Heavy-to-light decay form factors on $N_f = 2+1+1$ HISQ ensembles

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Centro Andaluz de Física de Partículas Elementales

· Advances in lattice gauge theory, CERN, 31 July 2018 ·

Introduction

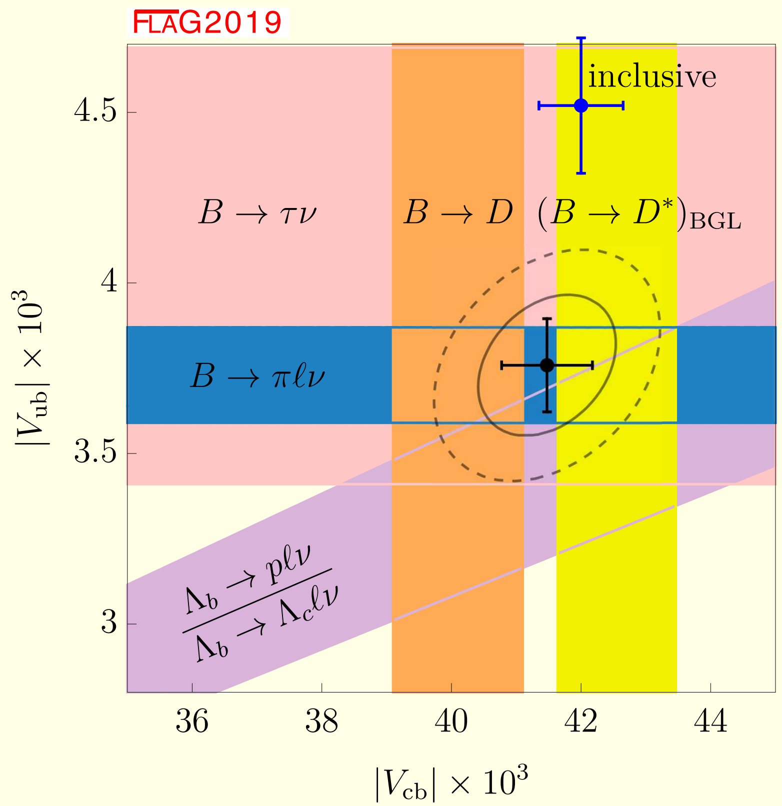
Precise determinations of CKM matrix elements:

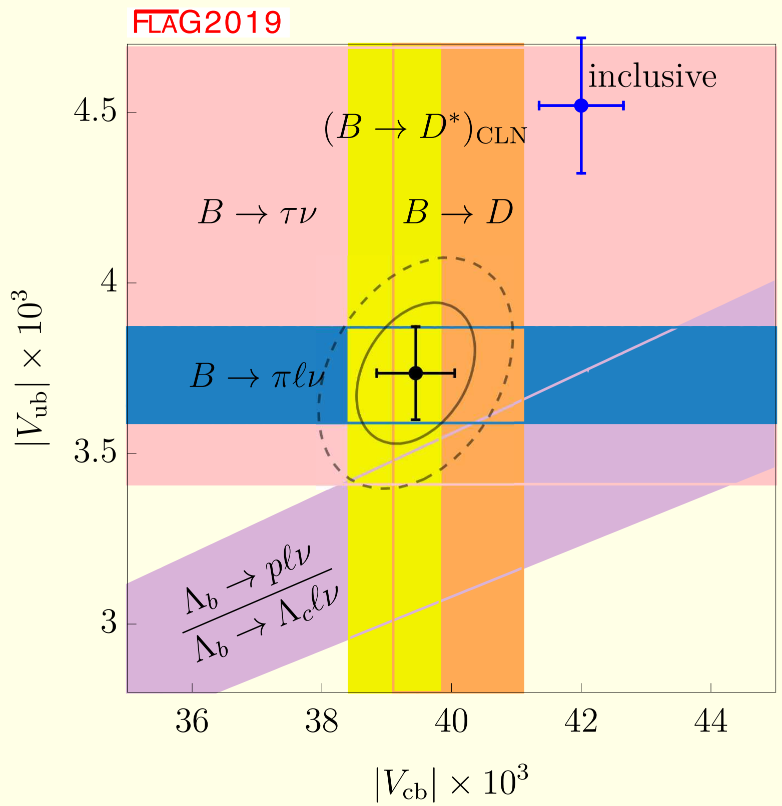
$$V_{CKM} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ & & B \to \pi\tau\nu,\, B_s \to K\ell\nu \\ & & \Lambda_b \to p\ell\nu \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ D \to \pi\ell\nu & D \to K\ell\nu & B_{(s)} \to D_{(s)} \left(D^*_{(s)}\right) \ell\nu \\ |V_{td}| & |V_{ts}| & |V_{tb}| \\ B \to \pi\ell\ell & B \to K\ell\ell & \end{pmatrix}$$

Tensions: Inclusive-Exclusive determinations of $|V_{ub}|$ and $|V_{cb}|$.

Introduction

Long-standing tension between exclusive and inclusive determinations of the CKM matrix elements $|V_{cb}|$ and $|V_{ub}|$ at the $\sim 3\sigma$ level.



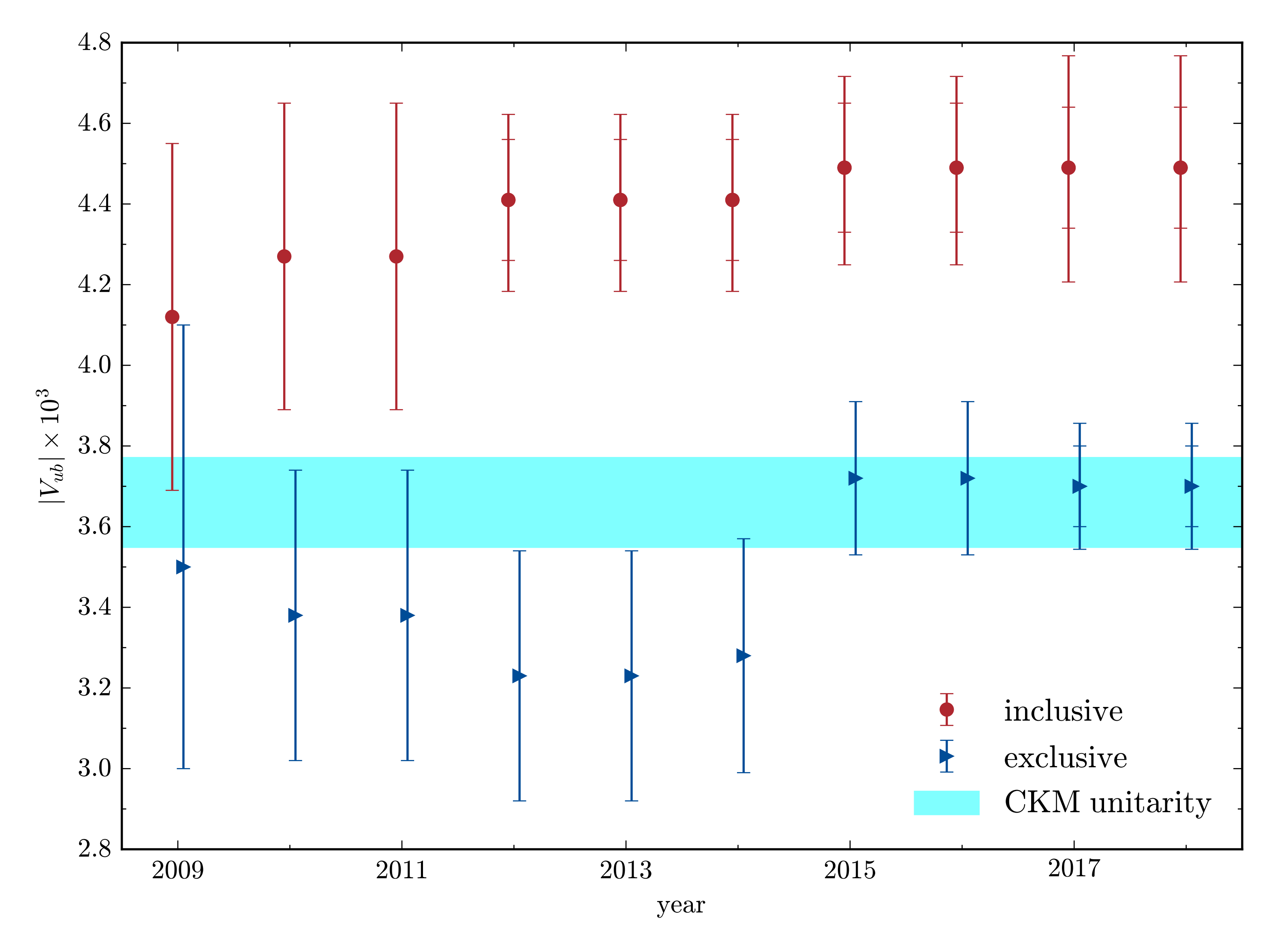


$|V_{cb}|^{B\to D^*}$ inclus.-exclus. tension not resolved by BGL vs CLN

(Belle (untagged) **1809.03290** and BaBar **1903.1002** results not included in plots)

From **Belle** 1809.03290 and **FNAL/MILC** 2014 $|V_{cb}|^{\rm CLN} = (38.4 \pm 0.9) \cdot 10^{-3}$
$|V_{cb}|^{\rm BGL} = (38.3 \pm 1.0) \cdot 10^{-3}$

Introduction: Status exclusive $|V_{ub}|$ extraction



Update of plot in 1711.08085. CKM unitarity band from CKMfitter

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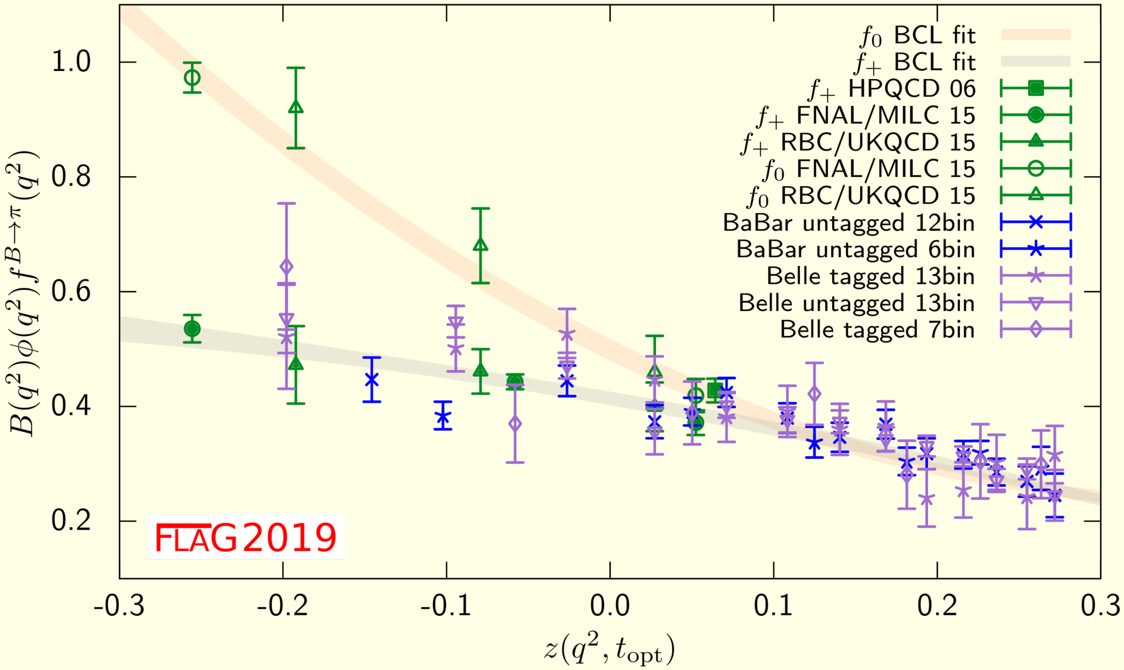

$|V_{ub}|$ **from** $B \to \pi l \nu$

Combined BCL fit to experim. and $N_f = 2+1$ lattice data on different q^2 regions

RBC/UKQCD, 1501.05373

FNAL/MILC, 1503.07839

HPQCD, hep-lat/0601021

$$|V_{ub}|^{FLAG2019} = 3.73(14) \cdot 10^{-3}$$

Good consistency between lattice and experimental shapes and commensurate errors

$$|V_{ub}|^{\rm inclusive, HFLAV2017} = (4.52 \pm 0.15^{+0.11}_{-0.14}) \cdot 10^{-3}$$ $\sim 3\sigma$ disagreement.

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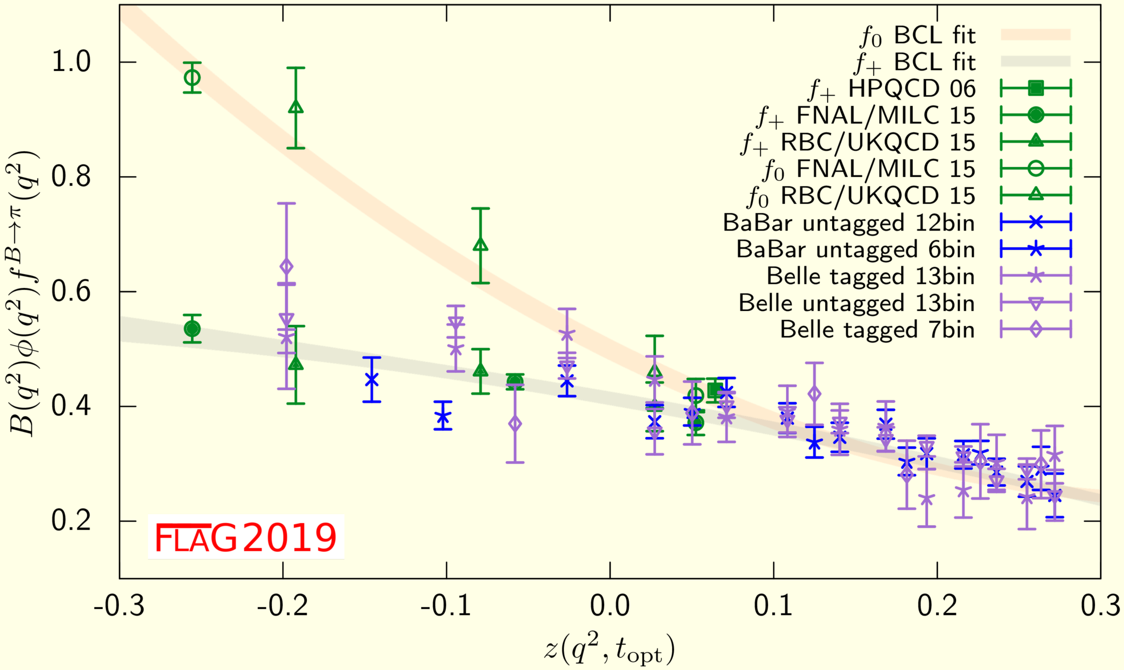

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Leptonic determinations

* Less precise (dominated by exp. errors on $\mathcal{B}(B \to \tau\nu)$)

* **BaBar** and **Belle** results don't agree very well.

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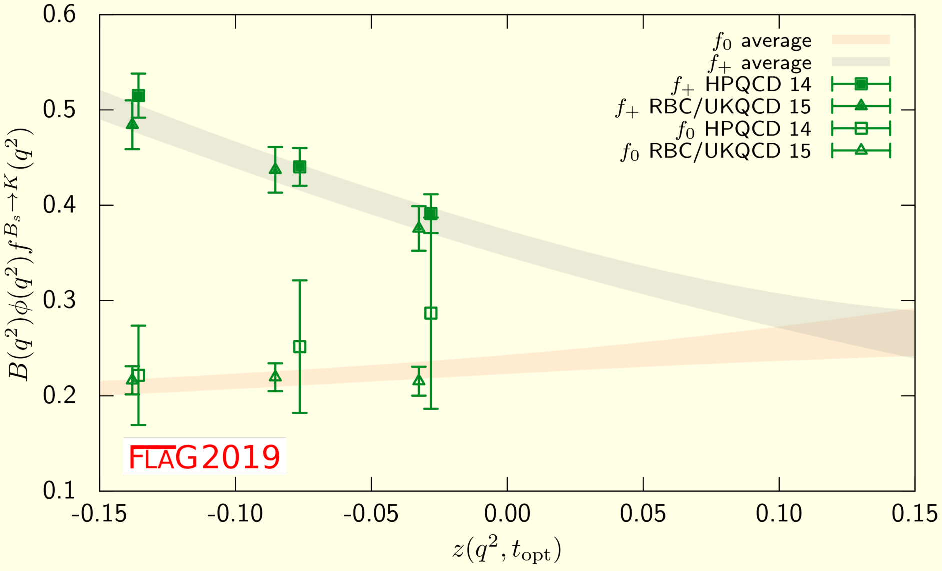
Important role for **Belle II** for both leptonic and semileptonic

Introduction: Status exclusive $|V_{ub}|$ extraction

Alternative way of getting $|V_{ub}|$: $B_s \to K\ell\nu$.



* Three LQCD calculations of the relevant form factors:

  HPQCD 1406.2279, **RBC/UKQCD** 1501.05373, **FNAL/MILC** 1901.02561

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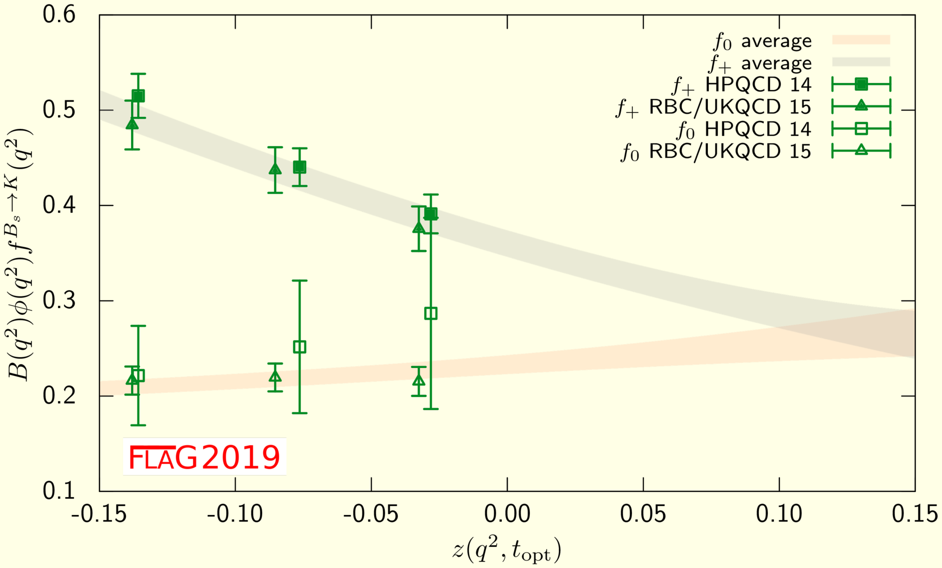

Also,

$$f_{0,+}(B_s \to K\ell\nu)/f_{0,+}(B_s \to D_s\ell\nu)$$

to get $|V_{ub}/V_{cb}|$

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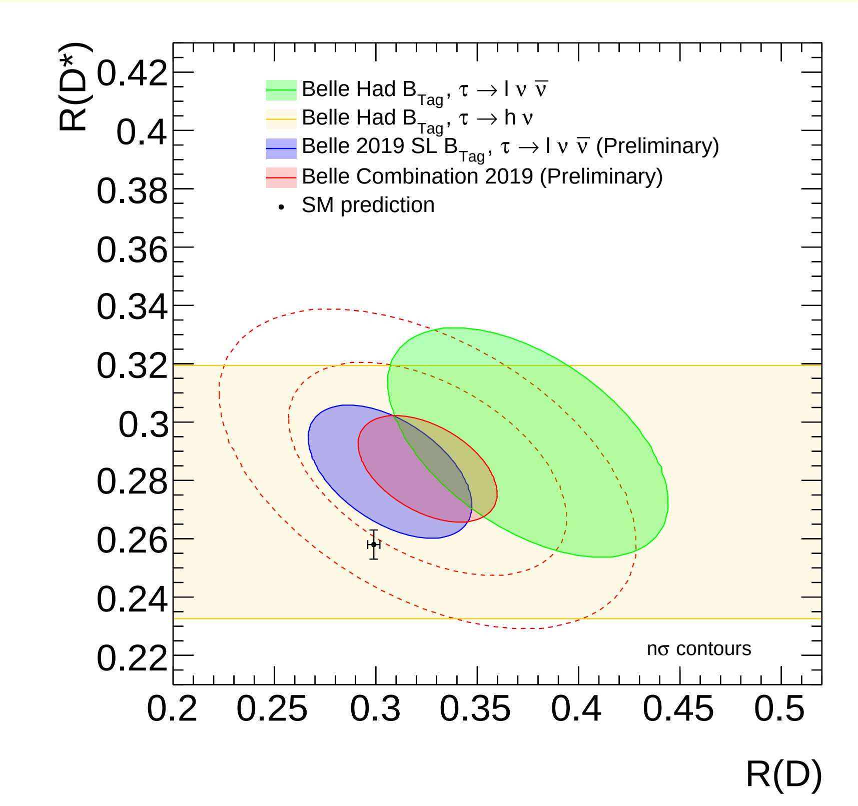
* Experimentally: Under investigacion by **LHCb**, expected to be measured at the $\Upsilon(5S)$ run at **Belle-II**

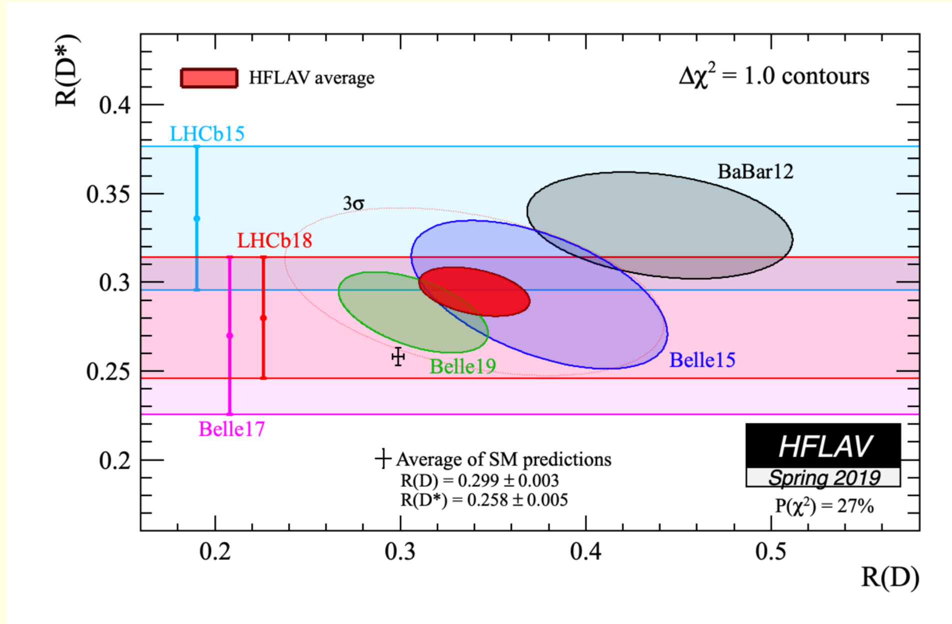
(maybe 5-10% precision for the decay rate at Belle-II)

Introduction: Lepton Flavor Universality tests

$$R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)} \tau \nu_\tau)}{\mathcal{B}(B \to D^{(*)} \ell \nu)}$$



Plot from **1904.08794**



Tension between **Belle** and **BaBar**

Belle 2019: $R(D) = 0.307 \pm 0.037 \pm 0.016$ (consistent with SM),
$R(D^*) = 0.283 \pm 0.018 \pm 0.014$

World average at $\sim 3\sigma$ from SM.

Introduction: b rare decays (FCNC)

Flavor-changing neutral currents $b \to q$ transitions are potentially sensitive to NP effects $B \to K^* \gamma$, $B \to K^{(*)} \ell^+ \ell^-$, $B \to \pi \ell^+ \ell^-$

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Sets of tensions between SM predicions and experimentally measured $b \to s \ell^+ \ell^-$ observables

Branching fraction measurements: $B^0 \to K^{*0} \mu^+ \mu^-$, $B^+ \to K^{(*)+} \mu^+ \mu^-$, $B_s \to \phi \mu^+ \mu^-$

Angular analyses: $B^+ \to K^{(*)+} \mu^+ \mu^-$, $B_s \to \phi \mu^+ \mu^-$

Tests of Lepton Flavour Universality (μ/e): $B^0 \to K^{*0} \mu^+ \mu^-$, $B^+ \to K^{(*)+} \mu^+ \mu^-$

Very small sensitivity to hadronic form factors $\sim 10^{-4}$

$$R_{K^{(*)}}(q^2_{min}, q^2_{max}) \equiv \frac{\int_{q^2_{min}}^{q^2_{max}} dq^2 d\mathcal{B}(B \to K^{(*)} \mu^+ \mu^-)}{\int_{q^2_{min}}^{q^2_{max}} dq^2 d\mathcal{B}(B \to K^{(*)} e^+ e^-)}$$

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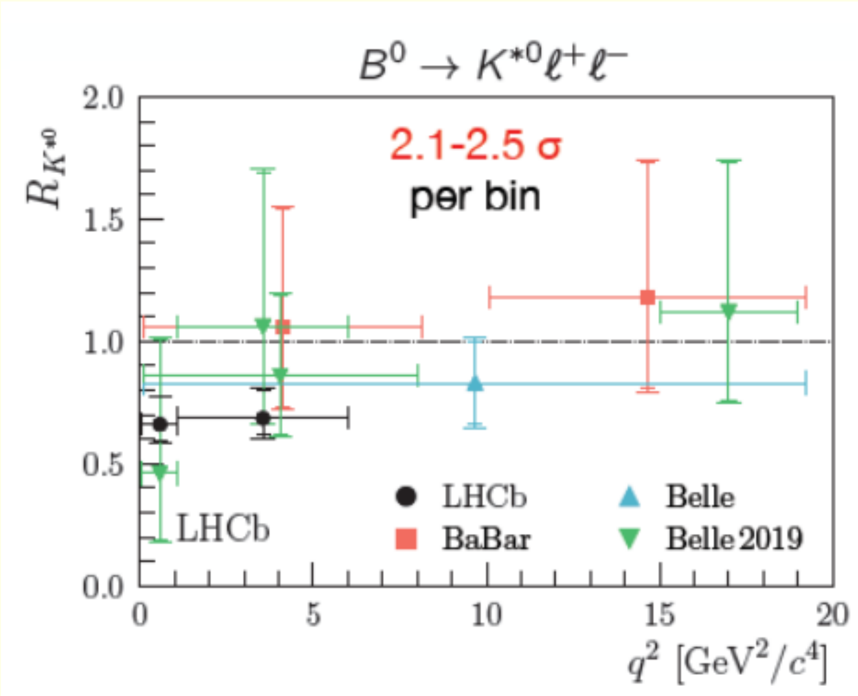
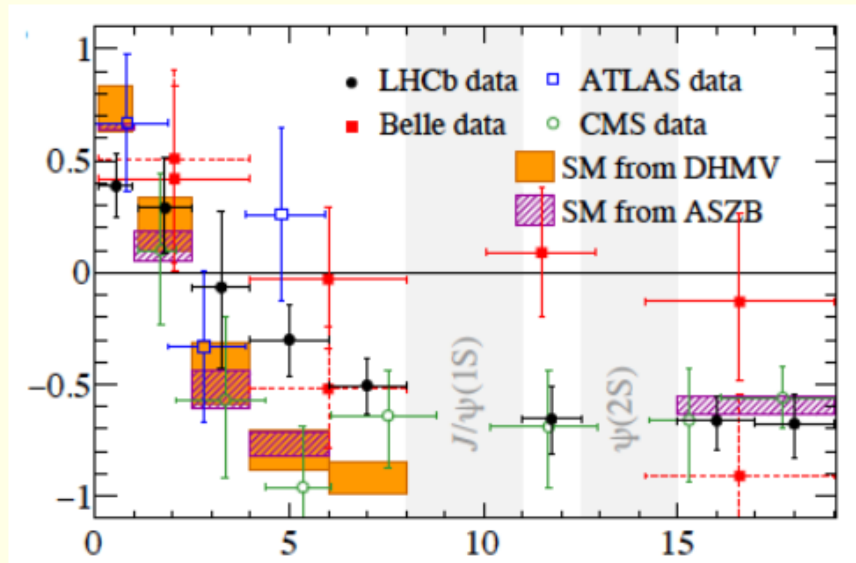


1904.02440 Belle preliminary

Angular Analysis (P_5')



LHCb finds 3.4σ, seems to be confirmed by Belle (ATLAS?) but not CMS

LHCb will reach $\sim 1.5\%$ precision for the branching fractions at both low and high q^2. **J. Albrecht et al** 1709.10308

Introduction: Neutral-current b decays

For $B \to P\ell\ell$, hadronic contributions are parametrized in terms of matrix elements of current (vector, axial and tensor) operators through three form factors

$$f_+ \;, f_0 \text{ (for } m_\ell \neq 0\text{) and } f_T$$

+ non-factorizable contributions

Allow the calculation of branching fractions, angular observables and LFUV quantities

Extract CKM matrix elements $|V_{td,ts}|$ or constrain Wilson coefficients C_9 and C_{10}.

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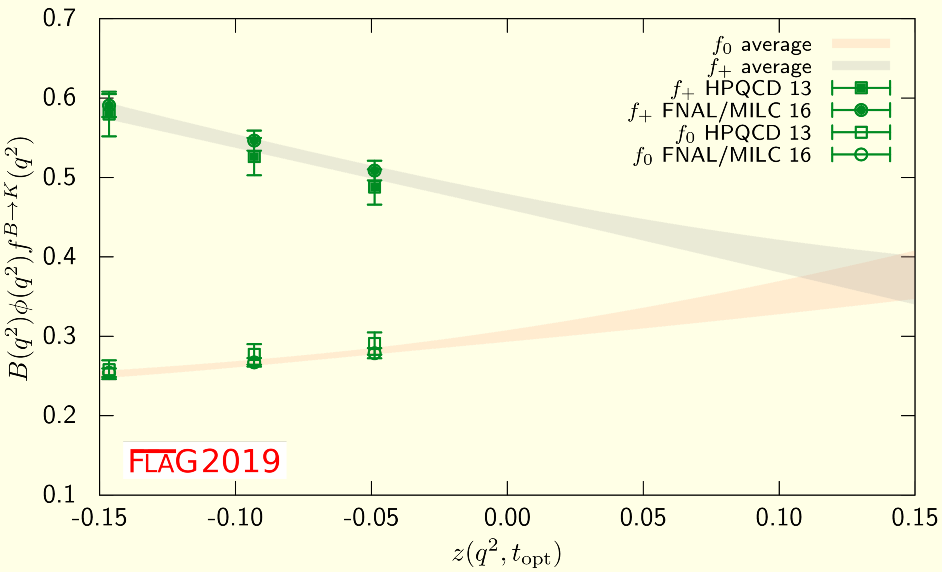
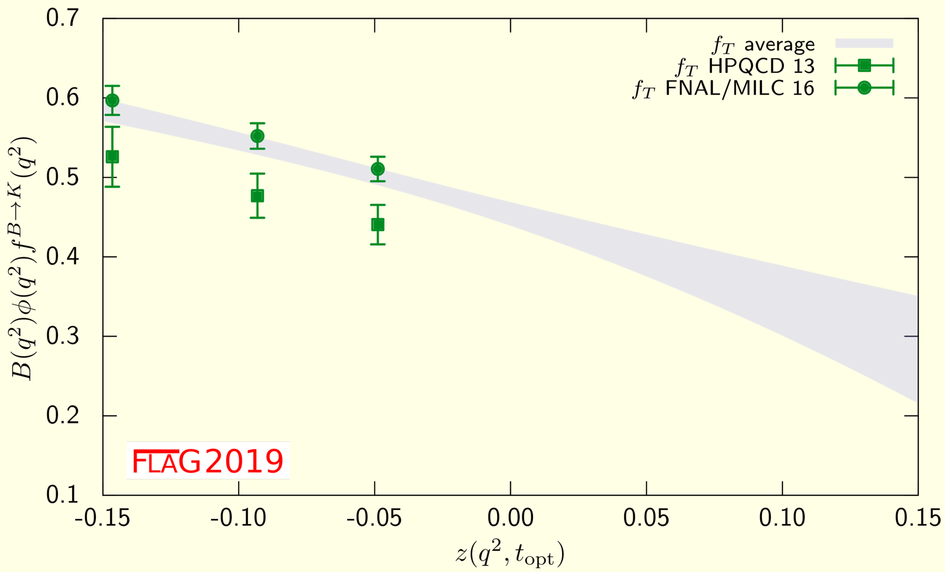
* Non-factorizable contributions under control? New physics or charm-loops?

* This talk: Form factors for $h \to l$ decays.

Current status: Form factors for $B \to K\ell^+\ell^-$

$B \to K\ell^+\ell^-$: **HPQCD** 1306.0434, 1306.2384, **FNAL/MILC**, 1509.06235





Overlapping ensemble sets (asqtad MILC $N_f = 2+1$) but different lattice actions:

HPQCD: NRQCD b + HISQ u, d, s

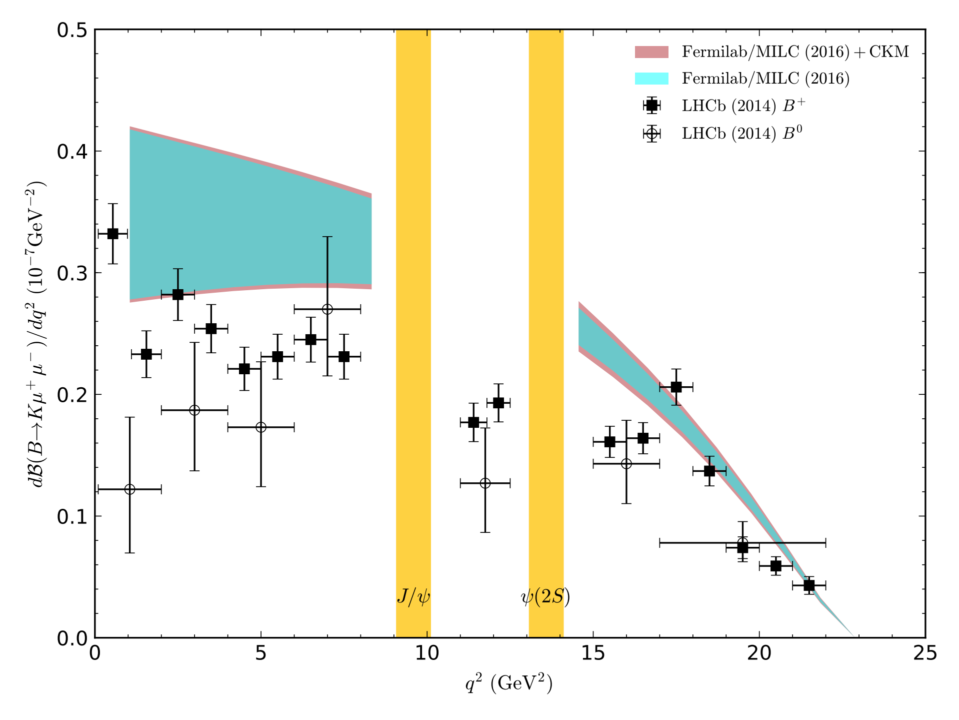
FNAL/MILC: Fermilab b + asqtad u, d, s

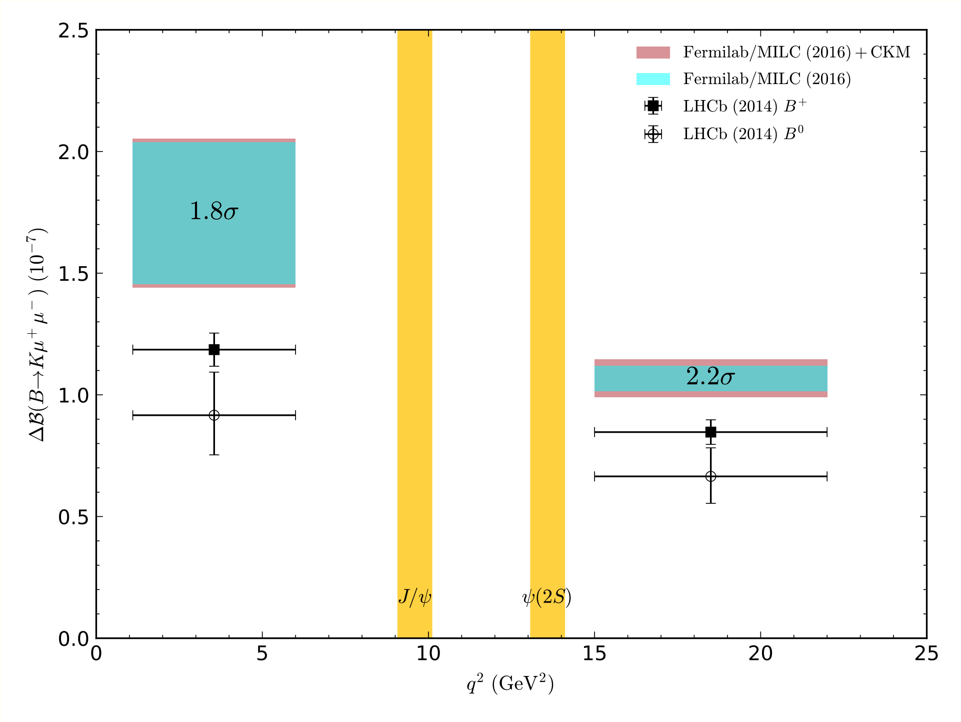
Consistent results for $f_{0,+,T}$, and with LCSR

Khodjamarian et al 1006.4945

Form factors for $B \to K\ell^+\ell^-$

From **D. Du et al** 1510.02349, **FNAL/MILC** 1509.06235 (non-factorizable contributions under control?)

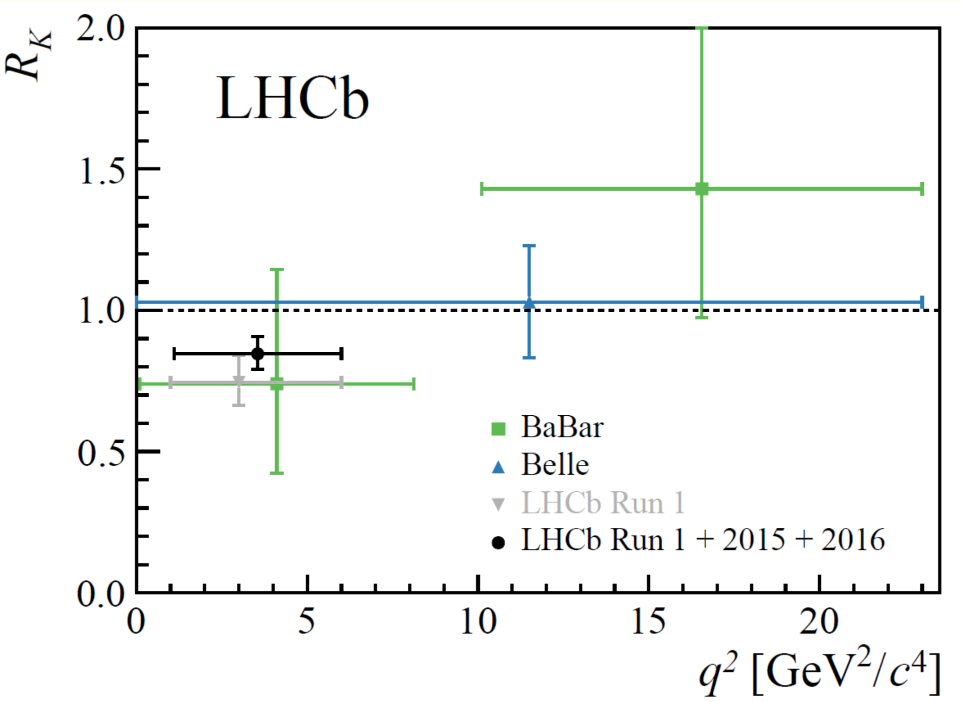




$1 - 2\sigma$ experiment-SM tensions.

focus on large bins above and below charmoninum resonances

$B \to K\ell^+\ell^-$: Lepton Flavor Universality Tests



$$(1 - R_K)^{\rm HPQCD} = 0.00074 \pm 0.00035\,, \quad (1 - R_{K^+})^{\rm FNAL/MILC} = 0.00050 \pm 0.00043$$

SM predictions for these ratios pretty insensitive to form factors and non-factor. contributions.

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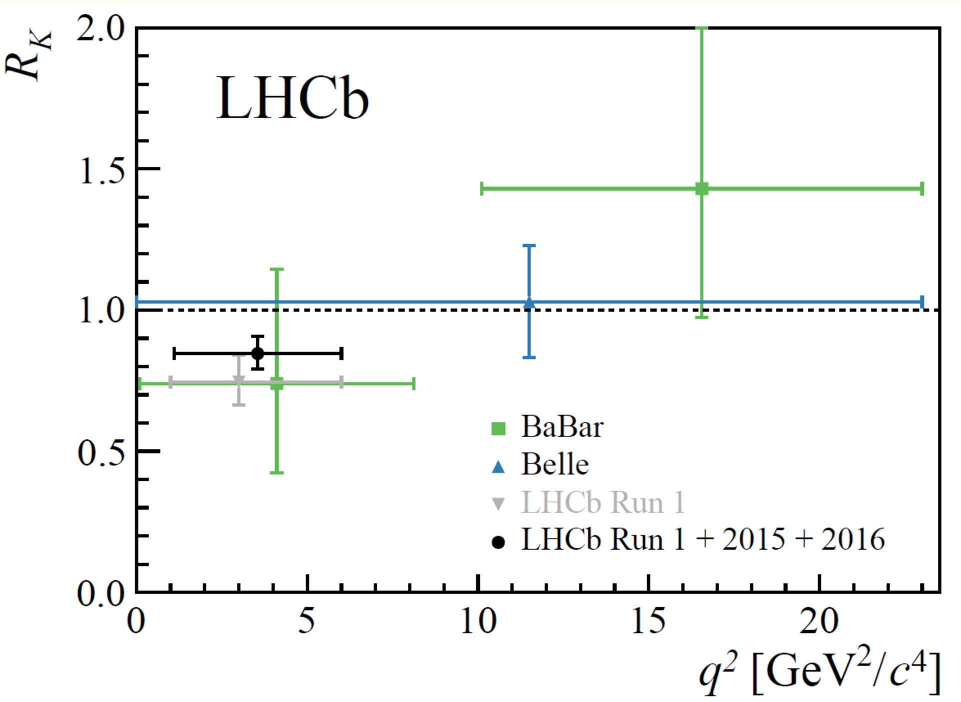

LHCb results

for $q^2 \in [1\ \mathrm{GeV}^2, 6\ \mathrm{GeV}^2]$

$$R_K^{old\ Run1} = 0.745^{+0.090}_{-0.074}(stat) \pm 0.036(syst)$$

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$$R_K^{2015+2016} = 0.928^{+0.089}_{-0.076}(stat)^{+0.020}_{-0.016}(syst)$$

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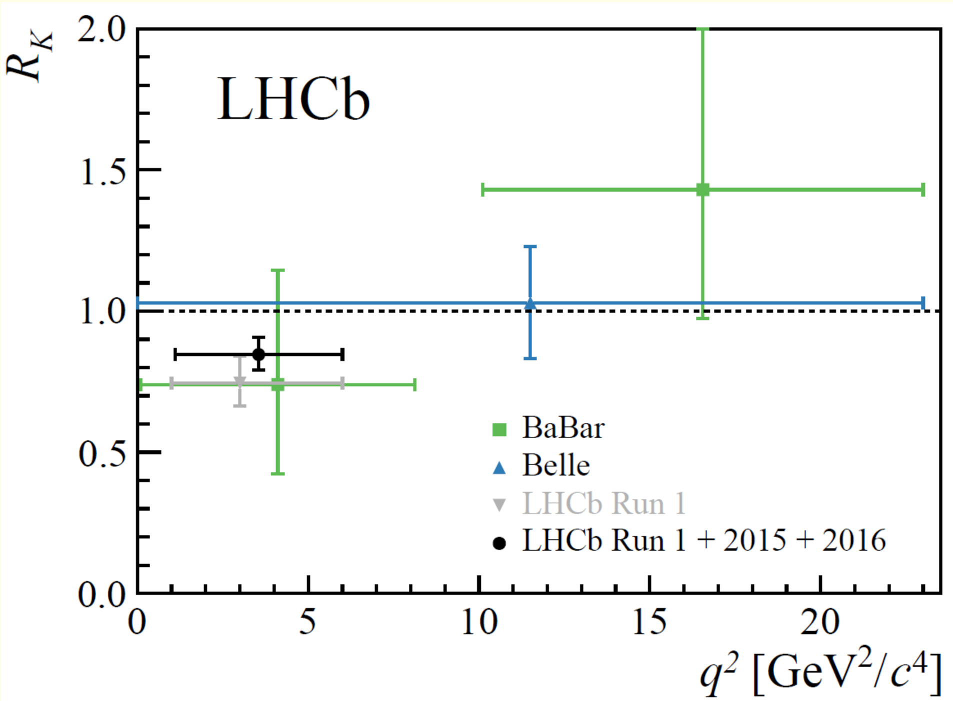

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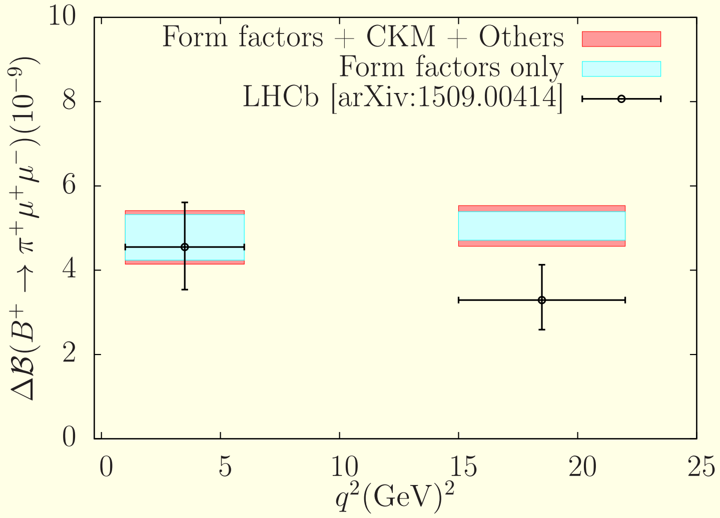
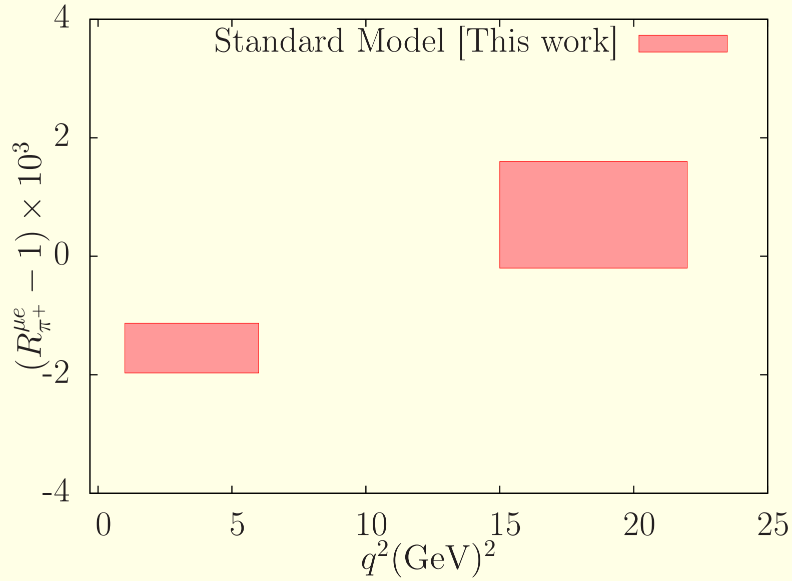
* LHCb expects a reduction by a factor of 4 by 2025.

Form factors for $B \to \pi \ell^+ \ell^-$

FNAL/MILC, 1507.01618, **D. Du et al.** 1510.02349 Take f_+ and f_0 from combined fit of lattice + experimental data for $B \to \pi \ell \nu$ (assume not significant NP effects at tree level).





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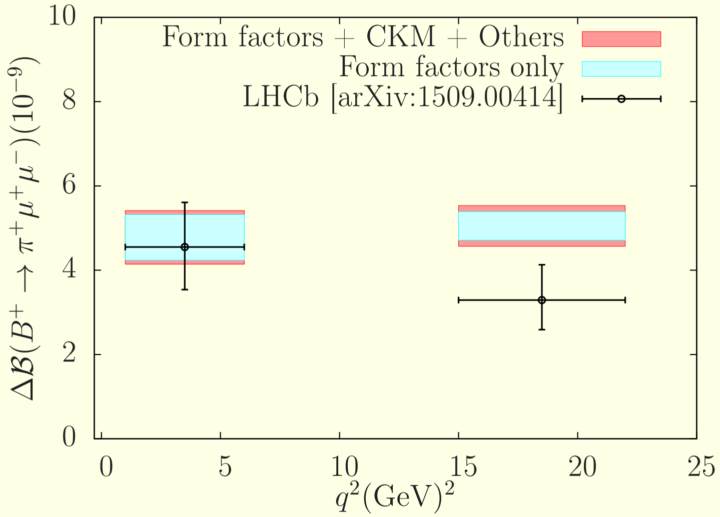

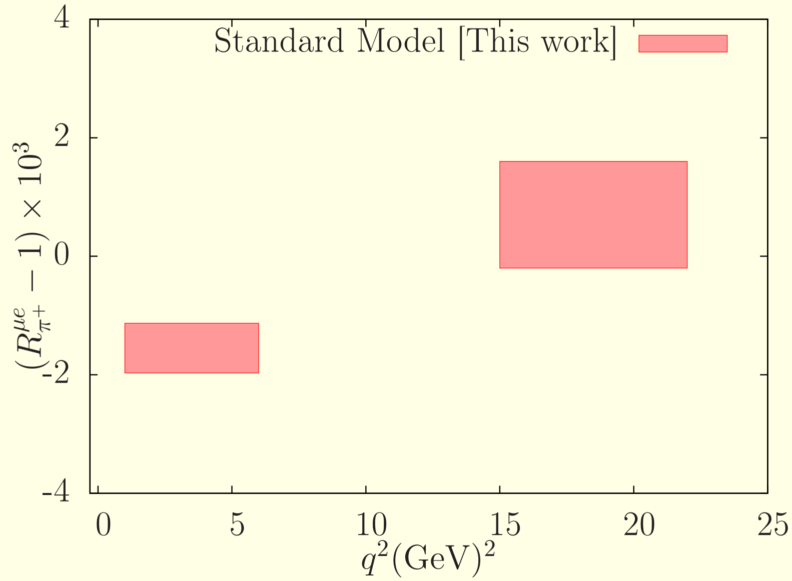

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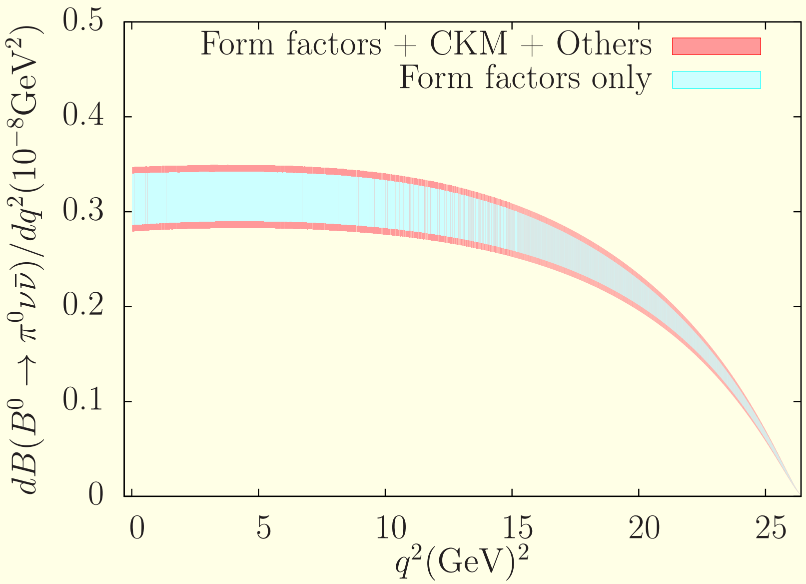
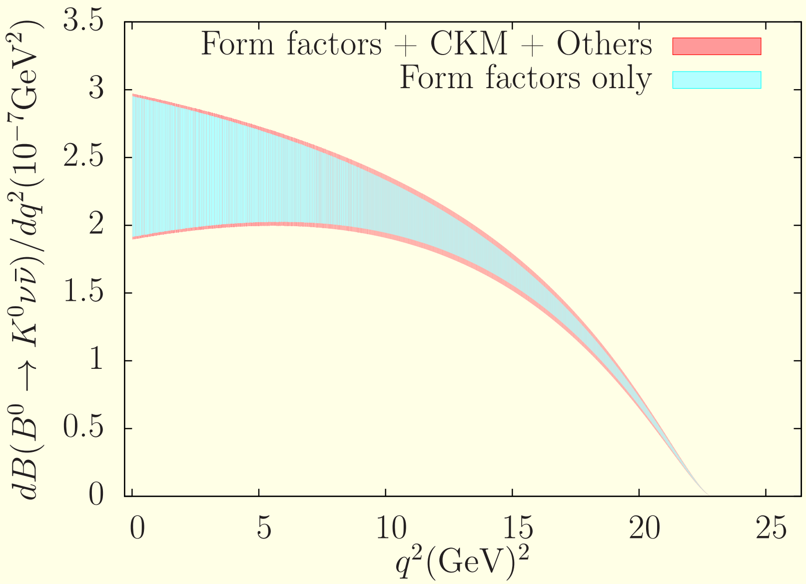
D. Du et al. 1510.02349 SM prediction for $R_\pi = \frac{\mathcal{B}(B \to \pi \tau \nu_\tau)}{\mathcal{B}(B \to \pi \ell \nu)} = 0.641(17)$.

Expected to be measured at **Belle-II**, possible to determine at **LHCb**

Rare semileptonic B decays to $\nu\bar{\nu}$ states

D. Du et al. 1510.02349 with FNAL/MILC form factors





Predictions for both neutral and charged channels: complementary information (also $|V_{td,ts}|$)

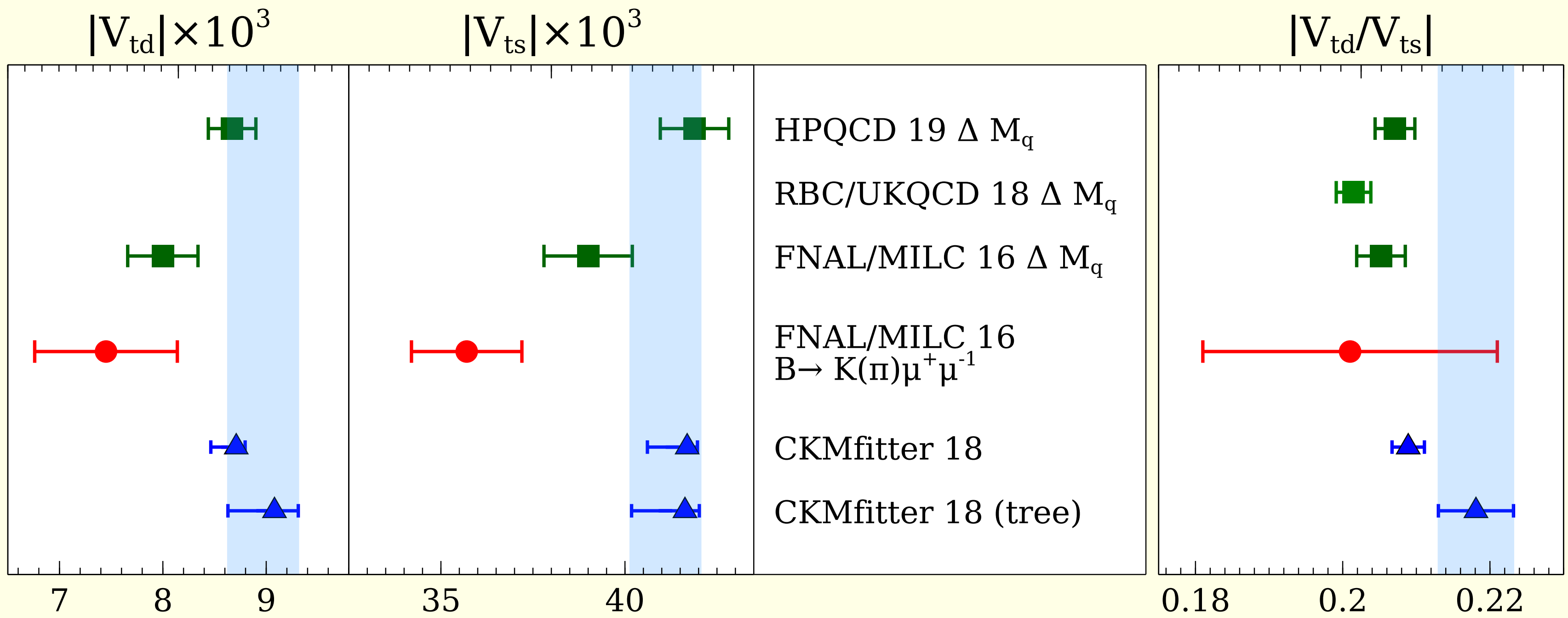
* Theoretically clean (no problem with charm LD contributions)

* Difficult to measure experimentally, Belle-II expected precision $\sim 10\%$ for $B \to K$

$$\mathcal{B}(B^0 \to \pi^0 \nu\bar{\nu}) \cdot 10^7 = 0.668(41)(49)(16)$$
$$\mathcal{B}(B^0 \to K^0 \nu\bar{\nu}) \cdot 10^7 = 40.1(2.2)(4.3)(0.9)$$

$$\mathcal{B}(B^+ \to \pi^+ \nu\bar{\nu}) \cdot 10^6 = 9.62(1)(92); \ \mathcal{B}(B^+ \to K^+ \nu\bar{\nu}) \cdot 10^6 = 4.94(52)(6)$$

Rare semileptonic B decays: CKM parameters



* B-mixing results **HPQCD** 1907.01025, **RBC/UKQCD** 1812.08791, **FNAL/MILC**, 1602.03560

* $B \to K(\pi)\mu^+\mu^-$ results from **D. Du et al**, 1510.02349

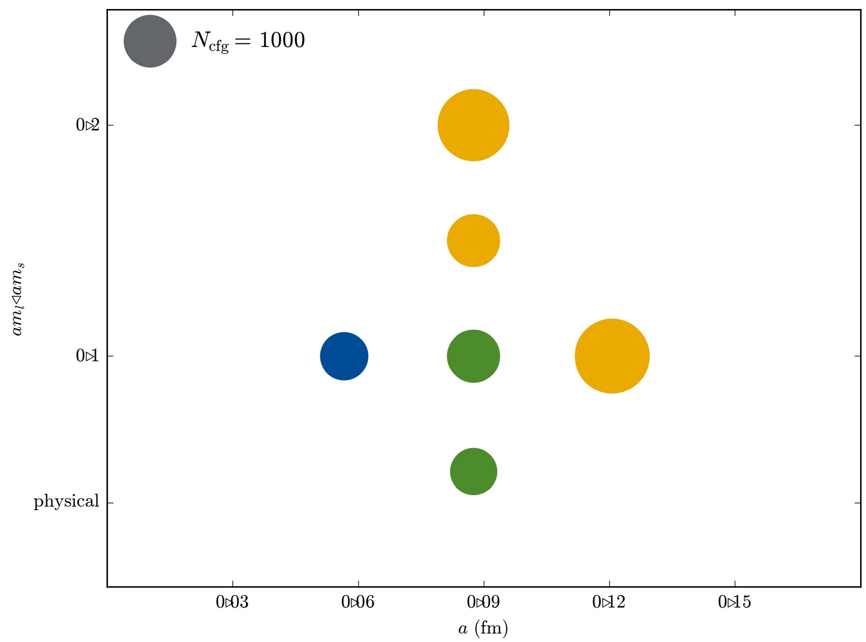
* Full/tree CKM unitarity results come from CKMfitter's fit 2018 using all inputs/only observable mediated at tree level of weak interactions.

Fermilab Lattice/MILC program for $b(c) \to s(d)$ decays

Form factors for $B_s \to K\ell\nu$

FNAL/MILC 1901.02561 on MILC asqtad $N_f = 2 + 1$ ensembles.

Valence sector: Fermilab b + asqtad l, s



Analysis led by Yuzhi Liu

* Errors: $\mathcal{O}(\alpha_s a^2), \mathcal{O}(\alpha_s a, a^2) f((m_b a)^2)$
* Scale set with r_1, with $r_1^{a=0} = 0.3117(22)$ fm
* Partially quenched: $m_s' \neq m_s$
* Lattice data $\in [17.4, 23.2]\,\mathrm{GeV}^2$

(Kaon momentum up to $\frac{2\pi}{N_s}(1, 1, 1)$

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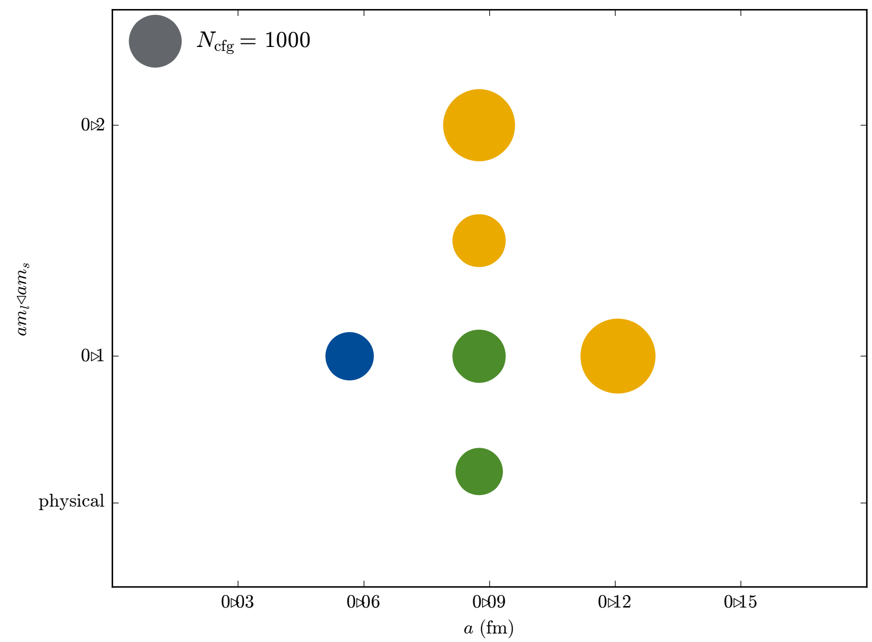

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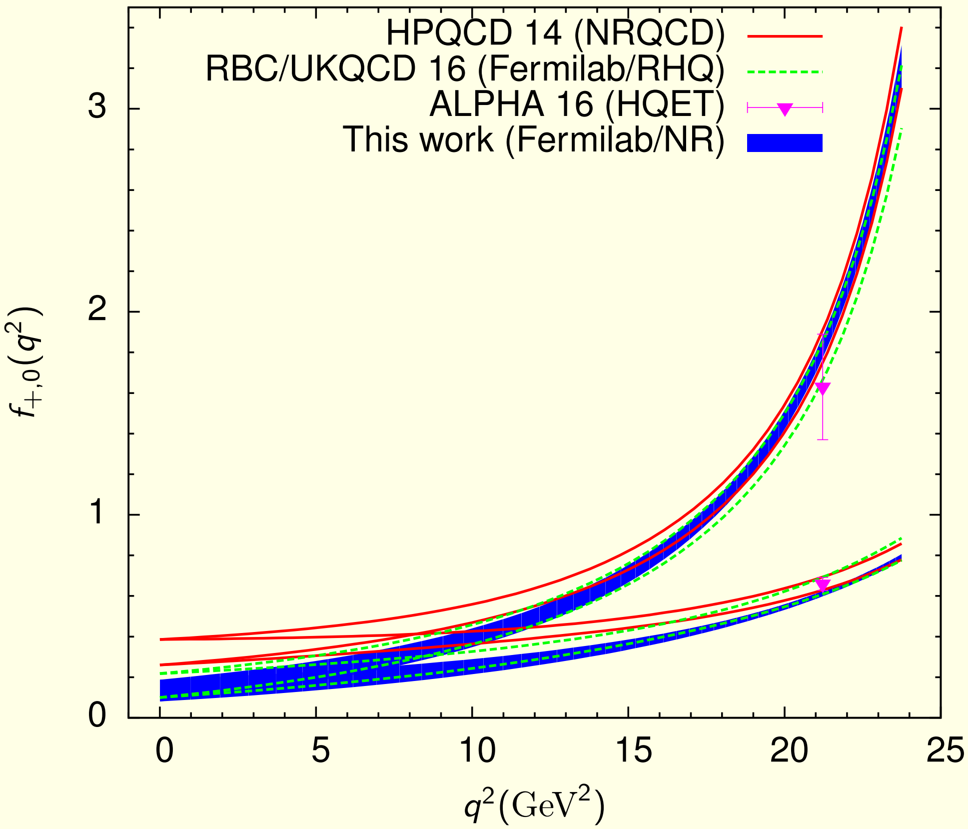
Chiral-continuum extrapolation with NLO HMrSChPT in SU(2) hard-kaon limit + NNLO analytic terms.

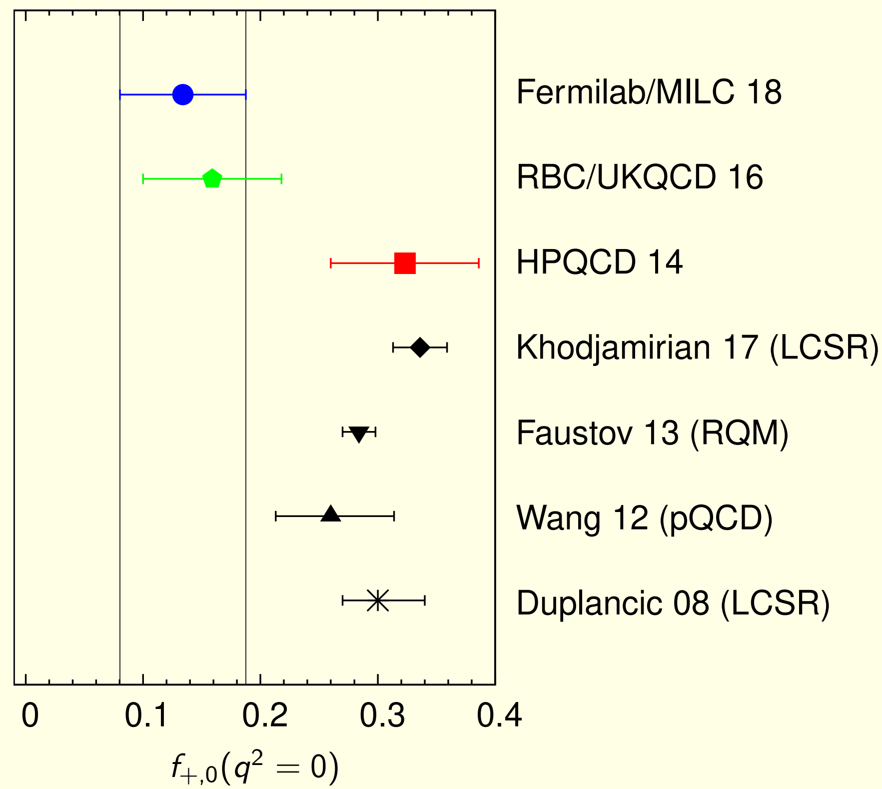
* Small adjustments to the physical m_b

Form factors for $B_s \to K\ell\nu$

Use BCL parametrization for $z-$expansion (with $K = 4$).

* Kinematic constraint $f_+(0) = f_0(0)$ enforced (without constraint, results satisfy $f_+(0) = f_0(0)$ within errors)





Tension with HPQCD (especially at low q^2). Good agreement with RBC/UKQCD.

Form factors for $B_s \to K\ell\nu$

Predictions for differential decay rates:

Ratios for LFU tests: $\Gamma(B_s \to K\tau\nu)/\Gamma(B_s \to K\mu\nu) = 0.836(34)$

Forward-backward asymmetry: (θ_l: angle between charged lepton and B)

$$A^{\ell}_{FB} = \int_0^1 \frac{d^2\Gamma}{dq^2 d\cos\theta_\ell} d\cos\theta_\ell - \int_{-1}^0 \frac{d^2\Gamma}{dq^2 d\cos\theta_\ell} d\cos\theta_\ell$$

$$\propto |p_K^2| \frac{m_\ell^2}{q^2} Re\left[f_+(q^2) f_0^*(q^2)\right]$$

Lepton polarization asymmetry:

$$A^{\ell}_{\rm pol} = \frac{d\Gamma^-/dq^2 - d\Gamma^+}{d\Gamma^-/dq^2 + d\Gamma^+} \propto f(|f_+(q^2)|, |f_0(q^2)|)$$

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Forward-backward asymmetry: (θ_l: angle between charged lepton and B)

$$A^{\ell}_{FB} = \int_0^1 \frac{d^2\Gamma}{dq^2 d\cos\theta_\ell} d\cos\theta_\ell - \int_{-1}^0 \frac{d^2\Gamma}{dq^2 d\cos\theta_\ell} d\cos\theta_\ell$$

$$\propto |p_K^2| \frac{m_\ell^2}{q^2} Re\left[f_+(q^2) f_0^*(q^2)\right]$$

Lepton polarization asymmetry:

$$A^{\ell}_{\rm pol} = \frac{d\Gamma^-/dq^2 - d\Gamma^+}{d\Gamma^-/dq^2 + d\Gamma^+} \propto f(|f_+(q^2)|, |f_0(q^2)|)$$

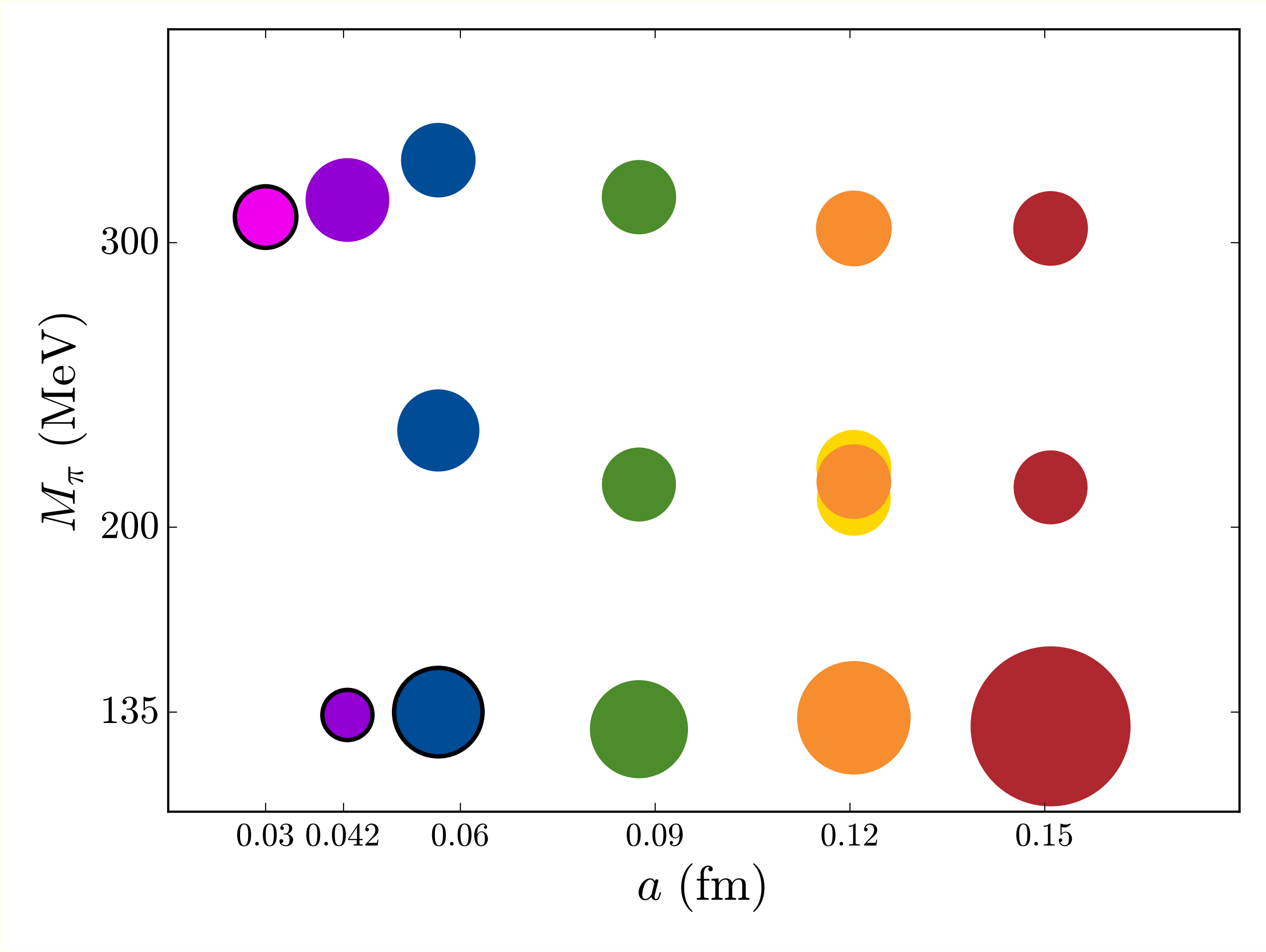
Also provides ratios of f_+ and f_0 for $B_s \to K\ell\nu$ and $B_s \to D_s\ell\nu$ as functions of q^2: useful for the determination of $|V_{ub}/V_{cb}|$.

$b(c) \to s(d)$ decays on MILC $N_f = 2+1+1$ HISQ ensembles

(in progress)

$b(c) \to s(d)$ decays on MILC $N_f = 2+1+1$ HISQ ensembles

* MILC $N_f = 2+1+1$ HISQ ensembles



$b(c) \to s(d)$ decays on MILC $N_f = 2+1+1$ HISQ ensembles

* MILC $N_f = 2+1+1$ HISQ ensembles
* Lüscher-Weisz, one-loop Symanzik and tadpole improved gauge action $\to \mathcal{O}(\alpha_s^2 a^2)$
* Valence l, s, c quarks are always described with HISQ action $\to \mathcal{O}(\alpha_s a^2)$
* Scale set with ω_0/a

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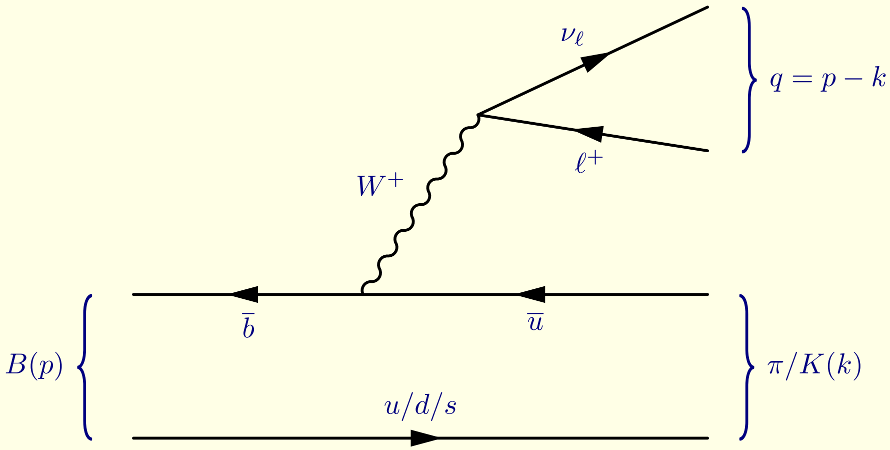
A Clover action with Fermilab interpretation for $b \to \mathcal{O}(\alpha_s a, a^2) f((m_b a)^2)$

B HISQ action for heavy quarks, $m_c \leq m_h \leq m_b \to \mathcal{O}(\alpha_s a^2) f((m_h a)^2)$

$B_{(s)} \to \pi(K)\ell\nu$: charged currents

Extraction of $|V_{ub}|$: $B \to \pi\ell\nu$ and $B_s \to K\ell\nu$.

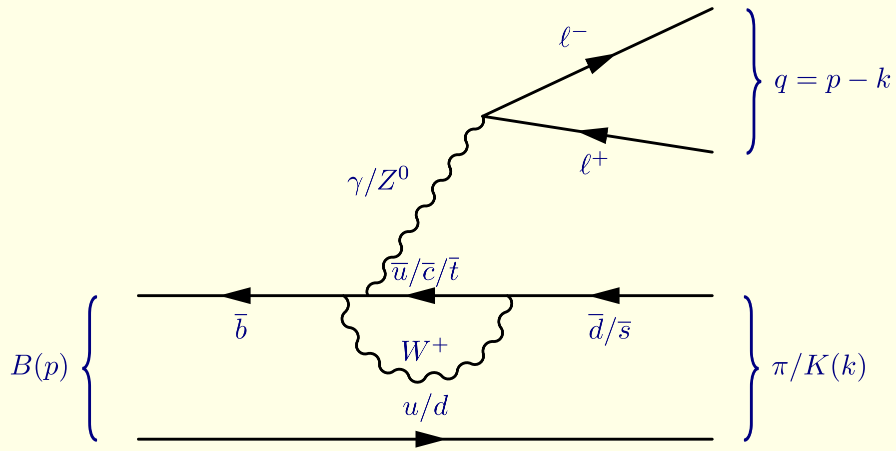


$$\frac{d\Gamma}{dq^2} = (\text{known})\,|V_{ub}|^2\,\left\{f_+(q^2), f_0(q^2)\right\}$$

$B \to \pi(K)\ell^+\ell^-$: flavour-changing neutral currents

Flavor-changing neutral currents $b \to q$ transitions are potentially sensitive to NP effects $B \to K^*\gamma$, $B \to K^*\ell^+\ell^-$,

$$B \to \pi(K)\ell^+\ell^-, B_s \to K\ell^+\ell^-$$



Most important contributions to all this type of decays are expected to come from matrix elements of current (vector, axial and tensor) operators

Need vector, f_+, scalar, f_0 and tensor, f_T form factors from LQCD

$$\frac{d\Gamma}{dq^2} = (\text{known})\, |V_{tb}V^*_{td(s)}|^2 \left\{f_+(q^2),\, f_0(q^2),\, f_T(q^2)\right\}$$

Form factors for $B_{(s)} \to K(\pi)$

Taking Lorentz and discrete symmetries into account:

$$\langle P(k)|\mathcal{V}^\mu|B(p)\rangle = f_+(q^2)\left(p^\mu + k^\mu - \frac{M_B^2 - M_P^2}{q^2}q^\mu\right) + f_0(q^2)\frac{M_B^2 - M_P^2}{q^2}q^\mu$$

$$\langle P(k)|\mathcal{S}|B(p)\rangle = f_0(q^2)\frac{M_B^2 - M_P^2}{m_b - m_q}$$

$$\langle P(k)|\mathcal{T}^{\mu\nu}|B(p)\rangle = f_T(q^2)\frac{2}{M_B + M_P}\left(p^\mu k^\nu - p^\mu k^\nu\right)$$

Form factors for $B_{(s)} \to K(\pi)$

Taking Lorentz and discrete symmetries into account:

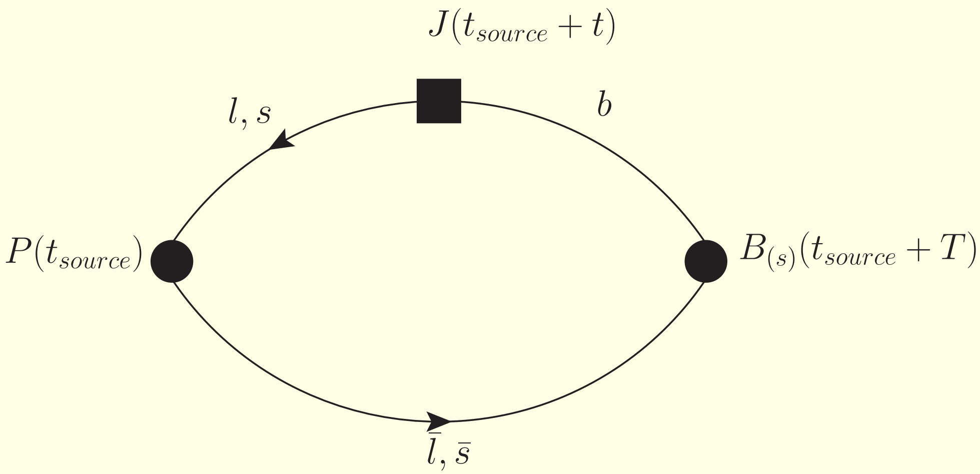
$$
\begin{aligned}
\langle P(k)|\mathcal{V}^\mu|B(p)\rangle &= f_+(q^2)\left(p^\mu + k^\mu - \frac{M_B^2 - M_P^2}{q^2}q^\mu\right) + f_0(q^2)\frac{M_B^2 - M_P^2}{q^2}q^\mu \\
&= \sqrt{2M_B}\left[k_\perp^\mu f_\perp(E_P) + v^\mu f_\parallel(E_P)\right], \quad v = p/M_B \\
\langle P(k)|\mathcal{S}|B(p)\rangle &= f_0(q^2)\frac{M_B^2 - M_P^2}{m_b - m_q} \\
\langle P(k)|\mathcal{T}^{\mu\nu}|B(p)\rangle &= f_T(q^2)\frac{2}{M_B + M_P}\left(p^\mu k^\nu - p^\mu k^\nu\right)
\end{aligned}
$$

and then

$$
\begin{aligned}
f_\perp(E_P) &= \frac{\langle P(k)|\mathcal{V}^i|B(p)\rangle}{\sqrt{2M_B}}\frac{1}{k^i} \\
f_\parallel(E_P) &= \frac{\langle P(k)|\mathcal{V}^0|B(p)\rangle}{\sqrt{2M_B}} \\
f_T(q^2) &= \frac{M_B + M_P}{\sqrt{2M_B}}\frac{\langle P(k)|\mathcal{T}^{0i}|B(p)\rangle}{\sqrt{2M_B}}\frac{1}{k^i}
\end{aligned}
$$

Correlation Functions

Ratios of 3- and 2-point correlation functions



Suppress oscillating and excited states

$$\bar{C}_3^{\mu(\nu)}(t, T; \boldsymbol{k}) \equiv \frac{e^{-E_P^{(0)} t} e^{-M_H^{(0)}(T-t)}}{8} \left[\frac{C_3^{\mu(\nu)}(t, T; \boldsymbol{k})}{e^{-E_P^{(0)} t} e^{-M_H^{(0)}(T-t)}} + \frac{C_3^{\mu(\nu)}(t+1, T; \boldsymbol{k})}{e^{-E_P^{(0)}(t+1)} e^{-M_H^{(0)}(T-t-1)}} + \frac{C_3^{\mu(\nu)}(t+2, T; \boldsymbol{k})}{e^{-E_P^{(0)}(t+2)} e^{-M_H^{(0)}(T-t-2)}} + T \to T+1 \right]$$

$$\bar{R}^{\mu(\nu)} \equiv \frac{\bar{C}_3^{\mu(\nu)}(t, T; \boldsymbol{k})}{\sqrt{\bar{C}_{2,P}(t; \boldsymbol{k}) \bar{C}_{2,H}(T-t; \boldsymbol{k})}} \sqrt{\frac{2E_P^{(0)}}{e^{-E_P^{(0)}} e^{-M_H^{(0)}(T-t)}}}$$

Correlation Functions

Ratios of 3- and 2-point correlation functions

Suppress oscillating and excited states:

$$\bar{C}_3^{\mu(\nu)}(t,T;\boldsymbol{k}) \equiv \frac{e^{-E_P^{(0)}t}e^{-M_H^{(0)}(T-t)}}{8} \left[\frac{C_3^{\mu(\nu)}(t,T;\boldsymbol{k})}{e^{-E_P^{(0)}t}e^{-M_H^{(0)}(T-t)}} + \frac{C_3^{\mu(\nu)}(t+1,T;\boldsymbol{k})}{e^{-E_P^{(0)}(t+1)}e^{-M_H^{(0)}(T-t-1)}} + \frac{C_3^{\mu(\nu)}(t+2,T;\boldsymbol{k})}{e^{-E_P^{(0)}(t+2)}e^{-M_H^{(0)}(T-t-2)}} + T \to T+1\right]$$

$$\bar{R}^{\mu(\nu)} \equiv \frac{\bar{C}_3^{\mu(\nu)}(t,T;\boldsymbol{k})}{\sqrt{\bar{C}_{2,P}(t;\boldsymbol{k})\bar{C}_{2,H}(T-t;\boldsymbol{k})}} \sqrt{\frac{2E_P^{(0)}}{e^{-E_P^{(0)}}e^{-M_H^{(0)}(T-t)}}}$$

$$\to F^{\mu(\nu)}\left[1 - F_P e^{-\Delta M_P t} - F_P e^{-\Delta M_H(T-t)} + \ldots\right] + \mathcal{O}\left(\Delta M_P^2, \Delta M_P \Delta M_H, \Delta M_H^2\right)$$

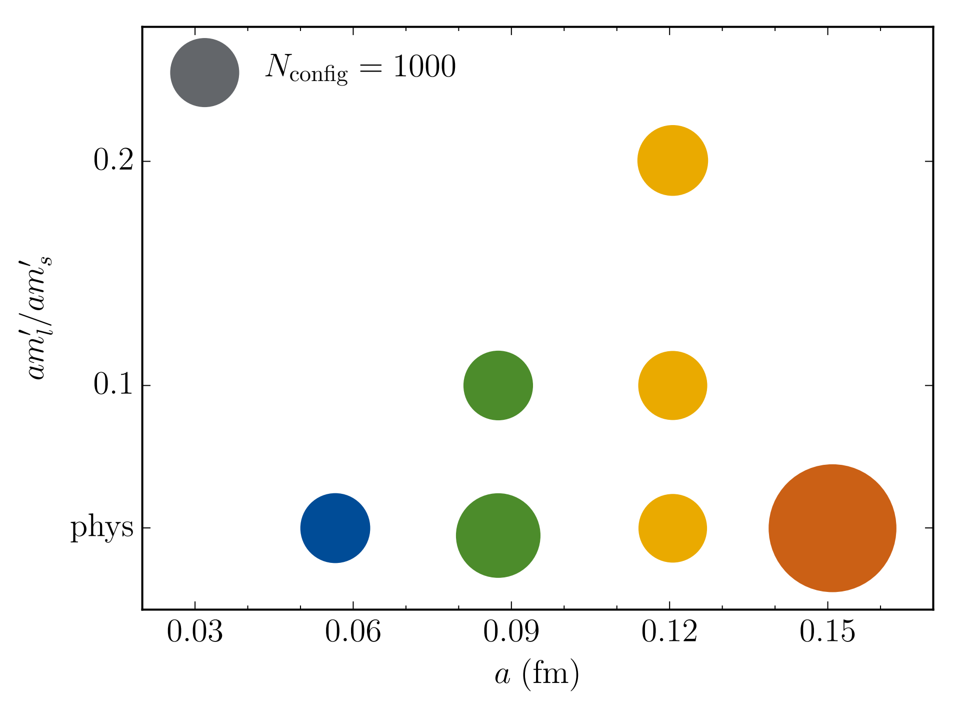
$$\begin{aligned}
f_\perp(E_P) &= Z_\perp \frac{F^i(\boldsymbol{k})}{k^i} \\
f_\parallel(E_P) &= Z_\parallel F^4(\boldsymbol{k}) \\
f_T(E_P) &= Z_T \frac{M_H + M_P}{\sqrt{2M_H}} \frac{F^{4i}(\boldsymbol{k})}{k^i}
\end{aligned}$$

$b \to s(d)$ decays on MILC $N_f = 2+1+1$ HISQ ensembles

A Fermilab b

Analysis led by Zech Gelzer

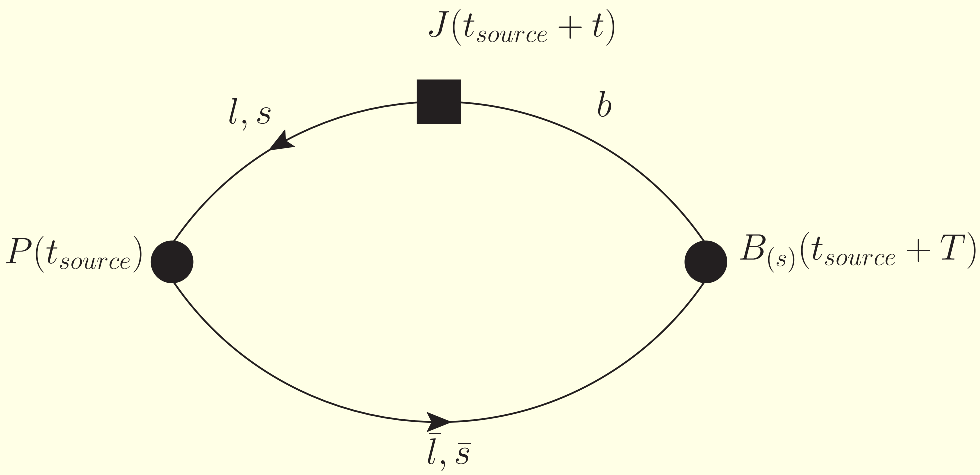
Simulation data



Parameters for physical-mass ensembles

$\approx a$(fm)	$N_s^3 \times N_t$	am'_l	am'_s	am'_c	k'_b	$N_{\text{conf}} \times N_{\text{sour}}$
0.15	$32^3 \times 48$	0.002426	0.06730	0.8447	0.07732	3630×8
0.12	$48^3 \times 64$	0.001907	0.05252	0.6382	0.08574	986×8
0.088	$64^3 \times 96$	0.0012	0.0363	0.432	0.09569	1535×8
0.057	$96^3 \times 192$	0.0008	0.022	0.260	0.10604	1027×8

Correlation Functions and Fits

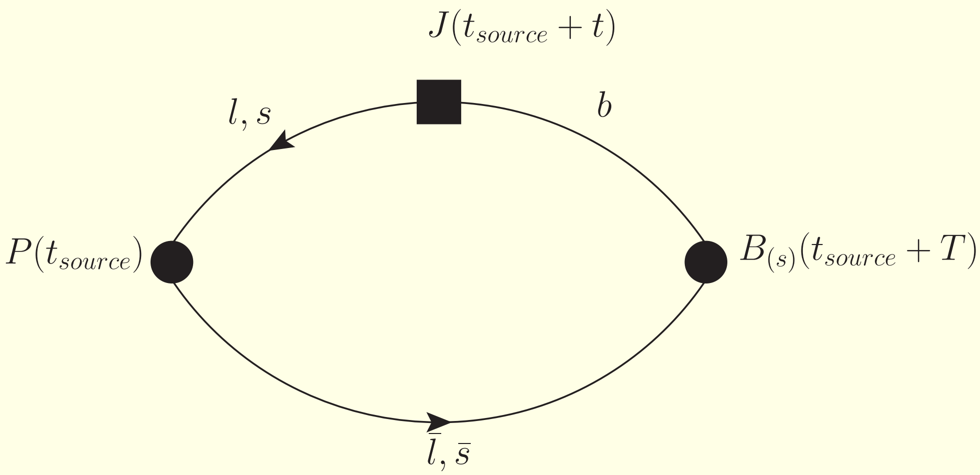


* $J = \mathcal{V}^{\mu}, \mathcal{T}^{0i}$
* Two values of T and 8 time sources.
* Light (HISQ) quarks sources: random wall.
* Heavy (Fermilab) quarks sources: local + 1S-smeared.
* P momenta generated up to $\boldsymbol{k} = (2, 2, 2) \times 2\pi/(aN_s)$ (7 values)

$$C_2^B(t;\mathbf{0}) = \sum_{\boldsymbol{x}} \left\langle \mathcal{O}_B(t,\boldsymbol{x})\, \mathcal{O}_B^{\dagger}(0,\mathbf{0}) \right\rangle , \quad C_2^P(t;\boldsymbol{k}) = \sum_{\boldsymbol{x}} \left\langle \mathcal{O}_P(t,\boldsymbol{x})\, \mathcal{O}_P^{\dagger}(0,\mathbf{0}) \right\rangle e^{-i\boldsymbol{k}\cdot\boldsymbol{x}},$$

$$C_3^{\mu(\nu)}(t,T;\boldsymbol{k}) = \sum_{\boldsymbol{x},\boldsymbol{y}} e^{i\boldsymbol{k}\cdot\boldsymbol{y}} \left\langle \mathcal{O}_P(0,\mathbf{0})\, J^{\mu(\nu)}(t,\boldsymbol{y})\, \mathcal{O}_B^{\dagger}(T,\boldsymbol{x}) \right\rangle$$

Correlation Functions and Fits



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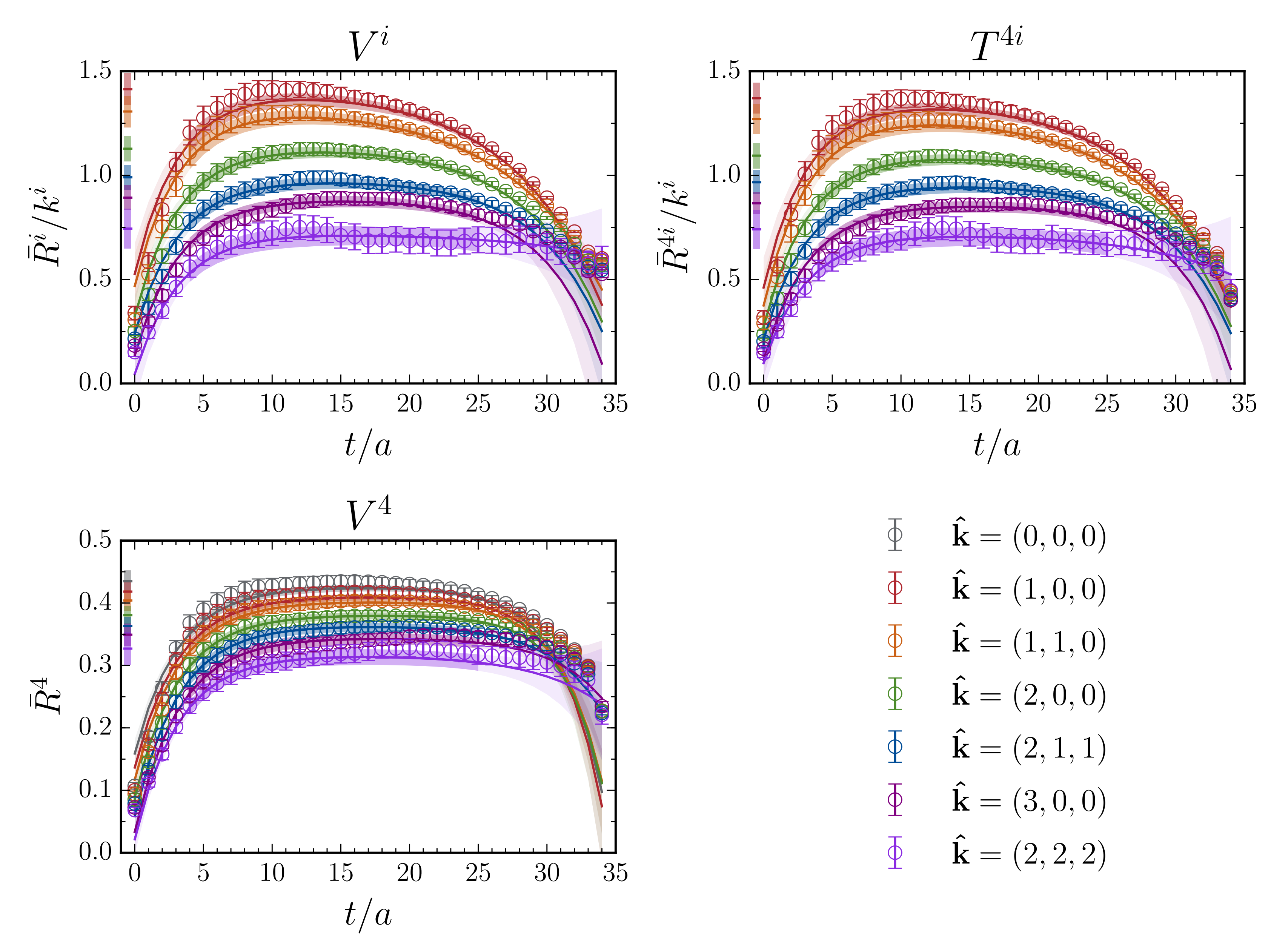
$$C_3^{\mu(\nu)}(t, T; \boldsymbol{k}) = \sum_{\boldsymbol{x},\boldsymbol{y}} e^{i\boldsymbol{k}\cdot\boldsymbol{y}} \left\langle \mathcal{O}_P(0, \mathbf{0})\, J^{\mu(\nu)}(t, \boldsymbol{y})\, \mathcal{O}_B^\dagger(T, \boldsymbol{x}) \right\rangle$$

* Mostly nonperturbative matching: $Z_J = \rho_J \sqrt{Z_{V^4_{bb}} Z_{V^4_{qq}}}$ with ρ_J calculated perturbat. at one loop and $Z_{V^4_{bb}}$, $Z_{V^4_{qq}}$ nonperturbatively.

  ** Introduce a blinding factor through the renormalization factors.

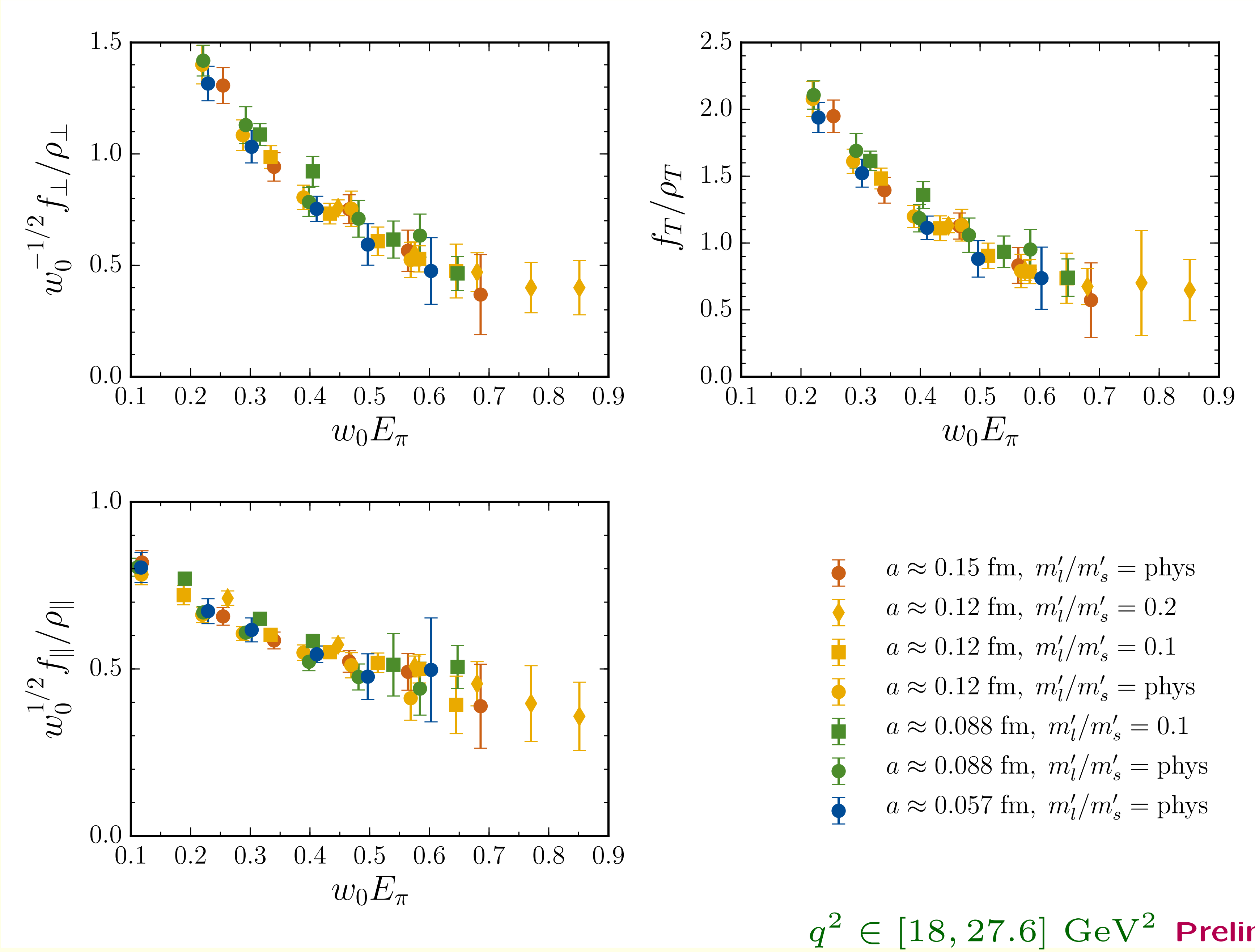
Correlators and Fits: $B \to K$ on phys. $a = 0.057$ fm

Form factors from direct (combined) fits to all correlation functions: **Preliminary**



(consistent with fits to ratios $\bar{R}$ of 3-point over 2-point functions)

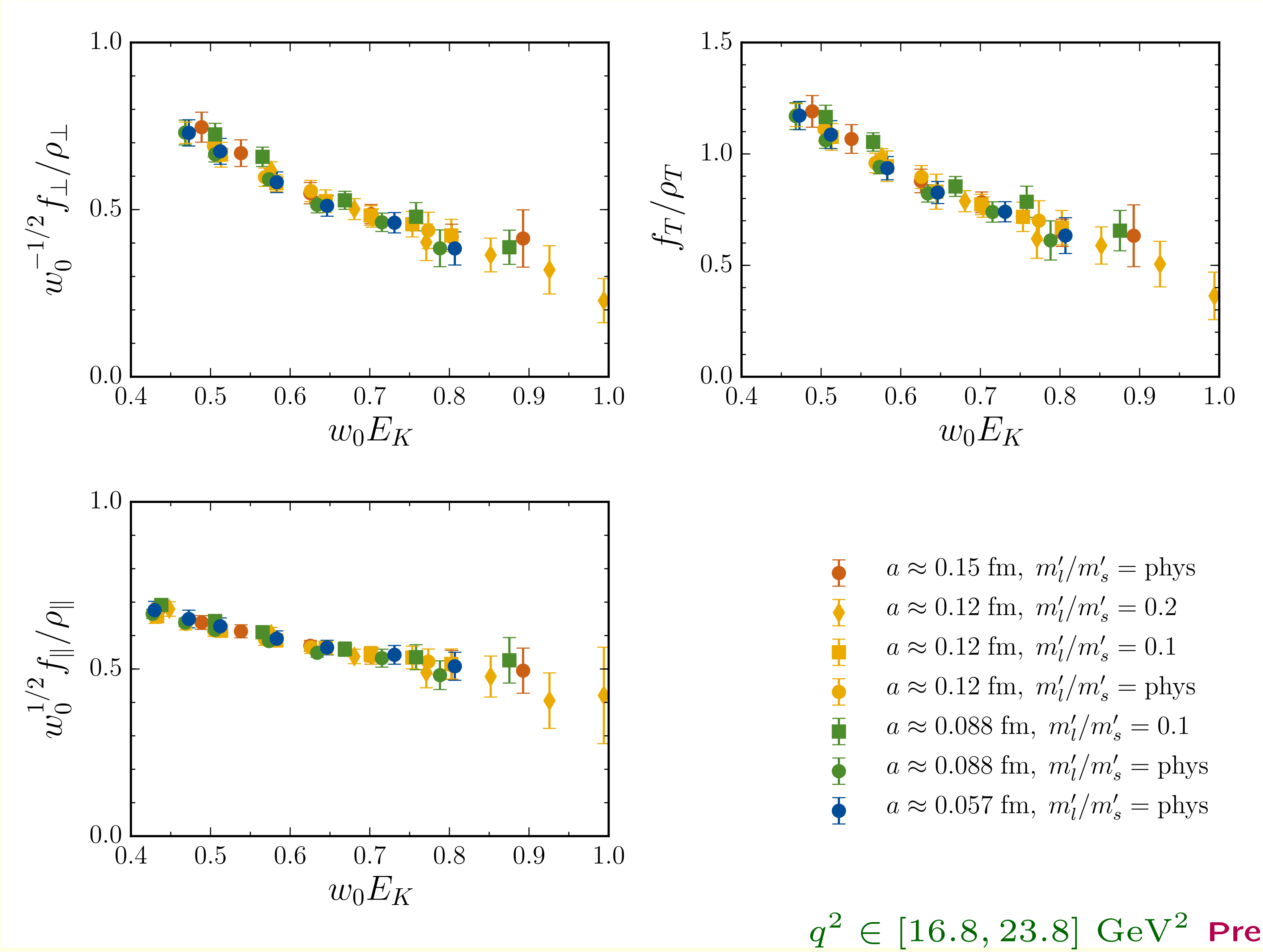
Form factors for $B \to \pi$ Preliminary



$q^2 \in [18, 27.6]$ GeV^2 **Preliminary**

Note: Correct renomalization ρ_J factors missing. Only $\sqrt{Z_{V^4_{bb}} Z_{V^4_{qq}}}$ included.

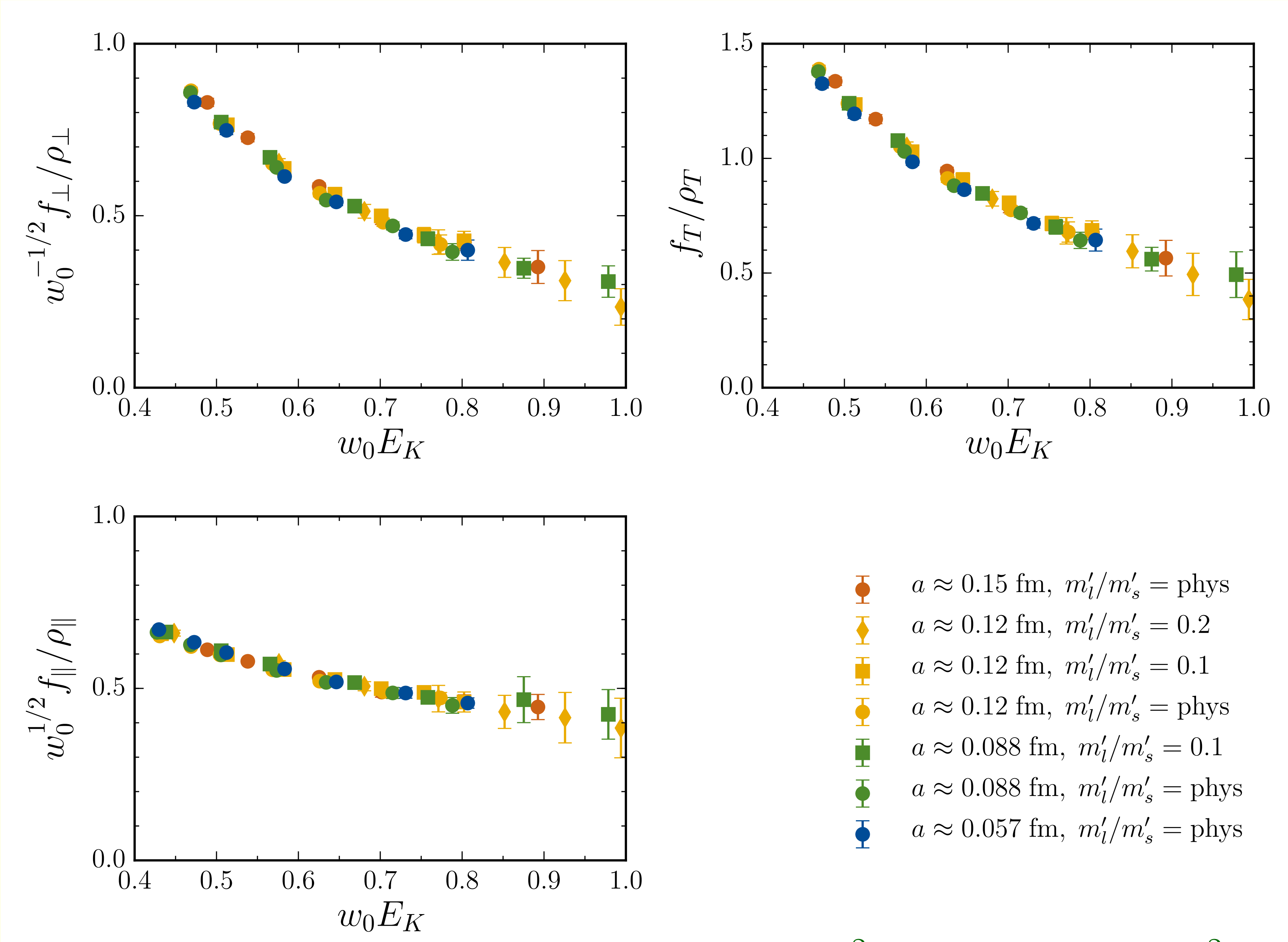
Form factors for $B \to K$ Preliminary



$q^2 \in [16.8, 23.8]$ GeV^2 **Preliminary**

Note: Correct renomalization ρ_J factors missing. Only $\sqrt{Z_{V^4_{bb}} Z_{V^4_{qq}}}$ included.

Form factors for $B_s \to K$ Preliminary



$q^2 \in [16.8, 23.8]$ GeV^2 **Preliminary**

Note: Correct renomalization ρ_J factors missing. Only $\sqrt{Z_{V^4_{bb}} Z_{V^4_{qq}}}$ included.

Chiral-continuum interp./extrap.: $B_s \to K$

We extrapolate the form factors to the continuum and interpolate to the physical quark masses using $SU(2)$ HMrSχPT

$$
\begin{aligned}
f_J &= f_J^{(0)} \times \left(1 + \delta f_J^{logs} + \delta f_J^{NLO} + \delta f_J^{N^2LO} + \ldots\right) \times \left(1 + \delta f_J^{b}\right) \\
f_J^{(0)} &= \frac{g_\pi}{f_\pi (E_P + \Delta_P^*)} \\
\delta f_J^{NLO} &= c_J^l \chi_l + c_J^s \chi_s + c_J^E \chi_E + c_J^{E^2} \chi_E^2 + c_J^{a^2} \chi_{a^2}
\end{aligned}
$$

* $\Delta_P^* = \left(M_{B^*}^2 - M_{B_s}^2 - M_P^2\right)/(2M_{B_s})$, where M_{B^*} is a 1^- or 0^+ mass.

* f_J^{logs}: nonanalytic functions of m_l, a.

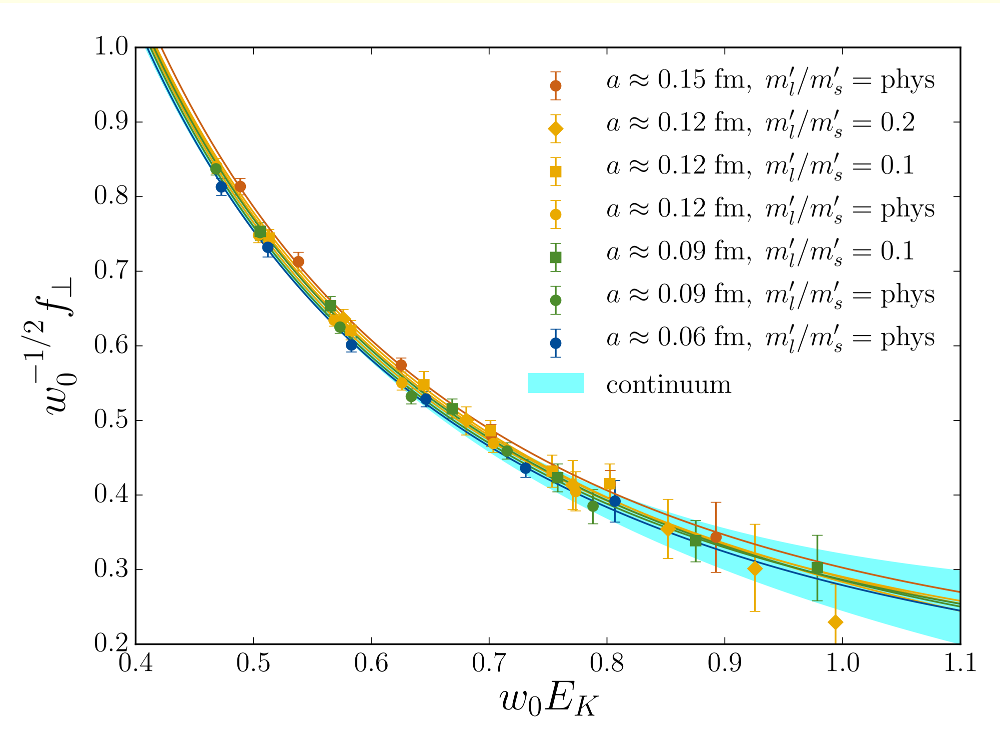
* f_J^b: b-quark discretization effects,

$$\mathcal{O}\left((a\Lambda)^2, \alpha_s a\Lambda, \alpha_s (a\Lambda)^2\right) \times \text{ mistmach functions } (am_b, \alpha_s) \times h_J^i.$$

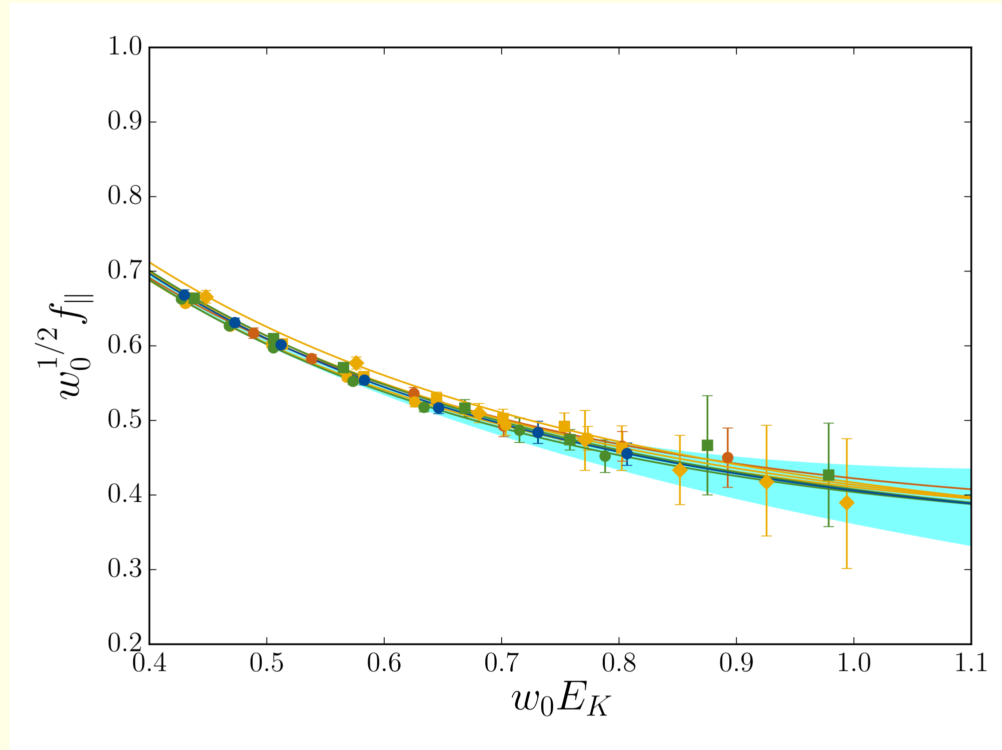
* Perturbative part of Z_J implemented with priors: $\tilde{\rho}_J = 1 + \tilde{\rho}_J^{(1)} \alpha_s + \tilde{\rho}_J^{(2)} \alpha_s^2$

Chiral-continuum interp./extrap.: $B_s \to K$

Preliminary



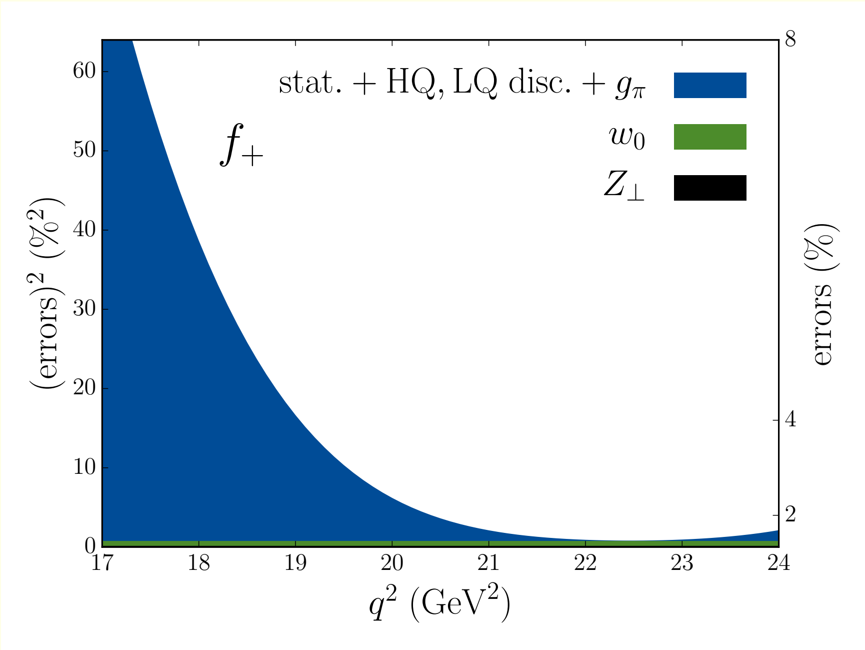
Preliminary

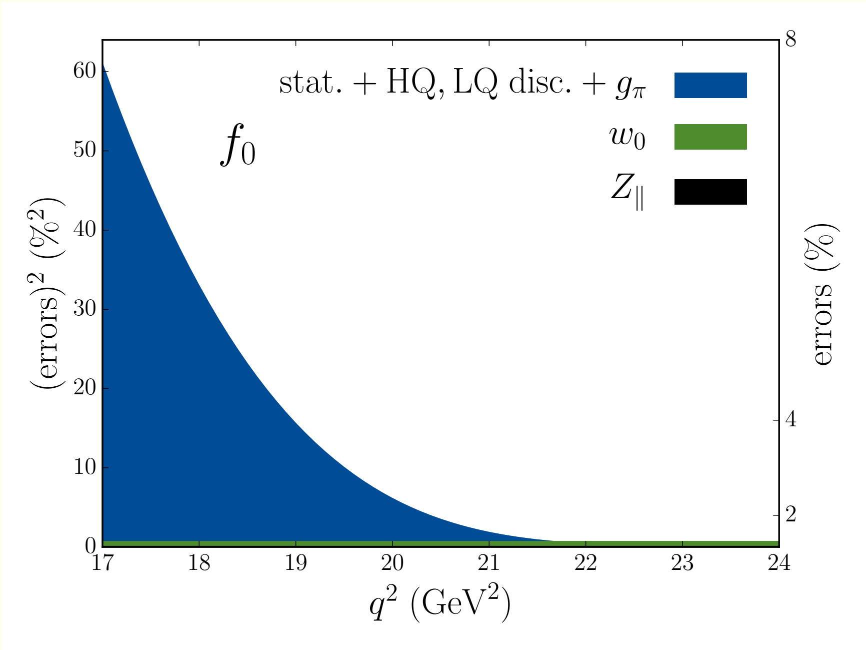


- $f_\perp$ and $f_\parallel$ fit simultaneously.
- Central fit: $NLO\ SU(2)$ HMrSχPT $+ N^2LO$ analytic terms.

Error budget for $B_s \to K$

Preliminary and missing perturbative ρ_J factors





Error budget for $B_s \to K$

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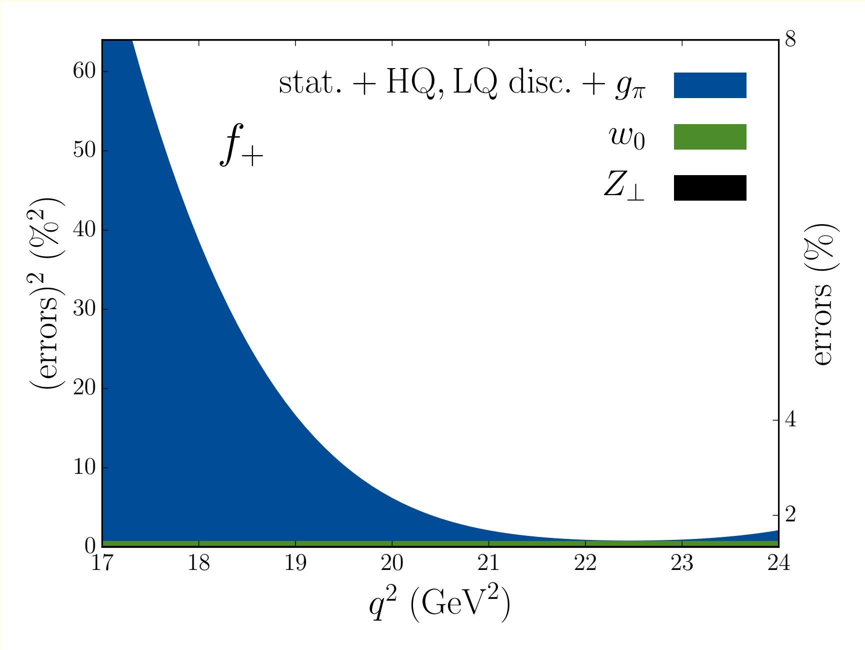

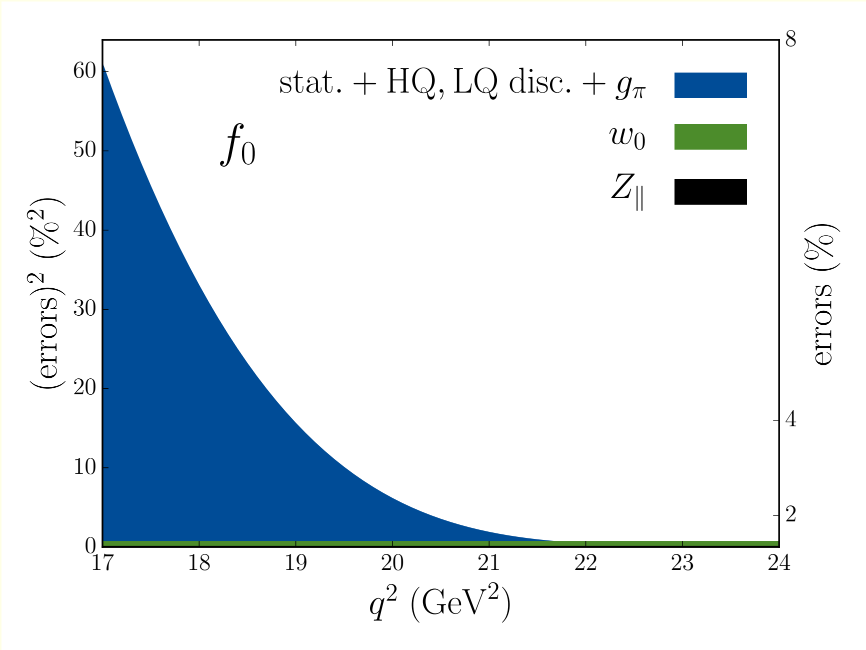

Compared to previous FNAL/MILC:

Similar $a \to$ similar statistics, smaller discretization (HISQ)

Physical m_l' ensembles $\to$ remove chiral extrapolation error

Outlook

On-going calculation of form factors f_0, f_+, f_T for $B \to \pi$, $B \to K$, $B_s \to K$ with Fermilab b and HISQ l, s, c on HISQ $N_f = 2+1+1$ MILC ensembles.

* 4 lattice spacings, 7 ensembles (including 4 with phys. masses)
* Mostly non-perturbative renormalization.
* Chiral+continuum fits: NLO `HMrSChPT` in SU(2) hard-kaon limit + NNLO analytic terms.

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* 4 lattice spacings, 7 ensembles (including 4 with phys. masses)
* Mostly non-perturbative renormalization.
* Chiral+continuum fits: NLO HMrSChPT in SU(2) hard-kaon limit + NNLO analytic terms.

Need to do

* Renormalization coefficients: calculate ρ_J, get $Z_{V^4_{bb,qq}}$ with better stat.
* z expansions and finalize systematic error budgets.
* Phenomenology: $|V_{ub}|$, $|V_{td}|$, $|V_{ts}|$, confront branching fractions and angular observables with experiment, make predictions for the not yet measured quantities.
* Correlated ratios for different processes

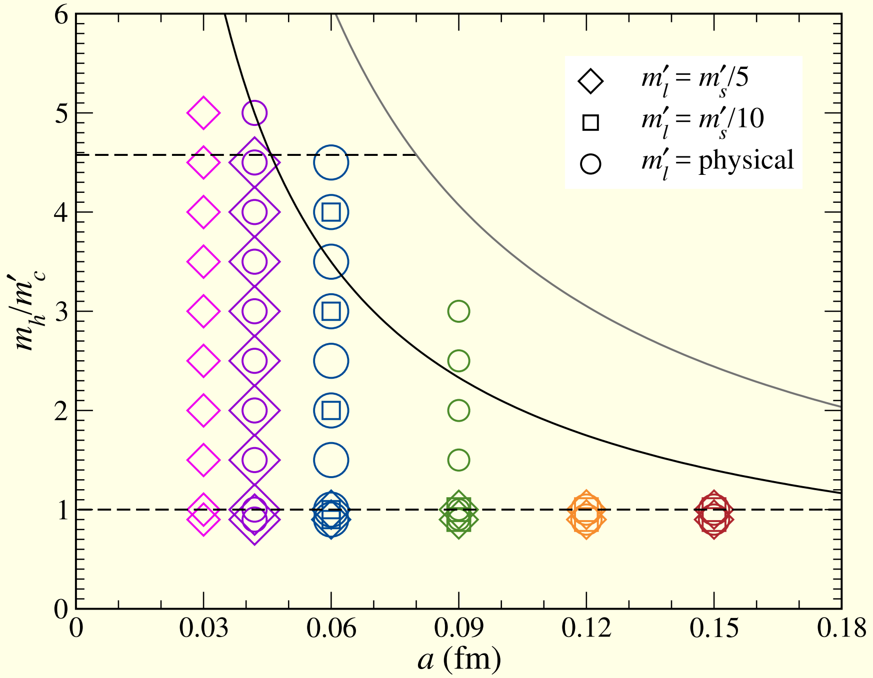
$h \to s(d)$ decays on MILC $N_f = 2+1+1$ HISQ ensembles

B HISQ heavy

Analysis led by William Jay

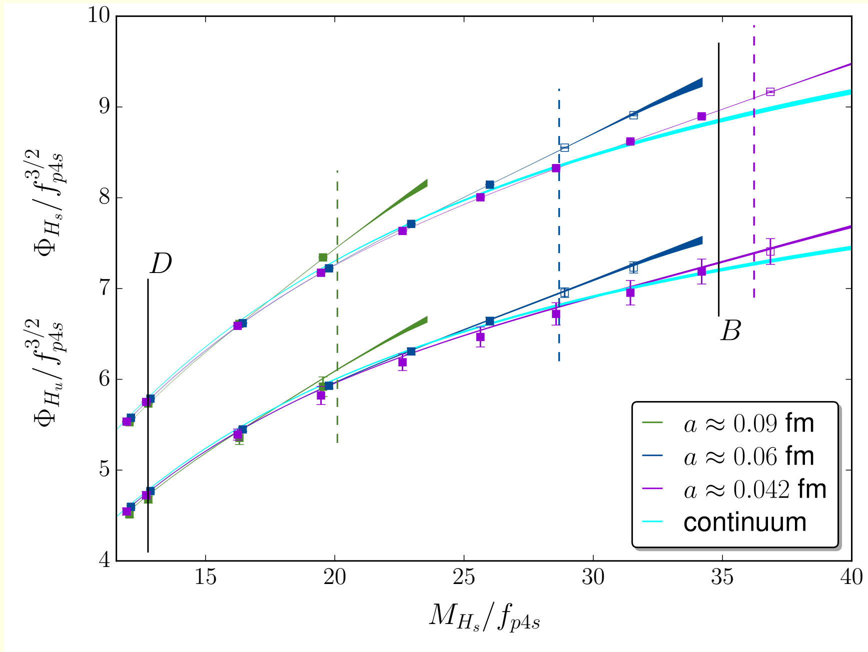
All-HISQ decay constants analysis

It is feasible to do B physics with HISQ: Decay constants



Avoid large lattice artifact including data with $am_h < 0.9$ (black solid line)

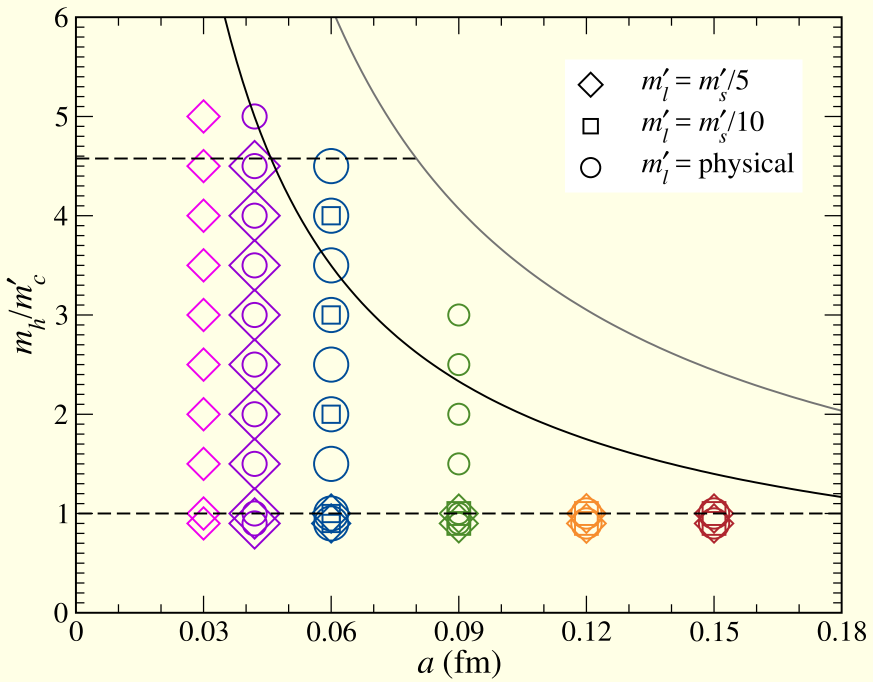
(physical mass ensembles)



Use HQET-inspired model for extrapolating to the B mass.

All-HISQ decay constants analysis

It is feasible to do B physics with HISQ: Decay constants



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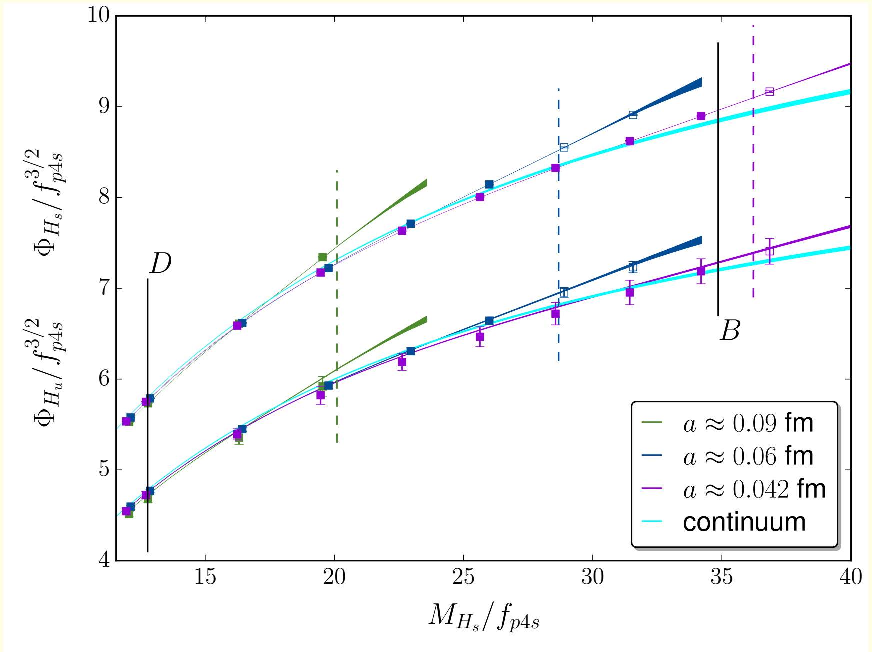


Use HQET-inspired model for extrapolating to the B mass.

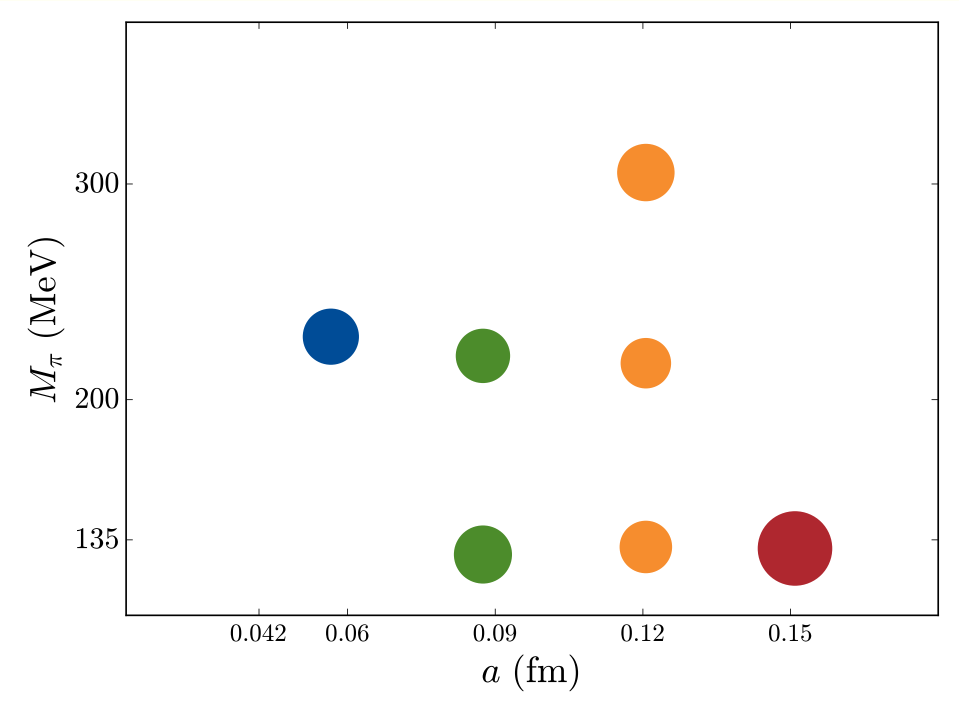
* Errors: 0.2-0.3% for c decay constants, 0.6-0.7% for b decay constants.

Largest systematic errors: choice of fit model (continuum extrapolation errors), correlator fits (excited-state contamination).

All-HISQ decay constants analysis

($f_{\pi,PDG}$ also important systematic for charmed decay constants)

* Controversy with EW radiative corrections needed to exract $|V_{ud}|$ from superallowed β decays: **Seng, Gorchtein, Patel, Ramsey-Musolf** 1807.10197, **Czarnecki, Marciano, Sirlin 1907.06737**



Simulation data

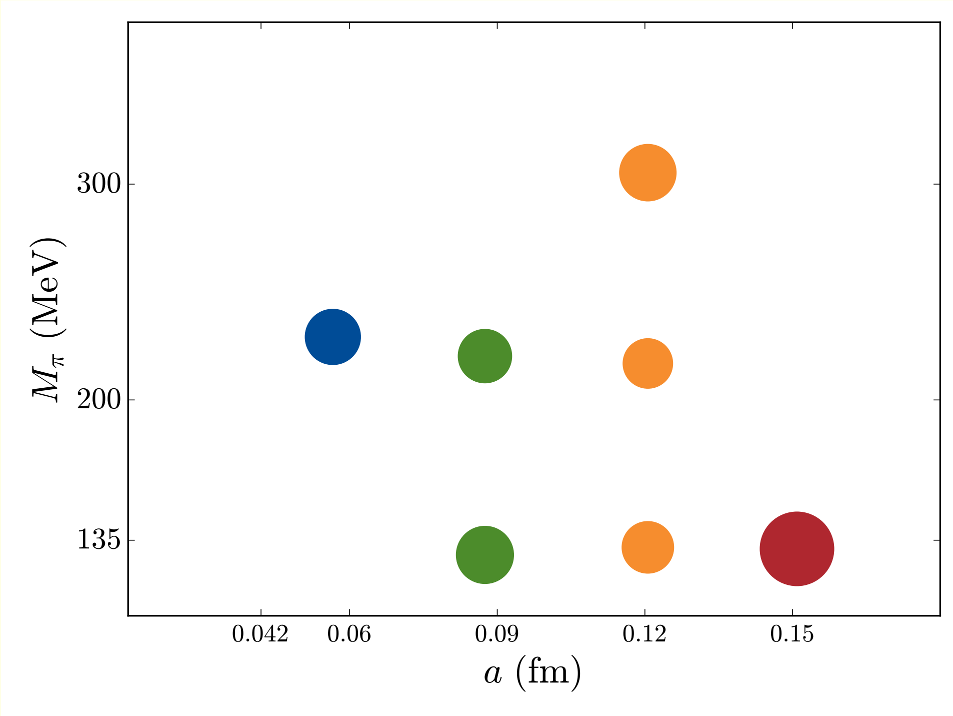
Data generated for all-HISQ heavy semileptonic project until middle July 2019

$$B \to K,\ B \to \pi,\ B_s \to K$$

(and $D \to K$, $D \to \pi$, $D_s \to K$)

$$B_{(s)} \to D_{(s)}$$

$\approx a(\mathrm{fm})$	$N_S^3 \times N_t$	am_l'	am_s'	am_c'	am_h/am_c
0.15	$32^3 \times 48$	0.002426	0.06730	0.8447	0.9, 1, 1.1
0.12	$24^3 \times 64$	0.0102	0.0509	0.635	0.9, 1, 1.4
0.12	$32^3 \times 64$	0.00507	0.0507	0.628	0.9, 1, 1.4
0.12	$48^3 \times 64$	0.001907	0.05252	0.6382	0.9, 1, 1.4
0.088	$48^3 \times 96$	0.00363	0.0363	0.430	0.9, 1, 1.5, 2, 2.5
0.088	$64^3 \times 96$	0.0012	0.0363	0.432	0.9, 1, 1.5, 2, 2.5
0.057	$64^3 \times 144$	0.0024	0.024	0.286	0.9, 1, 2, 3, 4



Simulation data

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$$B \to K,\ B \to \pi,\ B_s \to K$$

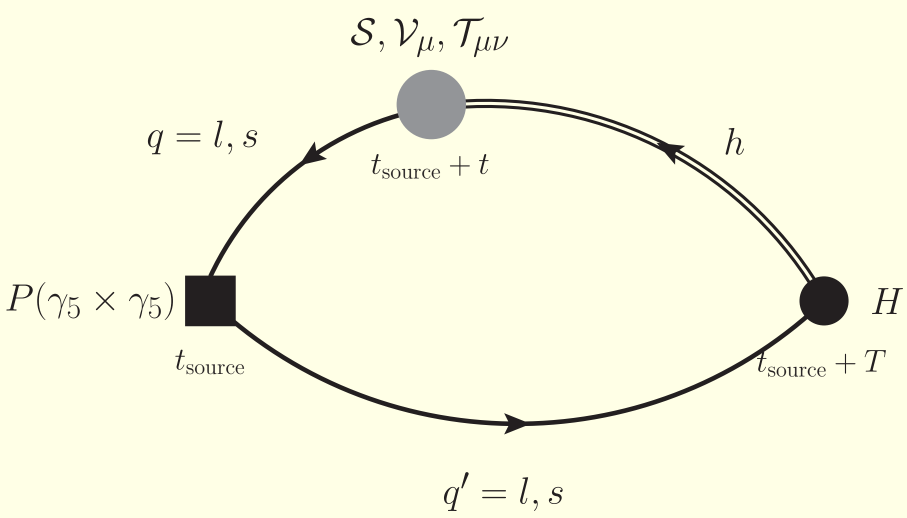
(and $D \to K$, $D \to \pi$, $D_s \to K$)

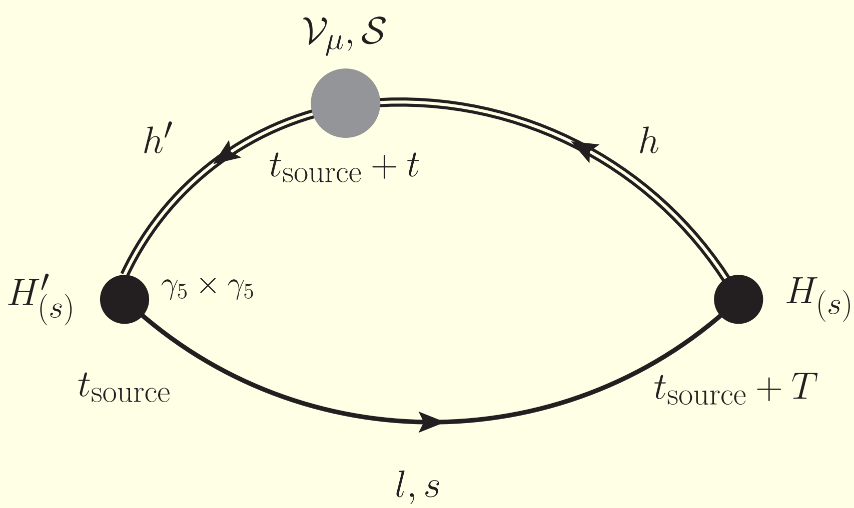
$$B_{(s)} \to D_{(s)}$$

Include partially-quenched data: fine-tuning light quark masses, isospin-breaking effects.

$\approx a(\mathrm{fm})$	$N_S^3 \times N_t$	am'_l	am'_s	am'_c	am_h/am_c
0.15	$32^3 \times 48$	0.002426	0.06730	0.8447	0.9, 1, 1.1
0.12	$24^3 \times 64$	0.0102	0.0509	0.635	0.9, 1, 1.4
0.12	$32^3 \times 64$	0.00507	0.0507	0.628	0.9, 1, 1.4
0.12	$48^3 \times 64$	0.001907	0.05252	0.6382	0.9, 1, 1.4
0.088	$48^3 \times 96$	0.00363	0.0363	0.430	0.9, 1, 1.5, 2, 2.5
0.088	$64^3 \times 96$	0.0012	0.0363	0.432	0.9, 1, 1.5, 2, 2.5
0.057	$64^3 \times 144$	0.0024	0.024	0.286	0.9, 1, 2, 3, 4

Correlation Functions





* Random wall sources.

* 4 values of T generated, 3 more being generated in some ensembles.

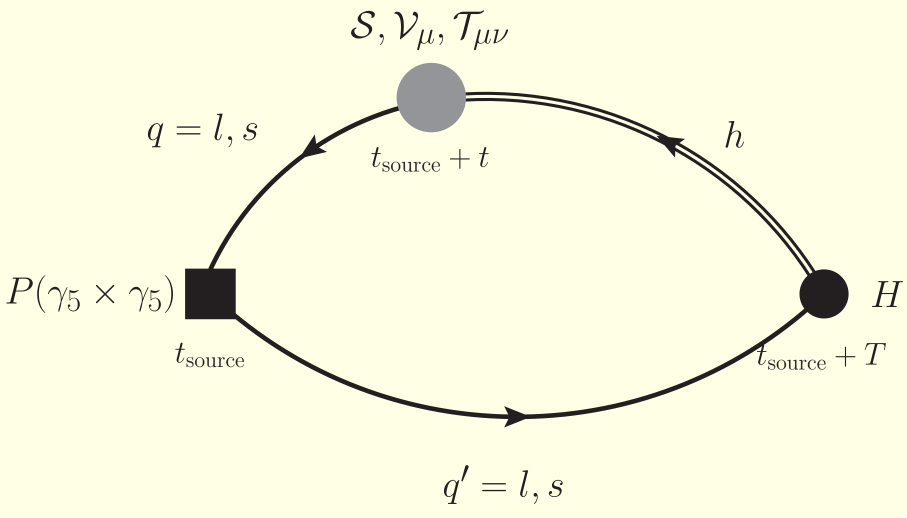
* 6-8 time sources.

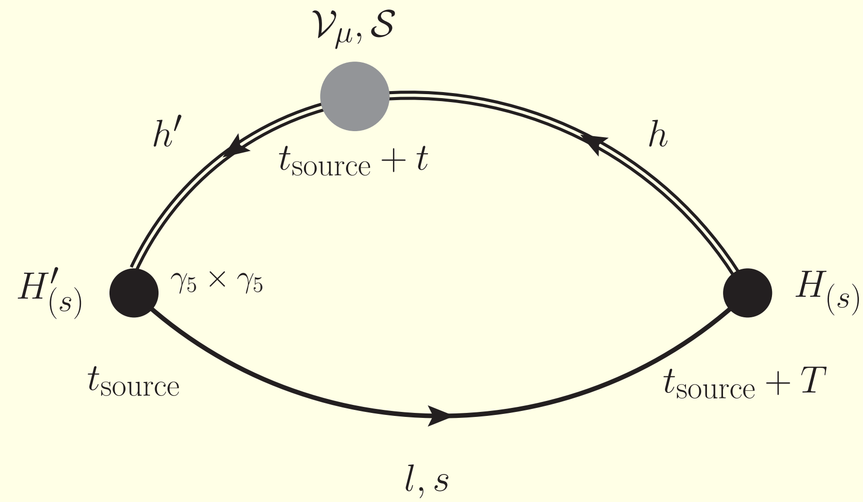
* **Local** scalar and temporal vector currents, **point-split** spatial vector currents.

** $\mathcal{S}$ and $\mathcal{V}_i$ are taste singlets $\rightarrow$ parent $H_{(s)}$ has spin-taste $\gamma_5 \times \gamma_5$ (Goldstone meson).

** $\mathcal{V}_0$ and $\mathcal{T}_{\mu\nu}$ have taste γ_0 $\rightarrow$ parent $H_{(s)}$ has spin-taste $\gamma_0\gamma_5 \times \gamma_0\gamma_5$ (non-Goldstone meson).

Correlation Functions





* P momenta data generated up to $\boldsymbol{k} = (4,0,0) \times 2\pi/(aN_s)$ (8 values)

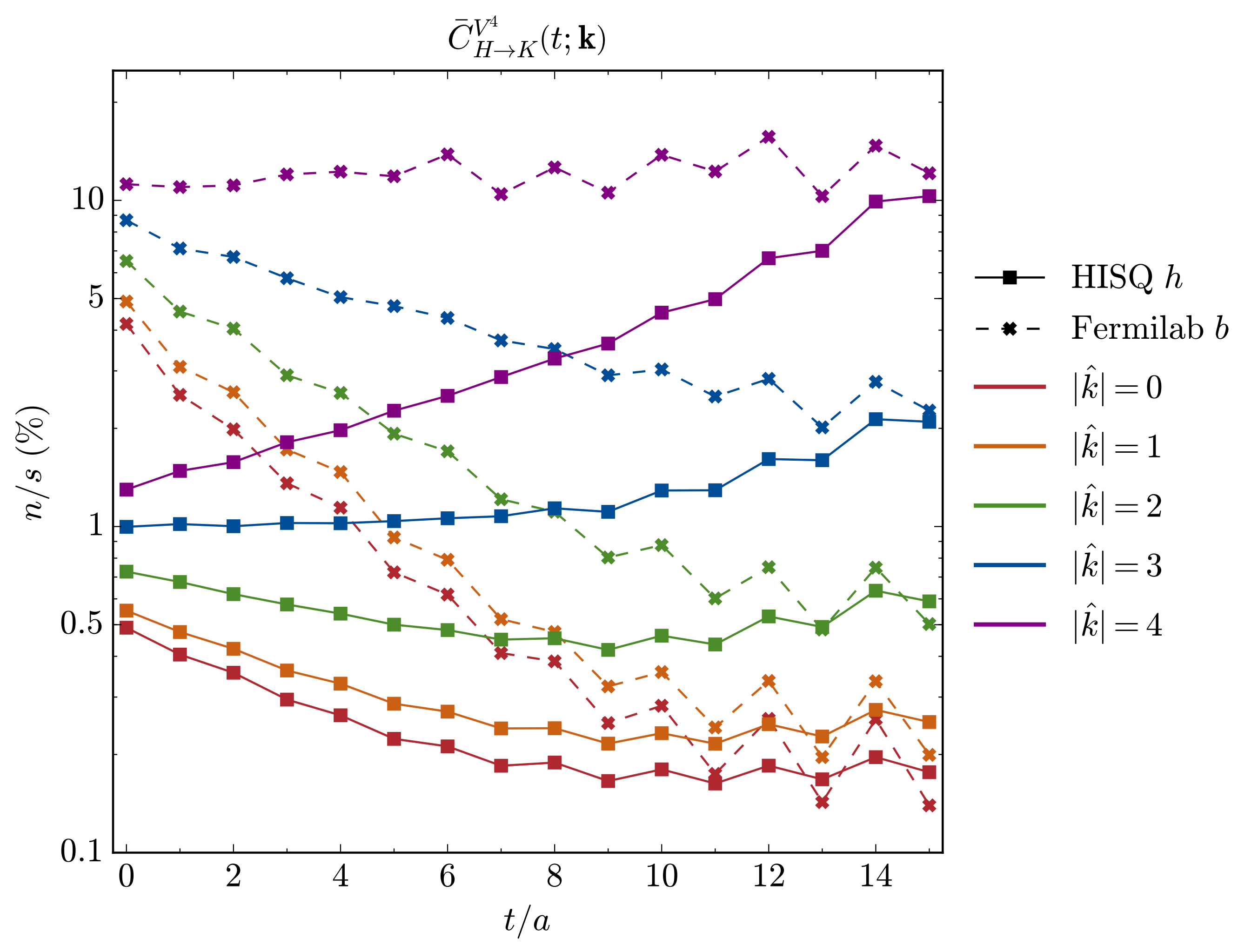
$$C_2^{H_{(s)}}(t;\boldsymbol{k}) = \sum_{\boldsymbol{x}} \left\langle \mathcal{O}_{H_{(s)}}(t,\boldsymbol{x})\, \mathcal{O}^\dagger_{H_{(s)}}(0,\boldsymbol{0}) \right\rangle e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}, \quad C_2^{P}(t;\boldsymbol{k}) = \sum_{\boldsymbol{x}} \left\langle \mathcal{O}_{P}(t,\boldsymbol{x})\, \mathcal{O}^\dagger_{P}(0,\boldsymbol{0}) \right\rangle e^{-i\boldsymbol{k}\cdot\boldsymbol{x}},$$

$$C_3^{\mu(\nu)}(t,T;\boldsymbol{k}) = \sum_{\boldsymbol{x},\boldsymbol{y}} e^{i\boldsymbol{k}\cdot\boldsymbol{y}} \left\langle \mathcal{O}_P(0,\boldsymbol{0})\, J^{\mu(\nu)}(t,\boldsymbol{y})\, \mathcal{O}^\dagger_{H_{(s)}}(T,\boldsymbol{x}) \right\rangle$$

$$\tilde{C}_3^{\mu}(t,T;\boldsymbol{k}) = \sum_{\boldsymbol{x},\boldsymbol{y}} e^{i\boldsymbol{k}\cdot\boldsymbol{y}} \left\langle \mathcal{O}_{H'_{(s)}}(0,\boldsymbol{0})\, J^{\mu}(t,\boldsymbol{y})\, \mathcal{O}^\dagger_{H_{(s)}}(T,\boldsymbol{x}) \right\rangle$$

Comparison of noise-to-signal at $a \approx 0.12\text{fm}$

Fermilab heavy b vs HISQ h



* Physical l, s and c masses
* Source-sink separation $T = 15, 16$.
* $m_h = 1.4 m_c$

Typical fit range:

$$\sim [2 - 13]$$

To suppress oscillating-state contributions for better visualization, an averaging scheme has been applied over neighboring time slices.

Extracting the form factors

Using the Ward identity $q_\mu \langle P|\mathcal{V}^\mu_{\rm lat}|H\rangle Z_{V^\mu_{\rm lat}} = (m_h - m_q)\langle P|\mathcal{S}|H\rangle$ and the definition of the form factors

$$
\begin{aligned}
f_0(q^2) &= \frac{m_h - m_q}{M_H^2 - M_P^2}\langle P|\mathcal{S}|H\rangle_{q^2} \quad \text{no renor. needed} \\
f_+(q^2) &= \frac{1}{2M_H}\,\frac{(M_H - M_P)(m_h - m_q)\langle P|\mathcal{S}|H\rangle - q^2 Z_{V^0}\langle P|\mathcal{V}^0|H\rangle}{\boldsymbol{k}^2} \\
&= \frac{1}{2M_H}\left[Z_{V^0}\langle P|\mathcal{V}^0|H\rangle + \frac{M_H - M_P}{k^i} Z_{V^i}\langle P|\mathcal{V}^i|H\rangle\right] \\
f_T(q^2) &= \frac{M_H + M_P}{\sqrt{2M_H}} Z_T \frac{\langle P|\mathcal{T}^{0i}|H\rangle}{\sqrt{2M_H}}
\end{aligned}
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* For the local temporal current, with both mesons at rest:

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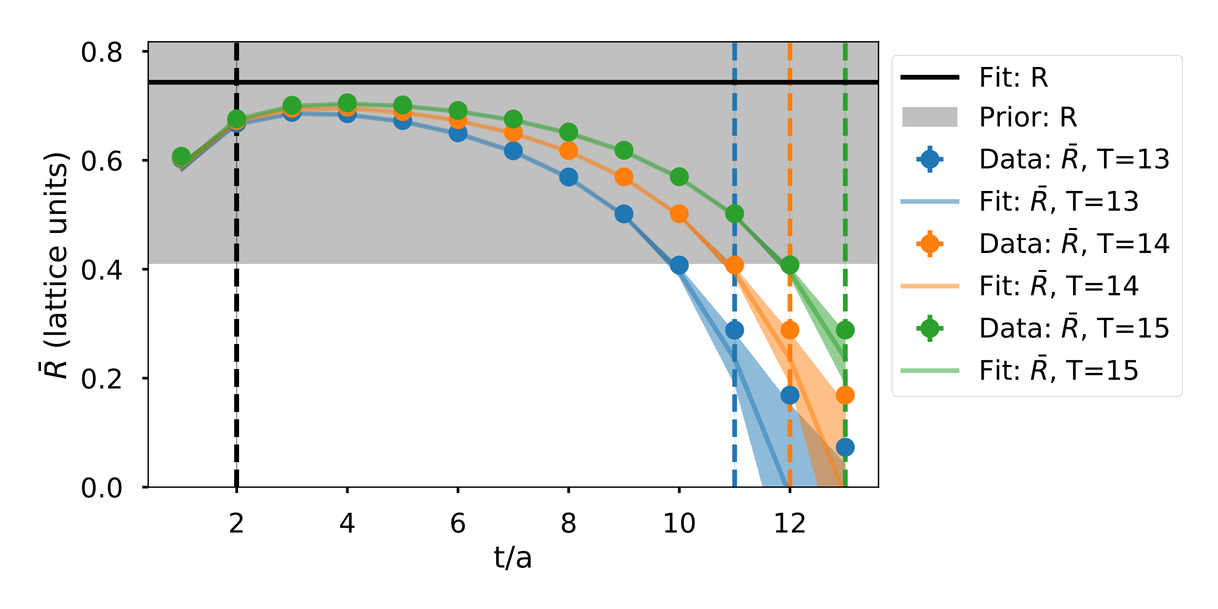
$$

* Renormalization factors Z_{V^i}, Z_T: Under investigation.
  ** First step: Mostly non-perturbative renormalization?

Correlation Functions and Fits

Example: $D \to \pi$ at $a \approx 0.12\mathrm{fm}$ with phys. quark masses

S correlation function for $\boldsymbol{k} = (1, 0, 0)$ (f_0)



Preliminary

* Combined correlated fit to 2-point and 3-point functions

(ratio $\bar{R}$ 3pt- and 2-point functions for visualization)

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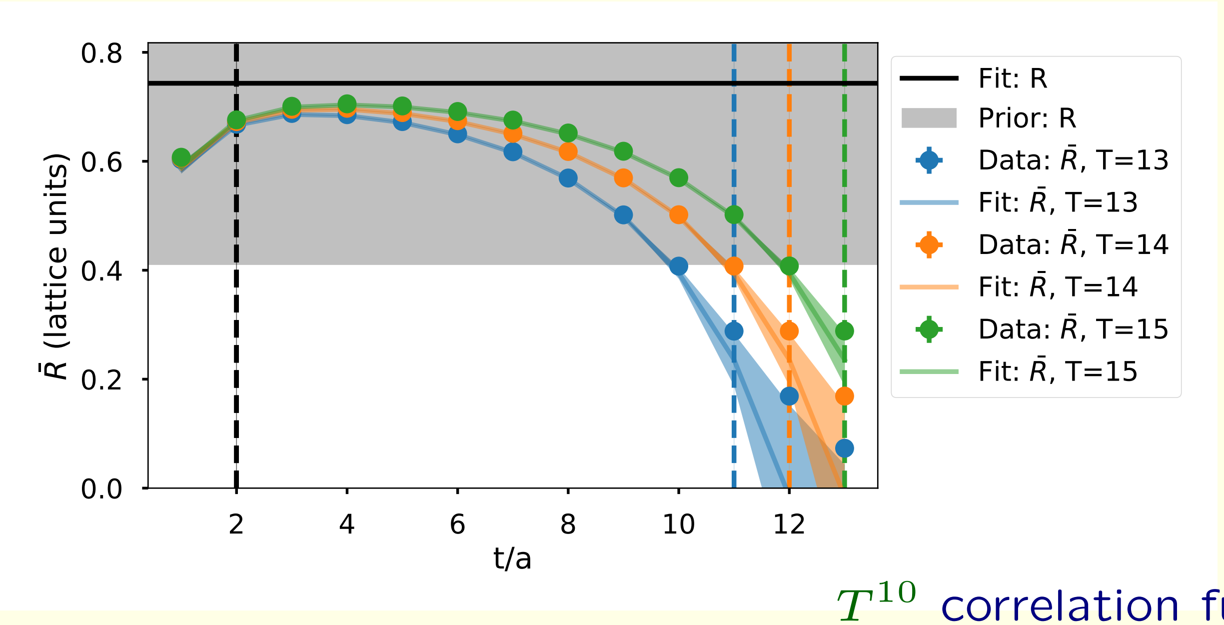

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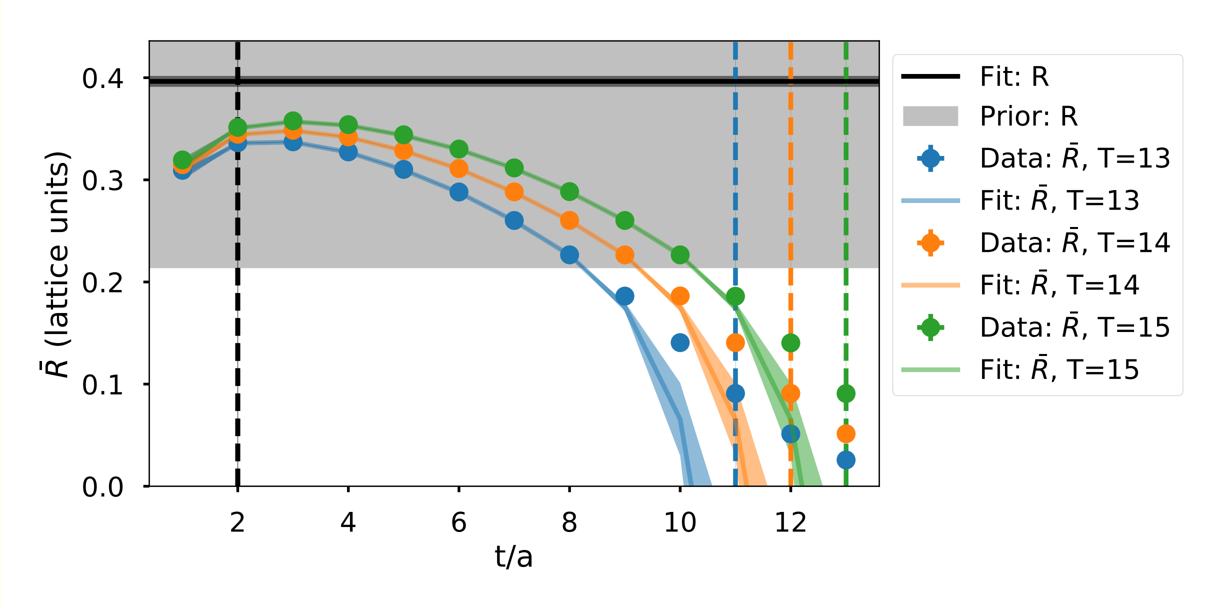
T^{10} correlation function for $\boldsymbol{k} = (1,0,0)$



* Similar results for all currents and most of the momenta.

* Add larger values of T: Better constrain of ground state contributions

Correlation Functions and Fits

Example: 3-point correlation function with $\mathcal{S}$ insertion and $\boldsymbol{k} = (1, 0, 0)$

$2 + 1$ states for π channel

and $4 + 2$ for D channel

Check stability

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$2 + 1$ states for π channel

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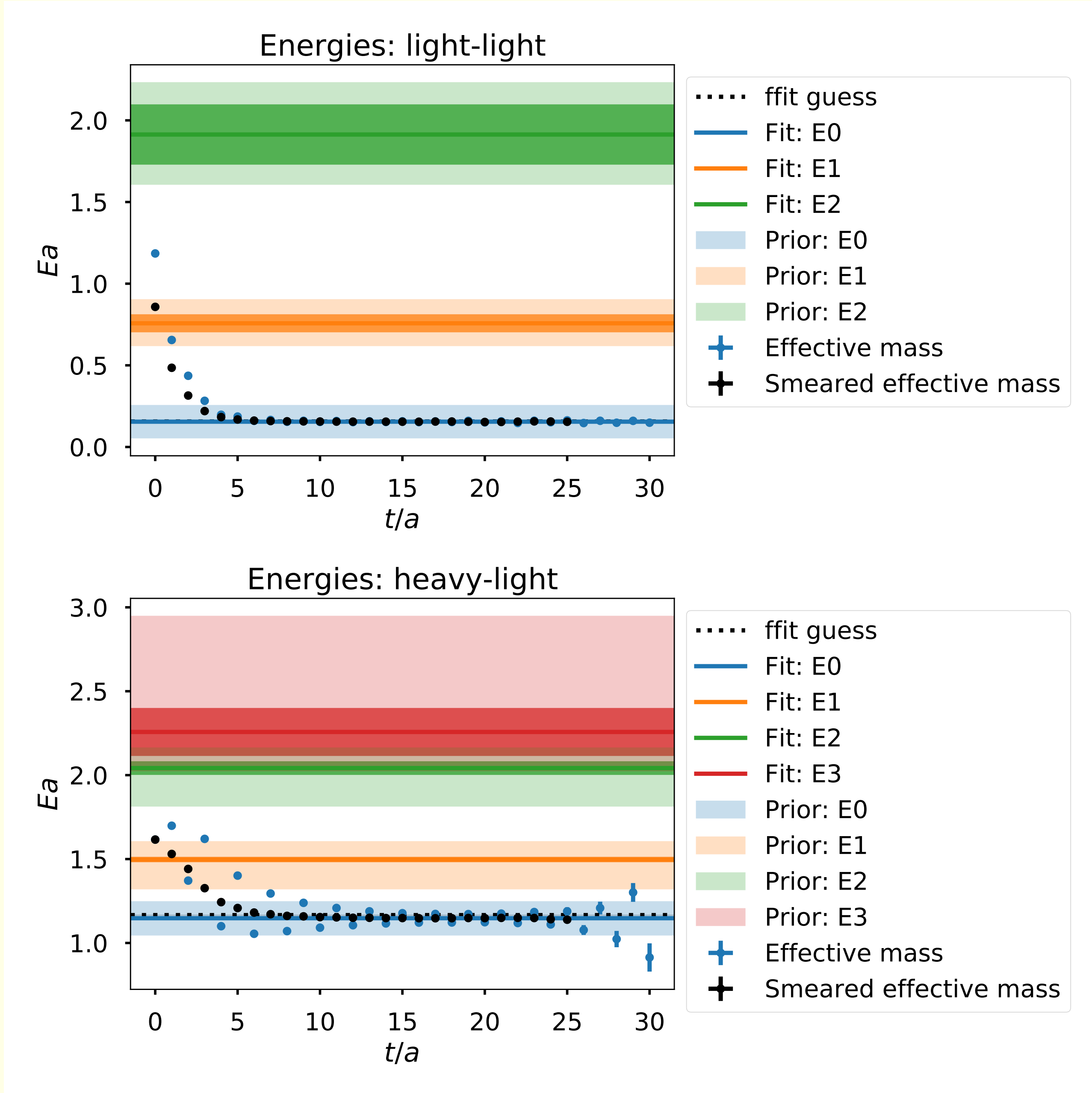
Check stability

* **Light bands:** broad priors
  (central value from 2-point fits)

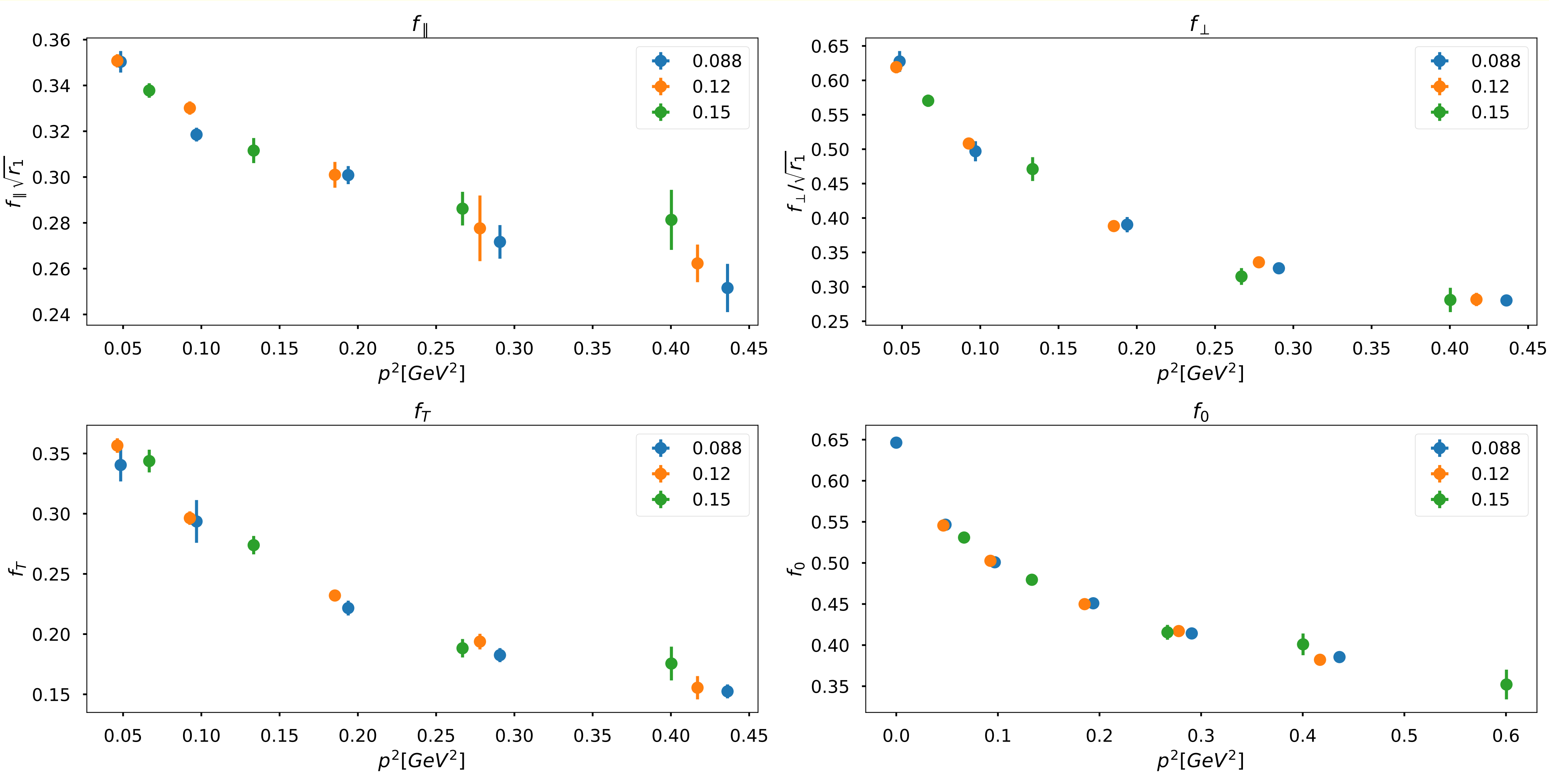
* **Dark bands:** (combined) fit values.



Preliminary: $D \to \pi$ form factors

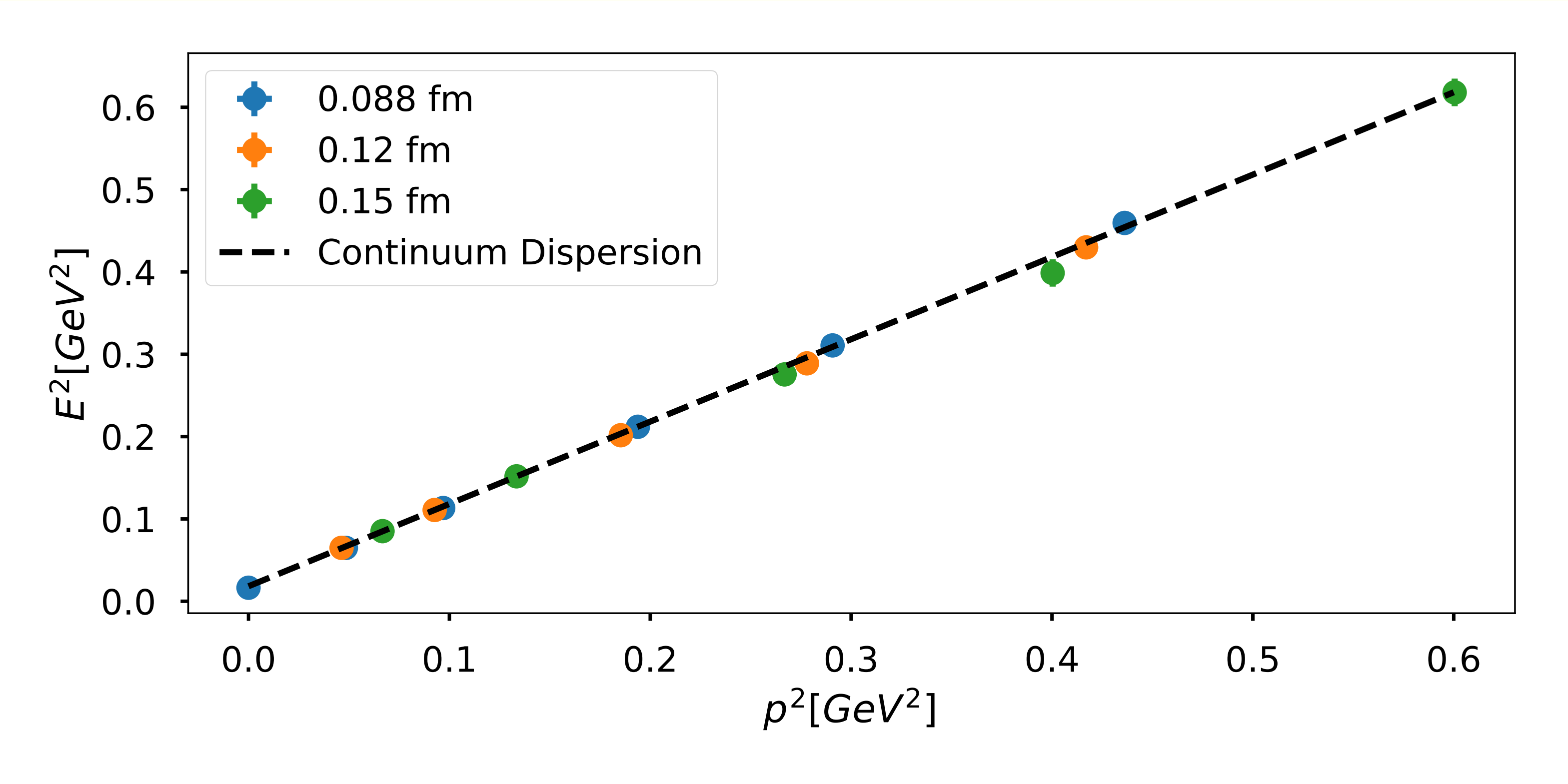
Physical masses for light and heavy masses $= 0.9 m_c$. Three lattice spacings $a \approx 0.088, 0.12, 0.15$ fm

Note: No renormalization included.



Preliminary: Pion dispersion relation

(for physical quark masses ensembles)



Conclusions and outlook

On-going calculation of form factors f_0, f_+, f_T for $H \to P$, $H \to H'$ processes with the HISQ action for all flavors on HISQ $N_f = 2+1+1$ MILC ensembles.

* So far: 4 lattice spacings, 7 ensembles (including 3 with phys. masses)
* Momenta up to $\boldsymbol{k} = (4,0,0) \times 2\pi/(aN_s)$: cover q^2 range for D semileptonic, down to $\sim 11\ \mathrm{GeV}^2$ B semileptonic.

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Next steps in the current analysis:

* Include larger source-sink separations: better determination of ground state.

* Optimize fitting methodology.

* Autocorrelations (plots in this talk, data binned by 10).

Conclusions and outlook

* Nonequilibrated topological charge effects.

For HISQ $N_f = 2+1+1$ MILC ensembles with smallest lattice spacings ($a \approx 0.042, 0.03$ fm), the topological charge Q is not properly sampled.

Correct the form factors in a similar way as we did for $K \to \pi\ell\nu$

$$f_+^{K\pi}(0)_{\rm corrected} = f_+^{K\pi}(0)_{\rm sampled} - \frac{1}{2\chi_T V}(f_+^{K\pi}(0))'' \left(1 - \frac{\langle Q^2\rangle_{\rm sample}}{\chi_T V}\right)$$

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* **Long term:** EM and isospin breaking effects.