STABILITY OF ISRAEL-STEWART THEORY IN THE PRESENCE OF DIFFUSION

Caio Brito, Gabriel Denicol

Universidade Federal Fluminense, Rio de Janeiro, Brazil

OBJECTIVES

i. Analyze the stability conditions in the presence of net-charge diffusion

ii. Verify the relation between causality and stability

iii. Investigate the possible values the coupling term can assume

iv. Obtain constraints for the relaxation times and coupling terms

RELATIVISTIC FLUID DYNAMICS

CONSERVATION LAWS

Energy and momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

Net-charge conservation

$$\partial_\mu N^\mu = 0$$

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - P\Delta^{\mu\nu} + \pi^{\mu\nu}$$

shear-stress tensor

$$N^\mu = n u^\mu + n^\mu$$

net-charge diffusion

ISRAEL-STEWART EQUATIONS

Net-charge diffusion: $\tau_n \dot{n}^{\langle\mu\rangle} + n^\mu = \kappa_n \nabla^\mu \alpha + \ell_{n\pi}\Delta^{\mu\nu}\nabla_\lambda \pi^\lambda_\nu + \cdots$

coupling terms

Shear-stress tensor: $\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \ell_{\pi n}\nabla^{\langle\mu} n^{\nu\rangle} + \cdots$

→ **Entropy production:** $\partial_\mu S^\mu \geq 0 \Longrightarrow \ell_{n\pi}\ell_{\pi n} \leq 0$

PERTURBATIONS AROUND EQUILIBRIUM

$$\varepsilon = \varepsilon_0 + \delta\varepsilon,\ n_B = \delta n_B,\ u^\mu = u_0^\mu + \delta u^\mu,$$
$$n^\mu = \delta n^\mu,\ \pi^{\mu\nu} = \delta\pi^{\mu\nu}.$$

Dimensionless variables

$$\hat{A} \equiv A[\tau_\eta] \quad \tau_\eta \equiv \frac{\eta}{\varepsilon + P}$$

Covariant variables

$$\Omega \equiv u_0^\mu k_\mu$$
$$\kappa^\mu \equiv \Delta_0^{\mu\nu} k_\nu$$

RESULTS

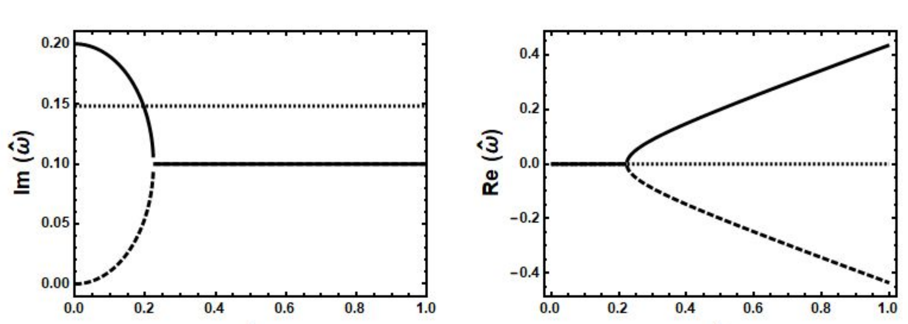
STABILITY CONDITIONS

$$\tau_\pi \geq \frac{2\eta}{\varepsilon + P} \qquad \tau_n \geq \frac{4\kappa_n T}{\varepsilon + P} \qquad |\ell_{n\pi}\ell_{\pi n}| \leq \frac{3}{2}\left(\tau_\pi - \frac{2\eta}{\varepsilon+P}\right)\left(\tau_n - \frac{4\kappa_n T}{\varepsilon + P}\right)$$

TRANSVERSE MODES

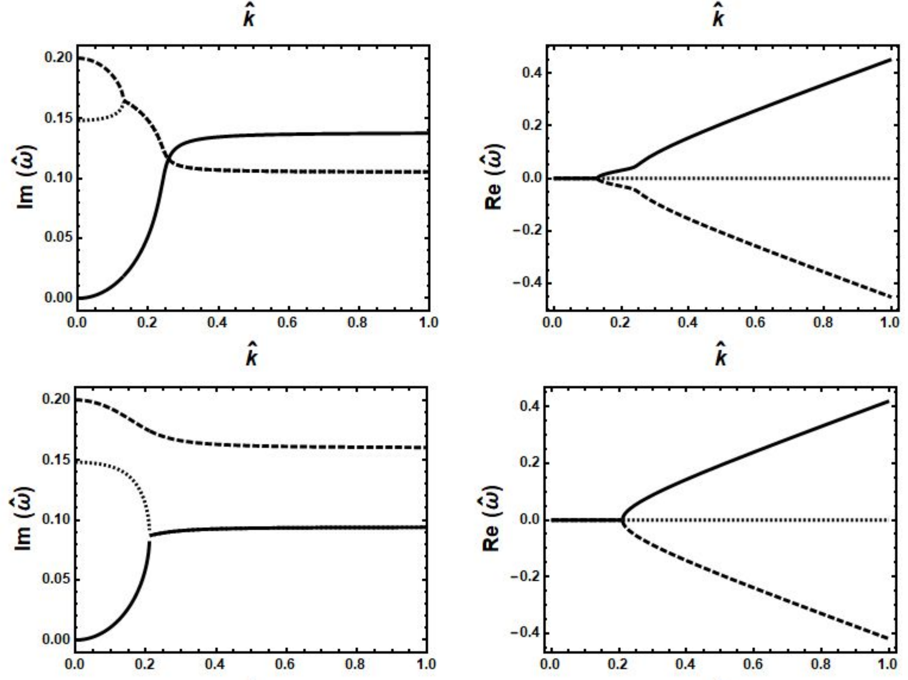
$$-\hat\tau_\pi\hat\tau_n\hat\Omega^3 + i(\hat\tau_\pi + \hat\tau_n)\hat\Omega^2 + \left[1 + \left(\hat\tau_n - \frac{1}{2}\hat\ell_{\pi n}\hat\ell_{n\pi}\right)\hat\kappa^2\right]\hat\Omega - i\hat\kappa^2 = 0$$

WITHOUT COUPLING



$\hat\ell_{n\pi}\hat\ell_{\pi n} = 0$

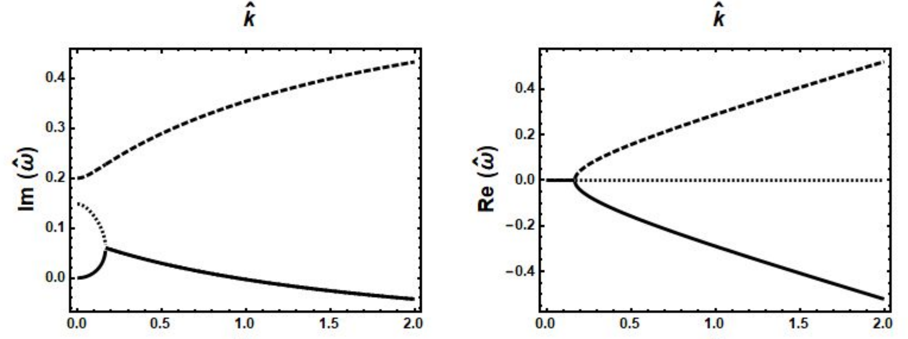
WITH COUPLING



$\hat\ell_{n\pi}\hat\ell_{\pi n} = -1$

$\hat\ell_{n\pi}\hat\ell_{\pi n} = 1$

UNSTABLE CASE

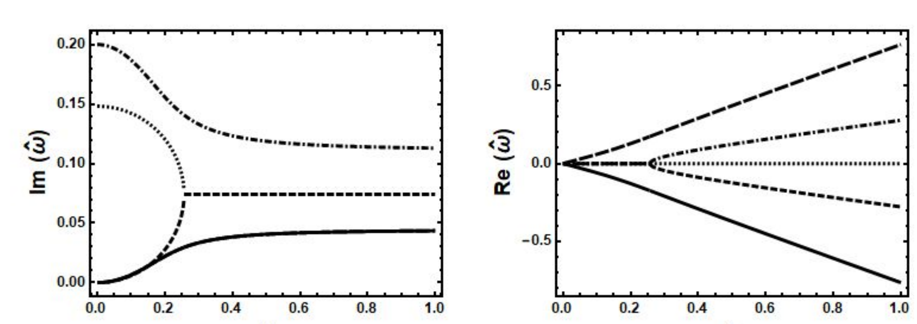


$\hat\ell_{n\pi}\hat\ell_{\pi n} = 10$

LONGITUDINAL MODES

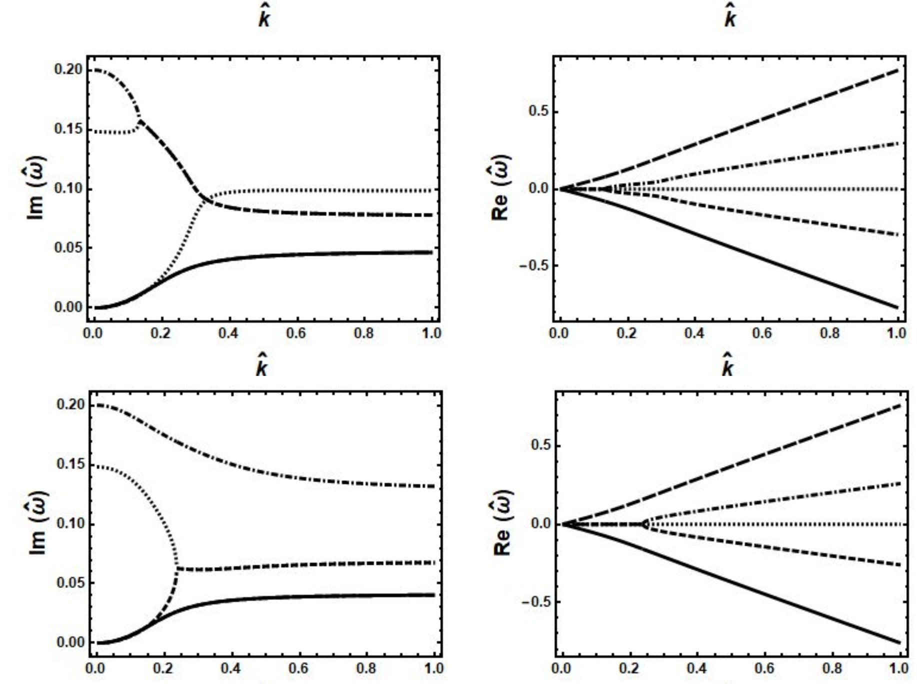
$$\left[\left(\hat\Omega^2 - \frac{1}{3}\hat\kappa^2\right)(i\hat\tau_\pi\hat\Omega + 1) - \frac{4}{3}i\hat\kappa^2\hat\Omega\right]\left[\hat\Omega(i\hat\tau_n\hat\Omega + 1) - i\hat\tau_\kappa\hat\kappa^2\right] - \frac{2}{3}\hat\ell_{\pi n}\hat\ell_{n\pi}\left(\hat\Omega^2 - \frac{1}{3}\hat\kappa^2\right)\hat\Omega\hat\kappa^2 = 0$$

WITHOUT COUPLING



$\hat\ell_{n\pi}\hat\ell_{\pi n} = 0$

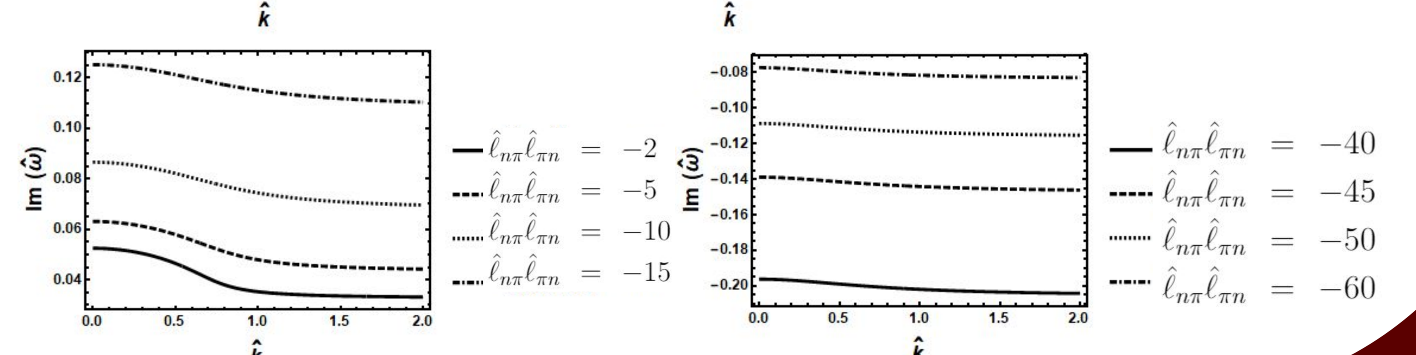
WITH COUPLING



$\hat\ell_{n\pi}\hat\ell_{\pi n} = -1$

$\hat\ell_{n\pi}\hat\ell_{\pi n} = 1$

UNSTABLE CASES



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ACKNOWLEDGEMENTS

CAPES

FAPERJ – Fundação Carlos Chagas Filho de Amparo à Pesquisa do Estado do Rio de Janeiro

CNPq – Conselho Nacional de Desenvolvimento Científico e Tecnológico