Status of the HVP contribution to $(g-2)_\mu$ from LQCD+QED

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August 1, 2019 – ALGT 2019, CERN

The magnetic moment

- The magnetic moment $\vec{\mu}$ determines the shift of a particle's energy in the presence of a magnetic field $\vec{B}$

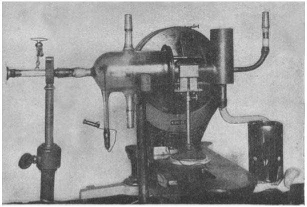
$$V = -\vec{\mu} \cdot \vec{B}$$

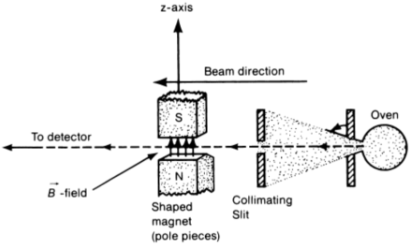
- The intrinsic spin $\vec{S}$ of a particle contributes

$$\vec{\mu} = g \left(\frac{e}{2m} \right) \vec{S}$$

with electric charge e, particle mass m, and Landé factor g.

Stern & Gerlach, 1922





$\vec{B} = 0$ $\vec{B} \neq 0$

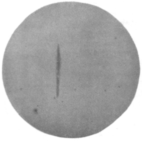


Fig. 2.

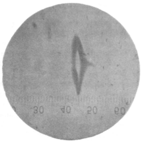


Fig. 3.

- Send silver atoms through non-uniform magnetic field, $\vec{F} = -\vec{\nabla} V$
- Atoms electrically neutral $\Rightarrow$ spin effects can dominate
- Silver has single 5s electron and fully filled shells below $\Rightarrow$ observe μ of the electron
- $\vec{B} \neq 0$: two distinct lines $\Rightarrow$ quantized spin, distance of lines $\Rightarrow$ g_e

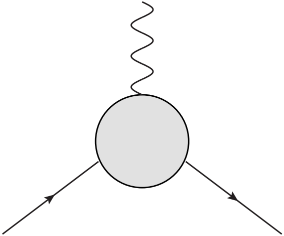
The anomalous magnetic moment

- 1924: Stern and Gerlach measured $g_e = 2.0(2)$
- 1928: Dirac shows that relativistic quantum mechanics yields $g_e = 2$
- 1947 (Phys. Rev. 72 1256, November 3): Kusch & Foley (Columbia) measure $g_e = 2.00229(8)$ in the Zeeman spectrum of gallium
- 1947 (Phys. Rev. 73 416, December 30): Schwinger calculates lowest-order radiative photon correction within quantum field theory (QFT): $g_e = 2 + \alpha/\pi = 2.00232\ldots$

  Define anomalous magnetic moment $a_e = (g_e - 2)/2$ exhibiting effects of QFT

The anomalous magnetic moment

- In QFT a can be expressed in terms of scattering of particle off a classical photon background



For external photon index μ with momentum q the scattering amplitude can be generally written as

$$(-ie)\left[\gamma_\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q^\nu}{2m}F_2(q^2)\right]$$

with $F_2(0) = a$.

Early measurements of a_μ

- Study of μ decays under varying magnetic field by Garwin, Lederman and Weinrich 1957 (Nevis Cyclotron, Columbia)

$$g_\mu = 2.0(2)$$

- Study of stopped muon precession by Garwin, Hutchinson, Penman, Shapiro 1960

$$a_\mu = 0.00113 + 0.00016 - 0.00012$$

- Crucial improvement (magic-momentum method) in CERN-3 experiment 1979

$$a_\mu = 0.001165924(9)\,.$$

Magic momentum method

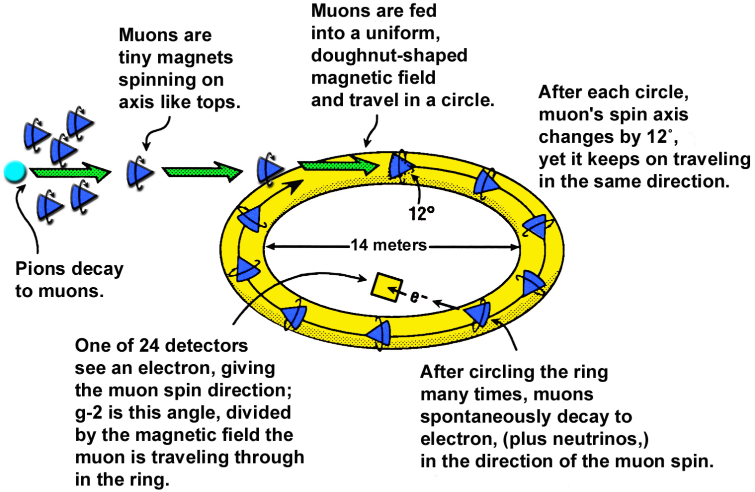
- Send muon in storage ring with uniform magnetic field, observe decays as function of time
- Measure difference of cyclotron frequency ω_C and spin rotation frequency ω_S directly with

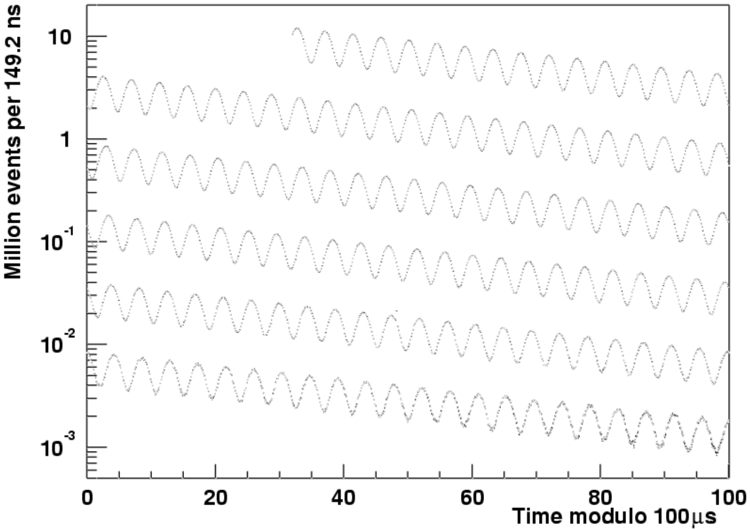
$$\vec{\omega_a} = \vec{\omega}_S - \vec{\omega}_C = -\frac{Qe}{m}\left[a_\mu \vec{B} - a_\mu \left(\frac{\gamma}{\gamma+1}\right)(\vec{\beta}\cdot\vec{B})\vec{\beta} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

  (Thomas 1927).
- Minimize uncertainty by tuning $\gamma^2 - 1 \approx 1/a_\mu$ or $p_\mu \approx 3.09$ GeV to suppress effect of electric field; treat $\vec{\beta}\cdot\vec{B}$ term as perturbation
- All experiments discussed in the following use this method

The BNL E821 experiment (2006)

http://www.g-2.bnl.gov/physics/index.html



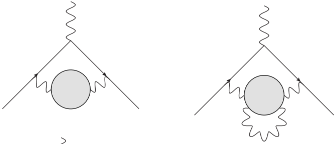


$$a_\mu^{\mathrm{E821}} = 0.00116592089(54)_{\mathrm{stat}}(33)_{\mathrm{sys}}$$

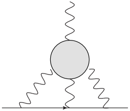
There is a tension of 3.7σ for the muon

$$a_\mu^{\mathrm{E821}} - a_\mu^{\mathrm{SM}} = 27.4\,\underbrace{(2.7)}_{\mathrm{HVP}}\,\underbrace{(2.6)}_{\mathrm{HLbL}}\,\underbrace{(0.1)}_{\mathrm{other}}\,\underbrace{(6.3)}_{\mathrm{E821}}\times 10^{-10}$$

Hadronic Vacuum Polarization (HVP)



Hadronic Light-by-Light (HLbL)



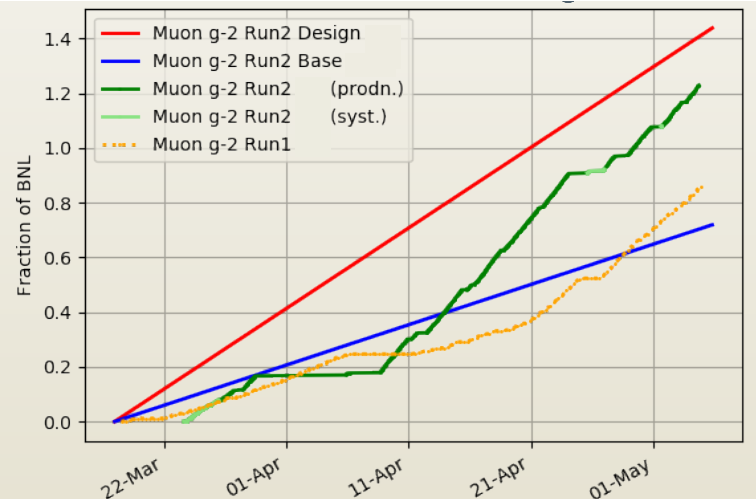
New experiment: Fermilab E989

$$a_\mu^{\text{E821}} - a_\mu^{\text{SM}} = 27.4\,\underbrace{(2.7)}_{\text{HVP}}\,\underbrace{(2.6)}_{\text{HLbL}}\,\underbrace{(0.1)}_{\text{other}}\,\underbrace{(6.3)}_{\text{E821}} \times 10^{-10}$$

$$\delta a_\mu^{\text{E989, } \mathbf{2019}} = 4.5 \times 10^{-10}\,, \qquad \delta a_\mu^{\text{E989, 2021}} = 1.6 \times 10^{-10}$$

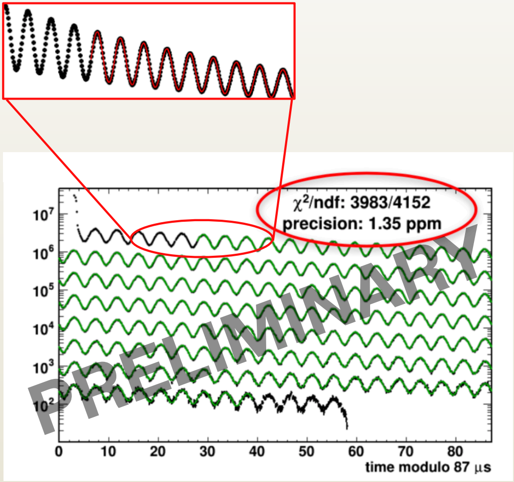
Need to improve uncertainties on HVP and HLbL contributions

Statistics Run 1 in 2018 and Run 2 in 2019 (talk by N. Tran at FPCP 2019):



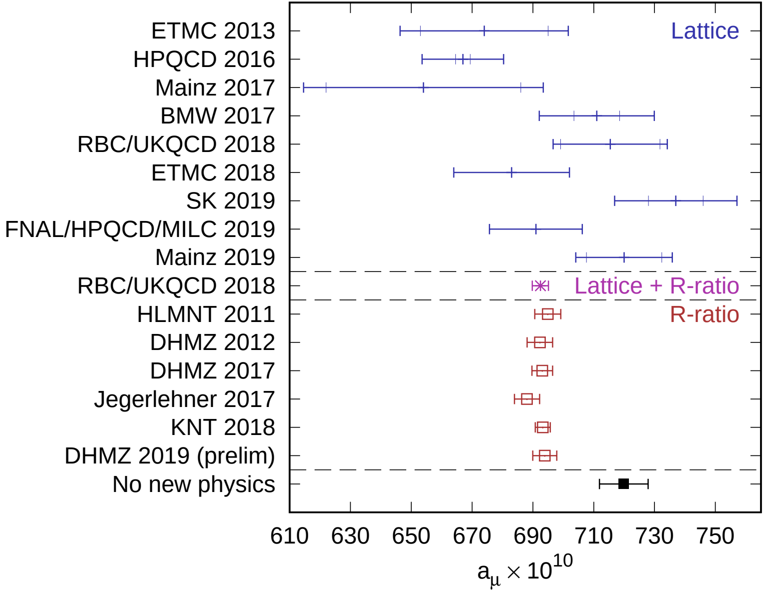
Run 1 fit (talk by N. Tran at FPCP 2019):

$$N(t) = N_0 e^{-t/\tau}\left[1 - A\cos\left(\omega_a t + \phi\right)\right]$$

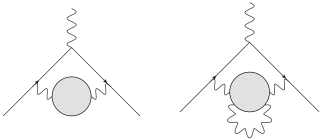


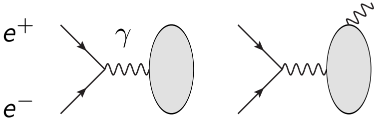
Relative unblinding of 6 analyzing groups successful!

Status of HVP determinations



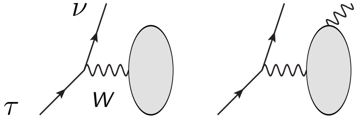
The HVP from dispersion relations





$$e^+e^- \to \mathrm{hadrons}(\gamma)$$

$$J_\mu = V_\mu^{I=1,I_3=0} + V_\mu^{I=0,I_3=0}$$



$$\tau \to \nu \mathrm{hadrons}(\gamma)$$

$$J_\mu = V_\mu^{I=1,I_3=\pm 1} - A_\mu^{I=1,I_3=\pm 1}$$

Knowledge of isospin-breaking corrections and separation of vector and axial-vector components needed to use τ decay data. (Talk by M. Bruno next week.)

Dispersive method - e^+e^- status

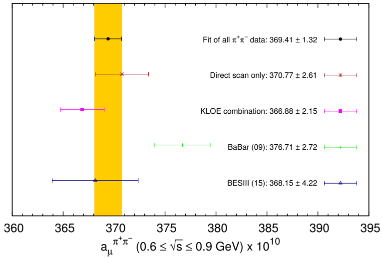
Recent results ($\times 10^{10}$) by Keshavarzi et al. 2018, Davier et al. 2017:

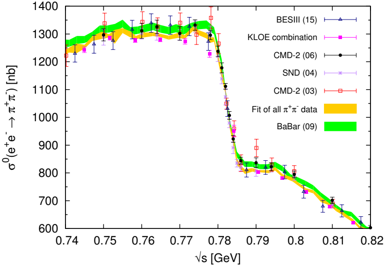
Channel	**This work (KNT18)**	**DHMZ17** [78]	**Difference**
Data based channels ($\sqrt{s} \leq 1.8$ GeV)			
$\pi^0\gamma$ (data + ChPT)	4.58 ± 0.10	4.29 ± 0.10	0.29
$\pi^+\pi^-$ (data + ChPT)	503.74 ± 1.96	507.14 ± 2.58	-3.40
$\pi^+\pi^-\pi^0$ (data + ChPT)	47.70 ± 0.89	46.20 ± 1.45	1.50
$\pi^+\pi^-\pi^+\pi^-$	13.99 ± 0.19	13.68 ± 0.31	0.31
...			
Total	693.3 ± 2.5	693.1 ± 3.4	0.2

Good agreement for total, individual channels disagree to some degree. Surprising since they use the same experimental input.

Dispersive method - e^+e^- status

Tension in 2π experimental input. BaBar and KLOE central values differ by $\delta a_\mu = 9.8(3.5) \times 10^{-10}$, compare to quoted total uncertainties of dispersive results of order $\delta a_\mu = 3 \times 10^{-10}$.





Conflicting input limits the precision and reliability of the dispersive results.
First-principles calculation to remove dependence on conflicting input data desirable.
(RBC/UKQCD 2018)

Looking for more data and insight: energy-scans update from CMD-3 in Novosibirsk and ISR updates from KLOE2, BaBar, Belle, BESIII and BelleII.

Talk by Zhang at EPS 2019 (DHMZ 2019 prelim):

Combined Results Fit [<0.6 GeV] + Data [0.6-1.8 GeV]

√s range [GeV]	a_μ^{had} [10^{-10}] All data	a_μ^{had} [10^{-10}] All but BABAR	a_μ^{had} [10^{-10}] All but KLOE
threshold - 1.8	$506.9 \pm 1.9_{total}$	$505.0 \pm 2.1_{total}$	$510.6 \pm 2.2_{total}$

⇒ The difference "All but BABAR" and "All but KLOE" = 5.6
to be compared with 1.9 uncertainty with "All data"

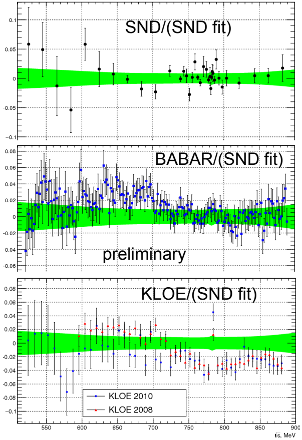
- The local error inflation is not sufficient to amplify the uncertainty
- Global tension (normalisation/shape) not previously accounted for
- Potential underestimated uncertainty in at least one of the measurements?
- Other measurements not precise enough and are in agreement with BABAR or KLOE

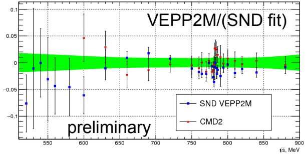
⇒ Given the fact we do not know which dataset is problematic, we decide to

- Add half of the discrepancy (2.8) as an additional uncertainty (correcting the local PDG inflation to avoid double counting)
- Take the mean value "All but BABAR" and "All but KLOE" as our central value

Talk by Druzhinin at EPS 2019 (SND experiment preliminary):

$$e^+e^- \to \pi^+\pi^-$$





$$0.53 < \sqrt{s} < 0.88 \text{ GeV}$$

	$a_\mu(\pi^+\pi^-) \times 10^{10}$
SND & VEPP-2000	$411.8 \pm 1.0 \pm 3.7$
SND & VEPP-2M	$408.9 \pm 1.3 \pm 5.3$
BABAR	$414.9 \pm 0.3 \pm 2.1$

Dispersive method - τ status

Experiment	$a_\mu^{\mathrm{had,LO}}[\pi\pi,\tau]$ (10^{-10}) $2m_{\pi^\pm} - 0.36$ GeV	$0.36 - 1.8$ GeV
ALEPH	$9.80 \pm 0.40 \pm 0.05 \pm 0.07$	$501.2 \pm 4.5 \pm 2.7 \pm 1.9$
CLEO	$9.65 \pm 0.42 \pm 0.17 \pm 0.07$	$504.5 \pm 5.4 \pm 8.8 \pm 1.9$
OPAL	$11.31 \pm 0.76 \pm 0.15 \pm 0.07$	$515.6 \pm 9.9 \pm 6.9 \pm 1.9$
Belle	$9.74 \pm 0.28 \pm 0.15 \pm 0.07$	$503.9 \pm 1.9 \pm 7.8 \pm 1.9$
Combined	$9.82 \pm 0.13 \pm 0.04 \pm 0.07$	$506.4 \pm 1.9 \pm 2.2 \pm 1.9$

Davier et al. 2013: $a_\mu^{\mathrm{had,LO}}[\pi\pi,\tau] = 516.2(3.5) \times 10^{-10}$ ($2m_\pi^\pm$ – 1.8 GeV)

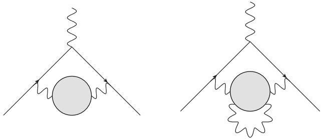
Compare to e^+e^-:

- $a_\mu^{\mathrm{had,LO}}[\pi\pi, e^+e^-] = 507.1(2.6) \times 10^{-10}$ (DHMZ17, $2m_\pi^\pm$ – 1.8 GeV)
- $a_\mu^{\mathrm{had,LO}}[\pi\pi, e^+e^-] = 503.7(2.0) \times 10^{-10}$ (KNT18, $2m_\pi^\pm$ – 1.937 GeV)

Here treatment of isospin-breaking to relate matrix elements of $V_\mu^{I=1,I_3=1}$ to $V_\mu^{I=1,I_3=0}$ crucial.

Can calculate from first-principles in lattice QCD+QED (Bruno, Izubuchi, CL, Meyer 2018)

Euclidean Space Representation



Starting from the vector current $J_\mu(x) = i\sum_f Q_f \overline{\Psi}_f(x)\gamma_\mu \Psi_f(x)$ we may write

$$a_\mu^{\mathrm{HVP\ LO}} = \sum_{t=0}^{\infty} w_t C(t)$$

with

$$C(t) = \frac{1}{3}\sum_{\vec{x}}\sum_{j=0,1,2}\langle J_j(\vec{x},t)J_j(0)\rangle$$

and w_t capturing the photon and muon part of the HVP diagrams.

The correlator $C(t)$ is computed in lattice QCD+QED at physical pion mass with non-degenerate up and down quark masses including up, down, strange, charm, and bottom quark contributions.

Statistical variance of correlator

$$\langle J(t)J(0)\rangle$$

is itself a correlation function

$$\sigma^2(t) = \langle J(t)^2 J(0)^2\rangle - \langle J(t)J(0)\rangle^2 \,.$$

While $C(t) \propto e^{-m_\rho t}$ (vector channel), $\sigma^2(t) \propto e^{-m_\pi t}$ (pseudoscalar channel). Therefore signal-to-noise is exponentially bad for large t.

C(t) is, however, very precise for shorter Euclidean times t (on order of $1-2$ fm)

Window method (RBC/UKQCD 2018)

We therefore also consider a window method

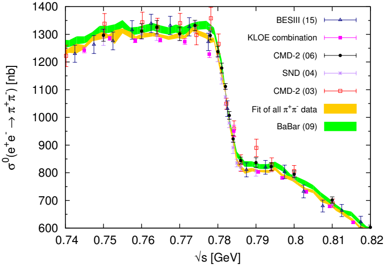
$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

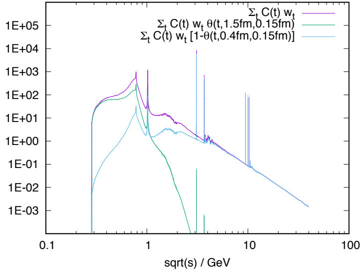
with

$$\begin{aligned}
a_\mu^{\mathrm{SD}} &= \sum_t C(t) w_t [1 - \Theta(t, t_0, \Delta)]\,, \\
a_\mu^{\mathrm{W}} &= \sum_t C(t) w_t [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)]\,, \\
a_\mu^{\mathrm{LD}} &= \sum_t C(t) w_t \Theta(t, t_1, \Delta)\,, \\
\Theta(t, t', \Delta) &= [1 + \tanh[(t - t')/\Delta]]/2\,.
\end{aligned}$$

In this version of the calculation, we use
$C(t) = \frac{1}{12\pi^2} \int_0^\infty d(\sqrt{s}) R(s) s e^{-\sqrt{s}t}$ with $R(s) = \frac{3s}{4\pi\alpha^2}\sigma(s, e^+e^- \to \mathrm{had})$
to compute a_μ^{SD} and a_μ^{LD} and Lattice QCD+QED for a_μ^{W}.

How does this translate to the time-like region?





Most of $\pi\pi$ peak is captured by window from $t_0 = 0.4$ fm to $t_1 = 1.5$ fm, so replacing this region with lattice data reduces the dependence on BaBar versus KLOE data sets.

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Editors' Suggestion

Calculation of the Hadronic Vacuum Polarization Contribution to the Muon Anomalous Magnetic Moment

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We present a first-principles lattice QCD + QED calculation at physical pion mass of the leading-order hadronic vacuum polarization contribution to the muon anomalous magnetic moment. The total contribution of up, down, strange, and charm quarks including QED and strong isospin breaking effects is $a_\mu^{\rm HVP\ LO} = 715.4(18.7) \times 10^{-10}$. By supplementing lattice data for very short and long distances with R-ratio data, we significantly improve the precision to $a_\mu^{\rm HVP\ LO} = 692.5(2.7) \times 10^{-10}$. This is the currently most precise determination of $a_\mu^{\rm HVP\ LO}$.

This method allows us to reduce HVP uncertainty over next years to $\delta a_\mu^{\rm LO\ HVP} \sim 1 \times 10^{-10}$, below Fermilab E989 uncertainty

Overview of individual contributions

Diagrams – Isospin limit

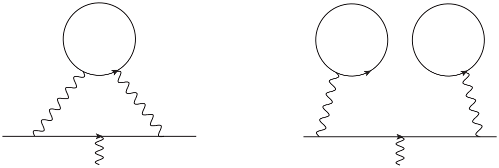
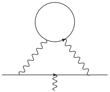
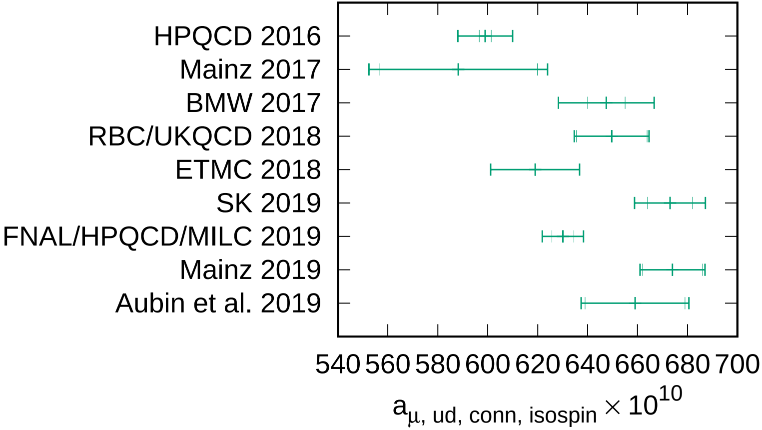
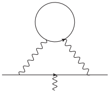
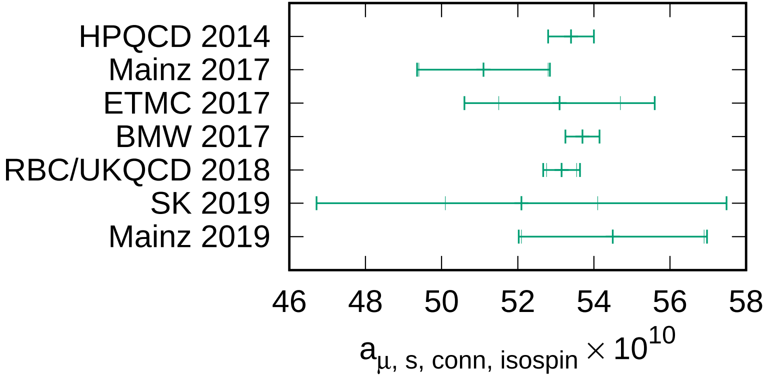


FIG. 1. Quark-connected (left) and quark-disconnected (right) diagram for the calculation of $a_\mu^{\mathrm{HVP\ LO}}$. We do not draw gluons but consider each diagram to represent all orders in QCD.




HPQCD 2016
Mainz 2017
BMW 2017
RBC/UKQCD 2018
ETMC 2018
SK 2019
FNAL/HPQCD/MILC 2019
Mainz 2019
Aubin et al. 2019
540 560 580 600 620 640 660 680 700
$a_{\mu,\text{ ud, conn, isospin}} \times 10^{10}$




HPQCD 2014
Mainz 2017
ETMC 2017
BMW 2017
RBC/UKQCD 2018
SK 2019
Mainz 2019
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52
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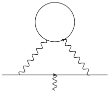
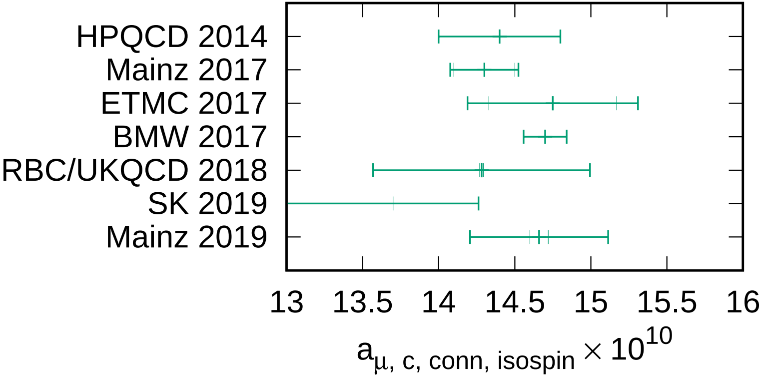
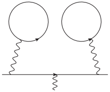
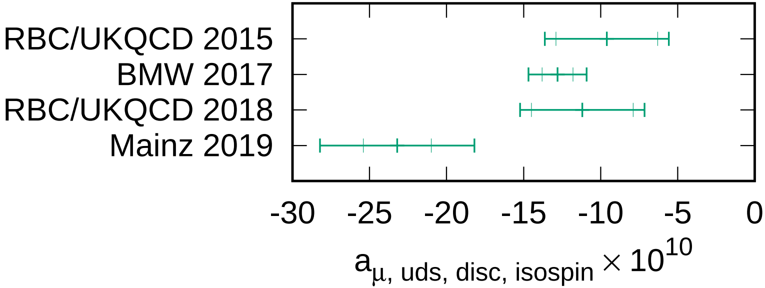
$a_{\mu,\ s,\ conn,\ isospin} \times 10^{10}$

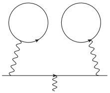



HPQCD 2014
Mainz 2017
ETMC 2017
BMW 2017
RBC/UKQCD 2018
SK 2019
Mainz 2019
13
13.5
14
14.5
15
15.5
16
$a_{\mu,\text{ c, conn, isospin}} \times 10^{10}$





RBC/UKQCD 2015
BMW 2017
RBC/UKQCD 2018
Mainz 2019
-30
-25
-20
-15
-10
-5
0
$a_{\mu,\,\text{uds, disc, isospin}} \times 10^{10}$



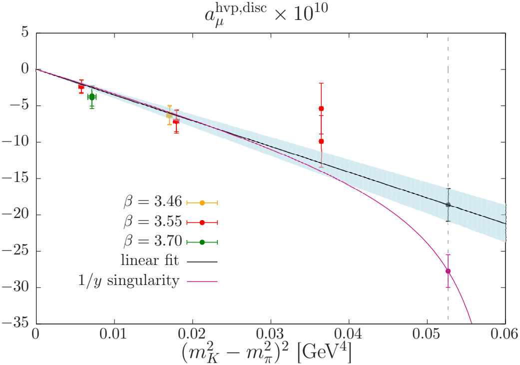
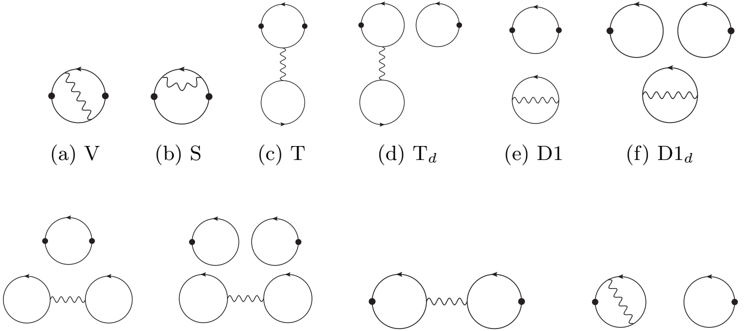


FIG. 9: Extrapolation of the disconnected contribution to a_μ^{hvp} in the SU(3)-breaking variable $\Delta_2 \equiv m_K^2 - m_\pi^2$. The data points for the local-local and the local-conserved discretizations are shown. A linear fit (straight black line), as well as a fit based on ansatz (30) are shown.

Mainz 2019: arXiv:1904.03120; better control of chiral extrapolation could be helpful

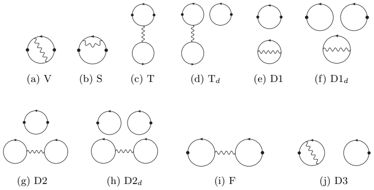
Diagrams – QED corrections

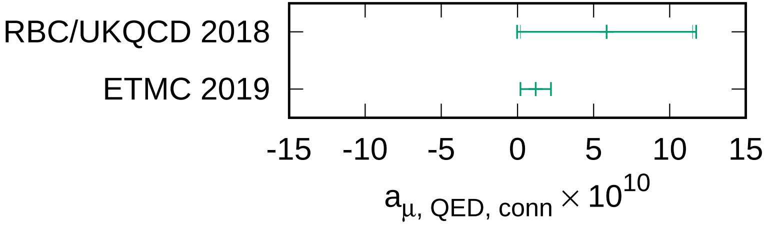


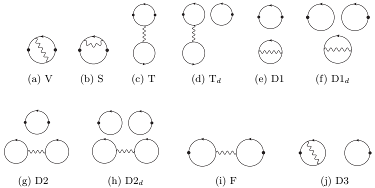
(a) V (b) S (c) T (d) T_d (e) D1 (f) $D1_d$

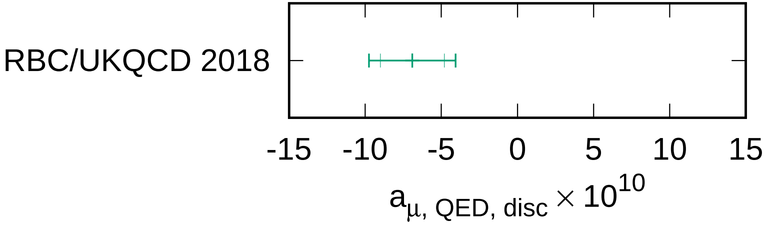
(g) D2 (h) $D2_d$ (i) F (j) D3

For diagram F we enforce exchange of gluons between the quark loops as otherwise a cut through a single photon line would be possible. This single-photon contribution is counted as part of the HVP NLO and not included for the HVP LO.

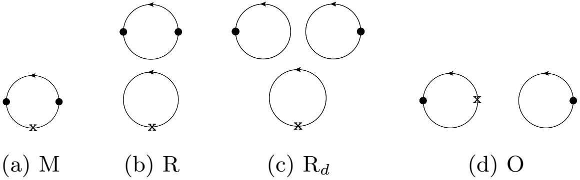

(a) V
(b) S
(c) T
(d) T_d
(e) D1
(f) $D1_d$
(g) D2
(h) $D2_d$
(i) F
(j) D3


RBC/UKQCD 2018
ETMC 2019
-15
-10
-5
0
5
10
15
$a_{\mu,\text{ QED, conn}} \times 10^{10}$

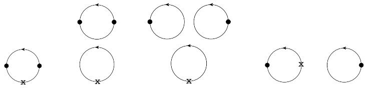

(a) V
(b) S
(c) T
(d) T_d
(e) D1
(f) $D1_d$
(g) D2
(h) $D2_d$
(i) F
(j) D3


RBC/UKQCD 2018
-15
-10
-5
0
5
10
15
$a_{\mu,\text{ QED, disc}} \times 10^{10}$

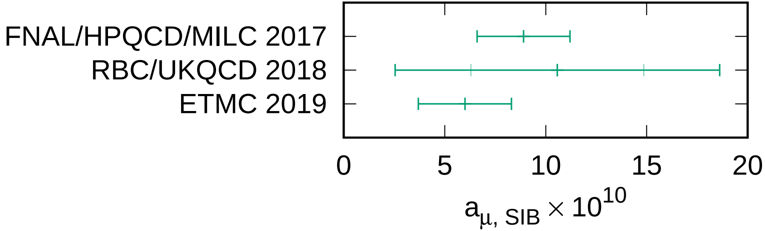
Diagrams – Strong isospin breaking



For the HVP R is negligible since $\Delta m_u \approx -\Delta m_d$ and O is SU(3) and $1/N_c$ suppressed.


x
x
x
x
x

(a) M (b) R (c) R_d (d) O


FNAL/HPQCD/MILC 2017
RBC/UKQCD 2018
ETMC 2019
0
5
10
15
20
$a_{\mu,\,SIB} \times 10^{10}$

Status of RBC/UKQCD effort

The pure lattice calculation of RBC/UKQCD 2018:

$$10^{10} \times a_\mu^{\mathrm{HVP\ LO}} = 715.4(18.7)$$
$$= 715.4(16.3)_{\mathrm{S}}(7.8)_{\mathrm{V}}(3.0)_{\mathrm{C}}(1.9)_{\mathrm{A}}(3.2)_{\mathrm{other}}$$

(S) statistics, (V) finite-volume errors, (C) the continuum limit extrapolation, (A) scale setting uncertainty;

other $\supset$ neglected diagrams for QED and SIB, estimate of bottom quark contribution

Statistical noise mostly from isospin symmetric light quark connected (14.2) and disconnected (3.3), QED (5.7), SIB (4.3)

RBC/UKQCD 2019 update (in preparation):

- Improved methodology
- A lot of new data

The RBC & UKQCD collaborations

BNL and BNL/RBRC

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Taku Izubuchi
Yong-Chull Jang
Chulwoo Jung
Meifeng Lin
Aaron Meyer
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni

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Ryan Abbot
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Christopher Kelly
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Julia Kettle
Michael Marshall
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Antonin Portelli
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James Richings
Chris Sachrajda

Stony Brook University

Jun-Sik Yoo
Sergey Syritsyn (RBRC)

Aaron & Mattia joined since 2018 paper

Improved methodology

Improved statistics and systematics – Bounding Method

BMW/RBC/UKQCD 2016

The correlator in finite volume

$$C(t) = \sum_n |\langle 0|V|n\rangle|^2 e^{-E_n t} .$$

We can bound this correlator at each t from above and below by the correlators

$$\tilde{C}(t; T, \tilde{E}) = \begin{cases} C(t) & t < T , \\ C(T)e^{-(t-T)\tilde{E}} & t \geq T \end{cases}$$

for proper choice of $\tilde{E}$. We can chose $\tilde{E} = E_0$ (assuming $E_0 < E_1 < \ldots$) to create a strict upper bound and any $\tilde{E}$ larger than the local effective mass to define a strict lower bound.

Improved Bounding Method

RBC/UKQCD 2018 (CL@KEK Feb 2018), A. Meyer's talk on Friday

Therefore if we had precise knowledge of the lowest $n = 0, \dots, N$ values of $|\langle 0|V|n\rangle|$ and E_n, we could define a new correlator

$$C^N(t) = C(t) - \sum_{n=0}^{N} |\langle 0|V|n\rangle|^2 e^{-E_n t}$$

which we could bound much more strongly through the larger lowest energy $E_{N+1} \gg E_0$. New method: do a GEVP study of FV spectrum to perform this subtraction.

Note: this avoids uncontrolled power-law errors in a simple GEVP reconstruction as discussed in the next talk.

Reduces statistical error of light quark contribution by more than a factor of 3.

Improved systematics – compute finite-volume effects from first-principles

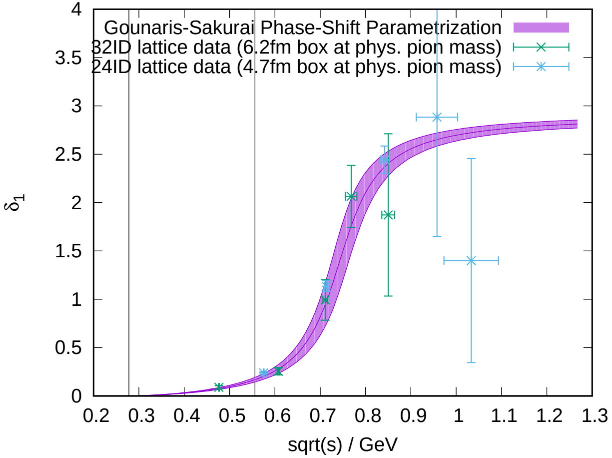
RBC/UKQCD study of QCD at **physical pion mass** at three different volumes:

$L = 4.66$ fm, $L = 5.47$ fm, $L = 6.22$ fm

Results for light-quark isospin-symmetric connected contribution:

- $a_\mu(L = 6.22 \text{ fm}) - a_\mu(L = 4.66 \text{ fm}) = 12.2 \times 10^{-10}$ (sQED), $21.6(6.3) \times 10^{-10}$ (lattice QCD)
- Need to do better than sQED in finite-volume

First constrain the p-wave phase shift from our $L = 6.22$ fm physical pion mass lattice:



$E_\rho = 0.766(21)$ GeV (PDG 0.77549(34) GeV)
$\Gamma_\rho = 0.139(18)$ GeV (PDG 0.1462(7) GeV)

GSL^2 finite-volume results compared to sQED and lattice

GSL^2 method of Meyer 2012

Results for light-quark isospin-symmetric connected contribution:

- FV difference between $a_\mu(L = 6.22\ \text{fm}) - a_\mu(L = 4.66\ \text{fm}) = 12.2 \times 10^{-10}$ (sQED), $21.6(6.3) \times 10^{-10}$ (lattice QCD), $20(3) \times 10^{-10}$ (GSL^2)
- GSL^2 prediction agrees with actual FV effect measured on the lattice, sQED is in slight tension, two-loop FV ChPT to be compared next Bijnens and Relefors 2017
- Use GSL^2 to update FV correction of Phys. Rev. Lett. 121, 022003 (2018): $a_\mu(L \to \infty) - a_\mu(L = 5.47\ \text{fm}) = 16(4) \times 10^{-10}$ (sQED), $22(1) \times 10^{-10}$ (GSL^2); sQED error estimate based on Bijnens and Relefors 2017, table 1.
- Compare also to Hansen-Patella 2019 1904.10010: $a_\mu(L \to \infty) - a_\mu(L = 5.47\ \text{fm}) \approx 14 \times 10^{-10}$, effect of neglected $e^{-\sqrt{2} m_\pi L}$ likely significant; see talk today!

Other improvements:

- HVP QED from re-analysis of HLbL point-source data (see also τ project, 1811.00508) reduces statistical noise by $\approx 10\times$ for V and S (See also talk by M. Bruno next week)
- Infinte-volume and continuum limit also for diagram V, S, and F
- First results for T, D1, and R; other sub-leading in preparation
- Global fit combined with calculation of mass derivatives gives much reduced uncertainty for diagrams M and O (connected and disconnected SIB)

New data set

Ensembles at physical pion mass:

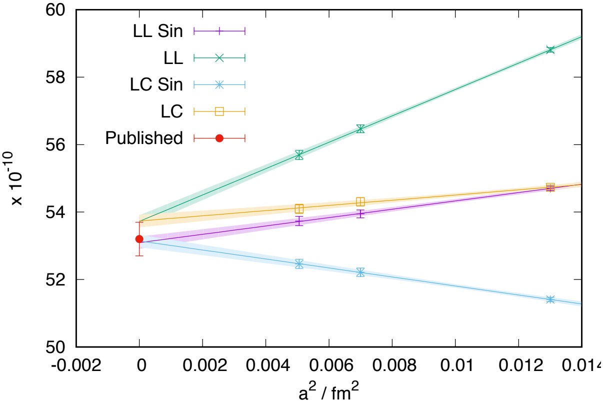
48I (1.73 GeV, 5.5fm), 64I (2.359 GeV, 5.4fm), 24ID (1 GeV, 4.7fm), 32ID (1 GeV, 6.2fm), 48ID (1 GeV, 9.3fm), 32IDf (1.37 GeV, 4.6fm)

RBC/UKQCD 2019 (data for light quarks, changes from 2018):

- A2A data for connected isospin symmetric: 48I (127 conf $\rightarrow$ 400 conf), 64I (160 conf $\rightarrow$ 250 conf), 24ID (new 130 conf, multi mass), 32ID (new 88 conf, multi mass)
- A2A data (tadpole fields) for disconnected: 48I (33 conf), 24ID (new 260 conf, multi mass), 32IDf (new 103 conf)
- QED and SIB corrections to meson and Ω masses, Z_V: 48I (30 conf) and 64I (new 30 conf)
- QED and SIB from HLbL point sources on 48I, 24ID, 32ID, 32IDf (on order of 20 conf each, 2000 points per config)
- Distillation data on 48I (33 conf), 64I (in progr.), 24ID (33 conf), 32ID (11 conf, multi-mass)
- New Ω mass operators (excited states control): 48I (130 conf)

Add $a^{-1} = 2.77$ GeV lattice spacing

- Third lattice spacing for strange data ($a^{-1} = 2.77$ GeV with $m_\pi = 234$ MeV with sea light-quark mass corrected from global fit):



- For light quark need new ensemble at physical pion mass. Started run on Summit Machine at Oak Ridge this year ($a^{-1} = 2.77$ GeV with $m_\pi = 139$ MeV).

Conclusions and Outlook

- Target precision for HVP is of $O(1 \times 10^{-10})$ in a few years; for now consolidate error at $O(3 \times 10^{-10})$
- Dispersive result from $e^+e^- \to$ hadrons right now is at 3×10^{-10} to 5×10^{-10} but limited by experimental tensions
- Two-pion channel from DHMZ17, KNT18 (e^+e^-) and DHMYZ13 (τ) are scattered by 12.5×10^{-10}

  Experimental updates and first-principles calculation of isospin-breaking corrections desirable. Combination of dispersive and lattice results can in short term lessen dependence on contested experimental data.
- RBC/UKQCD:
  - New methods to reduce statistical and systematic errors and a lot of additional data.
  - By end of this year, first-principles lattice result could have error of $O(5 \times 10^{-10})$