Computing isospin breaking corrections in massive QED on the lattice

Michele Della Morte

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UNIVERSITY OF SOUTHERN DENMARK

CP3

Collaborators: A. Bussone, T. Janowski, A. Walker Loud

Plan of the talk

Isospin symmetry

The formal N_f flavor QCD Lagrangian

$$L_{QCD}^{N_f} = \sum_{i=1}^{N_f} \overline{\psi}_i(\mathrm{i}(\gamma_\mu D^\mu) - m)\psi_i - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

in the case of degenerate up and down quarks, is invariant under SU(2) rotations in the (u-d) flavor space.

Isospin breaking (IB) has two sources

$m_u \neq m_d$ (strong IB)
$Q_u \neq Q_d$ (EM IB)

The separation makes sense classically. Renormalization effects induce a mass gap, even with bare degenerate masses ($\rightarrow$ scheme dependence).

IB is responsible for the neutron-proton mass splitting, whose value played an important role in nucleosynthesis and the evolution of stars [BMW, Science 347 (2015)].

More motivations

The 2016 FLAG review [Eur.Phys.J. C77 (2017) no.2, 112] (similar for 2019) gives

$$f_\pi = 130.2(8)\ \mathrm{MeV}\,, \qquad f_K = 155.7(7)\ \mathrm{MeV} \quad [N_f = 2+1]$$
$$f_D = 212(1)\ \mathrm{MeV}\,, \qquad f_{D_s} = 249(1)\ \mathrm{MeV} \quad [N_f = 2+1+1]$$

obtained in the isospin limit. EM corrections can be included following [Phys.Rev. D91 (2015) no.7, 074506 (Rome-Soton)]

These hadronic parameters are relevant for the extraction of CKM elements from purely leptonic decays. In that game the error is dominated by experiments, as opposed to the semileptonic case. [arXiv:1811.06364 (Rome-Soton)]

Well known 3σ tension in $(g-2)_\mu$

Future experiments will shrink the error! (Fermilab and J-PARC)

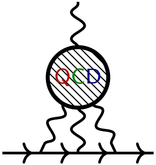
$\sigma\left(e^+ e^- \to \text{Had}\right)$-method still the most accurate
(includes all SM contributions)

Exp. data with space-like kin. allow for direct comparison with Lattice
[Carloni Calame et al. Phys. Lett. B746:325–329, 2015]

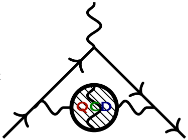
$3\sigma \simeq 4\%$ on a_μ^{HLO}

QED corrections $\approx 1\%$

$a_\mu^{\mathrm{HLbL}} =$



$= \quad O\left(e^7\right) \quad =$



$= a_\mu^{\mathrm{HLO}}(\alpha)$

Periodic boundary conditions (PBC)

$$\psi(x + L_\mu \hat{\mu}) = \psi(x)\ , \quad A_\mu(x + L_\nu \hat{\nu}) = A_\mu(x)$$

The Lagrangian with one fermion of charge 1 (and $e = 1$) invariant for

$$\begin{aligned} A_\mu(x) &\rightarrow A_\mu(x) + \partial_\mu \Lambda(x) \\ \psi(x) &\rightarrow e^{i\Lambda(x)}\psi(x) \\ \overline{\psi}(x) &\rightarrow \overline{\psi}(x)e^{-i\Lambda(x)} \end{aligned}$$

$\Lambda(x)$ does not need to be periodic

$$\Lambda(x + L_\mu \hat{\mu}) = \Lambda(x) + 2\pi r_\mu$$

The quantization in r_μ follows from the periodicity of the fermions. In general

$$\Lambda(x) = \Lambda^0(x) + 2\pi \left(\frac{r}{L}\right)_\mu x_\mu$$

with $\Lambda^0(x)$ periodic.

Let us consider the "large gauge transformations" defined by $\Lambda^0 = 0$

$$A_\mu(x) \rightarrow A_\mu(x) + 2\pi \frac{r_\mu}{L_\mu}\ , \quad \psi(x) \rightarrow \psi(x) e^{i2\pi \left(\frac{r}{L}\right)_\mu x_\mu}$$

they act as a finite volume shift symmetry on the gauge fields.

Considering now the correlator $\langle \psi(T/4, \underline{0}) \overline{\psi}(0, \underline{0}) \rangle$, it is clear that it vanishes as a consequence of invariance under large gauge transformations (choose $r_0$mod(4)=2).

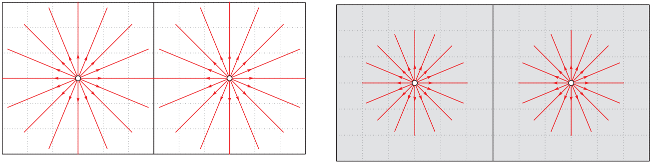
OK, let's gauge away the shift symmetry and require the 0-mode of A_μ to vanish

$$\int d^4x A_\mu(x) = 0$$

that is a non-local constraint, which cannot be imposed through a local gauge-fixing ! Not a derivative one at least .... We like those because gauge-independence of physical quantities is manifest.

Another way to look at the problem

Electric field of a point charge cannot be made periodic and continuous



$$Q = \int d^3x \rho(x) = \int d^3x \partial_i E_i(x) = 0$$

Introduce uniform, time-independent background current c_μ then

$$\int d^3x \rho(x) + \int d^3x c_0 = 0 \ ,$$

which allows to have a net charge.

Promoting c_μ to a field, the Lagrangian density is modified by a term

$$A_\mu(x)\int d^4(y)c_\mu(y)$$

whose EoM is $\int d^4xA_\mu(x)=0$. When enforcing this on each conf (not just on average) one obtains the QED_{TL} prescription used first in [Duncan et al.,Phys.Rev.Lett. 76 (1996)]. It is

- non-local
- without a Transfer matrix

An Hamiltonian formulation can be recovered adopting the QED_L prescription [Hayakawa and Uno, Prog.Theor.Phys. 120 (2008)], requiring

$$\int d^3xA_\mu(t,\underline{x})=0$$

(Imagine coupling a uniform but time-dependent current, as for charged particles propagators).

Both prescriptions

- Introduce some degree of non-locality (issues with renormalization ? O(a) improvement ? Mixing of IR and UV ?)
- Remove modes, which in the electroquenched approximation, would be un-constrained and cause algorithmic problems (wild fluctuations)

QED_L is to be preferred as it has a Transfer matrix. The 'quenched' modes should not play a role in the infinite-vol dynamics (fields vanish at infinity), so it is a matter of finite volume effects (see for example [Davoudi et al., arXiv:1810.05923] for studies in PT and numerically for scalar-QED).

Another natural approach:

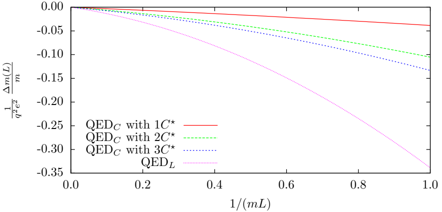
the quantization of the shift symmetry was due to BC for fermions. How about changing it to: [Lucini et al., JHEP 1602 (2016) 076] (C^* BC)

$$
\begin{aligned}
A_\mu(x + L_\nu \hat{\nu}) &= -A_\mu(x) = A^C_\mu(x) \\
\psi(x + L_\nu \hat{\nu}) &= \psi^C(x) = C^\dagger \overline{\psi}^T(x) \\
\overline{\psi}(x + L_\nu \hat{\nu}) &= -\psi(x)^T C \quad \text{with} \quad C^\dagger \gamma_\mu C = -\gamma_\mu^T
\end{aligned}
$$

Completely local, no zero-modes allowed, however at the price of violations of flavor and charge conservation (by boundary effects).

Also, SU(3) dynamical configurations need to be generated again.

It is useful to look at finite volume corrections, e.g. to point-like particles at $O(\alpha)$ ($1/L$ and $1/L^2$ universal) [Lucini et al., JHEP 1602 (2016) 076]



A PT-inspired approach [RM123, JHEP 1204 (2012) 124, Phys.Rev. D87 (2013) no.11, 114505]

Simpler in the case of strong IB:

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_{kin} + \mathcal{L}_m \\
&= \mathcal{L}_{kin} + \frac{m_u + m_d}{2}(\bar{u}u + \bar{d}d) - \frac{m_d - m_u}{2}(\bar{u}u - \bar{d}d) \\
&= \mathcal{L}_{kin} + m_{ud}\,\bar{q}q - \Delta m_{ud}\,\bar{q}\tau^3 q \\
&= \mathcal{L}_0 - \Delta m_{ud}\,\hat{\mathcal{L}}\,,
\end{aligned}$$

$$\begin{aligned}
\langle \mathcal{O} \rangle &\simeq \frac{\int D\phi\ \mathcal{O}\,(1 + \Delta m_{ud}\,\hat{S})\,e^{-S_0}}{\int D\phi\ \ (1 + \Delta m_{ud}\,\hat{S})\,e^{-S_0}} = \frac{\langle \mathcal{O} \rangle_0 + \Delta m_{ud}\,\langle \mathcal{O}\hat{S} \rangle_0}{1 + \Delta m_{ud}\,\langle \hat{S} \rangle_0} \\
&= \langle \mathcal{O} \rangle_0 + \Delta m_{ud}\,\langle \mathcal{O}\hat{S} \rangle_0\,,
\end{aligned}$$

Similarly, for QED corrections, one inserts $J_\mu(x)$ (and lattice tadpole) over 4dim vol in correlators evaluated in isospin-symm QCD.

+ One does not compute something tiny rather, derivatives wrt α and Δm_{ud}, which may be O(1)
+ Only renormalization in QCD needs to be discussed
= Still a zero-mode prescription for the explicit photon propagator is needed. Anyhow, much better control as the computation is fixed order in α.
– The expansion produces quark-disconnected diagrams ($\simeq$ those neglected in electroquenched).

$$L_{QED_m} = \frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}m_\gamma^2 A_\mu^2 + L_f = L_{Proca} + L_f$$

+ is renormalizable by power counting once the Feynman gauge is imposed through the Stückelberg mechanism **[see book by Zinn-Justin]**
+ it is local, softly breaks gauge symmetry and has a smooth $m_\gamma \to 0$ limit.
+ Clearly the shift-transformation is not a symmetry anymore. The mass term acts as an extra non-derivative gauge-fixing.

= It introduces a new IR scale on top of L. First one should take $L \to \infty$ and then $m_\gamma \to 0$.

+ Finite volume corrections are exponentially small, as long as $m_\gamma L \geq 4$ and $m_\gamma << m_\pi$.

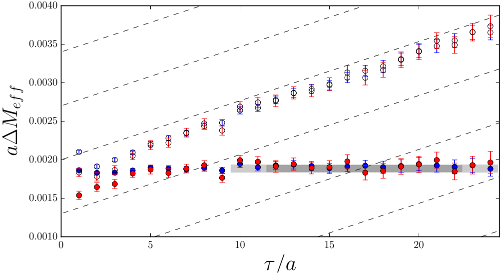
$V \to \infty$ and $m_\gamma \to 0$ limits; relation to 0-modes [M. Endres et al., LAT2015, Phys.Rev.Lett. 117 (2016)]

Consider the contribution of the 0-mode of A_0 to a charged correlator. To each quark-hop forward in time, in a hopping-expansion, is associated a factor $e^{i\frac{q}{V}\tilde{A}_0(0)}$ from the covariant derivative.

$$C_Q(t) \simeq e^{-Mt} \int d\tilde{A}_0 e^{i\frac{Q}{V}\tilde{A}_0(0)t} e^{-S(\tilde{A}_0)},$$

since the action for $\tilde{A}_0$ is gaussian, the result has a gaussian term in t

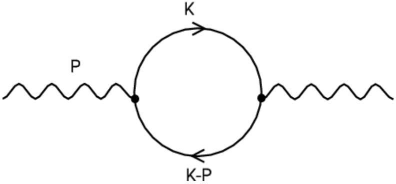
$$C_Q(t) \simeq e^{-Mt} e^{-xt^2} \qquad \text{with } x \propto \frac{1}{m_\gamma^2 V}$$



[$m_\gamma L \leq 1$]

Renormalization of the photon mass

- One is typically interested in $O(\alpha)$ corrections.
- The renormalization is multiplicative because in the massless limit one recovers gauge invariance and the mass term is not generated
- To leading order the only continuum diagram contributing is



that is absent is electroquenched theory (no quark loops coupled to photons), an so are the tadpoles. In full theory contributions $\propto m_\gamma^2$.

- $\Rightarrow$ In the electroquenched theory one only needs to scale am_γ with the lattice spacing to keep m_γ fixed.

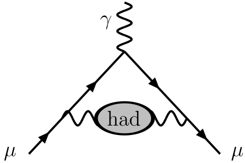
Why muon anomalous magnetic moment on the lattice

3.xxx sigmas discrepancy between theory and experiment [PDG]

$$a_\mu^{exp} = 1.16592091(63) \times 10^{-3}$$
$$a_\mu^{theo} = 1.16591803(50) \times 10^{-3}$$

[Jegerlehner and Nyffeler, 2009]

Contribution	Value	Error
QED incl. 4-loops+LO 5-loops	116 584 718.1	0.2
Leading hadronic vacuum polarization	6 903.0	52.6
Subleading hadronic vacuum polarization	-100.3	1.1
Hadronic light–by–light	116.0	39.0
Weak incl. 2-loops	153.2	1.8
Theory	116 591 790.0	64.6

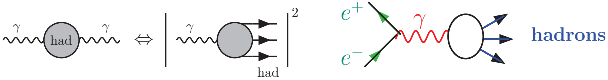


The *theory* number is obtained by estimating the hadronic contribution to the photon propagator from

The experimentally measured hadronic e^+e^- annihilation cross-section:

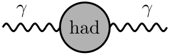
$$\text{DR}: \quad \Pi(k^2) - \Pi(0) = \frac{k^2}{\pi} \int_0^\infty ds \frac{\mathrm{Im}\Pi(s)}{s(s - k^2 - i\epsilon)}$$

$$+ \text{ optical theorem} \quad \mathrm{Im}\Pi(s) \propto s\sigma_{\mathrm{tot}}(e^+e^- \to \text{anything})$$



On the lattice, the Euclidean hadronic vacuum polarisation tensor is defined as

$$\Pi^{(N_f)}_{\mu\nu}(q) = i \int d^4x e^{iqx} \langle J^{(N_f)}_\mu(x) J^{(N_f)}_\nu(0) \rangle$$



Euclidean invariance and current conservation imply

$$\Pi^{(N_f)}_{\mu\nu}(q) = (q_\mu q_\nu - g_{\mu\nu} q^2) \Pi^{(N_f)}(q^2)$$

The relation between $\Pi^{(N_f)}_{\mu\nu}(q^2)$ and a_μ^{HLO} is **[E. De Rafael, 1994 and T. Blum, 2002]**

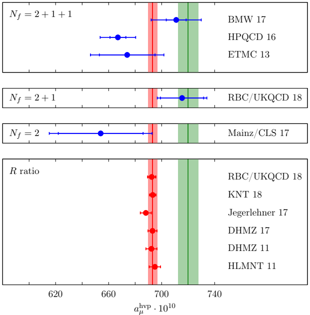
$$a_\mu^{HLO} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dq^2\, f(q^2) \hat{\Pi}(q^2)$$

with

$$f(q^2) = \frac{m_\mu^2 q^2 Z^3 (1 - q^2 Z)}{1 + m_\mu^2 q^2 Z^2}\,, \qquad Z = -\frac{q^2 - \sqrt{q^4 + 4 m_\mu^2 q^2}}{2 m_\mu^2 q^2}$$

and $\hat{\Pi}(q^2) = 4\pi^2 \left[\Pi(q^2) - \Pi(0)\right]$.

Recent results from [Meyer and Wittig, Prog.Part.Nucl.Phys. 104 (2019)]



IB breaking effects: [V. Gülpers et al., JHEP 1709 (2017) 153 and LAT18] using PT-method and QED_L, including leading disconnected contributions, $N_f = 2+1$, $a \simeq 0.12$ fm, with pysical pion mass:

$$a_\mu^{\rm QED,con} = 5.9(5.7)_S(1.1)_E(0.3)_C(1.2)_V(0.0)_A(0.0)_Z \times 10^{-10},$$

$$a_\mu^{\rm QED,\,disc} = -6.9(2.1)_S(1.3)_E(0.4)_C(0.4)_V(0.0)_A(0.0)_Z \times 10^{-10},$$

$$a_\mu^{\rm sIB} = 10.6(4.3)_S(1.3)_E(0.6)_C(6.6)_V(0.1)_A(0.0)_Z \times 10^{-10}.$$

Recently [RM123 1901.10462]: $\delta a_\mu^{HVP}(udsc) = 7.1(2.9) \cdot 10^{-10}$ ($m_\pi \simeq 210$ MeV, no disconnected, 3 lattice spacings)

Our contributions from

- *"Electromagnetic corrections to the hadronic vacuum polarization of the photon within QED_{L} and QED_{M}"*
  A. Bussone, M. Della Morte and T. Janowski. arXiv:1710.06024. EPJ Web Conf. 175 (2018) 06005.
- *"On the definition of schemes for computing leading order isospin breaking corrections"*
  A. Bussone, M. Della Morte, T. Janowski and A. Walker-Loud. arXiv:1810.11647.

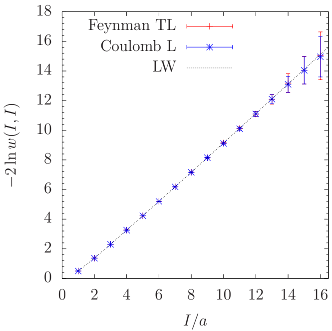
Wilson loops in pure U(1) gauge: $V = 32^4$

$$w_{\mu\nu}(I,I) = \exp\left(2e^2Q^2\left[C_\mu(I,0) - C_\nu(I,I\hat{\nu})\right]\right), \quad C_\mu(I,x) = ID(x) + \sum_{\tau=1}^{I-1}(I-\tau)D(x+\tau\hat{\mu})$$

$D(x)$ is the infinite lattice massless/massive scalar propagator in coordinate space

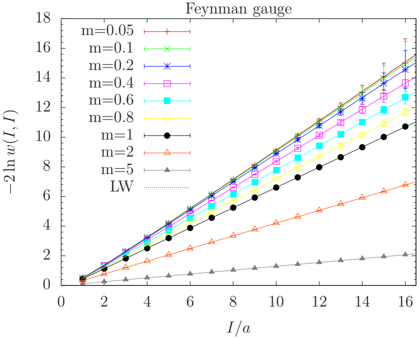
Lüscher-Weisz method

[Luscher and Weisz, Nucl. Phys. B 445 (1995) 429]



Borasoy-Krebs method

[Borasoy and Krebs Phys. Rev. D 72 (2005) 056003]



QCD ensembles

Goal: QED corrections to a_μ^{HLO} in QCD+qQED framework

Dynamical QCD cnfs generated by CLS with $N_f = 2$ degenerate flavors of non-perturbatively O(a) improved Wilson fermions

[Capitani et al. Phys. Rev. D 92 (2015) no.5, 054511]

$\beta = 5.2,\ c_{sw} = 2.01715,\ \kappa_c = 0.1360546,\ a[\mathrm{fm}] = 0.079(3)(2),\ L/a = 32$

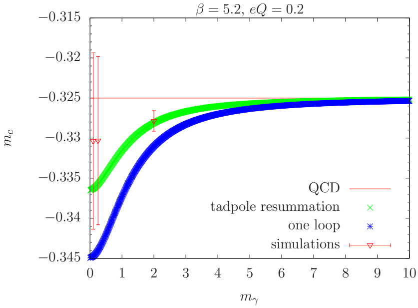
Run	κ	am_π	$m_\pi L$	m_π[MeV]
A3	0.13580	.1893(6)	6.0	473
A4	0.13590	.1459(6)	4.7	364
A5	0.13594	.1265(8)	4.0	316

QED inclusion shifts the critical mass!

Remark: 1% Net effect on m_c translates in O(100%) change in m_q (for $m_0 \simeq m_c^{\mathrm{QCD}}$)
Important for m_π and therefore HVP!

QCD+qQED ensembles

Inclusion of qQED with $\alpha = 1/137$ and physical charges $Q = 2/3, -1/3$



Run	am_γ	$am_{\pi^0=u\bar{u}}$	$am_{\pi^0=d\bar{d}}$	$am_{\pi^\pm}$
A3	0	.2549(9)	.2071(9)	.2330(9)
	0.1	.2556(7)	.2074(8)	.2337(8)
	0.25	.2553(7)	.2072(8)	.2331(8)
A4	0	.2240(8)	.1691(9)	.1994(9)
	0.1	.2252(9)	.1699(9)	.2005(9)
	0.25	.2246(8)	.1700(10)	.1998(9)
A5	0	.2105(7)	.1526(9)	.1849(8)
	0.1	.2114(7)	.1528(9)	.1856(8)
	0.25	.2111(7)	.1531(9)	.1852(8)

Pion masses going from 380 MeV to 640 MeV

Notice: m_c EM shift in A5 gives $m_{\pi^0=u\bar{u}}^{Q(C+E)D} \simeq 2m_\pi^{QCD}$

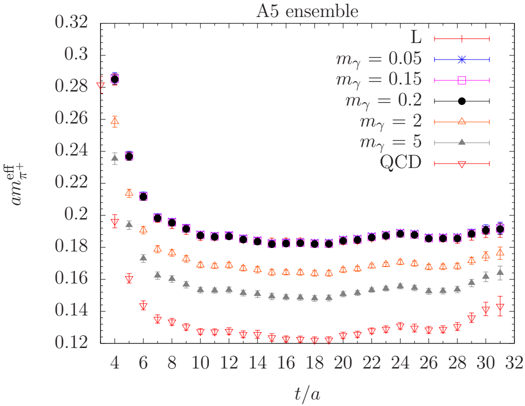
Notice: Matching between ensembles $m_{\pi^\pm}^{Q(C+E)D}(\mathrm{A5}) \simeq m_\pi^{QCD}(\mathrm{A3})$

Dependence on m_γ

For $m_\gamma = 0.1$ the coeff. of linear t-term in eff. energies is suppressed

$$(m_\gamma^2 V)^{-1} \simeq 5 \times 10^{-5}$$

not visible in the effective masses for $m_\gamma \in [0.05, 0.1, 0.15, 0.2, 0.25, 2, 5]$



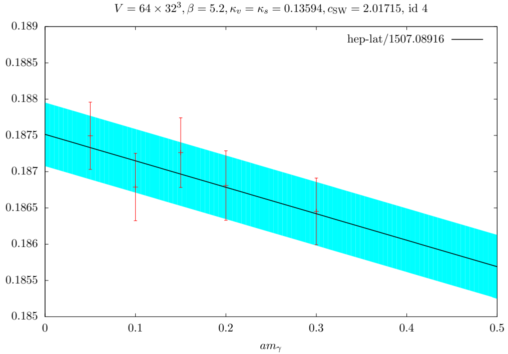
$\mathrm{QED_L}$ is consistent with $\mathrm{QED_M}$ for $m_\gamma \to 0$

Expectation:
photons decouple for $m_\gamma \to \infty$

[Appelquist and Carazzone Phys. Rev. D 11 (1975) 2856]

Our choices are $m_\gamma = 0.1, 0.25$

Blow up of plateau masses at small m_γ



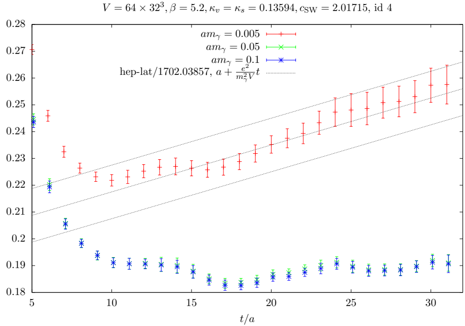
Prediction:
$M(\alpha, m_\gamma) - M(\alpha, 0) = -\frac{\alpha}{2} Q^2 m_\gamma$

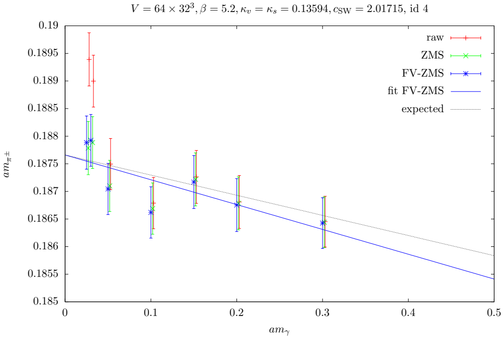
Also, in scalar QED we computed

$$M = M_{QCD} + c_{1,0}\alpha M_{QCD} + c_{1,1}\alpha m_\gamma \quad (M \text{ being } m_\pi)$$

Enders et al. give $c_{1,1} = -1/2$, which we confirm, however what is interesting is also the ratio $\frac{c_{1,1}}{c_{1,0}}$ (i.e. the relative size of the massive corrections compared to what we are after), which we found to be ≈ 2.
As a thumb rule one therefore wants $m_\gamma \lesssim \frac{m_\pi}{4}$.

Pushing m_γ down





One starts seeing linear terms in effective masses ...

So, we have two competing effects on m_γ giving upper and lower bounds on its value:

Lower bound. Suppose we want

$$\frac{a^2}{m_\gamma^2 V} \lesssim 5 \times 10^{-5} \quad (\beta = 5.2) \quad \text{or} \quad \frac{r_0^2}{m_\gamma^2 V} \lesssim 2 \times 10^{-3}$$

Let us turn this in a lower bound on L. Use $T = 2L$ and plug the upper bound $m_\gamma \lesssim \frac{m_\pi}{4}$:

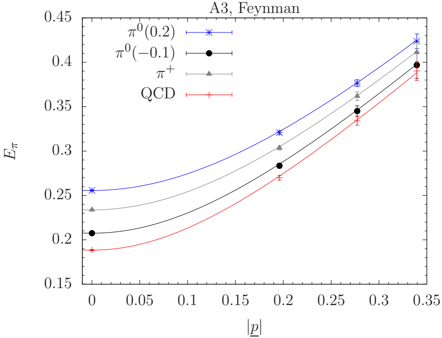
$$\frac{4r_0^2}{m_\pi^2 L^4} \lesssim 10^{-3} \;\Rightarrow\; m_\pi^4 L^4 \gtrsim 4 \times 10^3 (r_0 m_\pi)^2$$

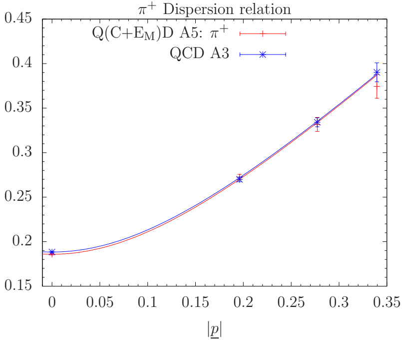
That goes on top of the $m_\pi L \gtrsim 5$ (QCD) FSE thumb rule.

At $m_\pi = 135$ MeV one gets from above $L \approx 6.8$ fm $\Rightarrow m_\pi L \approx 5$.

At $m_\pi = 400$ MeV one gets instead $L \approx 4$ fm $\Rightarrow m_\pi L \approx 8$.

Dispersion relation $m_\gamma = 0.1$





No stiffness in $|\underline{p}|$ **[Patella PoS LATTICE 2016 (2017)]**

$$- \quad E_{\mathrm{eff}}(t, \underline{p}) \overset{m_\gamma \to 0}{\simeq} \frac{(Q_u - Q_d)^2 e^2}{m_\gamma^2 V} t - \frac{\mathrm{d}}{\mathrm{d}t} \ln \langle \mathcal{O}(t, \underline{0}) \overline{\mathcal{O}}(0) \delta_{Q_{\mathrm{T}}, \mathbf{0}} \rangle_{\mathrm{TL}},$$

All the effective energies agree with the continuum curve (solid lines)

Charged pion mass in A3 QCD matches the one in A5 Q(C+E_{M})D

So far...

$m_\gamma \simeq 0.1$ seems to be a safe choice

- Negligible finite photon mass (and therefore volume) effects
- No subtle reduction to $\mathrm{QED_{TL}}$
- $\mathrm{QED_L}$ is consistent (for the spectrum and these parameters)

Pion masses in A5 Q(C+E)D "match" A3 QCD ones

- HVP depends strongly on pion masses
- Can give direct access to EM effects in the HVP

HVP

HVP tensor: $\Pi_{\mu\nu}(q) = \int d^4x\, e^{iq\cdot x} \langle V_\mu(x) V_\nu(0) \rangle$

Is the current still conserved in Q(C+E)D formal theory?

$$\mathbf{SU}(2)_{\rm L} \otimes \mathbf{SU}(2)_{\rm R} \otimes \mathbf{U}(1)_{\rm V}$$
$\downarrow$ explicit and spontaneous
$$\mathrm{QCD}: \mathbf{SU}(2)_{\rm V} \otimes \mathbf{U}(1)_{\rm V}$$
$\downarrow$ explicit
$$\mathrm{Q(C+E)D}: \mathbf{U}'(1)_{\rm V} \otimes \mathbf{U}(1)_{\rm V}$$

Combination of $\mathbf{1}$ and τ^3 in flavor is conserved

$$V_\mu(x) = \overline{\Psi}(x)\gamma_\mu \left[\frac{Q_u}{2}\left(\mathbf{1} + \tau^3\right) + \frac{Q_d}{2}\left(\mathbf{1} - \tau^3\right) \right] \Psi(x)$$

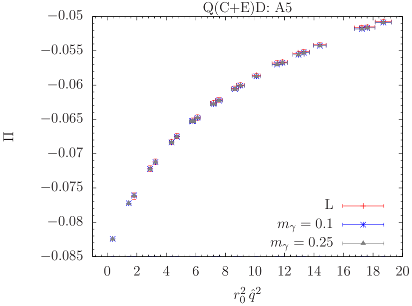
On the Lattice: 1-point-split current conservation implies $Z_V = 1$
no QED effects to take into account

For completeness:

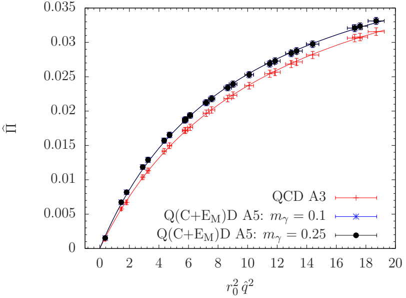
- Neglecting quark-disconnected diagrams
- Electroquenched approximation

Scalar HVP

Agreement between $\mathrm{QED_L}$ and $\mathrm{QED_M}$



Matching gives direct access to EM eff.



r_0/a as any other gluonic scale does not receive QED contributions in the quenched approximation

For completeness:

- ZMS modification [Bernecker and Meyer Eur. Phys. J. A 47 (2011) 148]
- Padé fit R_{10} to extract $\Pi(0)$ [Blum et al. JHEP 1604 (2016) 063]
- Point sources are used

Strategy to extract EM effects for a_μ

First strategy

- Fit scalar HVP in Q(C+E)D and compute a_μ
- Fit scalar HVP in QCD and compute a_μ
- After extrapolation to infinite volume, physical point and continumm take the difference between QCD and Q(C+E)D results

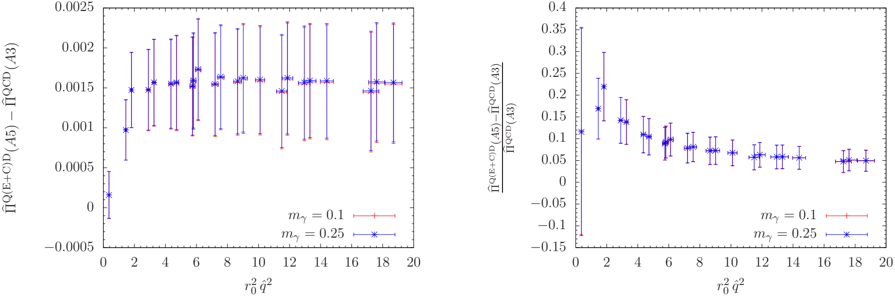
The effect can be washed out by the various systematics...

Second strategy

- Take $\widehat{\Pi}^{\mathrm{Q(C+E)D}} - \widehat{\Pi}^{\mathrm{QCD}} \equiv \delta\widehat{\Pi}$ at fixed pion masses
- Fit $\delta\widehat{\Pi}$ and plug it in $a_\mu^\delta = \int f(q)\delta\widehat{\Pi}$
- Extrapolate to infinite volume, physical point and continuum

Only one fit has to be performed to a slowly varying function

a_μ^δ PRELIMINARY estimates



There is a clear signal, integrating up to $r_0\hat{q}^2 \simeq 20$

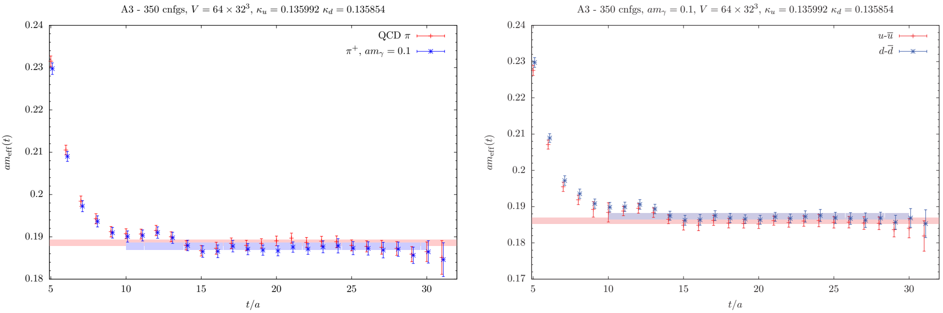
$$a_\mu^\delta \times 10^{10} = 21 \pm 9_{\rm stat}$$

[A. Bussone, MDM, T. Janowski, arXiv:1710.06024]

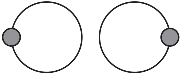
Still effects to quantify, e.g. in a and m_π (this could be large), so far $m_\pi \approx 460$ MeV, $a \approx 0.8$ fm ... Strong isospin breaking

We are changing a bit the strategy. Instead of matching valence (charged) pions between A5 in QCD+QED and A3 in QCD without changing κ, we now change the κ values on A3 once we switch on QED such that the charged pion masses match those of QCD only. That way valence and sea (charged) pions are matched (electroquenched !).

At the same time we require the unphysical $(m_{\pi^0}^{dd})^2$ and $(m_{\pi^0}^{uu})^2$ to be the same. For small enough masses, that allows to define a mass-degenerate point at $\alpha \neq 0$ **[MILC, arXiv:1807.05556]**

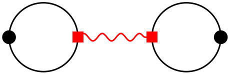


One more thing ...



Disconnected contributions for degenerate case in QCD only $\propto \left(\sum_{i=1}^{N_f} Q_f\right)^2$.

No longer true in QCD+QED. In the example all charges appear squared, no cancellations.



Indeed, RBC/UKQCD 17

$$a_\mu^{\text{QED,con}} = 5.9(5.7)_S(1.1)_E(0.3)_C(1.2)_V(0.0)_A(0.0)_Z \times 10^{-10},$$

$$a_\mu^{\text{QED, disc}} = -6.9(2.1)_S(1.3)_E(0.4)_C(0.4)_V(0.0)_A(0.0)_Z \times 10^{-10},$$

The inidication is that IB corrections in a_μ are basically of 'strong' type only (to the extent separation makes sense ...)

We want to check this in $N_f = 2$ and with QED_M. For disconnected contributions we use [Giusti et al., arXiv:1903.10447] plus dilution schemes.

Separating EM from strong effects at LO [A. Bussone et al., 1810.11647], [D. Giusti et al., 1811.06364]

We have been imagining

$$O(m_u + m_d, \Delta m, \alpha) = O(m_u + m_d, 0, 0) + \alpha \left.\frac{\partial O}{\partial \alpha}\right|_{\alpha=0} + \Delta m \left.\frac{\partial O}{\partial \Delta m}\right|_{\Delta m=0}$$

1 Not an expansion in indep params; $(m_u \pm m_d) \equiv (m_u \pm m_d)(\alpha)$

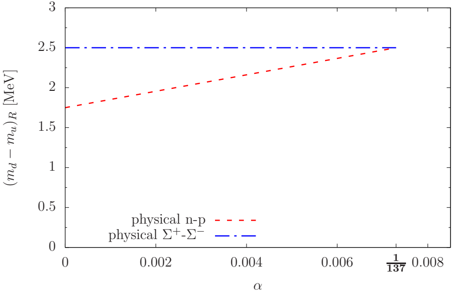
2 $\left.\frac{\partial O}{\partial \alpha}\right|_{\alpha=0}$ should be computed at the isosymmetric point. How is that "defined" at $\alpha \neq 0$?

3 Δm should be (e.g.) at $\alpha = 0$.
- How is that "defined" and how does it differ from $\Delta m(\alpha = 1/137)$?
- If difference is $O(\alpha)$ then by using 'physical' value one does a mistake $O(\alpha)$, i.e. as large as what is being computed (LO IB corrections).

[1] On the lattice we like hadronic schemes (less problems in renormalizing parameters). The π^0 is a Goldstone boson also at $\alpha \neq 0$, so it fixes the massless point. It has 'basically' no IB corr. at LO [Bijnens and Prades, hep-ph/9610360], just $\propto m_u + m_d$.

So let us fix $m_u + m_d$ by keeping the π^0 mass fixed $\forall\alpha$

For [2] and [3] we need to define Δm at $\alpha \neq 0$. E.g through $\Sigma^+ - \Sigma^-$ splitting or $n - p$ splitting [BMW, Science 347 (2015)]



By renormalizability of QCD+QED (not expanded) values agree at α_{phys}.

As a consequence of residual vector and axial transf., in a scheme (i) consistent with WI , which is also (ii) smooth in α (e.g. α-indep)

$$m_{u,i}(\alpha) = m_{u,i}(0) Z_{u,i}(\alpha)\ , \quad \text{and} \quad m_{d,i}(\alpha) = m_{d,i}(0) Z_{d,i}(\alpha)\ ,$$

with $Z_{X,i}(\alpha) = 1 + C_{X,i}\alpha + \cdots$. The mass on the rhs for example is the **renormalized** QCD mass in the i scheme. The splitting now reads

$$\begin{aligned} \Delta_i m(\alpha) &= \Delta_i m(0)\, Z_{d,i}(\alpha) + (Z_{d,i}(\alpha) - Z_{u,i}(\alpha)) m_{u,i}(0)\ , \\ &= \Delta_i m(0)\, (1 + C_{d,i}\alpha) + C_{(d-u)}\alpha m_u^i(0)\ . \end{aligned}$$

Using the fact that, *numerically, $\Delta m \simeq m_u$,* one obtains

$$\Delta_i m(\alpha) = \Delta_i m(0) + O(\alpha \Delta m)\ \ldots$$

Similarly, in such schemes $\Delta_1 m(0) = \Delta_2 m(0) + O(\alpha\Delta m) + O(\alpha^2)$. So, provided (i) and (ii), the ambiguity in using $\Delta m(1/137)$ instead of $\Delta m(0)$ is higher order in IB corrections.[3]

[2] In order to define the mass-symmetric point at $\alpha \neq 0$ one should require a quantity proportional to Δm up to quadratic IB corrections to vanish. E.g:

- Σ^+ - Σ^- splitting
- Unphysical $(m^{dd}_{\pi^0})^2$ - $(m^{uu}_{\pi^0})^2$ as done in [MILC, arXiv:1807.05556]

then the ambiguity in $\frac{\partial O}{\partial \alpha}\Big|_{\alpha=0}$ is at least linear in IB effect (higher order in $\alpha \frac{\partial O}{\partial \alpha}$).

We have neglected here e.g. the dependence of α_s (or a) on α. In the electroquenched approximation one should use a gluonic quantity (like r_0) to fix the relative scale. r_0/a is then independent from α

Conclusions

- Isospin effects need to be included beyond the point-like approx. for precision physics.
- QED with PBC not straightforward. Different approaches now producing many results. It is essential to compare them and very good that we have so many with different systematics.
  Lot of results for spectrum, decay rates and HVP (see FLAG 2019).
- I described an (early-stage) application of QED_M to the HVP for $(g-2)_\mu$.
- In preparing to go beyond that I collected a few thoughts on how to define a 'scheme' to compute and separate LO IB corrections.