Heavy-to-light decay form factors on $N_f = 2 + 1 + 1$ HISQ ensembles

Elvira Gámiz

(Lattice Fermilab and MILC Collaborations)
Introduction

Precise determinations of CKM matrix elements:

\[
V_{CKM} = \begin{pmatrix}
|V_{ud}| & |V_{us}| & |V_{ub}| \\
|V_{cd}| & |V_{cs}| & |V_{cb}| \\
|V_{td}| & |V_{ts}| & |V_{tb}|
\end{pmatrix}
\]

\[
B \to \pi \tau \nu, \quad B_s \to K \ell \nu \\
\Lambda_b \to p \ell \nu \\
D \to \pi \ell \nu, \quad D \to K \ell \nu, \quad B_{(s)} \to D_{(s)} \left( D_{(s)}^* \right) \ell \nu \\
B \to \pi \ell \ell, \quad B \to K \ell \ell
\]

**Tensions:** Inclusive-Exclusive determinations of \(|V_{ub}|\) and \(|V_{cb}|\).
Introduction

Long-standing tension between exclusive and inclusive determinations of the CKM matrix elements $|V_{cb}|$ and $|V_{ub}|$ at the $\sim 3\sigma$ level.

$|V_{cb}|^{B \rightarrow D^*}$ inclus.-exclus. tension not resolved by BGL vs CLN

(Belle (untagged) 1809.03290 and BaBar 1903.1002 results not included in plots)

From Belle 1809.03290 and FNAL/MILC 2014 $|V_{cb}|^{CLN} = (38.4 \pm 0.9) \cdot 10^{-3}$

$|V_{cb}|^{BGL} = (38.3 \pm 1.0) \cdot 10^{-3}$
Update of plot in 1711.08085. CKM unitarity band from CKMfitter
Introduction: Status exclusive $|V_{ub}|$ extraction

$|V_{ub}|$ from $B \rightarrow \pi l \nu$

Combined BCL fit to experim.
and $N_f = 2 + 1$ lattice data on different $q^2$ regions

$|V_{ub}|^{FLAG2019} = 3.73(14) \cdot 10^{-3}$

Good consistency between lattice and experimental shapes and commensurate errors

$|V_{ub}|^{inclusive, HFLAV2017} = (4.52 \pm 0.15^{+0.11}_{-0.14}) \cdot 10^{-3} \sim 3\sigma$ disagreement.
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- **RBC/UKQCD**, 1501.05373
- **FNAL/MILC**, 1503.07839
- **HPQCD**, hep-lat/0601021

Good consistency between lattice and experimental shapes and commensurate errors

$$|V_{ub}|^{FLAG2019} = 3.73(14) \cdot 10^{-3}$$

Leptonic determinations

* Less precise (dominated by exp. errors on $B(B \rightarrow \tau \nu)$)

* **BaBar** and **Belle** results don’t agree very well.
Introduction: Status exclusive \(|V_{ub}|\) extraction

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Leptonic determinations

* Less precise (dominated by exp. errors on \(B(B \to \tau\nu)\))

* \textbf{BaBar} and \textbf{Belle} results don’t agree very well.

Important role for \textbf{Belle II} for both leptonic and semileptonic
Introduction: Status exclusive $|V_{ub}|$ extraction

Alternative way of getting $|V_{ub}|$: $B_s \rightarrow K\ell\nu$.

* Three LQCD calculations of the relevant form factors:
  
  **HPQCD** 1406.2279, **RBC/UKQCD** 1501.05373, **FNAL/MILC** 1901.02561

* LQCD error smaller than for $B \rightarrow \pi$ form factors
Introduction: Status exclusive $|V_{ub}|$ extraction

Alternative way of getting $|V_{ub}|$: $B_s \rightarrow K\ell\nu$.

Also,

\[ f_0,+(B_s \rightarrow K\ell\nu)/f_0,+(B_s \rightarrow D_s\ell\nu) \]

to get $|V_{ub}/V_{cb}|$

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* LQCD error smaller than for $B \rightarrow \pi$ form factors

* Experimentally: Under investigacion by **LHCb**, expected to be measured at the $\Upsilon(5S)$ run at **Belle-II**

  (maybe 5-10% precision for the decay rate at Belle-II)
Introduction: Lepton Flavor Universality tests

\[ R(D^{(*)}) = \frac{\mathcal{B}(B \rightarrow D^{(*)} \tau \nu_{\tau})}{\mathcal{B}(B \rightarrow D^{(*)} \ell \nu)} \]

Tension between Belle and BaBar

Plot from 1904.08794

Belle 2019: \( R(D) = 0.307 \pm 0.037 \pm 0.016 \) (consistent with SM),
\( R(D^{*}) = 0.283 \pm 0.018 \pm 0.014 \)

World average at \( \sim 3\sigma \) from SM.
Introduction: $b$ rare decays (FCNC)

Flavor-changing neutral currents $b \rightarrow q$ transitions are potentially sensitive to NP effects $B \rightarrow K^* \gamma$, $B \rightarrow K^{(*)} \ell^+ \ell^-$, $B \rightarrow \pi \ell^+ \ell^-$
Introduction: \( b \) rare decays (FCNC)

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Sets of tensions between SM predictions and experimentally measured \( b \to s \ell^+ \ell^- \) observables

Branching fraction measurements: \( B^0 \to K^{*0} \mu^+ \mu^-, B^+ \to K^{(*)+} \mu^+ \mu^-, B_s \to \phi \mu^+ \mu^- \)

Angular analyses: \( B^+ \to K^{(*)+} \mu^+ \mu^-, B_s \to \phi \mu^+ \mu^- \)

Tests of Lepton Flavour Universality (\( \mu/e \)): \( B^0 \to K^{*0} \mu^+ \mu^-, B^+ \to K^{(*)+} \mu^+ \mu^- \)

Very small sensitivity to hadronic form factors \( \sim 10^{-4} \)

\[
R_{K^{(*)}}(q^2_{min}, q^2_{max}) \equiv \frac{\int_{q^2_{min}}^{q^2_{max}} dq^2 d\mathcal{B}(B \to K^{(*)} \mu^+ \mu^-)}{\int_{q^2_{min}}^{q^2_{max}} dq^2 d\mathcal{B}(B \to K^{(*)} e^+ e^-)}
\]
Introduction: Rare decays (FCNC)

Lepton Flavour Universality Tests

Angular Analysis ($P_5'$)

LHCb finds $3.4\sigma$, seems to be confirmed by Belle (ATLAS?) but not CMS

$LHCb$ will reach $\sim 1.5\%$ precision for the branching fractions at both low and high $q^2$. J. Albrecht et al 1709.10308

1904.02440 Belle preliminary
Introduction: Neutral-current $b$ decays

For $B \to P\ell\ell$, hadronic contributions are parametrized in terms of matrix elements of current (vector, axial and tensor) operators through three form factors

$$f_+, f_0 \text{ (for } m_\ell \neq 0\text{) and } f_T$$

+ non-factorizable contributions

Allow the calculation of branching fractions, angular observables and LFUV quantities

Extract CKM matrix elements $|V_{td,ts}|$ or constrain Wilson coefficients $C_9$ and $C_{10}$. 
Introduction: Neutral-current $b$ decays

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Extract CKM matrix elements $|V_{td,ts}|$ or constrain Wilson coefficients $C_9$ and $C_{10}$.

* Non-factorizable contributions under control? New physics or charm-loops?

* This talk: Form factors for $h \rightarrow l$ decays.
Current status: Form factors for \( B \to K\ell^+\ell^- \)

\[ B \to K\ell^+\ell^- : \text{HPQCD 1306.0434, 1306.2384, FNAL/MILC, 1509.06235} \]

Overlapping ensemble sets (asqtad MILC \( N_f = 2 + 1 \)) but different lattice actions:

**HPQCD**: NRQCD \( b + \text{HISQ} \ u, d, s \)\n
**FNAL/MILC**: Fermilab \( b + \text{asqtad} \ u, d, s \)

Consistent results for \( f_0, +, T \), and with LCSR

Khodjamarian et al 1006.4945
Form factors for $B \to K\ell^+\ell^-$

From D. Du et al 1510.02349, FNAL/MILC 1509.06235 (non-factorizable contributions under control?)

$1 - 2\sigma$ experiment-SM tensions.

focus on large bins above and below charmonium resonances
$B \to K\ell^+\ell^-$: Lepton Flavor Universality Tests

\[(1 - R_K)^{\text{HPQCD}} = 0.00074 \pm 0.00035, \quad (1 - R_{K^+})^{\text{FNAL/MILC}} = 0.00050 \pm 0.00043\]

SM predictions for these ratios pretty insensitive to form factors and non-factor contributions.
$B \to K\ell^+\ell^-$: Lepton Flavor Universality Tests

**LHCb results**

for $q^2 \in [1 \text{ GeV}^2, 6 \text{ GeV}^2]$

- $R_K^{\text{old Run1}} = 0.745^{+0.090}_{-0.074} (\text{stat}) \pm 0.036 (\text{syst})$
- $R_K^{\text{new Run1}} = 0.717^{+0.083}_{-0.071} (\text{stat})^{+0.017}_{-0.016} (\text{syst})$
- $R_K^{2015+2016} = 0.928^{+0.089}_{-0.076} (\text{stat})^{+0.020}_{-0.016} (\text{syst})$
- $R_K^{\text{RunI+2015+2016}} = 0.846^{+0.060+0.014}_{-0.054-0.016}$

$(1 - R_K)^{\text{HPQCD}} = 0.00074 \pm 0.00035$,

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$(1 - R_K^+)^{\text{LHCb 2019}} = 0.154^{+0.060}_{-0.054} (\text{stat})^{+0.014}_{-0.016} (\text{syst})$

compatible/tension with SM at $2.5\sigma$

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compatible/tension with SM at 2.5\( \sigma \)

SM predictions for these ratios pretty insensitive to form factors and non-factor contributions.

* LHCb expects a reduction by a factor of 4 by 2025.
**Form factors for** $B \rightarrow \pi \ell^+ \ell^-$

**FNAL/MILC, 1507.01618, D. Du et al. 1510.02349**

Take $f_+$ and $f_0$ from combined fit of lattice + experimental data for $B \rightarrow \pi \ell \nu$ (assume not significant NP effects at tree level).

The largest error is the one from the form factors.
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**D. Du et al. 1510.02349** SM prediction for $R_\pi = \frac{\mathcal{B}(B \to \pi \tau \nu_\tau)}{\mathcal{B}(B \to \pi \ell \nu)} = 0.641(17)$.

Expected to be measured at Belle-II, possible to determine at LHCb.
Rare semileptonic $B$ decays to $\nu \bar{\nu}$ states

D. Du et al. 1510.02349 with FNAL/MILC form factors

Predictions for both neutral and charged channels: complementary information (also $|V_{td,ts}|$)

* Theoretically clean (no problem with charm LD contributions)

* Difficult to measure experimentally, Belle-II expected precision $\sim 10\%$ for $B \rightarrow K$

$$\mathcal{B}(B^0 \rightarrow \pi^0 \nu \bar{\nu}) \cdot 10^7 = 0.668(41)(49)(16)$$

$$\mathcal{B}(B^0 \rightarrow K^0 \nu \bar{\nu}) \cdot 10^7 = 40.1(2.2)(4.3)(0.9)$$

$$\mathcal{B}(B^+ \rightarrow \pi^+ \nu \bar{\nu}) \cdot 10^6 = 9.62(1)(92); \; \mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu}) \cdot 10^6 = 4.94(52)(6)$$
Rare semileptonic $B$ decays: CKM parameters

| $|V_{td}| \times 10^3$ | $|V_{ts}| \times 10^3$ | $|V_{td}/V_{ts}|$ |
|---------------------|---------------------|---------------------|
| HPQCD 19 $\Delta M_q$ |
| RBC/UKQCD 18 $\Delta M_q$ |
| FNAL/MILC 16 $\Delta M_q$ |
| FNAL/MILC 16 $B \to K(\pi)\mu^+\mu^-$ |
| CKMfitter 18 |
| CKMfitter 18 (tree) |

*B-mixing results* HPQCD 1907.01025, RBC/UKQCD 1812.08791, FNAL/MILC, 1602.03560

*B $\to K(\pi)\mu^+\mu^-$ results from D. Du et al, 1510.02349

*Full/tree CKM unitarity results come from CKMfitter’s fit 2018 using all inputs/only observable mediated at tree level of weak interactions.*
Fermilab Lattice/MILC program for $b(c) \rightarrow s(d)$ decays
Form factors for $B_s \to K\ell\nu$

FNAL/MILC 1901.02561 on MILC asqtad $N_f = 2 + 1$ ensembles.
Valence sector: Fermilab $b +$ asqtad $l, s$

Analysis led by Yuzhi Liu

* Errors:
  $\mathcal{O}(\alpha_s a^2), \mathcal{O}(\alpha_s a, a^2)f((m_b a)^2)$

* Scale set with $r_1$, with
  $r_1^{a=0} = 0.3117(22) \text{ fm}$

* Partially quenched: $m'_s \neq m_s$

* Lattice data
  $\in [17.4, 23.2] \text{ GeV}^2$
  (Kaon momentum up to $\frac{2\pi}{N_s} (1, 1, 1)$)
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# Chiral-continuum extrapolation with NLO HMrSChPT in SU(2)
hard-kaon limit $+$ NNLO analytic terms.

* Small adjustments to the physical $m_b$
Form factors for $B_s \rightarrow K\ell\nu$

# Use BCL parametrization for $z$–expansion (with $K = 4$).

* Kinematic constraint $f_+(0) = f_0(0)$ enforced (without constraint, results satisfy $f_+(0) = f_0(0)$ within errors).

Tension with HPQCD (especially at low $q^2$). Good agreement with RBC/UKQCD.
Form factors for $B_s \rightarrow K\ell\nu$

# Predictions for differential decay rates:

Ratios for LFU tests: $\Gamma(B_s \rightarrow K\tau\nu)/\Gamma(B_s \rightarrow K\mu\nu) = 0.836(34)$

Forward-backward asymmetry: ($\theta_\ell$: angle between charged lepton and $B$)

$$A_{\ell FB} = \int_0^1 \frac{d^2\Gamma}{dq^2 d\cos\theta_\ell} d\cos\theta_\ell - \int_{-1}^0 \frac{d^2\Gamma}{dq^2 d\cos\theta_\ell} d\cos\theta_\ell$$

$$\propto |p_K^2| \frac{m_\ell^2}{q^2} Re \left[ f_+(q^2)f_0^*(q^2) \right]$$

Lepton polarization asymmetry:

$$A_{\ell pol} = \frac{d\Gamma^-/dq^2 - d\Gamma^+}{d\Gamma^-/dq^2 + d\Gamma^+} \propto f(|f_+(q^2)|, |f_0(q^2)|)$$
Form factors for $B_s \to K\ell\nu$

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$$A_{pol}^\ell = \frac{d\Gamma^-/dq^2 - d\Gamma^+}{d\Gamma^-/dq^2 + d\Gamma^+} \propto f(|f_+(q^2)|, |f_0(q^2)|)$$

# Also provides ratios of $f_+$ and $f_0$ for $B_s \to K\ell\nu$ and $B_s \to D_s\ell\nu$ as functions of $q^2$: useful for the determination of $|V_{ub}/V_{cb}|$. 
$b(c) \rightarrow s(d)$ decays on MILC $N_f = 2 + 1 + 1$ HISQ ensembles

(in progress)
\[ b(c) \rightarrow s(d) \] decays on MILC \( N_f = 2 + 1 + 1 \) HISQ ensembles

* MILC \( N_f = 2 + 1 + 1 \) HISQ ensembles
$b(c) \rightarrow s(d)$ decays on MILC $N_f = 2 + 1 + 1$

HISQ ensembles

* MILC $N_f = 2 + 1 + 1$ HISQ ensembles

* Lüscher-Weisz, one-loop Symanzik and tadpole improved gauge action $\rightarrow \mathcal{O}(\alpha_s^2 a^2)$

* Valence $l, s, c$ quarks are always described with HISQ action $\rightarrow \mathcal{O}(\alpha_s a^2)$

* Scale set with $\omega_0/a$
\[ b(c) \to s(d) \] \text{decays on MILC } N_f = 2 + 1 + 1 \text{ HISQ ensembles}

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* Valence \( l, s, c \) quarks are always described with HISQ action \( \to \mathcal{O}(\alpha_s a^2) \)

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A Clover action with Fermilab interpretation for \( b \to \mathcal{O}(\alpha_s a, a^2) f((m_b a)^2) \)

B HISQ action for heavy quarks, \( m_c \leq m_h \leq m_b \to \mathcal{O}(\alpha_s a^2) f((m_h a)^2) \)
$B_{(s)} \rightarrow \pi(K)\ell\nu$: charged currents

Extraction of $|V_{ub}|$: $B \rightarrow \pi\ell\nu$ and $B_s \rightarrow K\ell\nu$.

\[
\frac{d\Gamma}{dq^2} = (\text{known}) \ |V_{ub}|^2 \ \{ f_+(q^2), f_0(q^2) \}
\]
$B \rightarrow \pi(K)\ell^+\ell^-$: flavour-changing neutral currents

Flavor-changing neutral currents $b \rightarrow q$ transitions are potentially sensitive to NP effects $B \rightarrow K^*\gamma$, $B \rightarrow K^*\ell^+\ell^-$, $B \rightarrow \pi(K)\ell^+\ell^-$, $B_s \rightarrow K\ell^+\ell^-$.

Most important contributions to all this type of decays are expected to come from matrix elements of current (vector, axial and tensor) operators.

Need vector, $f_+$, scalar, $f_0$ and tensor, $f_T$ form factors from LQCD

$$\frac{d\Gamma}{dq^2} = (\text{known}) \ |V_{tb}V_{td(s)}^*|^2 \left\{ f_+(q^2), f_0(q^2), f_T(q^2) \right\}$$
Form factors for $B_{(s)} \rightarrow K(\pi)$

Taking Lorentz and discrete symmetries into account:

$$\langle P(k) | V^\mu | B(p) \rangle = f_+(q^2) \left( p^\mu + k^\mu - \frac{M_B^2 - M_P^2}{q^2} q^\mu \right) + f_0(q^2) \frac{M_B^2 - M_P^2}{q^2} q^\mu$$

$$\langle P(k) | S | B(p) \rangle = f_0(q^2) \frac{M_B^2 - M_P^2}{m_b - m_q}$$

$$\langle P(k) | T^{\mu \nu} | B(p) \rangle = f_T(q^2) \frac{2}{M_B + M_P} \left( p^\mu k^\nu - p^\mu k^\nu \right)$$
Form factors for $B_{(s)} \rightarrow K(\pi)$

Taking Lorentz and discrete symmetries into account:

$$\langle P(k) | \mathcal{V}^\mu | B(p) \rangle = f_+(q^2) \left( p^\mu + k^\mu - \frac{M_B^2 - M_P^2}{q^2} q^\mu \right) + f_0(q^2) \frac{M_B^2 - M_P^2}{q^2} q^\mu$$

$$= \sqrt{2M_B} \left[ k^\mu f_\perp(E_P) + v^\mu f_\parallel(E_P) \right], \quad v = p/M_B$$

$$\langle P(k) | S | B(p) \rangle = f_0(q^2) \frac{M_B^2 - M_P^2}{m_b - m_q}$$

$$\langle P(k) | \mathcal{T}^{\mu \nu} | B(p) \rangle = f_T(q^2) \frac{2}{M_B + M_P} \left( p^\mu k^\nu - p^\nu k^\mu \right)$$

and then

$$f_\perp(E_P) = \frac{\langle P(k) | \mathcal{V}^i | B(p) \rangle}{\sqrt{2M_B}} \frac{1}{k^i}$$

$$f_\parallel(E_P) = \frac{\langle P(k) | \mathcal{V}^0 | B(p) \rangle}{\sqrt{2M_B}}$$

$$f_T(q^2) = \frac{M_B + M_P}{\sqrt{2M_B}} \frac{\langle P(k) | \mathcal{T}^{0i} | B(p) \rangle}{\sqrt{2M_B}} \frac{1}{k^i}$$
Correlation Functions

Ratios of 3- and 2-point correlation functions

\[ C_3^{\mu(\nu)}(t, T; k) \equiv \frac{e^{-E_P(0)t} e^{-M_H(0)(T-t)}}{8} \]

\[ \bar{R}^{\mu(\nu)} \equiv \frac{\bar{C}_3^{\mu(\nu)}(t, T; k)}{\sqrt{\bar{C}_{2,P}(t; k)\bar{C}_{2,H}(T - t; k)}} \sqrt{\frac{2E_P(0)}{e^{-E_P(0)} e^{-M_H(0)(T-t)}}} \]

Suppress oscillating and excited states
Correlation Functions

Ratios of 3- and 2-point correlation functions

Suppress oscillating and excited states:

\[
\bar{C}_3^{\mu(\nu)}(t, T; k) \equiv \frac{e^{-E_P^0 t} e^{-M_H^0 (T-t)}}{8} \left[ \frac{C_3^{\mu(\nu)}(t, T; k)}{e^{-E_P^0 t} e^{-M_H^0 (T-t)}} + \frac{C_3^{\mu(\nu)}(t + 1, T; k)}{e^{-E_P^0 (t+1)} e^{-M_H^0 (T-t-1)}} \right. \\
+ \left. \frac{C_3^{\mu(\nu)}(t + 2, T; k)}{e^{-E_P^0 (t+2)} e^{-M_H^0 (T-t-2)}} + T \to T + 1 \right]
\]

\[
\bar{R}^{\mu(\nu)} \equiv \frac{\bar{C}_3^{\mu(\nu)}(t, T; k)}{\sqrt{\bar{C}_{2,P}(t; k)\bar{C}_{2,H}(T-t; k)}} \left[ \frac{2E_P^0}{e^{-E_P^0} e^{-M_H^0 (T-t)}} \right]
\]

\[
\to F^{\mu(\nu)} [1 - F_P e^{-\Delta M_P t} - F_P e^{-\Delta M_H (T-t)} + \ldots] + \mathcal{O} \left( \Delta M_P^2, \Delta M_P \Delta M_H, \Delta M_H^2 \right)
\]

\[
f_{\perp}(E_P) = Z_{\perp} \frac{F^i(k)}{k^i}
\]

\[
f_{\parallel}(E_P) = Z_{\parallel} F^4(k)
\]

\[
f_T(E_P) = Z_T \frac{M_H + M_P}{\sqrt{2M_H}} \frac{F^{4i}(k)}{k^i}
\]
\[ b \to s(d) \text{ decays on MILC } N_f = 2 + 1 + 1 \text{ HISQ ensembles} \]

Analysis led by Zech Gelzer
Simulation data

Parameters for physical-mass ensembles

<table>
<thead>
<tr>
<th>$\approx a$ (fm)</th>
<th>$N_s^3 \times N_t$</th>
<th>$a m'_l$</th>
<th>$a m'_s$</th>
<th>$a m'_c$</th>
<th>$k'_b$</th>
<th>$N_{\text{conf}} \times N_{\text{sour}}$</th>
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<td>$64^3 \times 96$</td>
<td>0.0012</td>
<td>0.0363</td>
<td>0.432</td>
<td>0.09569</td>
<td>$1535 \times 8$</td>
</tr>
<tr>
<td>0.057</td>
<td>$96^3 \times 192$</td>
<td>0.0008</td>
<td>0.022</td>
<td>0.260</td>
<td>0.10604</td>
<td>$1027 \times 8$</td>
</tr>
</tbody>
</table>
Correlation Functions and Fits

\[
J(t_{\text{source}} + t)
\]

\[
P(t_{\text{source}})
\]

\[
B(s)(t_{\text{source}} + T)
\]

\[
\bar{l}, \bar{s}
\]

\[
l, s
\]

\[
\bar{b}
\]

\[
J = \mathcal{V}^\mu, \mathcal{T}^{0i}
\]

* Two values of \( T \) and 8 time sources.

* Light (HISQ) quarks sources: random wall.

* Heavy (Fermilab) quarks sources: local + 1S-smeared.

* \( P \) momenta generated up to

\[
k = (2, 2, 2) \times 2\pi / (aN_s) \quad (7 \text{ values})
\]

\[
C_2^B(t; 0) = \sum_{x} \left\langle \mathcal{O}_B(t, x) \mathcal{O}_B^\dagger(0, 0) \right\rangle, \quad C_2^P(t; k) = \sum_{x} \left\langle \mathcal{O}_P(t, x) \mathcal{O}_P^\dagger(0, 0) \right\rangle e^{-ik \cdot x},
\]

\[
C_3^{\mu(\nu)}(t, T; k) = \sum_{x, y} e^{ik \cdot y} \left\langle \mathcal{O}_P(0, 0) J^{\mu(\nu)}(t, y) \mathcal{O}_B^\dagger(T, x) \right\rangle
\]
Correlation Functions and Fits

\[ J(t_{\text{source}} + t) \]

\[ B(s)(t_{\text{source}} + T) \]

\[ P(t_{\text{source}}) \]

\[ B_{(s)}(t_{\text{source}} + T) \]

* \[ J = \mathcal{V}^\mu, \mathcal{T}^{0i} \]
* Two values of \( T \) and 8 time sources.
* Light (HISQ) quarks sources: random wall.
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* \( P \) momenta generated up to \( k = (2, 2, 2) \times 2\pi/(aN_s) \) (7 values)

\[
C^B_2(t; 0) = \sum_x \left< \mathcal{O}_B(t, x) \mathcal{O}^\dagger_B(0, 0) \right>, \quad C^P_2(t; k) = \sum_x \left< \mathcal{O}_P(t, x) \mathcal{O}^\dagger_P(0, 0) \right> e^{-i k \cdot x},
\]

\[
C^\mu(\nu)(t, T; k) = \sum_{x, y} e^{i k \cdot y} \left< \mathcal{O}_P(0, 0) J^{\mu(\nu)}(t, y) \mathcal{O}^\dagger_B(T, x) \right>
\]

* Mostly nonperturbative matching: \( Z_J = \rho_J \sqrt{Z_{V_{bb}}^4 Z_{V_{qq}}^4} \) with \( \rho_J \) calculated perturbat. at one loop and \( Z_{V_{bb}}^4, Z_{V_{qq}}^4 \) nonperturbatively.

** Introduce a blinding factor through the renormalization factors.
Correlators and Fits: $B \to K$ on phys. $a = 0.057$ fm

Form factors from direct (combined) fits to all correlation functions: Preliminary

(consistent with fits to ratios $\bar{R}$ of 3-point over 2-point functions)
Form factors for $B \to \pi$

$\rho_{J}^{2} \in [18, 27.6] \text{ GeV}^{2}$  Preliminary

Note: Correct renomalization $\rho_{J}$ factors missing. Only $\sqrt{Z_{V_{bb}^{4}} Z_{V_{qq}^{4}}}$ included.
Form factors for $B \rightarrow K$

Note: Correct renomalization $\rho_J$ factors missing. Only $\sqrt{Z_{bb}^4 Z_{qq}^4}$ included.
Form factors for $B_s \rightarrow K$

Note: Correct renormalization $\rho_J$ factors missing. Only $\sqrt{Z_{V_{bb}^4}Z_{V_{qq}^4}}$ included.
Chiral-continuum interp./extrap.: \( B_s \to K \)

We extrapolate the form factors to the continuum and interpolate to the physical quark masses using \( SU(2) \) HM\( r \)S\( \chi \)PT

\[
\begin{align*}
  f_J &= f_J^{(0)} \times \left( 1 + \delta f_J^{\log s} + \delta f_J^{NLO} + \delta f_J^{N^2LO} + \ldots \right) \times \left( 1 + \delta f_J^b \right) \\
  f_J^{(0)} &= \frac{g_\pi}{f_\pi (E_P + \Delta_P^*)} \\
  \delta f_J^{NLO} &= c_J^l \chi_l + c_J^s \chi_s + c_J^E \chi_E + c_J^{E^2} \chi_E^2 + c_J^{a^2} \chi_a^2
\end{align*}
\]

* \( \Delta_P^* = \left( M_{B^*}^2 - M_{B_s}^2 - M_P^2 \right) / (2M_{B_s}) \), where \( M_{B^*} \) is a 1\(^-\) or 0\(^+\) mass.

* \( f_J^{\log s} \): nonanalytic functions of \( m_l, a \).

* \( f_J^b \): \( b \)-quark discretization effects,

\[
\mathcal{O} \left( (a\Lambda)^2, \alpha_s a\Lambda, \alpha_s (a\Lambda)^2 \right) \times \text{mistmach functions } (a m_b, \alpha_s) \times h^i_J.
\]

* Perturbative part of \( Z_J \) implemented with priors: \( \tilde{\rho}_J = 1 + \tilde{\rho}_J^{(1)} \alpha_s + \tilde{\rho}_J^{(2)} \alpha_s^2 \)
Chiral-continuum interp./extrap.: $B_s \rightarrow K$

* $f_\perp$ and $f_\parallel$ fit simultaneously.

* Central fit: $NLO \ SU(2) \ \text{HM}r\text{S}\chi\text{PT} + N^2LO$ analytic terms.
Error budget for $B_s \rightarrow K$

Preliminary and missing perturbative $\rho_J$ factors

![Graphs showing error budget for $f_+$ and $f_0$](image)
Error budget for $B_s \rightarrow K$

Preliminary and missing perturbative $\rho_J$ factors

Compared to previous **FNAL/MILC**:

Similar $a \rightarrow$ similar statistics, smaller discretization (HISQ)

Physical $m'_i$ ensembles $\rightarrow$ remove chiral extrapolation error
Outlook

On-going calculation of form factors $f_0, f_+, f_T$ for $B \rightarrow \pi$, $B \rightarrow K$, $B_s \rightarrow K$ with Fermilab $b$ and HISQ $l, s, c$ on HISQ $N_f = 2 + 1 + 1$ MILC ensembles.

* 4 lattice spacings, 7 ensembles (including 4 with phys. masses)

* Mostly non-perturbative renormalization.

* Chiral+continuum fits: NLO HMxSChPT in SU(2) hard-kaon limit + NNLO analytic terms.
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* 4 lattice spacings, 7 ensembles (including 4 with phys. masses)

* Mostly non-perturbative renormalization.

* Chiral+continuum fits: NLO HMrSChPT in SU(2) hard-kaon limit + NNLO analytic terms.

Need to do

* Renormalization coefficients: calculate $\rho_J$, get $Z_{V_{bb,qq}}^A$ with better stat.

* $z$ expansions and finalize systematic error budgets.

* Phenomenology: $|V_{ub}|$, $|V_{td}|$, $|V_{ts}|$, confront branching fractions and angular observables with experiment, make predictions for the not yet measured quantities.

* Correlated ratios for different processes
$h \rightarrow s(d)$ decays on MILC $N_f = 2 + 1 + 1$ HISQ ensembles

B HISQ heavy

Analysis led by William Jay
All-HISQ decay constants analysis

It is feasible to do $B$ physics with HISQ: Decay constants

Avoid large lattice artifact including data with $a m_h < 0.9$ (black solid line)

Use HQET-inspired model for extrapolating to the $B$ mass.
**All-HISQ decay constants analysis**

It is feasible to do $B$ physics with HISQ: Decay constants

Avoid large lattice artifact including data with $a m_h < 0.9$ (black solid line)

* Errors: 0.2-0.3% for $c$ decay constants, 0.6-0.7% for $b$ decay constants.

Largest systematic errors: choice of fit model (continuum extrapolation errors), correlator fits (excited-state contamination).

Use HQET-inspired model for extrapolating to the $B$ mass.
All-HISQ decay constants analysis

\( f_{\pi,K}^{PDG} \) also important systematic for charmed decay constants

* Controversy with EW radiative corrections needed to extract \( |V_{ud}| \) from superallowed \( \beta \) decays: Seng, Gorchtein, Patel, Ramsey-Musolf 1807.10197, Czarnecki, Marciano, Sirlin 1907.06737
Simulation data

Data generated for all-HISQ heavy semileptonic project until middle July 2019

\[ B \to K, \ B \to \pi, \ B_s \to K \]

(and \( D \to K, \ D \to \pi, \ D_s \to K \))

\[ B_{(s)} \to D_{(s)} \]

<table>
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<tr>
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Simulation data

Data generated for all-HISQ heavy semileptonic project until middle July 2019

\[ B \to K, \ B \to \pi, \ B_s \to K \]

(and \( D \to K, \ D \to \pi, \ D_s \to K \))

\[ B_s^{(s)} \to D_s^{(s)} \]

Include partially-quenched data: fine-tuning light quark masses, isospin-breaking effects.

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Correlation Functions

* Random wall sources.

* 4 values of $T$ generated, 3 more being generated in some ensembles.

* 6-8 time sources.

* **Local** scalar and temporal vector currents, **point-split** spatial vector currents.

  **$S$ and $\mathcal{V}_i$ are taste singlets $\rightarrow$ parent $H_{(s)}$ has spin-taste $\gamma_5 \times \gamma_5$ (Goldstone meson).**

  **$\mathcal{V}_0$ and $\mathcal{T}_{\mu\nu}$ have taste $\gamma_0$ $\rightarrow$ parent $H_{(s)}$ has spin-taste $\gamma_0\gamma_5 \times \gamma_0\gamma_5$ (non-Goldstone meson).**
Correlation Functions

\[ C_{2}^{H_{(s)}}(t; \mathbf{k}) = \sum_{x} \left\langle \mathcal{O}_{H_{(s)}}(t, x) \mathcal{O}_{H_{(s)}}^{\dagger}(0, 0) \right\rangle e^{-i\mathbf{k} \cdot \mathbf{x}}, \quad C_{2}^{P}(t; \mathbf{k}) = \sum_{x} \left\langle \mathcal{O}_{P}(t, x) \mathcal{O}_{P}^{\dagger}(0, 0) \right\rangle e^{-i\mathbf{k} \cdot \mathbf{x}}, \]

\[ C_{3}^{\mu(\nu)}(t, T; \mathbf{k}) = \sum_{x, y} e^{i\mathbf{k} \cdot \mathbf{y}} \left\langle \mathcal{O}_{P}(0, 0) J_{\mu(\nu)}(t, y) \mathcal{O}_{H_{(s)}}^{\dagger}(T, x) \right\rangle \]

\[ \tilde{C}_{3}^{\mu}(t, T; \mathbf{k}) = \sum_{x, y} e^{i\mathbf{k} \cdot \mathbf{y}} \left\langle \mathcal{O}_{H'_{(s)}}(0, 0) J_{\mu}(t, y) \mathcal{O}_{H_{(s)}}^{\dagger}(T, x) \right\rangle \]

* \( P \) momenta data generated up to \( \mathbf{k} = (4, 0, 0) \times 2\pi/(aN_{S}) \) (8 values)
Comparison of noise-to-signal at $a \approx 0.12\text{fm}$

**Fermilab heavy $b$ vs HISQ $h$**

To suppress oscillating-state contributions for better visualization, an averaging scheme has been applied over neighboring time slices.

* Physical $l$, $s$ and $c$ masses

* Source-sink separation $T = 15, 16$.  

* $m_h = 1.4m_c$

Typical fit range:

$\sim [2 - 13]$
Extracting the form factors

Using the Ward identity $q_\mu \langle P|\mathcal{V}_{\text{lat}}^\mu|H\rangle Z_{\mathcal{V}_{\text{lat}}} = (m_h - m_q)\langle P|S|H\rangle$ and the definition of the form factors

$$f_0(q^2) = \frac{m_h - m_q}{M_H^2 - M_P^2} \langle P|S|H\rangle_{q^2} \quad \text{no renor. needed}$$

$$f_+(q^2) = \frac{1}{2M_H} \left( \frac{(M_H - M_P)(m_h - m_q)\langle P|S|H\rangle - q^2 Z_{V^0}\langle P|V^0|H\rangle}{k^2} \right)$$

$$= \frac{1}{2M_H} \left[ Z_{V^0}\langle P|V^0|H\rangle + \frac{M_H - M_P}{k^i} Z_{V^i}\langle P|V^i|H\rangle \right]$$

$$f_T(q^2) = \frac{M_H + M_P}{\sqrt{2M_H}} Z_T \frac{\langle P|T^0_i|H\rangle}{\sqrt{2M_H}}$$
Extracting the form factors

Using the Ward identity $q_\mu \langle P|\mathcal{V}_{\text{lat}}^\mu|H\rangle Z_{V_{\text{lat}}} = (m_h - m_q)\langle P|S|H\rangle$ and the definition of the form factors

\[
f_0(q^2) = \frac{m_h - m_q}{M_H^2 - M_P^2} \langle P|S|H\rangle q^2 \quad \text{no renor. needed}
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\[
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\]

* For the local temporal current, with both mesons at rest:

\[
Z_{V_0} \langle P|\mathcal{V}_0|H\rangle_{q^2_{\text{max}}} = \frac{m_h - m_q}{M_H - M_P} \langle P|S|H\rangle_{q^2_{\text{max}}}
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\]

* Renormalization factors \( Z_{V^i}, Z_T \): Under investigation.

** First step: Mostly non-perturbative renormalization?
Correlation Functions and Fits

Example: $D \rightarrow \pi$ at $a \approx 0.12\text{fm}$ with phys. quark masses

$S$ correlation function for $\mathbf{k} = (1, 0, 0) \ (f_0)$

Preliminary

* Combined correlated fit to 2-point and 3-point functions
  (ratio $\bar{R}$ 3pt- and 2-point functions for visualization)
Correlation Functions and Fits

Example: $D \rightarrow \pi$ at $a \approx 0.12\text{fm}$ with phys. quark masses

$S$ correlation function for $k = (1, 0, 0)$ ($f_0$)

$T^{10}$ correlation function for $k = (1, 0, 0)$

* Similar results for all currents and most of the momenta.

* Add larger values of $T$: Better constrain of ground state contributions.

Preliminary

* Combined correlated fit to 2-point and 3-point functions

(ratio $\bar{R}$ 3pt- and 2-point functions for visualization)
**Correlation Functions and Fits**

**Example:** 3-point correlation function with $S$ insertion and $k = (1, 0, 0)$

$2 + 1$ states for $\pi$ channel
and $4 + 2$ for $D$ channel

Check stability
Correlation Functions and Fits

**Example:** 3-point correlation function with $S$ insertion and $k = (1, 0, 0)$

$2 + 1$ states for $\pi$ channel

and $4 + 2$ for $D$ channel

Check stability

* **Light bands:** broad priors
  (central value from 2-point fits)

* **Dark bands:** (combined) fit values.
Preliminary: $D \rightarrow \pi$ form factors

Physical masses for light and heavy masses $= 0.9m_c$. Three lattice spacings $a \approx 0.088, 0.12, 0.15$ fm

Note: No renormalization included.
Preliminary: Pion dispersion relation

(for physical quark masses ensembles)
Conclusions and outlook

On-going calculation of form factors $f_0, f_+, f_T$ for $H \rightarrow P$, $H \rightarrow H'$ processes with the HISQ action for all flavors on HISQ $N_f = 2 + 1 + 1$ MILC ensembles.

* So far: 4 lattice spacings, 7 ensembles (including 3 with phys. masses)

* Momenta up to $k = (4, 0, 0) \times 2\pi/(aN_s)$: cover $q^2$ range for $D$ semileptonic, down to $\sim 11$ GeV$^2$ $B$ semileptonic.
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* Good behaviour of dispersion relation
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* Good behaviour of dispersion relation

Next steps in the current analysis:

* Include larger source-sink separations: better determination of ground state.

* Optimize fitting methodology.

* Autocorrelations (plots in this talk, data binned by 10).
Conclusions and outlook

* Nonequilibrated topological charge effects.

For HISQ \( N_f = 2 + 1 + 1 \) MILC ensembles with smallest lattice spacings \((a \approx 0.042, 0.03 \text{ fm})\), the topological charge \( Q \) is not properly sampled.

Correct the form factors in a similar way as we did for \( K \to \pi \ell \nu \)

\[
f_+^{K\pi}(0)_{\text{corrected}} = f_+^{K\pi}(0)_{\text{sampled}} - \frac{1}{2 \chi_T V} (f_+^{K\pi}(0))'' \left( 1 - \frac{\langle Q^2 \rangle_{\text{sample}}}{\chi_T V} \right)
\]

with \((f_+^{K\pi}(0))'' = d^2 f_+/d\theta^2 |_{\theta=0}\) and \(\chi_T = \langle Q \rangle/V\) the topological susceptibility.
Conclusions and outlook

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with $(f_+^{K\pi}(0))'' = d^2 f_+/d\theta^2|_{\theta=0}$ and $\chi_T = \langle Q \rangle / V$ the topological susceptibility.

* Following C. Bernard and D. Toussaint 1707.05430, use ChPT to study Q-dependence ($\theta$ dependence) of the form factor and obtain $(f_+^{K\pi}(0))''$ at LO:
Conclusions and outlook

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with $(f_{+}^{K\pi}(0))'' = \frac{d^2 f_+}{d\theta^2}|_{\theta=0}$ and $\chi_T = \langle Q \rangle / V$ the topological susceptibility.

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* Renormalization for $\mathcal{T}$ current.
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* **Long term**: EM and isospin breaking effects.