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UNIVERSITY OF
LIVERPOOL



The Cockcroft Institute
of Accelerator Science and Technology



SEVENTH FRAMEWORK
PROGRAMME

SiPM



EUROPEAN COOPERATION
IN SCIENCE AND TECHNOLOGY



Fast Advanced Scintillator Timing



SENSE

Ultimate Low-Light Level Sensor Development

Principles of Detection and Characteristics of Photosensors

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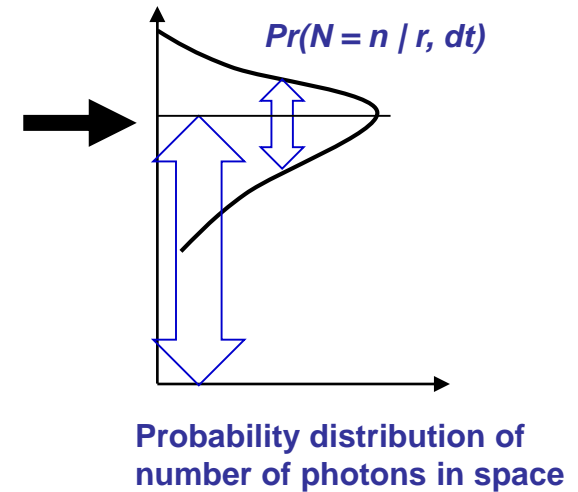
*National Research Nuclear University «MEPhI», Moscow, Russia
Laboratory of Silicon Photomultipliers*

Outline

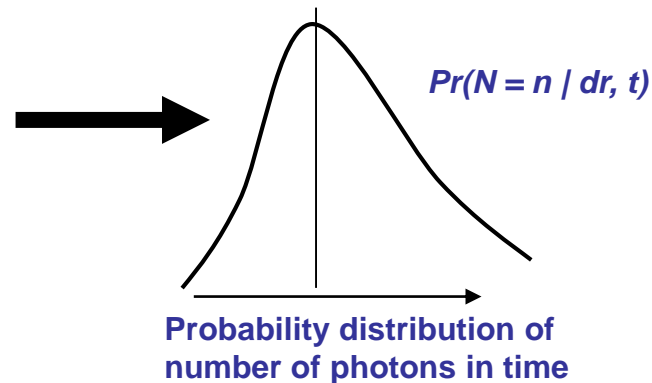
- My focus: probabilistic consideration of detection and detectors
 - ◆ Photon-number-resolving photodetectors
 - Silicon Photomultipliers (SiPM)

Flux of photons – stochastic process

Number of photons – random variable

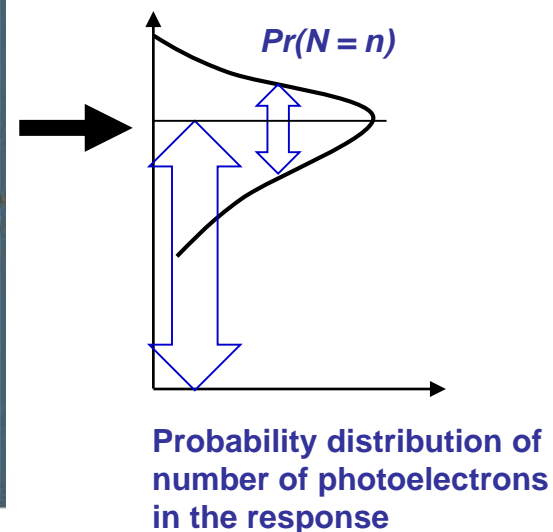
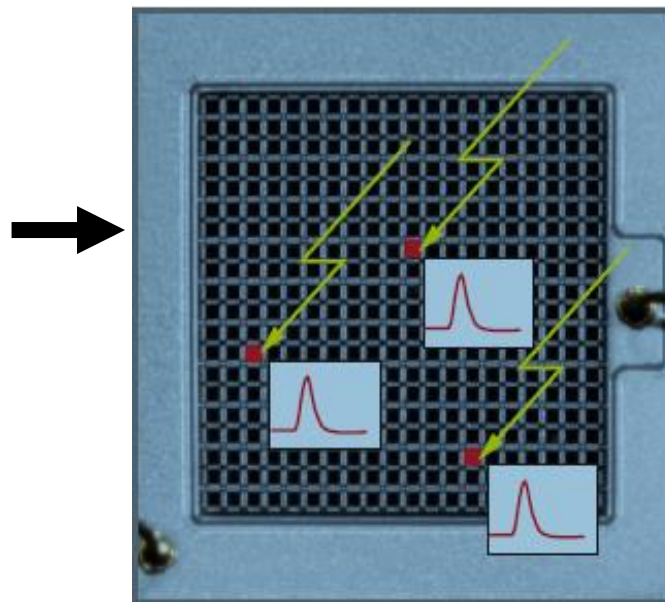
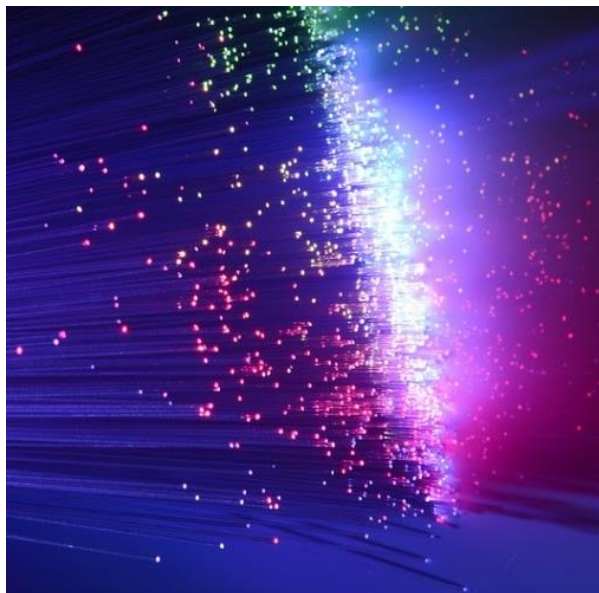


- Probability distribution of a random variable fully characterize the random variable



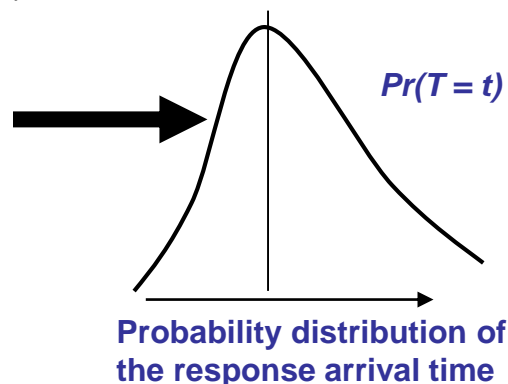
Detection of photons – stochastic process

Detector response (photoelectrons) – random variable



Statistical results of photodetection:

1. Random number of photoelectrons in a time interval (pulse duration) N
 \Rightarrow Energy resolution
2. Random time of the pulse arrival T
 \Rightarrow Time resolution



Spatial resolution \sim pixel / detector size (non-random) in the most of photodetector designs

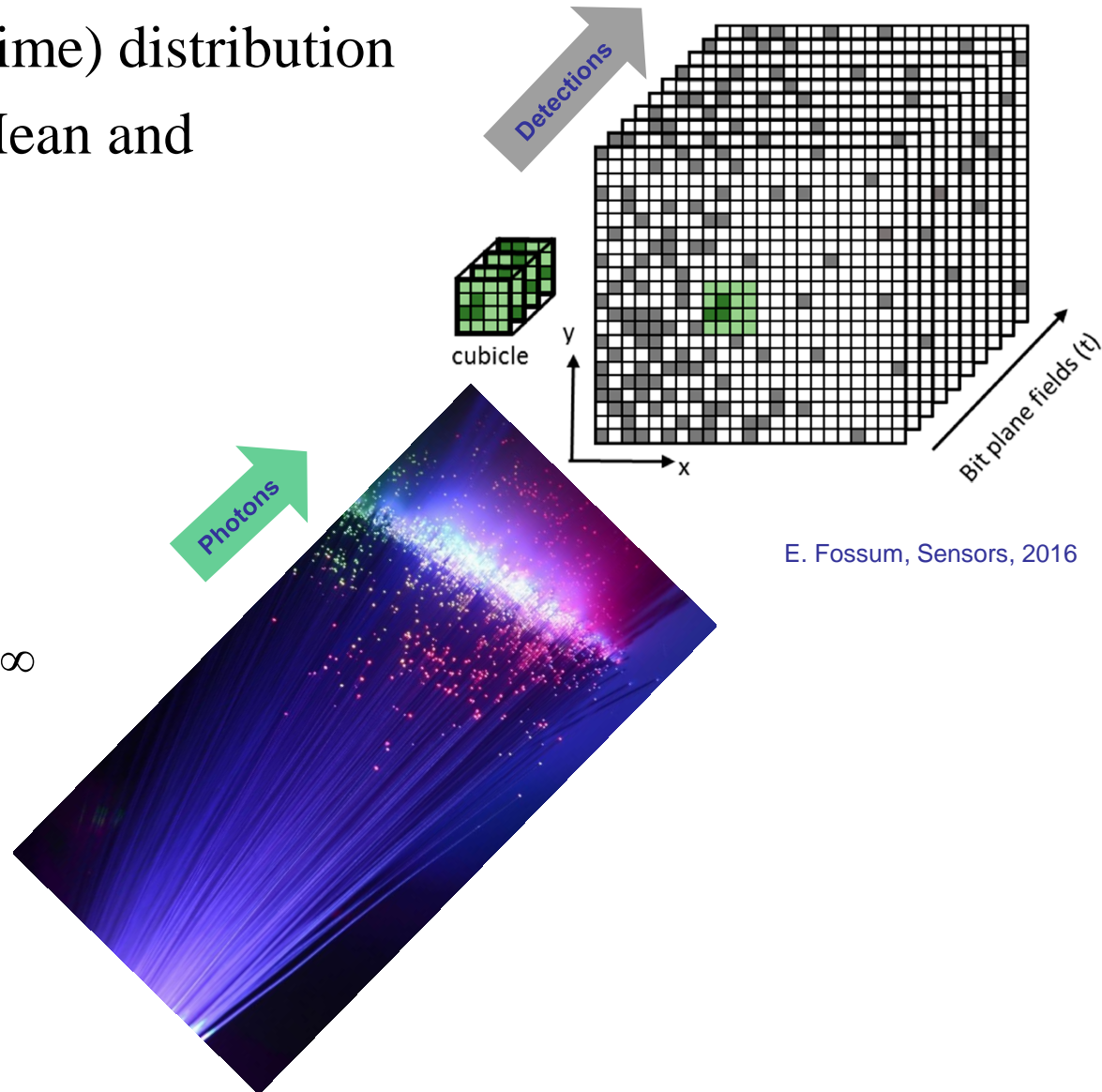
Ultimate photon detection at quantum level

To determine 3D (space-time) distribution of photons with precise Mean and lowest possible StdDev

- ◆ In numbers
- ◆ In time
- ◆ In space

☐ Ideally:

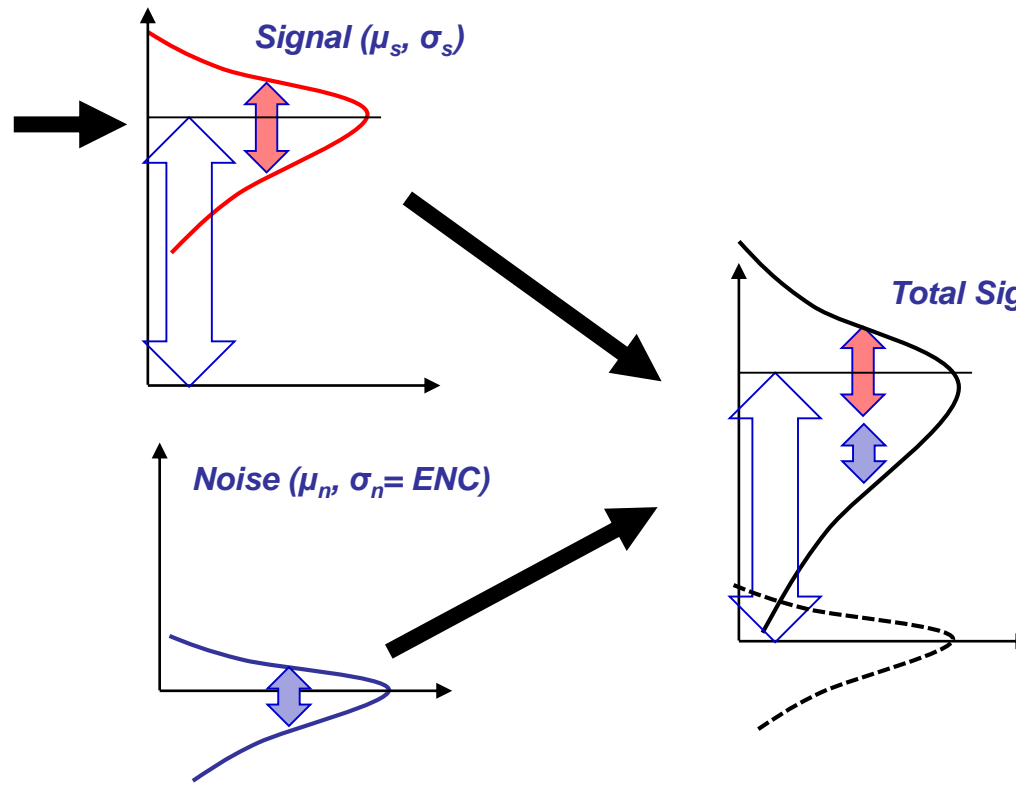
- ◆ From single photons to ∞
- ◆ Without losses
- ◆ Without noises



E. Fossum, Sensors, 2016

Input-dependent characteristics of output (SNR, RES) Internal characteristics of photodetection (DQE, ENF)

- SNR → DQE
- Resolution → ENF



Characteristics of output signal

Signal-to-Noise

$$SNR = \frac{\mu_T - \mu_n}{\sigma_T} = \frac{\mu_S}{\sigma_T} = \frac{\mu_S}{\sqrt{\sigma_S^2 + ENC^2}}$$

Resolution = Noise-to-Signal

$$RES = NSR = \frac{\sigma_T}{\mu_S} = \frac{\sqrt{\sigma_S^2 + ENC^2}}{\mu_S}$$

Characteristics of photodetection

Detective Quantum Efficiency

$$DQE = \frac{SNR_{out}^2}{SNR_{in}^2} = \frac{1}{ENF}$$

Total Excess Noise Factor

$$ENF = \frac{RES_{out}^2}{RES_{in}^2} = \frac{1}{DQE}$$

Resolution (RES) and Excess Noise Factor (ENF) of stochastic processes in photodetection

Total Excess Noise Factor

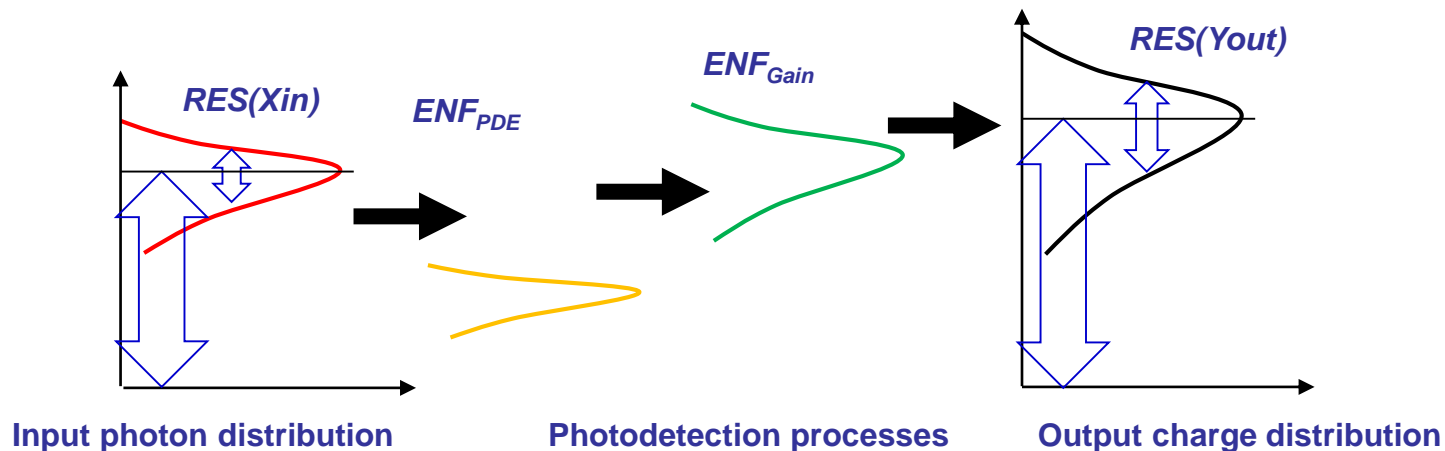
$$ENF(X_{in} = random) = \frac{RES_{out}^2}{RES_{in}^2}$$

Specific Excess Noise Factor

$$ENF(X_{in} = 1) = 1 + \frac{\sigma_{out}^2}{\mu_{out}^2}$$

- Resolution at output = Total ENF-times degraded Resolution at input
- Total ENF is a product of Specific ENFs

$$RES(Y_{out}) = RES(X_{in}) \cdot \sqrt{ENF_{total}} = RES(X_{in}) \cdot \sqrt{ENF_{process1} \cdot ENF_{process2} \dots}$$



Resolution of single photon requires detection with Gain

Input: single photon $\mu_{in} \equiv 1, \sigma_{in} \equiv 0$

Output: some total signal μ_T, σ_T

Total signal is a sum of photosignal (μ_s, σ_s) and electronic noise (0, ENC)

ENC - equivalent noise charge (electrons)

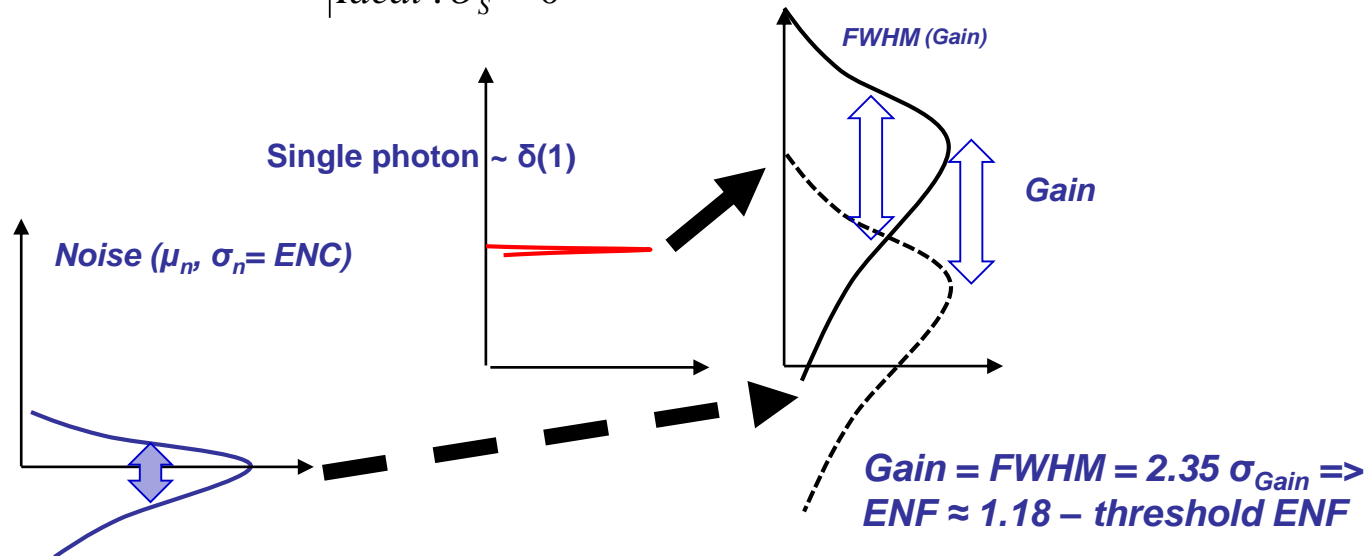
Single photon resolution demands for Gain:

Mean photosignal > Noise => Gain > ENC (FWHM)

$$RES = \frac{\sigma_T}{\mu_s} = \frac{\sqrt{\sigma_s^2 + ENC^2}}{\mu_s} \quad \left| \text{Ideal: } \sigma_s = 0 \right. \quad = \frac{ENC}{\mu_s} = \frac{ENC}{Gain}$$

ENC ~ 10⁴ @ BW ~ 1 GHz or Δt = 1 ns

ENC ~ 10² @ BW ~ 100 KHz or Δt = 10 μs

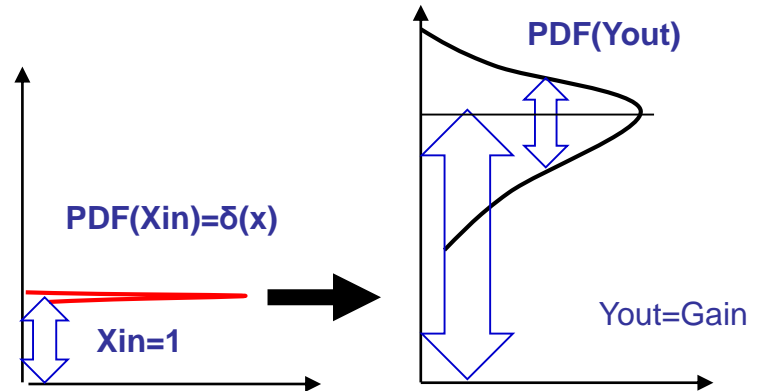


ENF of specific detection processes

☐ Multiplication ($X_{in}=1$)

Y_{out} of any distribution

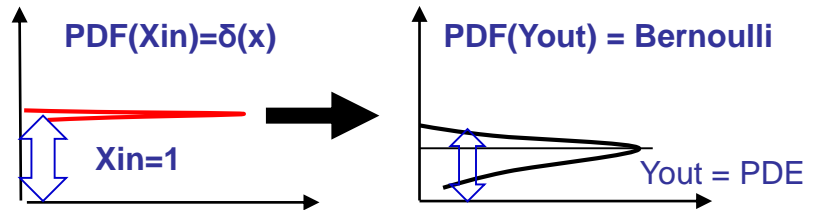
$$ENF_{gain} = 1 + \frac{\sigma_{gain}^2}{Gain^2}$$



☐ Photon detection ($X_{in}=1$)

Bernoulli distribution

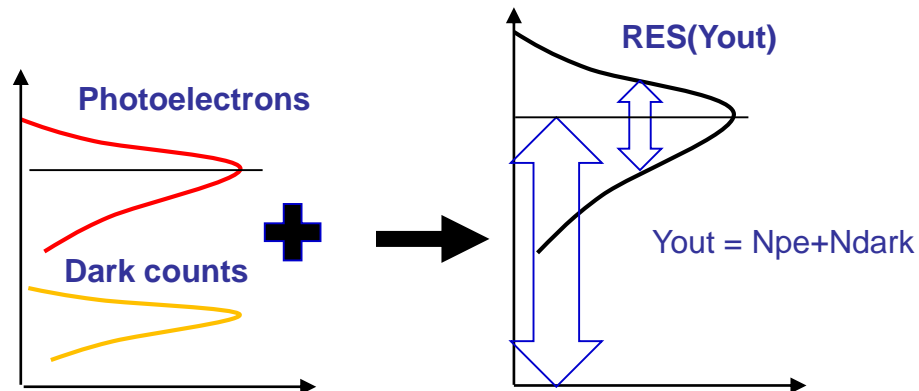
$$ENF_{pde} = 1 + \frac{PDE \cdot (1 - PDE)}{PDE^2} = \frac{1}{PDE}$$



☐ Dark counts ($X_{in}=N_{pe}$)

N_{pe} and N_{dark} ~ Poisson distribution

$$ENF_{dark} = 1 + \frac{N_{dark}}{N_{pe}}$$



Resolution of photodetector for Poisson photons

Incident photons - Poisson (N_{ph}); Output signal - electrons - (μ_{out}, σ_{out})

$$\mu_{out} = N_{ph} \cdot PDE \cdot Gain \quad \sigma_{out}^2 = N_{ph} \cdot PDE \cdot Gain^2 \cdot ENF + ENC^2$$

$$RES_{out} = \frac{\sigma_{out}}{\mu_{out}} = \sqrt{\frac{ENF}{N_{ph} \cdot PDE} + \frac{ENC^2}{(N_{ph} \cdot PDE \cdot Gain)^2}}$$

PDE – Photon Detection Efficiency

ENC – Equivalent Noise Charge, electrons

$ENC \sim 10^4$ @ $BW \sim 1GHz$

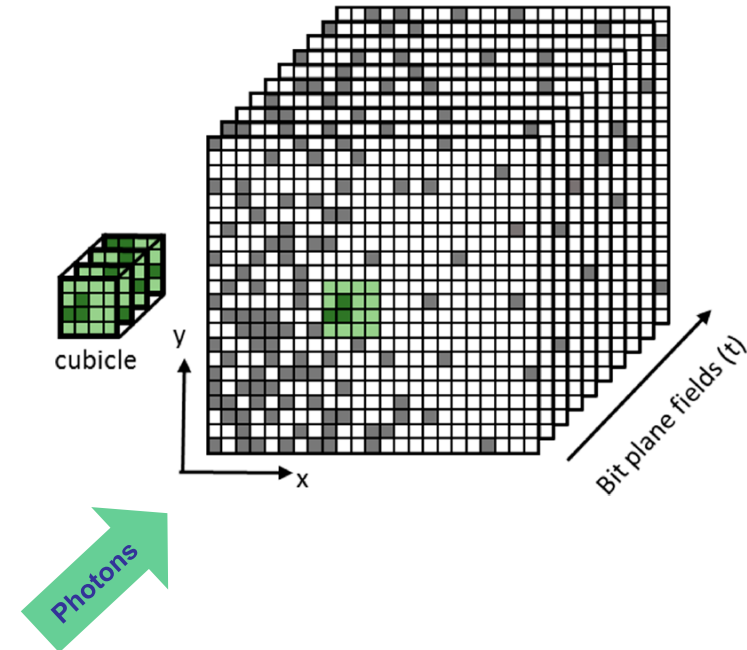
Contributions:

- ◆ Shot noise of photoelectrons
- ◆ Excess noise of multiplication
- ◆ Electronic noise ENC

✓ Ideal photon detector:

- ✓ $PDE = 100\%$
- ✓ $ENC / Gain \rightarrow 0$
- ✓ $ENF = 1$

$$RES_{out} = RES_{in} = \sqrt{\frac{1}{N_{ph}}}$$



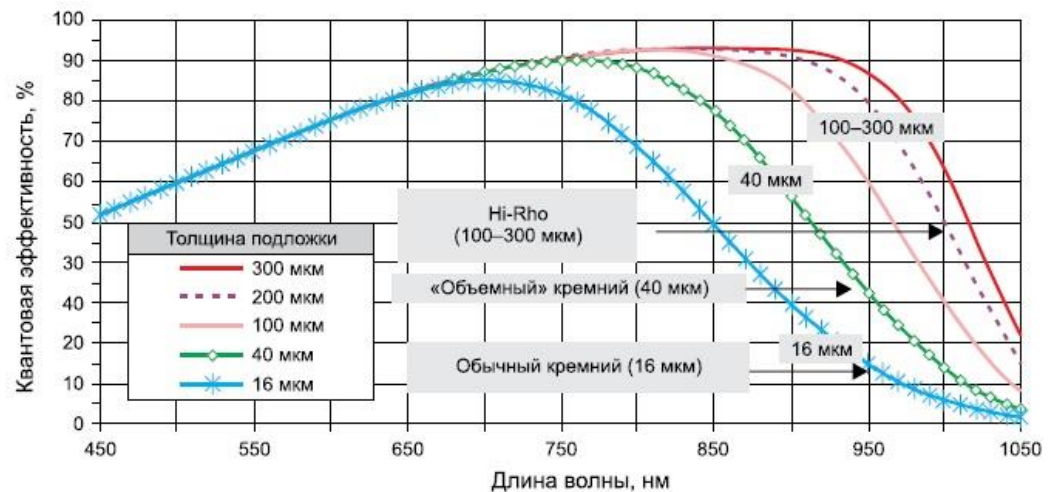
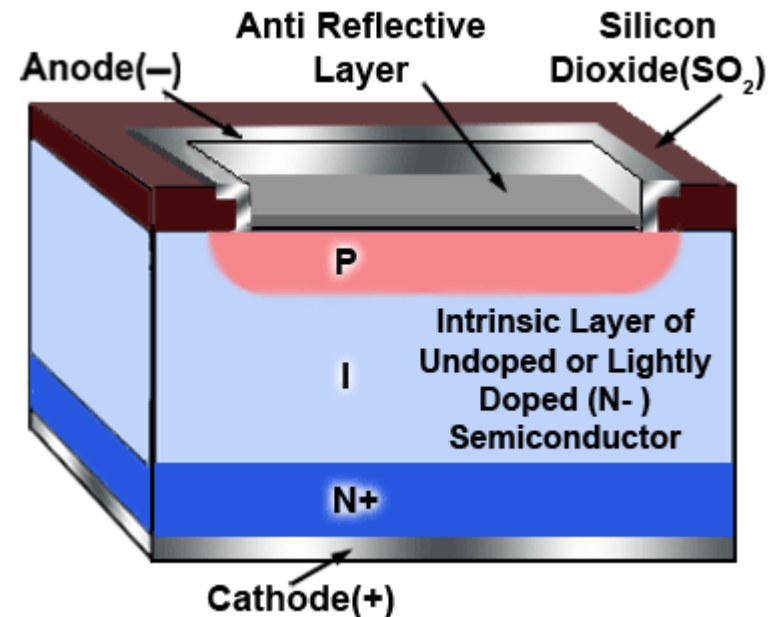
Optimization of Photodiode

$$RES_{out} = \frac{\sigma_{out}}{\mu_{out}} = \sqrt{\frac{ENF}{N_{ph} \cdot PDE} + \frac{ENC^2}{(N_{ph} \cdot PDE \cdot Gain)^2}}$$

✓ Focus on max QE and large signals

- ✓ $N_{ph} \gg ENC$
- ✓ $Gain = 1, ENF = 1$
- ✓ $PDE \sim 80\% - 90\%$,

$$RES(N_{ph}) = \sqrt{\frac{1}{N_{ph} \cdot PDE}}$$



Optimization of Avalanche Photodiode (APD)

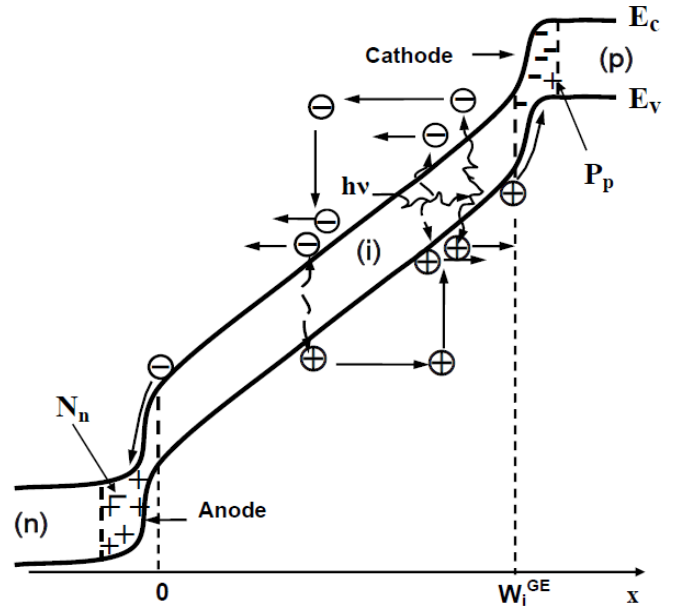
$$RES(N_{ph}) = \frac{\sigma_{out}}{\mu_{out}} = \sqrt{\frac{ENF}{N_{ph} \cdot PDE} + \frac{ENC^2}{(N_{ph} \cdot PDE \cdot Gain)^2}}$$

✓ **Focus on optimal balance: Gain vs ENF** $Gain$

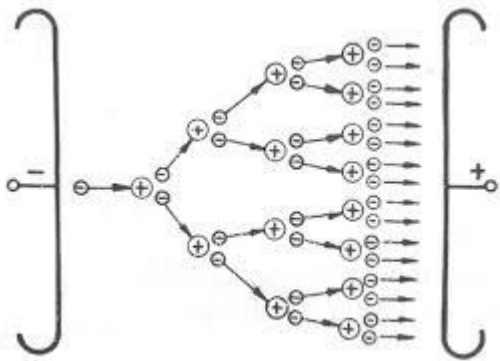
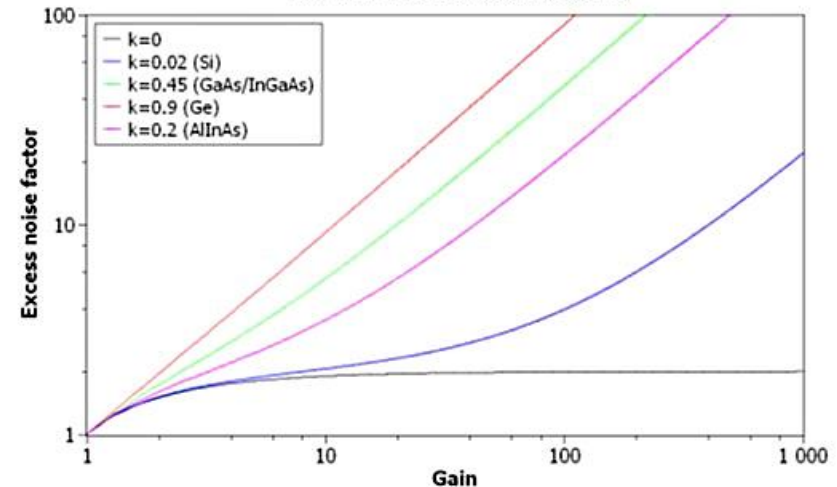
Conventional APD: $ENF(Gain) \sim Gain$

$$Min R \text{ at } Gain \sim \sqrt[3]{2 \frac{ENC^2}{N}} \Big|_{ENC=100, N=1} \sim 27$$

Optimal gain ~ 30 – typical practical value



Excess noise factor vs APD gain for various semiconductors



$$ENF_{gain} = 1 + \frac{\sigma_{gain}^2}{Gain^2}$$

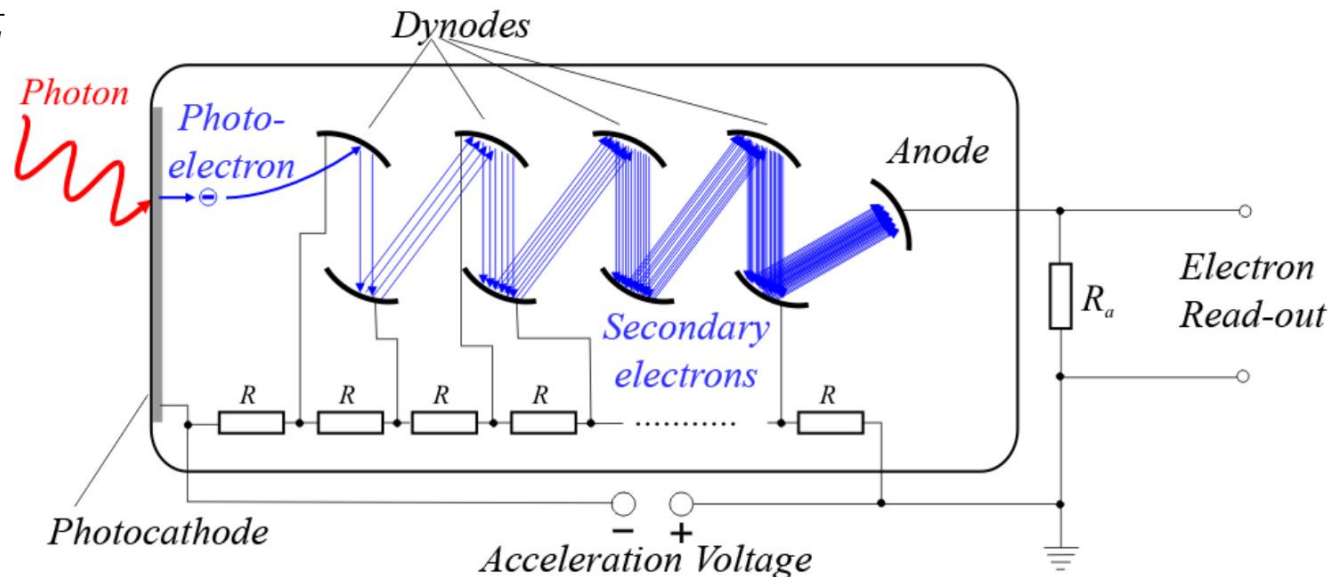
Optimization of vacuum photomultiplier

$$RES(N_{ph}) = \frac{\sigma_{out}}{\mu_{out}} = \sqrt{\frac{ENF}{N_{ph} \cdot PDE} + \frac{ENC^2}{(N_{ph} \cdot PDE \cdot Gain)^2}}$$

Focus on max QE:

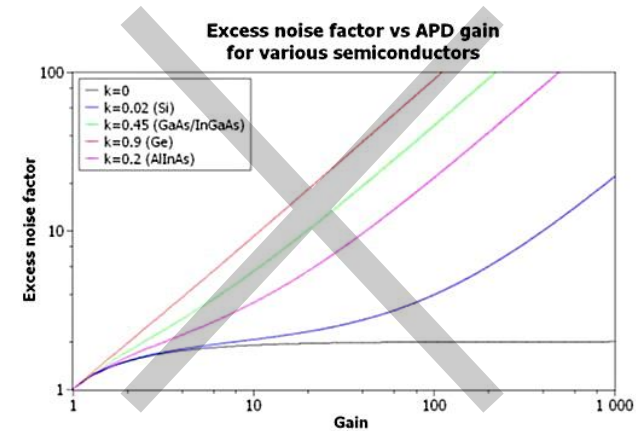
- ✓ $Gain \gg ENC$
- ✓ $ENF \approx 1.1 \dots 1.3$ - is determined by dynode emission of ~ 4 electrons $ENF \sim 1 + \frac{4}{4^2} \sim 1.25$
- ✓ $QE \sim 20\% - 30\%$

$$RES(N_{ph}) = \sqrt{\frac{ENF_{Gain}}{N_{ph} \cdot PDE}}$$



Optimization of Geiger mode APD array

$$RES(N_{ph}) = \frac{\sigma_{out}}{\mu_{out}} = \sqrt{\frac{ENF}{N_{ph} \cdot PDE} + \frac{ENC^2}{(N_{ph} \cdot PDE \cdot Gain)^2}}$$



- ✓ **1. Focus on triggering of Geiger breakdown** and binary detection of single photons

$$RES(N_{ph}) = \sqrt{\frac{1}{N_{ph} \cdot PDE}} \Big|_{N_{ph} \leq 1}$$

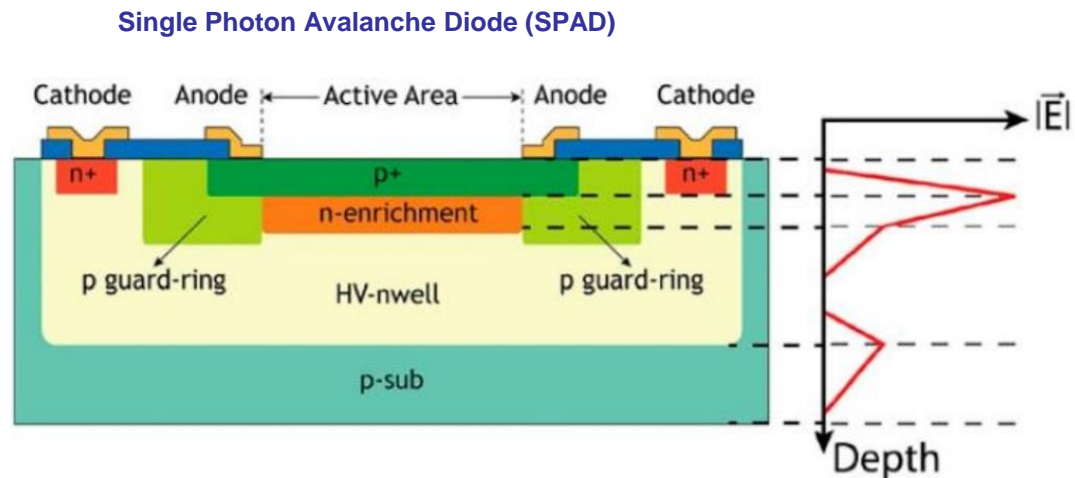
$$PDE = FF \cdot QE \cdot P_{AV}$$

FF – fill factor, geometric efficiency

$FF = \text{Active Area} / \text{Total Area}$

QE – quantum efficiency

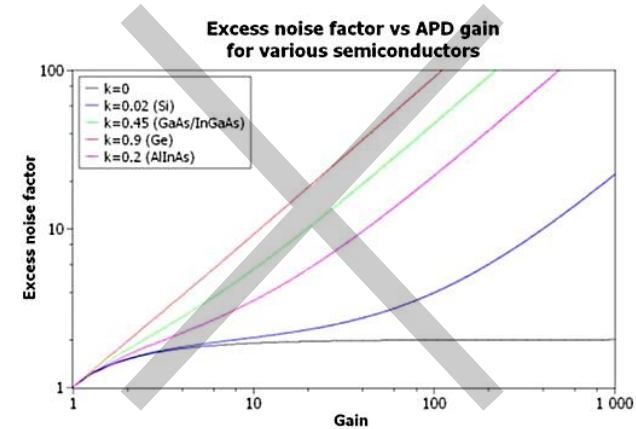
P_{AV} – probability of avalanche



A. Tosi, F. Zappa, MiSPiA: microelectronic single-photon 3D imaging arrays, Proc. SPIE (2013)

Optimization of Geiger mode APD array

$$RES(N_{ph}) = \frac{\sigma_{out}}{\mu_{out}} = \sqrt{\frac{ENF}{N_{ph} \cdot PDE} + \frac{ENC^2}{(N_{ph} \cdot PDE \cdot Gain)^2}}$$



- ✓ **2. Focus on active quenching of Geiger breakdown** (electronics => **inactive** area)

$$RES(N_{ph}) = \sqrt{\frac{1}{N_{ph} \cdot PDE}}$$

$$PDE = FF \cdot QE \cdot P_{AV}$$

FF – fill factor, geometric efficiency

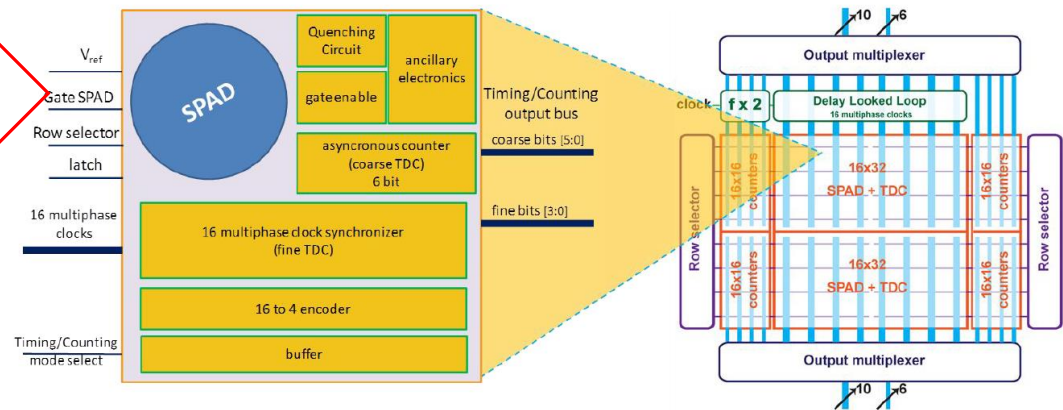
$FF = \text{Active Area} / \text{Total Area}$

QE – quantum efficiency

P_{AV} – probability of avalanche

FF = 3%

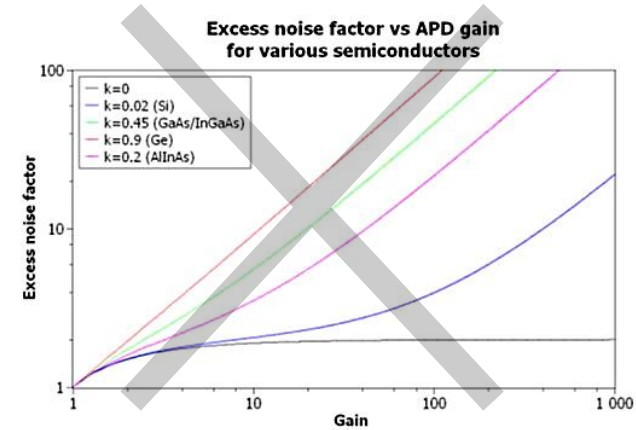
Single Photon Avalanche Diode (SPAD)



A. Tosi, F. Zappa, MiSPiA: microelectronic single-photon 3D imaging arrays, Proc. SPIE (2013)

Silicon Photomultiplier

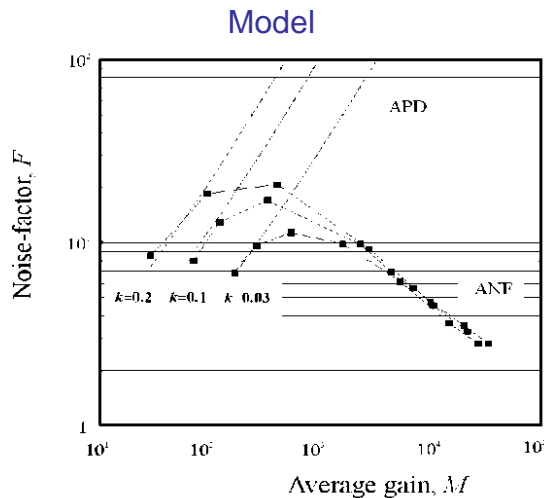
$$RES(N_{ph}) = \frac{\sigma_{out}}{\mu_{out}} = \sqrt{\frac{ENF}{N_{ph} \cdot PDE} + \frac{ENC^2}{(N_{ph} \cdot PDE \cdot Gain)^2}}$$



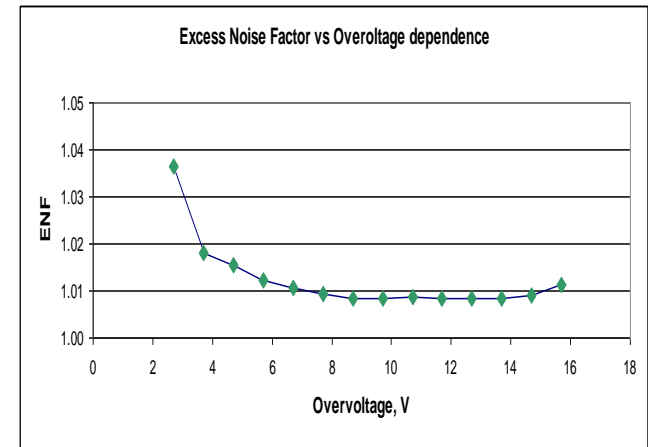
✓ Focus on high Gain low excess noise Geiger breakdown

- ✓ $Gain \gg ENC$
- ✓ $ENF_{Gain} \rightarrow 1$
- ✓ $PDE \rightarrow max$

$$RES(N_{ph}) = \sqrt{\frac{ENF}{N_{ph} \cdot PDE}}$$



Experiment



[Model] D. Shushakov, V. Shubin, Proc. SPIE (1995)

[Experiment] K. Linga, E. Godik, J. Krutov, D.A. Shushakov, V. Shubin, S. Vinogradov, E. Levin, Proc. SPIE (2006)

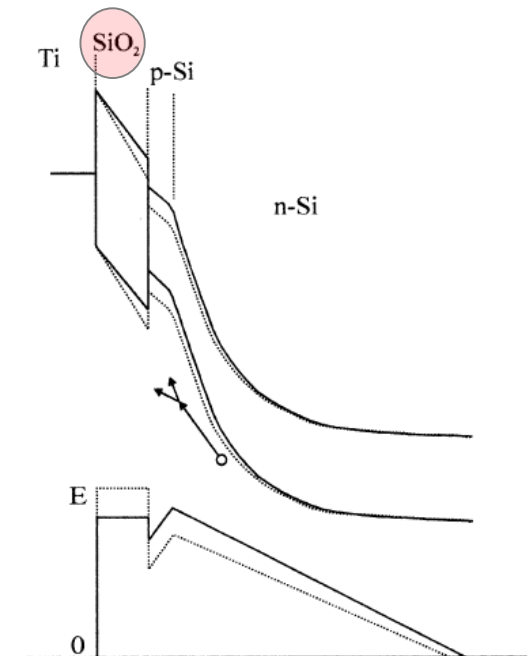
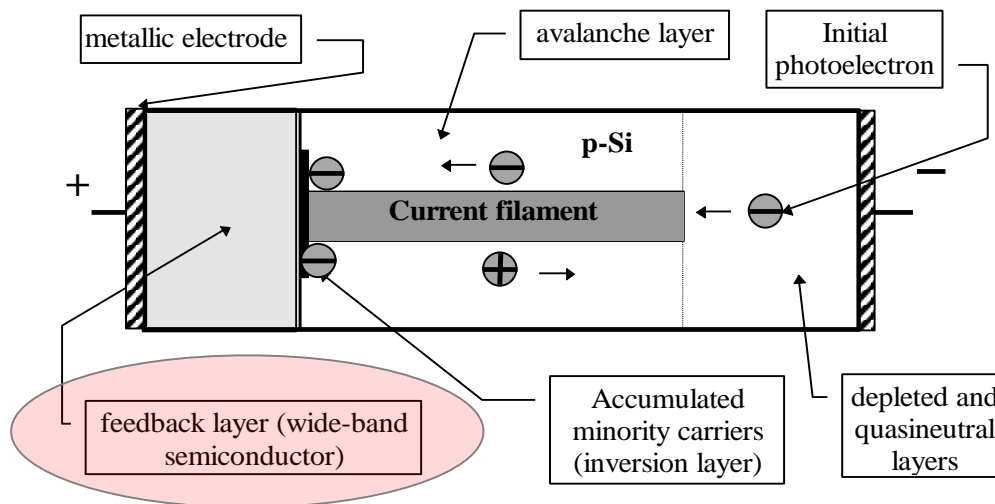
SiPM concept:

Basic studies at Lebedev Physical Institute (1970s-1990s)

■ Avalanche with Negative Feedback (ANF) concept:

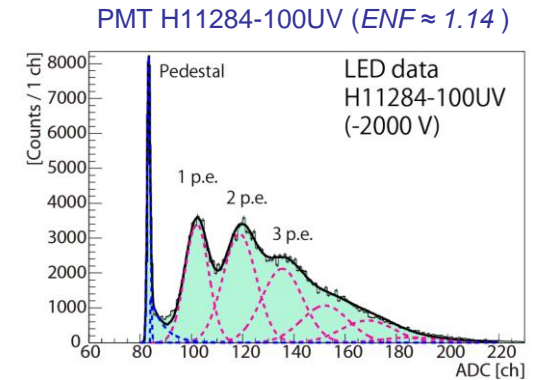
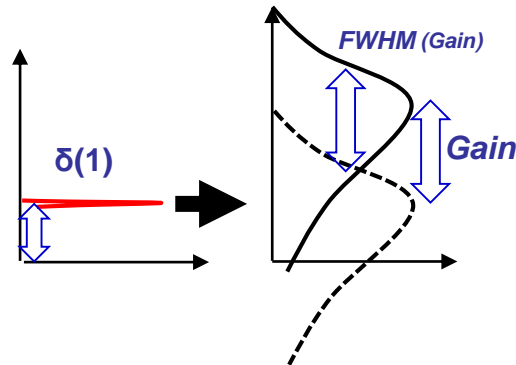
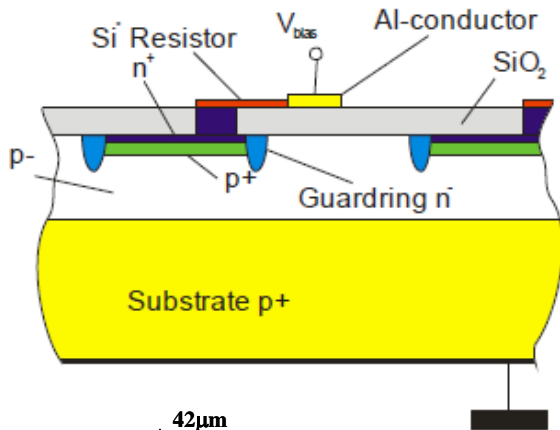
- ◆ Avalanche development by strong positive feedback
→ Geiger mode avalanche breakdown with high overvoltage
- ◆ Avalanche quenching by strong negative feedback
→ Passive quenching by built-in elements (ANF APD)
 - Capacitive (feedback ~ avalanche charge)
 - Resistive (feedback ~ avalanche current)

ANF
=>
High gain
+
Low noise
+
Fast time



V. Shubin, D. Shushakov, Encyclopedia of Optical Engineering 2003; DOI: 10.1081/E-EOE 120009727

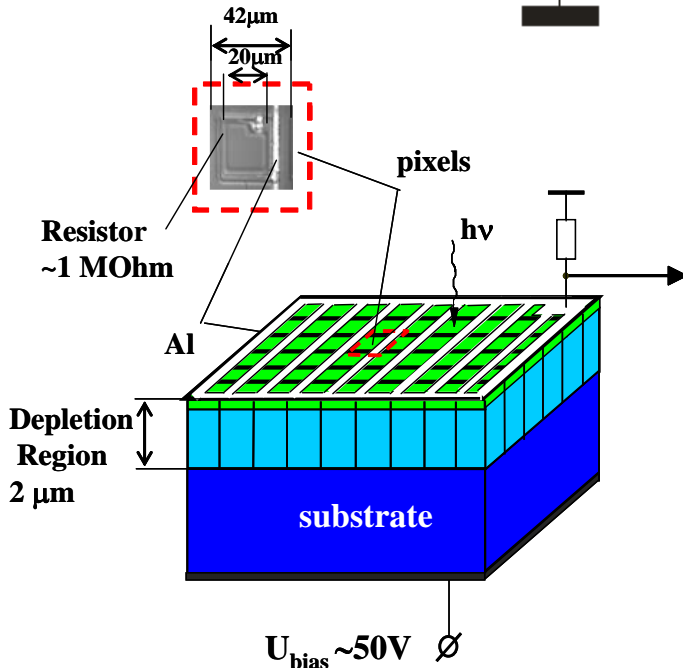
Кремниевый фотоумножитель Silicon Photomultiplier (SiPM)



$$Gain = FWHM = 2.35 \sigma_{Gain} \Rightarrow$$

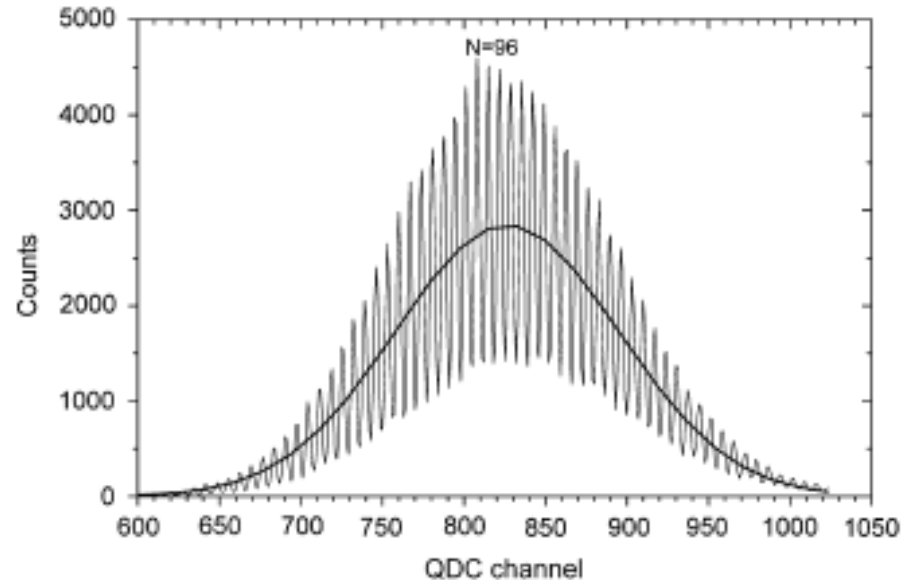
$ENF \approx 1.18$ - порог разрешения 1 фотона

T. Cogami et al, NIMA 2016



SiPM, B. Dolgoshein et al., MEPhI / Pulsar, 2000-2003

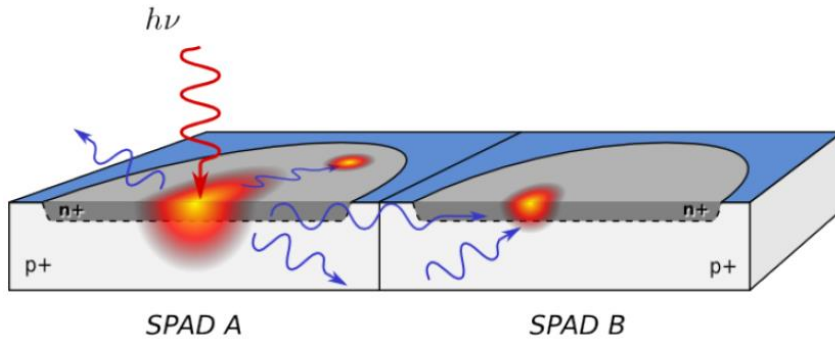
SiPM (MEPhI/Pulsar)



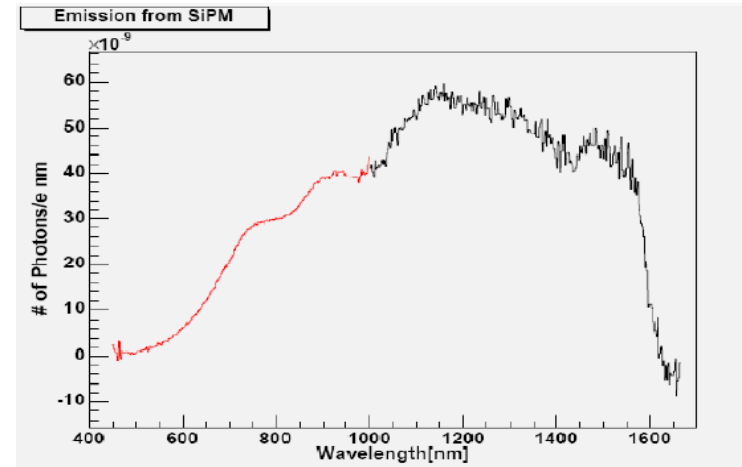
R. Mirzoyan et al., NDIP, 2008

Correlated avalanche events: SiPM crosstalk (CT) \rightarrow ENF_{CT}

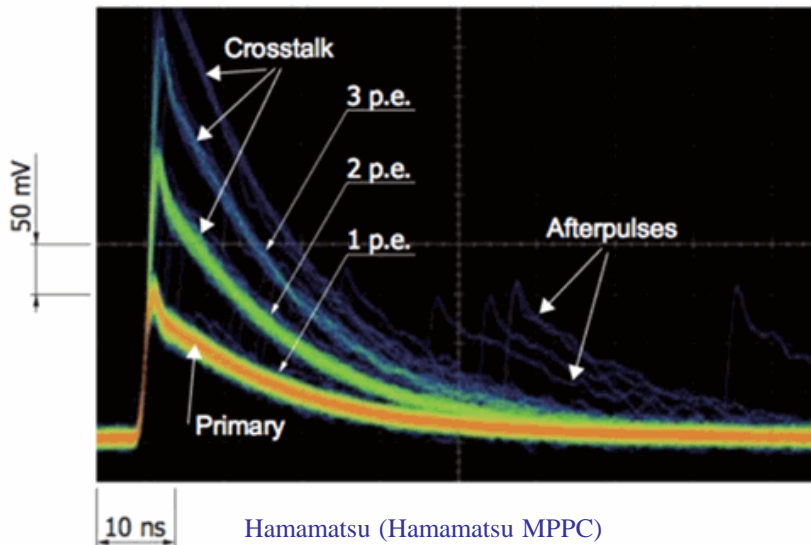
Hot carrier photon emission



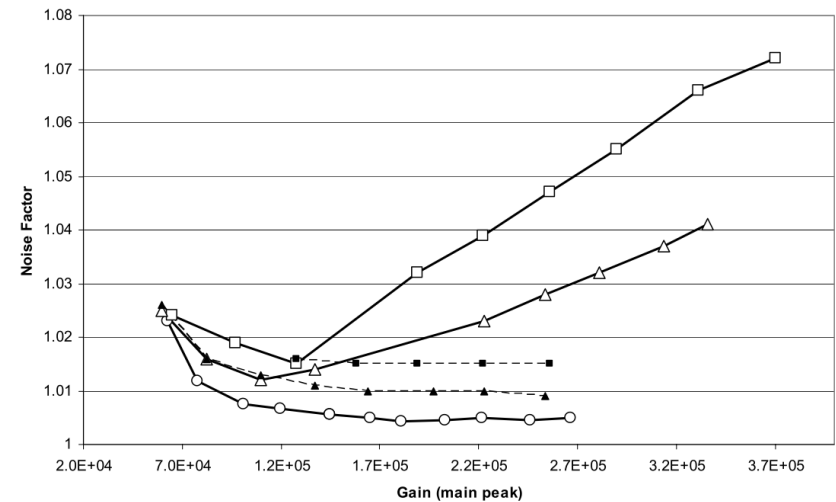
A. Lacaita et al., IEEE TED, 1993



R. Mirzoyan, NDIP, 2008



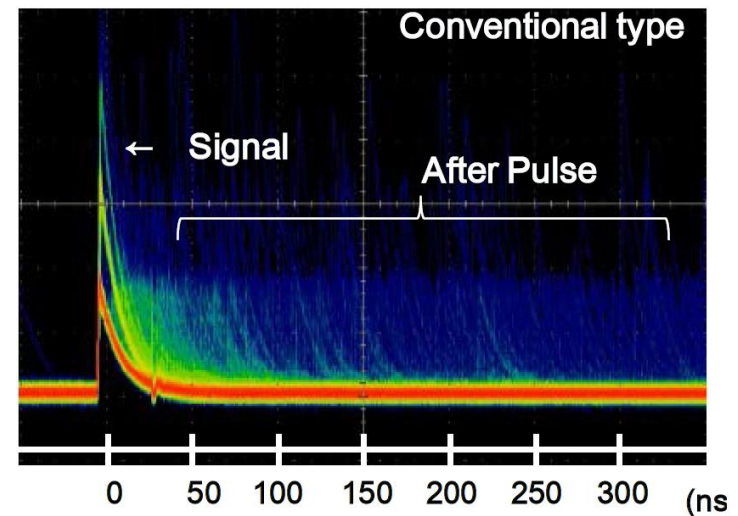
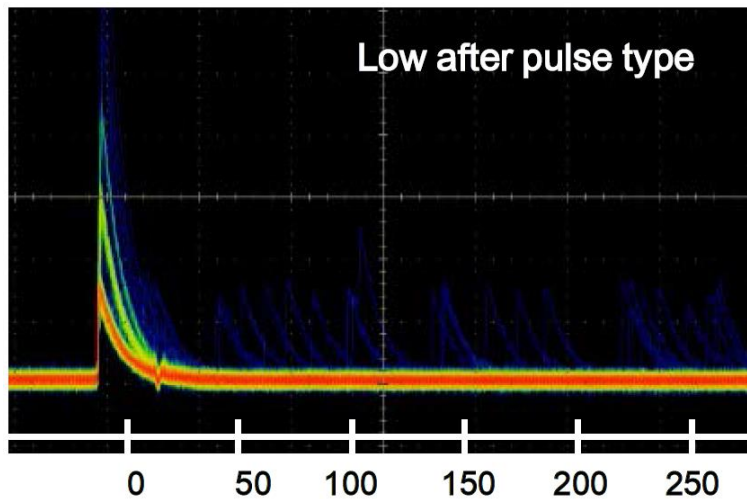
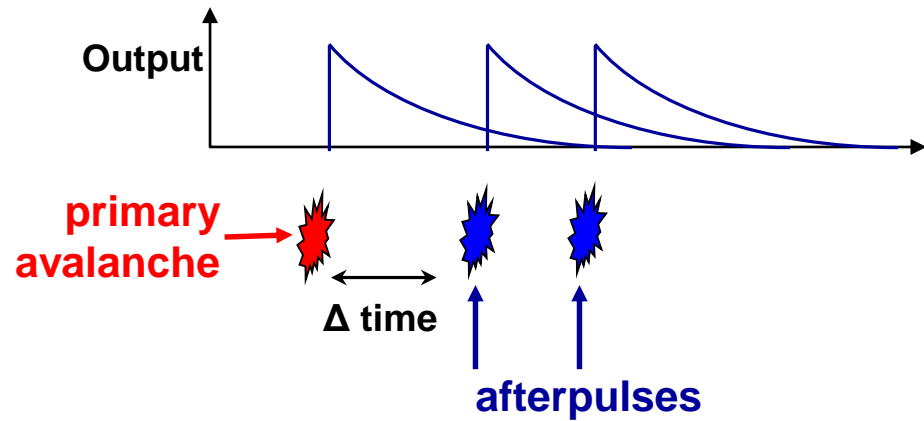
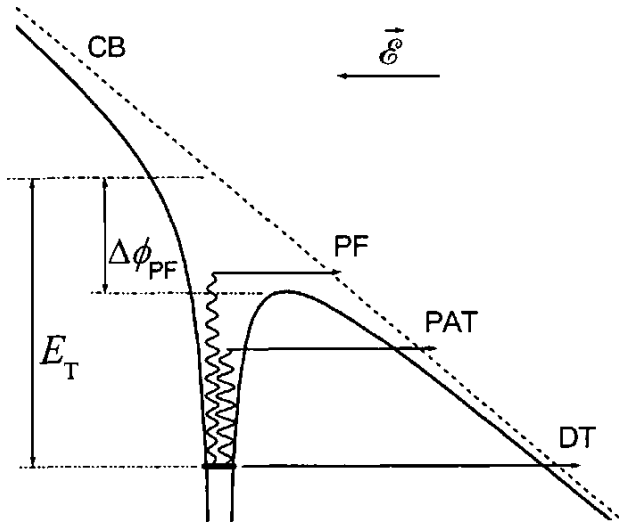
Hamamatsu (Hamamatsu MPPC)



K. Linga et al, Proc. SPIE (2006)

Correlated avalanche events: SiPM afterpulsing (AP) \rightarrow ENF_{AP}

- Afterpulsing: electron/hole trapping – emission – new avalanche



SiPM nonlinearity: Random losses $\rightarrow ENF_{NL}$

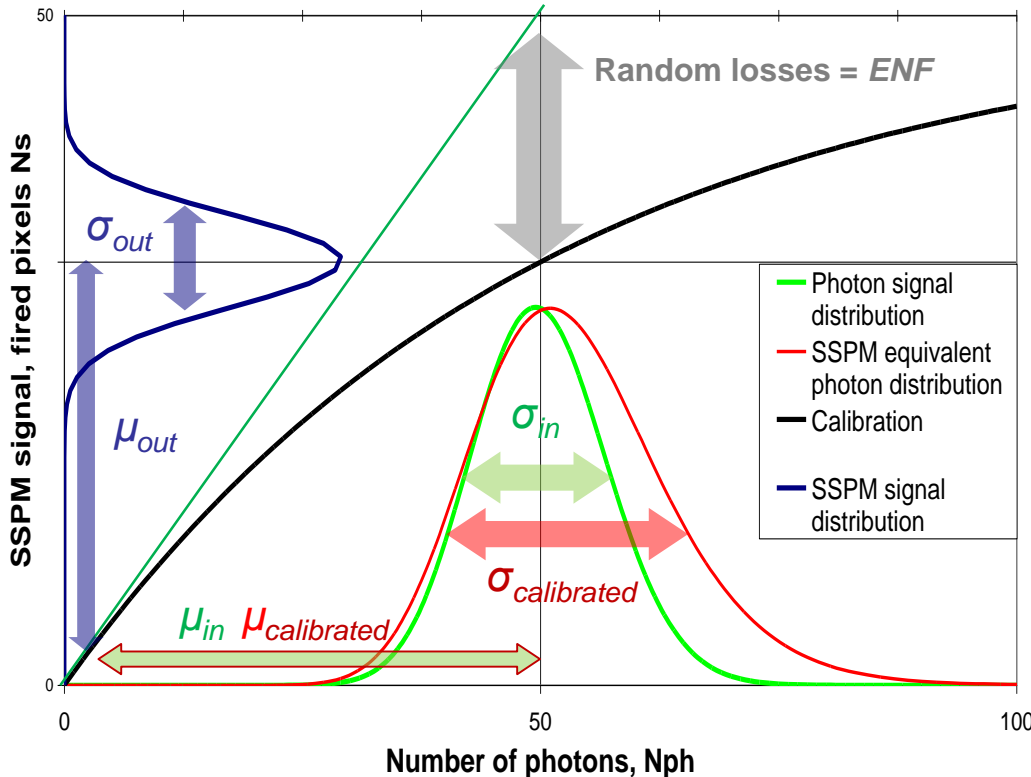
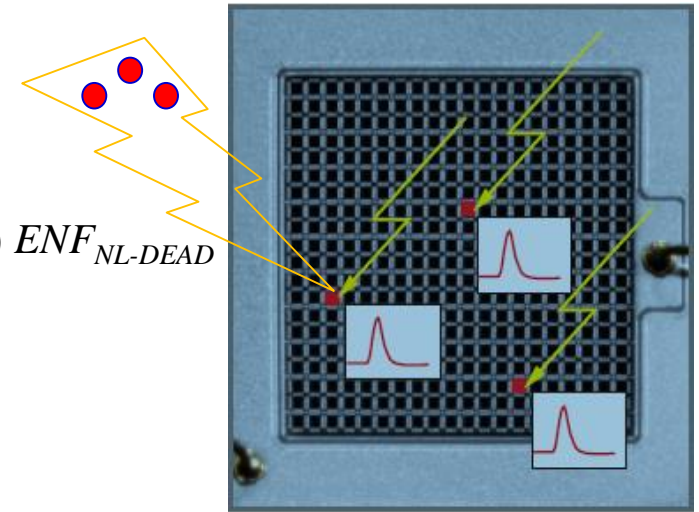
Нелинейность \Rightarrow потери фотонов $\Rightarrow ENF_{NL}$

$ENF \Rightarrow$ деградация разрешения Nph

Нелинейность SiPM:

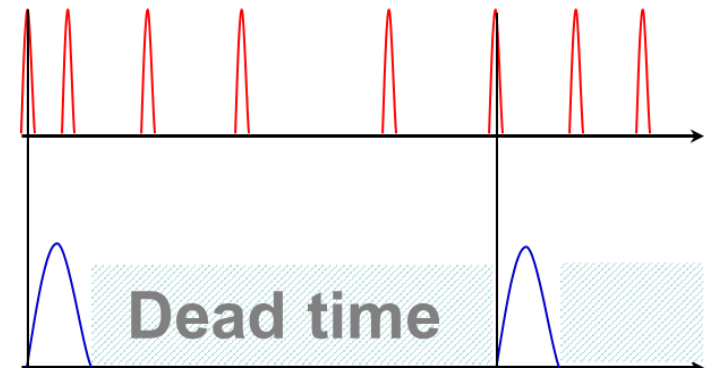
- ◆ 1) Из-за ограниченного числа ячеек ENF_{NL-PIX}
- ◆ 2) Из-за мертвого времени (восстановления ячейки) $ENF_{NL-DEAD}$

1) По ячейкам



2) По мертвому времени

Photoelectron mean arrival rate per pixel = λ



Detective Quantum Efficiency (DQE)

Resolution

$$RES_{out} = \frac{\sigma_{out}}{\mu_{out}} = \sqrt{\frac{ENF}{N_{ph} \cdot PDE} + \frac{ENC^2}{(N_{ph} \cdot PDE \cdot Gain)^2}}$$

SNR

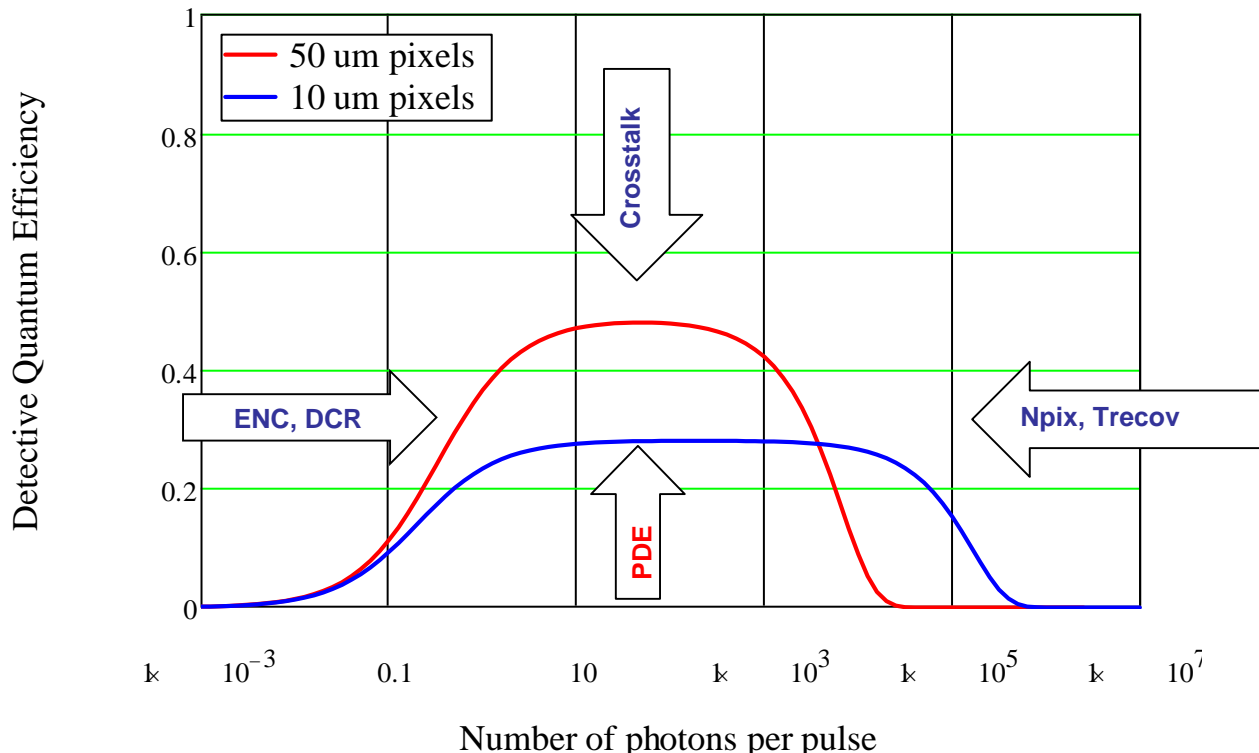
$$SNR_{out} = \frac{\mu_{out}}{\sigma_{out}}$$

DQE

$$DQE = \frac{SNR_{out}^2}{SNR_{in}^2}$$

SiPM (and others with Gain >> ENC):

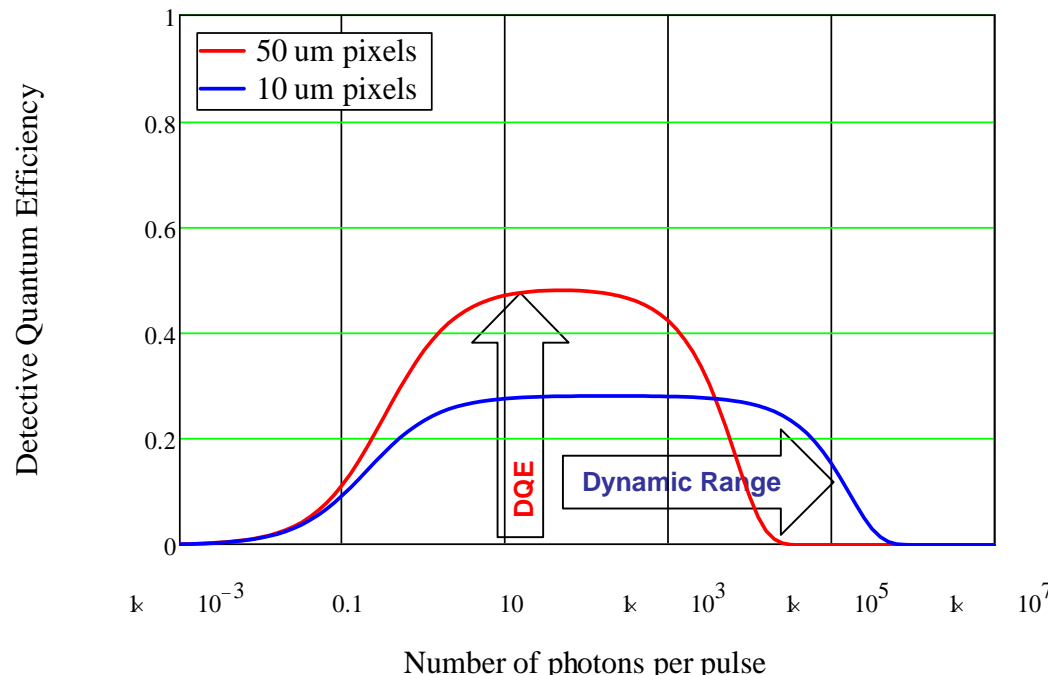
$$DQE = \frac{PDE}{ENF} = \frac{FF \cdot QE \cdot P_{AV}}{ENF_{Gain} \cdot ENF_{CT} \cdot ENF_{AP} \cdot ENF_{NL} \cdot ENF_{DCR}}$$



Main trends in SiPM R&D

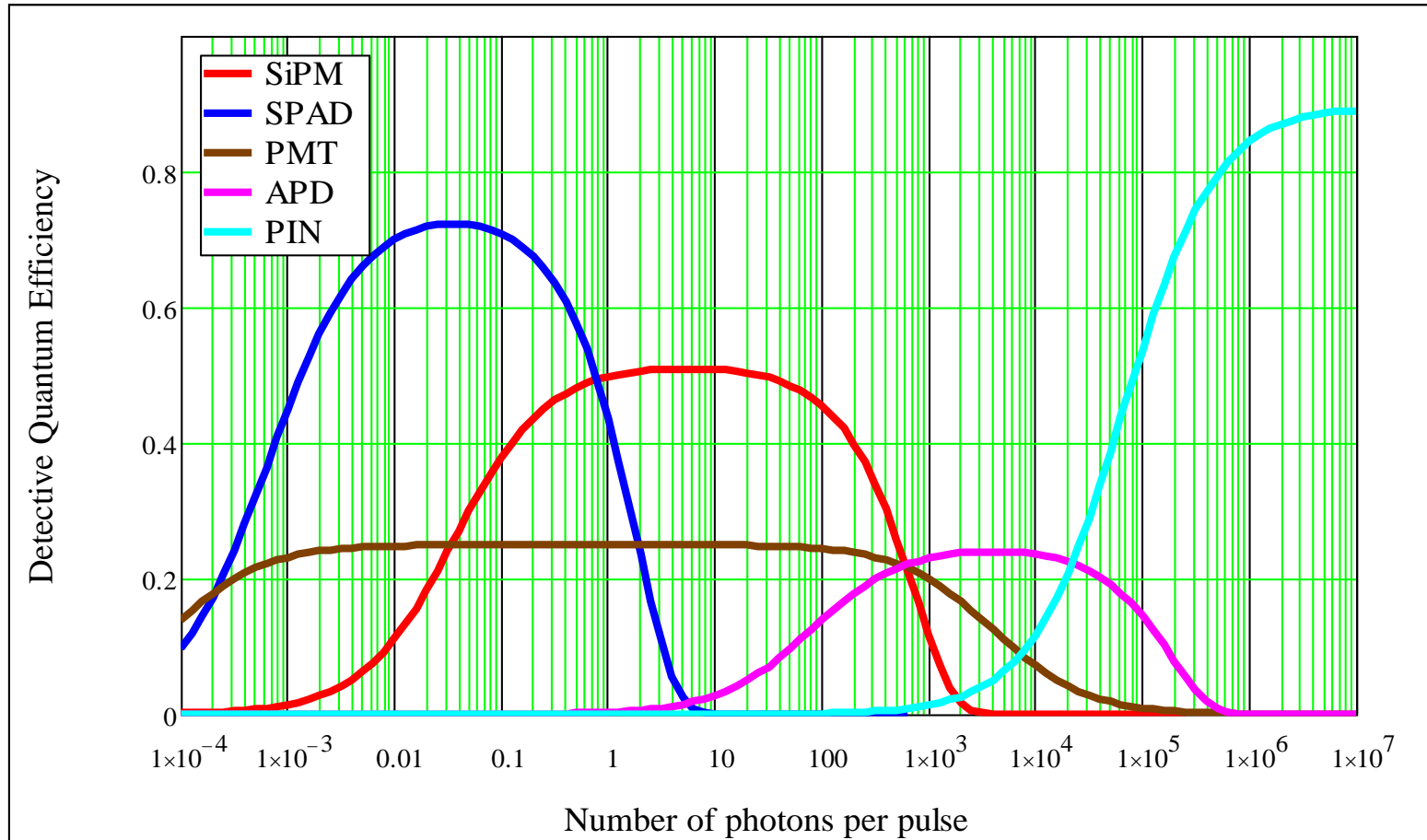
1. Max $QE, P_{AV} \Rightarrow \max$ PDE & DQE – beneficial for any applications and designs
2. High DQE (\sim in a limited Dynamic Range)
 - «Big cells», “Scientific SiPM”
3. High Dynamic Range (\sim with a limited DQE)
 - «Small cells», “HDR / UHDR SiPM”

$$DQE = \frac{PDE}{ENF} = \frac{FF \cdot QE \cdot P_{AV}}{ENF_{Gain} \cdot ENF_{CT} \cdot ENF_{AP} \cdot ENF_{NL} \cdot ENF_{DCR}}$$



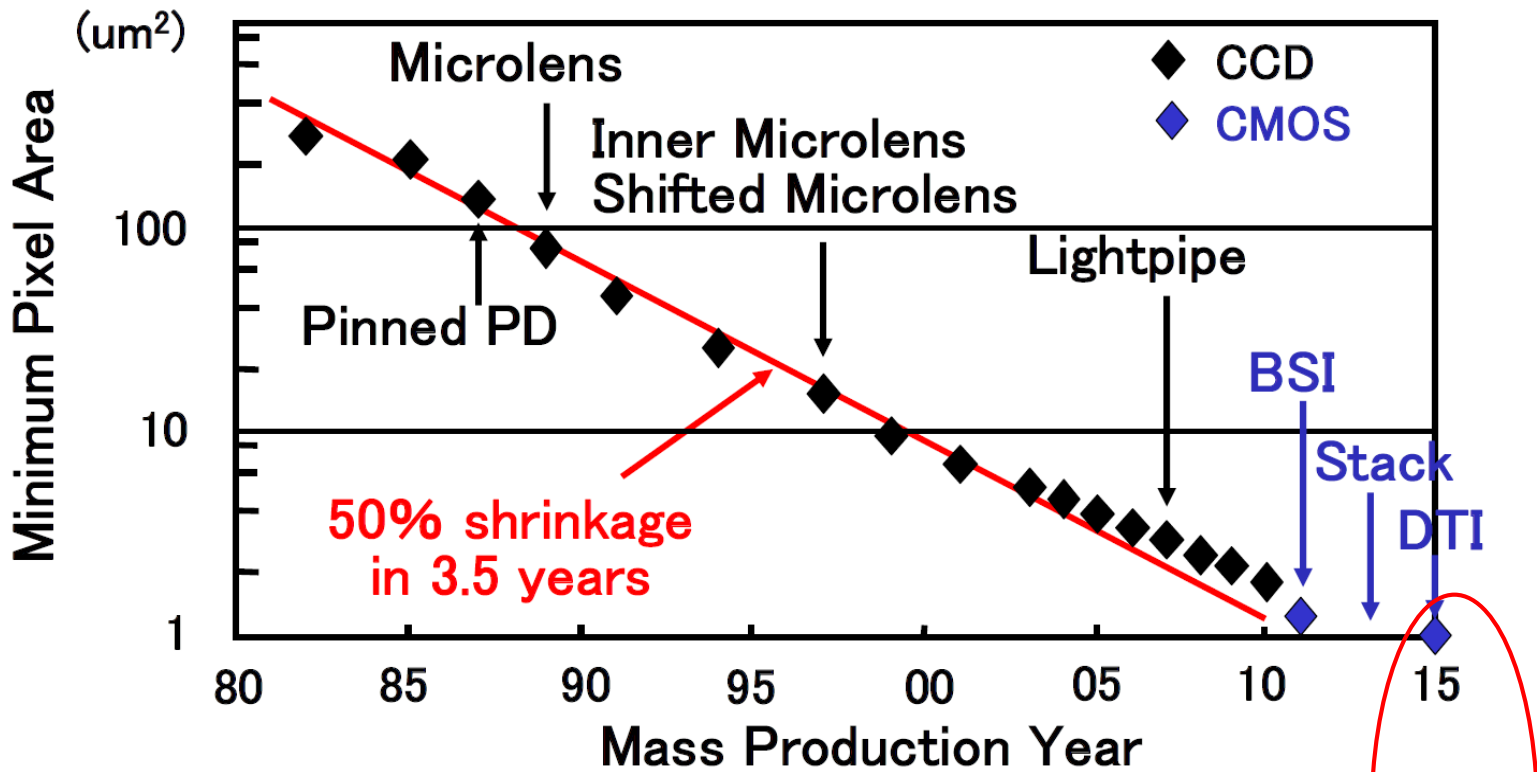
DQE of photodetectors: Map of competitiveness

- Detection of 10 ns pulse @ max spectral sensitivity



Evolution of CCD/CMOS imagers

- Shrinkage speed becomes slower recently.
- In 2017, 0.9 um pixel began to be mass produced.



- In 2015, several organization reported low noise $< 0.3 e\text{-rms}$

N. Teranishi, Recent Progresses of Visible Light Image Sensors, Cern Detect. Semin. (2018).

$$RMS_{noise} = \frac{\sqrt{kTC}}{q} [electrons] \quad Si\text{-photodiode of } 1 \times 1 \times 10 \text{ um}^3 \Rightarrow RMS_{noise} \sim 1.3 e$$

Pinned Photodiode (PPD) and modern CMOS imagers

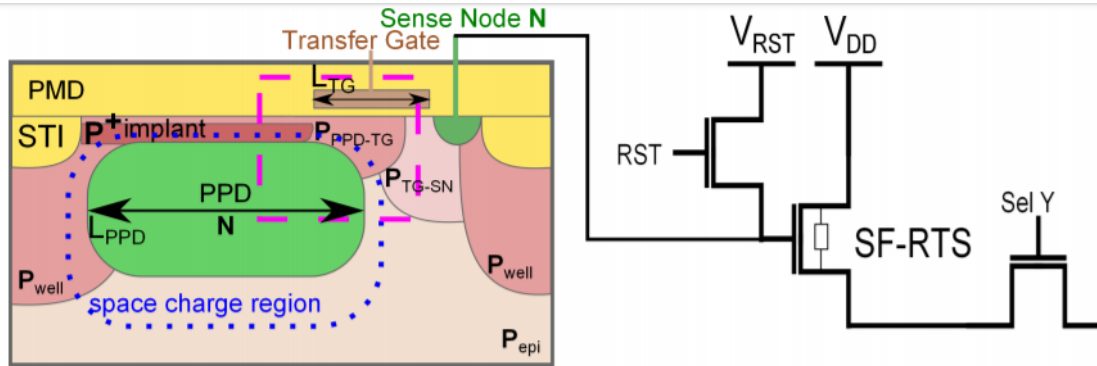
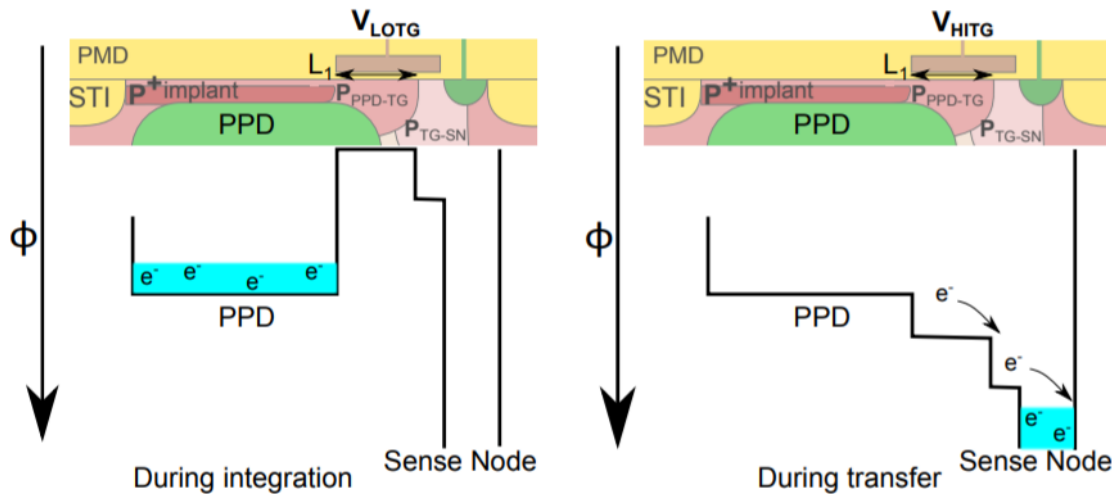
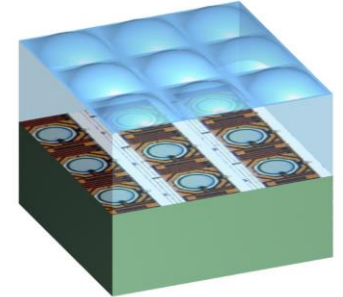


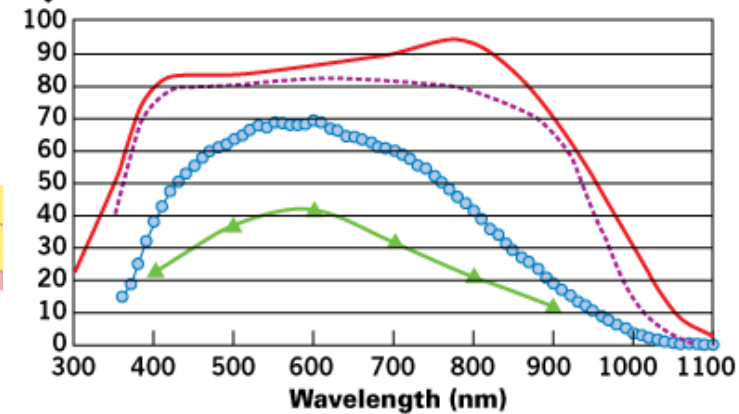
Fig. 2. Pixel architecture and cross section of the pinned photodiode used as a reference. L_{TG} is the transfer gate length and L_{PPD} is the PPD length.



Fill Factor Recovery: Microlenses



Quantum efficiency (%)



CMOS QE:

triangles – front-side illumination without microlenses,
 circles – front-side illumination with microlenses,
 dashed line – back-side illumination imager,
 solid line – back-side illumination imager IMEC

Y. Bai, et al "Silicon CMOS imaging technologies",
 Proc. SPIE 2008

Quantum Image Sensor: photon-number-resolving CMOS imager (Eric Fossum)

- ☐ Sensing node size < 500 nm !!!
- ☐ Capacitance < 1 fF
- ☐ Gain $\sim C/q \sim 1\text{mV}/e$
- ☐ Noise $\sim 0.15 e$ RMS
- ☐ $\Rightarrow \text{ENF} < 1.02$



FIGURE 2. A Dartmouth QIS test chip contains 20 different 1 Mjot QIS arrays and was fabricated by TSMC in a modified 45/65 nm 3D-stacked BSI CIS process.

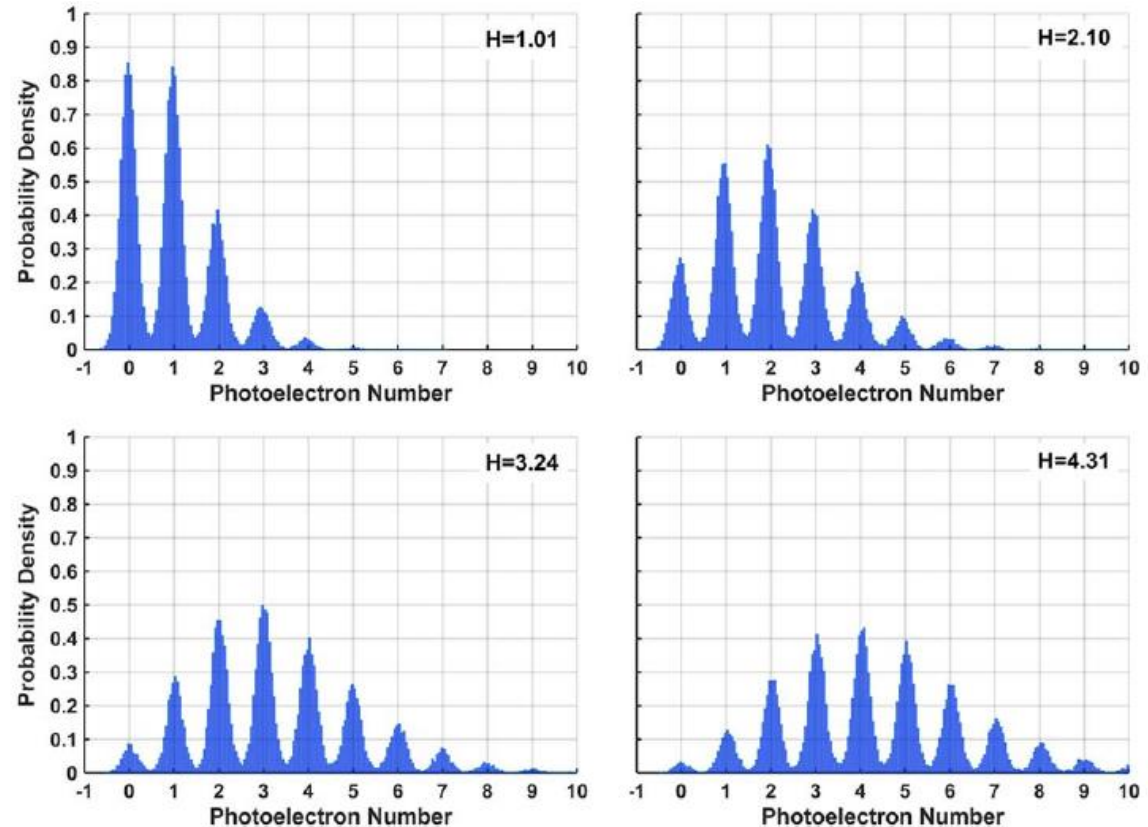


Fig. 4. Experimental demonstration of photoelectron counting.

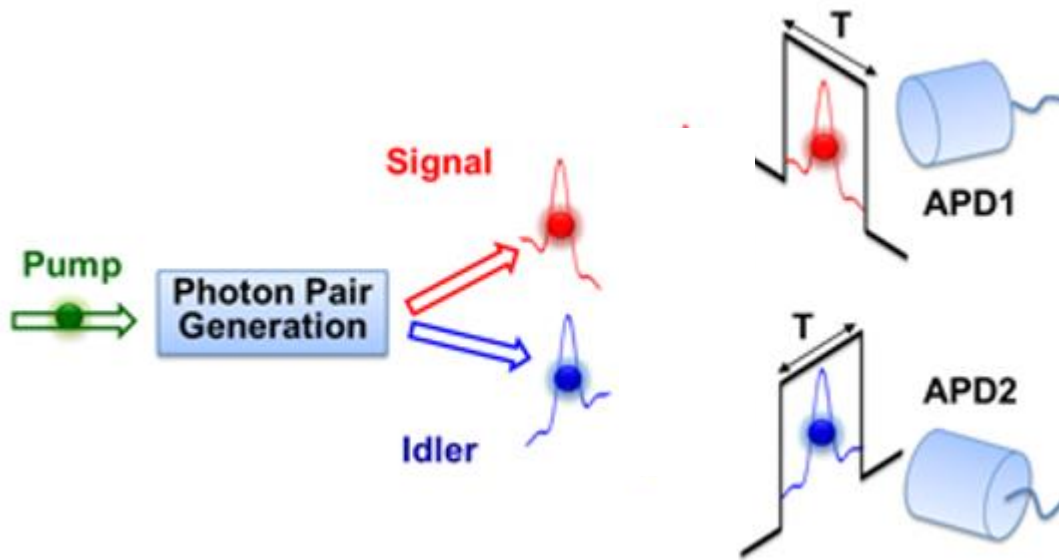
J. Ma et al., Photon-number-resolving megapixel image sensor at room temperature without avalanche gain, C
E. Fossum, Advances in Detectors: The Quanta image sensor (QIS): Making every photon count, Laser Focus

Characterization of photon-number-resolving detectors

- Characterization of SiPM and photon-number-resolving detectors

Characterization: PDE

- Photon Detection Efficiency \sim probability to detect single photon
- Absolute calibration of PDE of your photodetector
 - 1. “Single photon-on-demand” source or
 - 2. “Paired photons” down conversion technique (Alan Migdal, NIST)



Detection of paired photons

$$N(\text{det } 1) = N_{\text{pairs}} \cdot PDE_1$$

$$N(\text{det } 2) = N_{\text{pairs}} \cdot PDE_2$$

$$N(\text{coin}12) = N_{\text{pairs}} \cdot PDE_1 \cdot PDE_2$$

$$\Rightarrow PDE_1 = \frac{N(\text{coin}12)}{N(\text{det } 2)}$$

$$\Rightarrow PDE_2 = \frac{N(\text{coin}12)}{N(\text{det } 1)}$$

Characterization: PDE

- ☐ Photon Detection Efficiency ~ probability to detect single photon
- ☐ Reference calibration of PDE of your photodetector
 - ◆ 3. Calibrated photodetector and uncalibrated Poisson light source
 - From metrology center (e.g. NIST) (PIN or others e.g. SiPM)
 - Zero-peak method is recognized as the best for SiPM
 - Attention to integration time - TBD

Poisson photons μ_N with dark counts μ_D

DUT :

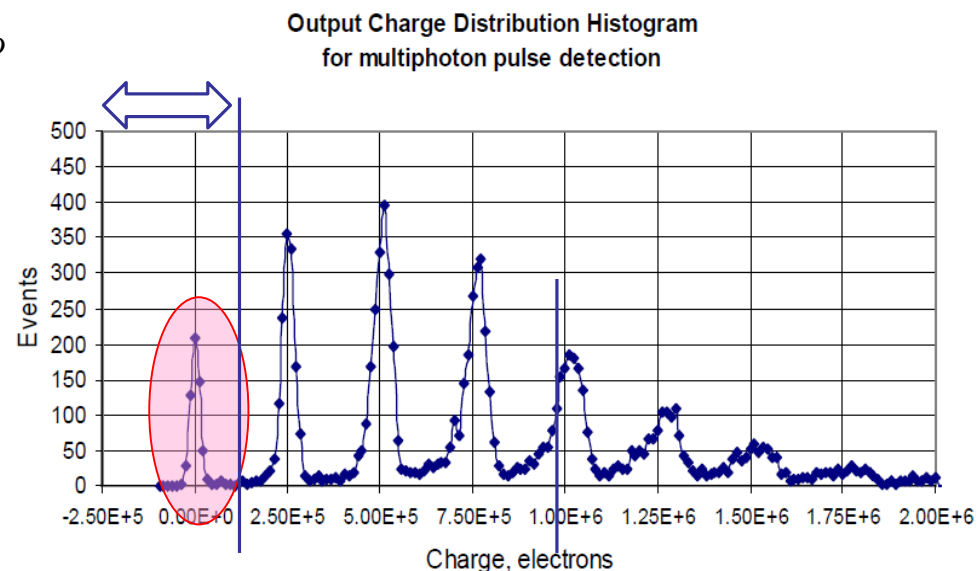
$$P_{DUT}(N_{N+D} = 0) = e^{-(\mu_N PDE_{DUT} + \mu_D)}$$

$$P_{DUT}(N_D = 0) = e^{-\mu_D}$$

$$PDE_{DUT} = -\frac{1}{\mu_N} \ln \left[\frac{P_{DUT}(N_{N+D} = 0)}{P_{DUT}(N_D = 0)} \right]$$

Reference detector:

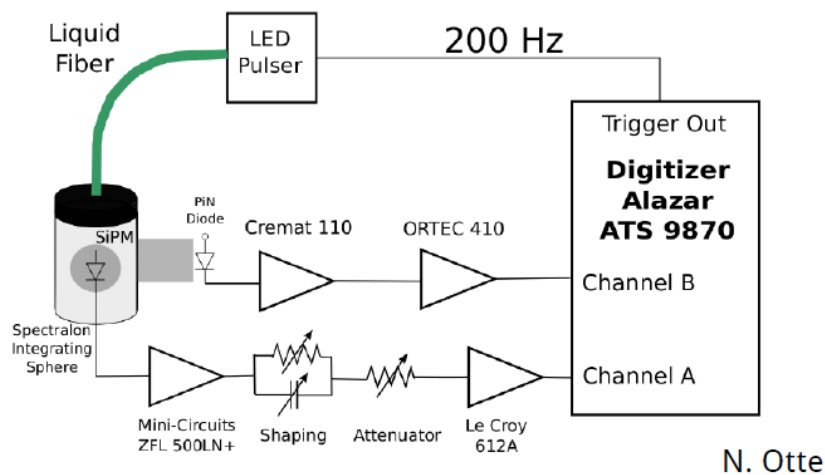
$$\mu_N = -\frac{1}{PDE_{REF}} \ln \left[\frac{P_{REF}(N_{N+D} = 0)}{P_{REF}(N_D = 0)} \right]$$



Characterization: PDE

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Swapping Sensors vs. Optical Splitter

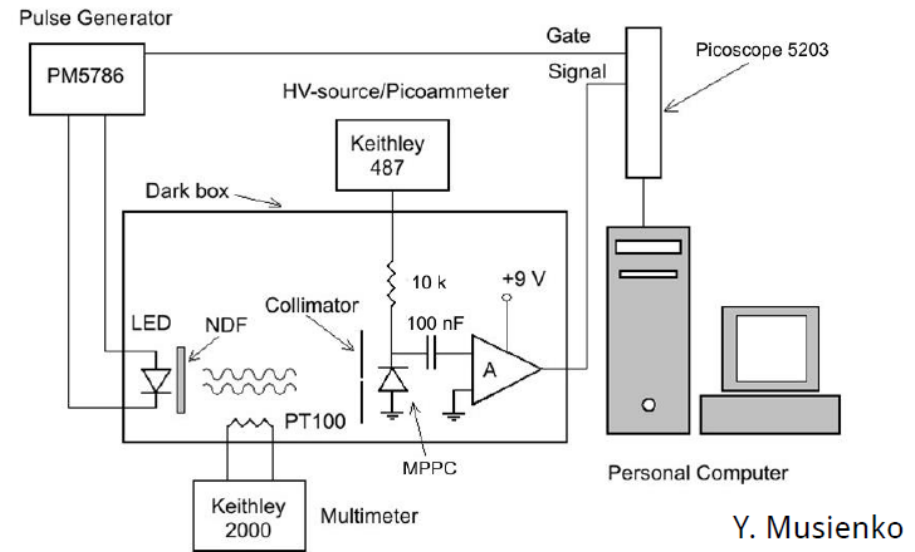


Pro:

- Measure reference and DUT simultaneously

Contra:

- Possible wavelength dependent splitting ratio
- Photons can trickle out over long time from integrating sphere



Pro:

- no beam splitter

Contra:

- reference and DUT need to be measured in sequence
→ need a monitoring device

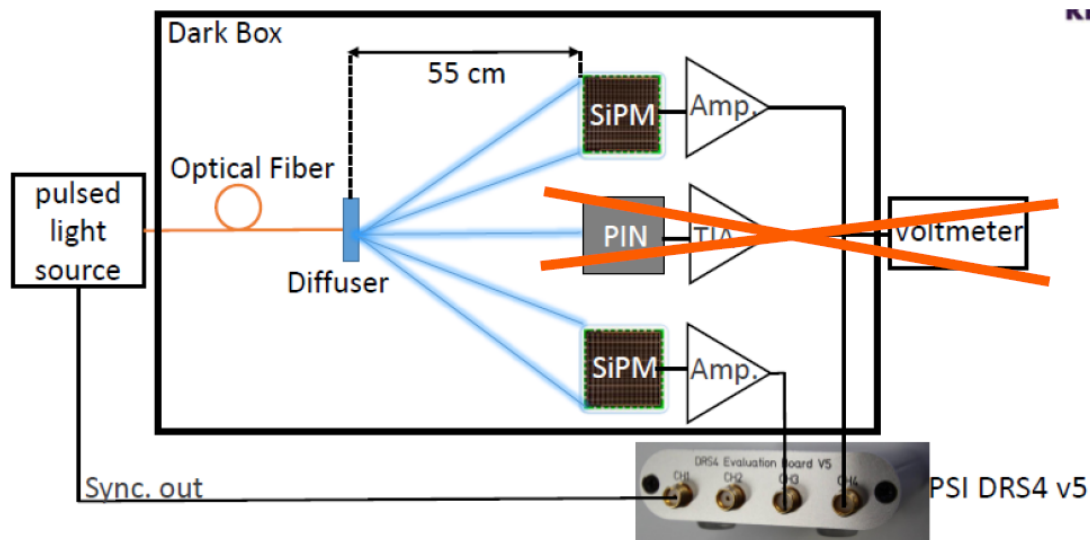
Characterization: PDE

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Summary of the Photon Detection Efficiency Working Group

Proposed Standard Setup

- Use calibrated SiPM as reference (i.e. no PIN diode) → splitting ratio of ~ 1
- Standard "PDE Box"

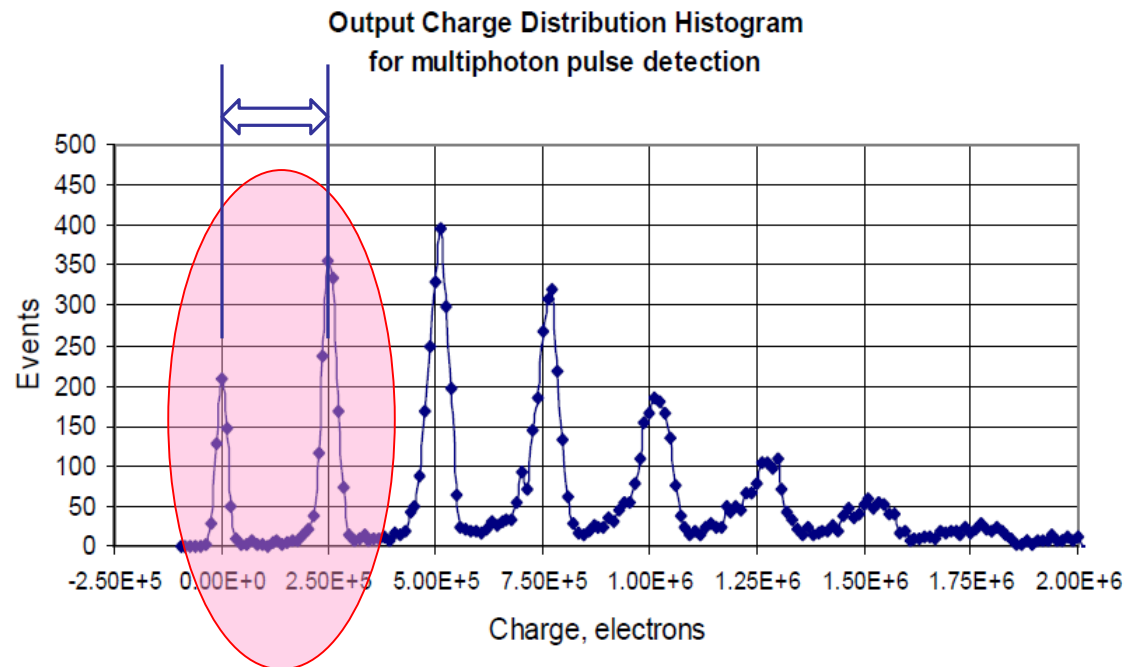
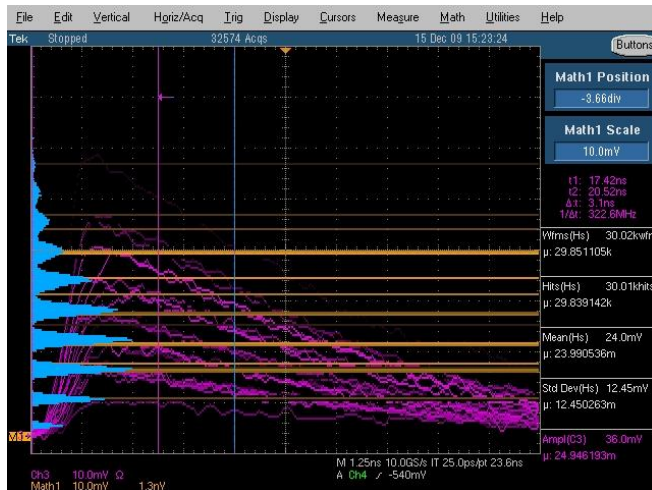


s. E. Engelmann

Nepomuk Otte

Characterization: Gain

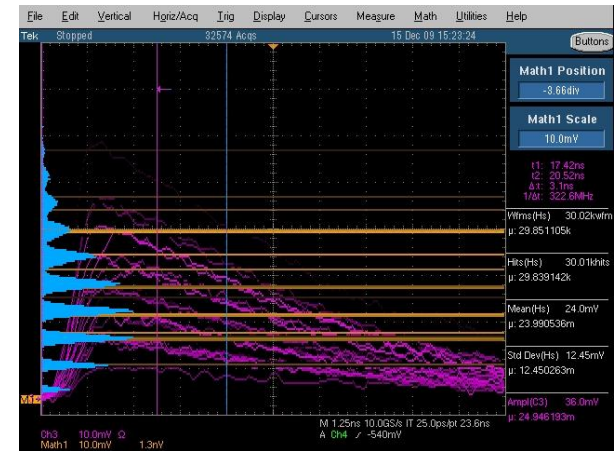
- Gain: Mean Gain and Variance of Gain (ENF)
- Mean = $\overline{Gain} = \mu_1 - \mu_0$ = distance between peaks (0-1, 1-2...)



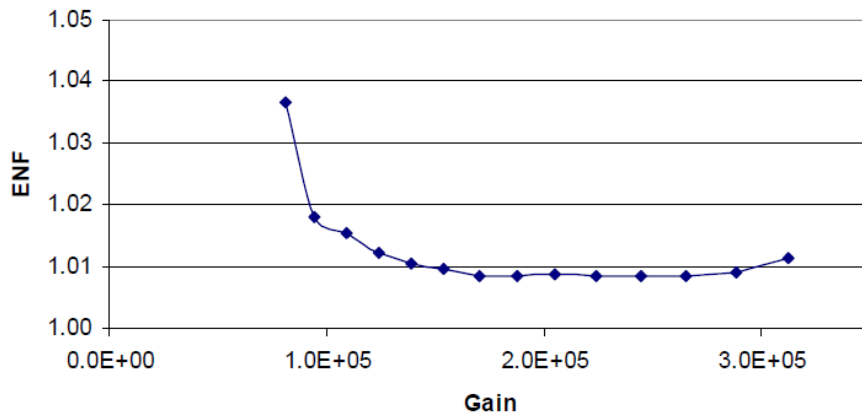
Characterization: ENF of Gain

$$ENF_{gain} = 1 + \frac{\sigma_{gain}^2}{Gain^2}$$

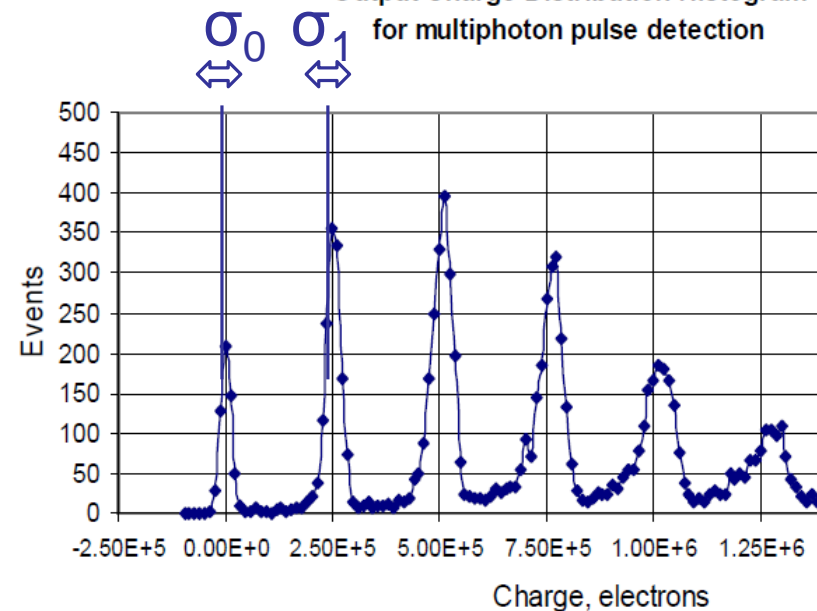
- Variance = $Var(Gain) = \sigma_{Gain}^2 = \sigma_1^2 - \sigma_0^2$
 - ◆ Correct if zero-peak dispersion is exclusively due to electronic noise
 - ◆ Baseline fluctuations of zero-peak due to preceding dark events (TBD)



Excess Noise Factor vs Gain dependence



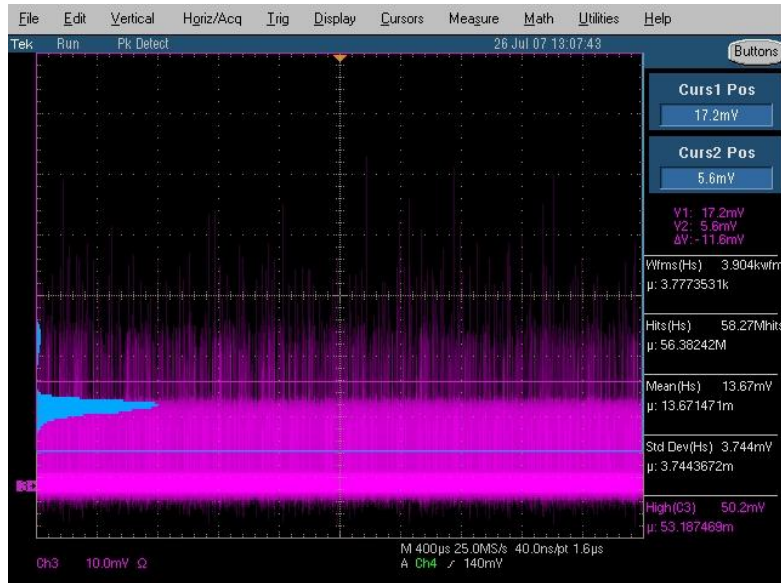
Output Charge Distribution Histogram for multiphoton pulse detection



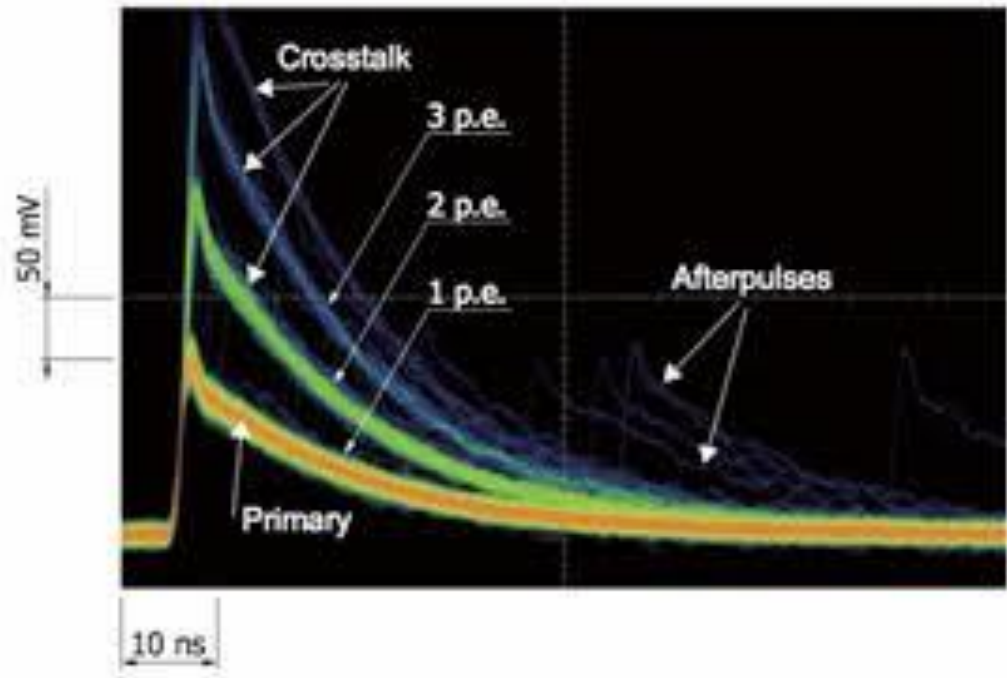
Characterization: crosstalk

- Crosstalk events initiated by a single primary avalanche event

- Counting mode



- Triggering mode



Characterization: crosstalk initiated by non-random primary

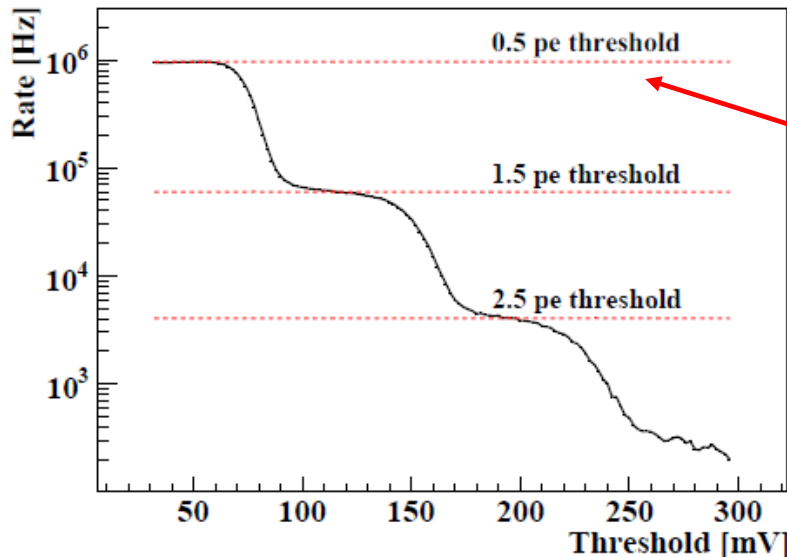
■ Crosstalk initiated by non-random primary =>

All events are crosstalk
except some extra
dark counts =>
to be subtracted as Poisson
events (if not negligible)

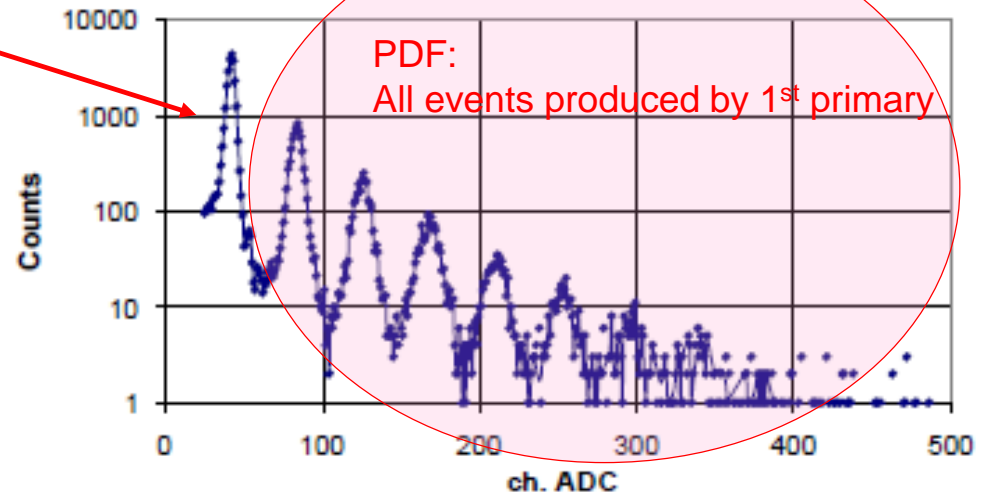
$$Pct = \frac{\sum Ndrk(>1)}{Ndrk(1)} \quad \text{Dark amplitude histogram}$$

$$Pct = \frac{DCR(>1.5)}{DCR(>0.5)} \quad \text{DCR vs threshold}$$

$$p = 1 - \left[1 - \frac{DCR_{1.5}}{DCR_{0.5}} \right] \cdot \exp(DCR_{0.5} \cdot \tau)$$



SES MEPI/PULSAR APD, U=57.5V, T=-28 C



P. Eckert et al, nima.2010.03.169].

Characterization: crosstalk initiated by Poisson primaries

■ Crosstalk initiated by Poisson

primaries =>

Photo- and CT events in the 1st

peak are independent

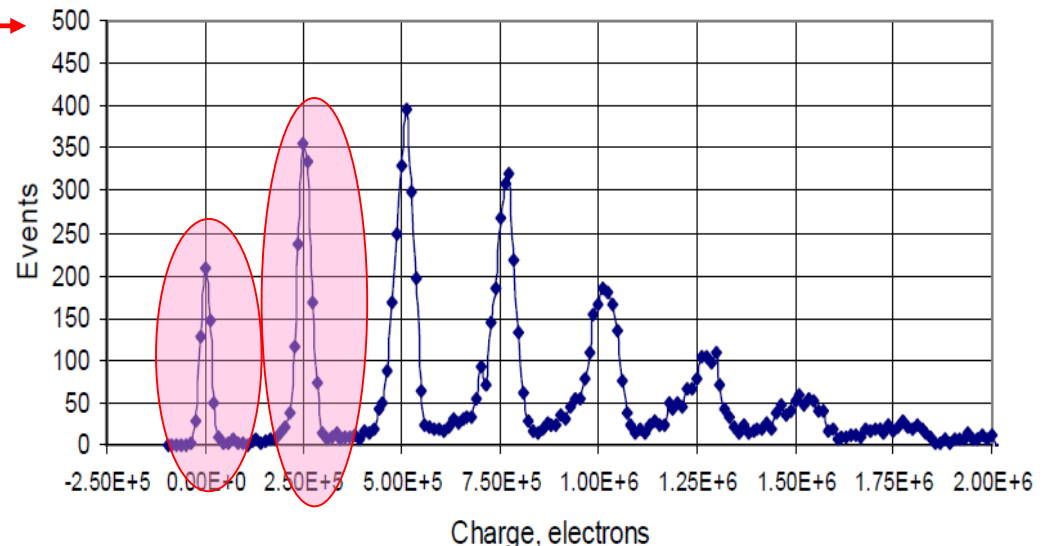
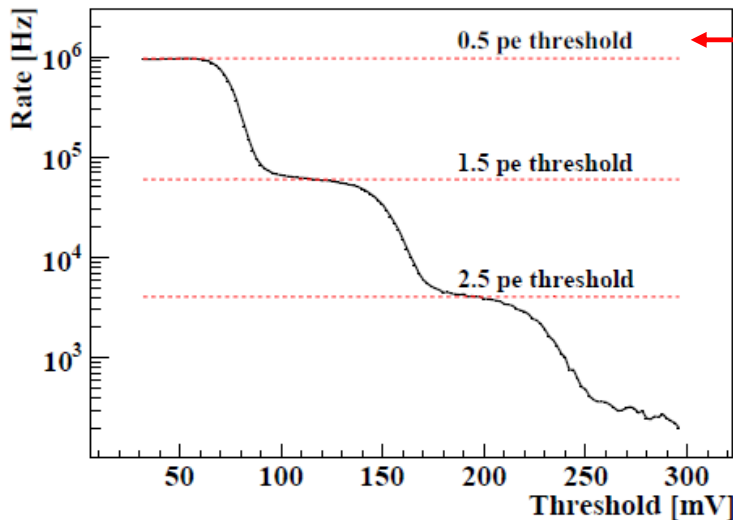
$$P(1) = P_{pois}(1) \cdot P_{ct}(0)$$

Multiphoton histogram

$$P(1) = P_{poisson}(1) \cdot (1 - P_{ct})$$

$$P_{ct} = 1 - \frac{P(1)}{P_{poisson}(1)}$$

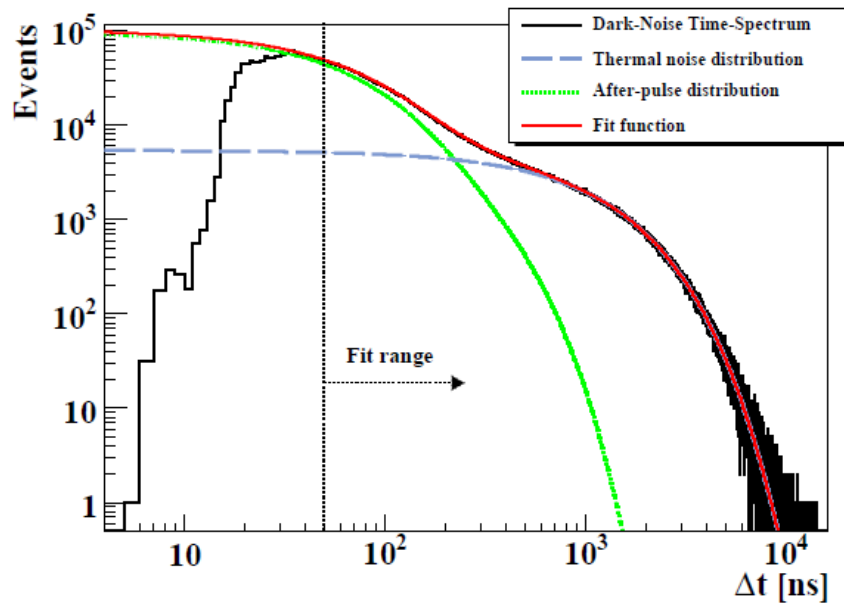
Output Charge Distribution Histogram
for multiphoton pulse detection



P. Eckert et al, nima.2010.03.169].

Characterization: afterpulsing by histogram of interarrival times

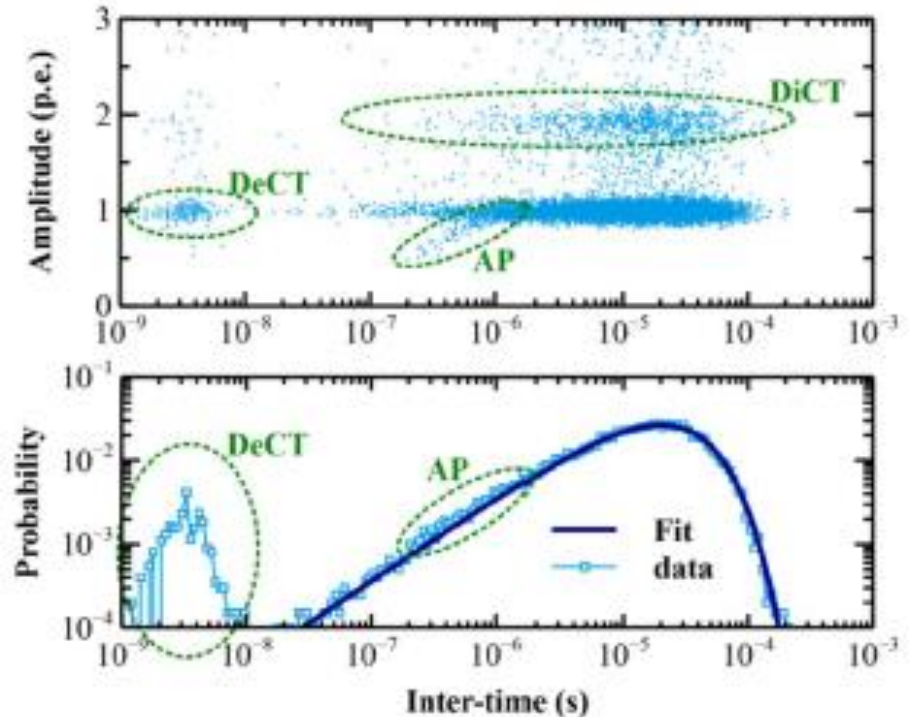
- Common-sense analysis of correlated events
- Contributions are fitted separately in specific time frames
 - ◆ Correctness of expressions as a sum of fast + slow AP (?order statistics?)



P. Eckert et al, NIMA 2010

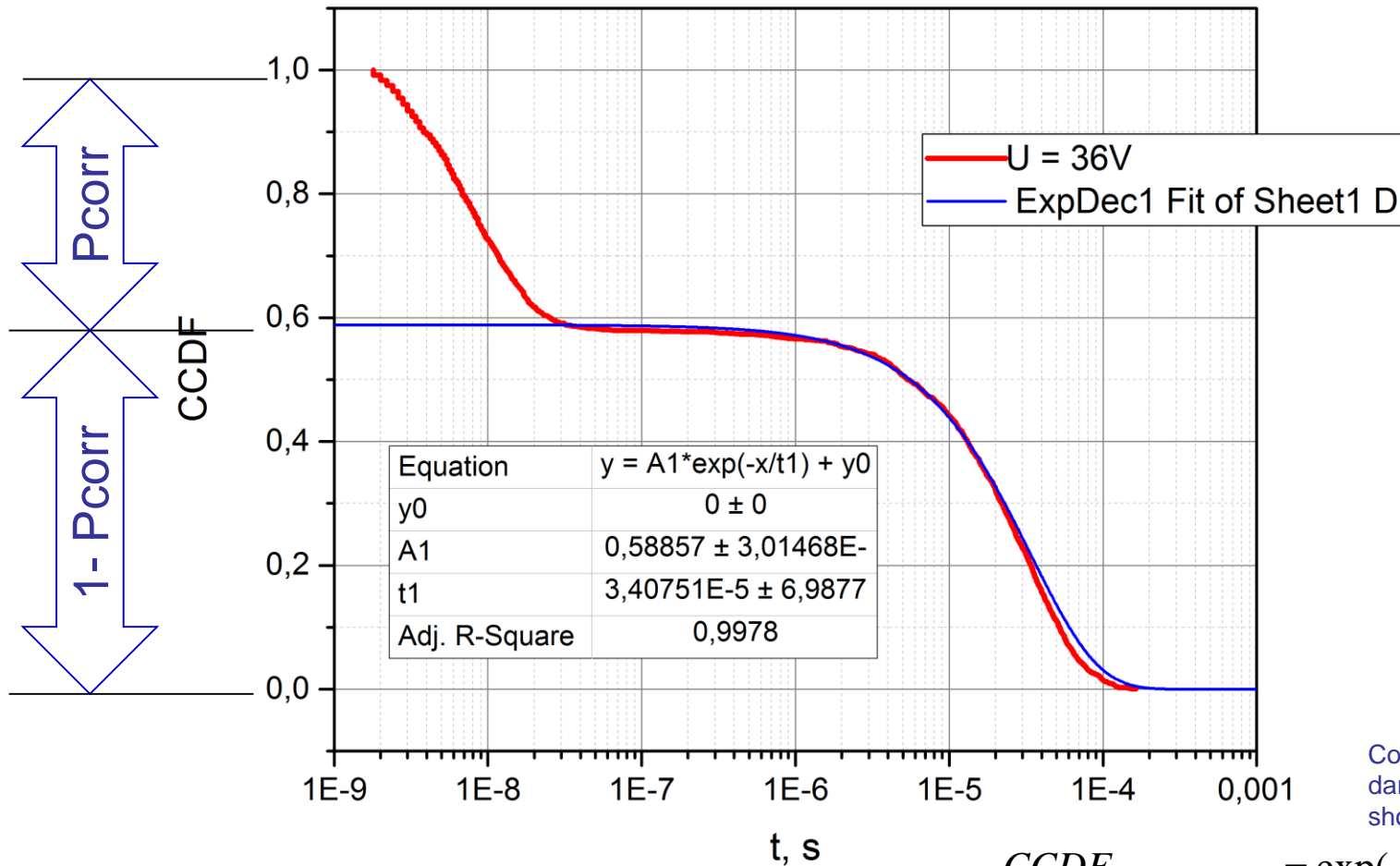
$$n_{tp}(\Delta t) = N_{tp}/\tau_{tp} \cdot e^{-\frac{\Delta t}{\tau_{tp}}}$$

$$n_{ap}(\Delta t) = N_{apf}/\tau_{apf} \cdot e^{-\frac{\Delta t}{\tau_{apf}}} + N_{aps}/\tau_{aps} \cdot e^{-\frac{-\Delta t}{\tau_{aps}}}$$



F. Acerbi et al, TNS 2016

Characterization: afterpulsing by cumulative distribution of interarrival times



Correction on lost dark counts at Tmax should be applied:

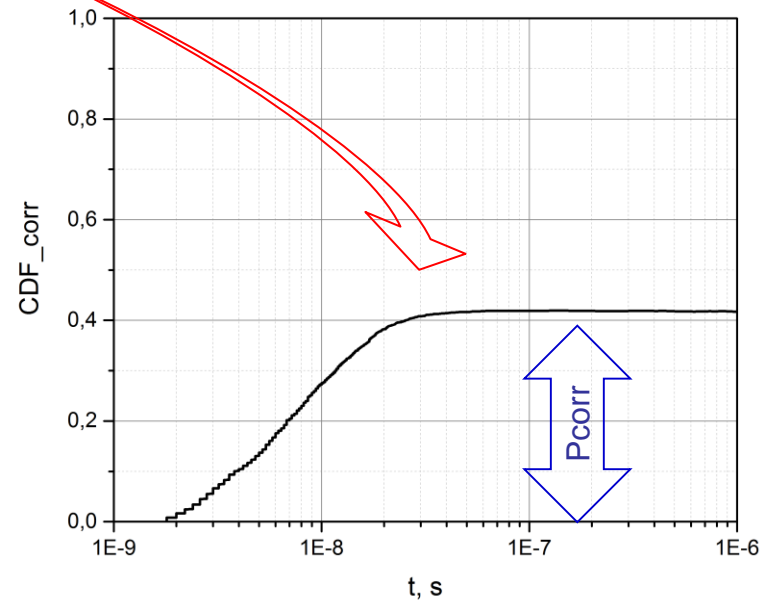
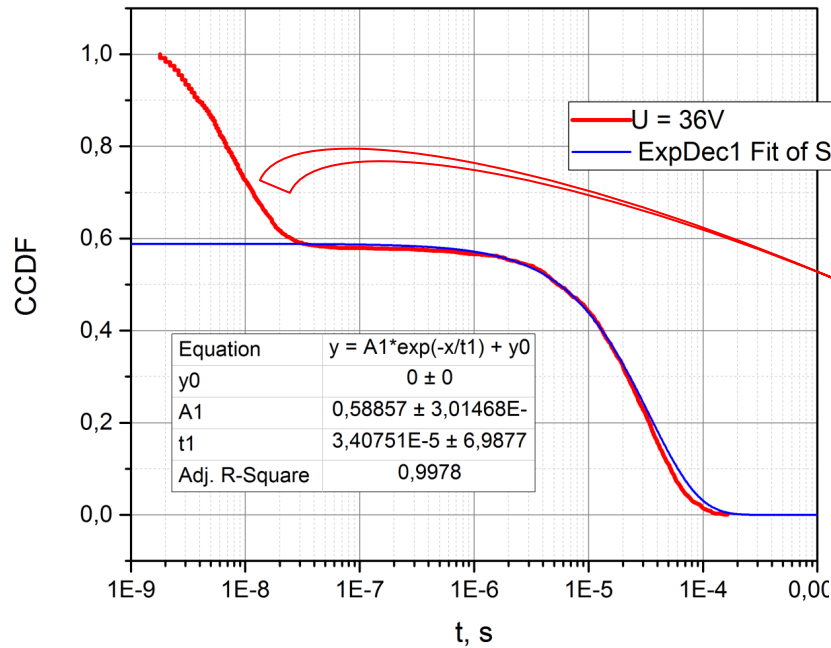
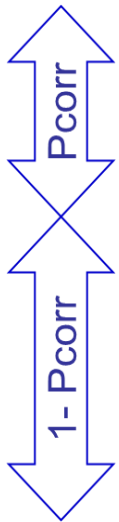
$$CCDF_{dark_correction} = \exp(-DCR \cdot T_{max})$$

S. Vinogradov, NSS/MIC 2016

Experimental example – courtesy of E. Popova, D. Philippov... , NSS/MIC 2016

Characterization: afterpulsing by cumulative distribution of interarrival times

$$F_{corr}(t, P_{corr}) = 1 - (1 - F_{total}(t, N_{ph}, DCR)) \cdot \exp(DCR \cdot t)$$



Now we are ready to play with correlated event CDF or PDF applying any models and analysis of timing behavior.

Total number of afterpulsing events could be of the geometric distribution with P_{corr} .

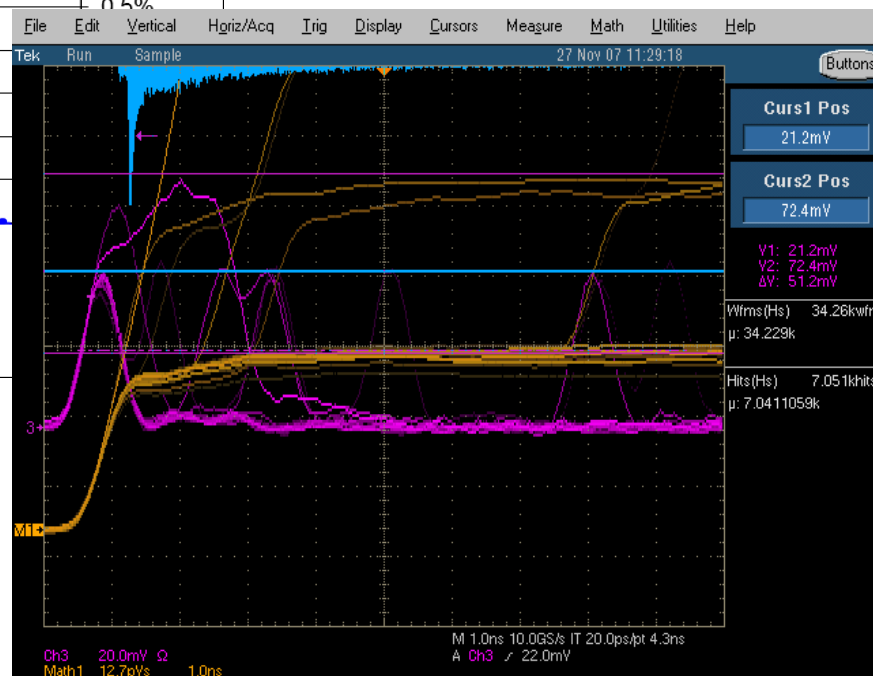
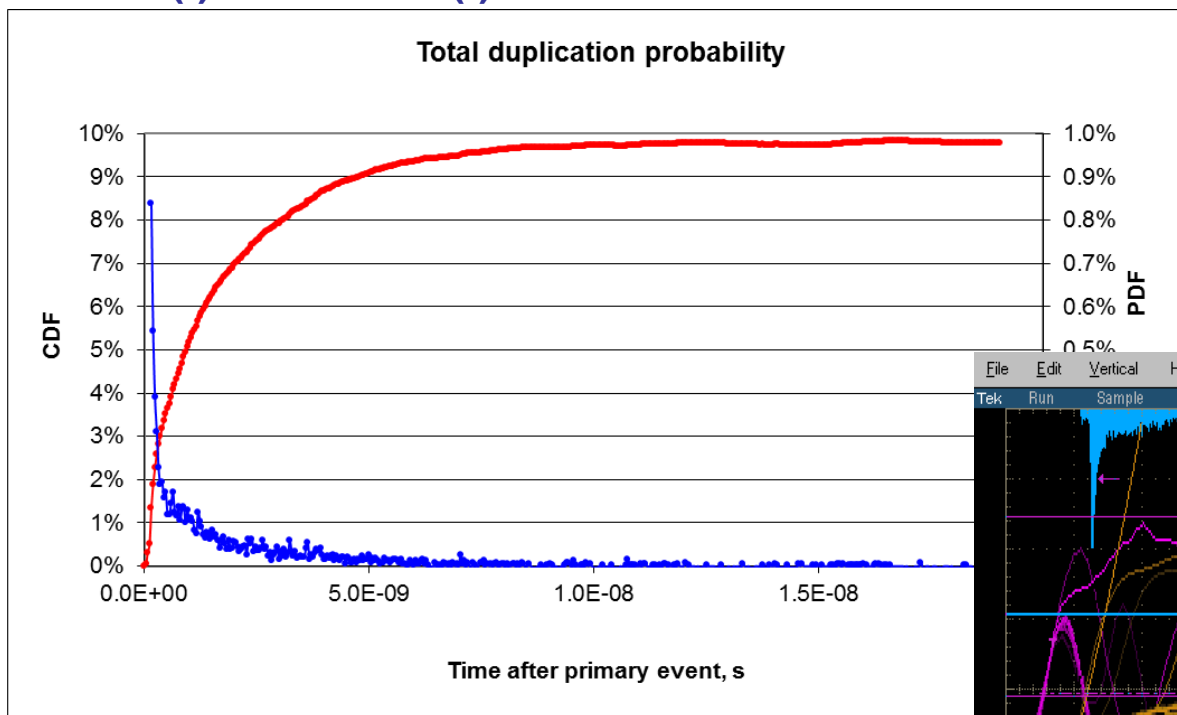
S. Vinogradov, NSS/MIC 2016

Experimental example – courtesy of E. Popova, D. Philippov , NSS/MIC 2016

Joint characterization of CT & AP by CCDF time distribution

Seamless measurement of CT+AP at short times:

Q(t) instead of I(t) measurement of time interval from primary to secondary event



$$1 - P_{total}(\Delta t) = (1 - P_{drk}(\Delta t)) \cdot (1 - P_{ap}(\Delta t))$$

$$P_{ap}(\Delta t) = 1 - \frac{1 - P_{total}(\Delta t)}{e^{-DCR \cdot \Delta t}} \quad CDF$$

$$\rho(\Delta t) = \frac{dP_{ap}(\Delta t)}{d\Delta t} \quad PDF$$



P.N. Lebedev Physical
Institute of the Russian
Academy of Science



Ultimate Low-Light Level Sensor Development

The end

Thank you for your attention!

Questions?
Objections?
Opinions?

...

vin@lebedev.ru