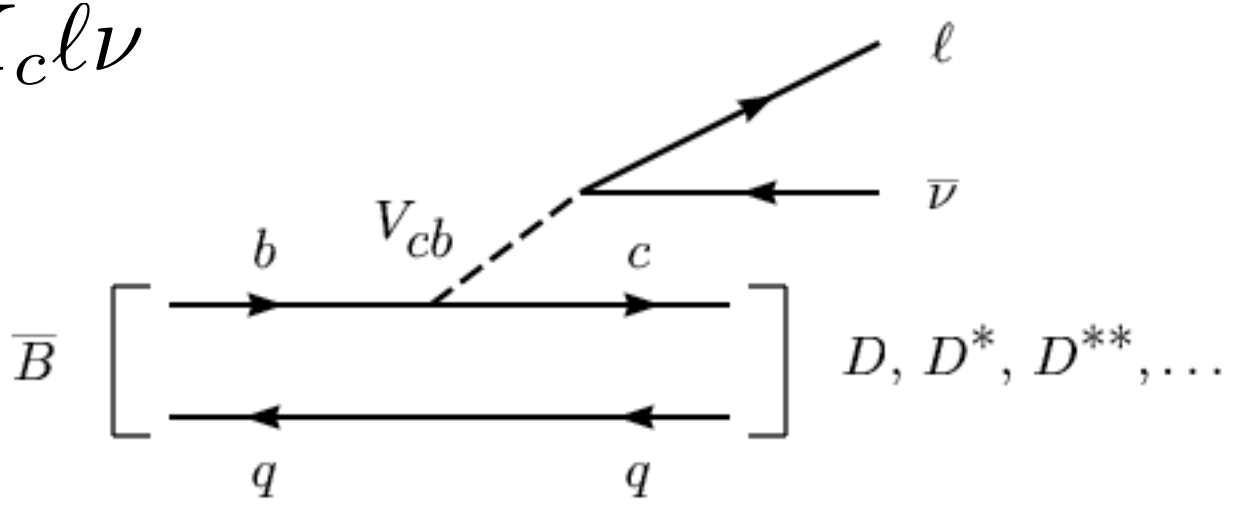
$$e^- N \to e^- X$$



- Bjorken scaling → parton model → PDF
- Resonance region is often ignored (too difficult).

Inclusive B meson decays

$$B \to X_c \ell \nu$$



- Perturbation theory is used with heavy quark expansion.
- Resonances make a problem, see below.

LQCD for inclusive processes

- Standard approach = OPE, then matrix element

$$\frac{m\langle\bar{q}q\rangle}{Q^4} \qquad \langle x \rangle \qquad \frac{\langle B|D_\perp^2|B\rangle}{m_B^2}$$

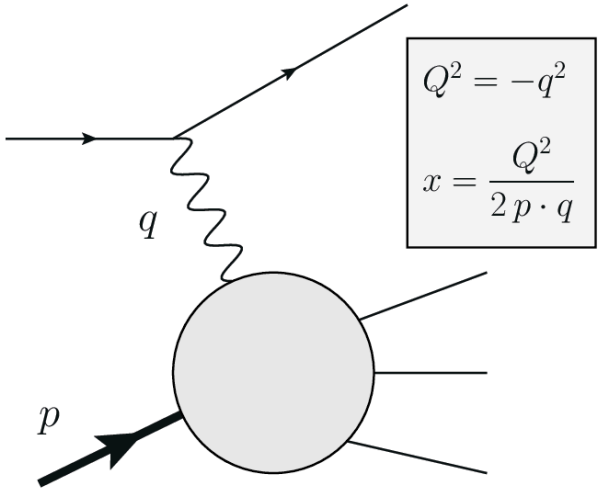
- Space-like quantity

DIS as an example:

see also, QCDSF, PRL 118, 242001 (2017)

(Deep) (In)elastic scattering

see also, QCDSF, PRL 118, 242001 (2017)
χQCD, 1906.05312



structure function

$$W_{\mu\nu} = \frac{1}{2}\sum_{\rm pol}\sum_{X}\langle N(p)|J_\mu|X(p_X)\rangle\langle X(p_X)|J_\nu|N(p)\rangle$$
$$\times\,(2\pi)^3\delta^{(4)}(p - p_X + q)$$

optical theorem: $W_{\mu\nu} = \frac{1}{\pi}{\rm Im}T_{\mu\nu}$

forward-scattering matrix element

$$T_{\mu\nu} = \frac{1}{2}\sum_{\rm pol}\int d^4x\, e^{iqx}\langle N(p)|T\{J_\mu(x)J_\nu(0)\}|N(p)\rangle$$

Can we calculate this on the lattice?

$$T_{\mu\nu} = \frac{1}{2}\sum_{\rm pol}\int d^4x\, e^{iqx}\langle N(p)|T\{J_\mu(x)J_\nu(0)\}|N(p)\rangle$$

- with a fixed 4-momentum *q*

On the Euclidean lattice, one may start from

$$M_{\mu\nu}(t) \equiv \int d^3\mathbf{x}\, e^{-i\mathbf{q}\cdot\mathbf{x}}\langle N(\mathbf{p})|J_\mu(\mathbf{x},t)J_\nu(\mathbf{0},0)|N(\mathbf{p})\rangle$$

to obtain

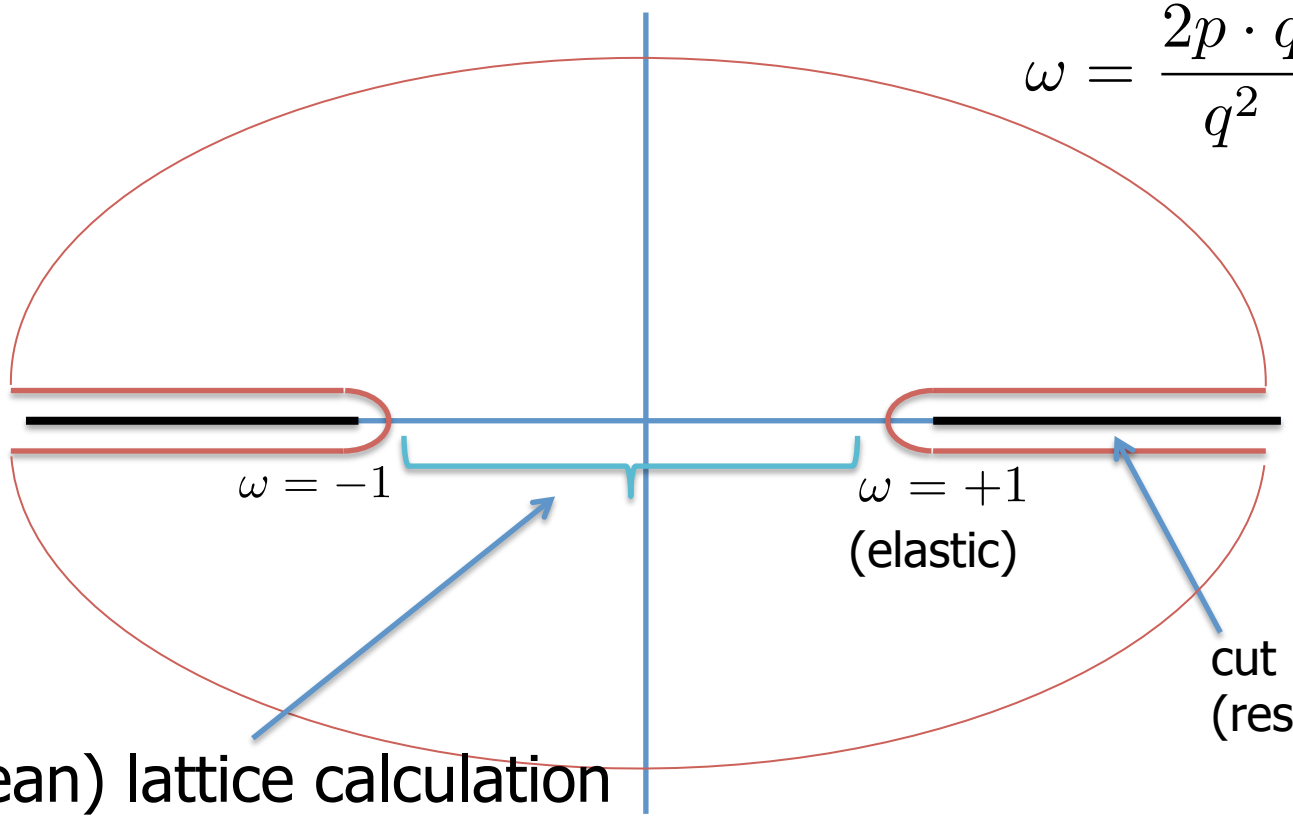
$$T_{\mu\nu}(p,q) = \int_0^\infty dt\, e^{q^0 t} M_{\mu\nu}(t)$$

Analytic continuation is allowed until one encounters any singularity.

$$T_{\mu\nu}(p,q)$$

$$\omega = \frac{2p\cdot q}{q^2} = \frac{2Mq^0}{-Q^2}$$

fixed Q^2



(Euclidean) lattice calculation only in the region $|\omega|<1$. (no singularity)

Contour integral (schematically…)

$$T(\omega) = \int_{-\infty}^{-1} d\omega' \frac{W(\omega')}{\omega' - \omega} + \int_{1}^{\infty} d\omega' \frac{W(\omega')}{\omega' - \omega}$$

$$= 2\omega \int_{1}^{\infty} d\omega' \frac{W(\omega')}{\omega'^2 - \omega^2}$$

$\omega'=1/x$

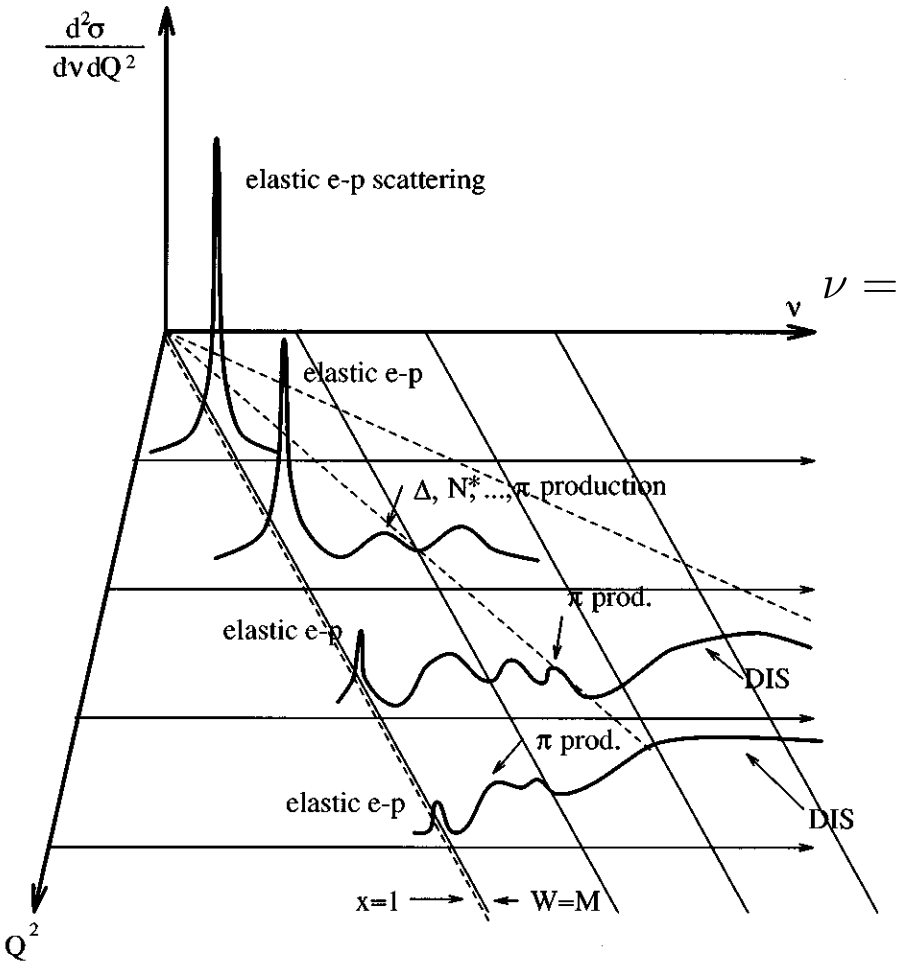
$$= 2\omega \int_{0}^{1} dx \frac{W(x)}{1 - (\omega x)^2}$$

$$= 2 \sum_{n=0}^{\infty} \omega^{2n+1} \int_{0}^{1} dx\, x^{2n} W(x)$$

moments of PDF

Calculation of T(w) at w=0 (and its derivatives) gives arbitrary moments of structure functions.

Badelek, Kwiecinski, RMP 68, 445 (1996)



$$\nu = \frac{p \cdot q}{M} = \frac{Q^2}{2M}\omega$$

Advantages:

- Theoretically clean (only analyticity is used).
- Can obtain the maximum set of info (arbitrary moments) at once…, in principle.
- Can apply to low Q^2 structure functions, for which PDF is no longer a valid description.

- Also related to the Cottingham sum rule, for (M_p-M_n).
- Also related to the two-photon exchange contrib to the Lamb shift.

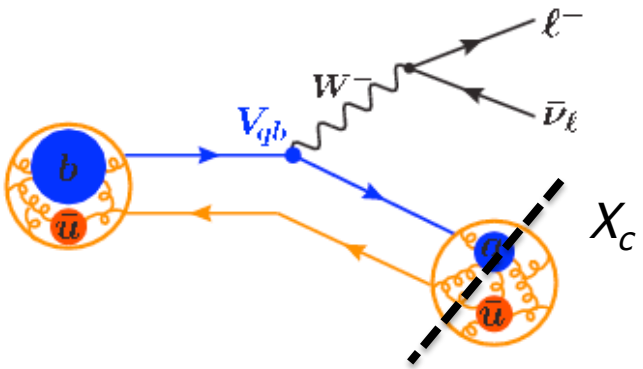
Disadvantages:

- Four-point function is computationally challenging (many contractions), but, see χQCD, arXiv:1906.05312
- S/N would be bad due to nucleon prop on both ends.

Inclusive semileptonic B decays

B meson semileptonic decays

- Determination of $|V_{xb}|$
  - $|V_{cb}|$, in particular
  - BR ~ 10% (e) + 10% (μ)



  - exclusive = decays to individual final states (D, D*, …)
  - inclusive = sum over all possible final states including charm

Formulation is very similar to DIS.

Partial decay rate:

$$d\Gamma \sim |V_{cb}|^2 l^{\mu\nu} W_{\mu\nu}$$

Structure function:

$$W_{\mu\nu} = \sum_X (2\pi)^3 \delta^4(p_B - q - p_X) \frac{1}{2M_B} \langle B(p_B)|J_\mu^\dagger(0)|X\rangle\langle X|J_\nu(0)|B(p_B)\rangle$$

sum over all final states

Optical theorem: $-\frac{1}{\pi}\mathrm{Im}T_i = W_i$

analytic function of q^2 and $v.q$

Forward scattering matrix element:

$$T_{\mu\nu} = i\int d^4x e^{-iqx} \frac{1}{2M_B} \langle B|T\{J_\mu^\dagger(x)J_\nu(0)\}|B\rangle$$

$$T_{\mu\nu} = i\int d^4x e^{-iqx}\frac{1}{2M_B}\langle B|T\{J_\mu^\dagger(x)J_\nu(0)\}|B\rangle$$

analytic function of q^2 and $v.q$

$$\int_{-\infty}^{(v\cdot q)_{\max}} \frac{d(v\cdot q')}{\pi}\frac{\mathrm{Im}T(v\cdot q')}{v\cdot q' - v\cdot q} = T(v\cdot q)$$

$v\cdot q$ (with fixed q^2)

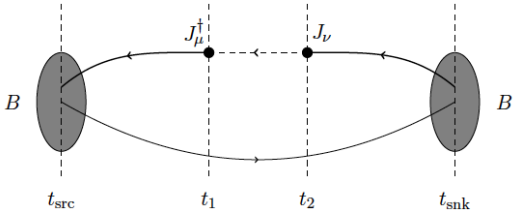
$= m_B - \omega$

physical cut $b \to c$

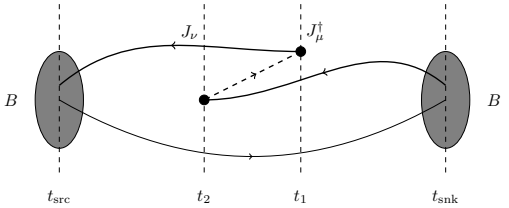
unphysical cut $b \to bb\bar{c}$

$\frac{1}{2M_B}(M_B^2 + q^2 - m_X^2)$

$\frac{1}{2M_B}((2M_B + M_X)^2 - q^2 - M_B^2)$



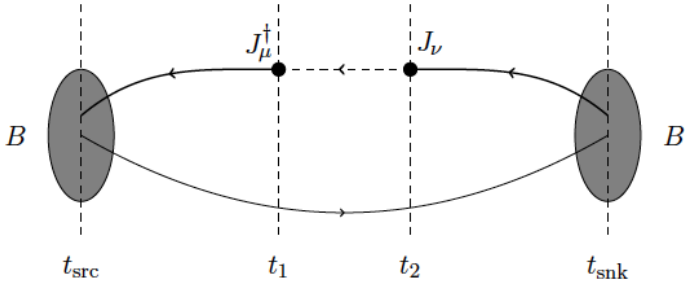
"unphysical kinematics": calculable on the lattice



SH, PTEP 2017, 053B03.

Forward-scattering ME on the lattice

1. Calculate the four-point function:



2. Extract the matrix element by taking a ratio to two-points

$$C^{JJ}_{\mu\nu}(t;\mathbf{q}) = \int d^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} \frac{1}{2M_B} \langle B(\mathbf{0}) | J^\dagger_\mu(\mathbf{x},t)\, J_\nu(0) | B(\mathbf{0}) \rangle$$

3. "Fourier (or Laplace) transform" with arbitrary ω

$$T^{JJ}_{\mu\nu}(\omega,\mathbf{q}) = \int_0^\infty dt\, e^{\omega t} C^{JJ}_{\mu\nu}(t;\mathbf{q})$$

→ $T_{\mu\nu}(v\cdot q, q^2)$ at $p_X=(\omega,-\boldsymbol{q})$, $q=(m_B-\omega,\boldsymbol{q})$

JLQCD lattice ensemble

- Mobius domain-wall fermion (2012~)
  - 2+1-flavor (uds)
  - chiral symmetry
    - residual mass < O(1 MeV)
  - lattice spacing : 1/a = 2.4, 3.6, 4.5 GeV
  - volume : L = 2.7 fm (32^3, 48^3, 64^3 lattices)
  - ud quark masses : m_π = 230, 300, 400, 500 MeV
  - statistics : 50-400 measurements
- Valence quarks
  - charm/bottom (MDW) + strange (MDW)
  - bottom is lighter than physical, $m_b = (1.25)^4\, m_c$
  - on Oakforest-PACS with

(3.4 GeV B_s meson)

β = 4.17, 1/a ~ 2.4 GeV, 32^3x64 (x12)

m_{ud}	m_π [MeV]	MD time
m_s = 0.030		
0.007	310	10,000
0.012	410	10,000
0.019	510	10,000
m_s = 0.040		
0.0035	230	10,000
0.0035 (48^3x96)	230	10,000
0.007	320	10,000
0.012	410	10,000
0.019	510	10,000

β = 4.35, 1/a ~ 3.6 GeV, 48^3x96 (x8)

m_{ud}	m_π [MeV]	MD time
m_s = 0.018		
0.0042	300	10,000
0.0080	410	10,000
0.0120	500	10,000
m_s = 0.025		
0.0042	300	10,000
0.080	410	10,000
0.0120	510	10,000

β = 4.47, 1/a ~ 4.6 GeV, 64^3x128 (x8)

0.0030	~ 300	10,000

Related works presented at Lattice 2019

- T. Kaneko (KEK), "$B \to D^{(*)} \ell \nu$ form factors from lattice QCD with relativistic heavy quarks"

- J. Koponen (KEK), "$B \to \pi \ell \nu$ form factors and $|V_{ub}|$ with Möbius domain wall fermions"

  (see also, B. Colquhoun, "$B \to \pi \ell \nu$ form factors with fully relativistic lattice QCD" @ KEK-FF 2019)

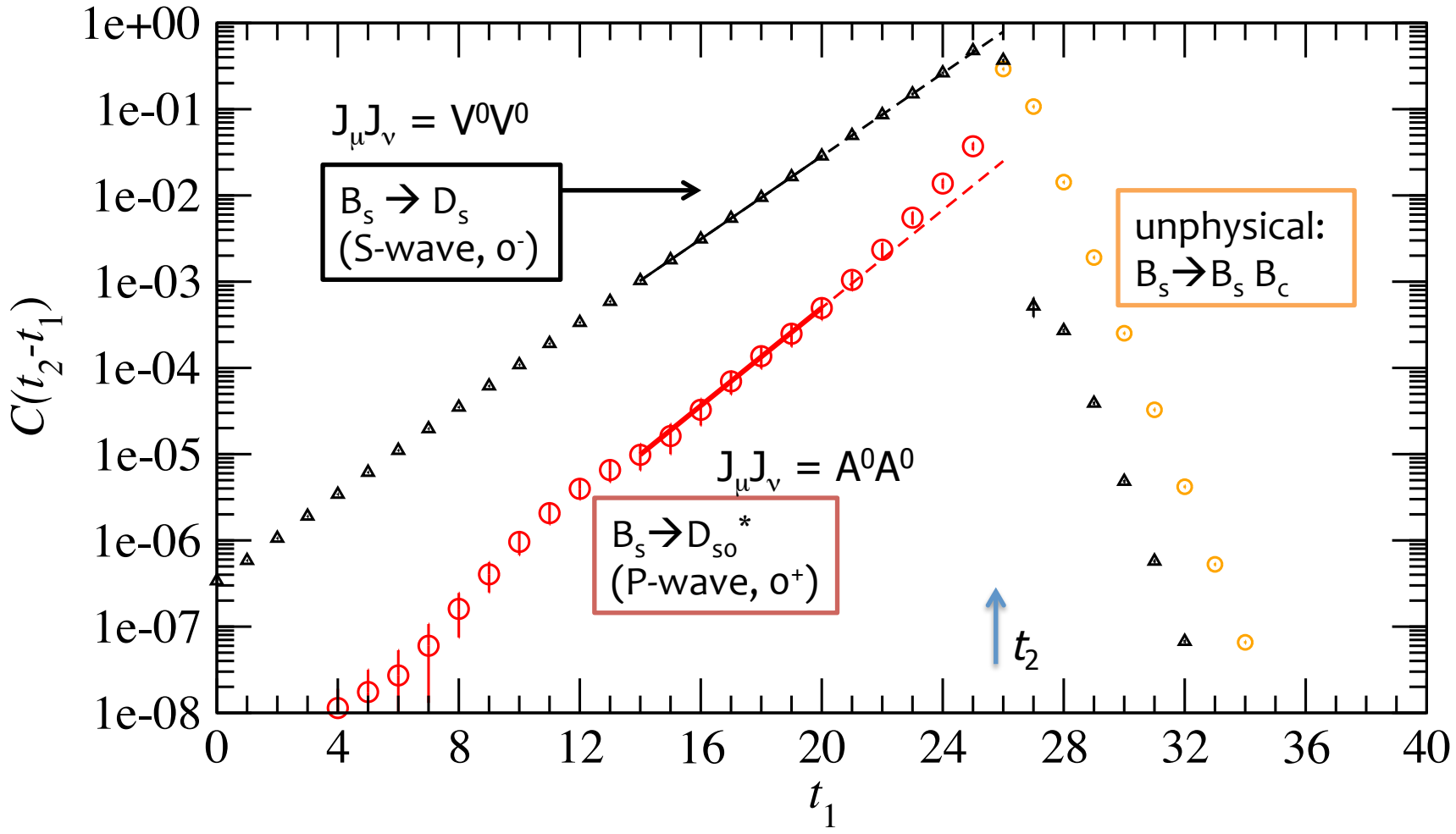
- T. Ishikawa (KEK/SOKENDAI), "Renormalization of bilinear and four-fermion operators through temporal moments"
- K. Nakayama (Osaka), "Charmonium contribution to $B \to K l^+ l^-$: testing the factorization approximation on the lattice"

- G. Bailas (KEK), "Study of intermediate states in the inclusive semi-leptonic $B \to X_c \ell \nu$ decay structure functions"

$$C^{JJ}_{\mu\nu}(t;\mathbf{q}) = \int d^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} \frac{1}{2M_B} \langle B(0)|J^\dagger_\mu(\mathbf{x},t)\,J_\nu(0)|B(0)\rangle$$

(t_1, t_2)

m_b=1.25^4m_c , zero recoil (**q**=**0**) 1/a = 3.6 GeV



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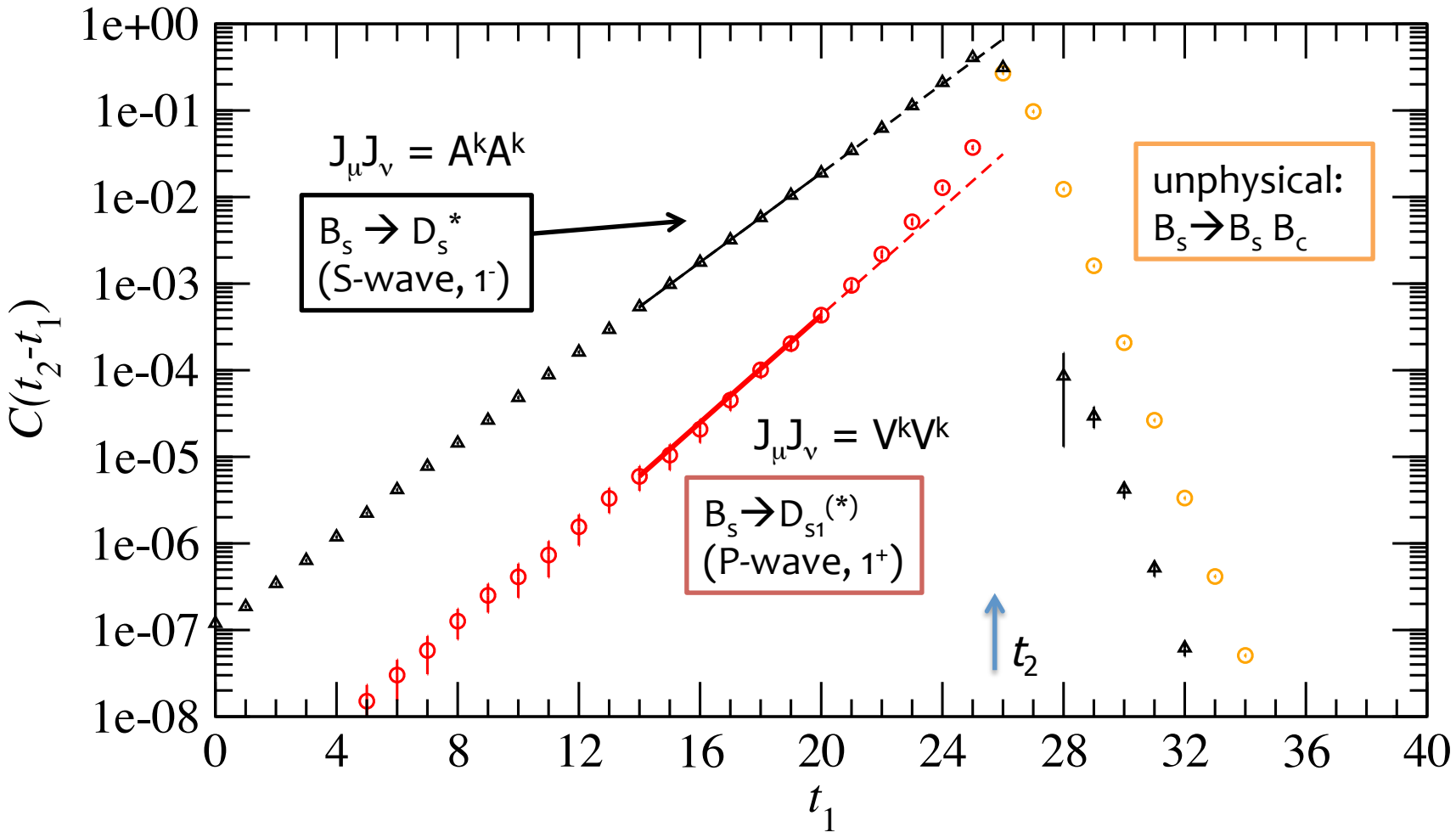
t_1 t_2

m_b=1.25^4m_c , zero recoil (**q**=**0**) 1/a = 3.6 GeV



“Fourier (or Laplace) transform”

- Time direction can be analytically continued to the time-like region with energy ω:

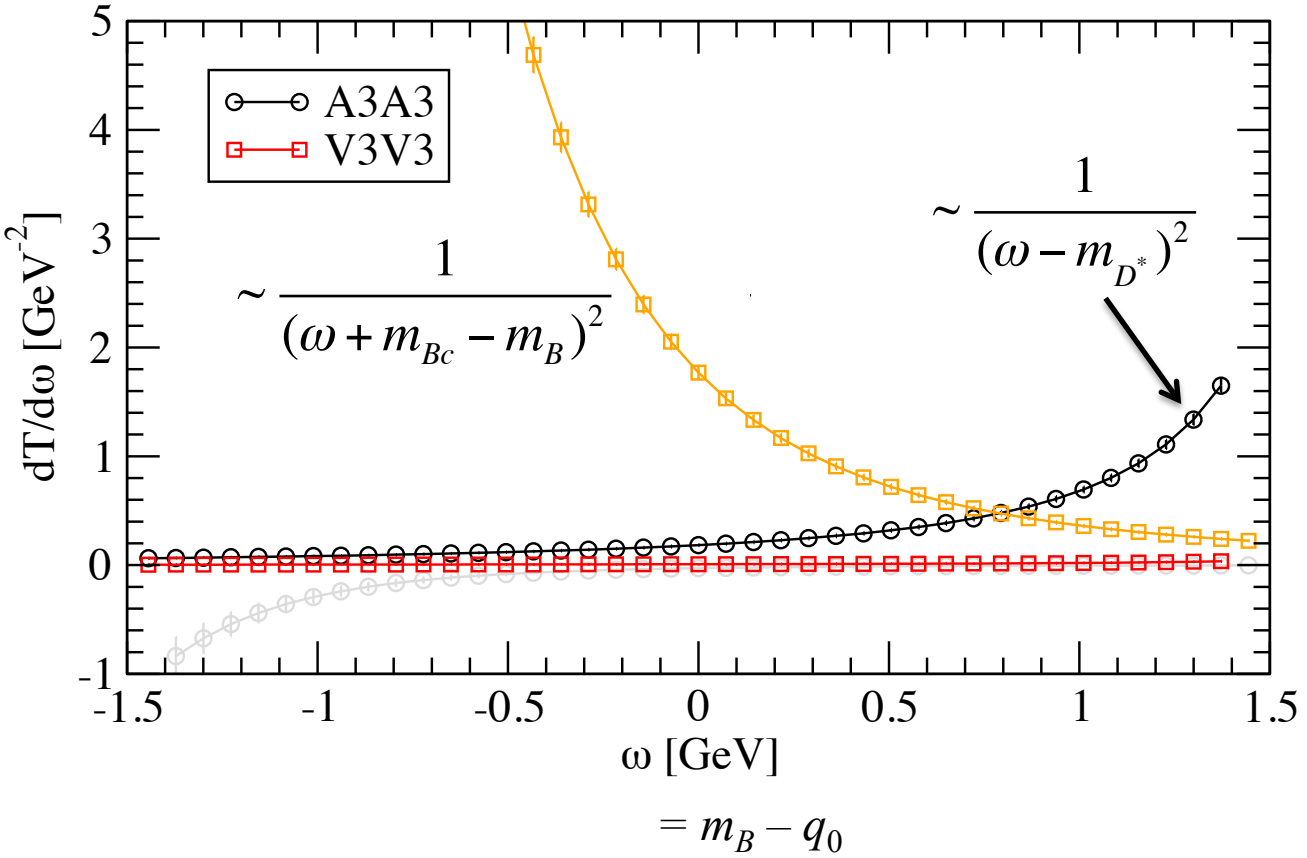
$$e^{ip_0 t} \to e^{\omega t} \quad : \quad T^{JJ}_{\mu\nu}(\omega, \mathbf{q}) = \int_0^\infty dt\, e^{\omega t} C^{JJ}_{\mu\nu}(t; \mathbf{q})$$

  - Can be understood by a Taylor expansion in p_0, and then reconstruct with ip_0=ω.
  - Only below any singularity: pole, cut, …
  - Obviously, $e^{-mt} \xrightarrow{F.T.} \dfrac{1}{\omega - m}$: sum of the poles

$$T^{JJ}_{\mu\nu}(\omega, \mathbf{q}) = \int_0^\infty dt\, e^{\omega t} C^{JJ}_{\mu\nu}(t; \mathbf{q})$$

m_b=1.25[4]m_c , zero recoil (**q**=**0**) $1/a$ = 3.6 GeV

derivative to avoid divergence (contact term)

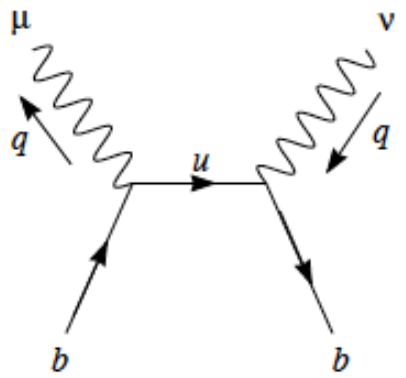


$= m_B - q_0$

Understood by perturbation theory?

(Based on discussions with P. Gambino)

- Heavy Quark Expansion (tree-level formulae):
  Blok, Koyrakh, Shifman, Vainshtein, PRD49, 3356 (1994).
  Manohar, Wise, PRD49, 1310 (1993).
  Falk, Ligeti, Neubert, Nir, PLB326, 145 (1994)
  Balk, Korner, Pirjol, Schilcher, ZP C64, 37 (1994).

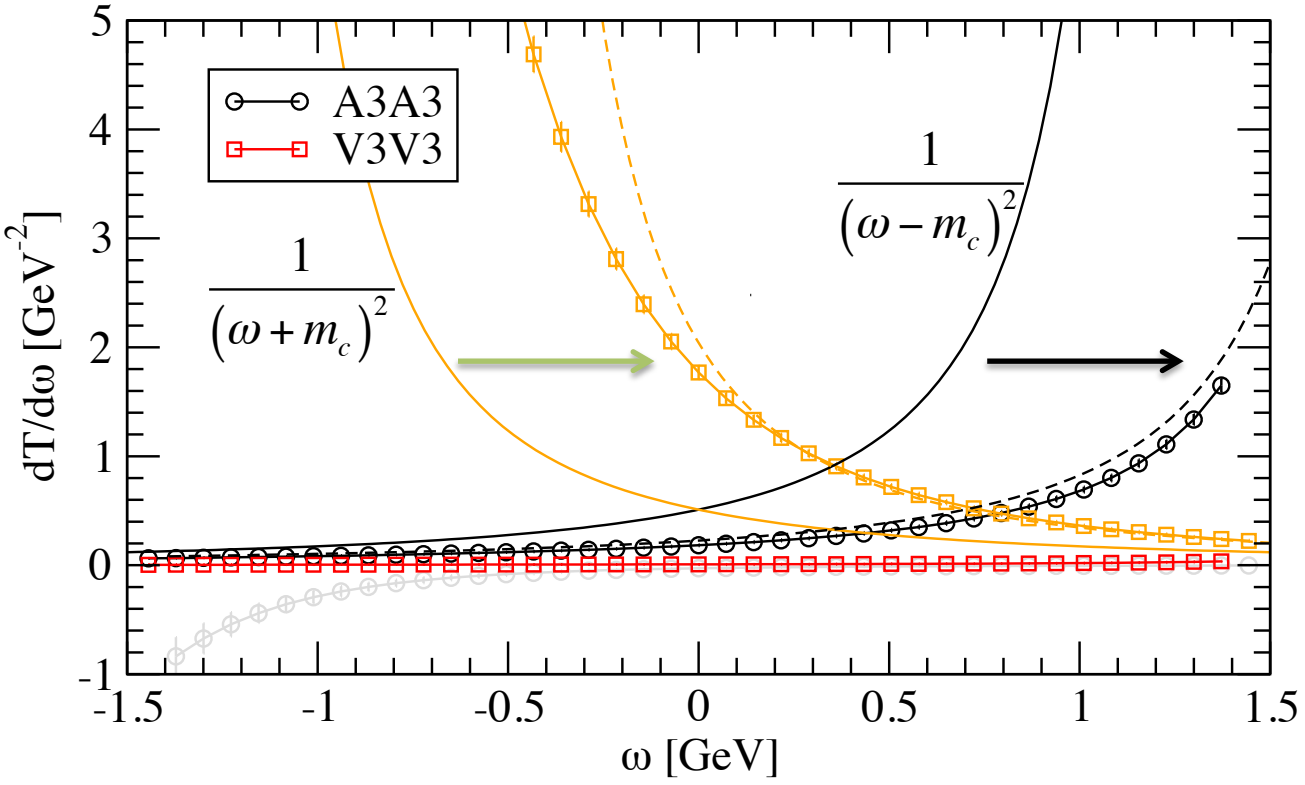


- Expand $\dfrac{1}{m_b \not{v} - \not{q} + \not{k} - m_c}$ in small *k*.

- Zero-recoil limit ($V_k V_K$ or $A_k A_k$ channel, leading order)

$$T_1^{VV} = -\frac{\omega - m_c}{\omega^2 - m_c^2}, \quad T_1^{AA} = -\frac{\omega + m_c}{\omega^2 - m_c^2} \qquad (\omega = m_B - q_0)$$

… pole at $\omega = -m_c$ ($V_k V_k$) or $\omega = m_c$ ($A_k A_k$)



Non-perturbative effect: shift due to $m_c \rightarrow m_D$
or to $m_c \rightarrow m_{Bc}$-m_B

One-loop calculation in progress (Gambino, SH)

Comparison with Experiment?

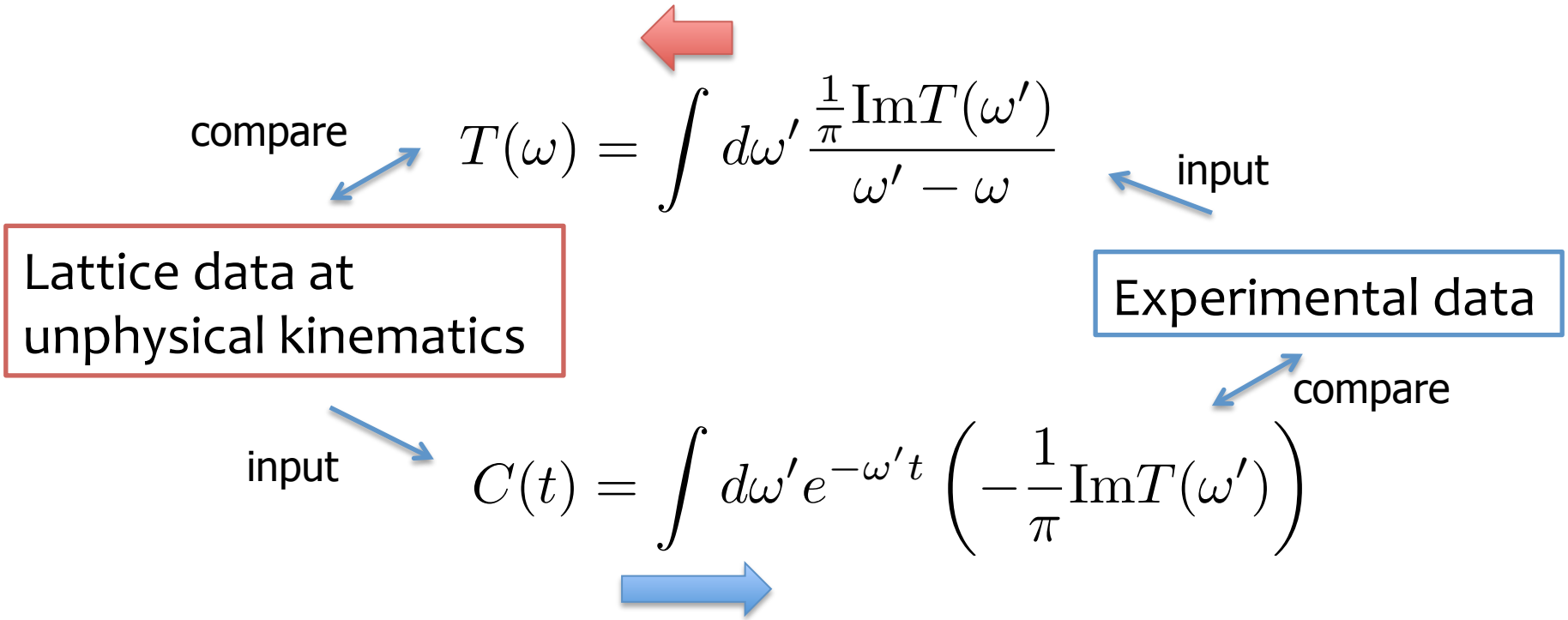
- Need the Cauchy's integral, which corresponds to the moments

$$\left\langle \frac{1}{(m_B - E_X)^n} \right\rangle$$

- Previously, only $\langle E_X^n \rangle$ are analyzed.

- Need to supplement the region $q^0 = m_B - E_X < 0$ using perturbation theory. (So, not entirely non-perturbative, unfortunately.)

Inverse problem, or not?



inverse problem: Backus-Gilbert method, etc
Hansen, Meyer, Robaina, PRD96, 094513 (2017)

see also, Tantalo's talk

Inverse problem, or not?

- What do you really want to achieve?
  - Full reconstruction of Im T ? (→ *ill-posed problem*)
  - Determination of PDF? (→ *Not an observable anyway*)

- To me, more important to do
  - controlled comparison with experimental data
  - together with perturbative QCD

  → Why not the structure function, even at unphysical kinematics!

Decomposition into final states

What are we actually observing?

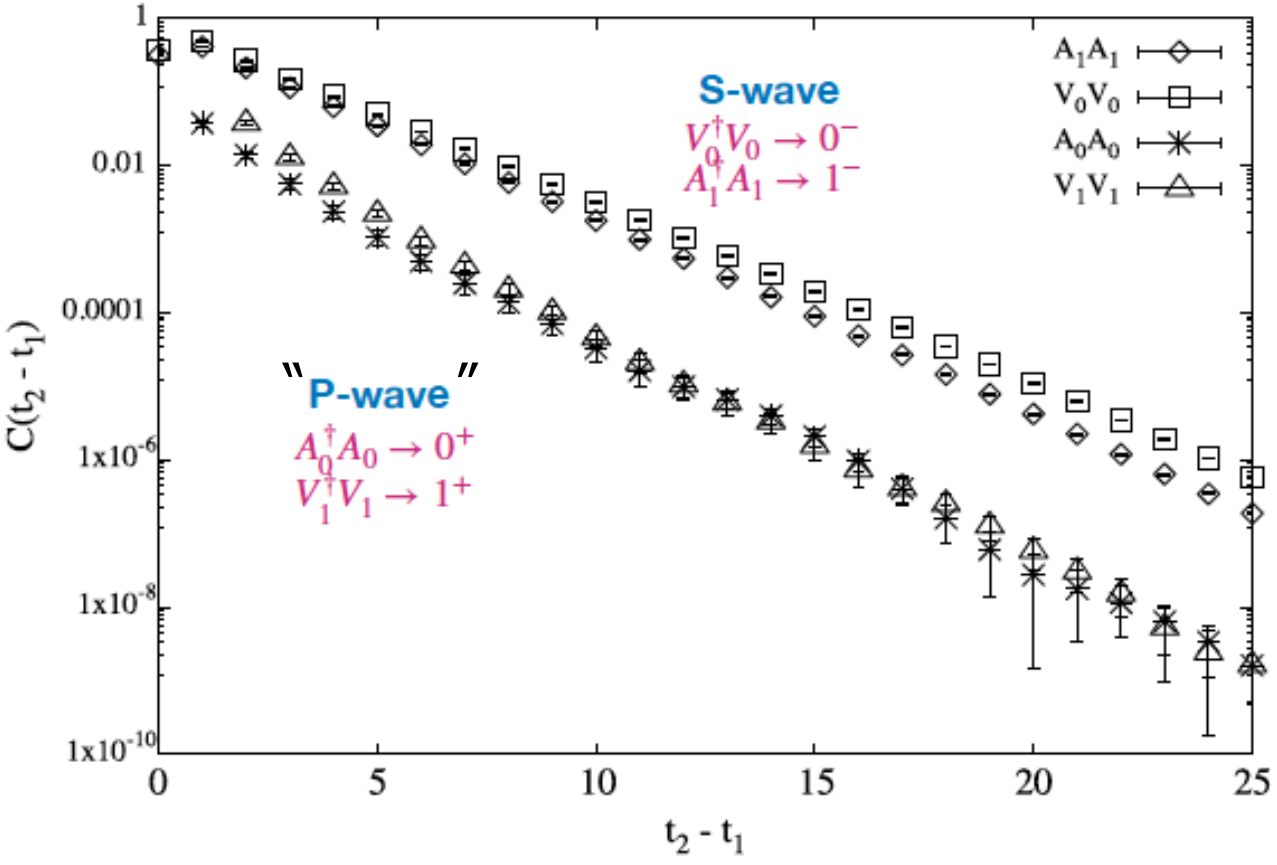
$$C^{JJ}_{\mu\nu}(t;\mathbf{q}) = \int d^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} \frac{1}{2M_B} \langle B(\mathbf{0})|J^\dagger_\mu(\mathbf{x},t)J_\nu(0)|B(\mathbf{0})\rangle$$

$$C^{JJ}_{\mu\nu}(t;\mathbf{0}) = \frac{1}{2M_B}\sum_X \langle B(\mathbf{0})|J^\dagger_\mu|X(\mathbf{0})\rangle \frac{e^{-E_X t}}{2E_X} \langle X(\mathbf{0})|J_\nu|B(\mathbf{0})\rangle$$

all possible states with the quantum number chosen by the currents

Zero-recoil: different parity eigenstates isolated

Bailas @ Lattice 2019

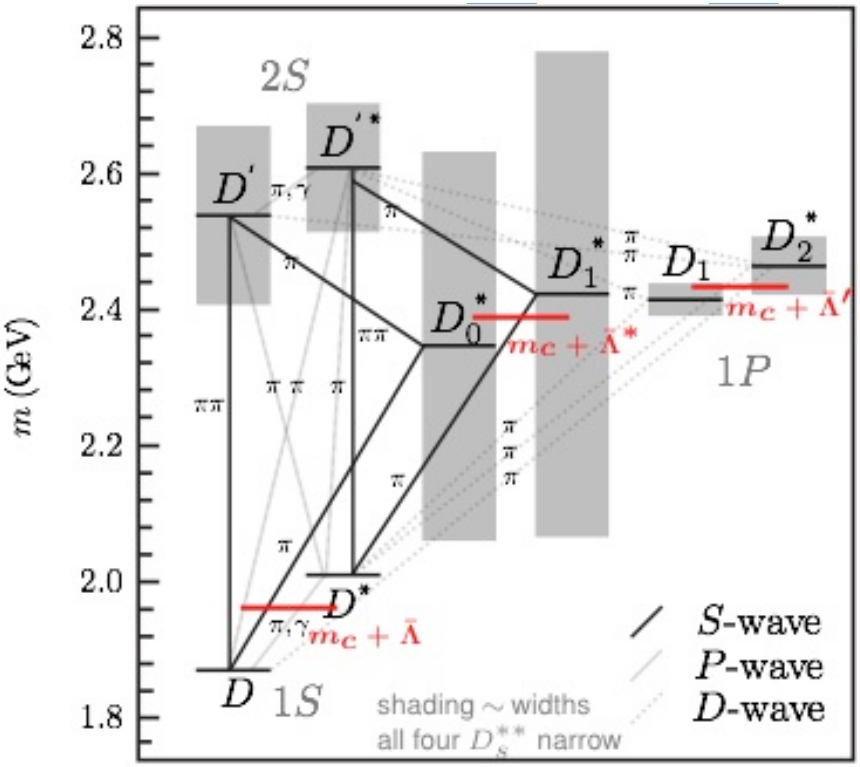


P-wave *D* mesons

Heavy quark symmetry: heavy quark spin decouples

- s_l = 1/2 or 3/2 for light degrees of freedom（ ½ + 1 ）

→ two doublets (D_0^*, D_1^*), (D_1, D_2^*)



“1/2 vs 3/2 puzzle” (Uraltsev et al.)

Bigi et al., EPJC 52, 975 (2007).

heavy quark limit, sum rule, quark model

theoretical estimates :

$$\Gamma(B \to D^{**}_{1/2}\ell\nu) \ll \Gamma(B \to D^{**}_{3/2}\ell\nu)$$

experimental data :

$$\Gamma(B \to D^{**}_{1/2}\ell\nu) \approx \Gamma(B \to D^{**}_{3/2}\ell\nu)$$

Are the broad states really P-wave?

Bernlochner, Ligeti, Turczyk, PRD85, 094033 (2012).

Bjorken and Uraltsev sum rules

Uraltsev, PLB 501, 86 (2001).
Le Yaouanc, Oliver, Raynal, PRD67, 114009 (2003).

- Heavy quark limit
- from B → ($D^{(*)}$, D^{**}) → B

$$T^{\mu\nu}(q,v,u) = i\int d^4x\, e^{-iq\cdot x}\langle B(u)|T[J^\mu(x)J^{\nu\dagger}(0)]|B(v)\rangle$$

slope of the Isgur-Wise function ($B\to D^{(*)}$)

BJ
$$\rho^2 - \frac{1}{4} = 2\sum_m |\tau^{(m)}_{3/2}(1)|^2 + \sum_n |\tau^{(n)}_{1/2}(1)|^2$$

Uraltsev
$$\frac{1}{4} = \sum_m |\tau^{(m)}_{3/2}(1)|^2 - \sum_n |\tau^{(n)}_{1/2}(1)|^2$$

zero-recoil (w=v.v'=1) form factors for $B\to D^{**}$

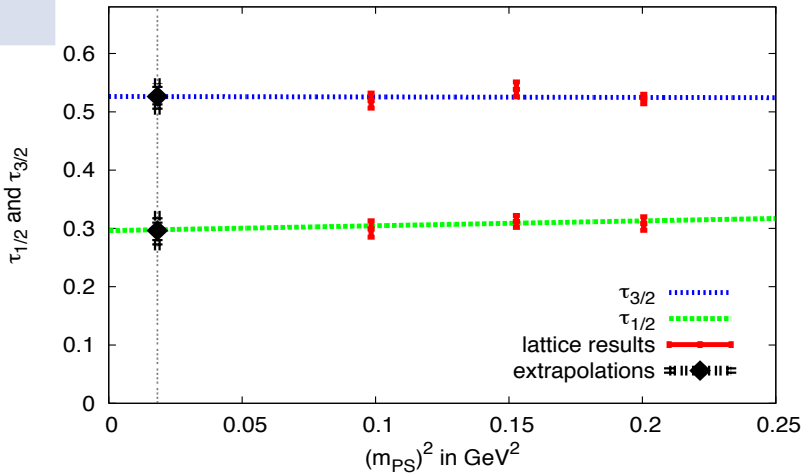
$$\rho^2 - \frac{1}{4} = 2\sum_m |\tau_{3/2}^{(m)}(1)|^2 + \sum_n |\tau_{1/2}^{(n)}(1)|^2$$

$$\frac{1}{4} = \sum_m |\tau_{3/2}^{(m)}(1)|^2 - \sum_n |\tau_{1/2}^{(n)}(1)|^2 \quad \Rightarrow \quad \tau_{3/2}^{(0)}(1) > \tau_{1/2}^{(0)}(1)$$

Previous estimates:

	$\tau_{3/2}^{(0)}(1)$	$\tau_{1/2}^{(0)}(1)$
SR	~0.6	~0.32
quark model	0.54	0.22
lattice	0.53(2)	0.30(3)

Blossier, et al., JHEP 0906 (2009) 022.
- HQET with derivative operators



Contributions from intermediate states

4pt func: $$C^{JJ}_{\mu\nu}(t;0) = \frac{1}{2M_B}\sum_X \langle B(0)|J^\dagger_\mu|X(0)\rangle \frac{e^{-E_X t}}{2E_X}\langle X(0)|J_\nu|B(0)\rangle$$

MEs:
(zero recoil)

$$\frac{1}{\sqrt{m_{D_0^*} m_B}}\langle D_0^*|A^0|B\rangle = 2g_+(1),$$

$$\frac{1}{\sqrt{m_{D_1^*} m_B}}\langle D_1^*|V^k|B\rangle = g_{V_1}(1)\,\epsilon^{*k},$$

$$\frac{1}{\sqrt{m_{D_1} m_B}}\langle D_1|V^k|B\rangle = f_{V_1}(1)\,\epsilon^{*k}$$

form factors
$g_+(1)$, $g_{V_1}(1)$, $f_{V_1}(1)$

$$C^{A^0A^0}(t) = |g_+(1)|^2 e^{-m_{D_0^*}t},$$

$$C^{V^kV^k}(t) = \frac{|g_{V_1}(1)|^2}{4} e^{-m_{D_1^*}t} + \frac{|g_{f_1}(1)|^2}{4} e^{-m_{D_1}t}$$

1/M expansion

Leibovich, Ligeti, Stewart, Wise, PRD57, 308 (1998)

zero-recoil form factors $g_+(1)$, $g_{V1}(1)$, $f_{V1}(1)$

- vanish in the heavy quark limit
- next order in 1/M

$$g_+(1) = \zeta(1)\left[-\frac{3}{2}(\varepsilon_c+\varepsilon_b)(\bar{\Lambda}^*-\bar{\Lambda})+\cdots\right],$$

$$g_{V_1}(1) = \zeta(1)\left[(\varepsilon_c-3\varepsilon_b)(\bar{\Lambda}^*-\bar{\Lambda})+\cdots\right],$$

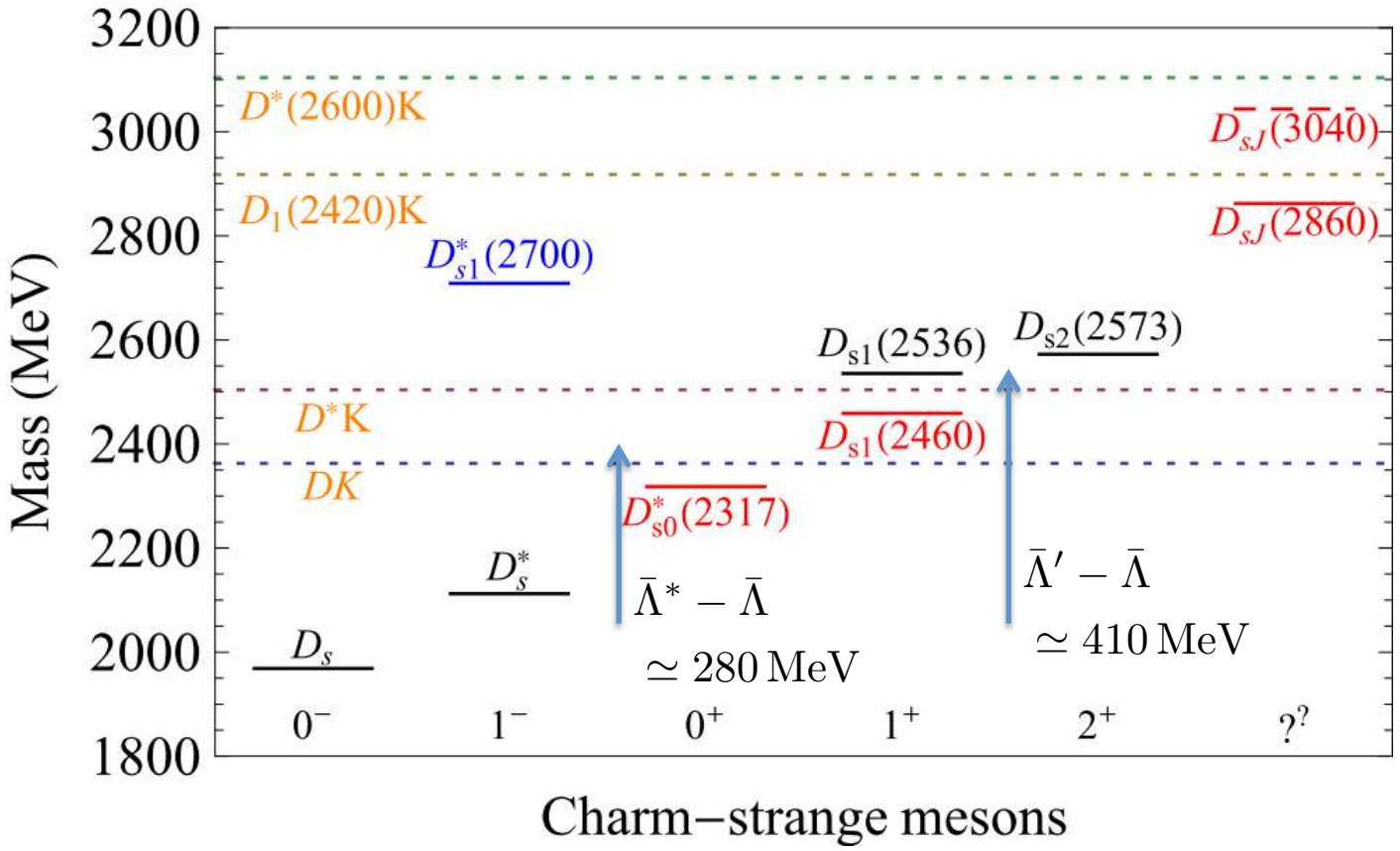
$$f_{V_1}(1) = \tau(1)\left[-\frac{8}{\sqrt{6}}\varepsilon_c(\bar{\Lambda}'-\bar{\Lambda})+\cdots\right]$$

~ Isgur-Wise func: $\zeta(1)$, $\tau(1)$

$1/2m_c$

mass difference between S-wave and P-wave

charmed-strange meson



Results

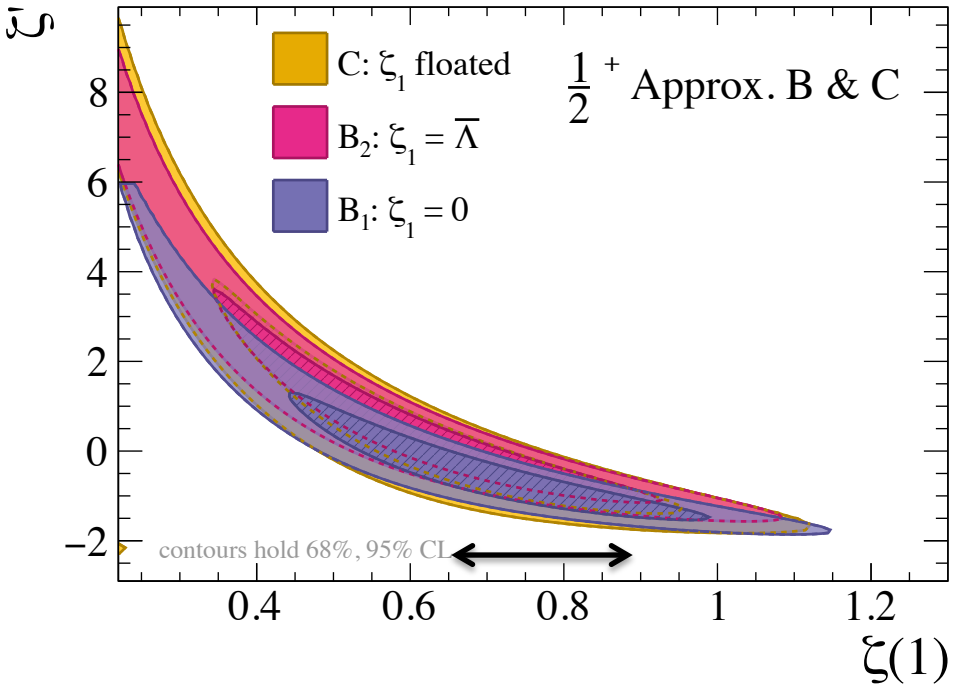
Bailas @ Lattice 2019

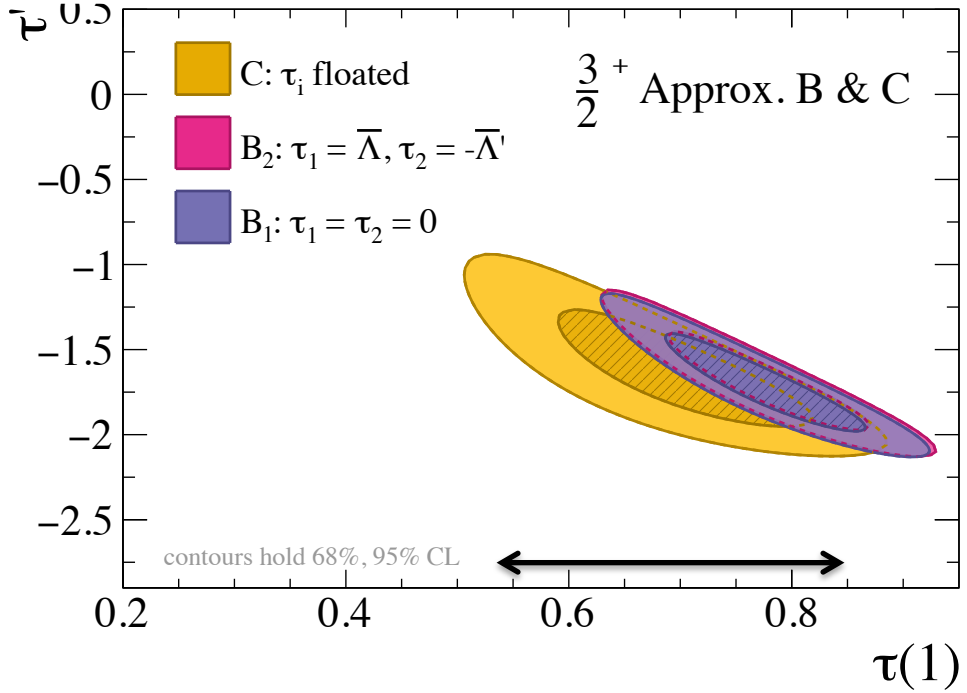
- assuming 1/M expansion
- "pole mass" for 1/m
- excitation energy from exp't

$$\zeta(1) = 0.77(13)$$
$$\tau(1) = 0.69(15)$$

$$\tau^{(0)}_{1/2}(1) = \frac{1}{2}\zeta(1) \quad = 0.45(7)$$

$$\tau^{(0)}_{3/2}(1) = \frac{1}{\sqrt{3}}\tau(1) = 0.39(7)$$



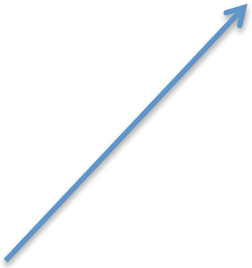


Bernlochner, Ligeti, PRD95, 014022 (2017)

- fit of experimental values
- with some approximations

Comparison to other estimates

	$\tau_{3/2}^{(0)}(1)$	$\tau_{1/2}^{(0)}(1)$
SR	~0.6	~0.32
quark model	0.54	0.22
lattice	0.53(2)	0.30(3)
this work	0.45(7)	0.39(6)



Blossier, et al., JHEP 0906 (2009) 022.
- HQET

Atoui et al., EPJC 75, 376 (2015).
- propagating, through 1/M
- signal not clear enough…

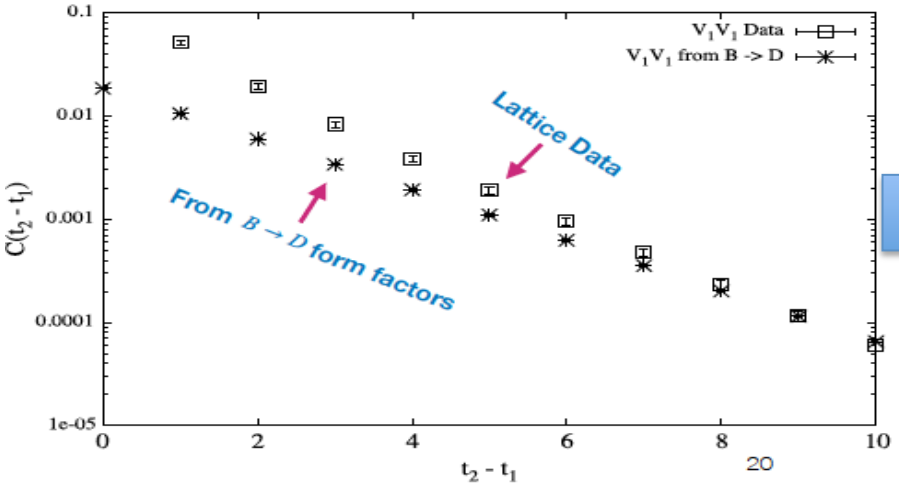
Too small to satisfy the sum rule?

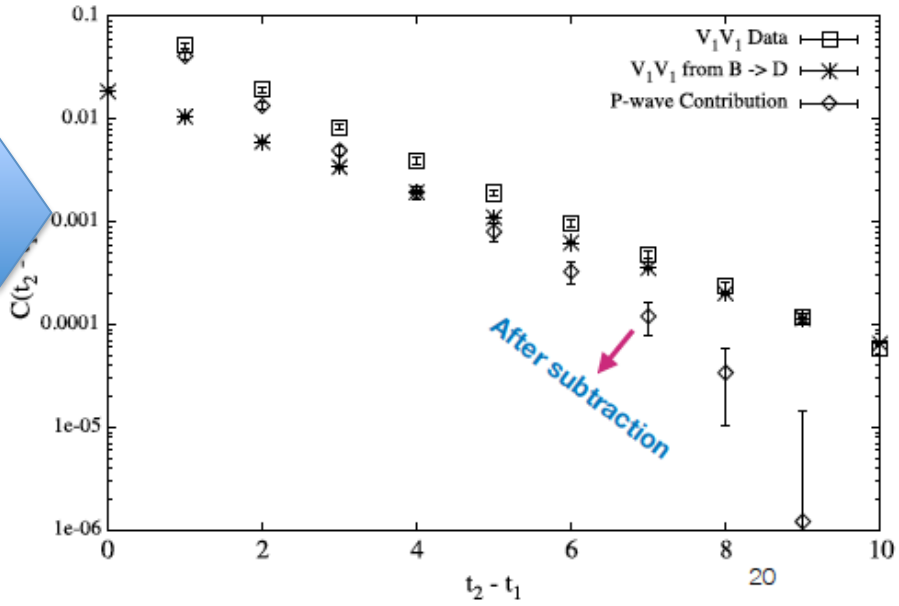
$$0.050(78) \approx |\tau_{3/2}^{(0)}(1)|^2 - |\tau_{1/2}^{0}(1)|^2$$

$\neq$ 1/4

Non-zero recoil: not protected by parity
→ subtraction of S-wave is necessary

Bailas @ Lattice 2019





large errors for the form factors...

Summary

- Inclusive processes

  (B decays, (not-so) DIS, …)

  – Well-defined controlled comparison between experiment and QCD is possible.

  – No worry about duality.

- Test with B decays on-going

  – More work also on pQCD side

  – Decomposition into individual states looks reasonable