Pion Scattering to $(g-2)_\mu$ at Physical Pion Mass

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August 2, 2019

Advances in Lattice Gauge Theory 2019

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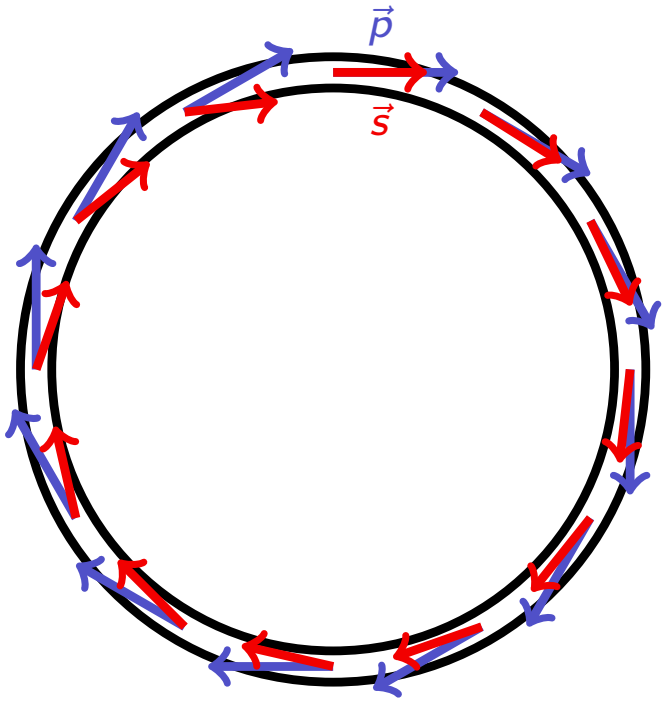
Outline

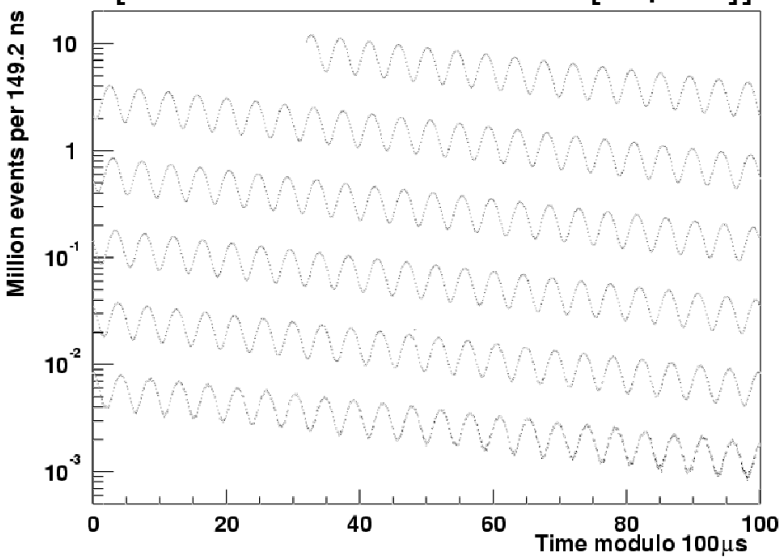
- Muon $g-2$ Experiment
  - Motivation from muon $g-2$
  - Tensions in $\pi\pi$ Scattering
  - Error Budget and LQCD Strategy
- Correlation Function Spectrum & Overlap
  - Lattice Parameters
  - GEVP Spectrum & Overlaps
  - $\pi\pi$ Scattering Phase Shift
  - 4π Correlation Functions
- Bounding Method and the Muon HVP
  - Correlation Function Reconstruction
  - (Improved) Bounding Method
  - Results
- $\pi\pi$ Scattering Phase Shifts
  - $I = 2, \ell = 0$
  - $I = 1, \ell = 1$
  - $I = 2, \ell = 2$
- Conclusions/Outlook

Introduction

Muon Anomalous Magnetic Moment Experiment



[Bennett *et al.*, 0602035[hep-ex]]



See talk by C.Lehner for detailed introduction

High-precision experiment of spin precession
relative to momentum direction in storage ring

Anomalous frequency $\omega_a = \frac{g-2}{2}\frac{eB}{m} = a_\mu \frac{eB}{m}$

Sensitive to new physics, and also discrepant with experiment!

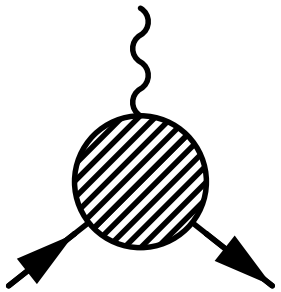
Fermilab Muon $g-2$ Experiment

Experiment has come a long way (and so has theory!)

Aiming for a $4\times$ improvement in uncertainty over the BNL result

Muon $g-2$ Theory Error Budget

Contribution	Value $\times 10^{10}$	Uncertainty $\times 10^{10}$
QED	11 658 471.895	0.008
EW	15.4	0.1
HVP LO	692.5	2.7
HVP NLO	−9.84	0.06
HVP NNLO	1.24	0.01
Hadronic light-by-light	10.5	2.6
Total SM prediction	11 659 181.7	3.8
BNL E821 result	11 659 209.1	6.3
Fermilab E989 target		≈ 1.6



Experiment-Theory difference is 27.4(7.3) $\implies$ 3.7σ tension!

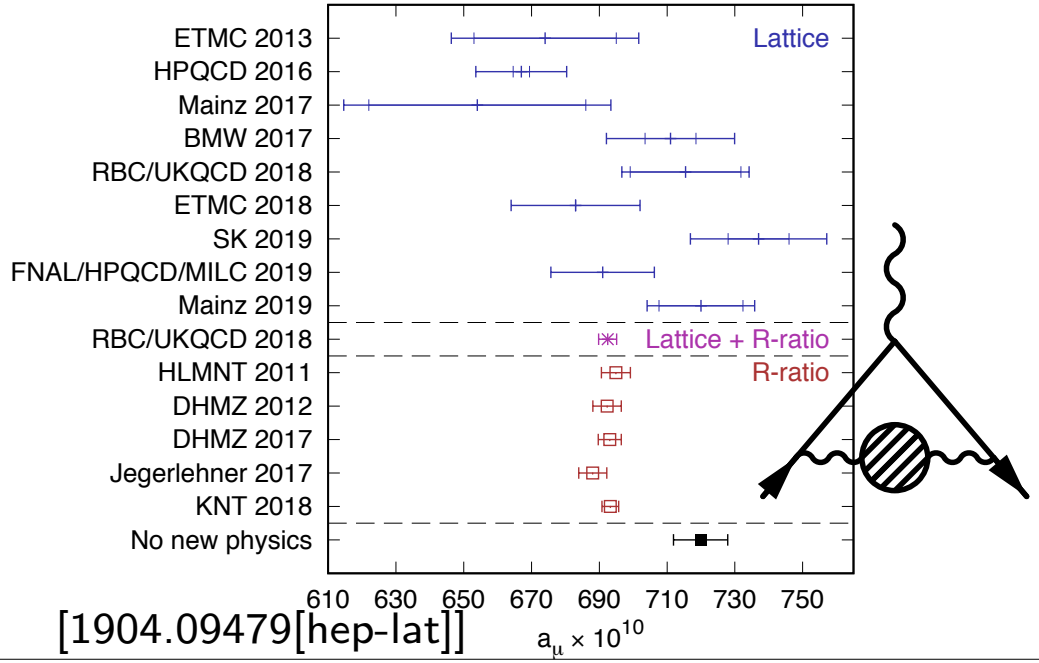
Target measurement:
Hadronic Vacuum Polarization (HVP)

$\implies$

Lattice results have larger uncertainty, but systematically improve

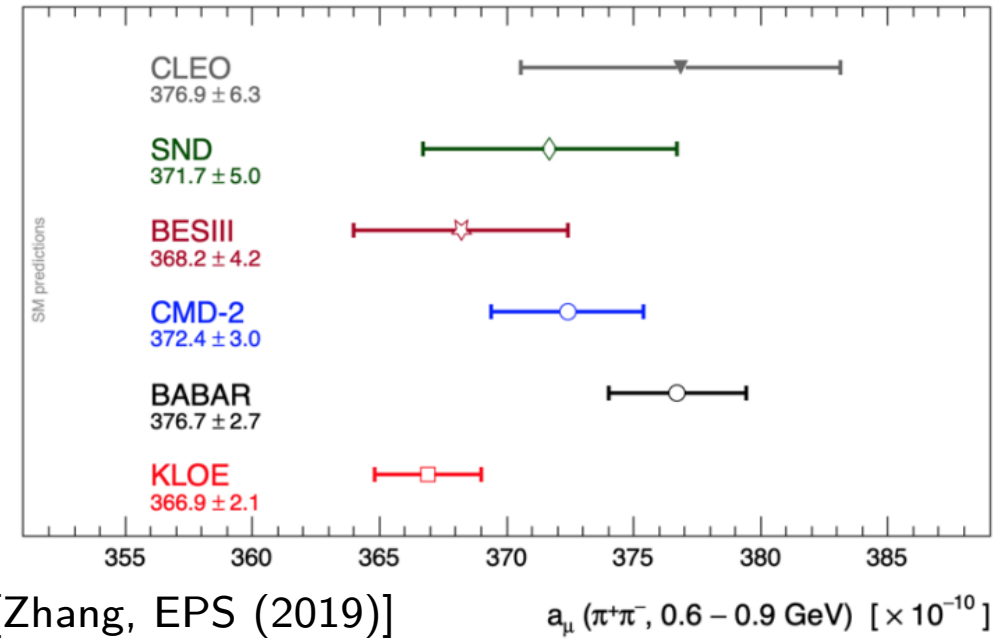
$\implies$

Dispersive ("R-ratio") results more precise, but static

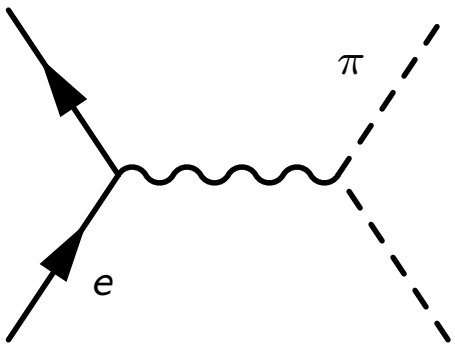


[1904.09479[hep-lat]]

Tensions in Experiment



[Zhang, EPS (2019)]



R-ratio data for $ee \to \pi\pi$ exclusive channel, $\sqrt{s} = 0.6 - 0.9\ \mathrm{GeV}$ region
Tension between most precise measurements (BABAR/KLOE)
R-ratio a_μ^{HVP} uncertainty $<$ difference in this channel

Avoid tension by computing precise lattice-only estimate of a_μ^{HVP}
Use lattice QCD to inform experiment, resolve discrepancy

Error Budget

$a_\mu^{\text{ud, conn, isospin}}$	$202.9(1.4)_{\rm S}(0.2)_{\rm C}(0.1)_{\rm V}(0.2)_{\rm A}(0.2)_{\rm Z}$	$649.7(14.2)_{\rm S}(2.8)_{\rm C}(3.7)_{\rm V}(1.5)_{\rm A}(0.4)_{\rm Z}(0.1)_{\rm E48}(0.1)_{\rm E64}$
$a_\mu^{\text{s, conn, isospin}}$	$27.0(0.2)_{\rm S}(0.0)_{\rm C}(0.1)_{\rm A}(0.0)_{\rm Z}$	$53.2(0.4)_{\rm S}(0.0)_{\rm C}(0.3)_{\rm A}(0.0)_{\rm Z}$
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[Blum *et al.*, (2018)]

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Full program of computations to reduce uncertainties:

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Compute QED contribution

Improve lattice spacing determination

Error Budget

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$a_\mu^{\text{QED, disc}}$	$-0.2(0.1)_{\rm S}(0.0)_{\rm C}(0.0)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(0.0)_{\rm E}$	$-6.9(2.1)_{\rm S}(0.4)_{\rm C}(1.4)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(1.3)_{\rm E}$
a_μ^{SIB}	$0.1(0.2)_{\rm S}(0.0)_{\rm C}(0.2)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(0.0)_{\rm E48}$	$10.6(4.3)_{\rm S}(0.6)_{\rm C}(6.6)_{\rm V}(0.1)_{\rm A}(0.0)_{\rm Z}(1.3)_{\rm E48}$
$a_\mu^{\text{udsc, isospin}}$	$231.9(1.4)_{\rm S}(0.2)_{\rm C}(0.1)_{\rm V}(0.3)_{\rm A}(0.2)_{\rm Z}(0.0)_{\rm M}$	$705.9(14.6)_{\rm S}(2.9)_{\rm C}(3.7)_{\rm V}(1.8)_{\rm A}(0.4)_{\rm Z}(2.3)_{\rm L}(0.1)_{\rm E48}$ $(0.1)_{\rm E64}(0.0)_{\rm M}$
$a_\mu^{\text{QED, SIB}}$	$0.1(0.3)_{\rm S}(0.0)_{\rm C}(0.2)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(0.0)_{\rm E}(0.0)_{\rm E48}$	$9.5(7.4)_{\rm S}(0.7)_{\rm C}(6.9)_{\rm V}(0.1)_{\rm A}(0.0)_{\rm Z}(1.7)_{\rm E}(1.3)_{\rm E48}$
$a_\mu^{\text{R-ratio}}$	$460.4(0.7)_{\rm RST}(2.1)_{\rm RSY}$	
a_μ	$692.5(1.4)_{\rm S}(0.2)_{\rm C}(0.2)_{\rm V}(0.3)_{\rm A}(0.2)_{\rm Z}(0.0)_{\rm E}(0.0)_{\rm E48}$ $(0.0)_{\rm b}(0.1)_{\rm c}(0.0)_{\overline{\rm S}}(0.0)_{\overline{\rm Q}}(0.0)_{\rm M}(0.7)_{\rm RST}(2.1)_{\rm RSY}$	$715.4(16.3)_{\rm S}(3.0)_{\rm C}(7.8)_{\rm V}(1.9)_{\rm A}(0.4)_{\rm Z}(1.7)_{\rm E}(2.3)_{\rm L}$ $(1.5)_{\rm E48}(0.1)_{\rm E64}(0.3)_{\rm b}(0.2)_{\rm c}(1.1)_{\overline{\rm S}}(0.3)_{\overline{\rm Q}}(0.0)_{\rm M}$

TABLE I. Individual and summed contributions to a_μ multiplied by 10^{10}. The left column lists results for the window method with $t_0 = 0.4$ fm and $t_1 = 1$ fm. The right column shows results for the pure first-principles lattice calculation. The respective uncertainties are defined in the main text.

[Blum *et al.*, (2018)]

Full program of computations to reduce uncertainties:

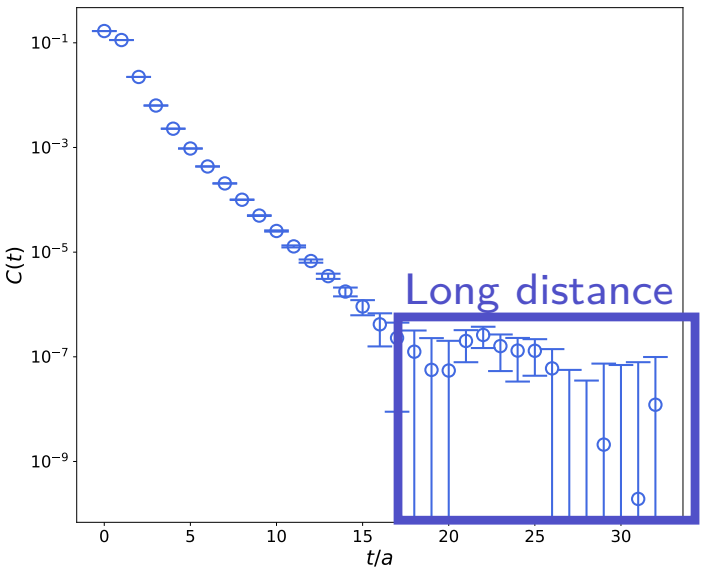
Reduce statistical uncertainties on light connected contribution

Compute QED contribution

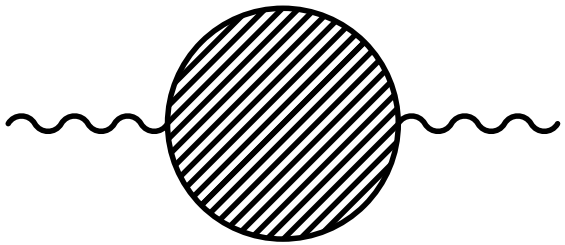
Improve lattice spacing determination

Finite volume and continuum extrapolation study

Exclusive Channels in the HVP



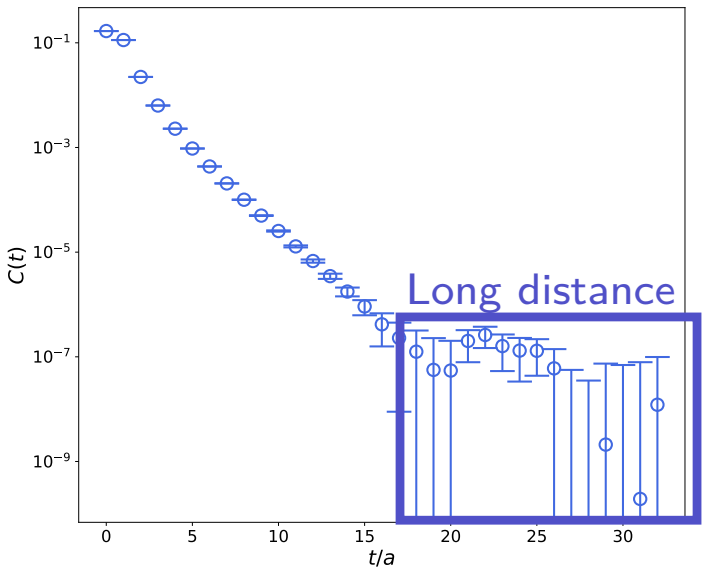
$$C(t) = \frac{1}{3}\sum_i \left\langle \left[\bar{\psi}\gamma_i\psi\right]_t \left[\bar{\psi}\gamma_i\psi\right]_0 \right\rangle$$



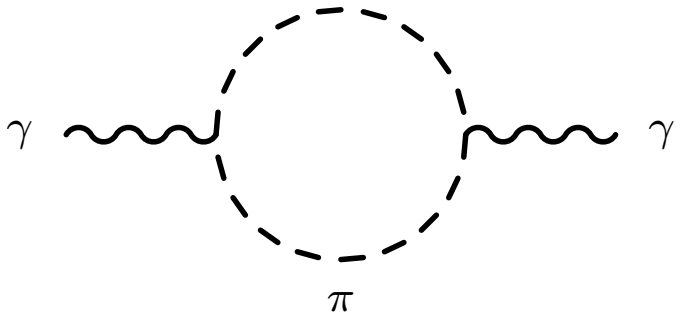
Correlator has large statistical error in long-distance region,
but contributions from high energy states are exponentially suppressed

Long distance correlator dominated by two-pion states,
but overlap of vector current with two-pion states is minimal

Exclusive Channels in the HVP



$$C(t) = \frac{1}{3}\sum_i \left\langle \left[\bar{\psi}\gamma_i\psi\right]_t \left[\bar{\psi}\gamma_i\psi\right]_0 \right\rangle$$
$$\approx \sum_n \left| \langle\Omega|\bar{\psi}\gamma_i\psi|n\rangle \right|^2 e^{-E_n t}$$



Correlator has large statistical error in long-distance region,
but contributions from high energy states are exponentially suppressed

Long distance correlator dominated by two-pion states,
but overlap of vector current with two-pion states is minimal

Strategy:

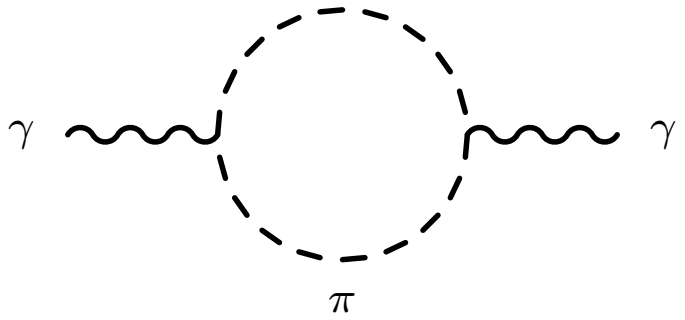
- Construct & measure operators that overlap strongly with $\pi\pi$ states
- Correlate these operators with the local vector current
- a_μ^{HVP} computed by integrating with time-momentum representation kernel,
  $a_\mu^{HVP} = \sum_t w_t C(t)$ [Bernecker et al., 1107.4388 [hep-lat]]

Pion Scattering

Extraction of $\pi\pi$ exclusive channels requires many operators

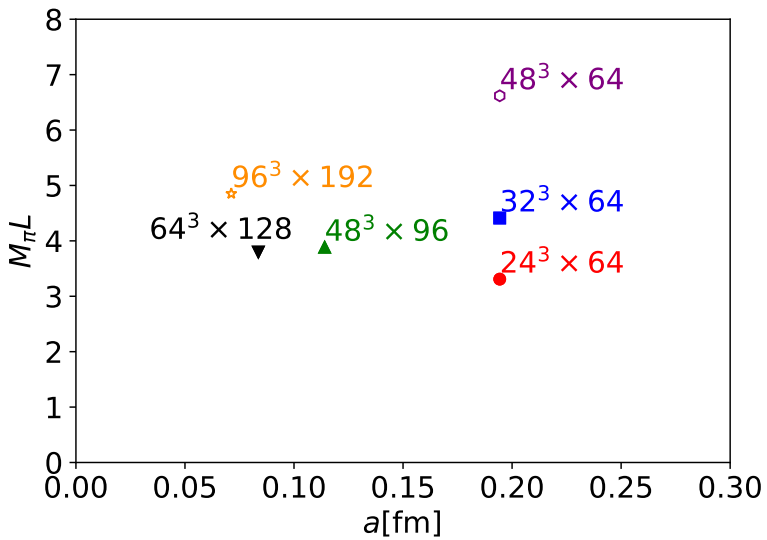
Operator basis generated with distillation, perambulators

- Can access pion scattering phase shifts with data
- Study ρ resonance region at physical pion mass
- Low 4π threshold, study effects on spectrum, HVP



LQCD Computation Setup

Computation Details



Computed on $2+1$ flavor Möbius Domain Wall Fermions for valance and sea, M_π at physical value on all ensembles

Computations using distillation setup with N_{eig} eigenvectors

Results in this talk include four ensembles:

- "24ID": $24^3 \times 64$ (4.8 fm), $a \approx 0.194$ fm ≈ 1.015 GeV^{-1}
- "32ID": $32^3 \times 64$ (6.2 fm), $a \approx 0.194$ fm ≈ 1.015 GeV^{-1}
- "48I": $48^3 \times 96$ (5.5 fm), $a \approx 0.114$ fm ≈ 1.730 GeV^{-1}
- "64I": $64^3 \times 128$ (5.3 fm), $a \approx 0.084$ fm ≈ 2.359 GeV^{-1}

Future work including other ensembles for better control of extrapolations

Operators

Operators constructed in $I = 1$, P-wave channel to impact upon HVP_μ

Designed to have strong overlap with specific target states,
but all operators unavoidably couple to all states in HVP spectrum

Vector current operators:

- Local $\mathcal{O}_{J_\mu} = \sum_x \bar{\psi}(x)\gamma_\mu\psi(x)$, $\mu \in \{1, 2, 3\}$
- Smeared $\mathcal{O}_{j_\mu} = \sum_{xyz} \bar{\psi}(x)f(x-z)\gamma_\mu f(z-y)\psi(y)$

2π operators with $\mathcal{O}_n$ given by $\vec{p}_\pi \in \frac{2\pi}{L} \times \{(1,0,0),(1,1,0),(1,1,1),(2,0,0)\}$

$$\mathcal{O}_n = \left|\sum_{xyz} \bar{\psi}(x)f(x-z)e^{-i\vec{p}_\pi\cdot\vec{z}}\gamma_5 f(z-y)\psi(y)\right|^2$$

Also test a 4π operator with $\vec{p}_\pi = \frac{2\pi}{L} \times (1,0,0)$:

$$\mathcal{O}_{4\pi} = \left|\sum_{xyz} \bar{\psi}(x)f(x-z)e^{-i\vec{p}_\pi\cdot\vec{z}}\gamma_5 f(z-y)\psi(y)\right|^2 \left|\sum_{xy} \bar{\psi}(x)f(x-y)\gamma_5\psi(y)\right|^2$$

Correlators arranged in a $N \times N$ symmetric matrix:

$\otimes$	$\mathcal{O}_{J_\mu}$	$\mathcal{O}_{j_\mu}$	$\mathcal{O}_{2\pi}$	$\mathcal{O}_{4\pi}$
$\mathcal{O}_{J_\mu}$	$C_{J_\mu J_\mu}$	$C_{J_\mu j_\mu}$	$C_{J_\mu 2\pi}$	$C_{J_\mu 4\pi}$
$\mathcal{O}_{j_\mu}$		$C_{j_\mu j_\mu}$	$C_{j_\mu 2\pi}$	$C_{j_\mu 4\pi}$
$\mathcal{O}_{2\pi}$			$C_{2\pi 2\pi}$	$C_{2\pi 4\pi}$
$\mathcal{O}_{4\pi}$				$C_{4\pi 4\pi}$

$\rightarrow C(t)$

Generalized EigenValue Problem (GEVP)

Generalized EigenValue Problem to estimate overlap with vector current & energies

$$C(t)\, V = C(t + \delta t)\, V\, \Lambda(\delta t)$$

$$\Lambda_{nn}(\delta t) \sim e^{+E_n \delta t}\,, \quad V_{im} \propto \langle \Omega | \mathcal{O}_i | m \rangle$$

$C(t)$ is the matrix of correlation functions from previous slide

Compute at fixed δt, vary t: plateau for large t

From result, reconstruct exponential dependence of local vector correlation function

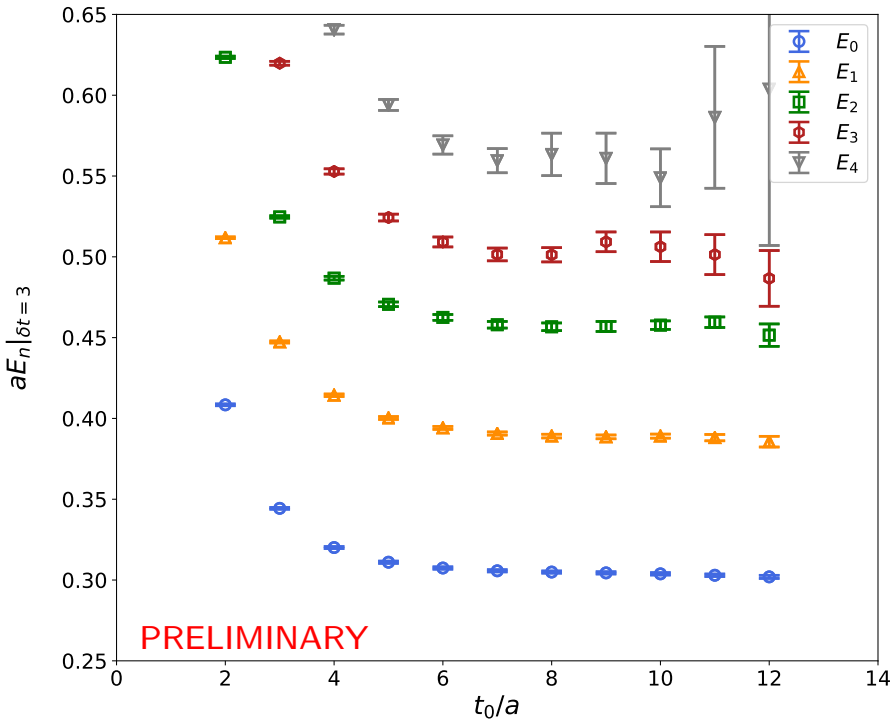
$$C_{ij}^{\text{latt.}}(t) = \sum_{n}^{N} \langle \Omega | \mathcal{O}_i | n \rangle \langle n | \mathcal{O}_j | \Omega \rangle\, e^{-E_n t}$$

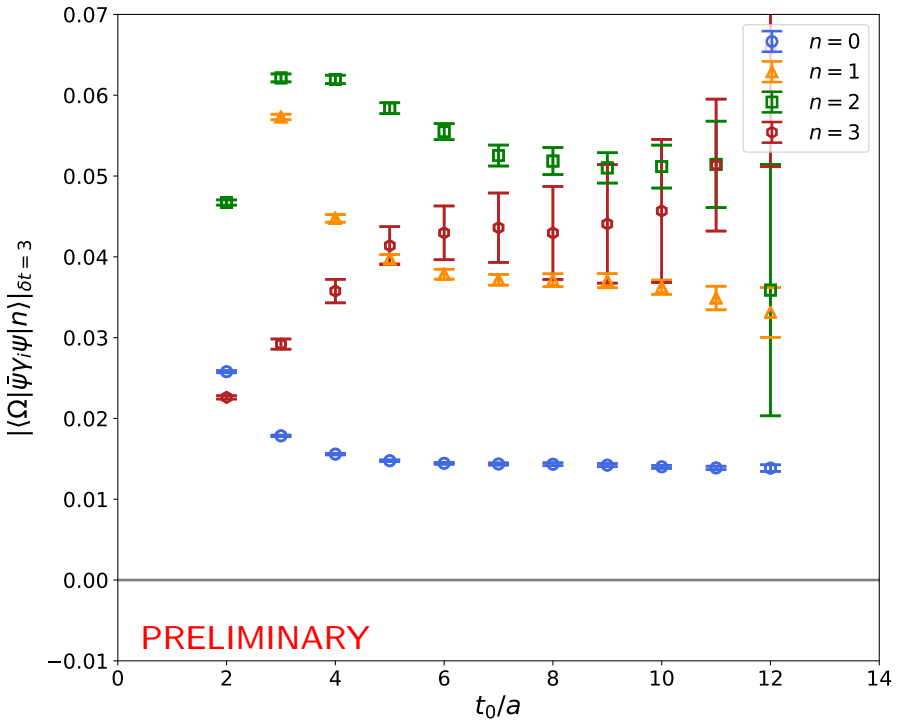
In theory, infinite number of states contribute to correlation function

In practice, only finite N necessary to model correlation function

$\implies$ finite GEVP basis is sufficient

GEVP Results - $J_\mu + 2\pi$ Operators only





6-operator basis on 48I ensemble: local+smeared vector, $4\times(2\pi)$

Data points from solving GEVP at fixed δt

$$C(t_0)\,V = C(t_0+\delta t)\,V\,\Lambda(\delta t)\,, \quad \Lambda_{nn}(\delta t) \sim e^{+E_n \delta t}$$

Excited state contaminations decay as $t_0, \delta t \to \infty$
moving right on plot $\implies$ asymptote to lowest states' spectrum & overlaps

Left: Spectrum; Right: Overlap with local vector current

Group Theory & Contraction Engine

An automated group theory engine has been an integral part of RBC-UKQCD's automated setup for two-pion diagrams in exclusive channel study

Group Theory & Contraction Engine

An automated group theory engine has been an integral part of RBC-UKQCD's automated setup for two-pion diagrams in exclusive channel study

Code builds a text representation of operators by performing tensor products and irrep decompositions of lattice operators with arbitrary spin & momentum

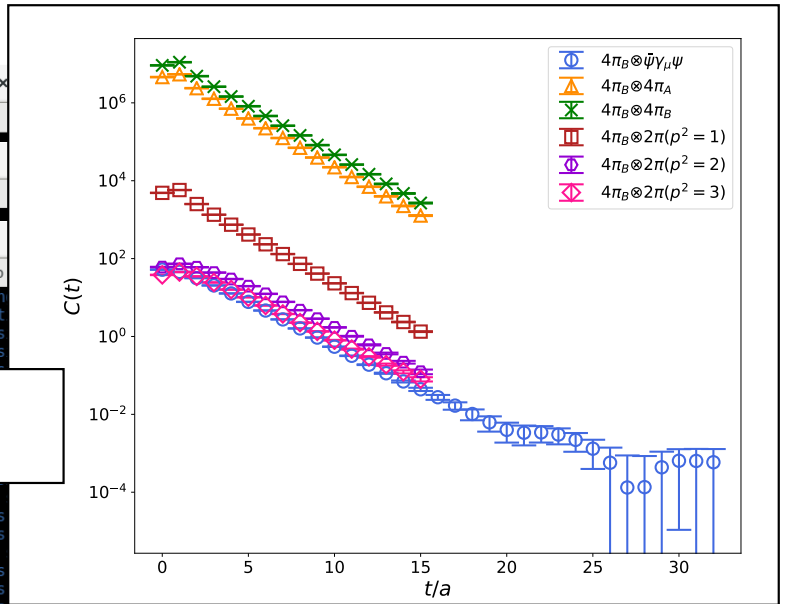
```
ameyer@ssh02:/home/sdcc/u/ameyer
File  Edit  View  Search  Terminal  Help
daughter 4: 110_a1+ [1, 1, 0]
 daughter 0: 100_a1+ [0, 0, 1]
   0 100_a1+ [0, 0, 2]
   1 000_a1+ [0, 0, 0]
   6 110_a2+ [1, 1, 0]
   7 110_a1- [1, 1, 0]
   8 110_a2- [1, 1, 0]
 daughter 3: 210_aa+ [2, 1, 0]
   0 100_a1+ [0, 0, 2]
   1 100_a2+ [0, 0, 2]
   2 100_bb0 [0, 0, 2]
   3 110_a1+ [1, 1, 0]
   4 110_a2+ [1, 1, 0]
   5 210_aa+ [3, 1, 0]
   6 211_aa+ [1, 1, 2]
   7 211_aa- [1, 1, 2]
   8 110_a1+ [2, 2, 0]
   9 110_a2+ [2, 2, 0]
 daughter 4: 111_aa+ [1, 1, 1]
   0 110_a1+ [1, 1, 0]
   1 110_a2- [1, 1, 0]
   2 211_aa+ [1, 1, 2]
 daughter 5: 111_bb0 [1, 1, 1]
   0 110_a1+ [1, 1, 0]
   1 110_a2+ [1, 1, 0]
   2 110_a1- [1, 1, 0]
   3 110_a2- [1, 1, 0]
   4 211_aa+ [1, 1, 2]
   5 211_aa- [1, 1, 2]
daughter 5: 110_a2+ [1, 1, 0]
 daughter 0: 100_a1+ [0, 0, 1]
   0 100_a1+ [0, 0, 2]
   1 000_a1+ [0, 0, 0]
```

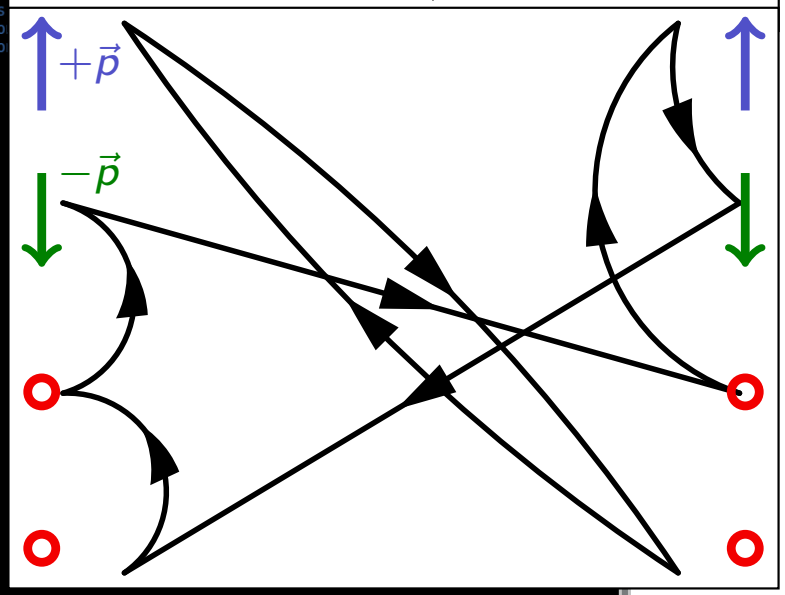
```
ameyer@ssh01:/home/sdcc/u/ameyer
MOM [1,0,0] t
U t local
DBAR t local
GAMMA 5
U t local
UBAR t local
GAMMA 5
U t local

FACTOR 0.0559016994375
 UBAR t local
 GAMMA 5
 MOM [1,0,0] t
 D t local
 UBAR t local
 GAMMA 5
 MOM [-1,0,0] t
 U t local
 DBAR t local
 GAMMA 5
 U t local
 UBAR t local
 GAMMA 5
 U t local

# 000_t1- p=0,0,0 LG_index=0

FACTOR -0.0559016994375
 UBAR t local
 GAMMA 5
 MOM [-1,0,0] t
 D t local
 DBAR t local
```

```
/u/ameyer
# group 12 / 16 with 16 elements simplified to 8 elements
# group 13 / 16 with 128 elements simplified to 64 elements
# group 14 / 16 with 320 elements simplified to 160 elements
# group 15 / 16 with 128 elements simplified to 64 elements
# 974 term(s) after simplification with heuristics
# 974 term(s) after simplification
#

FACTOR 3.43207780157573 0
BEGINTRACE
GAMMA 5
MOM [1,0,0] t
LIGHT t t0
GAMMA 5
LIGHTBAR t0 t
ENDTRACE
BEGINTRACE
GAMMA 5
MOM [-1,0,0] t0
LIGHT t0 t0
GAMMA 5
LIGHTBAR t0 t
GAMMA 5
MOM [-1,0,0] t
LIGHTBAR t t
GAMMA 5
LIGHTBAR t t
GAMMA 5
LIGHT t t0
GAMMA 5
MOM [1,0,0] t0
LIGHT t0 t0
ENDTRACE

FACTOR 3.43207780157573 0
BEGINTRACE
GAMMA 5
MOM [-1,0,0] t
                                                    60,1          0%
```

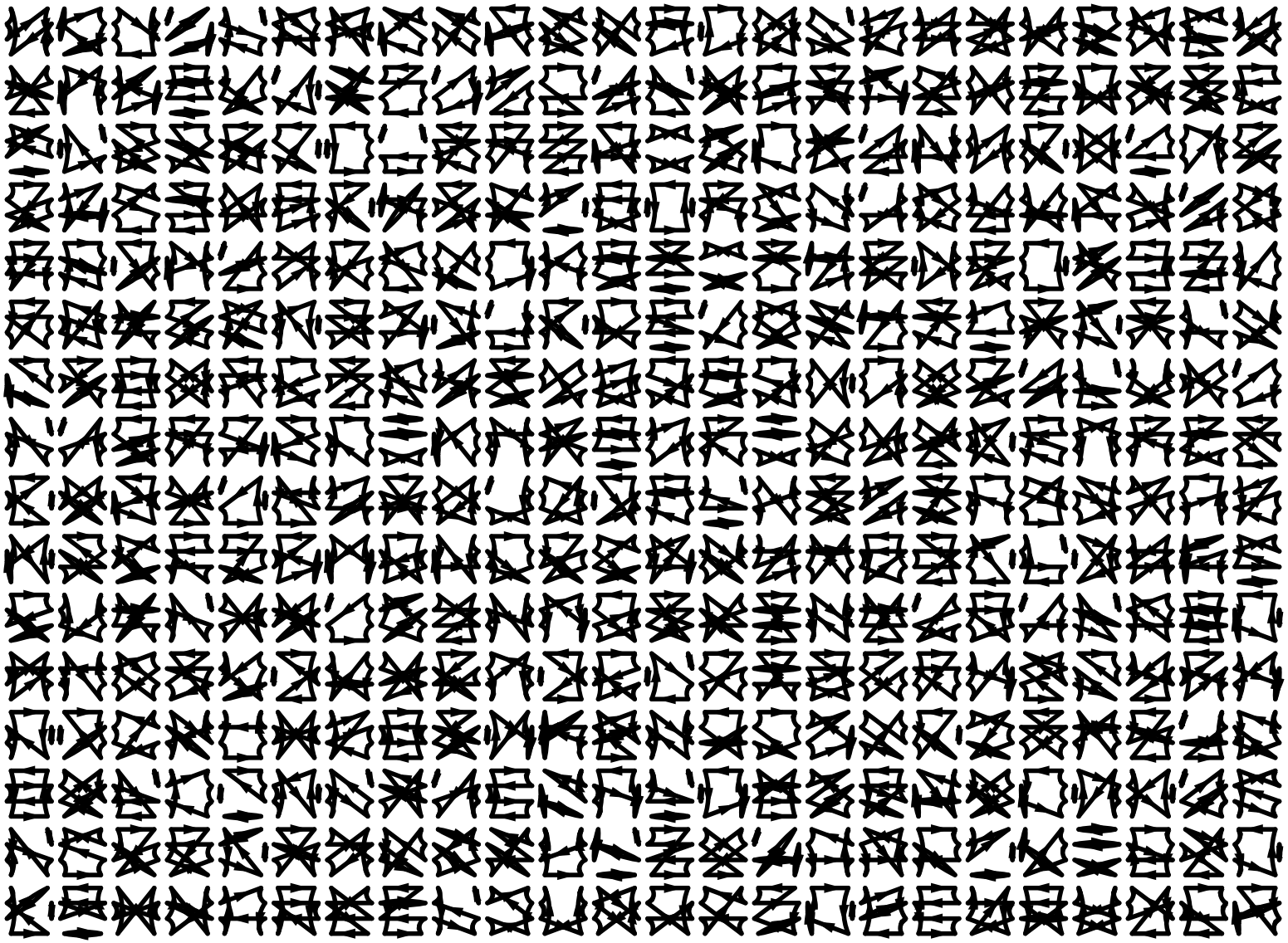
Group Theory & Contraction Engine

This has resulted in a world-first computation of 4π to 4π correlation functions in $I = 1$ channel

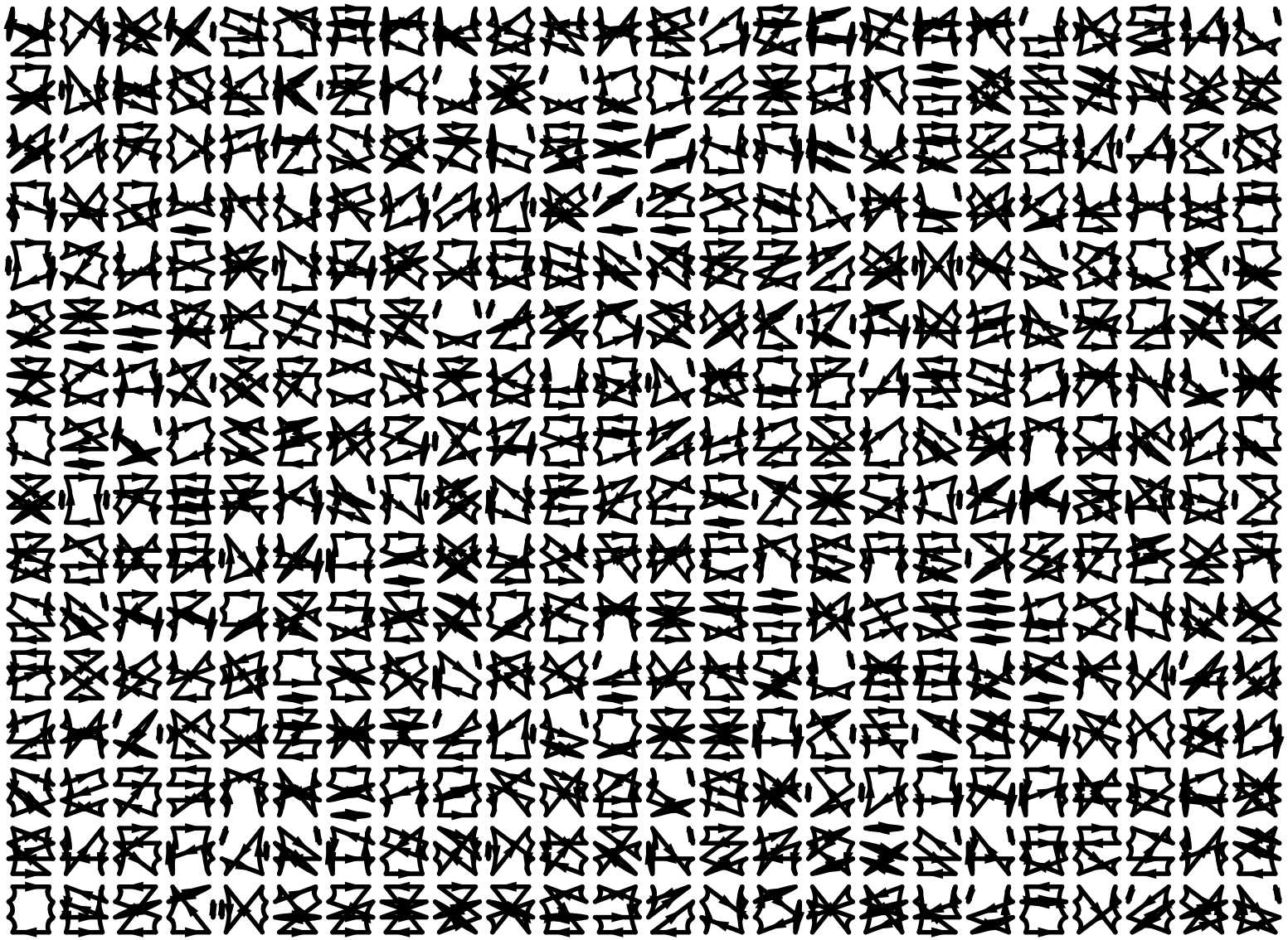




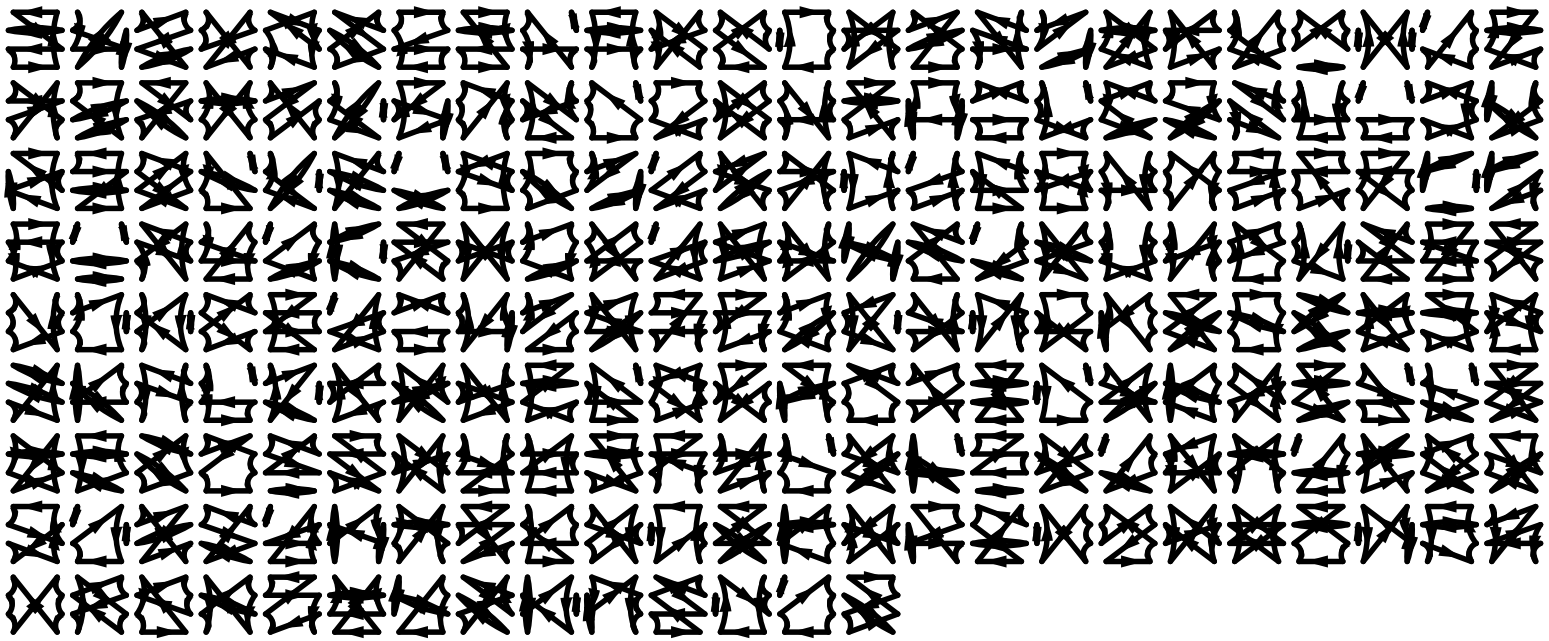
4π Contractions



4π Contractions cont...



4π Contractions cont... cont...



From 4π to $N\pi$ (and Beyond)

Group theory engine handles bosonic & fermionic octahedral irreps, any momentum
Easily build N-particle operators from single-particle building blocks

Strong need for multiparticle operators, as seen over past two weeks:

- Difficulty extracting scattering states without bilocal operators
- Large multiplicities of $X\pi + N$ states in nucleon correlators
- When simulating close to physical point, multiparticle thresholds are relevant

Nucleon matrix elements vital for neutrino community (discussion?)
Incredible work done by many people, could start tackling these problems soon

ameyer@ssh01:/home/sdcc/u/ameyer

File Edit View Search Terminal Help

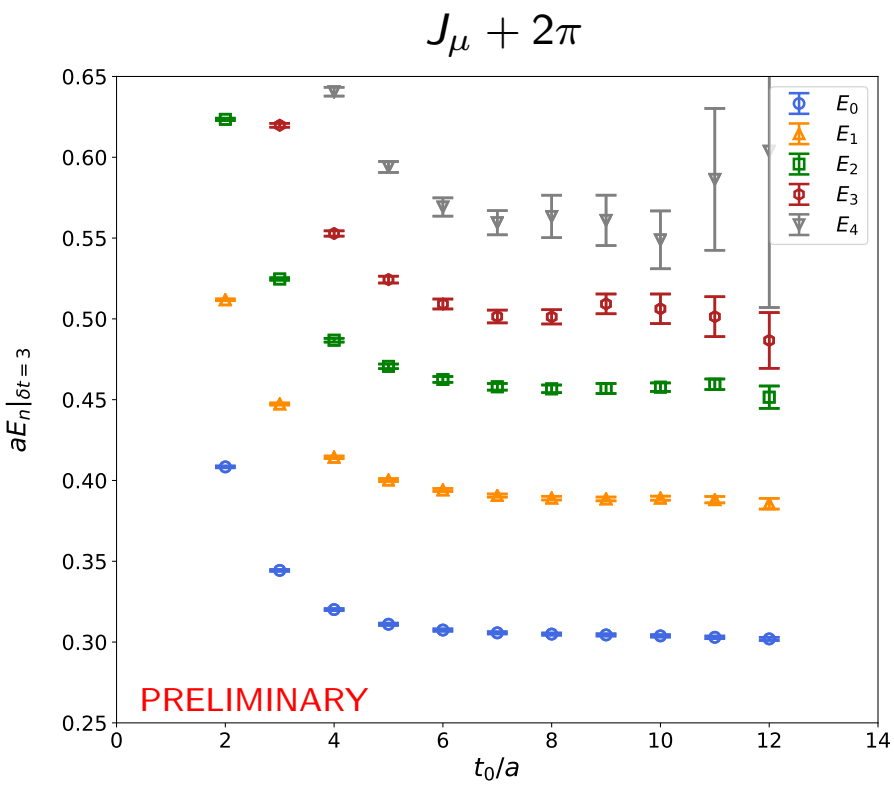
```
## nucleon operator
## uses A_1^+ bilinear

FACTOR 0.70710678118654746
 UTRANSPOSE t1
 GAMMA 5
 GAMMA_CHARGE
 EPSILON [-1,0,0] t1 c1
 D t1
 FREEBAR c1
 GAMMA_PARITY_POSITIVE
 U t1
 DBAR t0
 GAMMA 5
 MOM [1,0,0] t0
 D t0

FACTOR -0.70710678118654746
 UTRANSPOSE t1
 GAMMA 5
 GAMMA_CHARGE
 EPSILON [-1,0,0] t1 c1
 D t1
 FREEBAR c1
 GAMMA_PARITY_POSITIVE
 U t1
 UBAR t0
 GAMMA 5
 MOM [1,0,0] t0
 U t0

FACTOR 0.70710678118654746
 UTRANSPOSE t1
 GAMMA 5
 GAMMA_CHARGE
 EPSILON [0,-1,0] t1 c1
 D t1
 FREEBAR c1
 GAMMA_PARITY_POSITIVE
 U t1
```

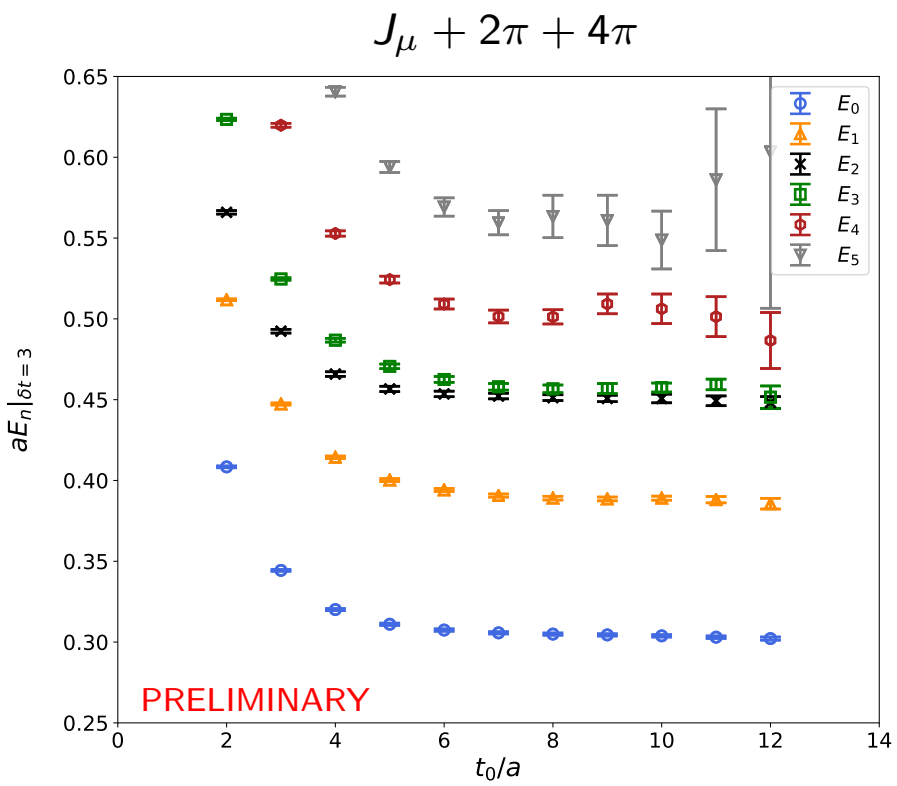
ameyer@ssh01:/home/sdcc/u/ameyer

File Edit View Search Terminal Help

```
# group 39 / 42 with 1 elements simplified to 1 elements
# group 40 / 42 with 1 elements simplified to 1 elements
# group 41 / 42 with 1 elements simplified to 1 elements
# 36 term(s) after simplification with heuristics
# 36 term(s) after simplification
#

FACTOR -0.707106781186547 0
BEGINTENSORTRACE c0 c1
BEGINTRACE
GAMMA 5
GAMMA_CHARGE
EPSILON [-1,0,0] t1 c1
LIGHT t1 t0
EPSILONCONJUGATETRANSPOSE [0,0,0] t0 c0
GAMMA_CHARGETRANSPOSE
GAMMATRANSPOSE 5
LIGHTTRANSPOSE t1 t0
ENDTRACE
BEGINFREETRACE
GAMMA_PARITY_POSITIVE
LIGHT t1 t0
GAMMA 5
MOM [1,0,0] t0
LIGHT t0 t0
GAMMA_PARITY_POSITIVE
ENDFREETRACE c1 c0
ENDTENSORTRACE c0 c1

FACTOR -0.707106781186547 0
BEGINTENSORTRACE c0 c1
BEGINTRACE
GAMMA 5
GAMMA_CHARGE
EPSILON [0,-1,0] t1 c1
LIGHT t1 t0
```

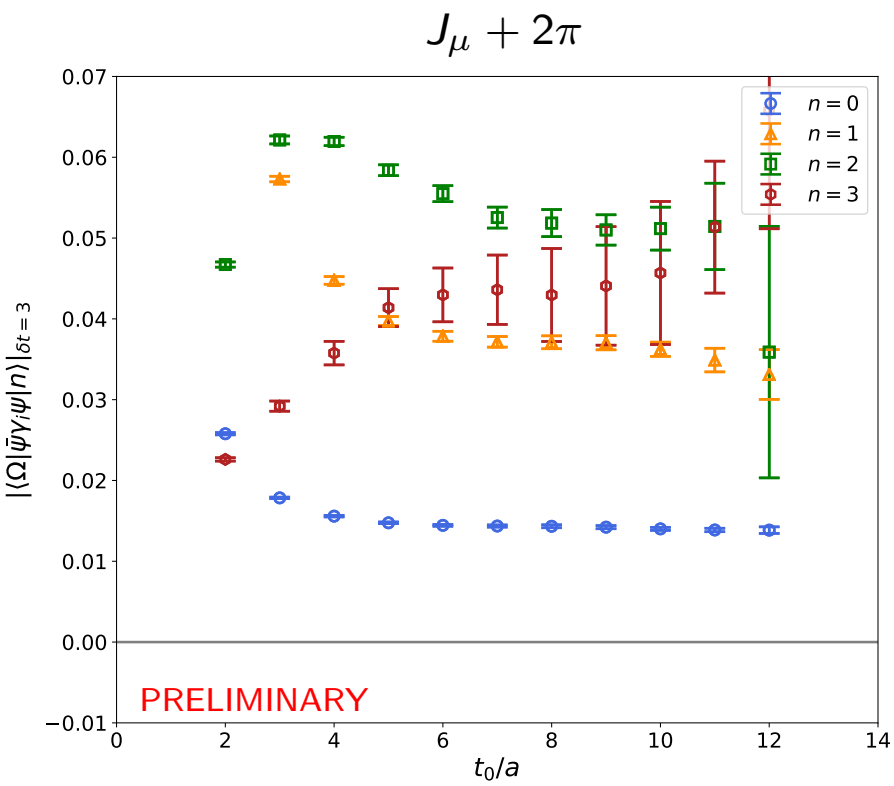
GEVP Results - 4π Operators

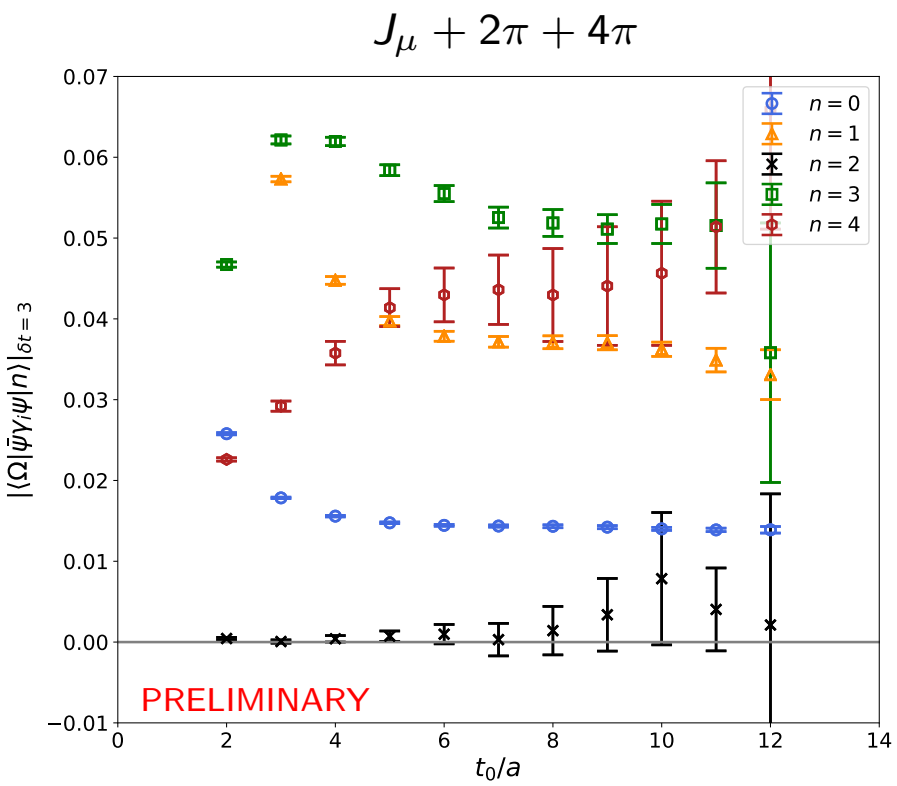




Breakdown of formalism for phase shifts +FVC could occur at 4π threshold
Compute $2\pi \to 4\pi$, $4\pi \to 4\pi$ correlation functions and check explicitly
Spectrum unaffected by inclusion of 4π operator, but state is resolvable

GEVP Results - 4π Operators





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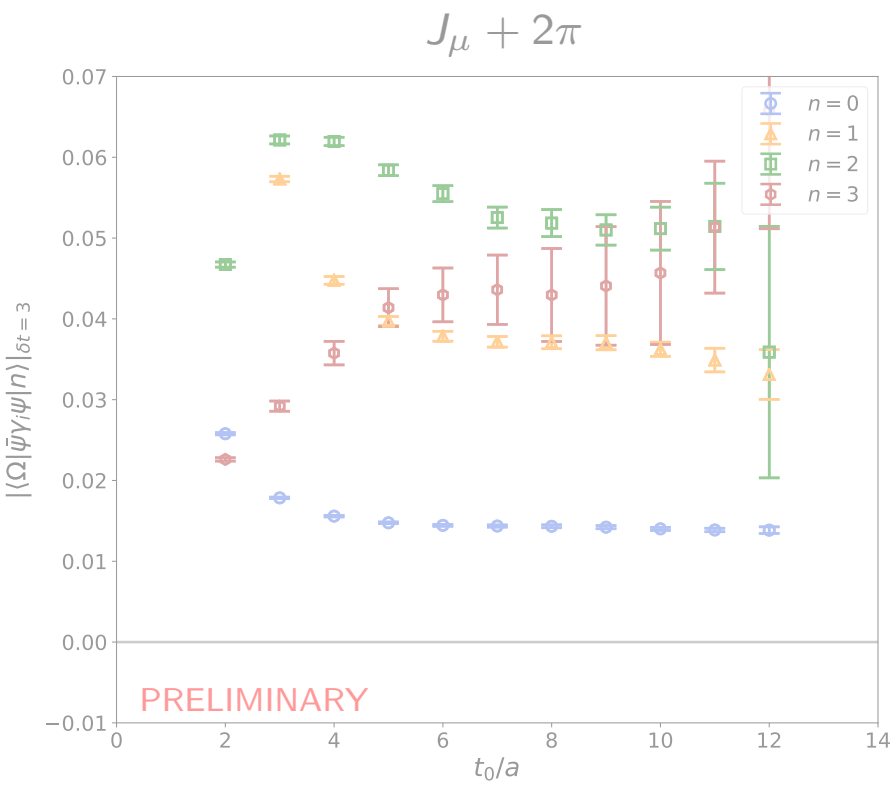
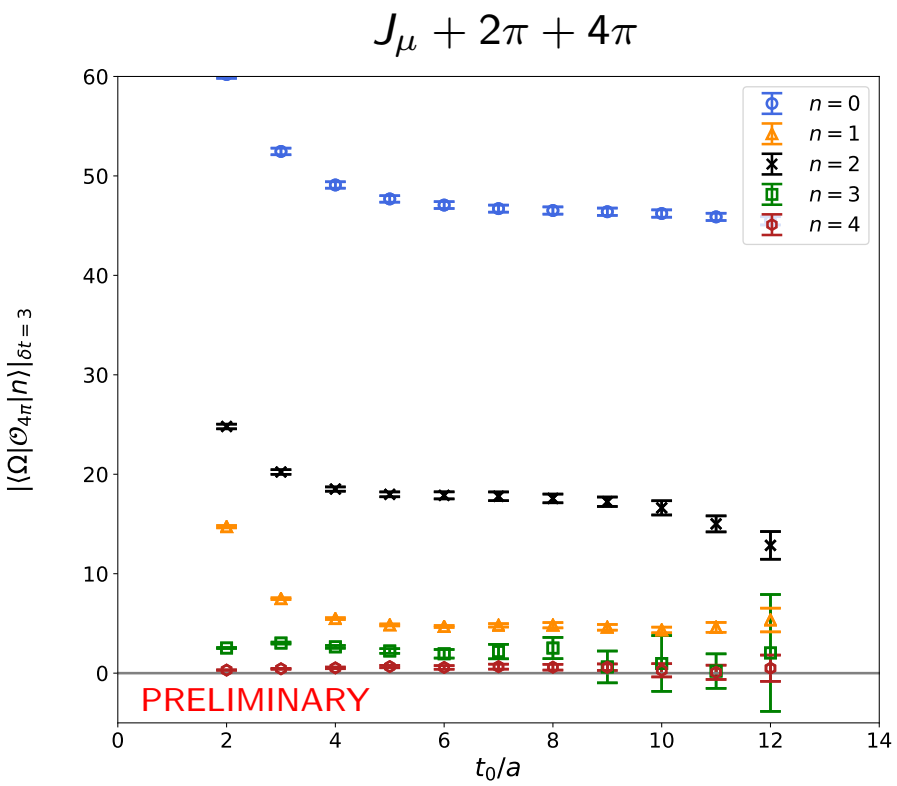
Overlap of 4π state with local vector current unresolvable

GEVP Results - 4π Operators





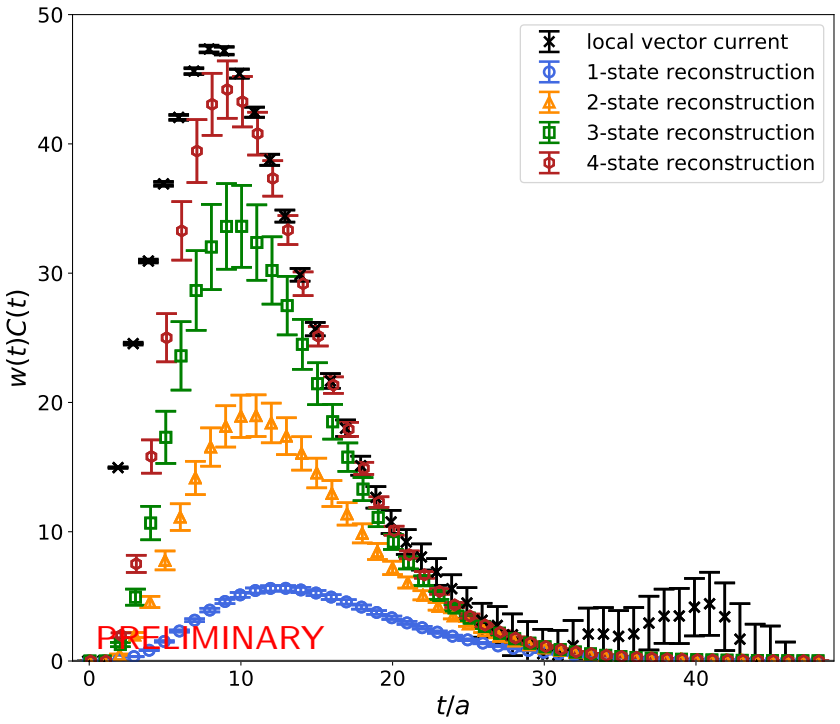
Breakdown of formalism for phase shifts +FVC could occur at 4π threshold
Compute $2\pi \to 4\pi$, $4\pi \to 4\pi$ correlation functions and check explicitly
Spectrum unaffected by inclusion of 4π operator, but state is resolvable

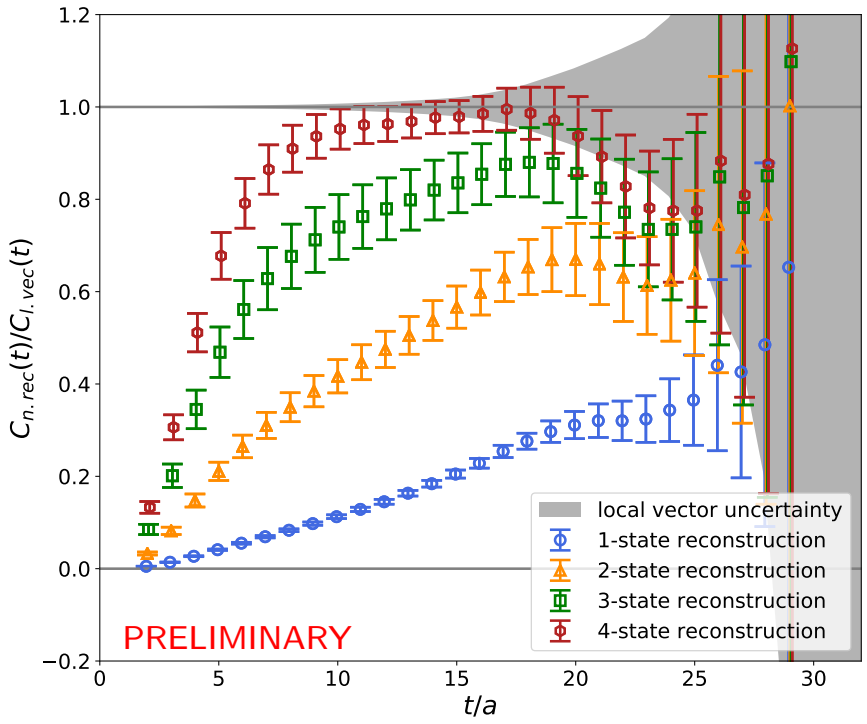
Overlap of 4π state with local vector current unresolvable

Overlap of state with 4π operator significant
$\implies$ 4π state safely negligible in local vector current

Correlator Reconstruction and Bounding

Correlation Function Reconstruction - 48I





Plotted: (weight kernel) $\times$ (correlation function); integral $\rightarrow a_\mu^{HVP}$

GEVP results to reconstruct long-distance behavior of
local vector correlation function needed to compute connected HVP

Explicit reconstruction good estimate of correlation function at long-distance,
missing excited states at short-distance

More states $\implies$ better reconstruction, can replace $C(t)$ at shorter distances

Improved Bounding Method

Use known results in spectrum to make a precise estimate of upper & lower bound on a_μ^{HVP} [RBC (2017)]

$$\widetilde{C}(t; t_{\max}, E) = \begin{cases} C(t) & t < t_{\max} \\ C(t_{\max})e^{-E(t-t_{\max})} & t \geq t_{\max} \end{cases}$$

Upper bound: $E \leq E_0$, lowest state in spectrum

Lower bound: $E \geq \log[\frac{C(t_{\max})}{C(t_{\max}+1)}]$

BMW Collaboration [K.Miura, Lattice2018] takes $E \to \infty$

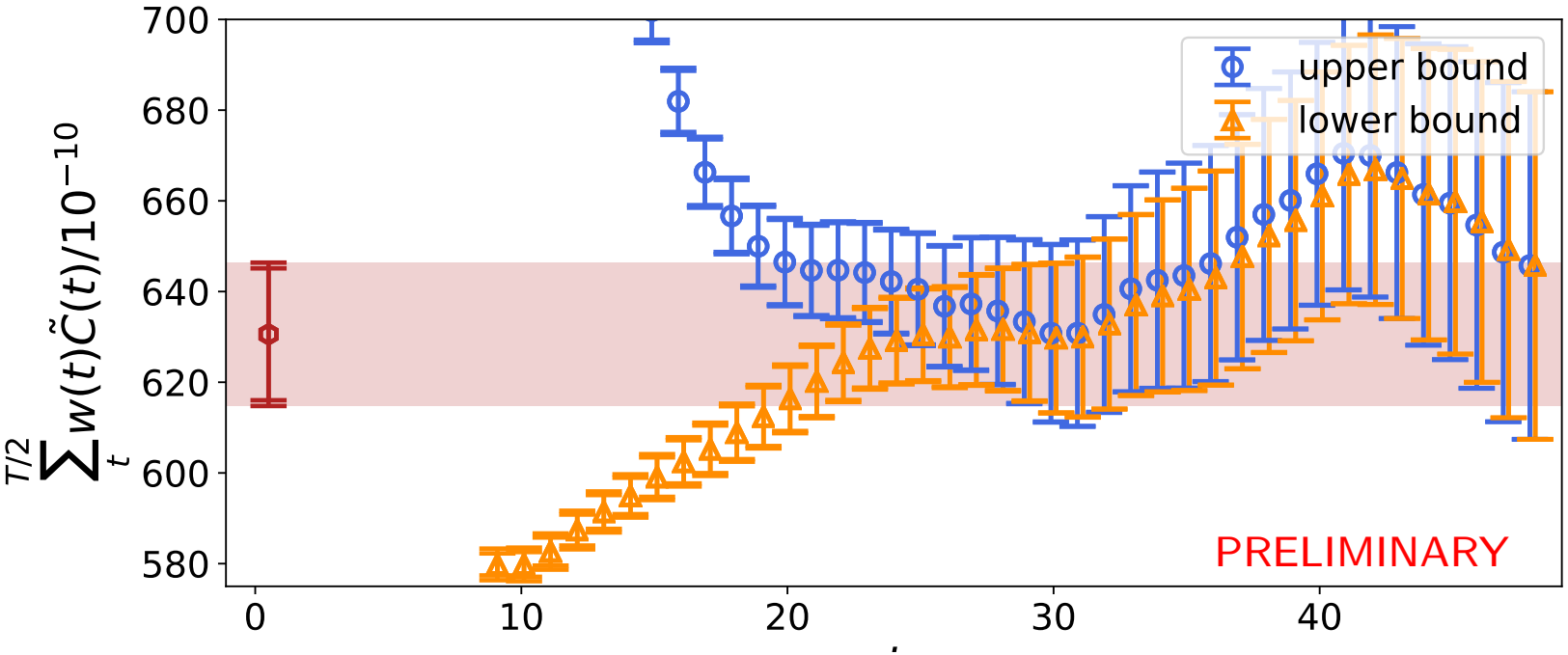
With good control over lower states in spectrum from exclusive reconstruction, improve bounding method [RBC/UKQCD 2018 (CL@KEK Feb 2018)]:

Replace $C(t) \to C(t) - \sum_n^N |c_n|^2 e^{-E_n t}$ and apply bounding procedure for $a_\mu - \delta a_\mu$

$\implies$ Long distance convergence now $\propto e^{-E_{N+1}t}$, lower bound falls faster
$\implies$ Smaller overall contribution from neglected states

After bounding, add back $\delta a_\mu = \sum_{t=t_{\max}}^{\infty} w_t \sum_n^N |c_n|^2 e^{-E_n t}$

Bounding Method Results - 48I

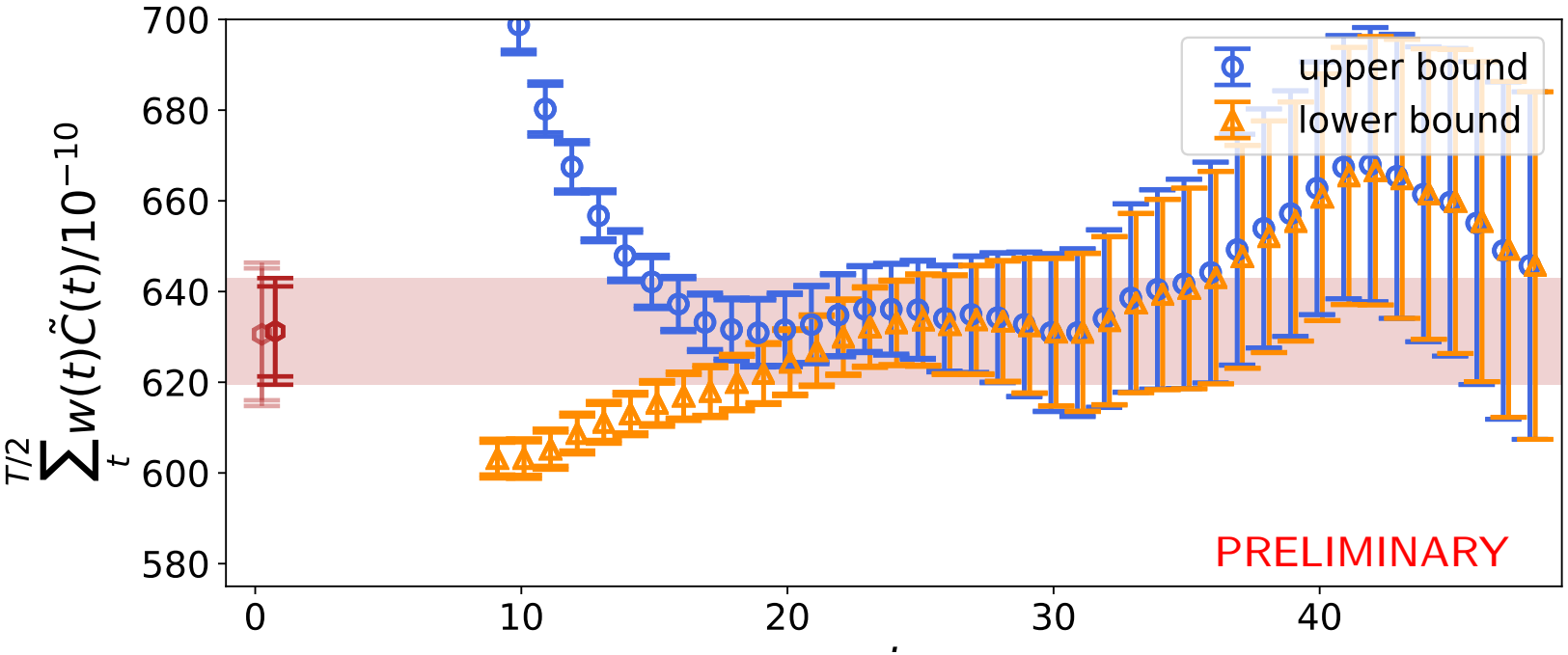


No bounding method: $a_\mu^{HVP} = 646(38)$

Bounding method $t_{\max} = 3.3$ fm, no reconstruction: $a_\mu^{HVP} = 631(16)$

Bounding method gives factor of 2 improvement over no bounding method

Bounding Method Results - 48I



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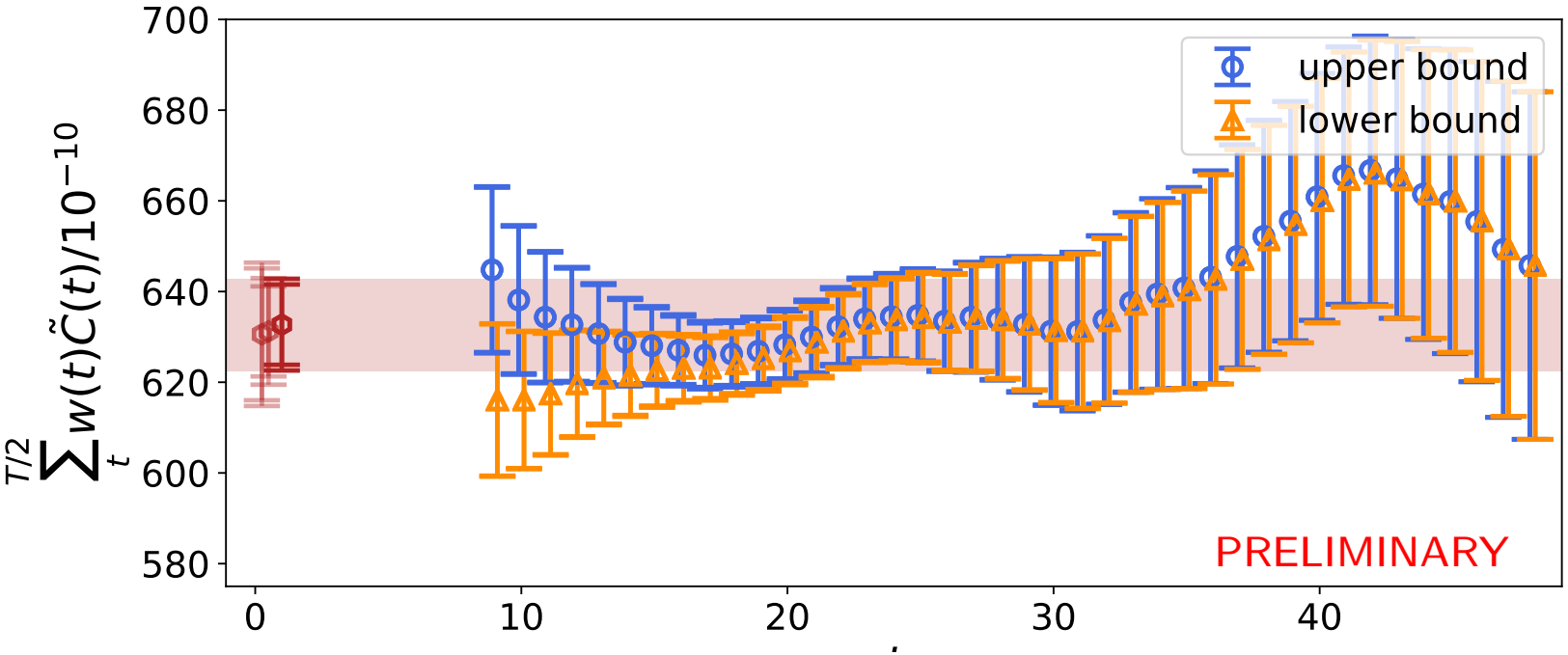
Bounding Method Results - 48I



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Bounding method $t_{\max} = 3.3$ fm, no reconstruction:	$a_\mu^{HVP} = 631(16)$
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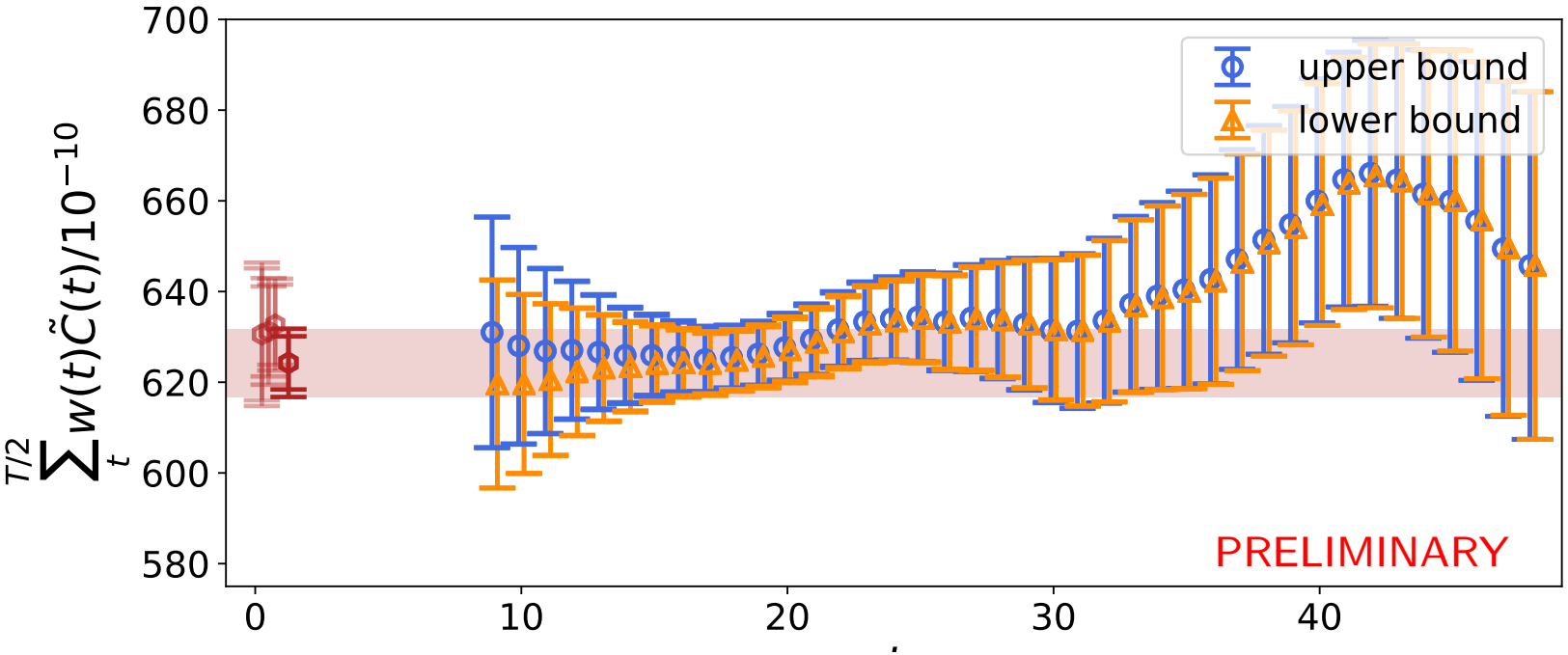
Bounding method $t_{\max} = 2.9$ fm, 2 state reconstruction:	$a_\mu^{HVP} = 633(10)$

Bounding method gives factor of 2 improvement over no bounding method

Bounding Method Results - 48I



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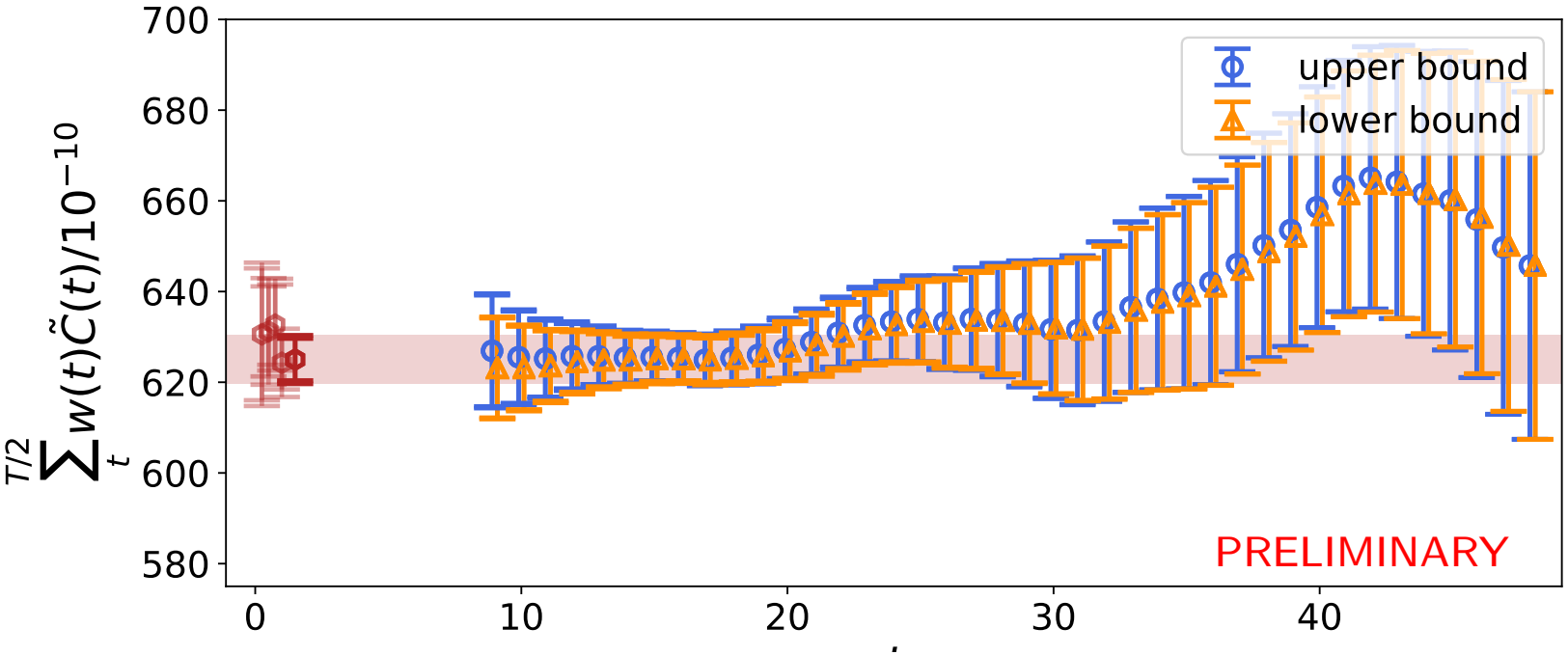
Bounding method $t_{\max} = 2.2$ fm, 3 state reconstruction:	$a_\mu^{HVP} = 624.3(7.5)$

Bounding method gives factor of 2 improvement over no bounding method

Bounding Method Results - 48I



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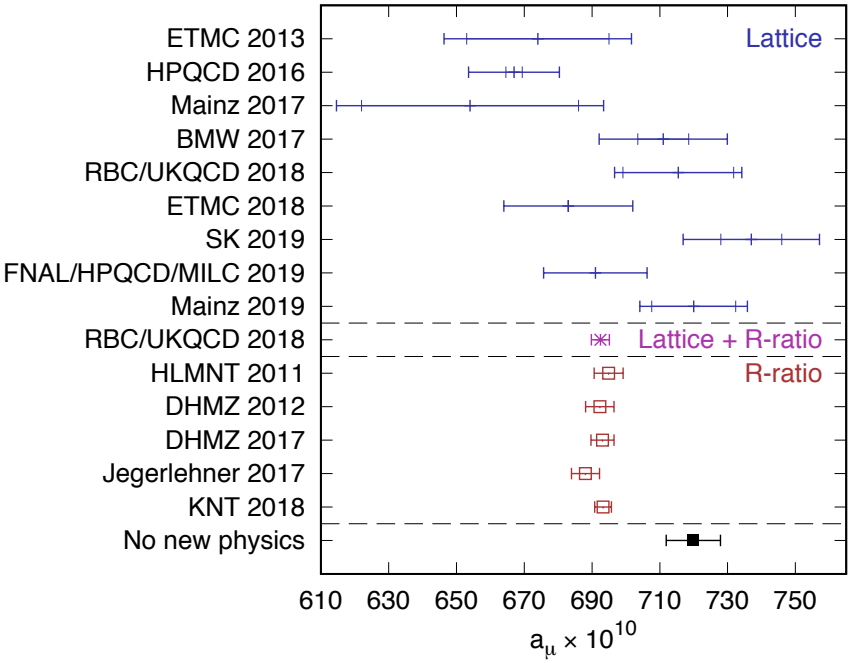
Bounding method $t_{\max} = 2.2$ fm, 3 state reconstruction:	$a_\mu^{HVP} = 624.3(7.5)$
Bounding method $t_{\max} = 1.8$ fm, 4 state reconstruction:	$a_\mu^{HVP} = 625.0(5.4)$

Bounding method gives factor of 2 improvement over no bounding method
Improving the bounding method increases gain to factor of 7, including systematics
Improvement should make all-lattice computation of a_μ^{HVP}
competitive with R-ratio by 2020

Error Budget and Timeline



Update to RBC-UKQCD calculation including exclusive study in preparation

$\implies$ could expect precision improvement $\times 3$, error on a_μ^{HVP} at 5×10^{-10}
$\implies$ to be included in $g-2$ Theory WP before release of Fermilab first results

Further reduction will require full RBC-UKQCD program of computations

Work on the exclusive channel study using bounding method has led to world-first estimation of finite volume corrections to a_μ^{HVP} at physical M_π

Complete analysis with full suite of systematic improvements ongoing
$\implies$ precision improvement $\times 10$ over original, target error on a_μ^{HVP} at 1×10^{-10}

Compare to dispersive $(3-5) \times 10^{-10}$

$\pi\pi$ Scattering Phase Shifts

$\pi\pi$ Scattering Phase Shifts in LQCD

Phase shifts in continuum, infinite volume classified by partial wave, isospin channel
Bose symmetry over $\pi\pi$ states: $(-1)^{L+I} = +1$

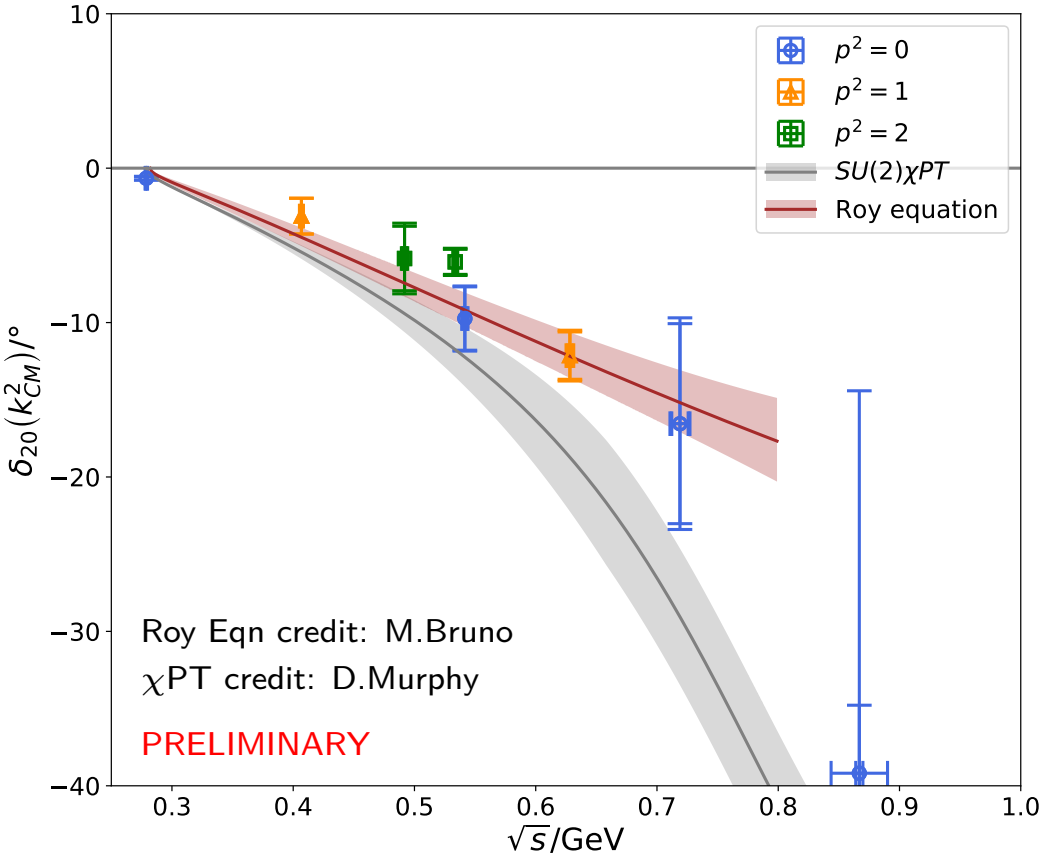
$\Longrightarrow$

Angular momentum conservation violated by lattice symmetries
Our analysis assumes all but lowest partial wave negligible

$$\det[e^{2i\delta(k)} - U(k)] = 0 \overset{\ell=0}{\Longrightarrow} \tan\delta(q) = \frac{q\pi^{3/2}}{\mathcal{Z}(1, q^2)}, \quad q = \frac{kL}{2\pi}$$

$\vec{p}\left(\frac{L}{2\pi}\right)$	Iso.	LQCD irrep	Cont. $L \leq 4$
$(0,0,0)$	2	A_1^+	$0, 4$
		E^+	$2, 4$
		T_2^+	$2, 4$
$(0,0,0)$	1	T_1^-	$1, 3$
$(1,0,0)$	2	A_1	$0, 2, 4$
		B_1	$2, 4$
		B_2	$2, 4$
		E_2	$2, 4$
$(1,1,0)$	2	A_1	$0, 2, 4$
		A_2	$2, 4$
		B_1	$2, 4$
		B_2	$2, 4$

$I = 2$ $\ell = 0$ Phase Shift (48I)



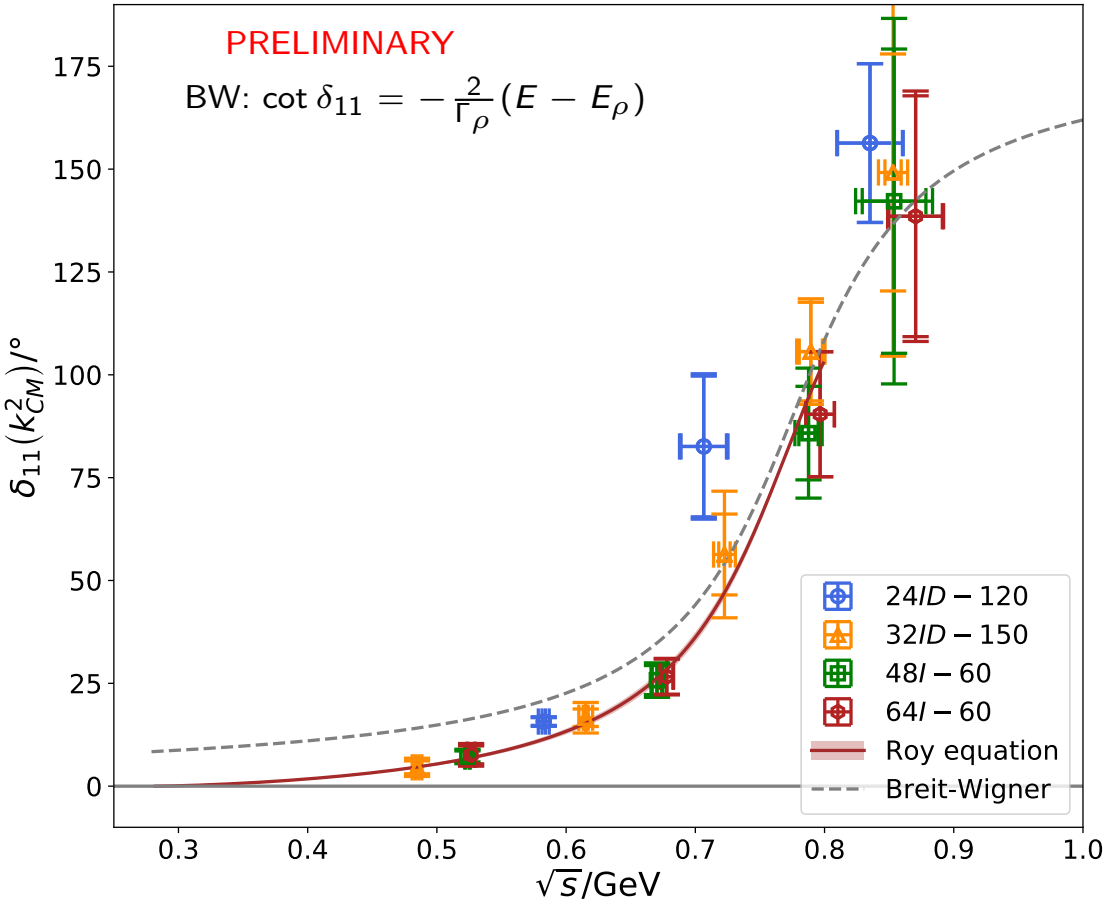
No resonances in $I = 2 \implies$ clean

Data requires subtraction periodic BC effects in time
$\implies$ handled through dedicated computation of 2π matrix elements

Breakdown of SU(2)χPT expected at around 500 MeV

Good agreement with pheno

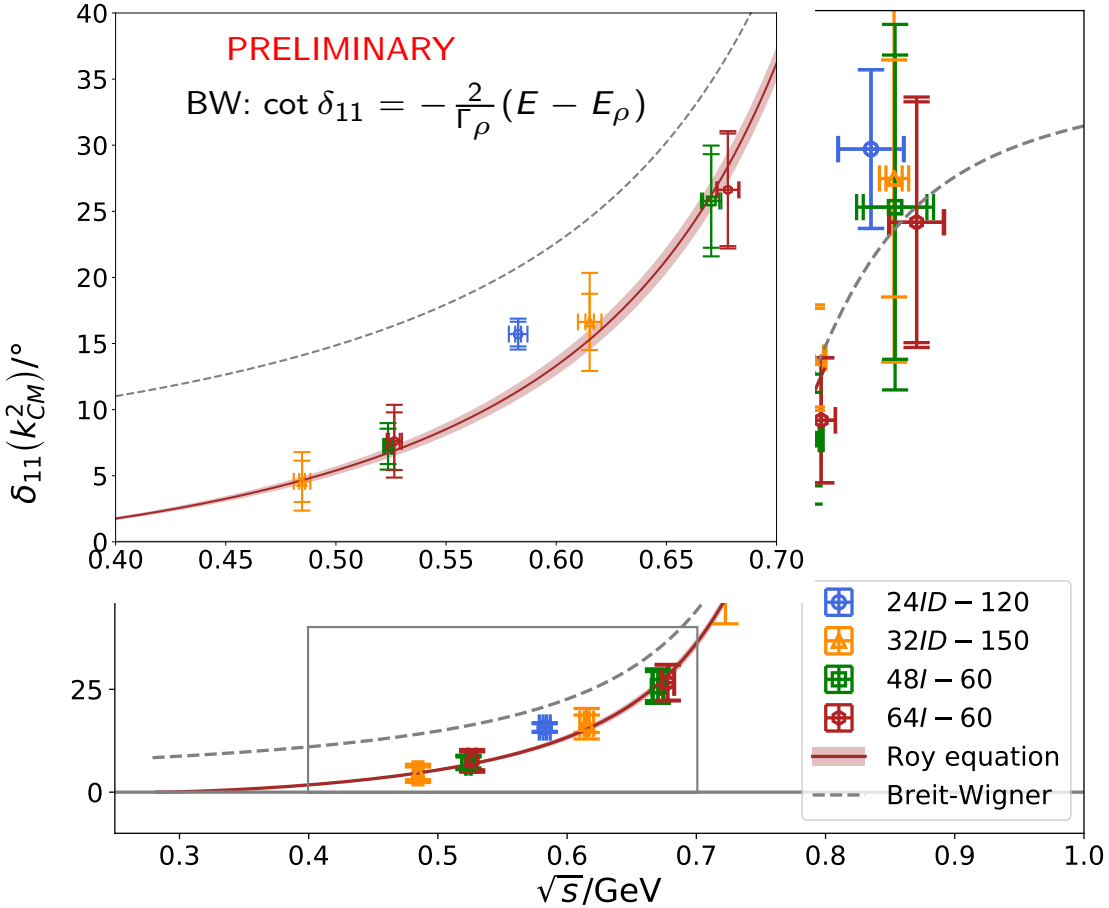
Phase Shift



Compute $\pi\pi$ scattering phase shifts in $I = 1$ channel from spectrum
Statistics + systematics included

Compare to simple Breit-Wigner parametrization and pheno (courtesy of M.Bruno)
Good agreement with pheno for 32ID, 48I, 64I
24ID: remnant excited state contaminations, still to be removed

Phase Shift



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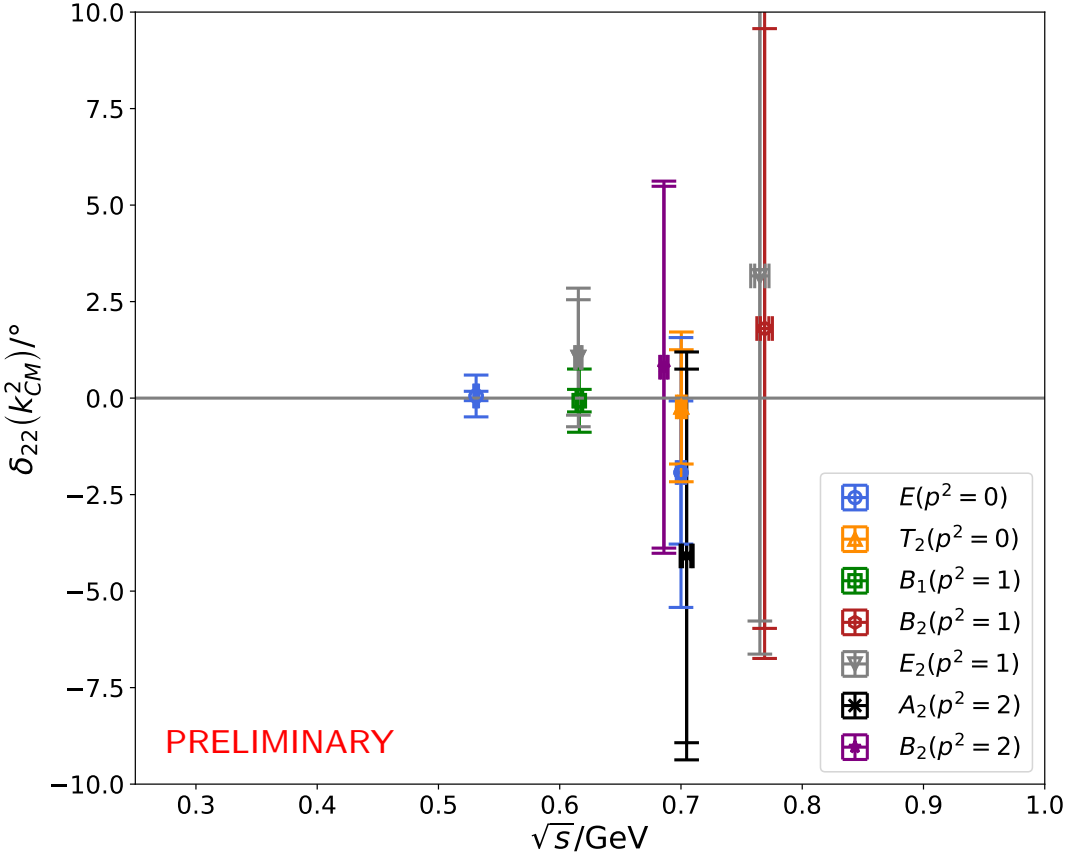
$I = 2$ $\ell = 2$ Phase Shift (48I)



Other lattice irreps probe higher partial waves

Phase shift small, unresolved $\implies$ limit only

Conclusions

Conclusions

Pion scattering is a simple playground to learn about multiparticle scattering in LQCD while addressing a_μ^{HVP}:

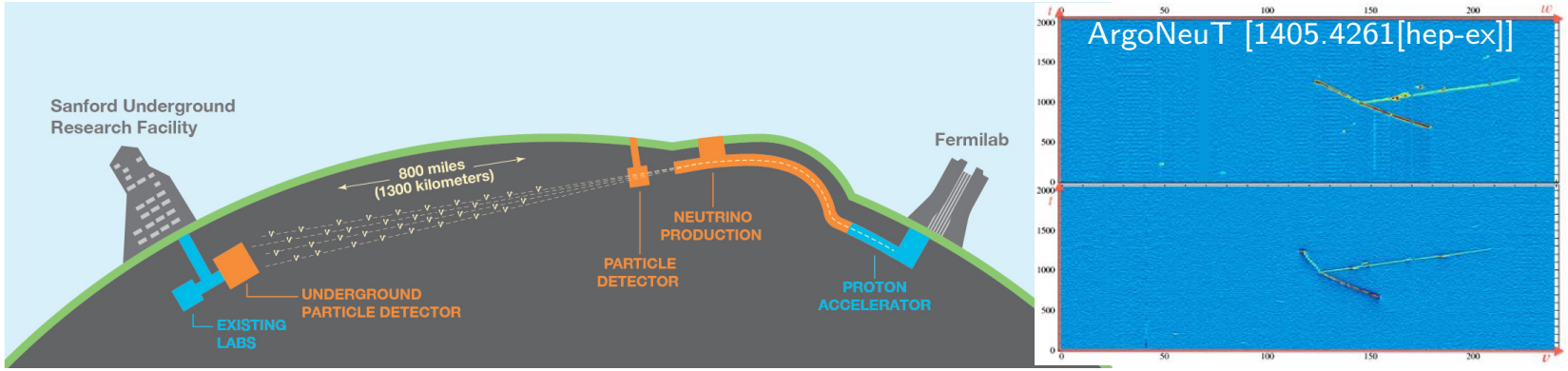
- Computed $2\pi \to 4\pi$, $4\pi \to 4\pi$ correlation functions to show explicitly that 4π state has negligible effect on HVP
- Study of exclusive channels able to significantly reduce statistical uncertainty on an all-lattice computation of muon HVP
  $\implies$ factor of 4 more statistics on 48I available now
  $\implies$ expect to reach precision of $O(5 \times 10^{-10})$ by the end of year
  $\implies$ can target $O(1 \times 10^{-10})$ for all-lattice calculation
- Part of ongoing lattice study to address all lattice systematics in RBC+UKQCD HVP computation (see talk by C.Lehner)
- Computed scattering phase shifts in $I = 1, 2$
  $\implies$ first calculation of ρ resonance at physical M_π!
- Scattering phase shifts to appear as part of series of papers by RBC+UKQCD

With experience from $\pi\pi$ scattering, will move on to tackle more challenging problems with $N\pi$ states and beyond

Thank you!

BACKUP

Neutrino Oscillation Experiments



Neutrino oscillation experiments are of great interest to the physics community, seeking to do a high-precision measurement of neutrino oscillation parameters

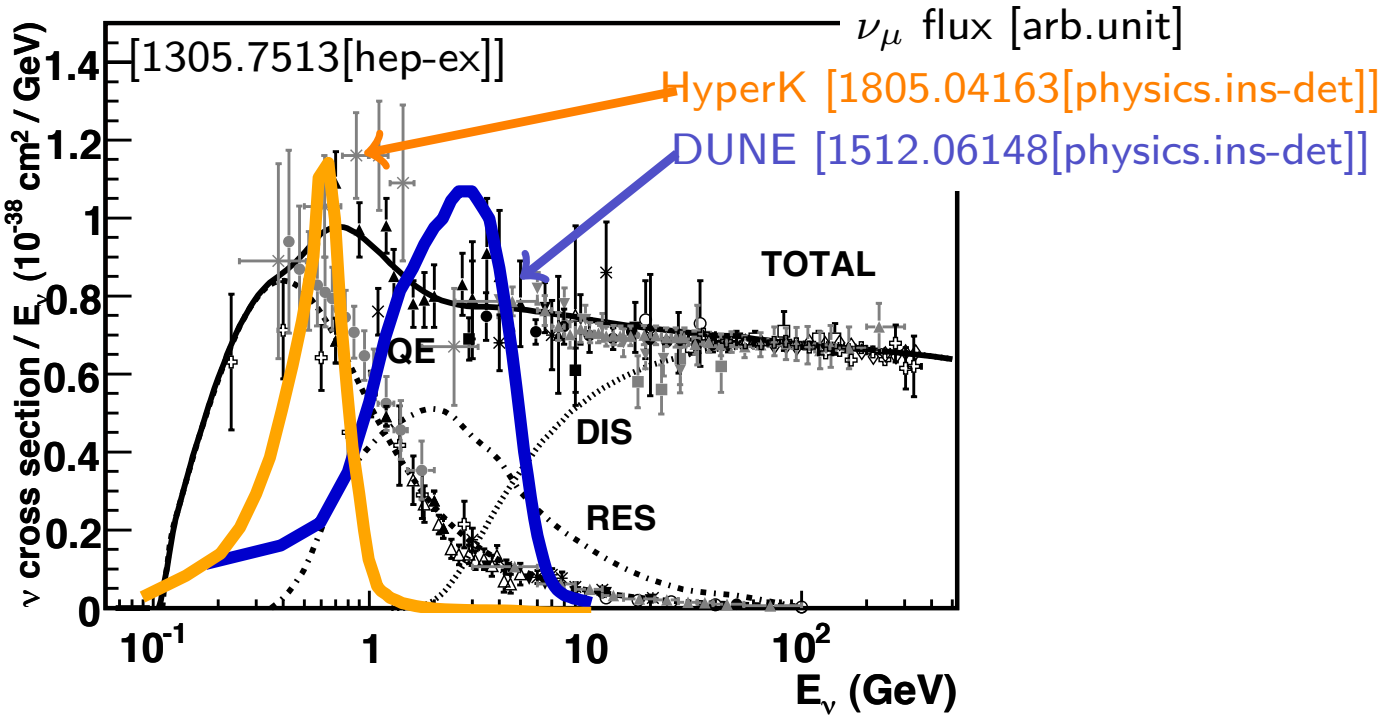
Large program of research goals addressing fundamental physics questions:

- Precision measurements of oscillation parameters, including δ_{CP} & sign$[\Delta m^2_{13}]$
- Measure supernova neutrinos
- Search for proton decay

DUNE due to start installation of first detector in 2022, data collection in 2026

This experiment sets a timescale for making theory contributions!

Flux ⊗ Cross Section

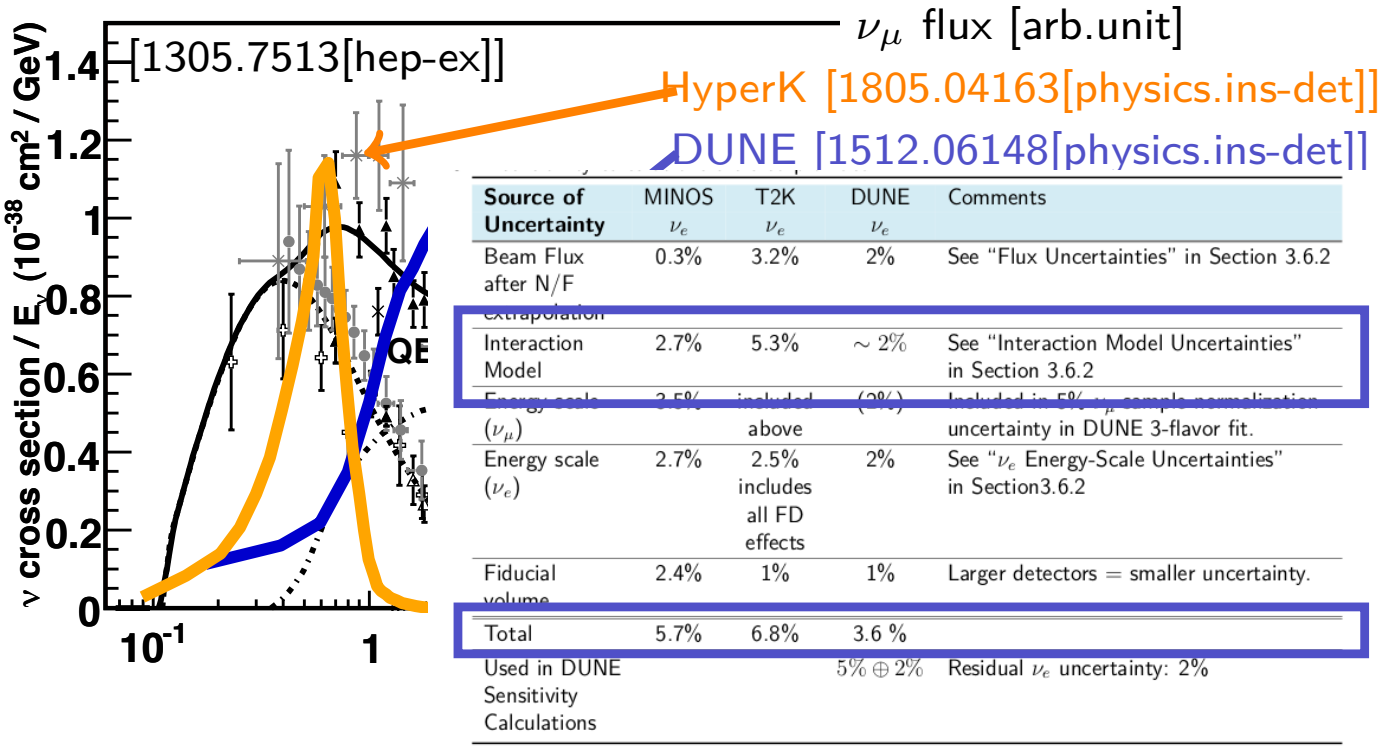


Neutrino interactions classified by their interaction products

3 general classes of interaction types:
Quasielastic (QE), Resonance (Res), Deep Inelastic Scattering (DIS)

DUNE events will be a mixture of all three; $\sim \frac{1}{3}$ events will be resonant

Flux ⊗ Cross Section



Source of Uncertainty	MINOS ν_e	T2K ν_e	DUNE ν_e	Comments
Beam Flux after N/F extrapolation	0.3%	3.2%	2%	See "Flux Uncertainties" in Section 3.6.2
Interaction Model	2.7%	5.3%	$\sim 2\%$	See "Interaction Model Uncertainties" in Section 3.6.2
Energy scale (ν_μ)	2.5%	included above	(2%)	Included in 5% ν_μ sample normalization uncertainty in DUNE 3-flavor fit.
Energy scale (ν_e)	2.7%	2.5% includes all FD effects	2%	See "ν_e Energy-Scale Uncertainties" in Section3.6.2
Fiducial volume	2.4%	1%	1%	Larger detectors = smaller uncertainty.
Total	5.7%	6.8%	3.6 %	
Used in DUNE Sensitivity Calculations			$5\% \oplus 2\%$	Residual ν_e uncertainty: 2%

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3 general classes of interaction types:
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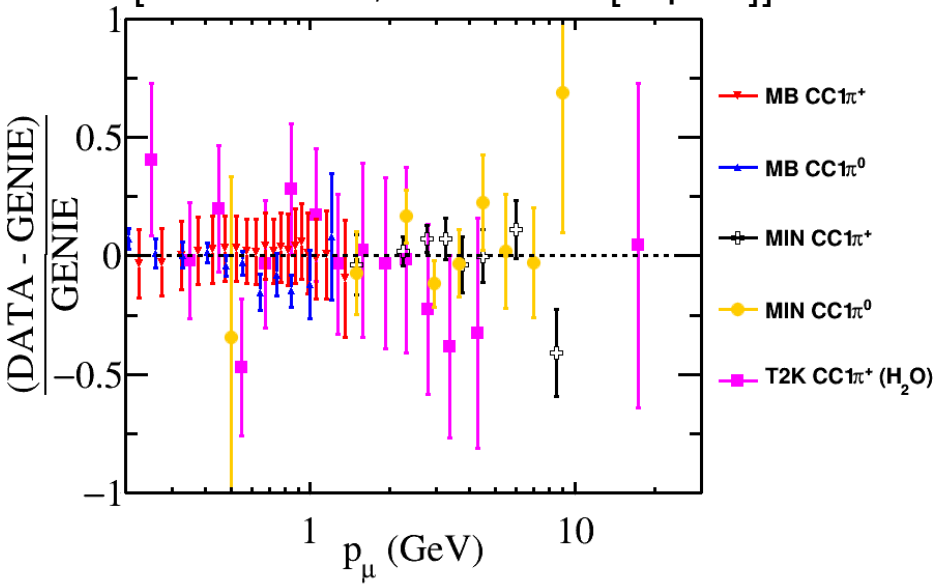
DUNE events will be a mixture of all three; $\sim \frac{1}{3}$ events will be resonant

A dominant contribution to systematics in DUNE will be cross section uncertainties

$\Longrightarrow$ Stringent requirements on cross section uncertainties, lots at stake
$\Longrightarrow$ QE was dominant before; coming under control now $\Longrightarrow$ next step is resonant

Discrepancies with Monte Carlo

[Mahn *et al.*,1803.08848[hep-ex]]



Resonant interactions typically result in CC1π (charge current with 1 pion) final states

Current state of affairs for CC1π interactions is confusing
Lepton kinematics under control, consistent and agree well with Monte Carlo (GENIE)

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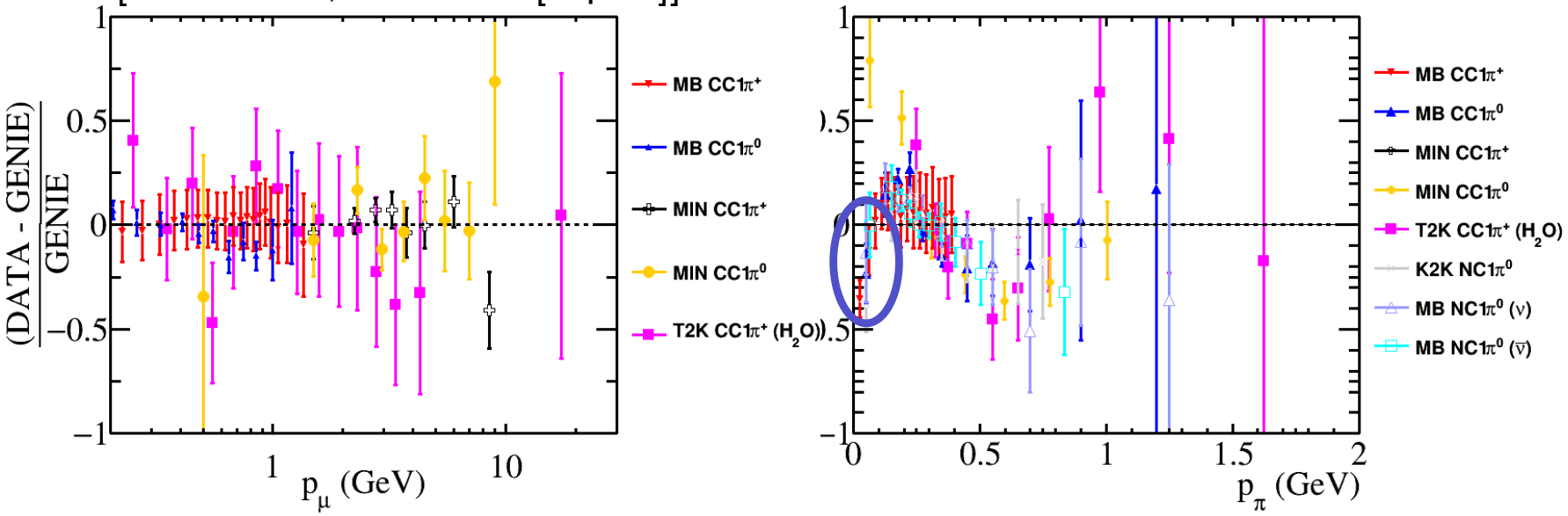

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Current state of affairs for CC1π interactions is confusing
Lepton kinematics under control, consistent and agree well with Monte Carlo (GENIE)

Pion kinematics systematically disagree with shape
Difficult to change pion kinematics without breaking other data

$\Longrightarrow$ Need another handle on pion kinematics!

Ideally a high-statistics *H* or *D* bubble chamber experiment; not likely to happen. . .

Nucleon to Delta Transitions

Knowledge of the $N \to \Delta$ transition is vital for neutrino oscillation experiments:

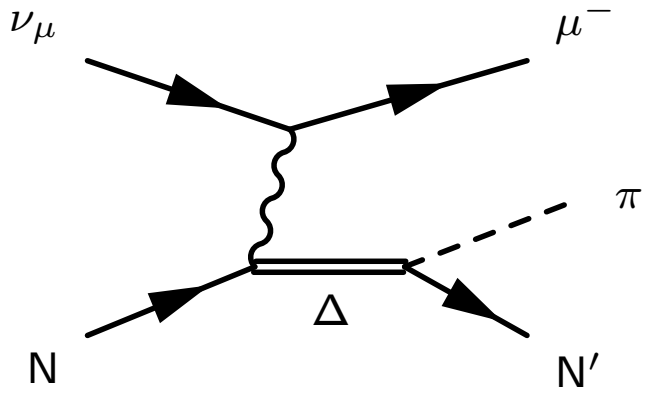
- Resonance region for neutrino scattering dominated by transitions to Δ baryon
- Four $N \to \Delta$ transition form factors, only C_{5A} is relevant
- Assuming a 10% uncertainty on normalization of C_{5A}:

  $\times 1/2$ for near/far detector ratio
  $\times 1/3$ for resonant/total interactions

  $\implies$ $1 - 2\%$ total theory uncertainty on oscillation observables

Compared to DUNE total error budget of $1 - 3\%$
$\implies$ 10% knowledge of C_{5A} norm would saturate the entire error budget of DUNE

Added difficulty: transition amplitudes needed to analyze same data that is used to constrain transition amplitudes $\implies$ likely underestimated uncertainties



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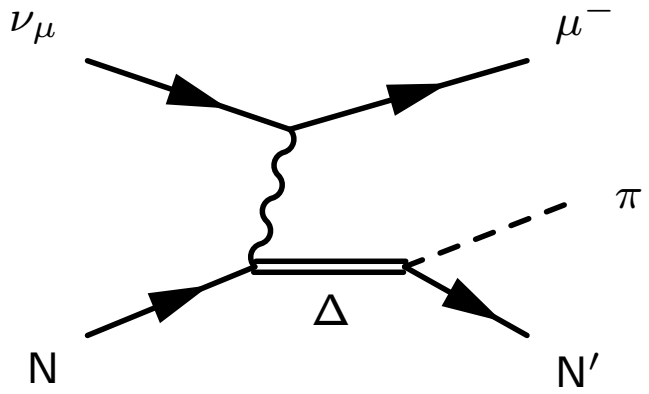
Room for improvement with Lattice QCD:

- Independent measurement to break theory/experiment degeneracy
- Providing realistic uncertainty estimates
- Improving constraints on form factor & amplitude uncertainty

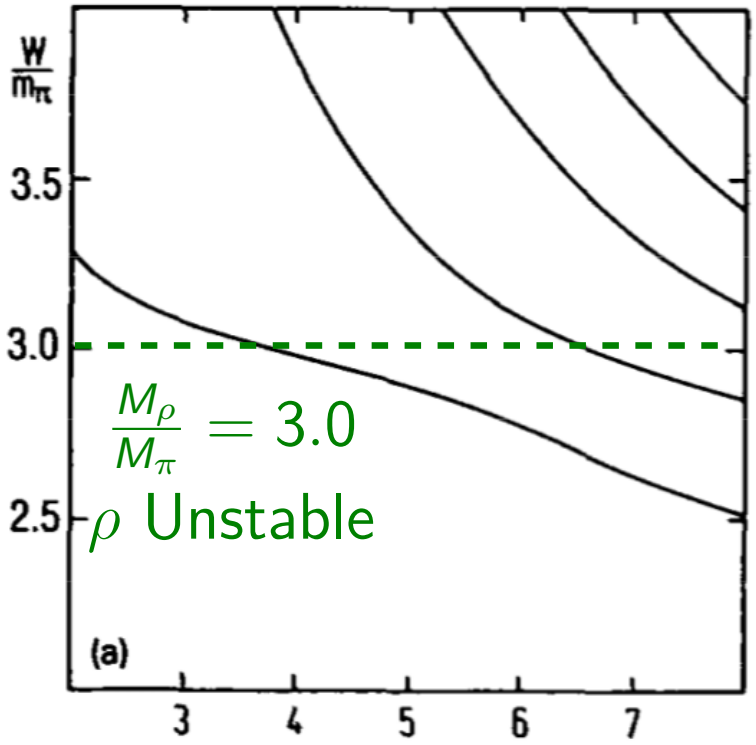


Multiparticle States in Lattice QCD

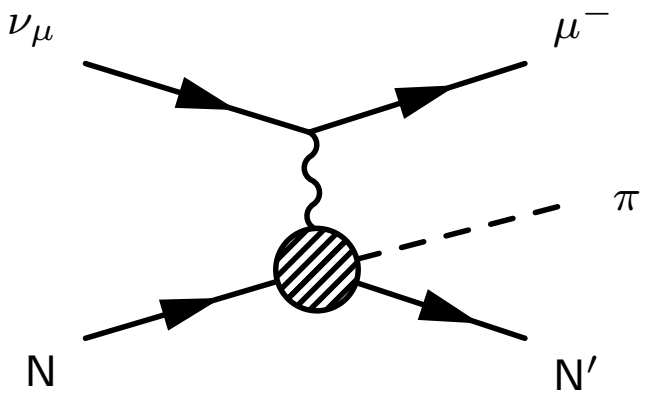
Extracting properties of resonances in LQCD is complicated by finite volume:

- Resonance properties must be extracted from decay product states
- Particles in multiparticle states are not asymptotic states
- Need to enforce energy & momentum conservation with discretized momenta
- Energy eigenstates are superpositions of states with definite particle content
  $\Longrightarrow$ avoided level crossings & complicated spectrum

Argument can be turned on its head:
Finite volume as a probe of multiparticle scattering states



[Nuc.Phys.B 364, p237.]



Neutrino Physics to $g-2$

Studying $N \to \Delta$ resonances requires study of $N\pi$ final states

Technically challenging:

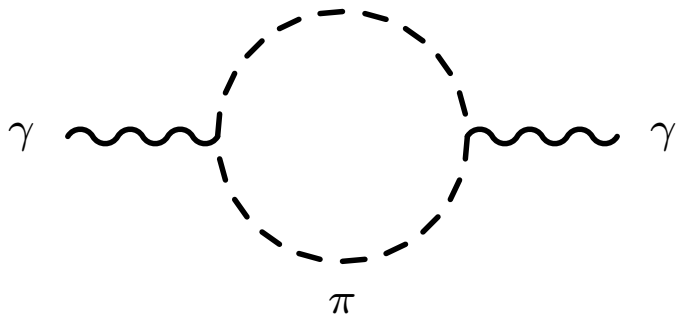
- Fermionic spin states, unequal masses
- Many Wick contractions $\implies$ large computational cost
- Signal to noise degrades rapidly
- More than two particle scattering states, e.g. $N\pi\pi$, open up quickly

This is a long-term goal that we are developing the methodology to address

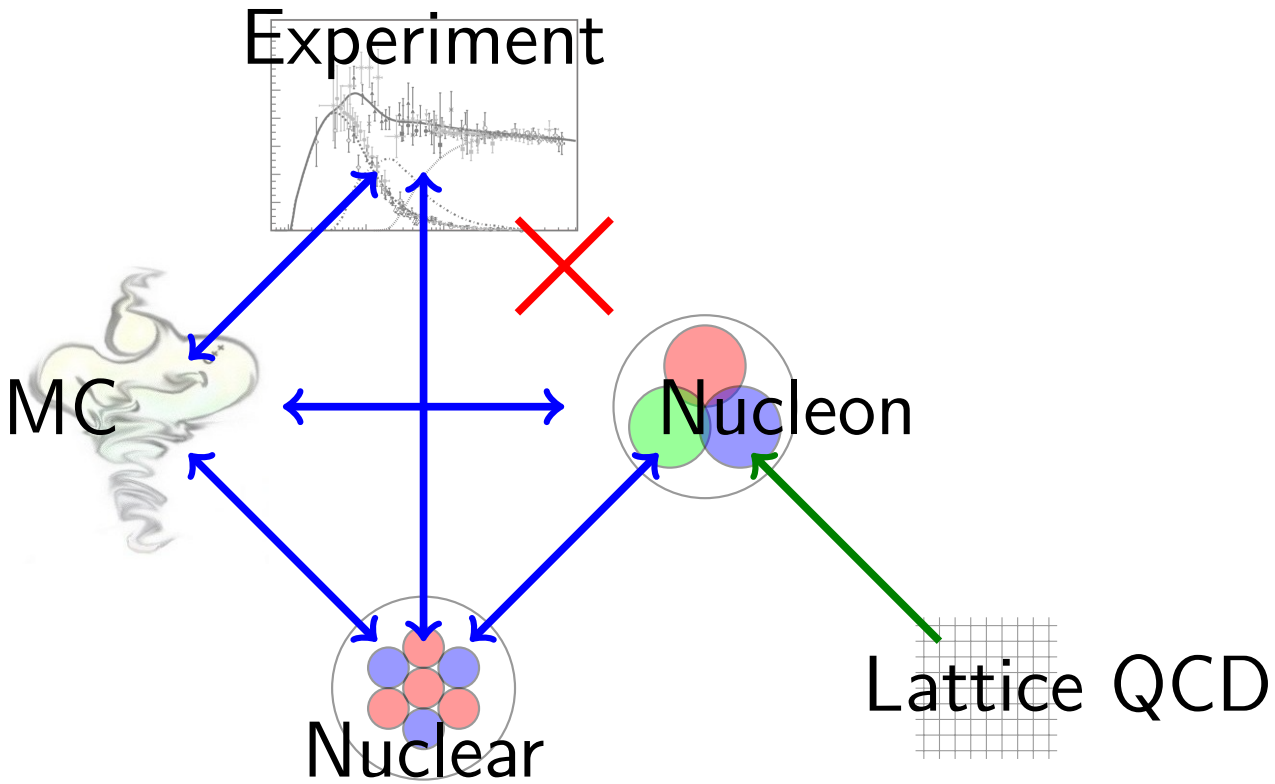
In the short term, use this formalism to address a simpler but timely physics question:
$\implies$ $\pi\pi$ scattering in the isospin-1 channel $\implies$ $(g-2)_\mu$ HVP contribution

This is an ideal starting point:

- Simplest channel with resonance
- Above issues significantly mitigated/eliminated
- Consistency checks against other LQCD calculations & experiment available



Lattice QCD Checklist



Lattice QCD is ideal tool for filling in missing pieces

To have the greatest impact, must satisfy the checklist:

- Process is important for meeting experimental goals ✓
- Current precision not sufficient ✓
- Difficult/impractical to measure experimentally ✓
- Accessible to Lattice QCD ✓