Hadron structure from the CLS $N_f = 2 + 1$ ensembles

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RQCD Collaboration

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Outline

Aim to determine nucleon structure observables with all the main systematics under control.

⋆ $N_f = 2 + 1$ CLS ensembles.

⋆ Excited state contamination.

⋆ Octet baryon mass spectrum.

⋆ Isovector baryon charges.

⋆ Nucleon axial, induced pseudoscalar and pseudoscalar form factors.

⋆ Use of a ChPT inspired fit to deal with large excited state contributions to the induced pseudoscalar and pseudoscalar form factors.

Preliminary!

RQCD

Mass spectrum: Gunnar S. Bali, Wolfgang Söldner, ...
Isovector charges: Wolfgang Söldner, Simon Weishäupl, Thomas Wurm, ...
Form factors: Michael Gruber, Philipp Wein, Thomas Wurm, ...

Nucleon isovector charges and form factors

Charged currents ($n \to p$): $\bar{u}\Gamma d$

Neutral currents ($p \to p$): $\bar{u}\Gamma u$, $\bar{d}\Gamma d$, $\bar{s}\Gamma s$

Isospin limit: $\langle p|\bar{u}\Gamma d|n\rangle = \langle p|\bar{u}\Gamma u - \bar{d}\Gamma d|p\rangle = \langle n|\bar{d}\Gamma d - \bar{u}\Gamma u|n\rangle$

Also for $\Gamma = \gamma_\mu$: $\langle p|\bar{u}\gamma_\mu d|n\rangle = \langle p|J_{em}|p\rangle - \langle n|J_{em}|n\rangle$ etc.

Isovector form factors:

$$\Gamma = \gamma_\mu \qquad F_1(Q^2)\gamma_\mu + \frac{F_2(Q^2)}{2m_N}\sigma_{\mu\nu}Q^\nu \qquad \overset{Q^2\to 0}{\rightsquigarrow} 1$$

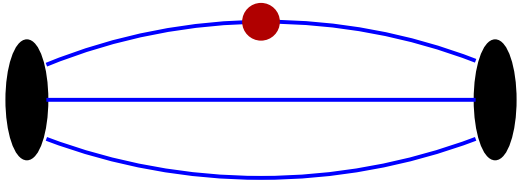
$$\Gamma = \gamma_\mu\gamma_5 \qquad G_A(Q^2)\gamma_\mu\gamma_5 - i\gamma_5\frac{\tilde{G}_P(Q^2)}{2m_N}Q_\mu \qquad \rightsquigarrow g_A$$

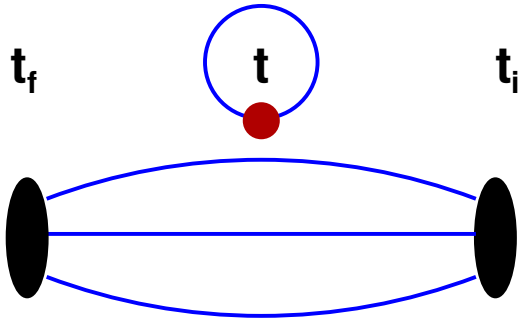
$$\Gamma = \sigma_{\mu\nu} \qquad G_T(Q^2)\sigma_{\mu\nu} \qquad \rightsquigarrow g_T$$

$$\Gamma = \mathbb{1} \qquad G_S(Q^2)\mathbb{1} \qquad \rightsquigarrow g_S$$

$$\Gamma = \gamma_5 \qquad G_P(Q^2)\gamma_5 \qquad \rightsquigarrow g_P$$

General considerations: $\langle N|\bar{q}\Gamma q|N\rangle$





Isovector: only connected quark line diagrams in the isosymmetric limit.
Isoscalar combinations also contain disconnected contributions.

Systematics:

- Excited state pollution.
- Renormalisation+ improvement: for $\vec{p} = \vec{p}' = 0$/some operators/actions $c_J = 0$ or $b_J = 0$ $J^{\overline{\mathrm{MS}}}(\mu) = Z^{\overline{\mathrm{MS}},\mathrm{latt}}(a\mu, g^2)\left[(1 + b_J a m_q)J^{\mathrm{latt}} + c_J a J_1^{\mathrm{latt}}\right]$
- Volume: exponentially suppressed $\sim e^{-LM_\pi}$, $LM_\pi > 4$.
- Discretisation effects: $\mathcal{O}(a)$ or $\mathcal{O}(a^2)$.
- Physical point extrapolation: ChPT (inspired) $M_\pi \to M_\pi^{\mathrm{phys}}$.

Excited states

Spectral decomposition (forward limit, zero momentum):

$$C_{2pt}(t_f) = Z_N e^{-E_N t_f} + Z_{N_1} e^{-E_{N_1} t_f} + \ldots = Z_N e^{-E_N t_f} \left[1 + \frac{Z_{N_1}}{Z_N} e^{-\Delta E_{N_1} t_f} + \ldots \right]$$

$$C_{3pt}(t_f, \tau) = Z_N \langle N(\vec{0}) | J | N(\vec{0}) \rangle e^{-E_N t_f} \times \left[1 + B_{01} (e^{-\Delta E_{N_1}(t_f - \tau)} + e^{-\Delta E_{N_1} \tau}) + B_{11} e^{-\Delta E_{N_1} t_f} + \ldots \right]$$

$Z_N = |\langle 0 | \mathcal{N} | N \rangle|^2$, $B_{10} = B_{01} \propto \sqrt{Z_{N_1}/Z_N} \langle N_1(\vec{0}) | J | N(\vec{0}) \rangle$, $B_{11} \propto (Z_{N_1}/Z_N) \langle N_1(\vec{0}) | J | N_1(\vec{0}) \rangle$

Ground state dominates for $t_f \gg \tau \gg 0$, noise increases with $e^{[m_N - \frac{3}{2} M_\pi] t_f}$.

Lowest states are multi-particle states for $M_\pi \sim M_\pi^{\mathrm{phys}}$ and $LM_\pi \sim 4$.

2pt and 3pt (zero momentum): $N_1 = N(\vec{0})\pi(\vec{0})\pi(\vec{0})$ or $N_1 = N(\vec{k})\pi(-\vec{k})$, ...

3-momentum transfer $\vec{q}$: $N_1 = N(\vec{0})\pi(\vec{q})$ or $N_1 = N(\vec{q})\pi(\vec{0})$, ...

Excited states

- ⋆ Optimise smearing of interpolator $\mathcal{N}$ to maximise $Z_N \gg Z_{N_i}$.
- ⋆ Use several $t_f \sim 0.7 - 1.4$ fm.
- ⋆ Include excited state terms in the fit.
- ⋆ Dominant excited state contribution also depends on $\langle N_i | J | N_j \rangle$.
- ⋆ More than one state may be significant.
- ⋆ ChPT has been used to provide some insight, e.g.
  - [Tiburzi,0901.0657,1503.06329] $N\pi$ excited state contribution to $G_A(0) = g_A$ (forward limit) in LO BChPT.
  - [Hansen,1610.03843] $N\pi$ excited state contribution to g_A, LO BChPT with finite volume interaction corrections a la Lellouch-Lüscher.
  - [Bär,1906.03652,1812.09191] $N\pi$ contributions to $G_A(Q^2)$, $\tilde{G}_P(Q^2)$ and $G_P(Q^2)$ computed in LO BChPT. Forward limit g_A, g_S, g_T [Bär,1606.09385].
  - [Meyer,1811.03360] $N\pi$ contributions to $G_A(Q^2)$, $\tilde{G}_P(Q^2)$ and $G_P(Q^2)$ computed to tree-level in BChPT.

$N_f = 2 + 1$ CLS ensembles

Coordinated Lattice Simulations (CLS): Berlin, CERN, Mainz, UA Madrid, Milano Bicocca, Münster, Odense, Regensburg, Rome I and II, Wuppertal, DESY-Zeuthen.

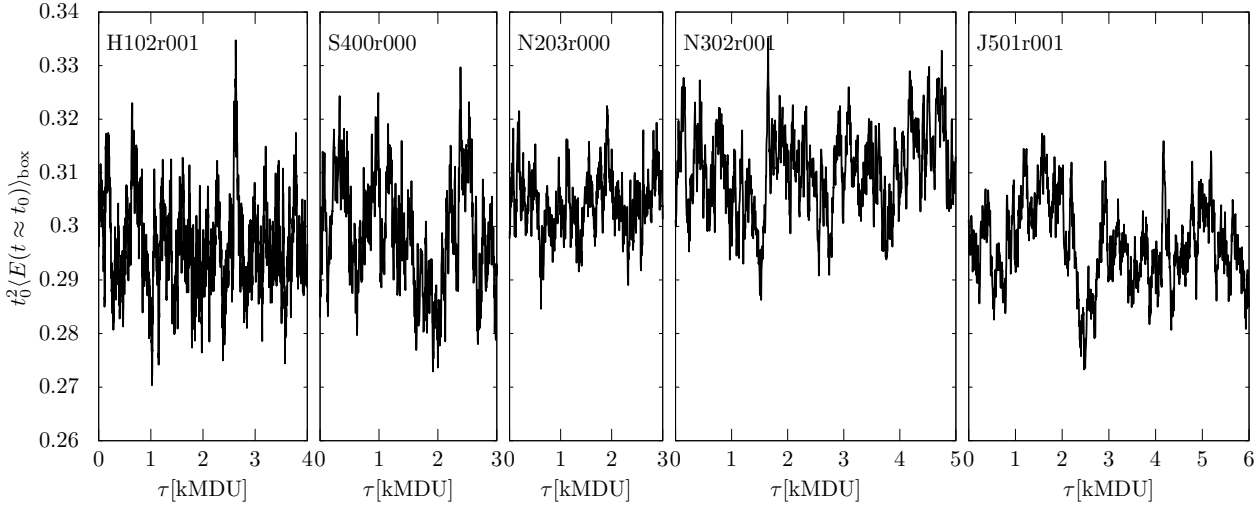
⋆ Non-perturbatively improved clover fermion action and tree-level Lüscher-Weisz gauge action.

⋆ Six (five) lattice spacings: $a = 0.1 - 0.04$ fm.

⋆ $LM_\pi \gtrsim 4$ and multiple spatial volumes.

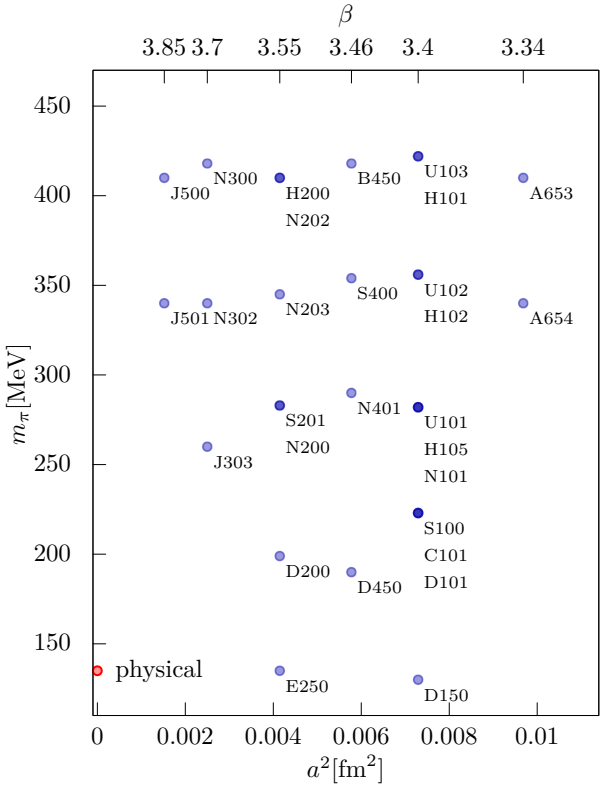
⋆ Mostly open boundary conditions in time.

Wilson flow action density, $t_0^2 E(t \approx t_0)$, $M_\pi \approx 340$ MeV, averaged over ≈ 1 fm slice.

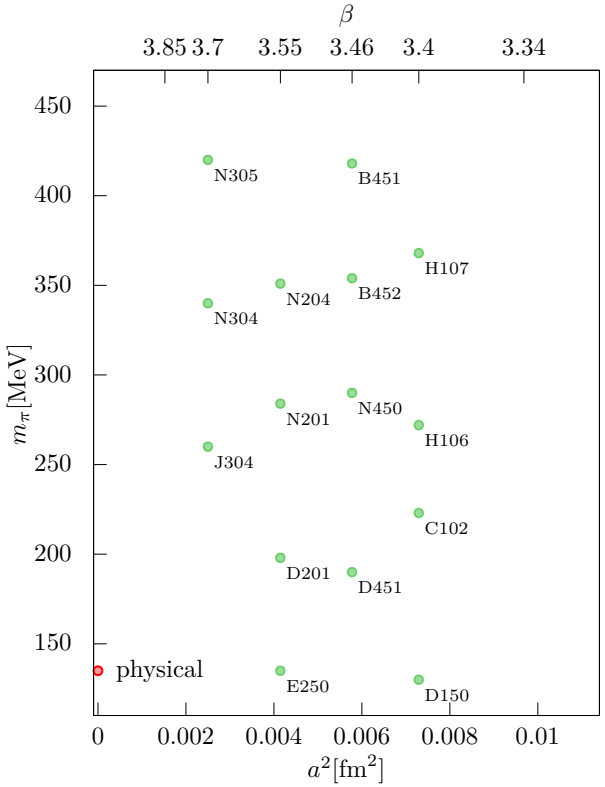


CLS ensembles: M_π vs a^2

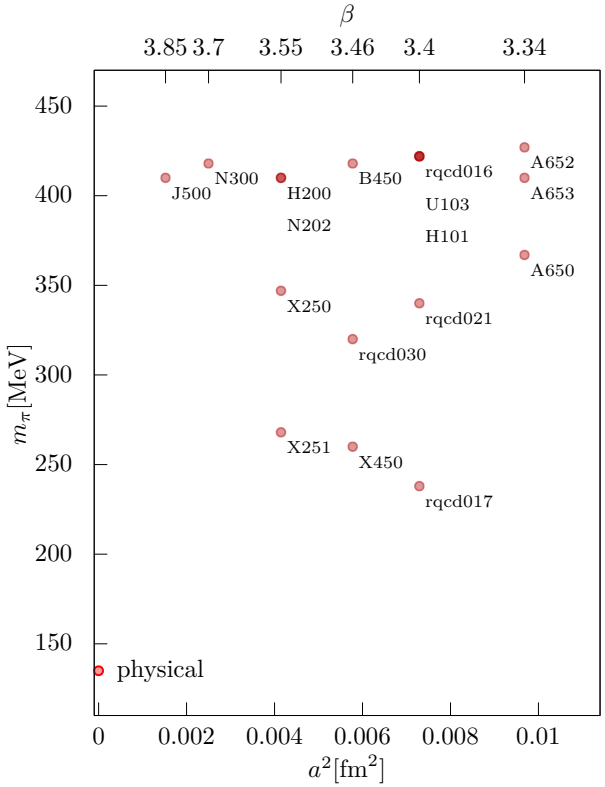
- ⋆ Three trajectories, physical point ensembles.
- ⋆ Typically 6000–10000 MDUs.



$2m_\ell + m_s = \text{const.}$

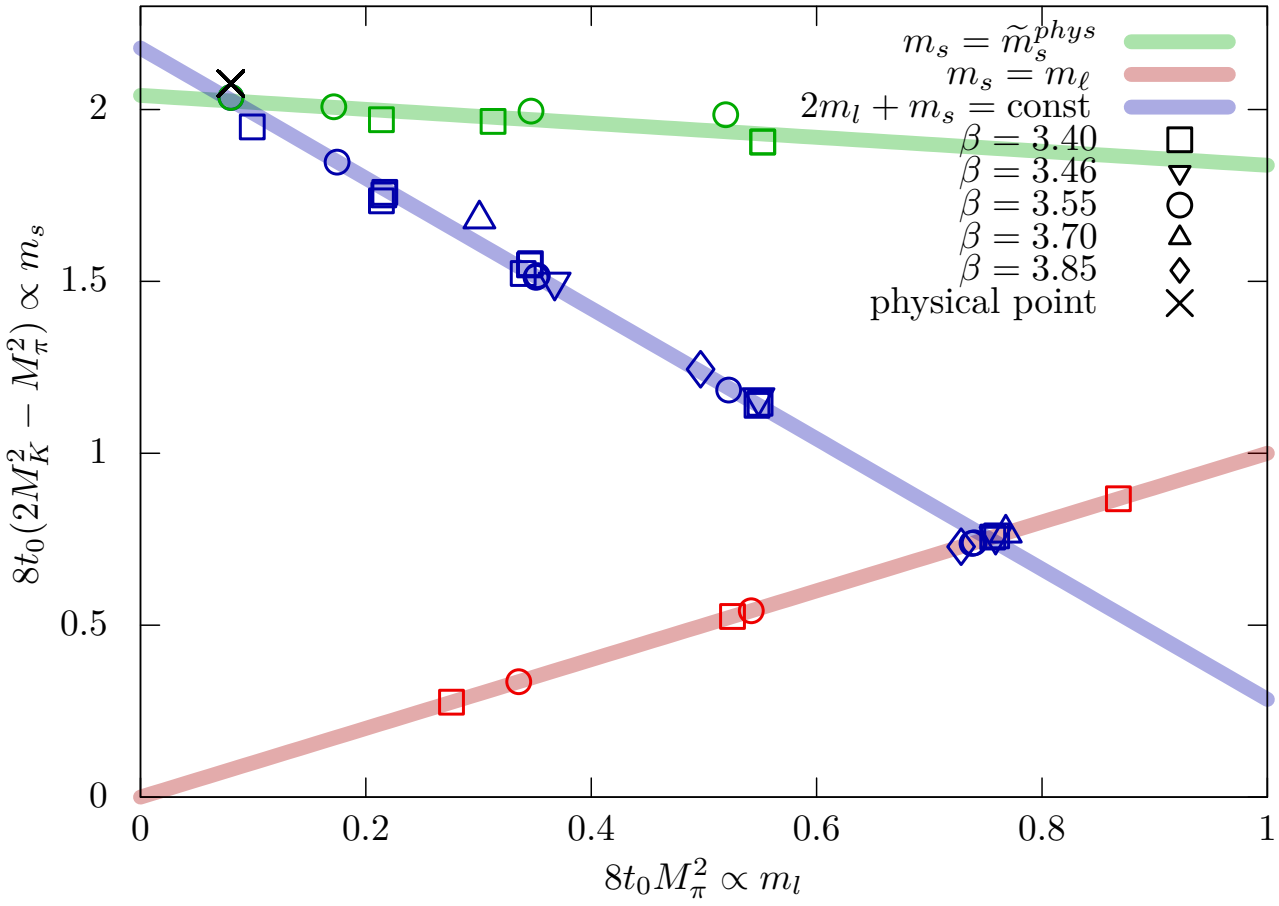


$m_s = \text{const.}$

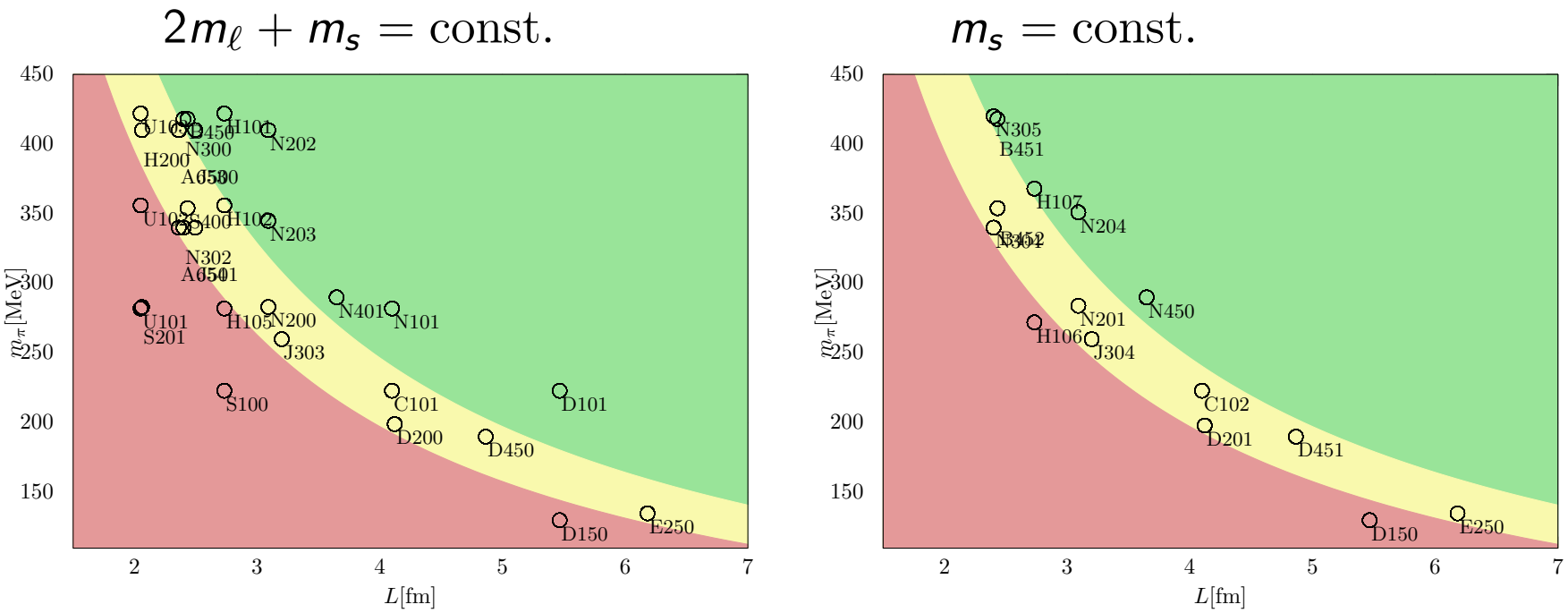


$m_\ell = m_s$

CLS ensembles: m_ℓ-m_s plane



CLS ensembles: spatial volume



$\mathbf{LM_\pi < 4}$ $\mathbf{4 \leq LM_\pi < 5}$ $\mathbf{LM_\pi \geq 5}$

Octet baryon spectrum: $B \in \{N, \Lambda, \Sigma, \Xi\}$

Perform fits to 2pt functions.

⋆ 'Optimised' smeared interpolators $\mathcal{N}$ with Wuppertal (Gaussian) smearing using APE smeared gauge transporters.

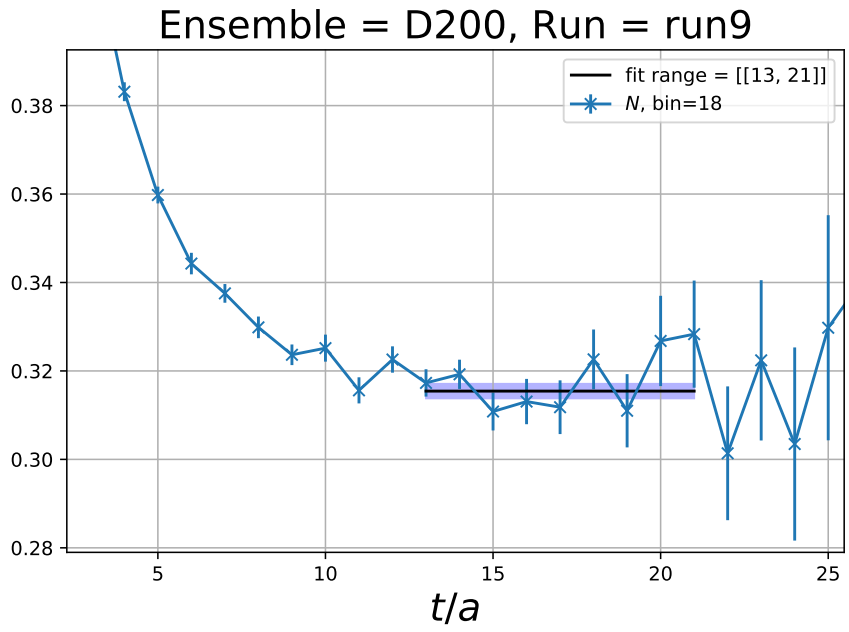
⋆ $\langle r^2 \rangle_{\Psi^2}^{1/2} \sim 0.6 - 0.8$ fm for $M_\pi = 410 - 200$ MeV.

⋆ Multiple measurements per configuration (4–20).

$t_{\min}$ for 1-state fit determined via 2-state fit. [ALPHA,1004.2661]

(Dependent on statistics)
$t_{\min} \sim 0.8 - 1.0$ fm.

Right: m_{eff} for nucleon with $M_\pi \sim 200$ MeV, $a = 0.064$ fm.



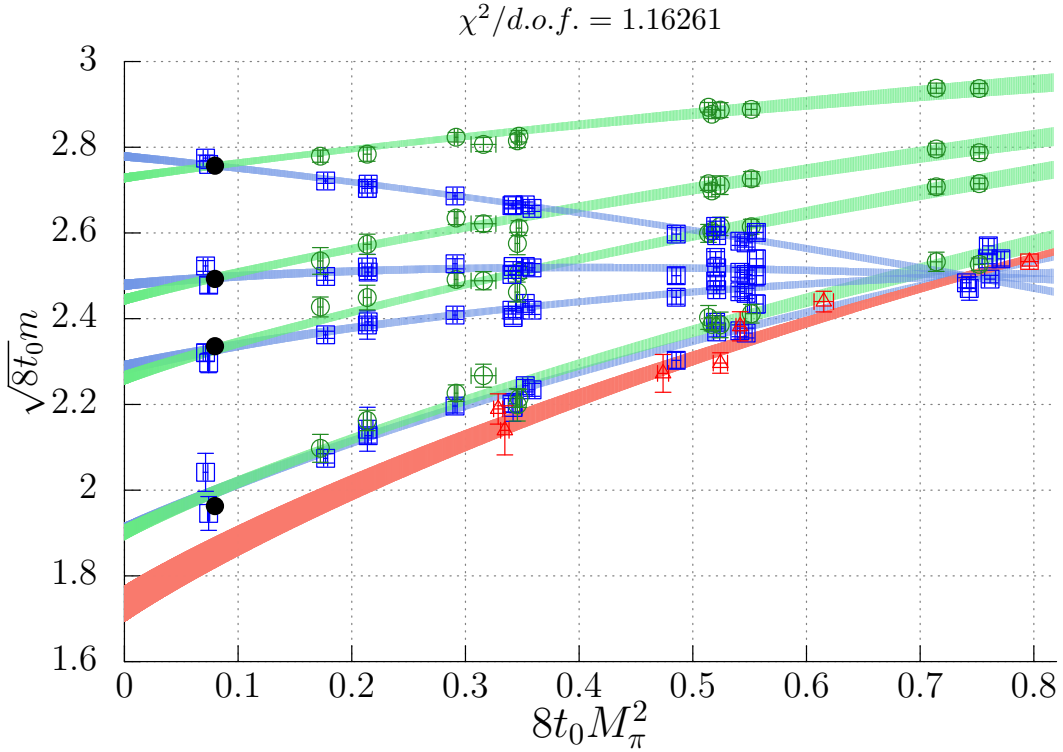
Mass extrapolations: $B \in \{N, \Lambda, \Sigma, \Xi\}$, **Preliminary**

Simultaneous fit to octet masses: (12 parameters)

$$\mathbb{m}_B(\mathbb{M}_\pi, \mathbb{M}_K, \mathbb{a}) = \mathbb{m}_B(\mathbb{M}_\pi, \mathbb{M}_K, 0)\left[1 + c\mathbb{a}^2 + \bar{c}\mathbb{a}^2\overline{\mathbb{M}}^2 + \delta c_B \mathbb{a}^2 \delta\mathbb{M}^2\right]$$

$\mathbb{m}_B = \sqrt{8t_0}m_B$, $\mathbb{a} = a/\sqrt{8t_0^*}$, $\overline{\mathbb{M}}^2 = (2\mathbb{M}_K^2 + \mathbb{M}_\pi^2)/3$, $\delta\mathbb{M}^2 = 2(\mathbb{M}_K^2 - \mathbb{M}_\pi^2)$

W. Söldner



Finite volume: $LM_\pi > 3.4$. $O(a^2)$ effects removed using fit. Similarly, data shifted to $\overline{\mathbb{M}}^2 = \overline{\mathbb{M}}^{2,\mathrm{phys}}$, $\overline{\mathbb{M}}^2_{\bar{s}s} = \overline{\mathbb{M}}^{2,\mathrm{phys}}_{\bar{s}s}$, $\overline{\mathbb{M}}^2_{\bar{s}s} = 2\mathbb{M}_K^2 - \mathbb{M}_\pi^2$.

Mass extrapolations

$O(p^3)$ (NNLO) covariant SU(3) BChPT: $(\mathbb{M}_{\eta_8}^2 = \overline{\mathbb{M}}^2 + \delta\mathbb{M}^2/3)$

$$\mathbb{m}_B(\mathbb{M}_\pi, \mathbb{M}_K, 0) = \mathbb{m}_0 + \overline{\mathbb{b}\mathbb{M}}^2 + \delta\mathbb{b}_B\delta\mathbb{M}^2 + \mathbb{g}_{B,\pi} f_O\left(\frac{\mathbb{M}_\pi}{\mathbb{m}_0}\right) + \mathbb{g}_{B,K} f_O\left(\frac{\mathbb{M}_K}{\mathbb{m}_0}\right) + \mathbb{g}_{B,\eta_8} f_O\left(\frac{\mathbb{M}_{\eta_8}}{\mathbb{m}_0}\right)$$

EOMS regularisation*: $f_O(x) = -2x^3[\sqrt{1-x^2/4}\arccos(x/2) + x\ln(x)/2]$

SU(3) constraints:

⋆ $\delta\mathbb{b}_B = \delta\mathbb{b}_{N,\Lambda,\Sigma,\Xi}$ determined by 2 parameters.

⋆ $\mathbb{g}_{B,\pi,K,\eta_8}$ known quadratic functions of the LECs F and D.

$m_\Xi^{\rm latt} = m_\Xi^{\rm ph}$ imposed to fix the scale: $\sqrt{8t_{0,\rm ph}} = 0.4128(22)(??)$ fm.

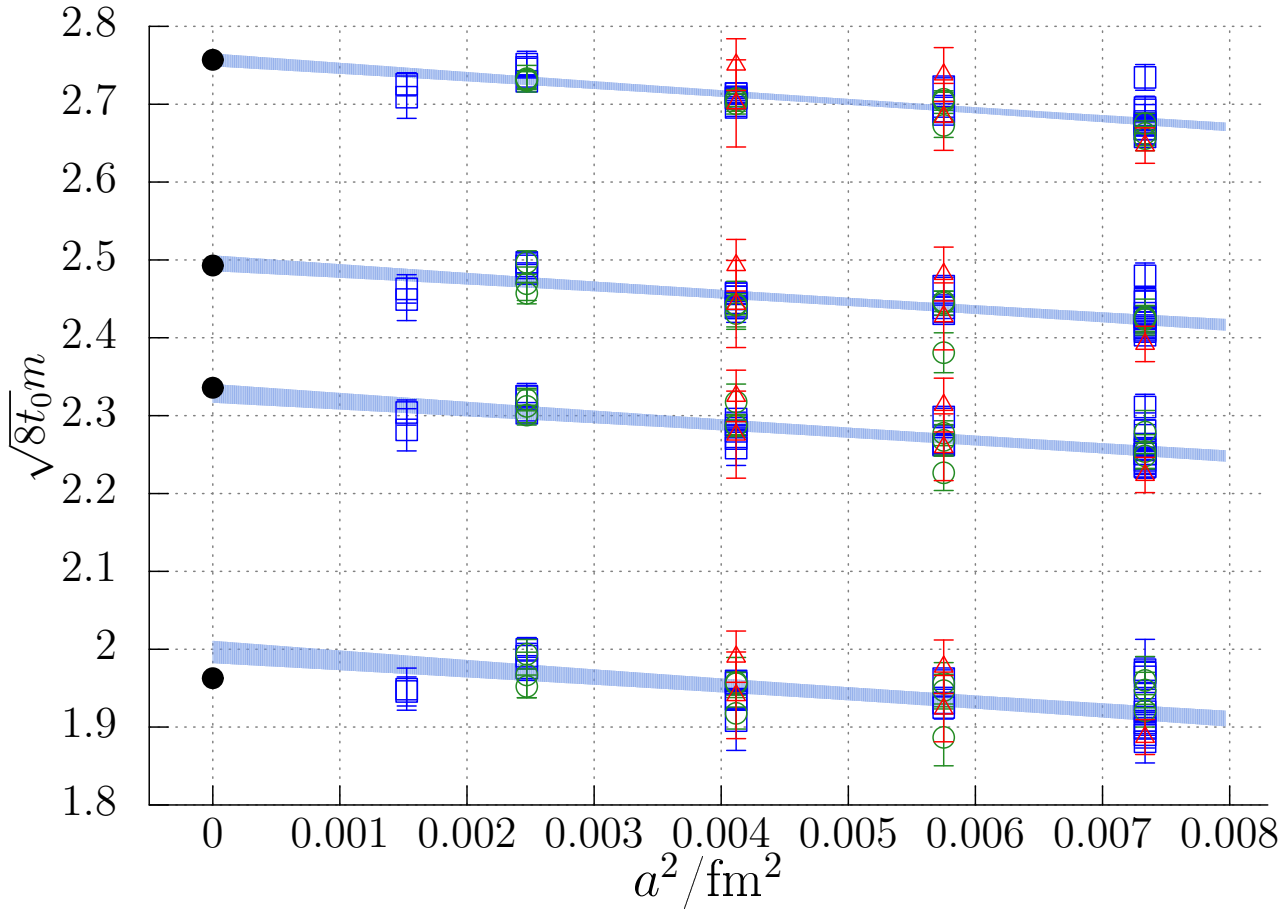
Compatible with $\sqrt{8t_{0,\rm ph}} = 0.413(6)$ fm from $F_\pi + 2F_K$ [ALPHA,1608.08900].

Preliminary. Still to try: SSE including decuplet baryons, Taylor expansion about the symmetric point, different mass/a cuts.

* Extended on mass shell scheme.

Mass extrapolations: discretisation effects

Data shifted to physical quark mass using the fit.



a^2 varied by a factor of 4.6.

σ terms: $\sigma_{q,B} = m_q \langle B|q\bar{q}|B\rangle$

Feynman-Hellmann theorem

$$\sigma_{\pi B} = m_u \frac{\partial m_B}{\partial m_u} + m_d \frac{\partial m_B}{\partial m_d} \approx M_\pi^2 \frac{\partial m_B}{\partial M_\pi^2}, \qquad \sigma_{sB} = m_s \frac{\partial m_B}{\partial m_s} \approx \frac{1}{2} M_{\bar{s}s}^2 \frac{\partial m_B}{\partial M_{\bar{s}s}^2}$$

$\sigma_{\pi B} = \sigma_{uB} + \sigma_{dB}$.

σ_s is not well determined as m_s is not varied near the physical point.

Pion sigma terms: **Preliminary**

$$\sigma_{\pi N} = 41(2)(2)(??)\ \mathrm{MeV} \qquad \sigma_{\pi \Lambda} = 29(2)(1)(??)\ \mathrm{MeV}$$

$$\sigma_{\pi \Sigma} = 23(1)(1)(??)\ \mathrm{MeV} \qquad \sigma_{\pi \Xi} = 13(1)(0)(??)\ \mathrm{MeV}$$

Compatible with [FLAG,1902.08191] average (FH+direct) for $N_f = 2+1$ of $\sigma_{\pi N} = 39.7(3.6)$ MeV.

[BMW-c,1510.08013] $\sigma_{\pi N} = 38(3)(3)$ MeV

Forward matrix elements: isovector charges g_A, g_S, g_T

⋆ Four source-sink separations $t_f \sim 0.7 - 1.2$ fm. $N_{\rm meas}$=1,2,3,4.

⋆ Renormalisation: $J^{\overline{\rm MS}}(\mu) = Z^{\overline{\rm MS},latt}(a\mu, g^2)\left[(1 + b_J a m_q) J^{\rm latt} + c_J a J_1^{\rm latt}\right]$.

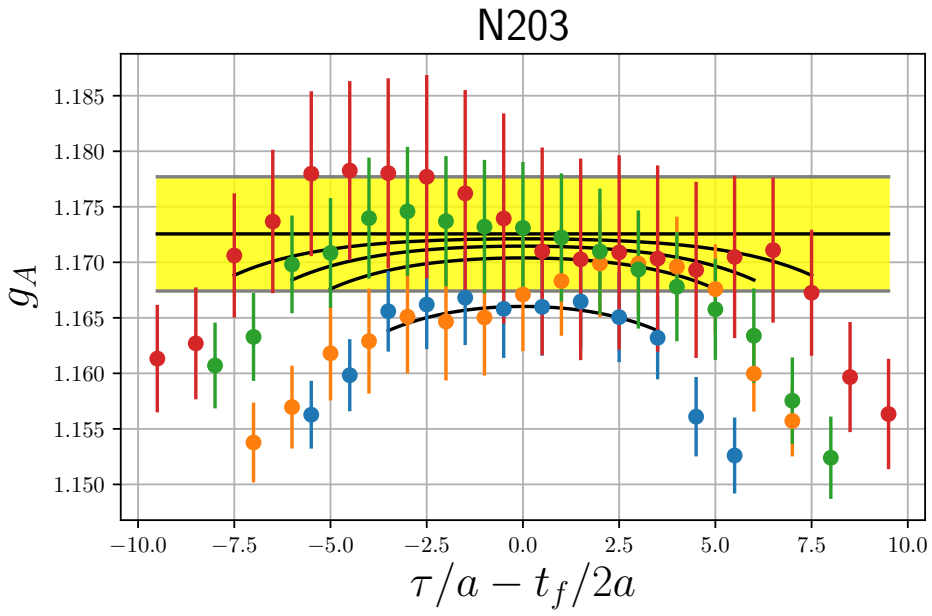
$c_S = 0$, c_A [ALPHA,1502.04999] and c_T only relevant for $\vec{q} \neq \vec{0}$.
$b_{A,S,T}$ [Korcyl,1607.07090]. $Z_{A,S,T}$ RI′-MOM S. Bürger, M. Göckeler, S. Piemonte, in preparation.

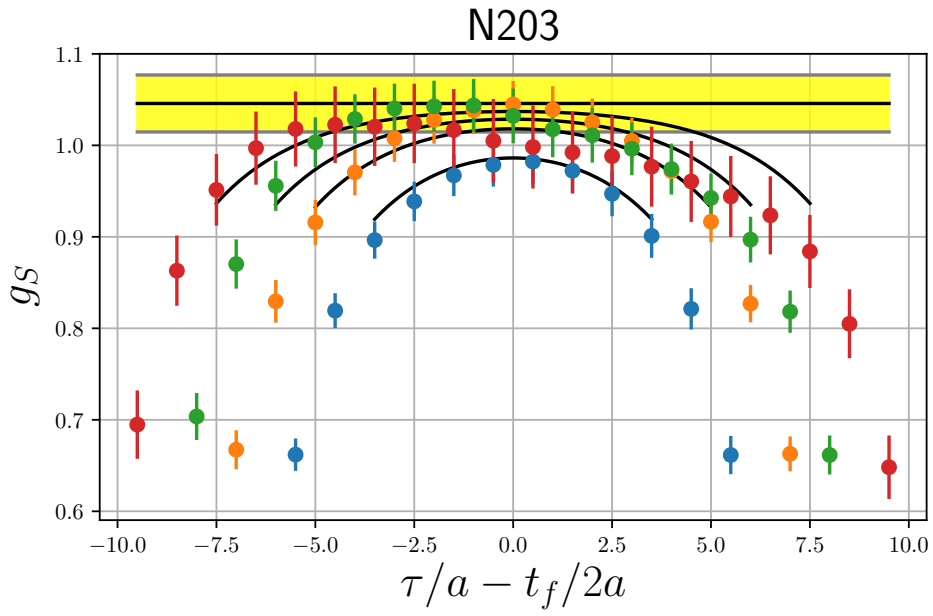
⋆ Fit ratio $C_{3pt}(t_f, \tau)/C_{2pt}(t_f) \to \langle N|J|N\rangle$

⋆ 2 state fit. Fit range variation included in final error.

Forward matrix elements: isovector charges g_A, g_S, g_T

$M_\pi \sim 350$ MeV, $a = 0.064$ fm.





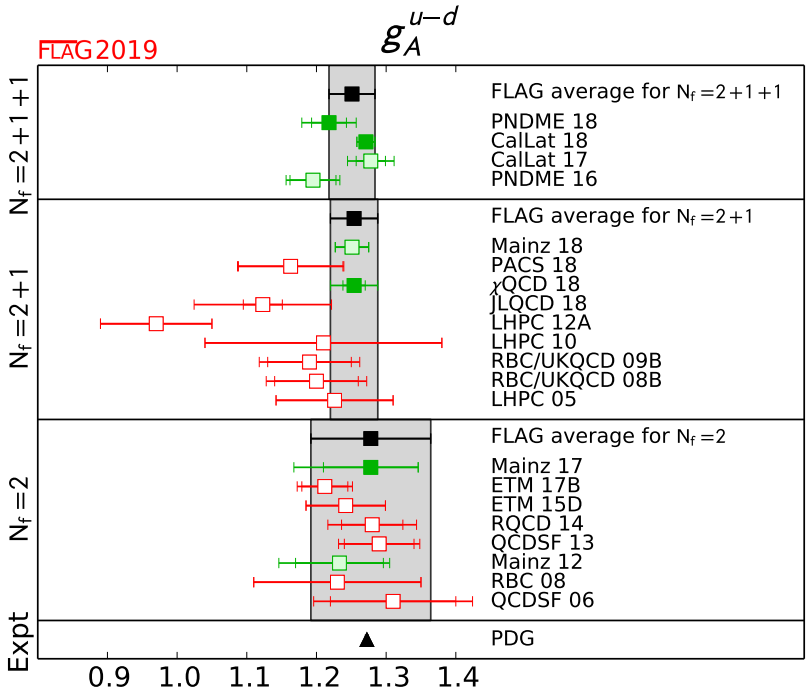
Axial charge: $g_A \equiv G_A(0)$

$\mathbf{g_A}/\mathbf{g_V} = 1.2724(23)$ PDG 2018 (assuming SM), $g_V = 1$ in the isospin limit.

Benchmark quantity: demonstration of lattice techniques, known to be sensitive to:

- excited state contamination,
- spatial volume,
- quark mass, ...

Right: [FLAG,1902.08191]



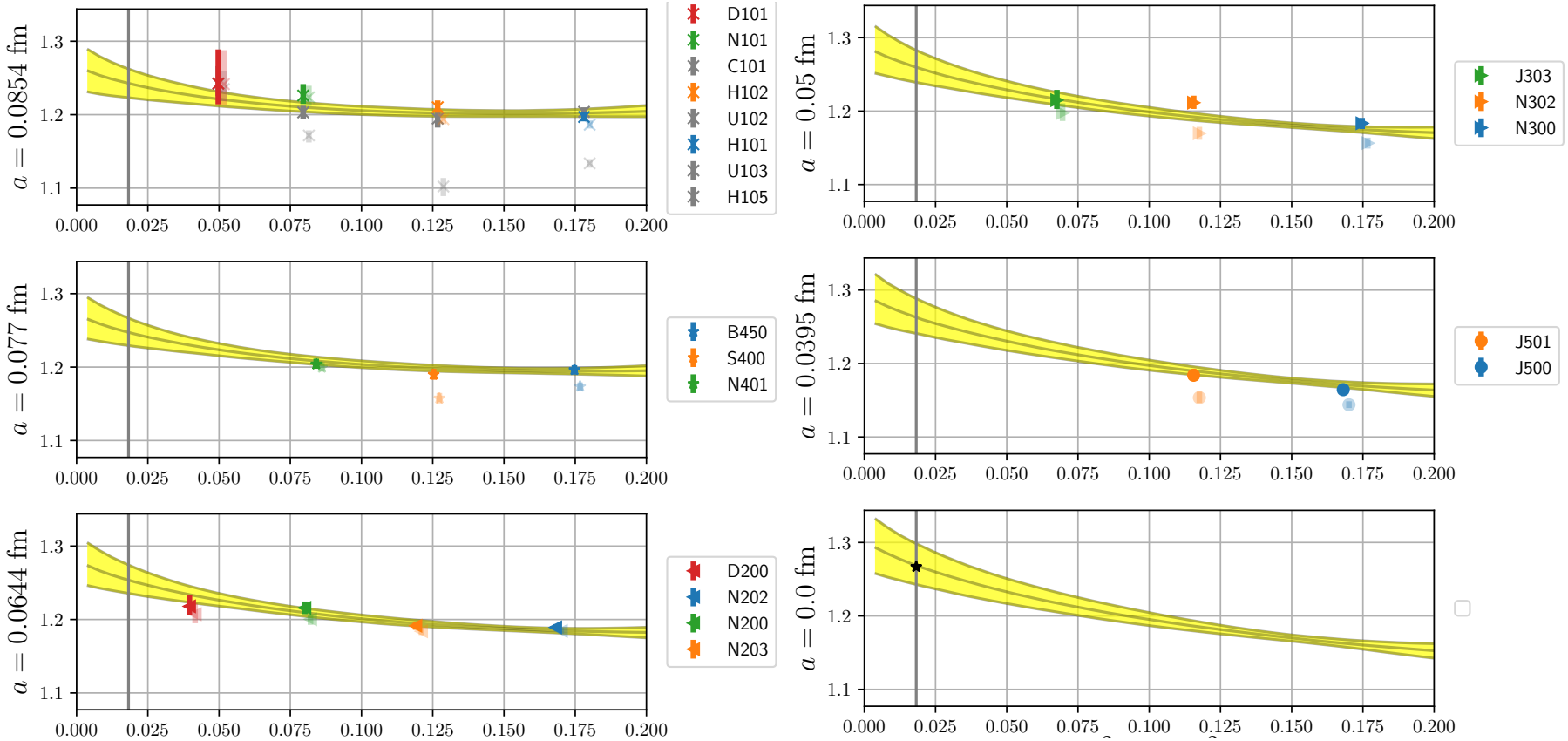
Axial charge: $g_A \equiv G_A(0)$

Analysis in progress: example of a, m_q, V extrapolation for $2m_\ell + m_s = \mathrm{const.}$ trajectory.

$$g_A(M_\pi, M_K, L, a) = g_A(M_\pi, M_K, L, 0)\left[1 + a^2\left(c_0 + c_1 M_\pi^2\right)\right]$$

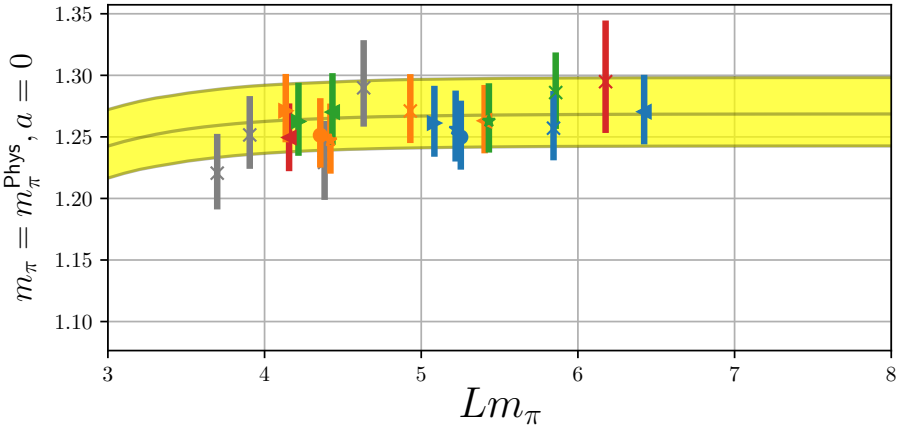
$$g_A(M_\pi, M_K, L, 0) = g_0 + g_1 M_\pi^2 + g_2 M_\pi^2 \ln M_\pi + g_3 M_\pi^2 e^{-LM_\pi}/\sqrt{LM_\pi}$$
$$+ g_4 M_K^2 \ln M_K + g_5 M_K^2 e^{-LM_K}/\sqrt{LM_K}$$

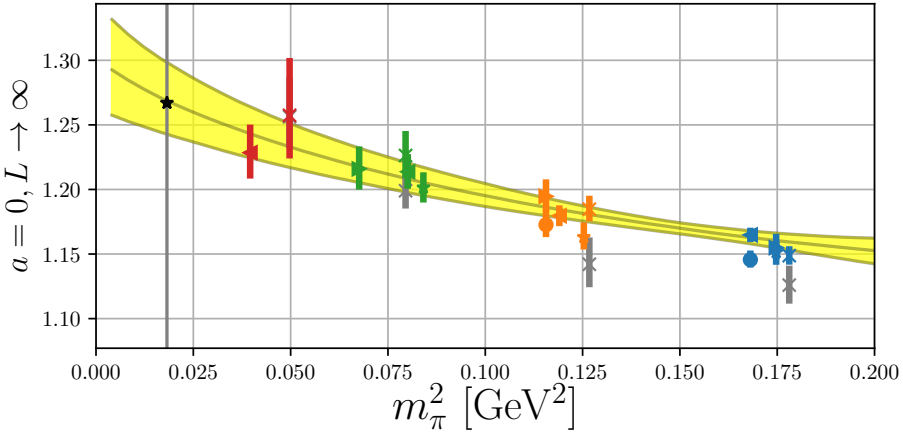
g_A vs M_π^2 (shifted in a and V, M_K depends on M_π): T. Wurm

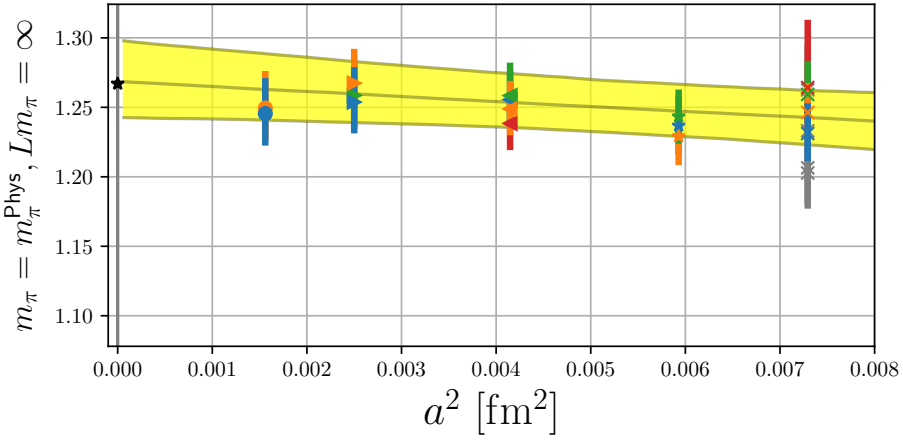


Axial charge: $g_A \equiv G_A(0)$

$g_A = 1.27(3)(??)$ $\quad \chi^2/\mathrm{d.o.f.} = 1.44$





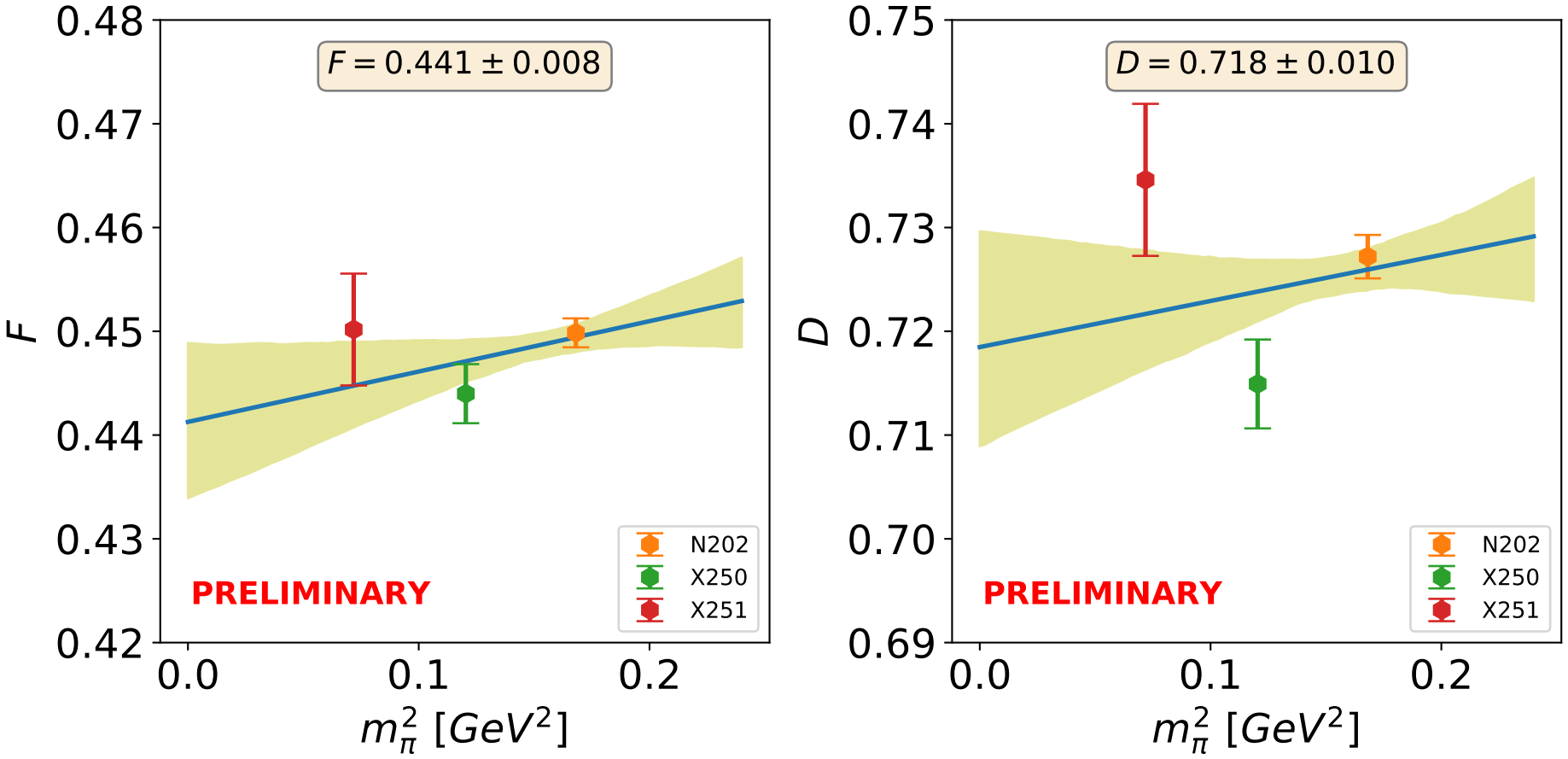


c.f. Mainz [Ottnad,1905.01291]: $g_A = 1.242(25)^{\mathrm{stat}}\left(^{+00}_{-31}\right)^{\mathrm{sys}}$.

SU(3) LECs F and D

In SU(3) flavour limit: $g_A^N = F + D$, $g_A^\Sigma = 2F$ and $g_A^\Xi = F - D$.

Extrapolation along $m_\ell = m_s$ trajectory. **Preliminary** S. Weishäupl



[Savanur,1901.00018] $'F' = 0.438(7)(6)$ and $'D' = 0.708(10)(6)$

Expt. hyperon semi-leptonic decays + SU(3) symmetry [Cabbibo,hep-ph/0307298]
$F = 0.463(8)$ and $D = 0.804(8)$.

Isovector charges: g_S and g_T

BSM contributions to β decay.

$$\mathcal{L}_{\rm CC} = -\frac{G_F^{(0)} V_{ud}}{\sqrt{2}} \; [\, \epsilon_S \; \bar{e}(1-\gamma_5)\nu_\ell \cdot \bar{u}d + \epsilon_T \; \bar{e}\sigma_{\mu\nu}(1-\gamma_5)\nu_\ell \cdot \bar{u}\sigma^{\mu\nu}(1-\gamma_5)d]$$

Studied in planned precision β decay expts. + LHC $pp \to e\nu + X$

Estimate of g_T:

Transverse spin $g_T = \int_{-1}^{1} \mathrm{d}x \, [\delta u(x) - \delta d(x)]$

$\delta q(-x) = -\delta \bar{q}(x)$

Phenomenological estimates from fits to SIDIS data $g_T \approx 1$, $\Delta g_T / g_T \gtrsim 25\%$.

Estimate of g_S:

CVC relation $\partial_\mu(\bar{u}\gamma_\mu d) = -i(m_u - m_d)\bar{u}d$ applied to $\langle p(p_{\rm f})|\bar{u}\gamma_\mu d|n(p_{\rm i})\rangle$

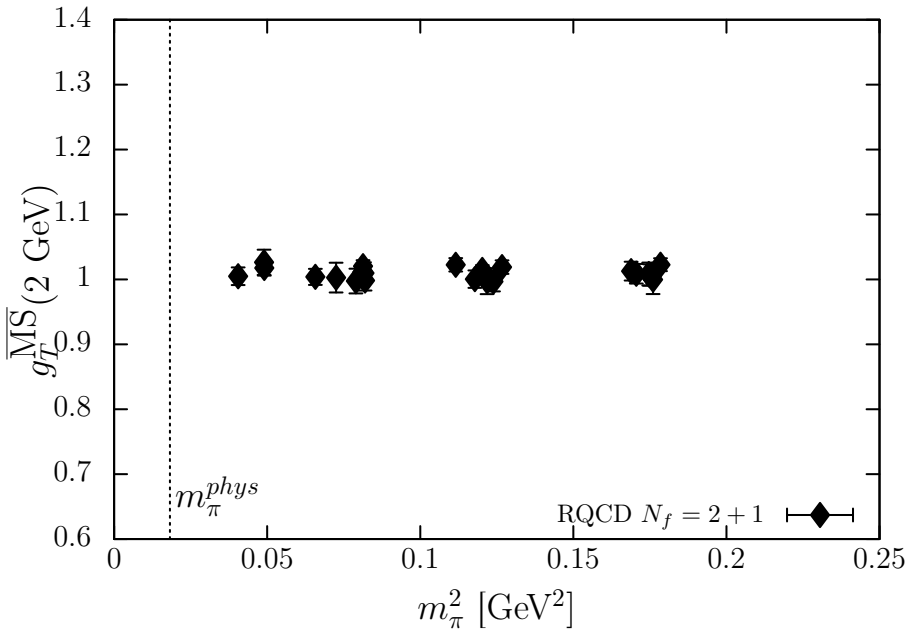
Forward limit: $(m_u - m_d)g_S = (m_p - m_n)^{\rm QCD}$.

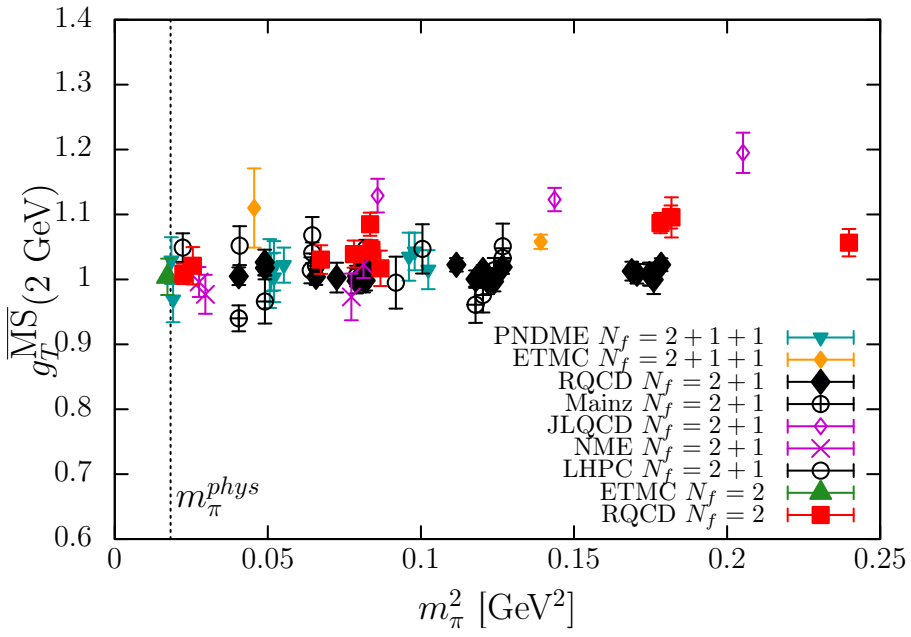
[Gonzalez-Alonso,1309.4434]:

Lattice estimates of $(m_u - m_d)$ and $(m_p - m_n)^{\rm QCD} \to g_S = 1.02(11)$.

g_T

Tensor charge well determined on the lattice.



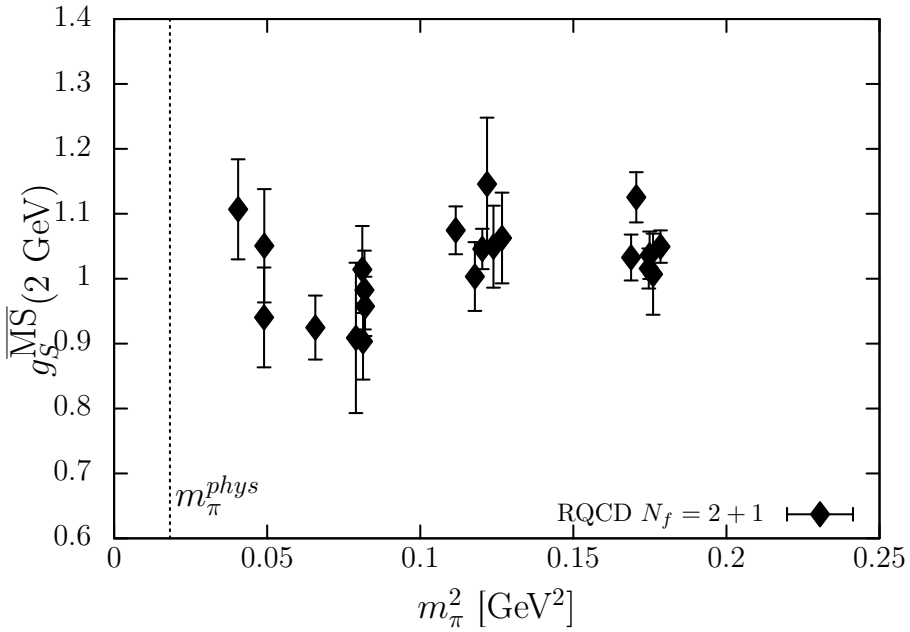


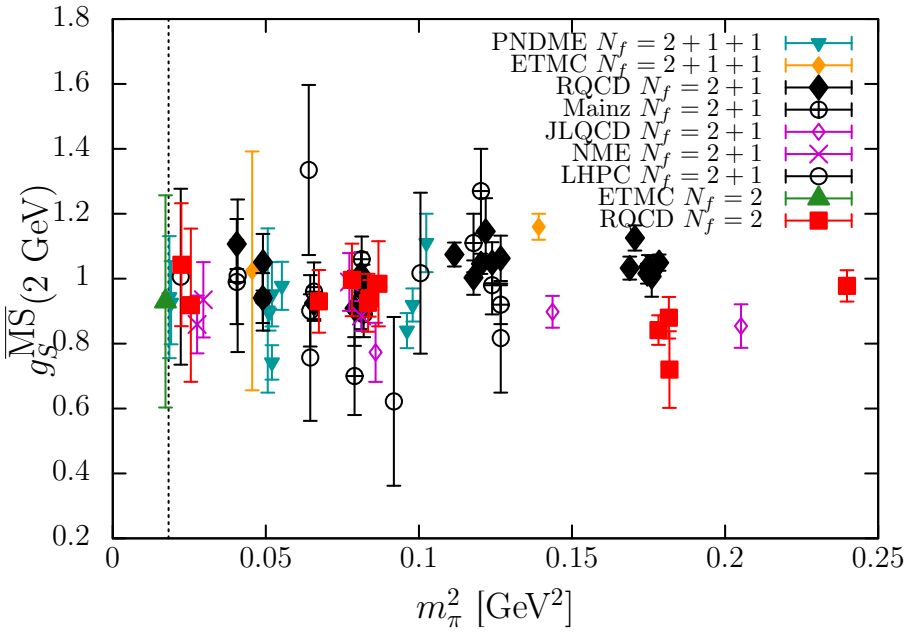
Preliminary $a \to 0$, $M_\pi \to M_\pi^{\rm phys}$, $L \to \infty$ extrapolation:
$g_T^{\overline{\rm MS}}(2\,{\rm GeV}) = 1.01(3)$.

c.f. Mainz [Ottnad,1905.01291]: $g_T^{\overline{\rm MS}}(2\,{\rm GeV}) = 0.965(38)^{\rm stat}(^{+13}_{-41})^{\rm sys}$.

[FLAG,1902.08191] $N_f = 2+1+1$,
$g_T^{\overline{\rm MS}}(2\,{\rm GeV}) = 0.989(32)(10)$ ([PNDME,1806.09006]).

g_S





Preliminary $a \to 0$, $M_\pi \to M_\pi^{\rm phys}$, $L \to \infty$ extrapolation: $g_S^{\overline{\rm MS}}(2\,{\rm GeV}) = 0.97(14)$.

c.f. Mainz [Ottnad,1905.01291]: $g_S^{\overline{\rm MS}}(2\,{\rm GeV}) = 1.13(11)^{\rm stat}(^{+07}_{-06})^{\rm sys}$.

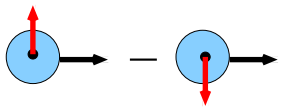
[FLAG,1902.08191] $N_f = 2+1+1$, $g_S^{\overline{\rm MS}}(2\,{\rm GeV}) = 1.02(8)(6)$ ([PNDME,1806.09006]).

Target of $10-15\%$ errors on g_S and g_T needed for constraining ϵ_S and ϵ_T [Bhattacharya,1110.6448] achieved.

Prediction for $(m_n - m_p)^{\rm QCD}$ via CVC relation $(m_u - m_d)g_S = (m_p - m_n)^{\rm QCD}$ [PNDME,1806.09006] find $\mathbf{(m_n - m_p)^{\rm QCD} = 2.63(27)}$ MeV using $(m_d - m_u) = 2.572(66)$ MeV from [MILC/Fermilab/TUMQCD,1802.04248].

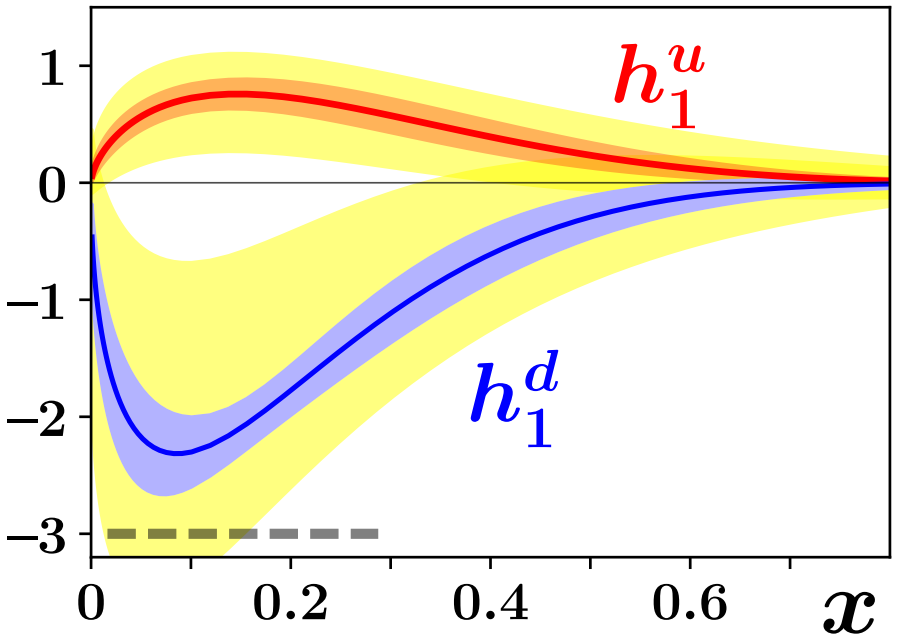
c.f. $(m_n - m_p)^{\rm QCD} = 2.52(29)$ **MeV** from $N_f = 1+1+1+1$ [BMW-c,1406.4088]

Global analysis of transversity with lattice constraints



Transverse spin: $g_T = \langle 1 \rangle_{\delta q - \delta \bar{q}} = \int_0^1 \mathrm{d}x \, [h_1^q(x,\mu^2) - h_1^{\bar{q}}(x,\mu^2)]$

PDFs $h_1^u(x) = \delta u(x)$, $h_1^d(x) = \delta d(x)$, assuming $\delta \bar{u} = \delta \bar{d} = \delta \bar{s} = \delta s$



From JAM: [H-W Lin,1710.09858]]: Fit SIDIS Collins asymmetry data ($A_{UT}^{\sin(\phi_h+\phi_s)}$) alone (yellow), and with $\delta u - \delta \bar{u} - \delta d + \delta \bar{d}$ constrained to $g_T = 1.01(6)$ (red+blue)

(average of [LHPC,1206.4527]], [PNDME,1606.07049]], [RQCD,1412.7336]])

Isovector form factors: $G_A(Q^2)$, $\tilde{G}_P(Q^2)$ and $G_P(Q^2)$

$$\langle N(\mathbf{p}_\mathrm{f})|A_\mu^{u-d}|N(\mathbf{p}_\mathrm{i})\rangle = \bar{u}_N(\mathbf{p}_\mathrm{f})\left[G_A(Q^2)\gamma_\mu - i\frac{\tilde{G}_P(Q^2)}{2m_N}Q_\mu\right]\gamma_5 u_N(\mathbf{p}_\mathrm{i})$$

$$\langle N(\mathbf{p}_\mathrm{f})|P^{u-d}|N(\mathbf{p}_\mathrm{i})\rangle = \bar{u}_N(\mathbf{p}_\mathrm{f})G_P(Q^2)\gamma_5 u_N(\mathbf{p}_\mathrm{i})$$

$q_\mu = p_{f,\mu} - p_{i,\mu}$, $Q^2 = -q_\mu q^\mu \geq 0$.

Forward limit: $G_A(Q^2) \to g_A$, (by extrapolation) $\tilde{G}_P(Q^2) \to \tilde{g}_P$, $G_P(Q^2) \to g_P$.

Shape at low Q^2, $\langle r_X^2\rangle = -6\frac{dG_X(Q^2)}{dQ^2}$: different probe $\to$ different radius.

$$G_X(Q^2) = G_X(0)\left[1 - \frac{1}{6}\langle r_X^2\rangle Q^2 + \ldots\right]$$

Parametrisation $\to$ dipole form, z expansion etc $\to \langle r_E^2\rangle$, $\langle r_A^2\rangle$.

PCAC relation and pion pole dominance

PCAC relation for nucleon matrix elements:

$$\mathbf{2\,m_q}\langle N(\vec{p}_f)|\mathbf{P}|N(\vec{p}_i)\rangle = \langle N(\vec{p}_f)|\partial_\mu \mathbf{A}_\mu|N(\vec{p}_i)\rangle + \mathcal{O}(a^2)$$

leads to

$$m_q G_P(Q^2) = m_N G_A(Q^2) - \frac{Q^2}{4m_N}\tilde{G}_P(Q^2)$$

Forward limit: $m_q G_P(0) = m_q g_p = m_N g_A$

Chiral limit: $\tilde{G}_P(Q^2) = 4m_N^2 G_A(Q^2)/Q^2$

Pion pole dominance (LO ChPT):

$$\tilde{G}_P(Q^2) = G_A(Q^2)\frac{4m_N^2}{Q^2 + M_\pi^2} + \text{corrections}$$

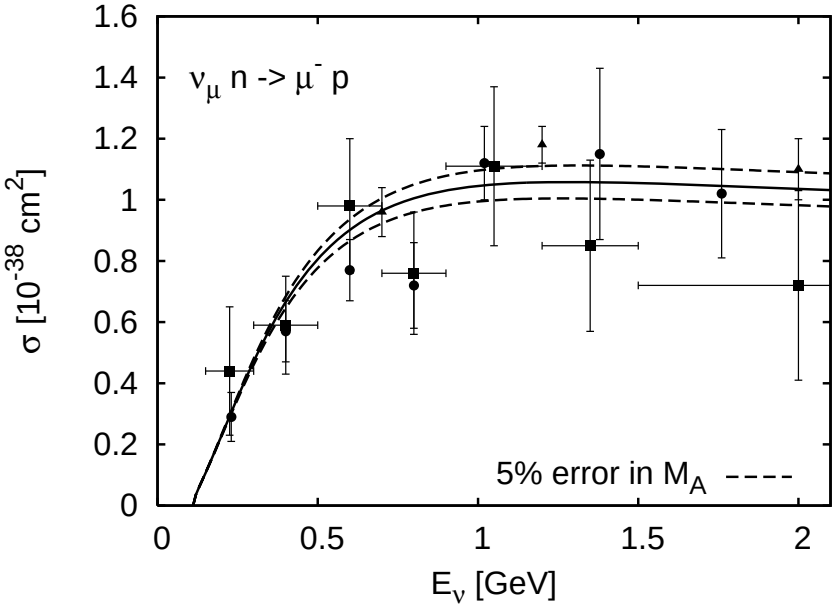
PCAC+pion pole dominance (PPD) $\rightarrow$ one independent form factor but PPD is an approximation.

Nucleon axial form factor $G_A(Q^2)$

Needed for neutrino oscillation experiments: Charged current quasielastic (CCQE) neutrino-nucleus interaction must be known to high precision.

Connecting quark - nucleon level: $G_A(Q^2)$ form factor.
nucleon - nucleus level: nuclear model.

Pre 1990 information on $\mathbf{G_A(Q^2)}$ extracted from $\bar{\nu}$-p and ν-d scattering. [Mosel,1602.00696]



$$G_A(Q^2) = \frac{g_A}{(1 + \frac{Q^2}{\mathbf{M_A}^2})^2}$$

Shown $M_A = 1.03(5)$ **GeV**

MiniBooNE: [Aguilar-Arevalo,1002.2680]

$M_A = 1.35(17)$ **GeV**

Systematics being explored including new analysis of old expt data:

$M_A = 1.01(24)$ **GeV** from z-expansion [Meyer,1603.03048].

Nucleon induced pseudoscalar form factor $\tilde{G}_P(Q^2)$

Not relevant for neutrino oscillations: enters cross-section with $m_{\rm lep}^2/m_N^2$.

Not well known from expt: muon capture $\mu^- p \to \nu_\mu n$ gives

$g_P^* = \frac{m_\mu}{2m_N}\tilde{G}_P(Q^2 = 0.88\, m_\mu^2) = 8.06(55)$ [MuCap,1210.6545]

Compatible with pion pole dominance.

Extracting the form factors

Consider ratio:

$$R_{\Gamma_i,\mathcal{O}} = \frac{C^{\mathcal{O}}_{3pt}(\vec{p}_f, \vec{p}_i, t_f, \tau)}{C_{2pt}(\vec{p}_f, t_f)} = \sqrt{\frac{Z_{\vec{p}_f}}{Z_{\vec{p}_i}}} \frac{E_{\vec{p}_f}}{E_{\vec{p}_f} + m_N} B_{\Gamma_i,\mathcal{O}}(\vec{p}_f, \vec{p}_i)\, e^{-(E_{\vec{p}_f} - E_{\vec{p}_i})\tau} + \ldots$$

Polarisation: $\Gamma_i = i\gamma_i\gamma_5$, overlap factor $Z_{\vec{p}} = |\langle 0|\mathcal{N}|N(\vec{p})\rangle|^2$

$$B_{\Gamma_i,A_\mu}(\vec{p}_f, \vec{p}_i) \propto \mathbf{G_A(Q^2)}\gamma_\mu - i\frac{\tilde{\mathbf{G}}_{\mathbf{P(Q^2)}}}{2m_N} Q_\mu \qquad B_{\Gamma_i,P}(\vec{p}_f, \vec{p}_i) \propto \mathbf{G_P(Q^2)}$$

Set-up: $\vec{p}_f = \vec{0}$, $\vec{q} = -\vec{p}_i$

$$R_{A_i\|\Gamma_i\perp\vec{q}} \propto G_A(Q^2) \qquad R_{A_i\|\Gamma_i\|\vec{q}} \propto (M_N + E_{\vec{q}})G_A(Q^2) - \frac{q_i^2}{2M_N}\tilde{G}_P(Q^2)$$

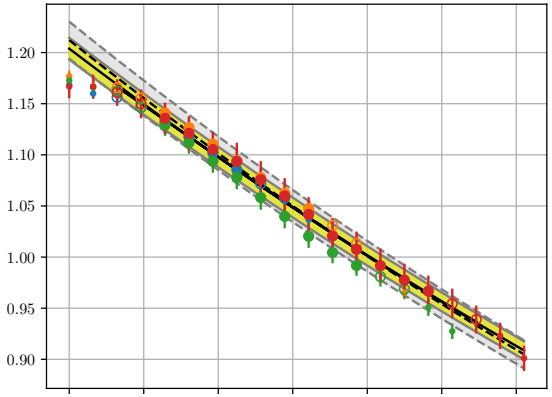
$$R_{A_4,\Gamma_i\|\vec{q}} \propto G_A(Q^2) + \frac{(M_N - E_{\vec{q}})}{2M_N}\tilde{G}_P(Q^2) \qquad R_{P,\Gamma_i\|\vec{q}} \propto G_P(Q^2)$$

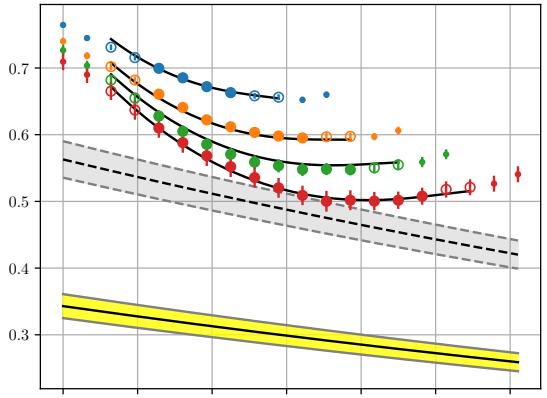
Extracting the form factors

2-state fit (**grey curve**) to R_{Γ_i,A_μ} and $R_{\Gamma_i,P}$ leads to large $\chi^2/\mathrm{d.o.f.}$

$M_\pi \sim 200$ MeV, $a = 0.064$ fm, $|\vec{q}| = 2\pi/(64a)$. T. Wurm

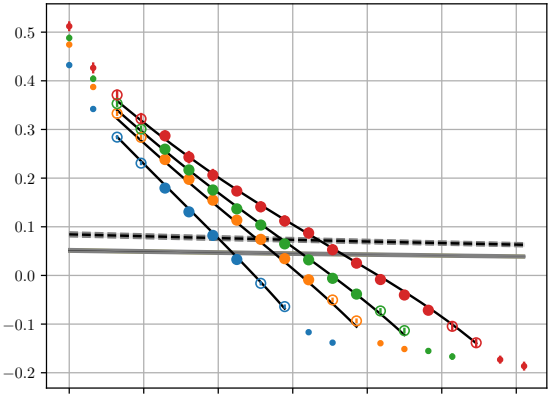
$R_{A_i \| \Gamma_i \perp \vec{q}}$

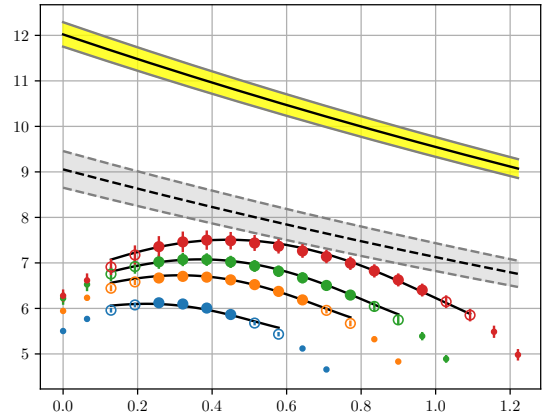




$R_{A_i \| \Gamma_i \| \vec{q}}$

$R_{A_4, \Gamma_i \| \vec{q}}$





$R_{P, \Gamma_i \| \vec{q}}$

$\tau/a - t_f/2a$ $\tau/a - t_f/2a$

Well known problem: e.g. [RQCD,1412.7336], [ETMC,1705.03399], [PNDME,1705.06834], [RQCD,1810.05569], [PACS,1811.07292].

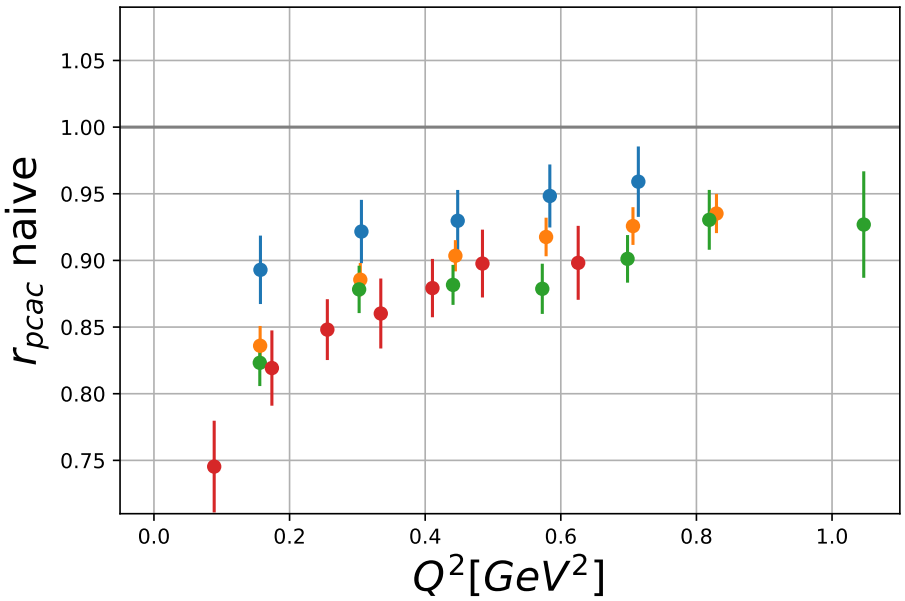
Usually $C_{3pt}^{A_4}$ omitted, χ^2 improves. Large excited state contributions to $\tilde{G}_P$, G_P.

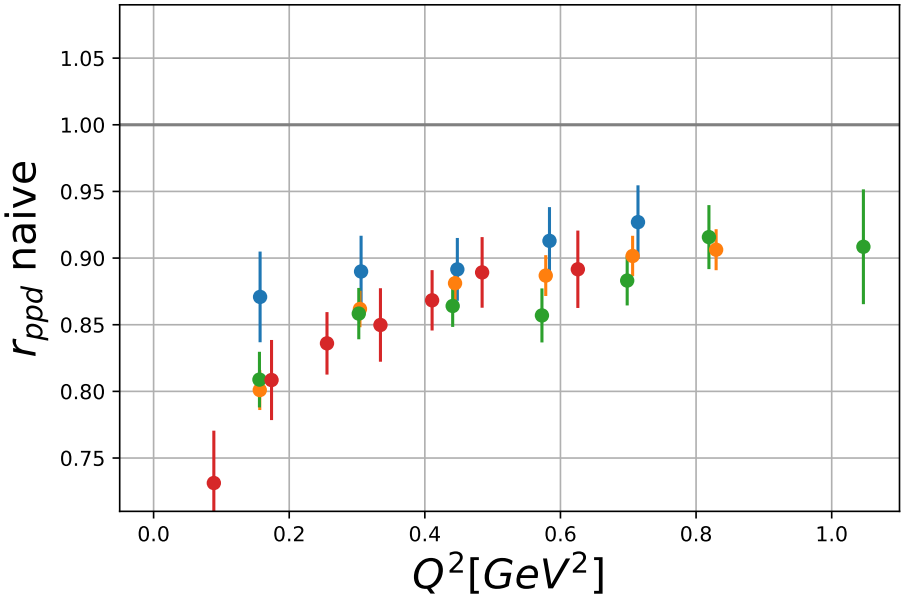
Test PCAC relation and pion pole dominance (PPD) with:

$$r_{PCAC} = \frac{m_q G_P(Q^2) + \frac{Q^2}{4m_N}\tilde{G}_P(Q^2)}{m_N G_A(Q^2)} = 1 + O(a^2) \qquad r_{PPD} = \frac{M_\pi^2 + Q^2}{4m_N^2}\frac{\tilde{G}_P(Q^2)}{G_A(Q^2)} = 1 + \ldots$$

M_π variation, $a = 0.064$ fm

T. Wurm

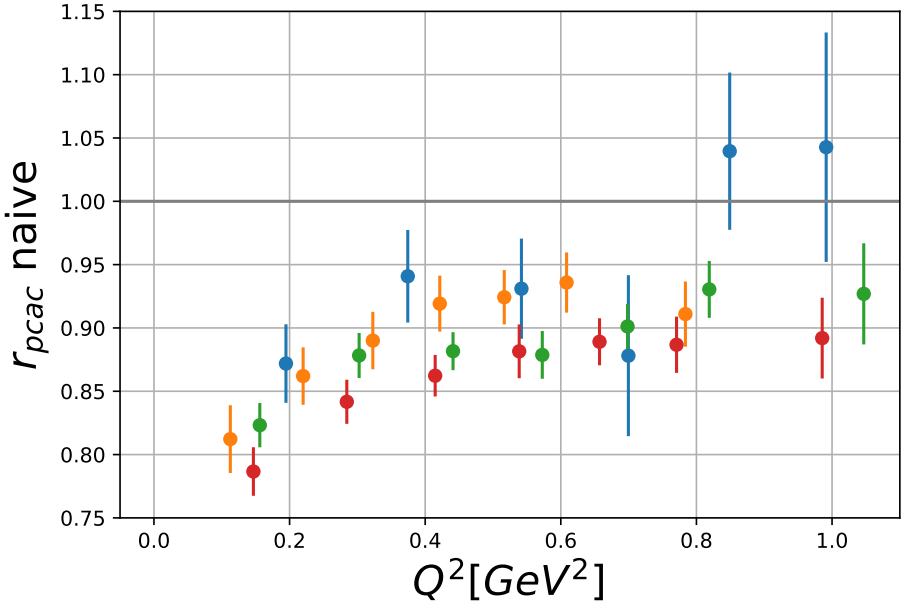


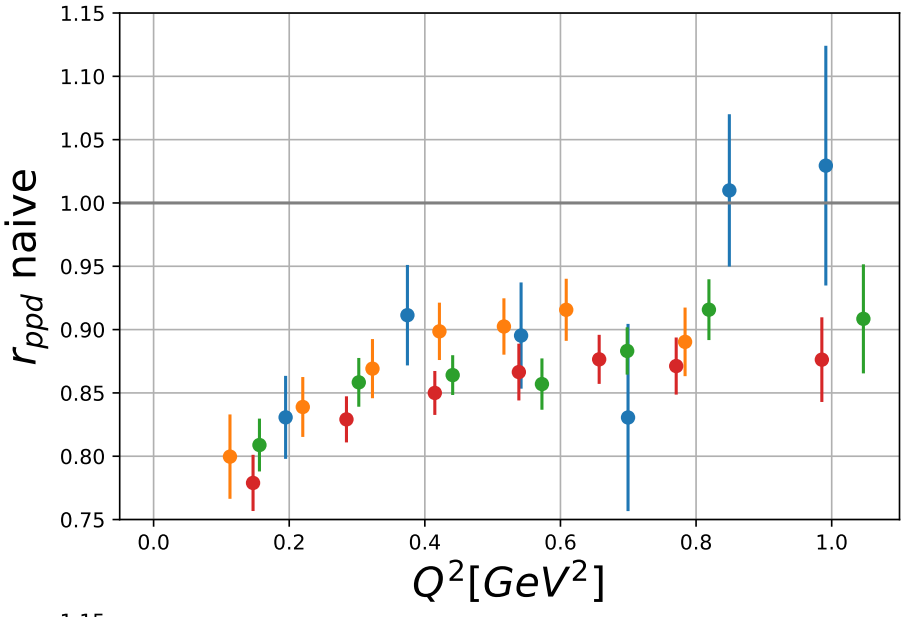


$a = 0.064$ fm, M_π: **Blue** 410 MeV, **Orange** 350 MeV, **Green** 280 MeV, **Red** 200 MeV.

a variation, $M_\pi \sim 280$ MeV

T. Wurm





$M_\pi \sim 280$ MeV, a: **Blue** 0.086 fm, **Orange** 0.076 fm, **Green** 0.064 fm, **Red** 0.05 fm,

[RQCD,1810.05569]: m_l^{PCAC} for $M_\pi \sim 290$ MeV determined from applying the PCAC relation to the pion 2pt function and to the nucleon 3pt functions are consistent in the continuum limit.

$N\pi$ excited state contributions

[Bär,1906.03652,1812.09191]: $N\pi$ contributions to C_{2pt} and C^J_{3pt} for $J = A_\mu$, P computed in leading one-loop order of SU(2) covariant ChPT.

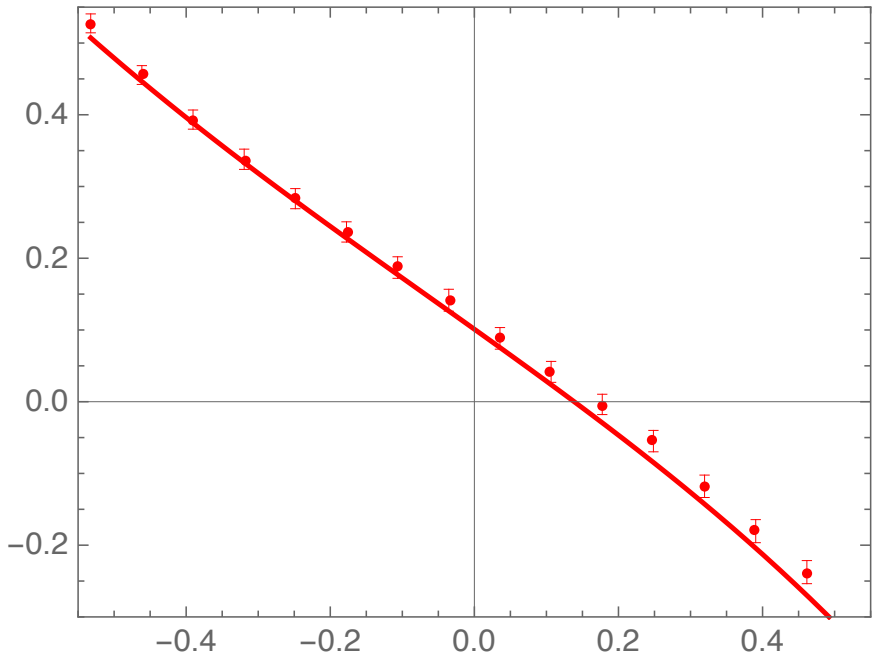
[Bär,1907.03284]

R'_4 vs $\tau/a - t_f/(2a)$

$t_f \to \infty$ $R'_4 \to \text{const.}$.

Data: [RQCD,1810.05569]:

$M_\pi \sim 150$ MeV, $a = 0.07$ fm, $t_f = 1.06$ fm, $\vec{p}_f = \vec{0}$, $|\vec{q}| = 2\pi/(64a)$



$$C^{A_4}_{3pt}(\vec{0}, \vec{p}_i = -\vec{q}, t_f, \tau) = C^{A_4}_{3pt,N}(\vec{q}, t_f, \tau) + C^{A_4}_{3pt,N\pi}(\vec{q}, t_f, \tau) = \mathcal{O}\left(\frac{1}{m_N}\right) + \mathcal{O}(1)$$

In this channel $N(\vec{0})\pi(-\vec{q}) \to N(\vec{0})$ enhanced, relative to $N(-\vec{q}) \to N(\vec{0})$.

Suggestion: correct lattice data with factor e.g. $G_P(Q^2, t_f, \tau) = G_P(Q^2)\left[1 + \epsilon_P(Q^2, t_f, \tau)\right]$

ChPT inspired fit

[Bär,1906.03652,1812.09191]: loop diagrams ($\Rightarrow$ a whole tower of $N\pi$ states) contribute to G_A while tree-level diagrams contribute to $\tilde{G}_P$ ($\Rightarrow N(\vec{0})\pi(-\vec{q})$).

Excited state contributions to $G_A(Q^2) \sim +5\%$ for $M_\pi = 140$ MeV, $M_\pi L = 3-6$, $t_f = 2$ fm, independent of $Q^2 \leq 0.25$ GeV^2. For $\tilde{G}_P(Q^2) = -10\%$ to -40% as Q^2 is decreased.

PCAC relation gives contributions to G_P.

Drawbacks:

$\star$ At LO wavefunction normalisation $Z_{N\pi} = Z_N$. Not expected with smearing unless $\langle r^2 \rangle^{1/2} \ll 1/m_\pi$.

$\star$ Applies to the plateau method.

Alternative:

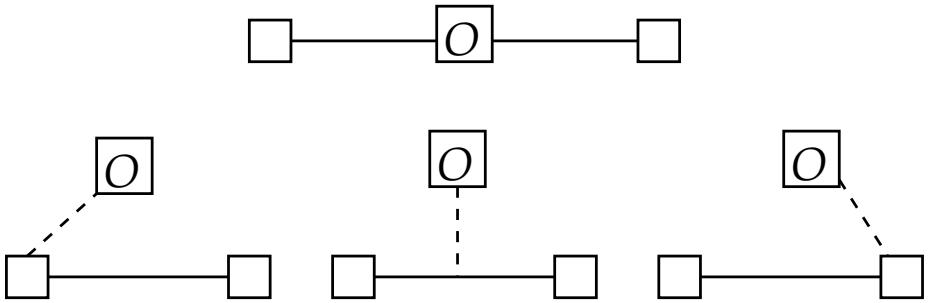
$\star$ Use tree-level ChPT to determine form of $N\pi$ contributions.

$\star$ Allow for dependence on the smearing of the interpolators.

$\star$ Avoid any constraints on the ground state.

ChPT inspired fit

Tree-level diagrams:



Top diagram:

$$\sim G_A \qquad \text{for } \mathcal{O} = A_\mu$$
$$=0 \qquad \text{for } \mathcal{O} = P$$

Bottom middle diagram

$$\sim \tilde{G}_P + \text{excited states} \qquad \text{for } \mathcal{O} = A_\mu$$
$$\sim G_P + \text{excited states} \qquad \text{for } \mathcal{O} = P$$

Other diagrams: only contribute to the excited states.

ChPT inspired fit

M. Gruber, P. Wein, T. Wurm

$$C_{3pt}^{\Gamma_\alpha, A_\mu}(\vec{p}_f, \vec{p}_i, t_f, \tau) = \frac{\sqrt{Z_{\vec{p}_f} Z_{\vec{p}_i}}}{2E_{\vec{p}_f} E_{\vec{p}_i}} e^{-E_{\vec{p}_f}(t_f-\tau)} e^{-E_{\vec{p}_i}\tau} \Big[B_{\Gamma_i, A_\mu}(\vec{p}_f, \vec{p}_i) \times$$

$$\left(1 + B_{10,A_\mu} e^{-\Delta E_{\vec{p}_f}(t-\tau)} + B_{01,A_\mu} e^{-\Delta E_{\vec{p}_i}\tau} + B_{11,A_\mu} e^{\Delta E_{\vec{p}_f}(t_f-\tau)} e^{-\Delta E_{\vec{p}_i}\tau}\right)$$

$$+ \frac{E_{\vec{p}_f}}{E_\pi} r_+^\mu c^{\vec{p}_f} q_\alpha e^{-\Delta E_{\vec{p}_f}^{N\pi}(t-\tau)} + \frac{E_{\vec{p}_i}}{E_\pi} r_-^\mu c^{\vec{p}_i} q_\alpha e^{-\Delta E_{\vec{p}_i}^{N\pi}(\tau)} \Big]$$

$$C_{3pt}^{\Gamma_\alpha, P}(\vec{p}_f, \vec{p}_i, t_f, \tau) = \frac{\sqrt{Z_{\vec{p}_f} Z_{\vec{p}_i}}}{2E_{\vec{p}_f} E_{\vec{p}_i}} e^{-E_{\vec{p}_f}(t_f-\tau)} e^{-E_{\vec{p}_i}\tau} \Big[B_{\Gamma_i, P}(\vec{p}_f, \vec{p}_i) \times$$

$$\left(1 + B_{10,P} e^{-\Delta E_{\vec{p}_f}(t-\tau)} + B_{01,P} e^{-\Delta E_{\vec{p}_i}\tau} + B_{11,P} e^{\Delta E_{\vec{p}_f}(t_f-\tau)} e^{-\Delta E_{\vec{p}_i}\tau}\right)$$

$$+ \frac{E_{\vec{p}_f}}{E_\pi} B_0 c^{\vec{p}_f} q_\alpha e^{-\Delta E_{\vec{p}_f}^{N\pi}(t-\tau)} + \frac{E_{\vec{p}_i}}{E_\pi} B_0 c^{\vec{p}_i} q_\alpha e^{-\Delta E_{\vec{p}_i}^{N\pi}(\tau)} \Big]$$

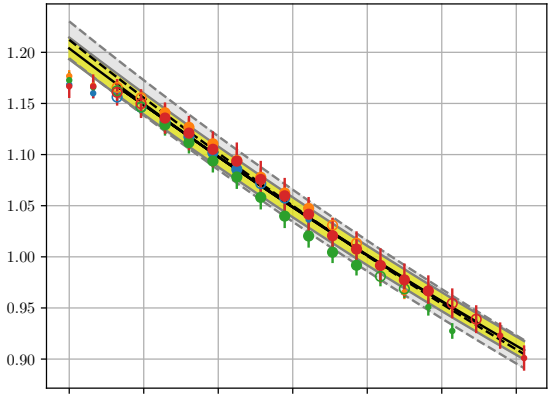
$\Delta E_{\vec{p}_f}^{N\pi} = E_\pi + E_{\vec{p}_f - \vec{q}} - E_{\vec{p}_f}$, $\Delta E_{\vec{p}_i}^{N\pi} = E_\pi + E_{\vec{p}_i + \vec{q}} - E_{\vec{p}_i}$, $\vec{p}_f = \vec{0}$
$r_\pm = (E_\pi, \pm\vec{q}$, $E_{\vec{p}} = \sqrt{m_N^2 + |\vec{p}|^2}$, $E_\pi = \sqrt{M_\pi^2 + |\vec{q}|^2}$,
$B_0 = -\langle \bar{u}u \rangle / F_0^2 \approx M_\pi/(2m_q)$.
$c^{\vec{p}_i}$ and $c^{\vec{p}_f}$ are the same for axial and pseudoscalar 3pt functions.

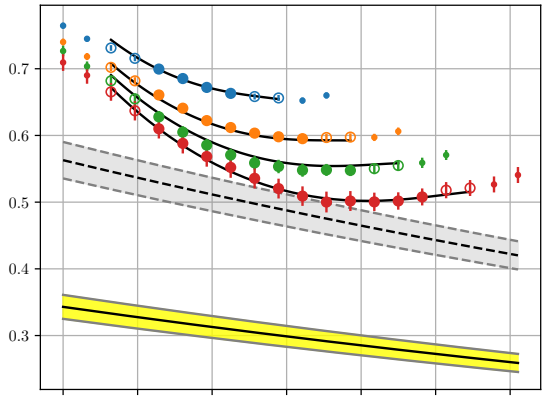
Extracting the form factors

Performing new fit (**yellow curve**) to R_{Γ_i,A_μ} and $R_{\Gamma_i,P}$ leads to reasonable $\chi^2/\mathrm{d.o.f.}$

$M_\pi \sim 200$ MeV, $a = 0.064$ fm, $|\vec{q}| = 2\pi/(64a)$. T. Wurm

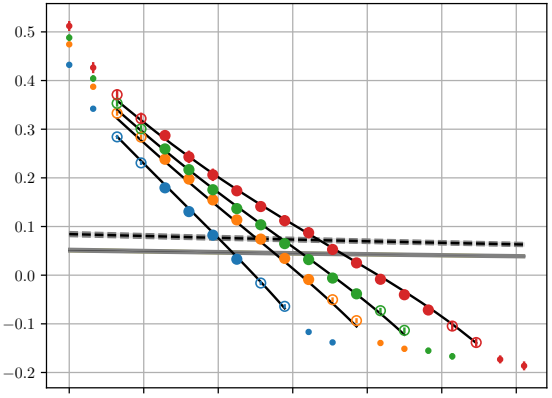
$R_{A_i \| \Gamma_i \perp \vec{q}}$

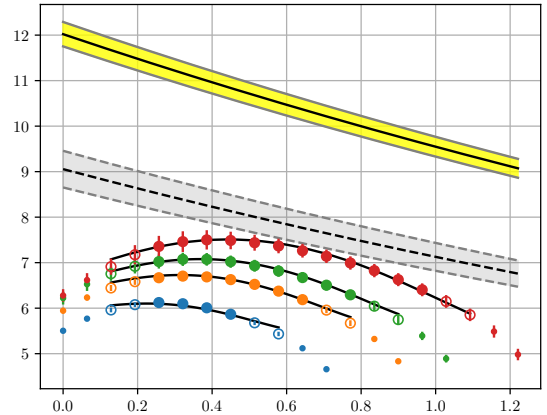




$R_{A_i \| \Gamma_i \| \vec{q}}$

$R_{A_4,\Gamma_i \| \vec{q}}$





$R_{P,\Gamma_i \| \vec{q}}$

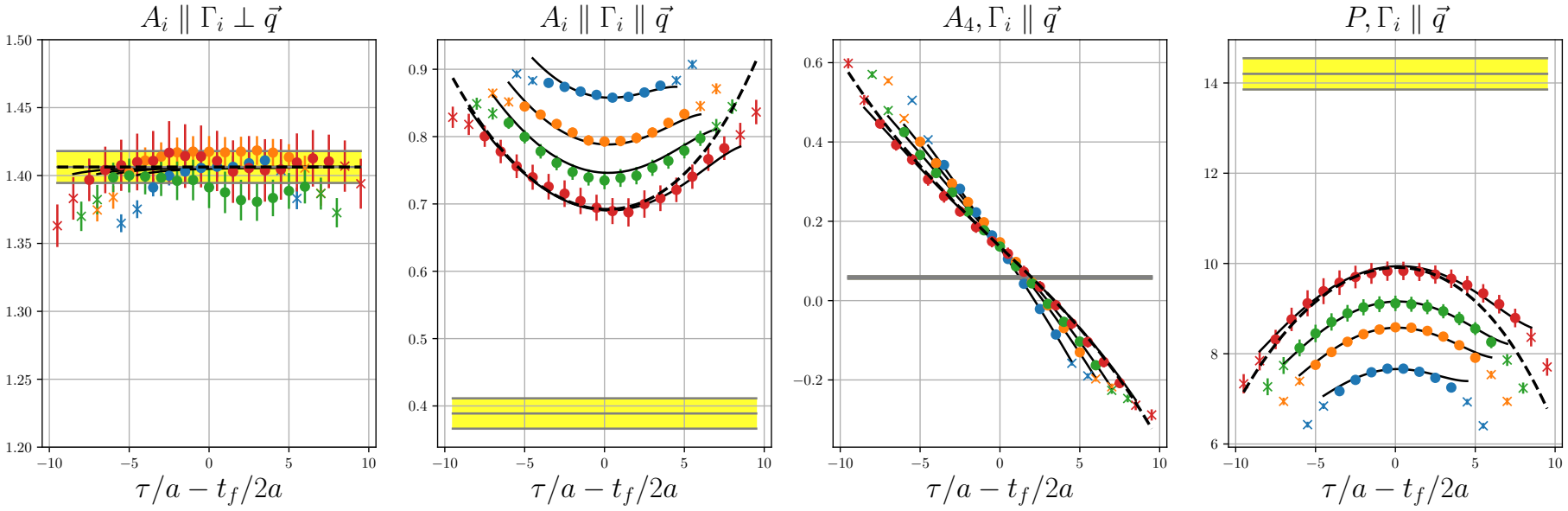
$\tau/a - t_f/2a$

$\tau/a - t_f/2a$

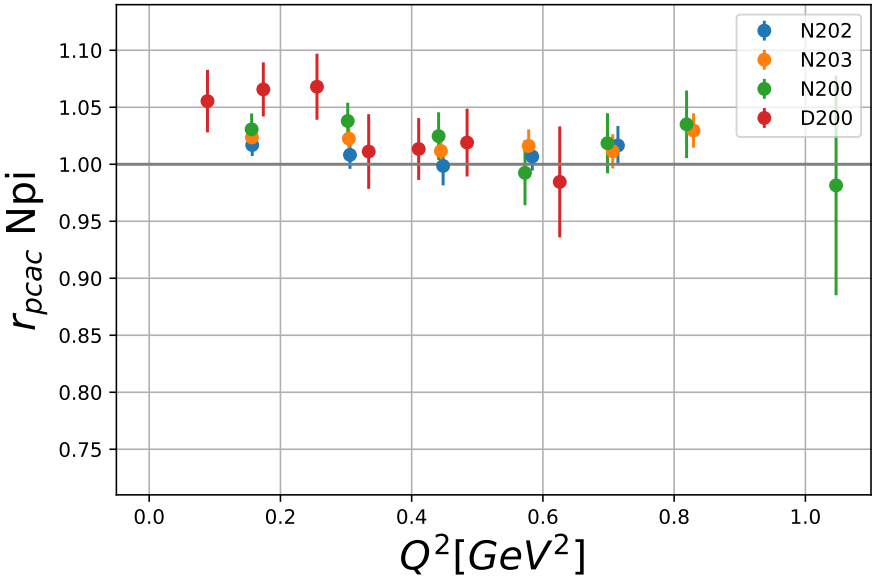
Extracting the form factors

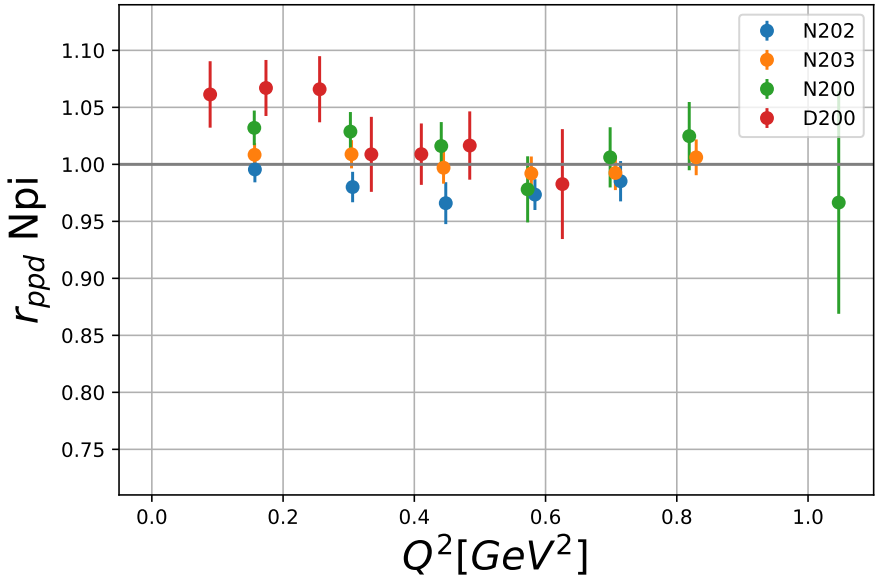
Modified R_{Γ_i, A_μ} and $R_{\Gamma_i, P}$, with $e^{-(E_{\vec{p}_i} - E_{\vec{p}_f})\tau}$ and other factors removed.

$M_\pi \sim 200$ MeV, $a = 0.064$ fm, $|\vec{q}| = 2\pi/(64a)$. T. Wurm

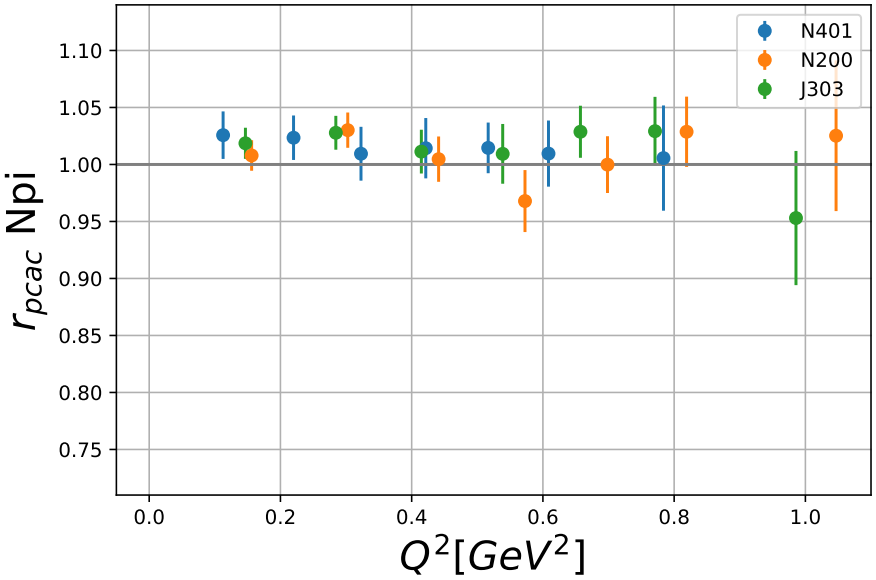


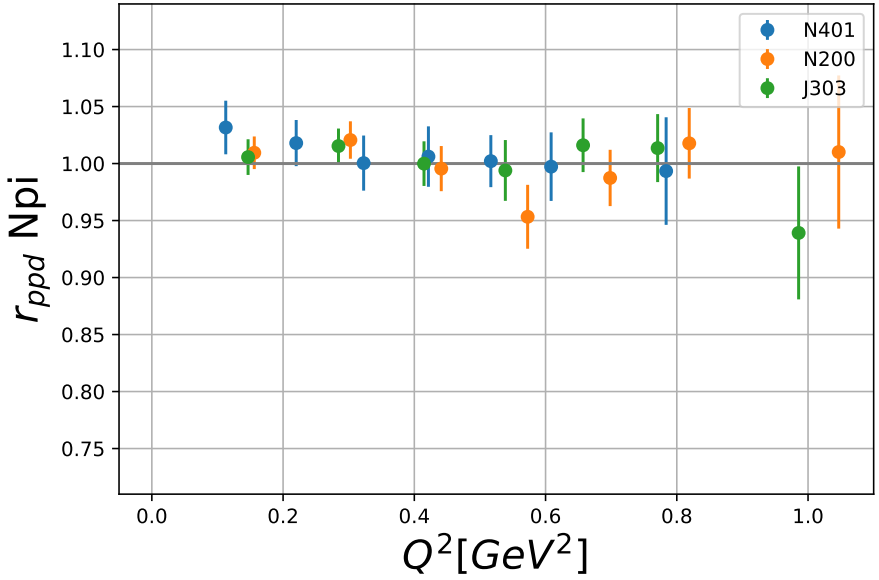
Testing the PCAC relation and pion pole dominance





$a = 0.064$ fm, M_π: **Blue** 410 MeV, **Orange** 350 MeV, **Green** 280 MeV, **Red** 200 MeV.





$M_\pi \sim 280$ MeV, a: **Blue** 0.076 fm, **Orange** 0.064 fm, **Green** 0.05 fm.

Extrapolation in a,m_q and V

Extrapolation in a, m_q and V as a function of Q^2.

Choose Q^2 parameterisation: in progress: $z-$expansion.

Dipole forms

$$G_A(Q^2) = \frac{g_A}{(1+Q^2/M_A^2)^2} \qquad \tilde{G}_P(Q^2) = \frac{1}{Q^2+M_\pi^2}\frac{\tilde{g}_p'}{(1+Q^2/M_{\tilde{P}^2})^2}$$

$$m_q G_P(Q^2) = \frac{1}{Q^2+M_\pi^2}\frac{g_p'}{(1+Q^2/M_{P^2})^2}$$

Using $m_q G_P$ means all form factors are renormalised with Z_A.

Consistent with the perturbative expectation at very large Q^2:

$$G_A(Q^2) \propto 1/Q^4 \qquad \tilde{G}_P(Q^2) \propto 1/Q^6 \qquad G_P(Q^2) \propto 1/Q^6$$

Fit form for $X \in \{M_A, g_A, M_P, g_P', M_{\tilde{P}}, \tilde{g}_P'\}$:

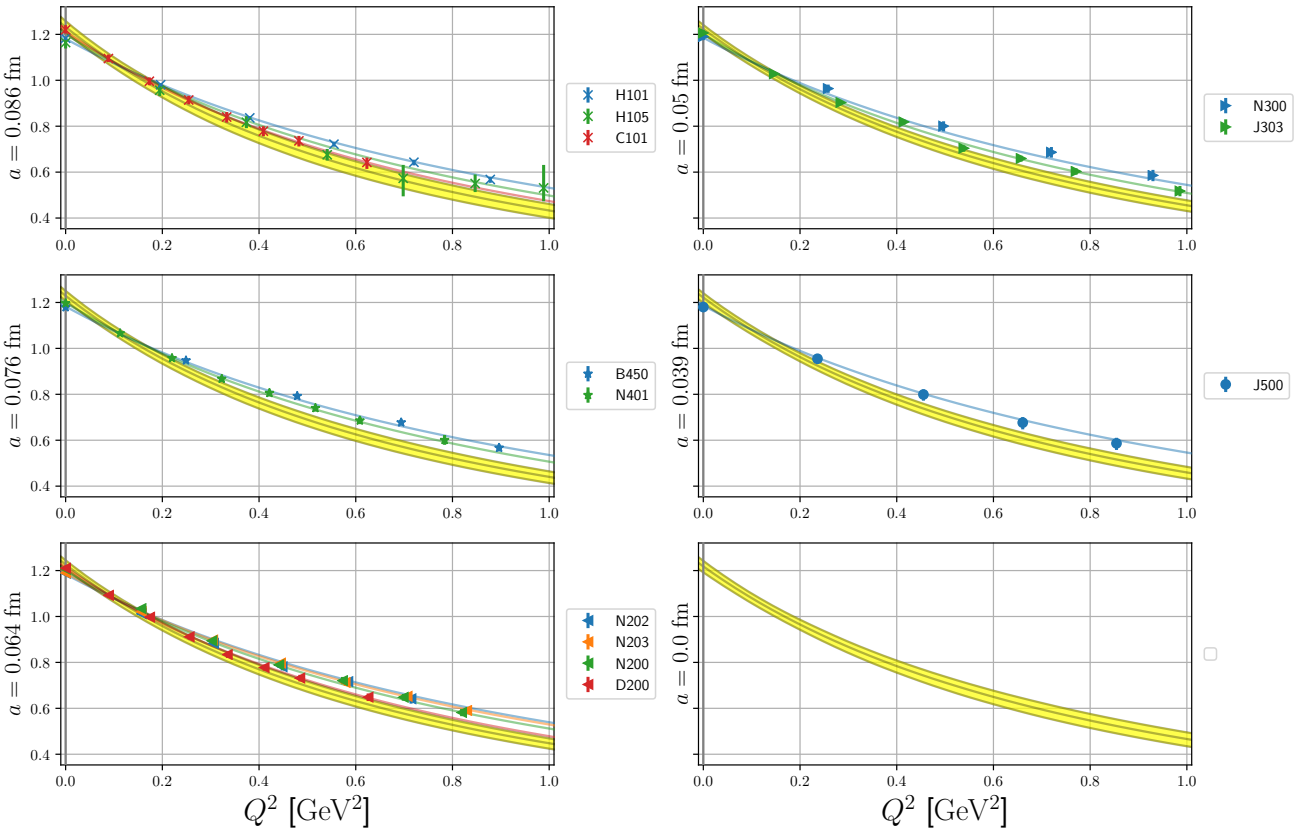
$$X(M_\pi, M_K, L, a) = \Big[x_0 + x_1 M_\pi^2 + x_2 M_\pi^2 \ln M_\pi + x_3 M_\pi^2 e^{-LM_\pi}/\sqrt{LM_\pi}$$

$$+x_4 M_K^2 + x_5 M_K^2 \ln M_K + x_6 M_K^2 e^{-LM_K}/\sqrt{LM_K}\Big] \times \left[1 + x_7 f a^2 + x_8 a^2 M_\pi^2 + x_9 a^2 M_K^2\right]$$

Extrapolation of $G_A(Q^2)$

- ⋆ Fit performed on a subset of the ensembles used for the charges.
- ⋆ $\chi^2/\mathrm{d.o.f.} = 1.31$, $G_A(0) = g_A \sim 1.2$, $\langle r_A^2 \rangle \sim 0.3$ fm^2, i.e. $\langle r_A^2 \rangle^{1/2} \sim 0.55$ fm.

T. Wurm

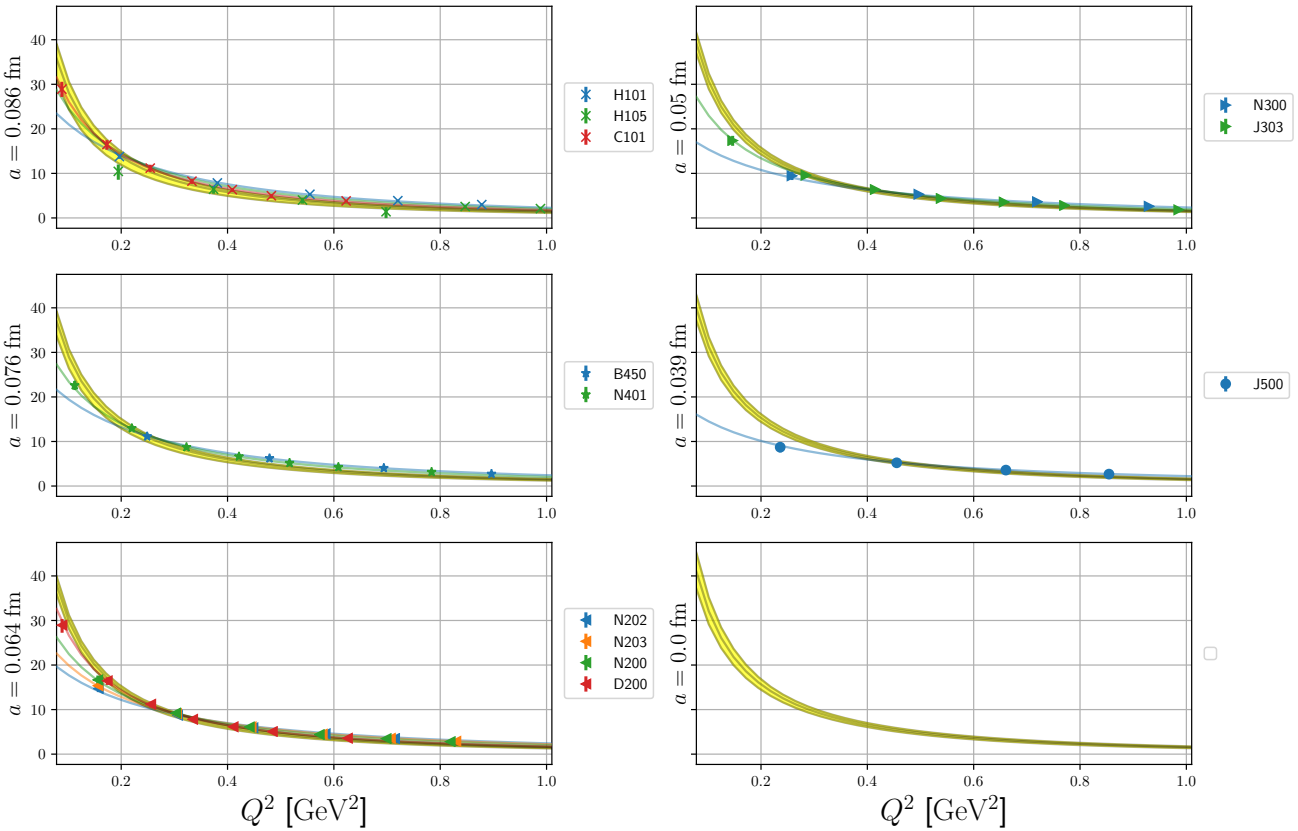


[ETMC,1705.03399], $\langle r_A^2 \rangle^{1/2} \sim 0.516(33)(14)$ fm, [PNDME,1705.06834], $\langle r_A^2 \rangle^{1/2} \sim 0.481(58)(62)$ fm, [PACS,1811.07292], $\langle r_A^2 \rangle^{1/2} \sim 0.647(22)(38)$ fm, [Mainz,1705.06186] $\langle r_A^2 \rangle^{1/2} \sim 0.600(60)(^{+133}_{-147})$ fm.

Extrapolation of $\tilde{G}_P(Q^2)$

$\star$ $\chi^2/\mathrm{d.o.f.} = 1.37$, $\tilde{G}_P(0) \sim 241$, $\langle r^2 \rangle \sim 12$ fm^2,
$g_P^* = \frac{m_\mu}{2m_N} \tilde{G}_P(Q^2 = 0.88\, m_\mu^2) \sim 8 - 9$

T. Wurm



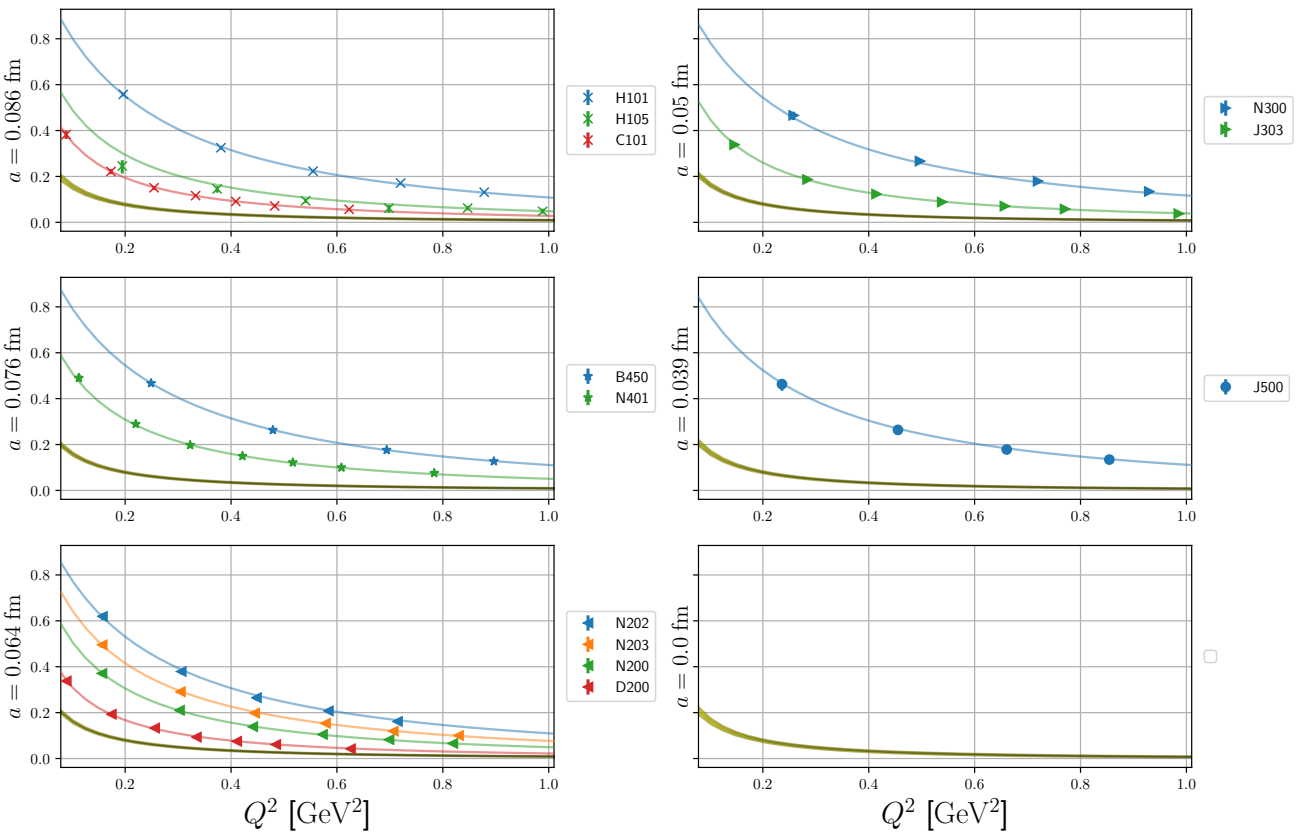
Previously: $g_P^* = 2.8(7)$ [RQCD,1810.05569],
$g_P^* = 7.7(1.8)^{+0.8}_{-2.0}$ [Mainz,1705.06186], $g_P^* = 4.44(18)$ [PNDME,1705.06834],
$\tilde{G}_P(0) = 165.62(9.82)(18.46)$ [ETMC,1705.03399]

Extrapolation of $m_q G_P(Q^2)$

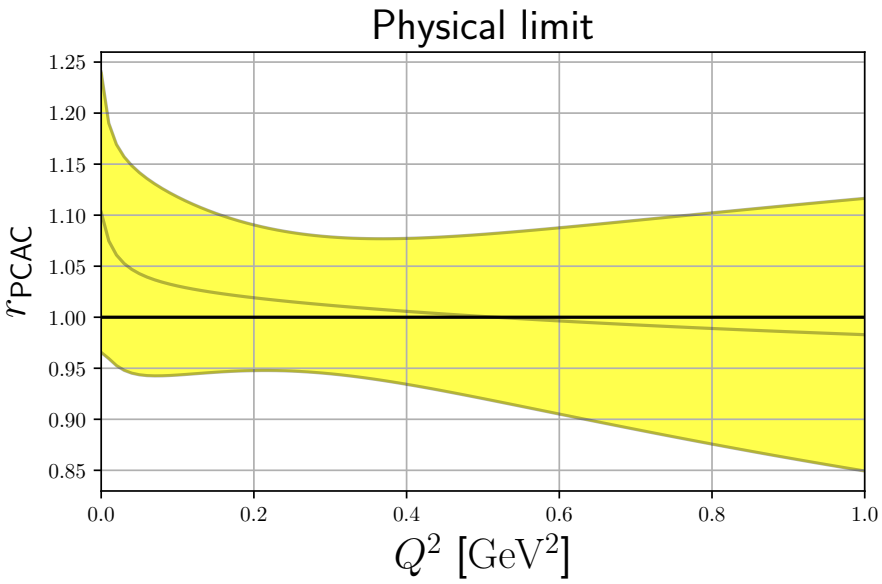
⋆ $\chi^2/\mathrm{d.o.f.} = 1.07$, $m_q G_P(0) \sim 1.26$, $\langle r^2 \rangle \sim 12$ fm^2.

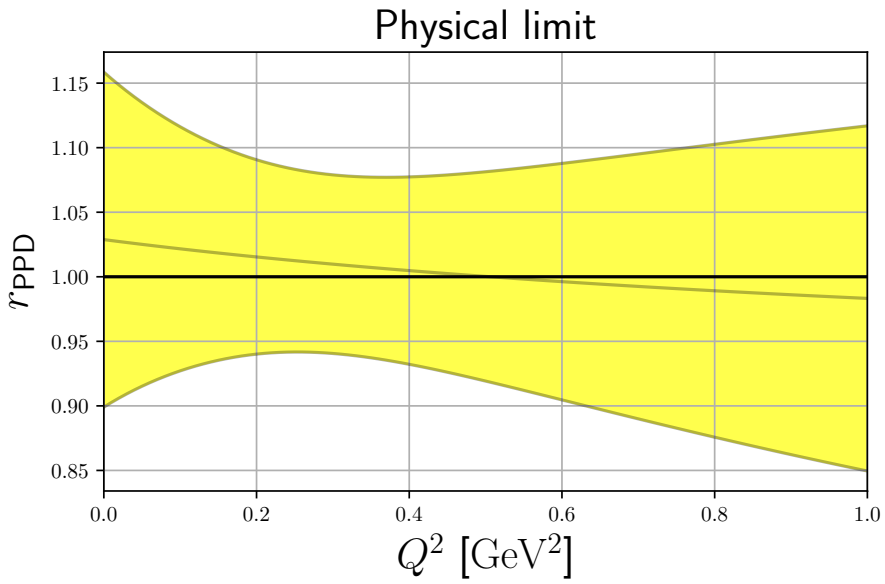
PCAC relation and expt.: $m_q G_P = m_N g_A = 1.19$.

T. Wurm



r_{PCAC} and r_{PPD} after extrapolation





Summary

⋆ Aim for determination of nucleon structure observables with all main systematics under control.

⋆ This is possible using the CLS ensembles with 5 lattice spacings, $M_\pi \sim 410 - 135$ MeV and $M_\pi L \gtrsim 4$.

⋆ Three trajectories allow the $m_l - m_s$ plane to be explored.

⋆ Excited state contributions to 2pt and 3pt functions in the forward limit are reasonably under control.

⋆ Reproduce octet baryon spectrum and results for the SU(3) LECs, $\sigma_{\pi N}$, charges and moments of PDFs: $g_{A,S,T}$, $\langle x \rangle_{u-d}$, $\langle x \rangle_{\Delta u - \Delta d}$, $\langle x \rangle_{\delta u - \delta d}$ to follow soon.

⋆ Form factors: well known problem of large excited state contributions to $\tilde{G}_P(Q^2)$, $G_P(Q^2)$. Relatively small contributions to $G_A(Q^2)$.

⋆ Utilising a ChPT inspired fit to the axial and pseudoscalar 3pt function (with no constraints made on the ground state), then the PCAC relation is respected. Approximate pion pole dominance is also found.