Towards a holographic description of cosmology on the lattice

Andreas Jüttner
UNIVERSITY OF Southampton

Advances in Lattice Gauge Theory 2019
CERN, 22 July 2019 — 9 August 2019

Image: ESA and Planck Collaboration

Collaboration

Edinburgh:
Luigi Del Debbio
Guido Cossu
Joseph Lee
Valentin Nourry
Antonin Portelli

Southampton:
Masanori Hanada
Andreas Jüttner
Ben Kitching-Morley
Kostas Skenderis

LLNL:
Pavlos Vranas

Waterloo:
Elizabeth Gould

Image: NASA

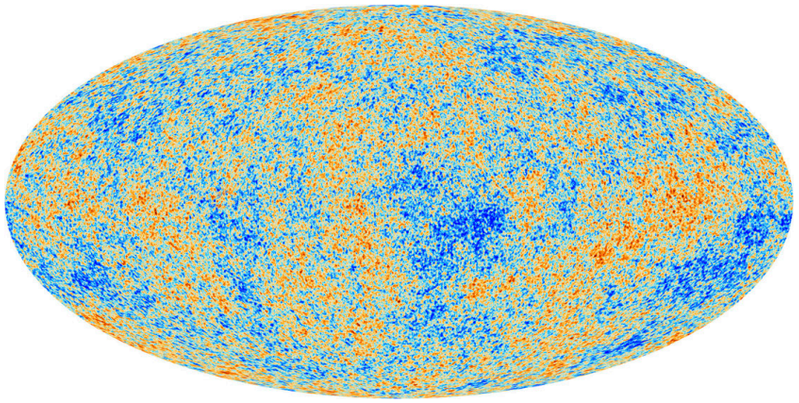
Cosmology

Problems in cosmology solved by inflation:

- Horizon problem
- Formation of structure
- Flatness problem

Famous testable prediction in many models of inflation:
power-law primordial power spectrum

$$\mathcal{P}(q) = \Delta_0^2 \left(\frac{q}{q_*} \right)^{n_s - 1}$$



ESA and the Planck Collaboration

Cosmology

The particle physics interpretation of inflation seems ad hoc:

- Fine tuning of inflation potential and inflaton
- Eternal inflation/initial conditions
- Effective theory — what about UV completion towards the very very early universe (initial singularity)?

Cosmology

The particle physics interpretation of inflation seems ad hoc:

- Fine tuning of inflation potential and inflaton
- Eternal inflation/initial conditions
- Effective theory — what about UV completion towards the very very early universe (initial singularity)?

The prospect of precision cosmological observations in the future motivates looking for more complete models or even first principles descriptions

Cosmology

Idea: **Quantum gravity description of early universe in terms of Holographic dual QFT**

Cosmological observables are mapped to correlation functions of the dual QFT

Maldacena JHEP 0305 (2003) 013
McFadden and Skenderis, PRD 81, 021301 (2010)

Cosmology

Idea: **Quantum gravity description of early universe in terms of Holographic dual QFT**

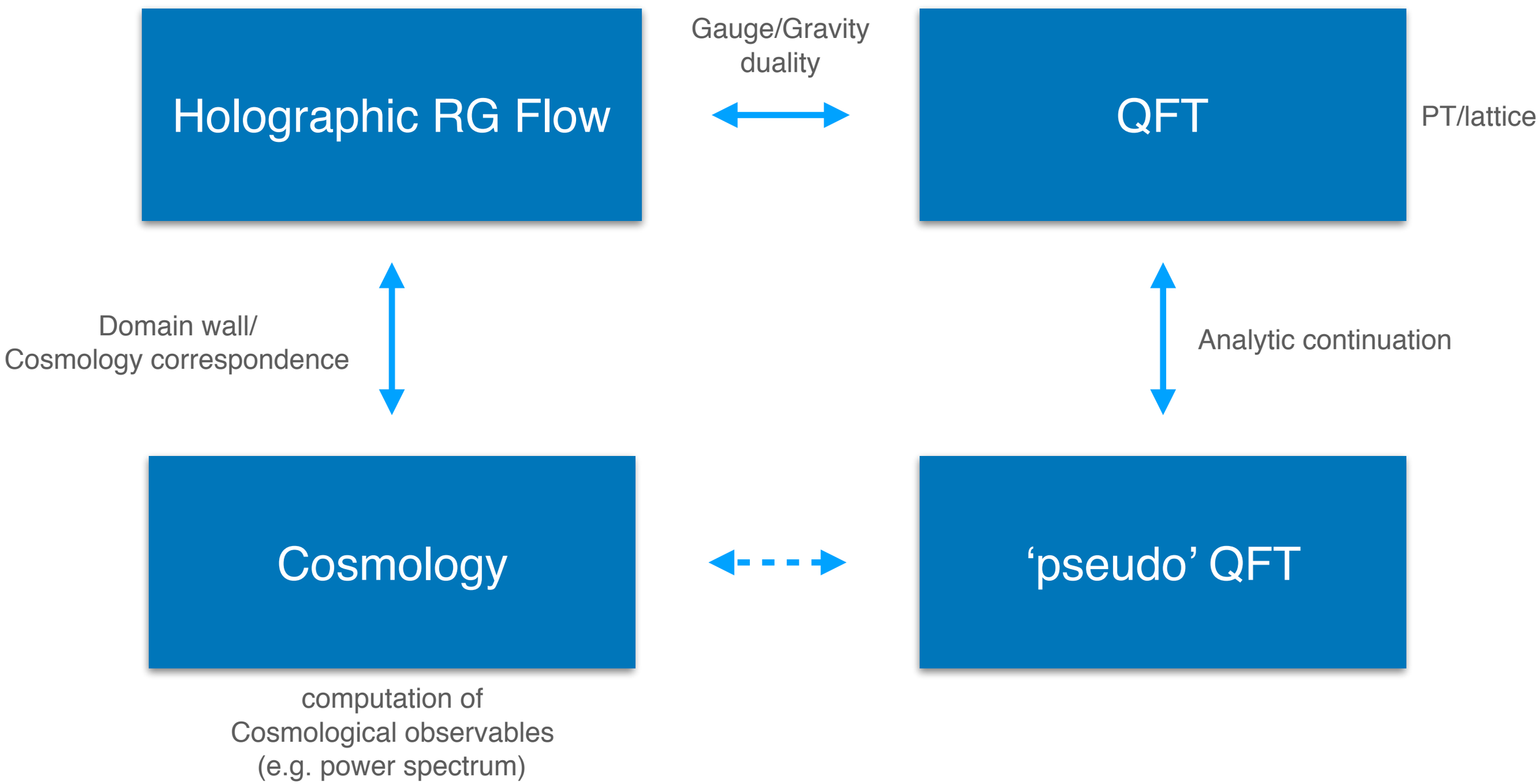
Cosmological observables are mapped to correlation functions of the dual QFT

Maldacena JHEP 0305 (2003) 013
McFadden and Skenderis, PRD 81, 021301 (2010)

Dual theory is 3d QFT with large-N scaling with massless scalars, fermions and gauge fields

Holographic Cosmology

McFadden and Skenderis, PRD 81, 021301 (2010)



Cosmology

Prediction for power spectrum from general ansatz for QFT:

$$S = \frac{1}{g_{\mathrm{YM}}^2} \int d^3x \mathrm{Tr} \left[\frac{1}{2} F_{ij} F^{ij} + (D\phi)^2 + \bar{\psi} D_i \gamma^i \psi + \mu \left(\bar{\psi} \psi \phi \right) + \lambda \phi^4 \right]$$

Cosmology

Prediction for power spectrum from general ansatz for QFT:

$$S = \frac{1}{g_{\mathrm{YM}}^2} \int d^3x \mathrm{Tr} \left[\frac{1}{2} F_{ij} F^{ij} + (D\phi)^2 + \bar{\psi} D_i \gamma^i \psi + \mu \left(\bar{\psi} \psi \phi \right) + \lambda \phi^4 \right]$$

CMB primordial power spectrum

$$\Delta_{\mathcal{R}}^2(q) = -\frac{q^3}{4\pi^2} \frac{1}{\langle T_{\mu\mu}(q) T_{\mu\mu}(-q) \rangle}$$

With $T_{\mu\mu}$ Energy-momentum tensor

Cosmology

Prediction for power spectrum from general ansatz for QFT:

$$S = \frac{1}{g_{\rm YM}^2} \int d^3x \mathrm{Tr} \left[\frac{1}{2} F_{ij} F^{ij} + (D\phi)^2 + \bar{\psi} D_i \gamma^i \psi + \mu \left(\bar{\psi} \psi \phi \right) + \lambda \phi^4 \right]$$

CMB primordial power spectrum

$$\Delta^2_{\mathcal{R}}(q) = -\frac{q^3}{4\pi^2} \frac{1}{\langle T_{\mu\mu}(q) T_{\mu\mu}(-q) \rangle}$$

With $T_{\mu\mu}$ Energy-momentum tensor

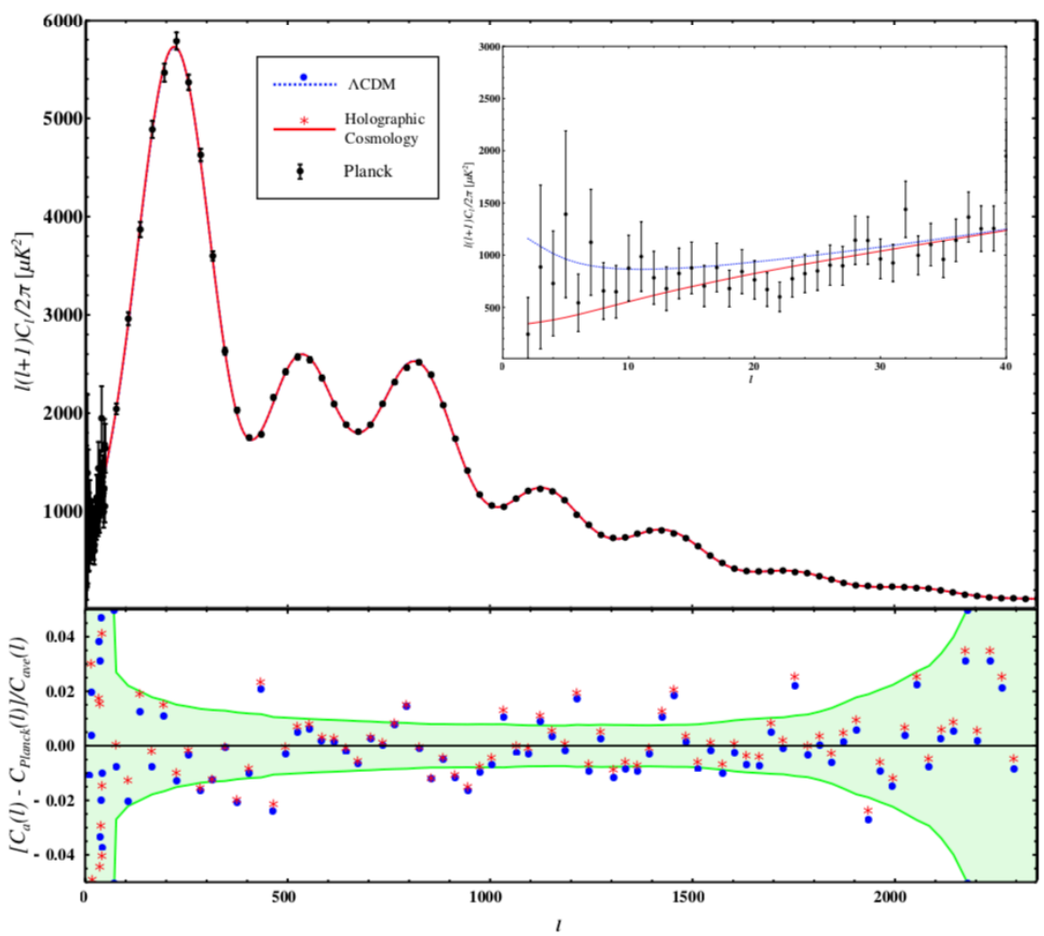
2-loop prediction for the power spectrum with 3 free parameters

McFadden and Skenderis, PRD 81, 021301 (2010)
JPCS 222, 012007 (2010), JCAP 05 (2011) 013,06 (2011) 030

$$\Delta^2_{\mathcal{R}}(q) = \frac{\Delta_0^2}{1 + \frac{g q_*}{q} \log \left| \frac{q}{\beta g q_*} \right|}$$

Holographic CMB spectrum

$$\Delta^2_{\mathcal{R}}(q) = \Delta^2_0 \left(\frac{q}{q_*}\right)^{n_s - 1} \quad \text{vs.} \quad \Delta^2_{\mathcal{R}}(q) = \frac{\Delta^2_0}{1 + \frac{g q_*}{q} \log\left|\frac{q}{\beta g q_*}\right|}$$



- PT prediction of Holographic Cosmology competitive with ΛCDM
- low-multipole region corresponds to strong-coupling in dual QFT
  → Lattice Holographic Cosmology

Afshordi et al. PRL 118, 041301 (2017)

Current study

Start with simplified model: **massless 3d scalar SU(N) matrix ϕ^4 theory**

$$\phi(x) = \phi^a(x) T^a$$

$$S = \frac{N}{g} \int d^3x \mathrm{Tr} \left((\partial_\mu \phi(x))^2 + (m^2 - m_c^2)\phi^2(x) + \phi^4(x) \right)$$

- $[\phi] = 1, [g] = 1$ (t'Hooft coupling, $g/N = g_{\mathrm{YM}}^2$)
- super-renormalisable
- log IR divergent in PT

Plan

Programme:

1. PT and non-PT study (is it IR divergent beyond PT?, determine critical properties, determine critical mass, …)
2. compute *n*-pt. functions (e.g. energy-momentum tensor)
3. make a first post-diction for the power-spectrum of the CMB and test against Planck data

Plan

Programme:

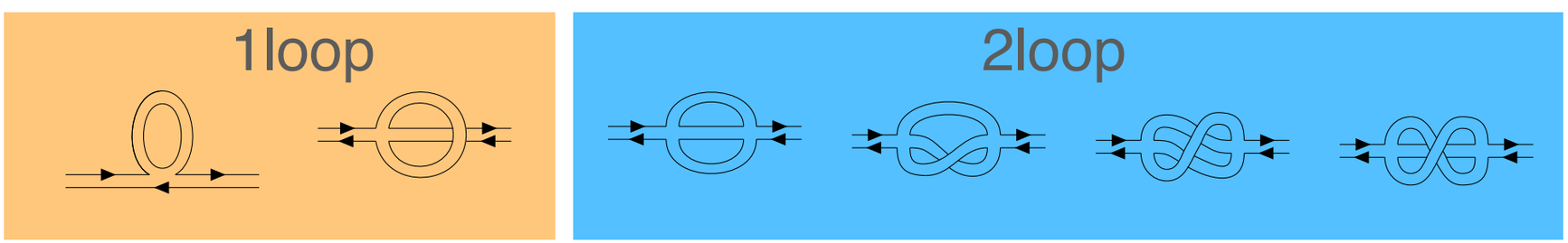
1. PT and non-PT study (is it IR divergent beyond PT?, determine critical properties, determine critical mass, …)
2. compute *n*-pt. functions (e.g. energy-momentum tensor)
3. make a first post-diction for the power-spectrum of the CMB and test against Planck data

Critical mass in NNLO lattice PT

$$S = \frac{N}{g}\int d^3x \mathrm{Tr}\left((\partial_\mu\phi(x))^2 + (m^2 - m_c^2)\phi^2(x) + \phi^4(x)\right)$$

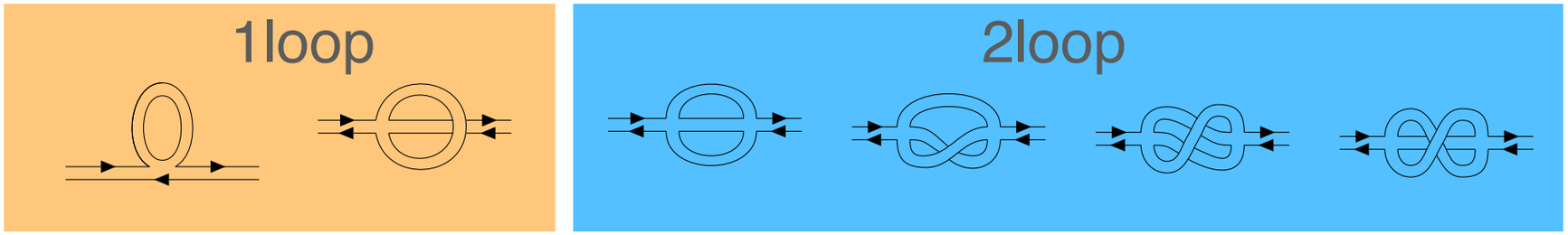
$$\phi(x) = \phi^a(x)T^a$$

$$S_i{}^j{}_k{}^l(p) = \frac{\delta_i{}^j - \frac{1}{N}\delta_i{}^j\delta_k{}^l}{\hat{p}^2 + m^2}$$



Critical mass in NNLO lattice PT

$$S = \frac{N}{g}\int d^3x \mathrm{Tr}\left((\partial_\mu\phi(x))^2 + (m^2 - m_c^2)\phi^2(x) + \phi^4(x)\right)$$



$$\frac{Z_0}{a} = \int_{-\pi/a}^{\pi/a} \frac{d^3k}{(2\pi)^3}\frac{1}{\hat{p}^2} \qquad Z_0 = 0.252731\ldots$$

$$D(p) = \int_{-\pi/a}^{\pi/a} \frac{d^3k}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\frac{1}{\hat{k}^2\,\hat{q}^2\,\hat{r}^2} \qquad r = -k - q - p$$

Critical mass in NNLO lattice PT

$$D(p) = \int_{-\pi/a}^{\pi/a} \frac{d^3k}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \frac{1}{\hat{k}^2\,\hat{q}^2\,\hat{r}^2}$$

By power counting and by explicit calculation $D(p)$ is IR-divergent $\sim \log(pa)$

This by itself could render the whole idea of Holographic Cosmology useless

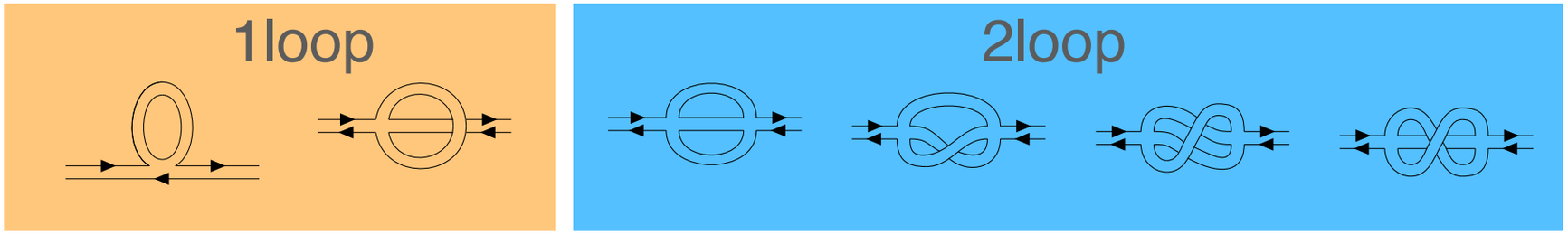
There are arguments suggesting that the theory might still be nonperturbatively IR-finite, so there is hope

Jackiw, Templeton PRD 23 1981
Applequist, Pisarski PRD 23 1981

In the examples studied there the IR-regulator was replaced by the dimensional coupling constant, g in our case

Critical mass in NNLO lattice PT

$$S = \frac{N}{g}\int d^3x \mathrm{Tr}\left((\partial_\mu\phi(x))^2 + (m^2 - m_c^2)\phi^2(x) + \phi^4(x)\right)$$



$$m_c^2 = -g\frac{Z_0}{a}\left(2 - \frac{3}{N^2}\right) + g^2 D\left(p = \Lambda_{\mathrm{IR}}\right)\left(1 - \frac{6}{N^2} + \frac{18}{N^4}\right)$$

- With IR-regularised 2loop integral $\Lambda_{\mathrm{IR}} = g/N$
- evaluate momentum sums using Vegas

Lattice study of scalar SU(N) matrix QFT

- Theory and observables implemented in Grid
- Ensemble generation on SKL cluster
  (STFC DiRAC Distributed Research utilizing Advanced Computing Science & Technology Facilities Council and University of Southampton Iridis5)
- O(100k) trajectories per ensemble

Simulation parameters:

N	2,3,4,5
g	0.1,0.2,0.3,0.5
L	16,32,64,128
m	many masses in vicinity of 2-loop m_c^2

Lattice study of scalar SU(N) matrix QFT

Magnetisation: $$M = \sum_x \phi(x)$$

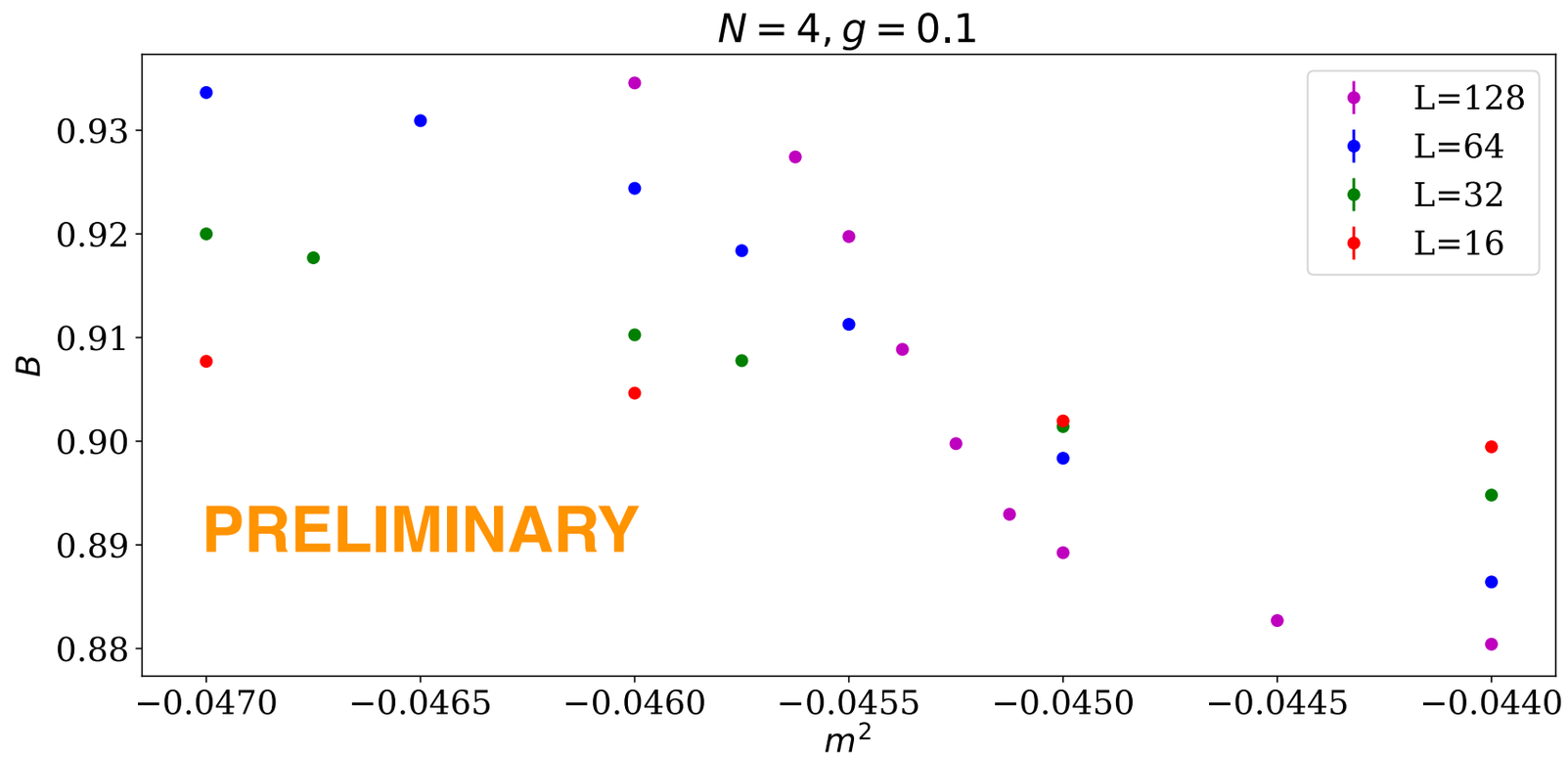
Binder cumulant: $$B = 1 - \frac{1}{N} \frac{\langle \mathrm{Tr}\left[M^4\right] \rangle}{\langle \mathrm{Tr}\left[M^2\right] \rangle^2}$$

Derivative of Binder cumulant:

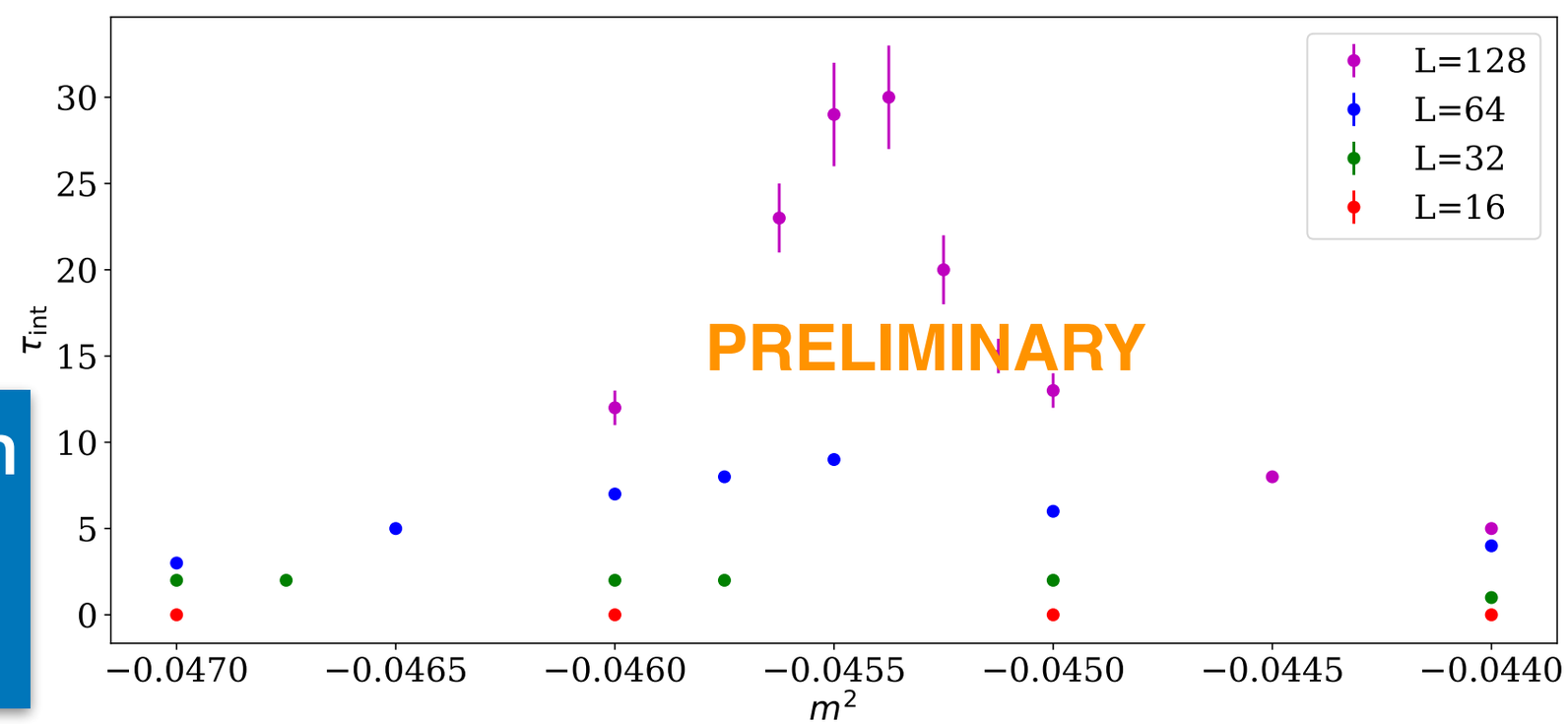
$$\frac{\partial B}{\partial m^2} = \frac{N}{g}(B-1)\left(\langle \mathrm{Tr}\left[\phi^2\right] \rangle - 2\frac{\langle \mathrm{Tr}\left[M^2\right] \mathrm{Tr}\left[\phi^2\right] \rangle}{\langle \mathrm{Tr}\left[M^2\right] \rangle} + \frac{\langle \mathrm{Tr}\left[M^4\right] \mathrm{Tr}\left[\phi^2\right] \rangle}{\langle \mathrm{Tr}\left[M^4\right] \rangle} \right)$$

A first look at the data

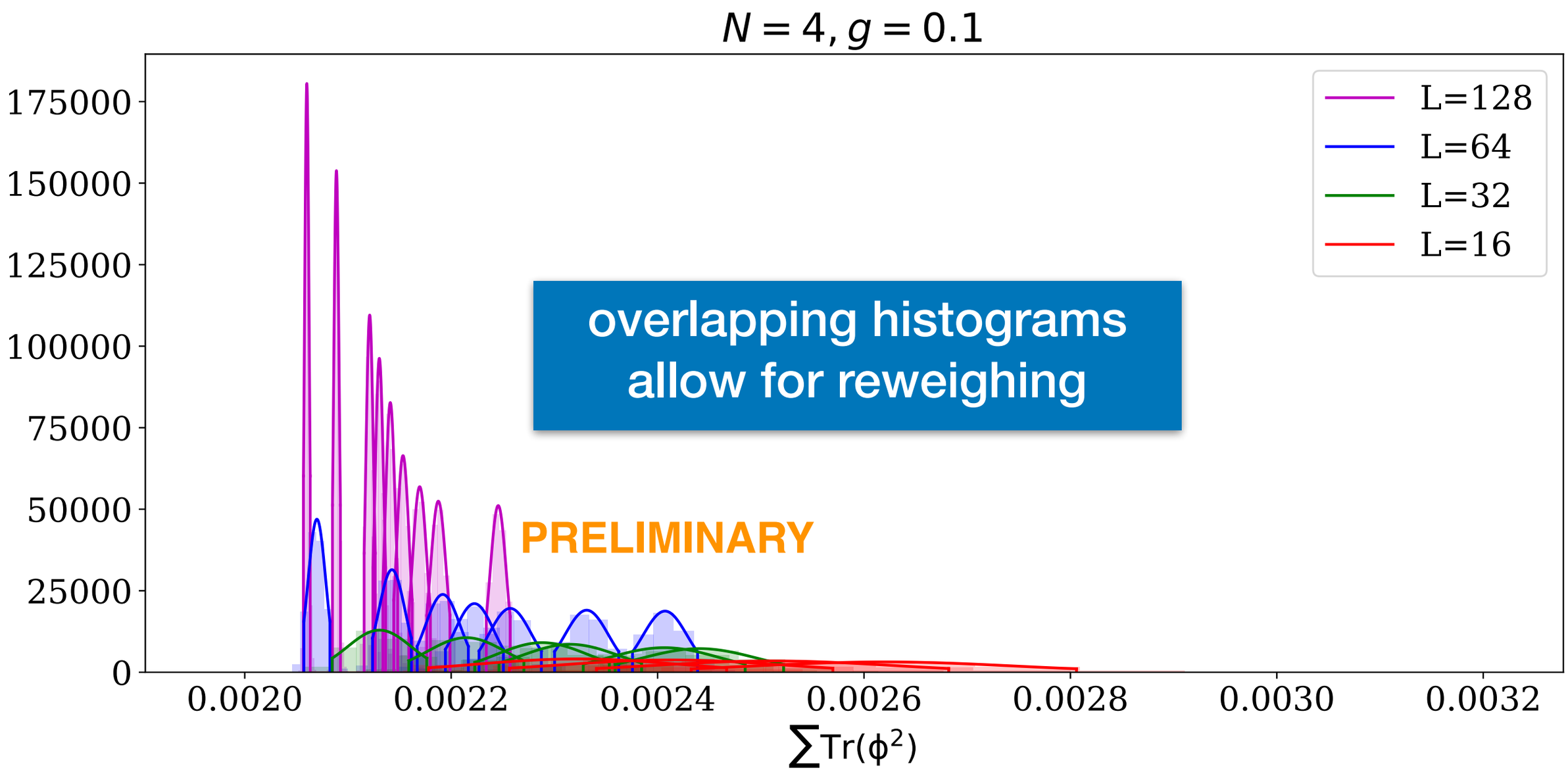
Binder cumulant in critical region for L=16,32,64,128



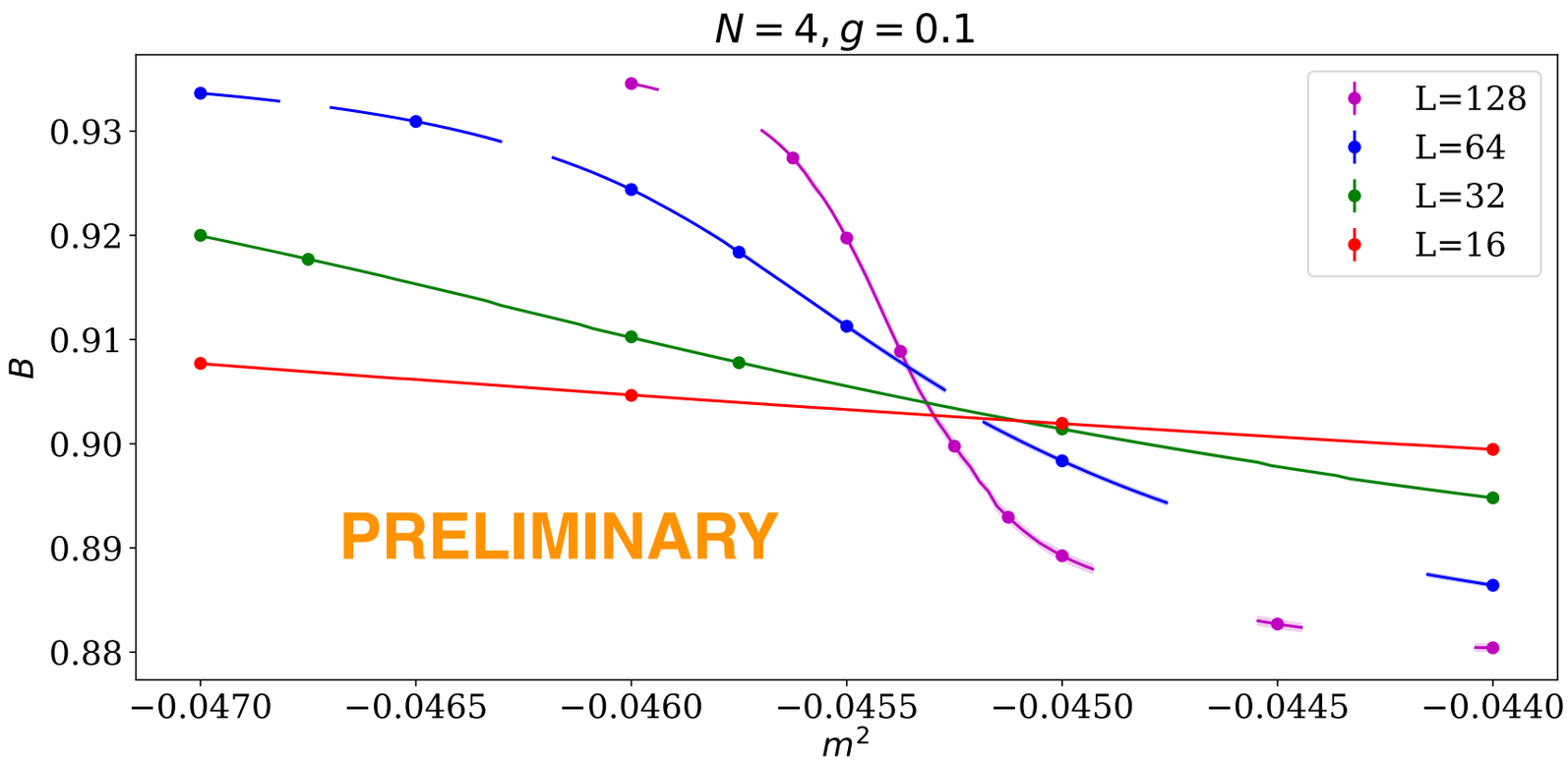
integrated autocorrelation time peaks in the region of the phase transition

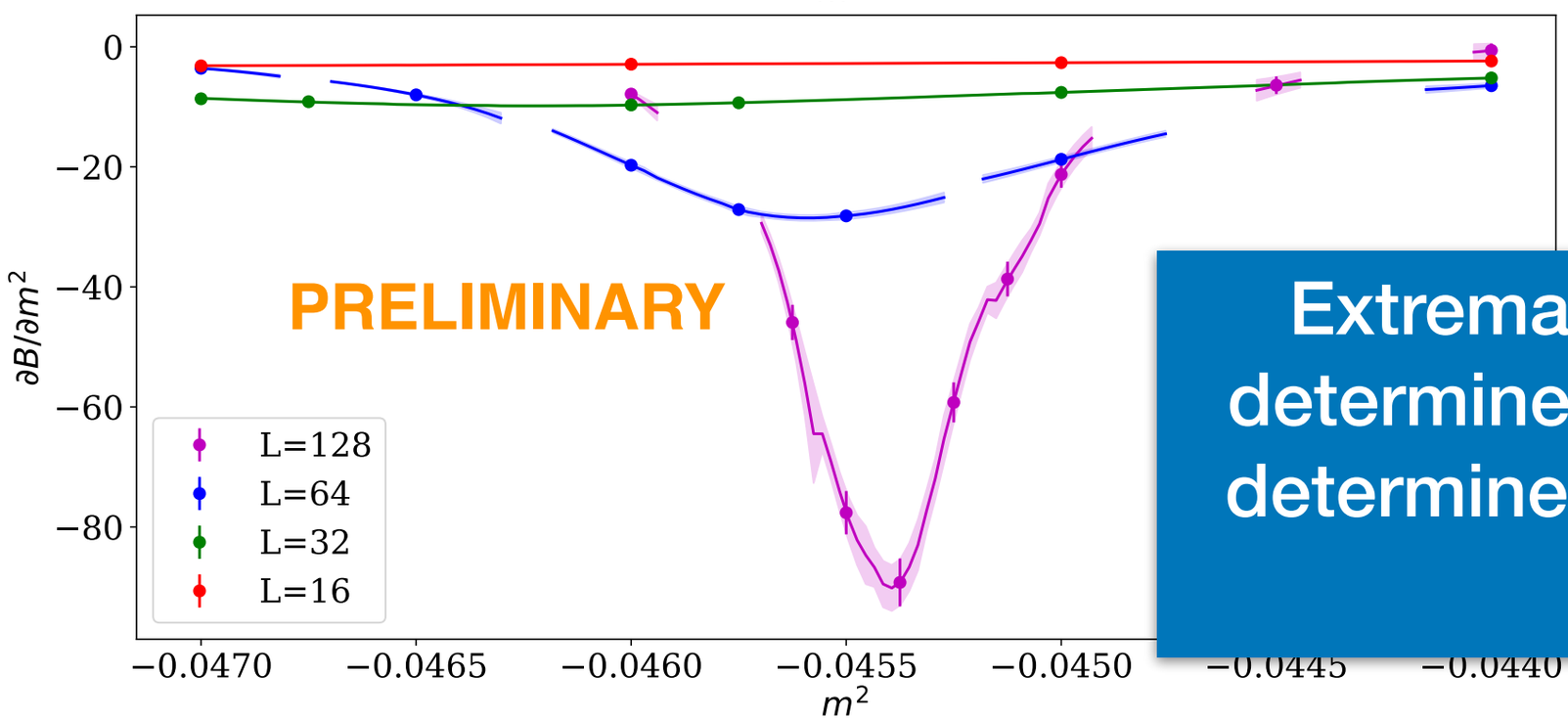


A first look at the data



A first look at the data





Extrema of 1st derivative determined under Bootstrap determine location of critical mass

Finite Size Scaling — EFT

Volume dependence of critical mass in effective theory:

$$S_{\text{eff.}}[M] = \frac{L^3 N}{g} \left(m^2 \, \text{Tr}\left[M^2\right] + \text{Tr}\left[M^4\right] \right)$$

$$\langle O[M] \rangle = \frac{1}{\mathcal{Z}_{\text{eff.}}} \int_{\mathfrak{su}(N)} dM \, O[M] \, e^{-S_{\text{eff.}}[M]}$$

Motivates model for global fit:

$$m_c^2(L) = m_c^2 + \alpha \, g^2 \, \frac{1}{(gL)^{1/(2/3)}}$$

At leading order in the EFT $\nu = \dfrac{2}{3}$

Finite size scaling study

Near the critical point

$$B(m^2) = f((m^2 - m_c^2)(gL)^{1/\nu})$$

$$B'(m^2) = (gL)^{1/\nu} f'((m^2 - m_c^2)(gL)^{1/\nu})$$

Our definition of m_{crit}:

$$B''(m_c^2(L)) = 0$$

i.e. where the 1st derivtive of the Binder cumulant peaks

Finite size scaling study

Near the critical point

$$B(m^2) = f((m^2 - m_c^2)(gL)^{1/\nu})$$

$$B'(m^2) = (gL)^{1/\nu} f'((m^2 - m_c^2)(gL)^{1/\nu})$$

Our definition of m_{crit}:

$$B''(m_c^2(L)) = 0$$

i.e. where the 1st derivtive of the Binder cumulant peaks

From this we obtain our ansatz for the FSS fit:

$$f''((m^2 - m_c^2)(gL)^{1/\nu}) \approx f''(0) + (gL)^{1/\nu} f'''(0)(m^2 - m_c^2)$$

$$m_c^2(L) = m_c^2 - \frac{f''(0)}{f'''(0)}(gL)^{-1/\nu}$$

Finite size scaling study

Ansätze for describing our data:

- Determine ν from fit to extremum of B'

$$B'(m_c^2(L))g^2 = \alpha_0(gL)^{1/\nu}$$

- Insert this into fit to position of extremum along m^2 to extrapolate critical mass to infinite volume limit

$$m_c^2(L) = m_c^2(g) + g^2\left(\beta_0 + \frac{\beta_1}{(gL)^{1/\nu}} + \beta_2(gL)^\rho\right)$$

Finite size scaling study

Ansätze for describing our data:

- Determine ν from fit to extremum of B'

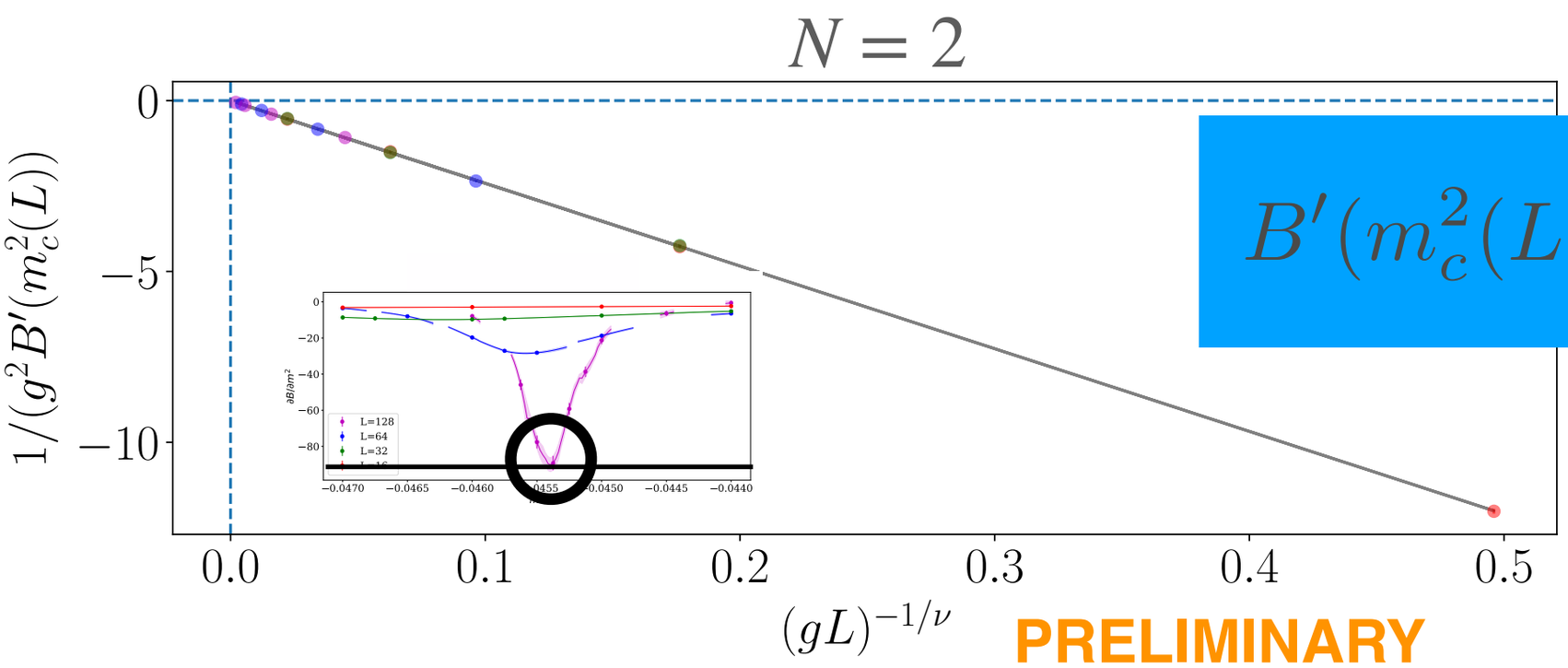
$$B'(m_c^2(L))g^2 = \alpha_0(gL)^{1/\nu}$$

- Insert this into fit to position of extremum along m^2 to extrapolate critical mass to infinite volume limit

$$m_c^2(L) = m_c^2(g) + g^2\left(\beta_0 + \frac{\beta_1}{(gL)^{1/\nu}} + \beta_2(gL)^\rho\right)$$

Use this ansatz and variations to extrapolate lattice data to infinite volume.
Determine critical mass and tune further simulations

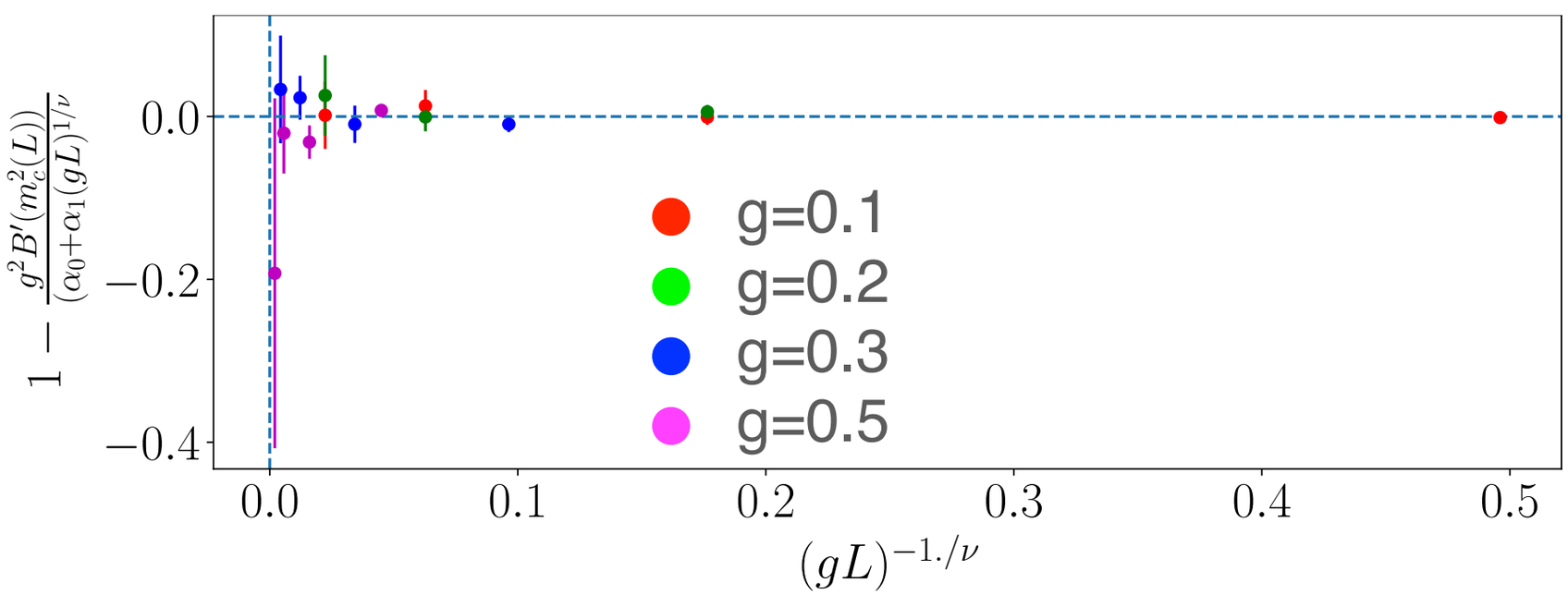
Global fit: Determination of ν



$$B'(m_c^2(L))g^2 = \alpha_0 (gL)^{1/\nu}$$

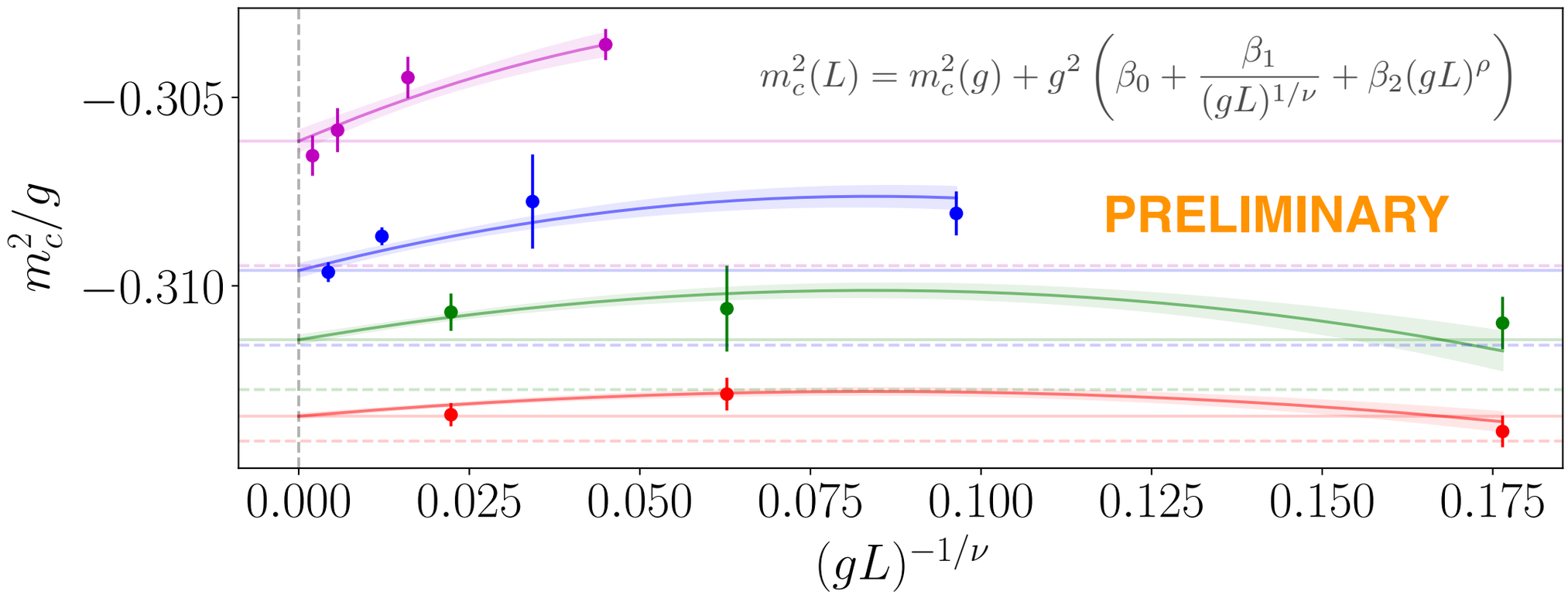
PRELIMINARY

Simple ansatz jointly describes data from g=0.1,0.2,0.3,0.5

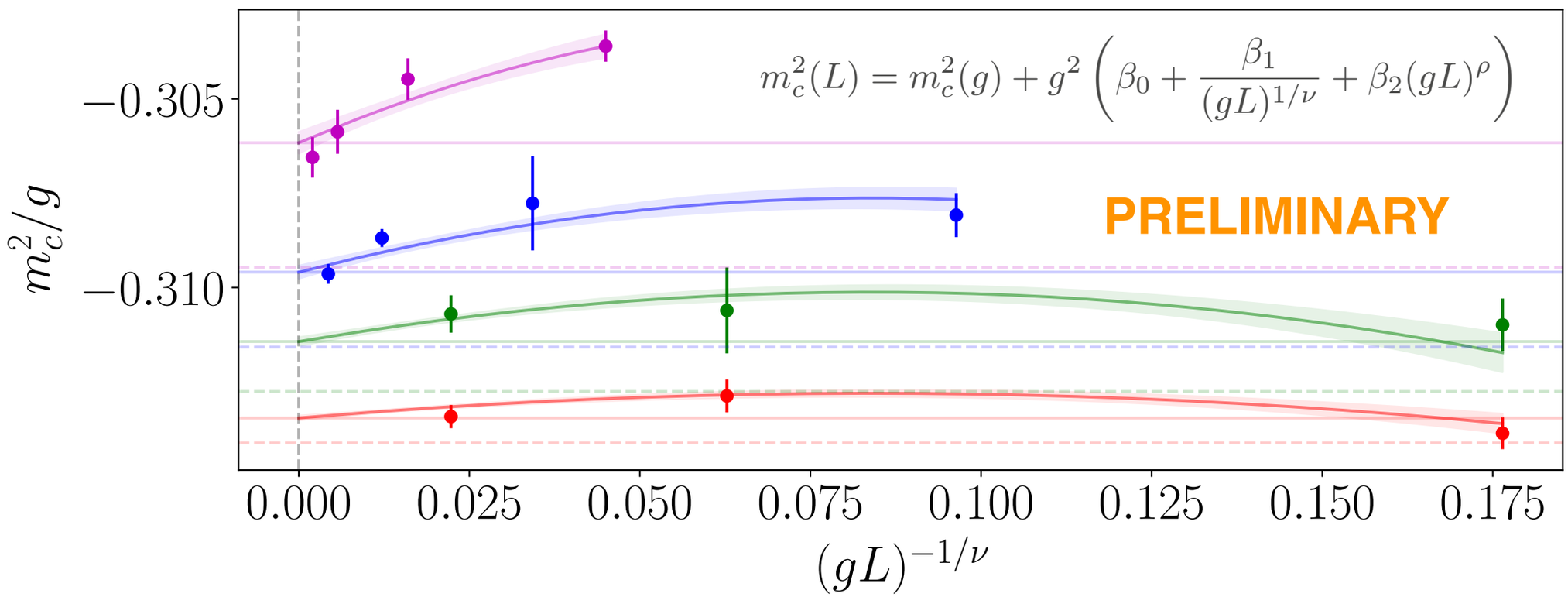


Preliminary result
$\nu_{N=2} = 0.670(2)$
(no systematics)
Close to 2/3!

Global fit: Determination of m_c^2



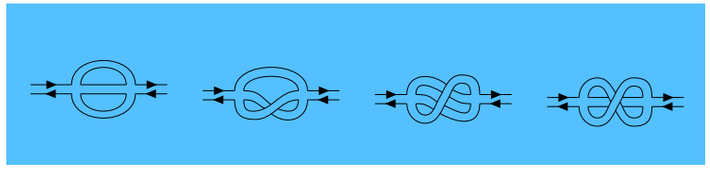
Global fit: Determination of m_c^2



- Example for global fit with $N = 2$ *(we fixed $\rho = 3$)*
- Fit describes data well and is stable under variation of ansatz
- We obtain precise predictions for critical mass needed to simulate massless theory

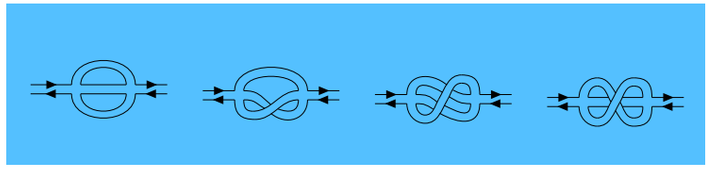
What about IR divergence?

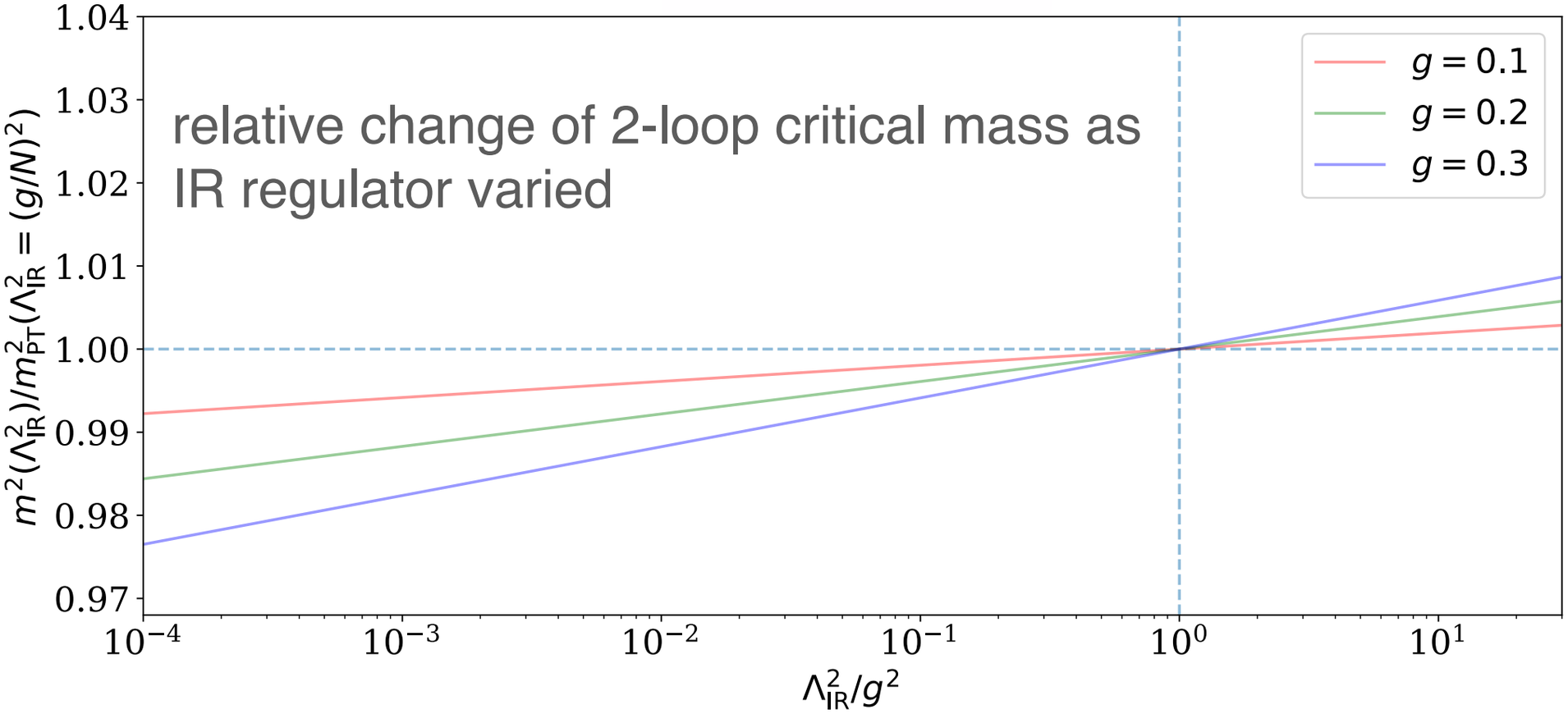
$$D(p = \Lambda_{\rm IR} = g/N) = \int_{-\pi/a}^{\pi/a} \frac{d^3k}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \frac{1}{\hat{k}^2\,\hat{q}^2\,\hat{r}^2} \qquad r = -k - q - p$$



What about IR divergence?

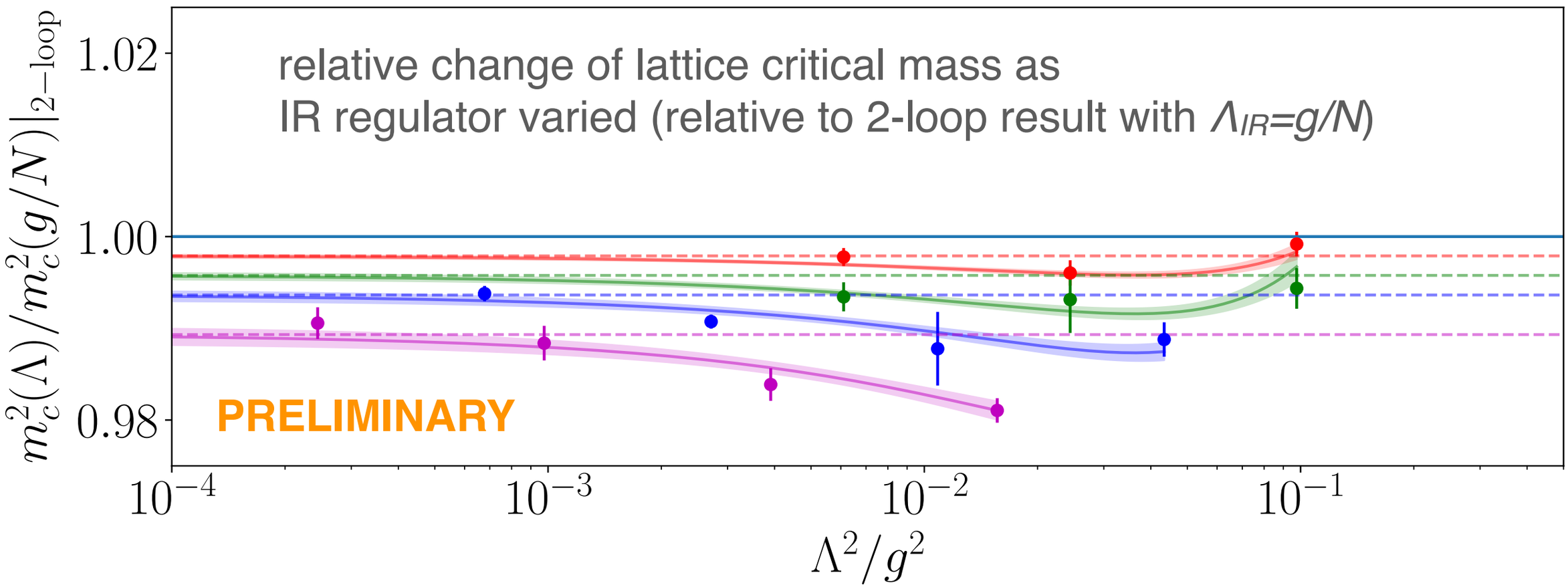
$$D(p = \Lambda_{\rm IR} = g/N) = \int_{-\pi/a}^{\pi/a} \frac{d^3k}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \frac{1}{\hat{k}^2\,\hat{q}^2\,\hat{r}^2} \qquad r = -k - q - p$$





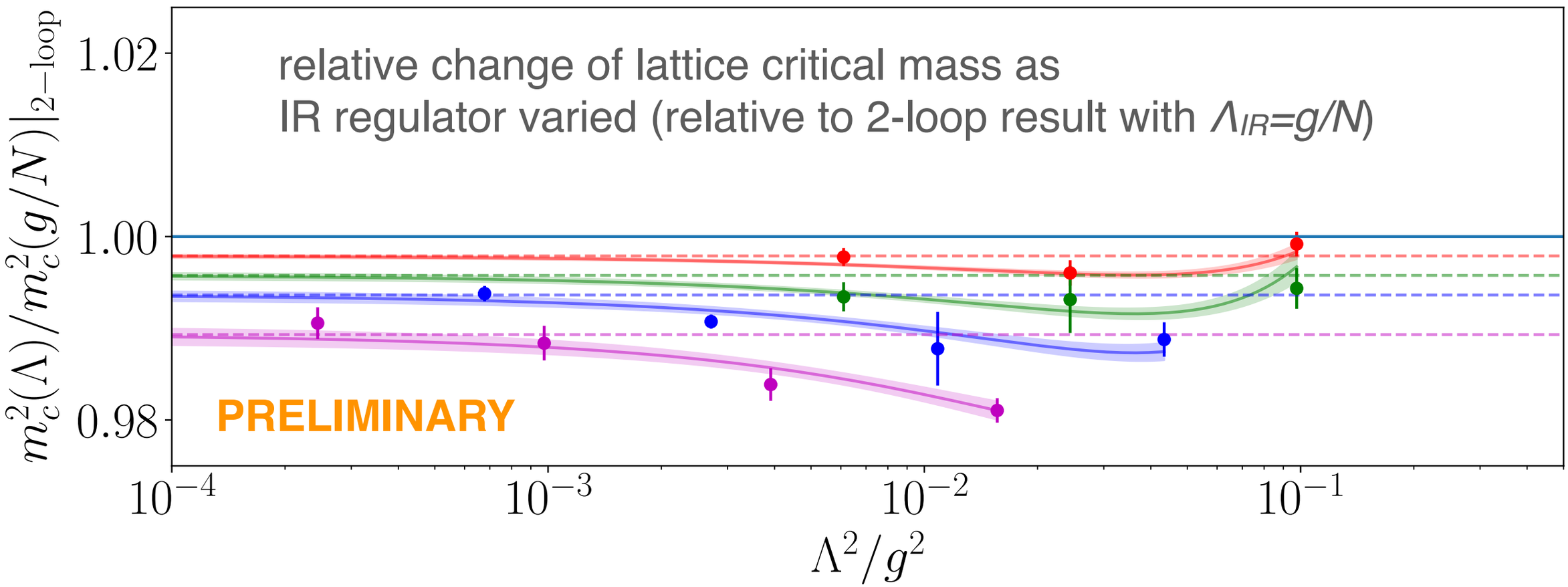
plot shows logarithmic divergence as IR-cutoff removed

What about IR divergence?



- 2-loop PT agrees with lattice results for critical mass at below-per-cent level when YM coupling used as IR regulator $\Lambda_{IR}=g/N$
- For $L\to\infty$ lattice data converges near $L=\infty$ PT — it does not seem to diverge

What about IR divergence?



Accumulating evidence for anticipated nonperturbative IR regularisation in superrenormalisable QFT

see Jackiw, Templeton PRD 23 1981
Applequist, Pisarski PRD 23 1981

Plan

Programme:

1. PT and non-PT study (is it IR divergent beyond PT?, determine critical properties, determine critical mass, …)
2. compute *n*-pt. functions (e.g. energy-momentum tensor)
3. make a first post-diction for the power-spectrum of the CMB and test against Planck data

Lattice EMT renormalisation

$$\Delta^2_{\mathcal{R}}(q) = -\frac{q^3}{4\pi^2}\frac{1}{\langle T_{\mu\mu}(q)T_{\mu\mu}(-q)\rangle}$$

$T_{\mu\nu}$ and $\langle T_{\mu\nu}T_{\rho\sigma}\rangle$ need to be renormalised

$$T^R_{\mu\nu} = \frac{N}{g}\mathrm{Tr}\left[2\bar{\delta}_\mu\phi\bar{\delta}_\nu\phi - \delta_{\mu\nu}\left(\bar{\delta}_\rho\phi\bar{\delta}_\rho\phi + (m^2 - \underline{c_3})\phi^2 + \phi^4\right)\right]$$

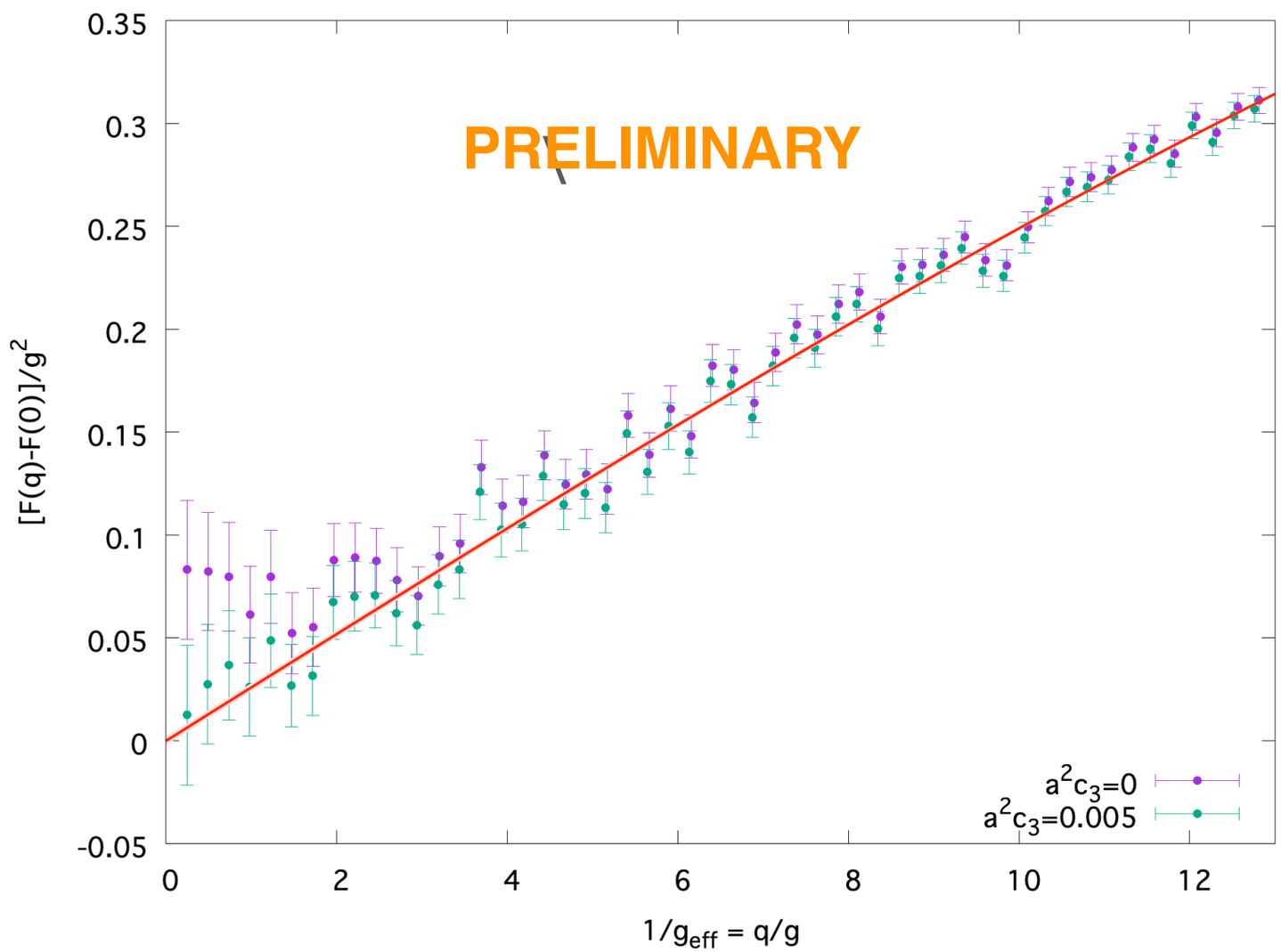
Caracciolo et al., NPB 309(4), 1988

- $T_{\mu\nu}$ mixes with $\mathrm{Tr}\phi^2$ — impose WI $q_\mu\langle T_{\mu\nu}\mathrm{Tr}\phi^2\rangle = 0$
- Subtract $1/a^3$, q/a^2, q^2/a divergences

Lattice EMT renormalisation — mixing

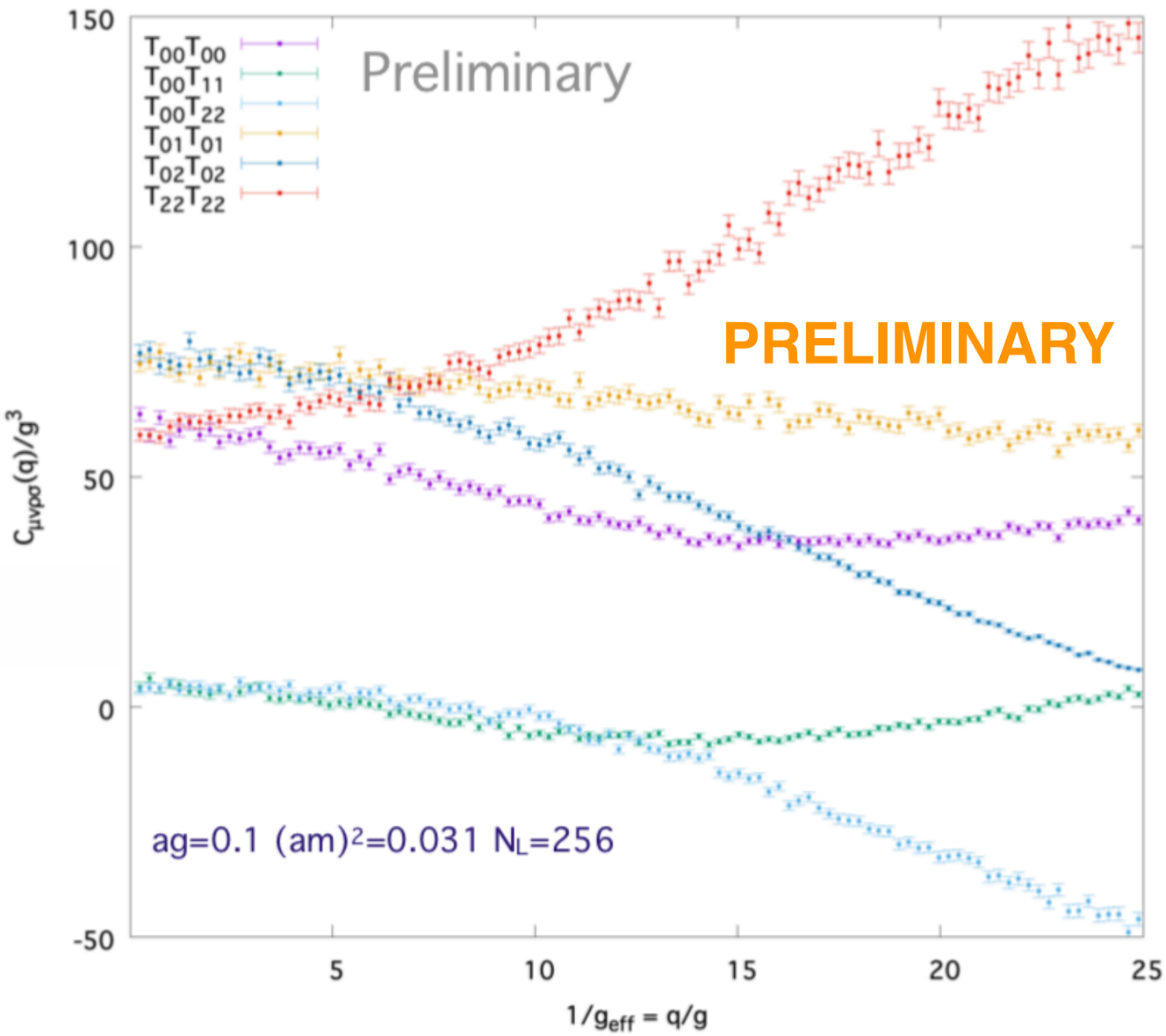
$$C^{(R)}_{\mu\nu}(q) = (F(q) - F(0))\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)$$

Determine c_3 from e.g. $C_{22}(q = (0,0,q)) = 0$



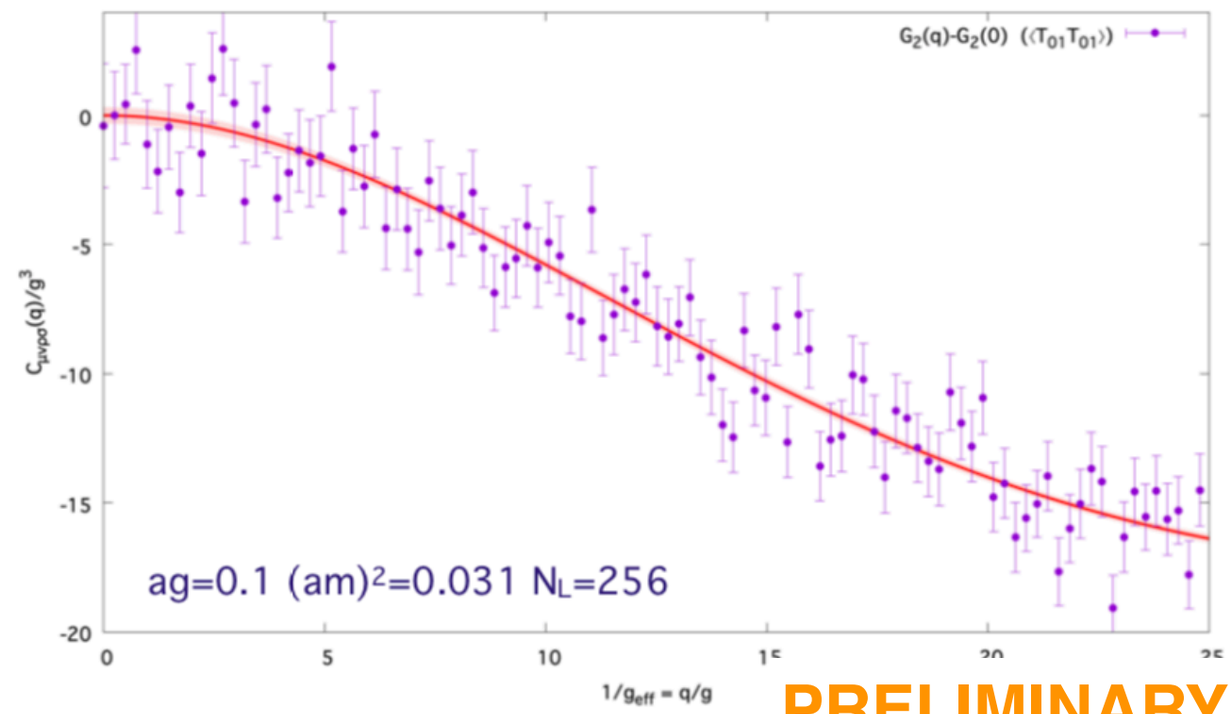
2pt function renormalisation

$$
\begin{aligned}
C_{\mu\nu\rho\sigma}(q) &= \langle \hat{T}^{(R)}_{\mu\nu}(q)\hat{T}^{(R)}_{\rho\sigma}(-q)\rangle \\
&= G_1(q)\delta_{\mu\nu}\delta_{\rho\sigma} \\
&\quad + G_2(q)\left(\delta_{\mu\rho}\delta_{\nu\sigma} + \delta_{\mu\sigma}\delta_{\nu\rho}\right) \\
&\quad + G_3(q)\left(\delta_{\mu\nu}q_\rho q_\sigma + \delta_{\rho\sigma}q_\mu q_\nu\right) \\
&\quad + G_4(q)\big(\delta_{\mu\rho}q_\nu q_\sigma. + \delta_{\mu\sigma}q_\nu q_\rho + \\
&\qquad \delta_{\nu\rho}q_\mu q_\sigma + \delta_{\nu\sigma}q_\mu q_\rho\big) \\
&\quad + G_5(a)q_\mu q_\nu q_\rho q_\sigma + H_1(q)\delta_{\mu\nu\rho\sigma} + \ldots
\end{aligned}
$$

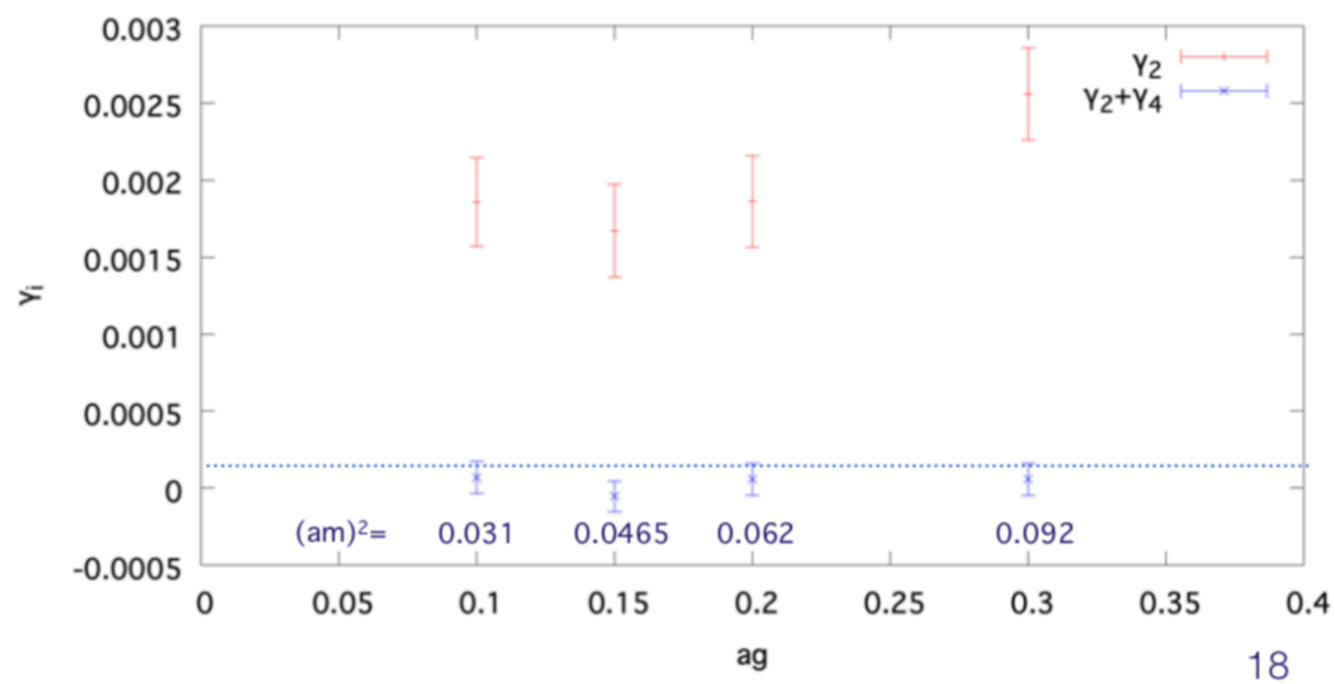


Lattice EMT renormalisation

$$G_i(q) - G_i(0) = \beta_i \frac{1}{a}\hat{q}^2 - \gamma_i \hat{q}^3$$



Verify WI after subtraction:



Conclusions

- Exciting project that covers the entire range of lattice QFT, Holography, theoretical and observational Cosmology
- Infinite volume critical masses determined in PT and non-perturbatively do agree very well if dimensional coupling used as PT IR regulator
- We see evidence accumulating for absence of IR divergence in 3d superrenormalisable QFT beyond PT
- Renormalisation of EMT and 2pt functions under way
- Todo: extrapolate to massless point, infinite volume and continuum limit and make first post-diction
- Consider gauged theory

Conclusions

- Exciting project that covers the entire range of lattice QFT, Holography, theoretical and observational Cosmology
- Infinite volume critical masses determined in PT and non-perturbatively do agree very well if dimensional coupling used as PT IR regulator
- We see evidence accumulating for absence of IR divergence in 3d superrenormalisable QFT beyond PT
- Renormalisation of EMT and 2pt functions under way
- Todo: extrapolate to massless point, infinite volume and continuum limit and make first post-diction
- Consider gauged theory

Really looking forward to look see Lattice contributing to cosmology in this novel and falsifiable way!