Early-Stage Shear Viscosity far from Equilibrium via Holography

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Motivation

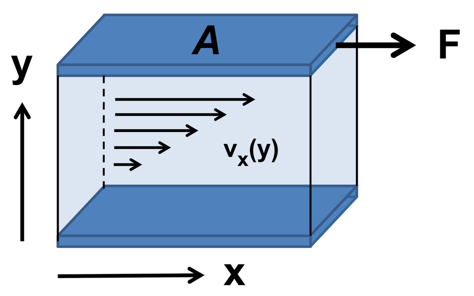
- The earliest stages of a particle collision are far from equilibrium and – at the same time – non-dilute and strongly coupled.
- The elliptic flow allows conclusions about the initial phase of a collision. It crucially depends on the shear viscosity η.
- We investigate the time-dependent evolution of η/s during the heating phase of a particle collision by the AdS/CFT correspondence.

Shear Viscosity

The quark-gluon plasma (QGP) in heavy-ion collisions shows nearly perfect fluidity: The shear viscosity η is small compared with the entropy density s, i.e. $\eta/s \sim 0.1$.
In hydrodynamics, the constitutive equation of the energy-momentum tensor with first-order non-ideal corrections reads

$$
\begin{aligned}
T_{\mu\nu} &= T^{\mathrm{IF}}_{\mu\nu} + T^{\mathrm{NIF}}_{\mu\nu} \\
&= (\epsilon\, u_\mu u_\nu + P\,\Delta_{\mu\nu}) - (2\eta\,\sigma_{\mu\nu} + \zeta\,(\nabla_\rho u^\rho)\,\Delta_{\mu\nu})
\end{aligned}
$$

with $\quad \Delta_{\mu\nu} \equiv g_{\mu\nu} + u_\mu u_\nu \qquad \sigma_{\mu\nu} \equiv \nabla_{\langle\mu} u_{\nu\rangle}.$



In the non-relativistic, flat-spacetime limit, T_{xy} is directly proportional to η: $-T_{xy} = F/A = 2\eta\,\sigma_{xy} = \eta\,\partial_y v_x$.

AdS/CFT Correspondence

The AdS/CFT correspondence identifies certain pairs of strongly coupled gauge theories at large gauge rank and supergravity on spacetimes with Anti-de Sitter asymptotics. The field theory lives on the "boundary" ($z = 0$) of the "bulk" spacetime.

A thermal state is dual to a black brane on the gravitational side, i.e. a black hole with an extended horizon. There is a direct relation between thermodynamic state variables on both sides.

Field theory	Black brane
T	$T_{\mathrm{H}} = \kappa/2\pi$
s	$S_{\mathrm{BH}} = A/4G_{\mathrm{N}}$
μ	$A_0\vert_{z\to 0}$

Expansion of bulk fields, ϕ, around the boundary encodes the sources, J, and one-point functions, $\langle\mathcal{O}(x)\rangle_J$, of the gauge theory as the leading and sub-leading coefficients. For instance,

$$\langle\mathcal{O}(x)\rangle_J = -\frac{\delta W}{\delta J(x)} \propto \phi_{(+)}(x).$$

Moreover, the boundary generating functional for connected n-point functions is represented as the gravity action:

$$W[J]\big|_{J=\phi_{(0)}} = S_{\mathrm{SUGRA}}[\phi]\big|_{\lim\limits_{z\to 0}(\phi(z,x)z^{\Delta-d})=\phi_{(0)}(x)}.$$

Based on these relations, the AdS/CFT correspondence yields a value of $\eta/s = 1/4\pi$ for a large class of strongly coupled theories.

Theoretical Description

On the field theory side, we extend the calculation of η/s to the far-from-equilibrium regime. We consider a sudden homogeneous energy deposition as a model for the heating phase during a heavy-ion collision.
This is realized on the gravity side by a Reissner-Nordström Vaidya spacetime in Einstein-Maxwell theory. The metric in infalling Eddington-Finkelstein coordinates reads

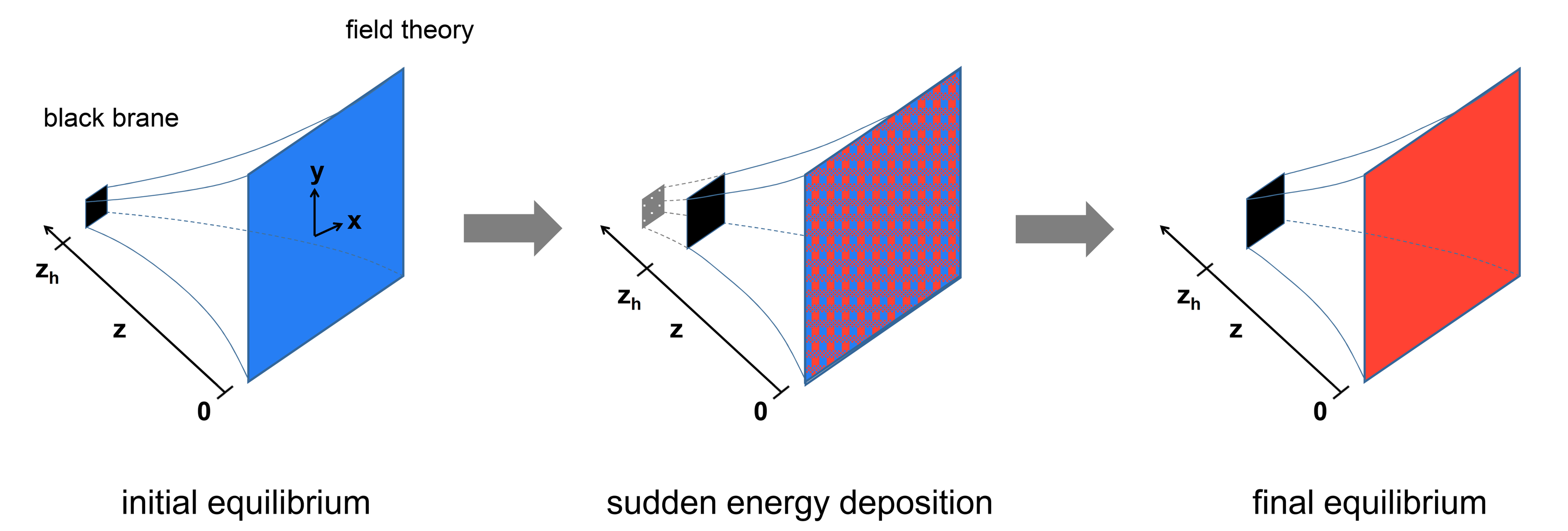
$$ds^2 = \frac{1}{\xi(v)^2 z^2}\left(-\left[f(v,\xi(v)z) + 2z\,\xi'(v)\right]\mathrm{d}v^2 - 2\xi(v)\,\mathrm{d}v\mathrm{d}z + \mathrm{d}x^2 + \mathrm{d}y^2\right)$$

with the blackening factor $f(v,z) = 1 - 2G_{\mathrm{N}}M(v)\,z^3 + G_{\mathrm{N}}Q(v)^2\,z^4$.
The mass function has a tanh profile. It determines the time scale and the temperature increase.

$$M(v) = m + m_{\mathrm{s}}\;\left(1 + \tanh\left[\left(v - t_{\mathrm{m,s}}\right)/\sigma_{\mathrm{m,s}}\right]\right)/2$$

The extension of the thermodynamic variables to the time-dependent regime is non-trivial. We use the holographic equation of state and translate the expressions for temperature and entropy density, T and s, into bulk degrees of freedom.

Holographic Approach



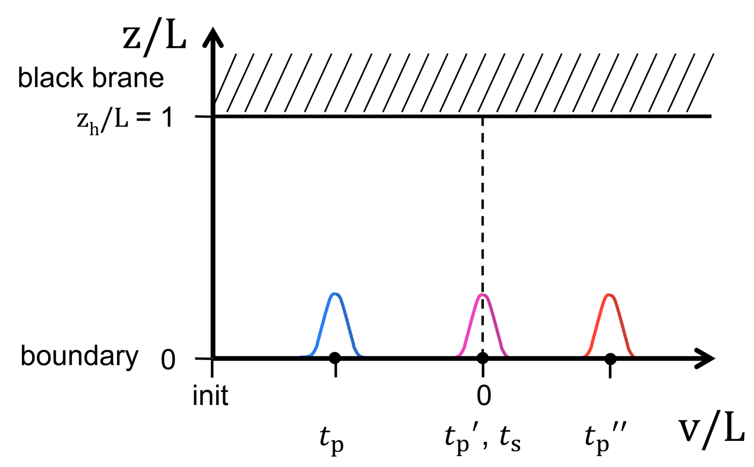
Far from equilibrium physics from the bulk and the boundary perspective.

Retarded Green's Function

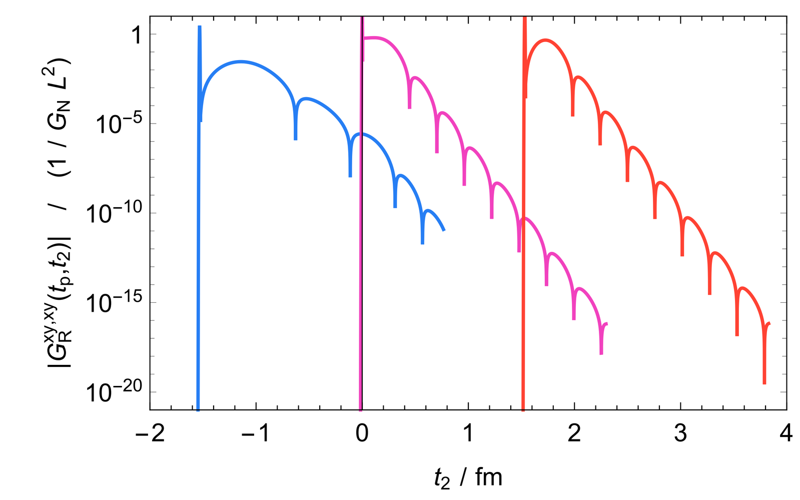
We linearize the Einstein field equations and solve them in the bulk spacetime. The solution of the metric fluctuation couples to the boundary energy-momentum tensor. One obtains the corresponding retarded Green's function in a linear response approach.

$$\langle T^{xy}(t_2)\rangle_h = \int \mathrm{d}\tau\; G_{\mathrm{R}}^{xy,xy}(\tau,\,t_2)\;\underbrace{h^{(0)}_{xy}(\tau)}_{=\epsilon\,\delta(\tau-t_{\mathrm{p}})} = \epsilon\; G_{\mathrm{R}}^{xy,xy}(t_{\mathrm{p}},\,t_2)$$

In a characteristic formulation, the solution is found numerically by the method of lines: A pseudospectral method is used on the spatial part, while a 4th order Runge-Kutta scheme evolves the system in time. The Dirac-pulse is represented by a Gaussian excitation, $h^{(0)}_{xy}(v) \simeq \frac{\epsilon}{\sqrt{2\pi}\,\sigma_{\mathrm{pv}}}\exp(-(v-t_{\mathrm{p}})/2{\sigma_{\mathrm{pv}}}^2)$.



Sketch of the setup. Null matter infall at t_{s}, perturbation $h^{(0)}_{xy}$ at different times t_{p}.

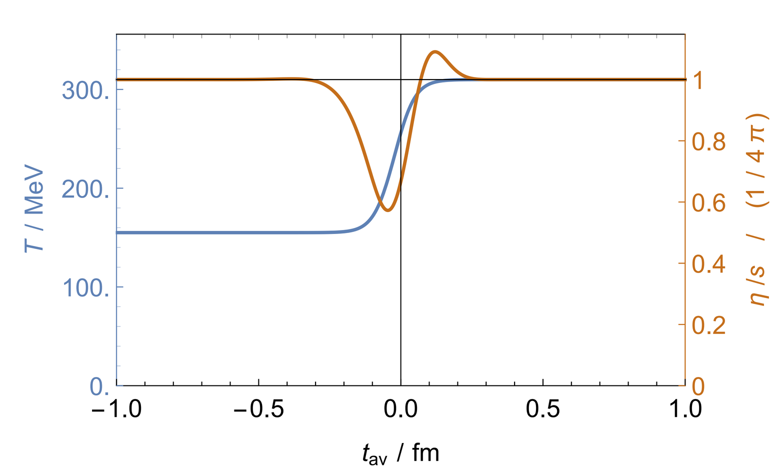


Retarded Green's function in position space. Note the scaling of the dominant quasi-normal mode with temperature.

The time-dependent retarded Green's function in momentum-space, $\tilde{G}_{\mathrm{R}}^{xy,xy}(t_{\mathrm{av}},\omega)$ arises by a Wigner transformation, where the reference time is $t_{\mathrm{av}} = (t_{\mathrm{p}} + t_2)/2$. The Kubo formula yields $\eta = -\lim\limits_{\omega\to 0}\frac{1}{\omega}\,\Im\,\tilde{G}_{\mathrm{R}}^{xy,xy}(t_{\mathrm{av}},\omega)$.

η/s far from Equilibrium

Typical parameters of a heating phase in RHIC collisions: Temperature increase of 155 MeV $\to$ 310 MeV within 0.3 fm.
Significant deviations during the far-from-equilibrium regime, reaching down to below 60% and up to 110% of the equilibrium value.



Evolution of temperature and of the shear viscosity to entropy density ratio.

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