Three-body scattering
Unitarity-based approach

Mikhail Mikhasenko

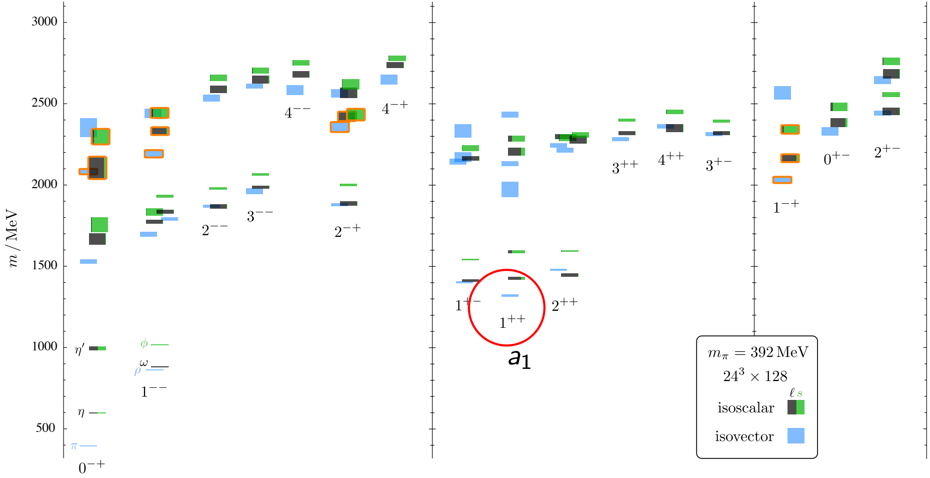
CERN, Switzerland

May 9$^{\text{th}}$, 2019

Content

Hadronic excitations from Lattice QCD

[Dudek et al., PRD 88, 094505 (2013)]



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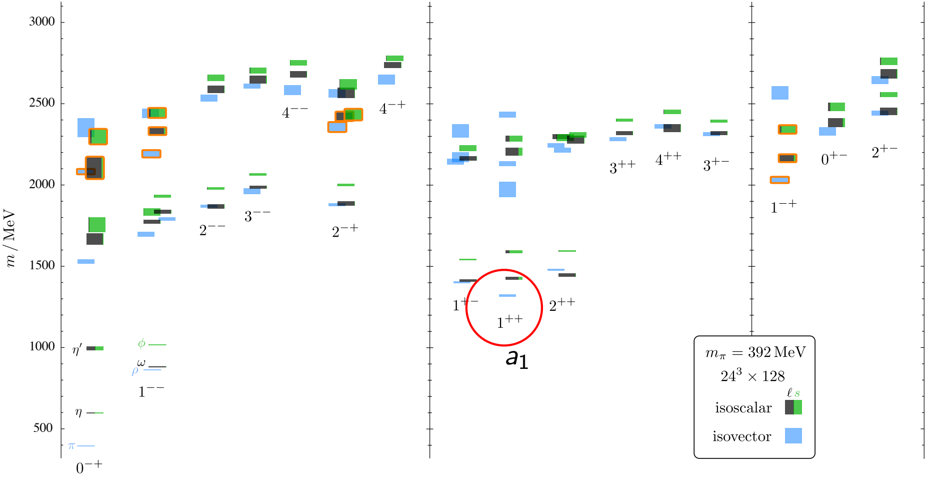
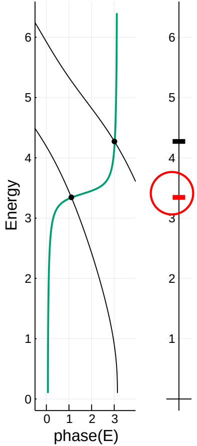


- no width
- mass is not well defined



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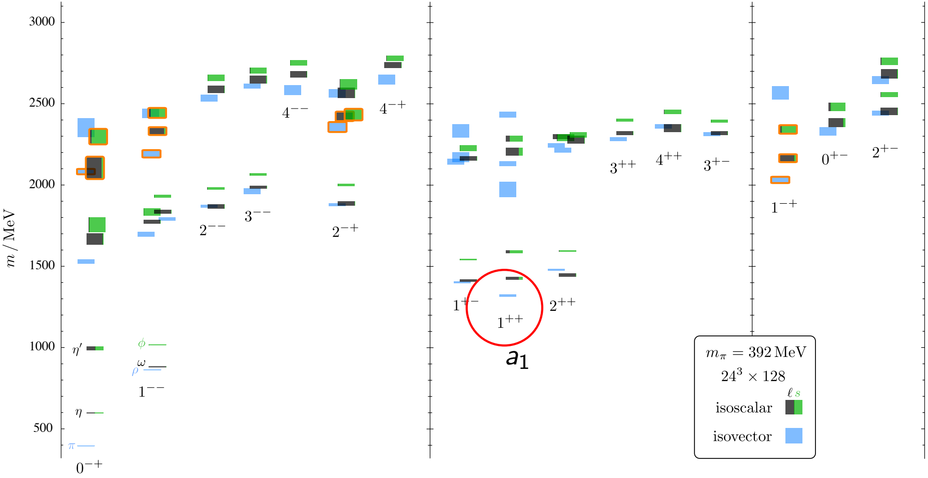

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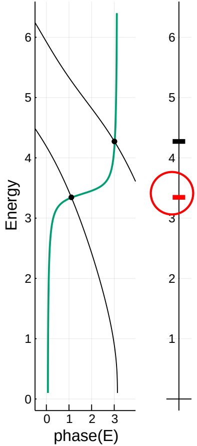


Meanwhile, the first $\rho\pi$ scattering – $I = 2$ [A.Woss, et al. JHEP 1807 (2018) 043]

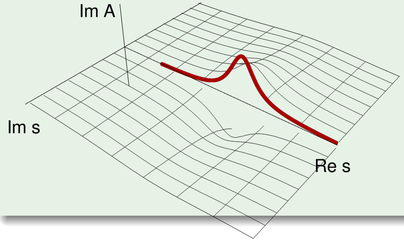
Unitarity of the scattering amplitude

unitarity cut, poles of resonances, dispersive relations

[books by Martin-Spearman, Collins, Gribov]

Resonances = Poles at the Complex plane

Example: Breit-Wigner amplitude



Features of the complex s plane:

- $s = E^2$ – the total inv.mass squared
- The Real axis $\rightarrow$ physical world
- The Imaginary axis $\rightarrow$ analytical continuation

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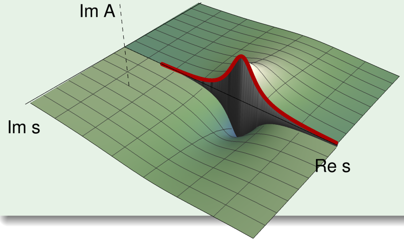

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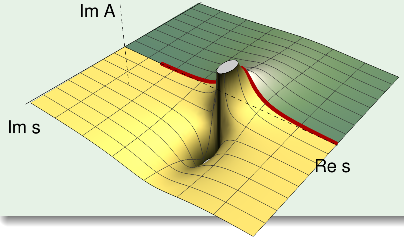

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Unitarity constraints for the **two-body** scattering

- $\hat{S}^\dagger \hat{S} = \hat{\mathbb{I}} \quad \hat{S} = \hat{\mathbb{I}} + i\hat{T}$ $\qquad \hat{T} - \hat{T}^\dagger = i\hat{T}^\dagger \hat{T}.$
- $T(s,t) = \left\langle p_1' p_2' \right| \hat{T} \left| p_1 p_2 \right\rangle$ $\qquad T(s,t) - T^\dagger(s,t) = \int \mathrm{d}\Phi_2\, T^\dagger(s,t')\, T(s,t'')$
- Partial-wave expansion . . .
- The final form $\qquad t_l - t_l^\dagger = t_l^\dagger\, \rho(s)\, t_l$

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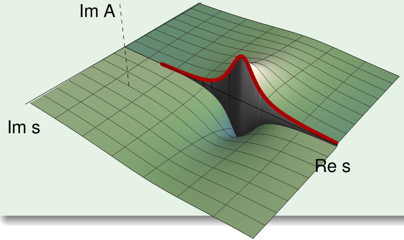

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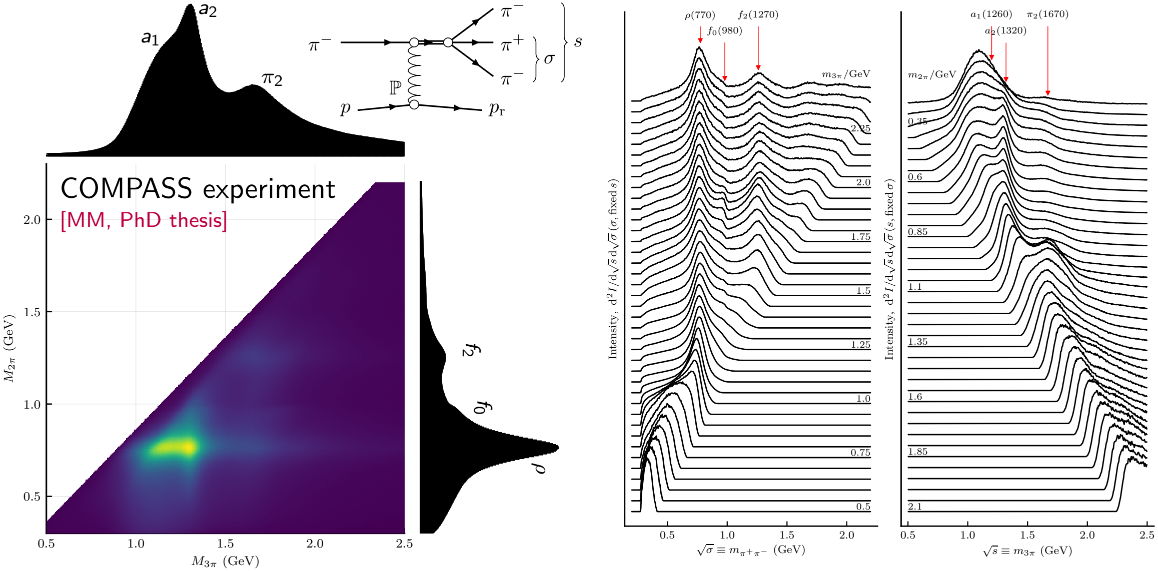
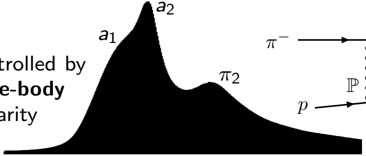
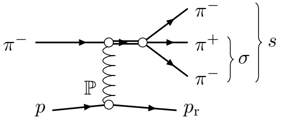
Three-particle interaction: resonances are everywhere

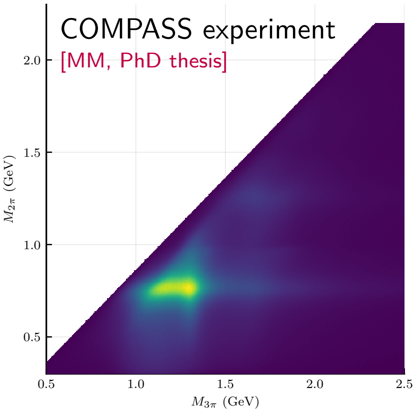


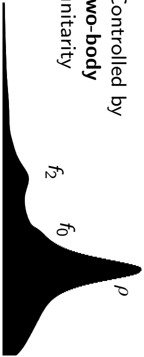
Three-particle interaction: resonances are everywhere

Controlled by **three-body** unitarity

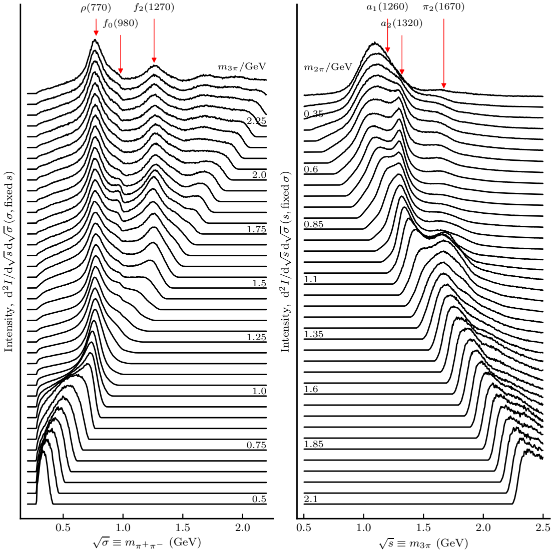








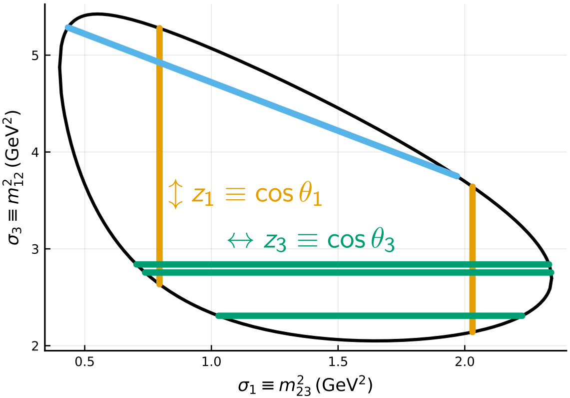
Controlled by **two-body** unitarity



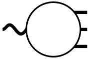
Three-body decay
Final-state interaction

isobar model, rescattering, ladder of exchanges

Three-body decay



Decay amplitude – $\langle p_1 p_2 p_3 | \hat{T} | p_0 \rangle$



$$= D^{J*}_{M\lambda}(\alpha,\beta,\gamma) F_\lambda(s,\sigma_1,\sigma_2)$$

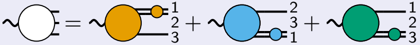
$$\xrightarrow[\text{scalars}]{} F(\sigma_1,\sigma_2)$$

Dalitz plot variables

- Subchannel resonances are bands.
- Angular distribution along the bands determined by angular momenta.

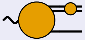
Partial-waves vs Isobar representation

Isobar representation



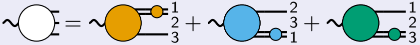
$$F(\sigma_1, \sigma_2) = F^{(1)}(\sigma_1, \sigma_2) + F^{(2)}(\sigma_1, \sigma_2) + F^{(3)}(\sigma_1, \sigma_2)$$

$$= \sum_{l}^{\text{few}} \sqrt{2l+1} P_l(z_1) a_l^{(1)}(\sigma_1) + \sum_{l}^{\text{few}} \sqrt{2l+1} P_l(z_2) a_l^{(2)}(\sigma_2) + \sum_{l}^{\text{few}} \sqrt{2l+1} P_l(z_3) a_l^{(3)}(\sigma_3).$$

Simple **model**:  $= a_l^{(i)}(\sigma_1) \to c^{(i)} \mathrm{BW}(\sigma_1) =$ .

Partial-waves vs Isobar representation

Isobar representation



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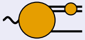
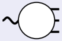
Simple **model**:  $= a_l^{(i)}(\sigma_1) \to c^{(i)}\mathrm{BW}(\sigma_1) =$ .

Partial-wave representation

 $= F(\sigma_1,\sigma_2) = \sum_l^{\infty} \sqrt{2l+1}P_l(z_1)f_l^{(1)}(\sigma_1)$

Why would someone do this? – theoretical constant to $f^{(1)}(\sigma_1)$ is straightforward.

Two-body unitarity and Khuri-Trieman model

Example of $f_0^{(1)}(\sigma_1)$ constraints:

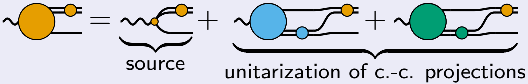
$$f_0^{(1)}(\sigma_1) = \underbrace{a_0^{(1)}(\sigma_1)}_{\text{same channel}} + \underbrace{\int_{-1}^{1} \frac{\mathrm{d}z_1}{2} \left(\sum_l \sqrt{2l+1} P_l(z_2) a_l^{(2)}(\sigma_2) + \sum_l \sqrt{2l+1} P_l(z_3) a_l^{(3)}(\sigma_3) \right)}_{\text{cross-channel(c.-c.) projections}}$$

Unitarity of $f_0^{(1)}(\sigma_1)$ – same RHC as $2 \to 2$ scattering amplitude, $\mathrm{BW}_0^{(1)}(\sigma_1)$

$\Rightarrow$ consistency relation the **direct term** and the **cross-channel projections**

$\Rightarrow$ $a_l^{(1)}(\sigma_1)$ obtains corrections from one seen in $2 \to 2$.

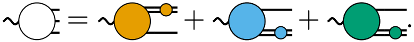
KT model: analytic continuation of **two-body** unitarity



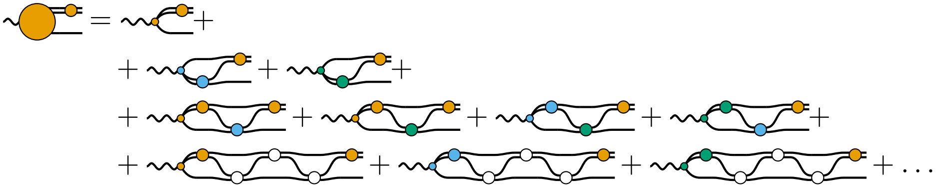
(the loop is a dispersive integral)

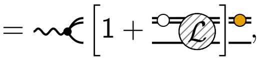
Diagramatic representation

Isobar representation with $a_l^{(i)}(\sigma_i) = \hat{a}_l^{(i)}(\sigma_i)\,\mathrm{BW}_l^{(i)}(\sigma_i)$

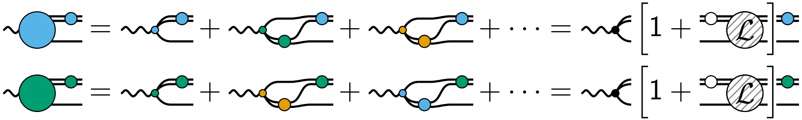


The amplitude prefactor is not constant: $a_l^{(i)}(\sigma_i) = c_l^i \mathrm{BW}_l^{(i)}(\sigma_i) + \dots$



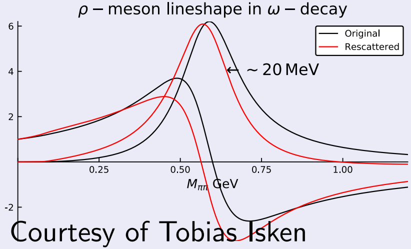


the ladder – a sum of all possible exchanges



Khuri-Treiman model in practice

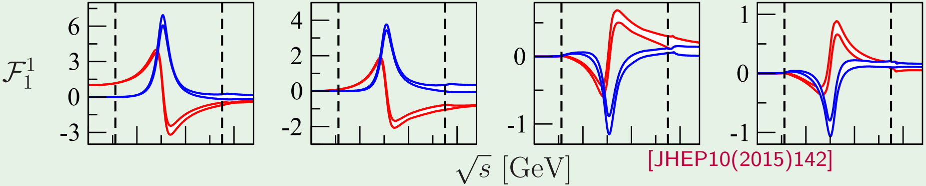
Light-meson decays



Courtesy of Tobias Isken

- $\eta \to 3\pi$ [Sebastian et al. (2011)], [P.Guo et al., JPAC, 2015], [Albaladejo, Moussallam (2017)]
- $\eta' \to \eta\pi\pi$ [Isken, Kubis (2017)]
- $\omega/\phi \to 3\pi$ [Niecknig, Kubis (2012)], [Danilkin et al., JPAC (2012)]
- $a_1 \to 3\pi$ [JPAC (in progress)]

Open charm, $D \to K\pi\pi$, [Niecknig, Kubis (2015)], [Moussallam, in progress (2017)]



[JHEP10(2015)142]

ρ-meson lineshape: direct and induced

Three-particle scattering

Three-body unitarity, Ladders and Resonances, short-range factorization

[arXiv:1904.11894]

Decomposition of the $3 \to 3$ scattering

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- **Particle pairing** (symmetrization or isobar decomposition), e.g. for the state of identical particles:

$$
\begin{aligned}
|p_1 p_2 p_3\rangle &= \frac{1}{3!}\left(|p_1\rangle \otimes |p_2\rangle \otimes |p_3\rangle + \text{symm.}\right) \\
&= \frac{1}{3}\sum_{a=1}^{3} |p_{a_1}\rangle \, \frac{|p_{a_2}\rangle \otimes |p_{a_3}\rangle + |p_{a_3}\rangle \otimes |p_{a_2}\rangle}{2} = \frac{1}{3}\sum_{a=1}^{3} |a\rangle \quad \text{– isobar-spectator states,}
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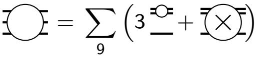

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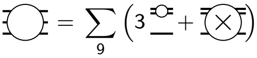

- Partial wave **expansion** (spin l in subchannels, spin j overall)

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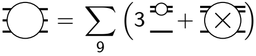
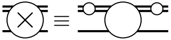


- Partial wave **expansion** (spin l in subchannels, spin j overall)
- **Amputation** of the last scattering bit

 $= t(\sigma),$

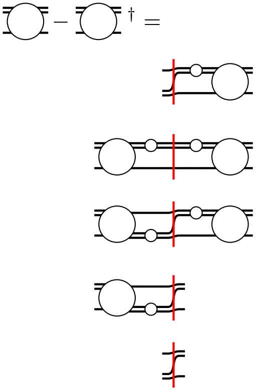
 $= t(\sigma')\,\mathcal{T}(\sigma', s, \sigma)\, t(\sigma),$

Three-body-unitarity constraint

[G.Fleming, Phys.Rev. 135 (1964)]

Three-body scattering amplitude must satisfy the integral equation

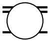
$$
\begin{aligned}
&\mathcal{T}(\sigma', s, \sigma) - \mathcal{T}^\dagger(\sigma', s, \sigma) = \\
&\quad 2i\,\frac{1}{\lambda_s^{1/2}(\sigma')}\frac{1}{8\pi}\int_{\sigma^-(\sigma',s)}^{\sigma^+(\sigma',s)} \mathrm{d}\sigma_3'\, t(\sigma_3')\mathcal{T}(\sigma_3', s, \sigma) \\
&\quad + \frac{i}{3}\int_{4m_\pi^2}^{(\sqrt{s}-m_\pi)^2} \frac{\mathrm{d}\sigma''}{2\pi}\mathcal{T}^\dagger(\sigma', s, \sigma'')\, t(\sigma'')t^\dagger(\sigma'')\,\rho(\sigma'')\rho_s(\sigma'')\,\mathcal{T}(\sigma'', s, \sigma) \\
&\quad + \frac{2i}{3}\frac{1}{(8\pi)^2}\iint_{\phi(\sigma_2'', s, \sigma_3'')>0} \frac{\mathrm{d}\sigma_2''\mathrm{d}\sigma_3''}{2\pi s}\,\mathcal{T}^\dagger(\sigma', s, \sigma_2'')t^\dagger(\sigma_2'')t(\sigma_3'')\mathcal{T}(\sigma_3'', s, \sigma) \\
&\quad + 2i\,\frac{1}{\lambda_s^{1/2}(\sigma)}\frac{1}{8\pi}\int_{\sigma^-(\sigma,s)}^{\sigma^+(\sigma,s)} \mathrm{d}\sigma_2\,\mathcal{T}^\dagger(\sigma', s, \sigma_2)t^\dagger(\sigma_2) \\
&\quad + 6i\,\frac{2\pi s}{\lambda_s^{1/2}(\sigma')\lambda_s^{1/2}(\sigma)}\,\theta^+(\phi(\sigma', s, \sigma)).
\end{aligned}
$$



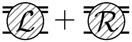
In a short form: [Aaron-Amada(TCP 2 (1977)), Mai et al.(EPJ A53 (2017)), Jackura et al.(EPJ C79 (2019))]:

$$
\mathcal{T} - \mathcal{T}^\dagger = \mathcal{D}\tau\mathcal{T} + \mathcal{T}^\dagger(\tau - \tau^\dagger)\mathcal{T} + \mathcal{T}^\dagger\tau^\dagger\mathcal{D}\tau\mathcal{T} + \mathcal{T}^\dagger\tau^\dagger\mathcal{D} + \mathcal{D},
$$

Splitting amplitude by the interaction range



$$\mathcal{T}(\sigma', s, \sigma) = \mathcal{L}(\sigma', s, \sigma) + \mathcal{R}(\sigma', s, \sigma).$$



Long-range part: exchange processes (on-shell) [Mai et al.(EPJ A53 (2017))]

- Infinite sum of the one-particle-exchange process

$$\mathcal{L} = \mathcal{B} + \mathcal{L}\tau\mathcal{B} = \mathcal{B} + \mathcal{B}\tau\mathcal{L}.$$

- $\mathcal{T} = \mathcal{L}$, $\mathcal{R} = 0$ already satisfy unitarity. Can it have resonances?

Short-range part: resonances [MM, Y.Wunderlich, et al. (JPAC) 1904.11894]

- Condition for $\mathcal{R}(\sigma', s, \sigma)$ is complicated
- However, simplified significantly if FSI is factorized from both sides:

$$\mathcal{R} \equiv (1 + \mathcal{L}\tau)\,\widehat{\mathcal{R}}\,(\tau\mathcal{L} + 1).$$

Unitarity constraint for the resonance kernel

Familiar form of the constraint (see two-body constraint at slide 4):

$$\begin{aligned}\widehat{\mathcal{R}} - \widehat{\mathcal{R}}^\dagger &= \widehat{\mathcal{R}}^\dagger (1 + \tau^\dagger \mathcal{L}^\dagger) \left[\tau - \tau^\dagger + \tau^\dagger \mathcal{D} \tau\right] (1 + \mathcal{L}\tau)\, \widehat{\mathcal{R}} \\ &= \widehat{\mathcal{R}}^\dagger \left[\tau - \tau^\dagger + \tau \mathcal{L} \tau - \tau^\dagger \mathcal{L}^\dagger \tau^\dagger\right] \widehat{\mathcal{R}}.\end{aligned}$$

Unitarity constraint for the resonance kernel

Familiar form of the constraint (see two-body constraint at slide 4):

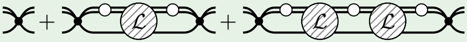
$$\widehat{\mathcal{R}} - \widehat{\mathcal{R}}^\dagger = \widehat{\mathcal{R}}^\dagger (1 + \tau^\dagger \mathcal{L}^\dagger) \left[\tau - \tau^\dagger + \tau^\dagger \mathcal{D} \tau \right] (1 + \mathcal{L}\tau) \widehat{\mathcal{R}}$$

$$= \widehat{\mathcal{R}}^\dagger \left[\tau - \tau^\dagger + \tau \mathcal{L} \tau - \tau^\dagger \mathcal{L}^\dagger \tau^\dagger \right] \widehat{\mathcal{R}}.$$

- K-matrix-like solution (*cf.* $K_{\text{d.f.}}$ [M.Hansen, S.Sharpe, PRD90 (2014), 116003])

$$\widehat{\mathcal{R}} = \mathcal{X} + \mathcal{X}(\tau + \tau\mathcal{L}\tau)\widehat{\mathcal{R}}$$

$$= \mathcal{X} + \mathcal{X}(\tau + \tau\mathcal{L}\tau)\mathcal{X} + \mathcal{X}(\tau + \tau\mathcal{L}\tau)\mathcal{X}(\tau + \tau\mathcal{L}\tau)\mathcal{X} + \ldots$$

=  + ...

Unitarity constraint for the resonance kernel

Familiar form of the constraint (see two-body constraint at slide 4):

$$\widehat{\mathcal{R}} - \widehat{\mathcal{R}}^\dagger = \widehat{\mathcal{R}}^\dagger (1 + \tau^\dagger \mathcal{L}^\dagger) \left[\tau - \tau^\dagger + \tau^\dagger \mathcal{D} \tau\right] (1 + \mathcal{L}\tau)\, \widehat{\mathcal{R}}$$

$$= \widehat{\mathcal{R}}^\dagger \left[\tau - \tau^\dagger + \tau \mathcal{L} \tau - \tau^\dagger \mathcal{L}^\dagger \tau^\dagger\right] \widehat{\mathcal{R}}.$$

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$$\widehat{\mathcal{R}} = \mathcal{X} + \mathcal{X}(\tau + \tau\mathcal{L}\tau)\, \widehat{\mathcal{R}}$$

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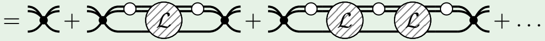
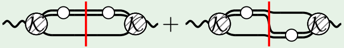


- Rescattering interpretation [MM, Y.Wunderlich, et al. (JPAC) 1904.11894]:

$$\widehat{\mathcal{R}} - \widehat{\mathcal{R}}^\dagger = \underbrace{\widehat{\mathcal{R}}^\dagger (1 + \tau^\dagger \mathcal{L}^\dagger)}_{\text{rescattering}} \overbrace{\left[\tau - \tau^\dagger + \tau^\dagger \mathcal{D}\tau\right]}^{\text{direct+crossed coupling}} \underbrace{(1 + \mathcal{L}\tau)\, \widehat{\mathcal{R}}}_{\text{rescattering}} \xrightarrow[\text{sort-of}]{}$$



Factorization of the resonance kernel

- Strictly, one more (weak) assumption – **Factorization**

$$\widehat{\mathcal{R}}(\sigma', s, \sigma) = k_f(\sigma')\widehat{\mathcal{R}}(s)\; k_i(\sigma) \quad \text{OR}$$
$$= \widehat{\mathcal{R}}_{00}(s) + \sigma'\widehat{\mathcal{R}}_{10}(s) + \widehat{\mathcal{R}}_{01}(s)\sigma + \sigma'\widehat{\mathcal{R}}_{11}(s)\sigma + \dots,$$

$\Rightarrow$ Unitarity requirement is algebraic!

$$\widehat{\mathcal{R}}(s) - \widehat{\mathcal{R}}^\dagger(s) = i\,\widehat{\mathcal{R}}^\dagger(s)\Sigma(s)\,\widehat{\mathcal{R}}(s),$$

with $\quad \Sigma \equiv \mathcal{K}^\dagger(\tau - \tau^\dagger)\mathcal{K} + \mathcal{K}^\dagger\tau^\dagger\mathcal{D}\tau\mathcal{K},$

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$\mathcal{K}$ is the modification of the isobar lineshape due to the rescattering

An approximate-three-body unitarity

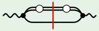
$$\widehat{\mathcal{R}}(s) = \frac{g^2}{m^2 - s - ig^2/2\,[\;\dots\;]}$$

contains effect of the subchannel-resonances **interference**

The quasi-two-body approximation

[J.Basdevant, Ed Berger, PRD19 (1979) 239]

$$\widehat{\mathcal{R}}(s) = \frac{g^2}{m^2 - s - ig^2/2\,[\;\dots\;]}$$



naively accounts for the subchannel-resonance decay

Analysis of $a_1(1260)$

subchannel-resonance interference, analytic continuation

[PRD98 (2018), 096021]

$a_1(1260)$ state – ground axial vector – isospin parter of ρ [PDG (2018)]

$a_1(1260)$ WIDTH

INSPIRE search

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
250 to 600	**OUR ESTIMATE**				
389 ± 29	**OUR AVERAGE** Error includes scale factor of 1.3.				
$430 \pm 24 \pm 31$		DARGENT	2017	RVUE	$D^0 \to \pi^-\pi^+\pi^-\pi^+$
$367 \pm 9\ ^{+28}_{-25}$	420k	ALEKSEEV	2010	COMP	$190\ \pi^- \to \pi^-\pi^-\pi^+ Pb'$
• • • We do not use the following data for averages, fits, limits, etc. • • •					
$410 \pm 31 \pm 30$		1 AUBERT	2007AU	BABR	$10.6\ e^+ e^- \to \rho^0\rho^\pm\pi^\mp\gamma$
520 - 680	6360	2 LINK	2007A	FOCS	$D^0 \to \pi^-\pi^+\pi^-\pi^+$
480 ± 20		3 GOMEZ-DUMM	2004	RVUE	$\tau^+ \to \pi^+\pi^+\pi^-\nu_\tau$
580 ± 41	90k	SALVINI	2004	OBLX	$\overline{p}\,p \to 2\,\pi^+ 2\,\pi^-$
460 ± 85	205	4 DRUTSKOY	2002	BELL	$B^{(*)}\ K^-\ K^{*0}$
$814 \pm 36 \pm 13$	37k	5 ASNER	2000	CLE2	$10.6\ e^+ e^- \to \tau^+\tau^-,\ \tau^- \to \pi^-\pi^0\pi^0\nu_\tau$

$a_1(1260)$ state – ground axial vector – isospin parter of ρ [PDG (2018)]

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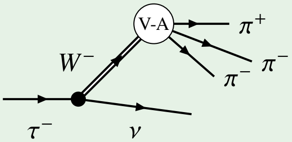
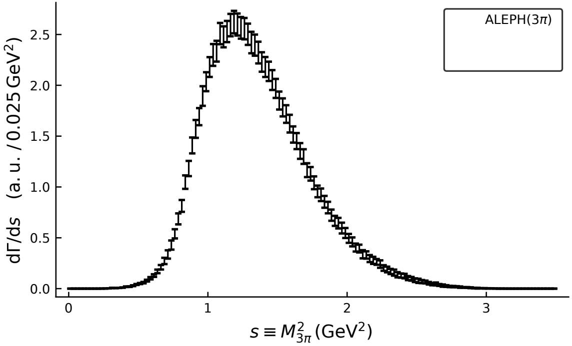
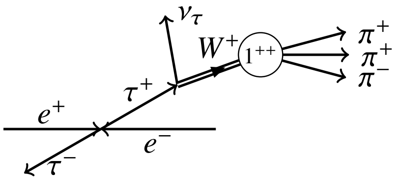
Clean $J^{PC} = 1^{++}$



- V-A: Vector (1^{--}) or Axial (1^{++})
- Isospin 1 due to the charge
- Negative G-parity ⇒ positive C-parity

Analysis of experimental (ALEPH measurements) [data from Phys.Rept.421 (2005)]





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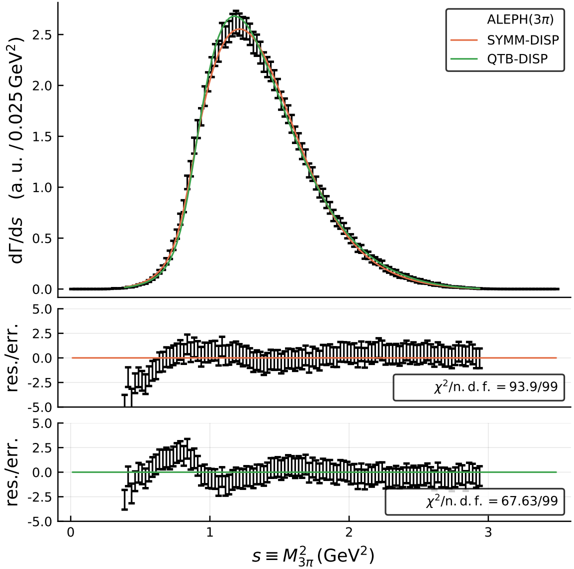
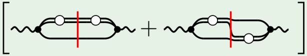
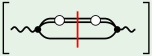


Two models of $\rho\pi$ scattering:

- SYMM-DISP: Approximate three-body unitarity (includes interference)

$\Sigma(s) =$ 

- QTB-DISP: Quasi-two-body unitarity

$\Sigma(s) =$ 

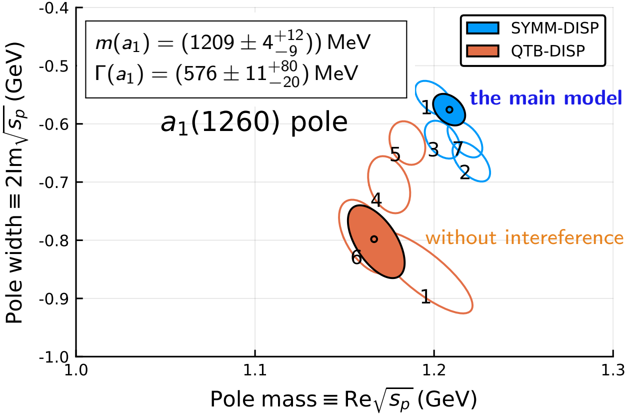
both neglect rescattering, $\mathcal{K} \to 1$.

Fit to the public data

- Stat. cov. matrix is used in the fit
- Syst. cov. matrix – in the bootstrap

Determination of the $a_1(1260)$ pole position

Non-trivial analvtic continuation [MM et al. (JPAC), PRD98 (2018), 096021]



#	Fit studies
1	$s < 2\ \mathrm{GeV}^2$
2	$R' = 3\ \mathrm{GeV}^{-1}$
3	$m'_\rho = m_\rho + 10$ MeV
4	$m'_\rho = m_\rho - 10$ MeV
5	$m'_\rho = m_\rho - 20$ MeV
6	$\Gamma'_\rho = \Gamma_\rho + 5$ MeV
7	$\Gamma'_\rho = \Gamma_\rho - 30$ MeV

- Large systematic uncertainties due to disregard of rescattering effects
- Effect of the subchannel-resonances interference is very important

Summary

- **Unitarity** is an important constraint
  - that guides the amplitude construction [Mai et al.(EPJ A53 (2017)), Jackura et al.(EPJ C79 (2019)), MM et al. (JPAC) 1904.11894]
  - is satisfied in a good FT consideration [R.Briceno, 1905.11188]
  - Separation between the short-range and the long-range is not unique [M.Doering et al., PLB681 (2009) 26-31]
  - Decomposition of the short-range is not unique [A.Jackura et al., 1905.12007]

- The **ladder** is a new phenomenon of the three-particle physics
  - sum of particle exchange diagrams
  - left-hand singularity in the physical region
  - genuine non-factorizable component
- The **resonance** part admits Factorization:
  - Effect of the **Ladder** is the common final-state interaction
  - Unitarity requirement casts to the familiar two-body-like form.

Summary

- **Unitarity** is an important constraint
  - that guides the amplitude construction [Mai et al.(EPJ A53 (2017)), Jackura et al.(EPJ C79 (2019)), MM et al. (JPAC) 1904.11894]
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Outlook

Better understanding of the exchange processes is needed:

$\Rightarrow$ studies of the final-state interaction in the decays

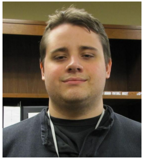
$\Rightarrow$ studies of the fix-target production data (COMPASS, GlueX)

$\Rightarrow$ **studies of the nice, clean scattering data from the lattice**

Thank you for attention

Thanks to my collaborators (JPAC group):

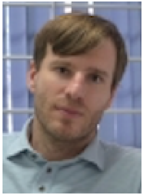

Yannick Wundelich


Andrew Jakura


Alessandro Pilloni


Vincent Mathieu


Miguel Albaladejo

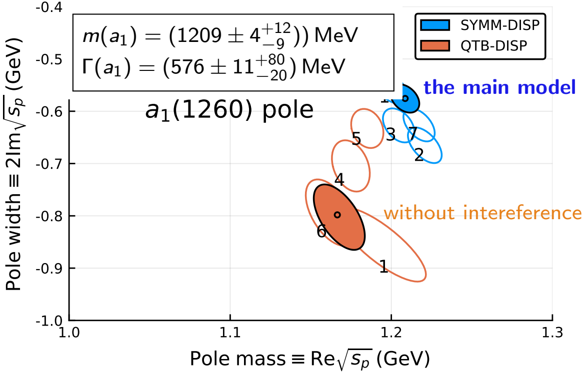

Cesar Fernandez


Bernhard Ketzer


Adam Szczepaniak

Backup

Systematic studies



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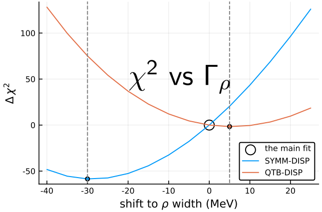
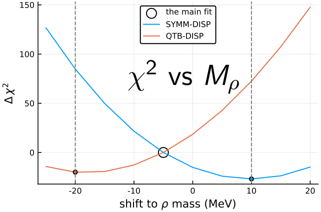
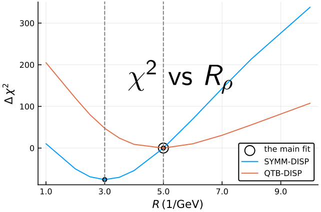






Tour to the complex plane

[MM (JPAC), PRD98 (2018), 096021]

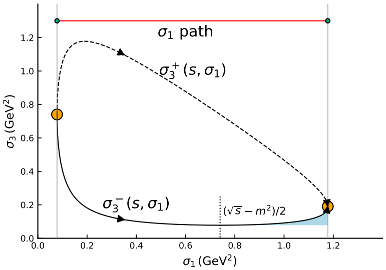
Analytical continuation

$$|t_{ll}^{-1}(s)| = \left| \frac{m^2 - s}{g^2} - i\left(\frac{\tilde{\rho}(s)}{2} + \rho(s)\right)\right|.$$

- Analytical continuation of $\rho(s)$: integral over the Dalitz plot for the complex energy

$$\rho(s) = \sum_{\lambda} \int \mathrm{d}\Phi_3 \left| f_\rho(\sigma_1) d_{\lambda 0}(\theta_{23}) - f_\rho(\sigma_3) d_{\lambda 0}(\hat{\theta}_3 + \theta_{12}) \right|^2$$

- Analytic contuation of ρ-meson decay amplitude f_ρ



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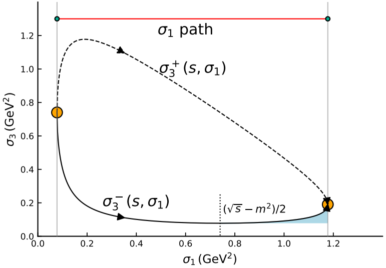

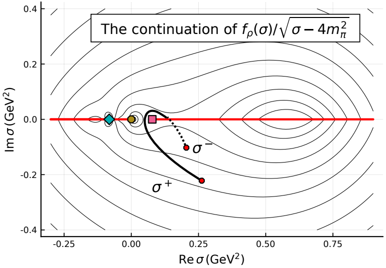

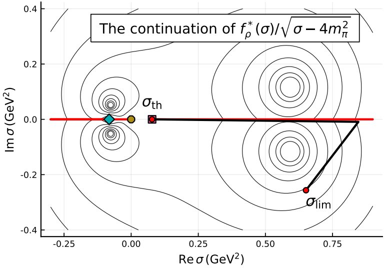

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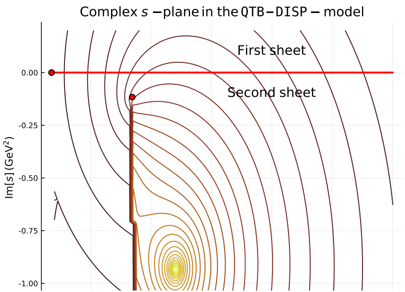

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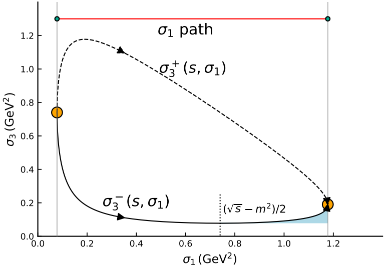

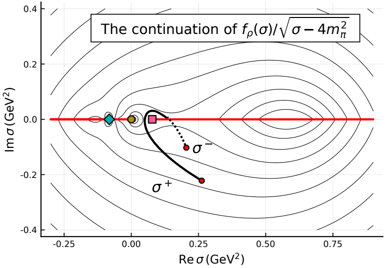

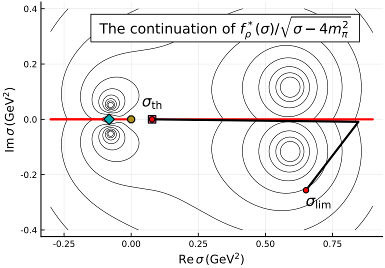

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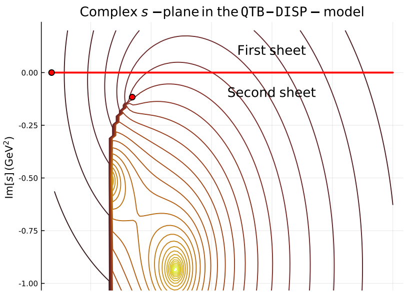

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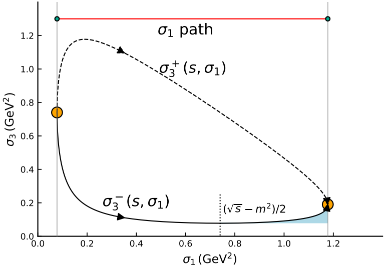
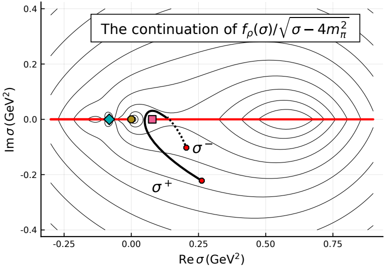
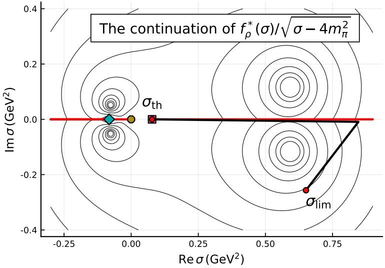
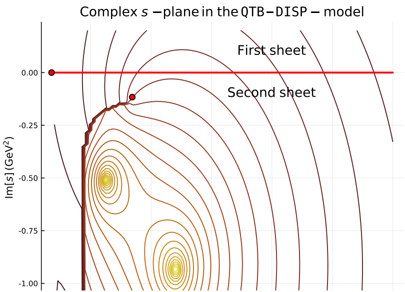
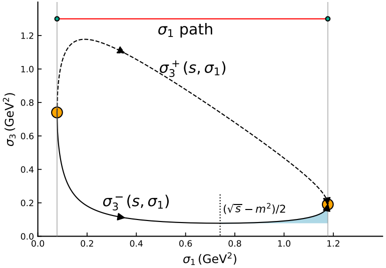






Tour to the complex plane





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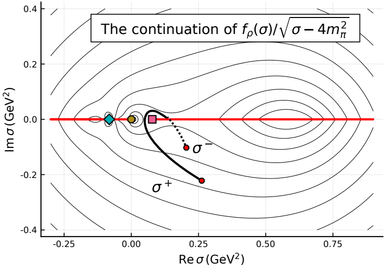
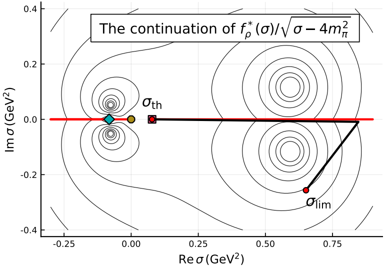
$$\rho(s) = \sum_{\lambda} \int \mathrm{d}\Phi_3 \left| f_\rho(\sigma_1) d_{\lambda 0}(\theta_{23}) - f_\rho(\sigma_3) d_{\lambda 0}(\hat{\theta}_3 + \theta_{12}) \right|^2$$

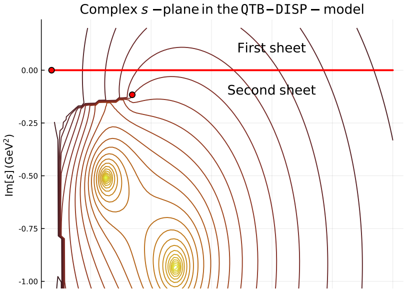
- Analytic contuation of ρ-meson decay amplitude f_ρ





Tour to the complex plane

[MM (JPAC), PRD98 (2018), 096021]



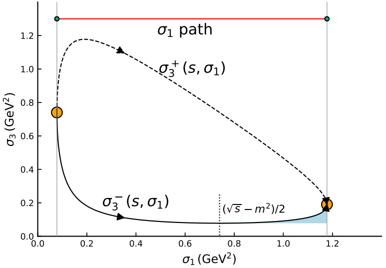
Analytical continuation

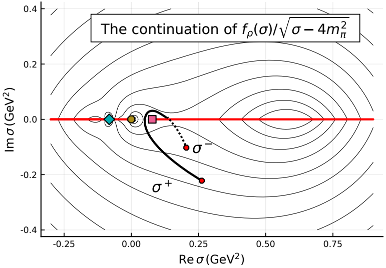
$$|t_{ll}^{-1}(s)| = \left| \frac{m^2 - s}{g^2} - i\left(\frac{\tilde{\rho}(s)}{2} + \rho(s)\right) \right|.$$

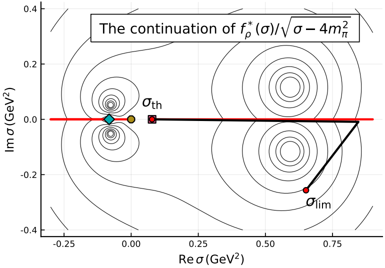
- Analytical continuation of $\rho(s)$: integral over the Dalitz plot for the complex energy

$$\rho(s) = \sum_{\lambda} \int \mathrm{d}\Phi_3 \left| f_\rho(\sigma_1) d_{\lambda 0}(\theta_{23}) - f_\rho(\sigma_3) d_{\lambda 0}(\hat{\theta}_3 + \theta_{12}) \right|^2$$

- Analytic contuation of ρ-meson decay amplitude f_ρ





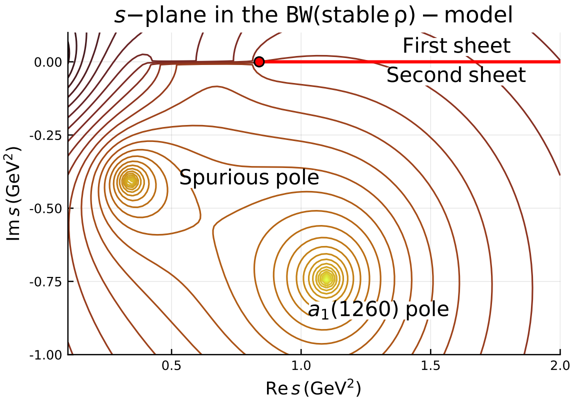


The spurious pole in the Breit-Wigner model

Energy dependent width, stable particles

$$t(s) = \frac{1}{m^2 - s - im\Gamma(s)}, \quad \Gamma(s) = \Gamma_0 \frac{p(s)}{p(m^2)} \frac{m}{\sqrt{s}}, \quad p(s) = \frac{\sqrt{(s-(m_1+m_2)^2)(s-(m_1-m_2)^2)}}{2\sqrt{s}}.$$

Example: $m_1 = 140\,\text{MeV},\ m_2 = 770\,\text{MeV},\ m = 1.26\,\text{GeV},\ \Gamma_0 = 0.5\,\text{GeV}$



Live demo