Simulating chiral anomalies in a box system

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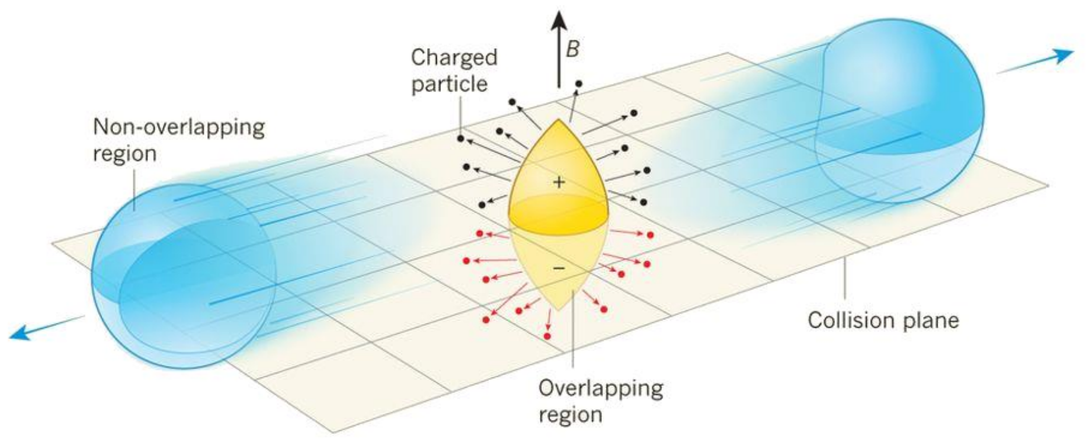
Motivation



$$\vec{J} = \frac{N_c}{2\pi^2\hbar^2}\mu_5 e\vec{B},$$
$$\vec{J}_5 = \frac{N_c}{2\pi^2\hbar^2}\mu e\vec{B}.$$

- Noncentral relativistic heavy-ion collisions could product a really huge strength of the magnetic field and a really high temperature which could restore the chiral symmetry. So it provides an ideal environment to study the chiral anomalies.
- There are two main chiral anomaly effects. One is the chiral magnetic effect induced by particles with unbalanced helicities. The other is the dual effect induced by particles with unbalanced charges, named as the chiral separation effect. These two effects are expressed as two equations for currents shown above.
- Both currents are proportional to the magnetic field, and they violate the P or CP symmetry.

Chiral Equations of motion

$$H = c\vec{\sigma}\cdot\vec{k}$$

Canonical equations

Spin EOM (SEOM)

$$\dot{\vec{r}} = c\vec{\sigma}$$
$$\dot{\vec{k}} = \dot{\vec{r}} \times qe\vec{B}$$
$$\dot{\vec{\sigma}} = c\frac{2}{\hbar}\vec{k}\times\vec{\sigma}$$

$$\vec{\sigma} \approx c\hat{k} - \frac{\hbar}{2k}\hat{k}\times\dot{\hat{k}}$$

Adiabatic approximation

Chiral EOM (CEOM)

$$\sqrt{G}\dot{\vec{r}} = \hat{k} + \hbar(c\vec{\Omega}\cdot\hat{k})qe\vec{B}$$
$$\sqrt{G}\dot{\vec{k}} = \hat{k}\times qe\vec{B}$$
$$\sqrt{G} = 1 + \hbar(qe\vec{B}\cdot c\vec{\Omega})$$

$$\vec{k} = \vec{p} - qe\vec{A},\ \vec{B} = \vec{\nabla}\times\vec{A},\ \vec{\Omega} = \frac{\vec{k}}{2k^3},\ \ \hat{k} = \vec{k}/k$$

- This is an intuitive semiclassical approach for spin-1/2 massless particles under an external magnetic field. k is the kinetic momentum and $\vec{\sigma}$ is the spin. By using canonical equation, the time evolutions of coordinates, momenta, and spins are obtained, which are called the spin equations of motion (SEOM). The second term of SEOM represents the Lorentz force.
- By using an adiabatic approximation for spin and substituting it into the SEOM, the chiral equations of motion (CEOM) were obtained.

Chiral Magnetic Wave

Interplay between the CME and the CSE

$$\mu/T \ll 1, \mu_5/T \ll 1$$

$$\rho \approx \frac{N_c T^2}{3\hbar^3}\mu,\ \rho_5 \approx \frac{N_c T^2}{3\hbar^3}\mu_5.$$

$$\left(\partial_t \pm v_p\partial_y - D_L\partial_y^2\right)\rho_{R/L} = 0$$

Initial condition

$$\rho_{R/L}(y,0) = \pm\frac{1}{2}A_c n\sin(\beta y)$$

Periodic Bound condition

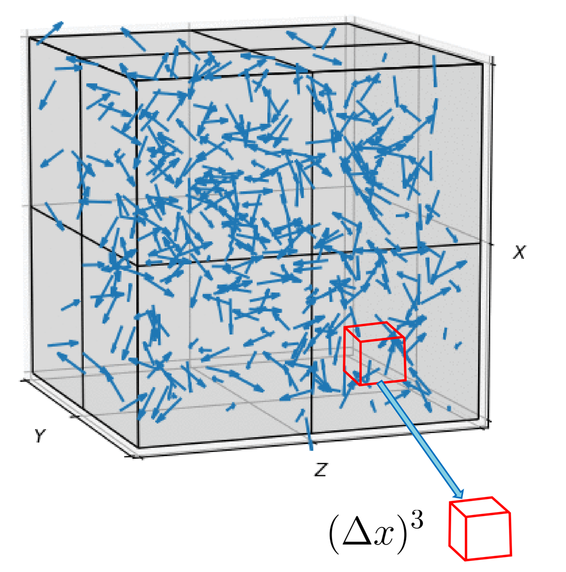
$$\rho_{R/L}|_{y=-l} = \rho_{R/L}|_{y=+l}$$

$$\rho_{R/L}(y,t) = \pm\frac{1}{2}A_c n e^{-D_L\beta^2 t}\sin[\beta(y\mp v_p t)]$$

$$\beta = \frac{\pi}{l}\quad n = \frac{N}{V}\quad v_p = \frac{3\hbar eB}{2\pi^2 T^2}\quad A_c = \frac{N_+ - N_-}{N_+ + N_-}$$

- Combining formulas for currents and densities as well as the continuity equation and Fick's law, the wave function for the chiral magnetic wave (CMW) can be obtained, D_l is the diffusion constant, and v_p is the phase velocity of the CMW.

Box System



$(\Delta x)^3$

- The chiral anomalies are studied in a cubic periodic box system, i.e., particles moving away from a side of box will come back from the opposite side.
- At the beginning, particles are initialized as uniformly in coordinate space, and momenta of each type particles are sampled according to the Fermi-Dirac distribution.
- The two-body scattering process and the Pauli blocking among particles were considered, and the blocking probability is $1-(1-f_3)(1-f_4)$.
f_3 and f_4 is the occupation number after collision.
- The stochastic method is applied for attempted collisions. This method divides the box into many small cubic boxes. Only particle pair in the same small box may collide with each other and the collision probability is calculated from

$$P_{22} = v_{\rm rel}\sigma_{22}\frac{\Delta t}{(\Delta x)^3},\quad v_{\rm rel} = s/(2E_1E_2).$$

- In CEOM, the occupation number is calculated by

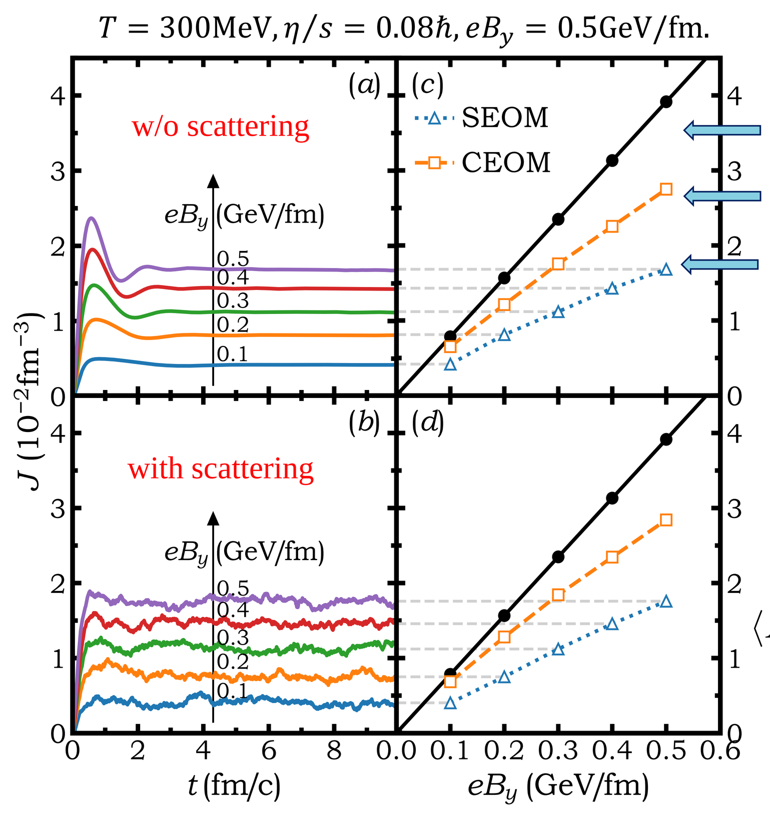
$$f = \frac{1}{1+e^{\frac{k-\mu_{qc}}{T}}},\ \mu_{qc} = q\mu + c\mu_5.$$

- In the SEOM scenario, the spin is initialized as the momentum direction multiplying c, and the Pauli blocking is considered for different spin states.

Results and Discussions

Currents

$T = 300$MeV, $\eta/s = 0.08\hbar$, $eB_y = 0.5$GeV/fm.



Theoretical limit

CEOM $0.3 < \sqrt{G} < 1.7$

SEOM Do not need truncation.

The $\sqrt{G}$ factor in CEOM leads to modification of the phase-space volume.

$$\int\frac{d^3k}{(2\pi\hbar)^3} \Rightarrow \int\sqrt{G}\frac{d^3k}{(2\pi\hbar)^3}$$

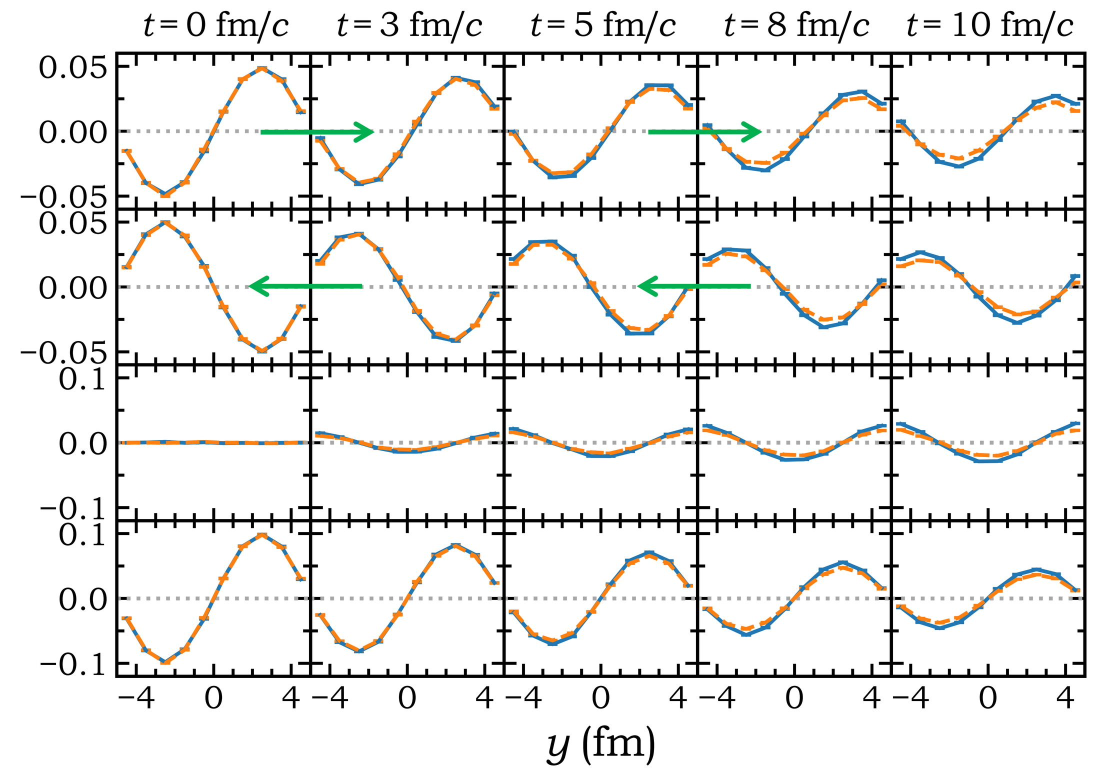
$$\langle A\rangle = \frac{\sum_i A_i\sqrt{G_i}}{\sum_i\sqrt{G_i}}\quad \sqrt{G}\text{ as a weight factor}$$

$$N_{C(+)} \neq N_{C(-)}, N_{Q(+)} = N_{Q(-)}$$

- The currents in SEOM are weaker than those in CEOM, and they are both lower than theoretical limits.
- The SEOM may have a relaxation time.

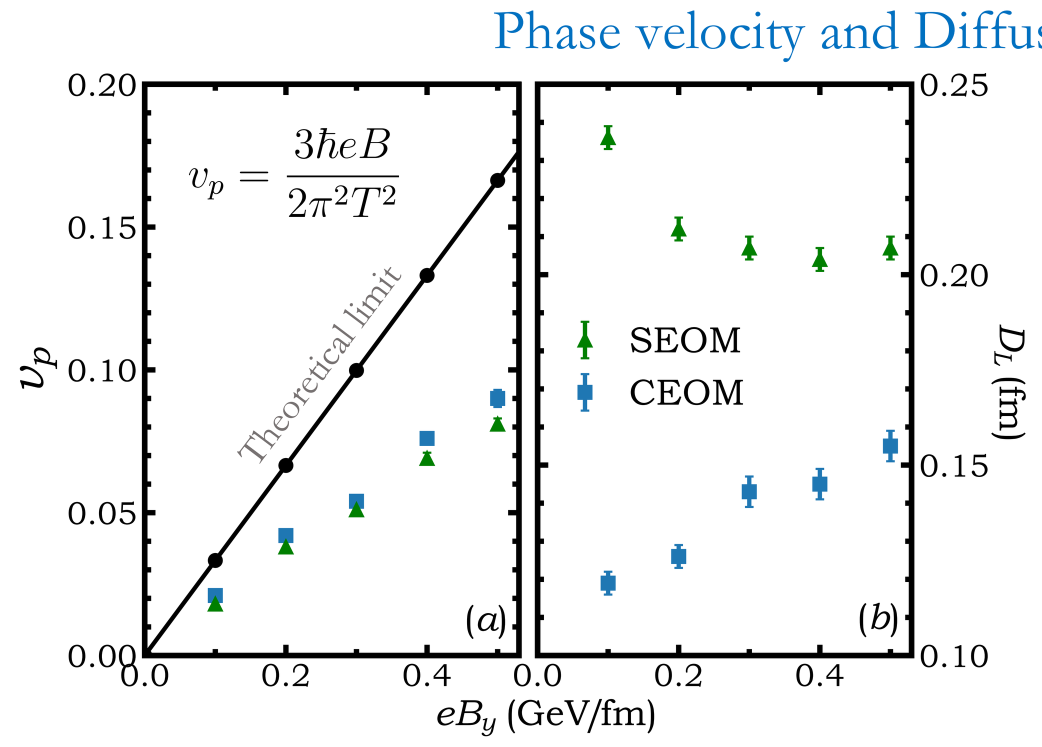
Chiral magnetic wave

- evolutions of various densities in the direction of the magnetic field.



dashed: SEOM solid: CEOM

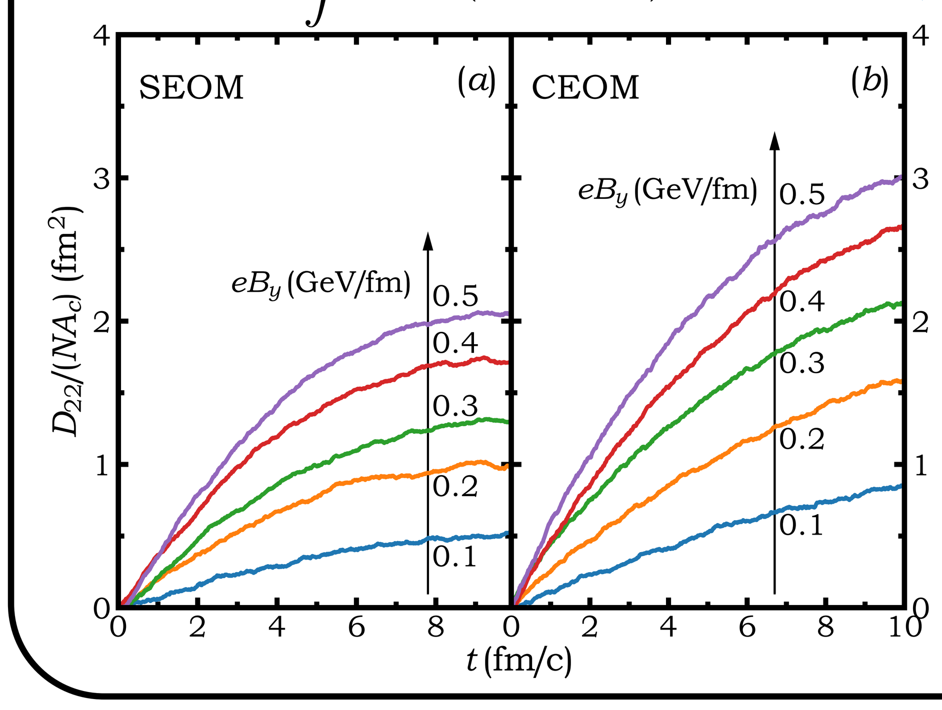
Phase velocity and Diffusion constant



- The velocities in CEOM are larger than those in SEOM, and they both are lower than theoretical limits.
- The tendencies of diffusion constants are different between two scenarios.

Electric quadrupole momentum

$$\mathcal{D}_{22} = \int\rho(\vec{r})\left(3y^2 - r^2\right)d^3r \quad\Rightarrow\quad \mathcal{D}_{22} = \frac{4A_c N}{\beta^2}e^{-D_L\beta^2 t}\sin(\beta v_p t)$$



- The electric quadrupole moment can be calculated from the charge density distribution.
- The electric quadrupole moment increases faster at lower temperatures, and faster in CEOM than in SEOM.

Summary

- The artificial truncation is needed for CEOM and underestimates the chiral effects compared to the theoretical limit.
- SEOM can be away from the artificial truncation but also leads to weaker chiral effects.
- The chiral magnetic wave in a box system can be described reasonably well with both CEOM and SEOM.

References

- [1] Wen-Hao Zhou and Jun Xu*, *Phys. Rev. C* 98, 044904 (2018).
- [2] Wen-Hao Zhou and Jun Xu*, *Phys. Lett. B* 798, (2019) 134932.

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