Parametrized equation of state with critical point and first-order phase transition

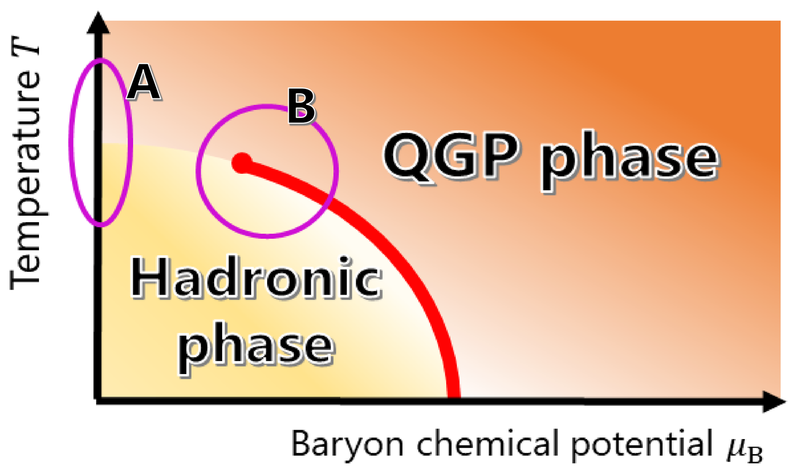
Yuto Yoshida (Sophia Univ.), Koichi Murase (Peking Univ.) and Tetsufumi Hirano (Sophia Univ.)
E-mail: y-yoshida-w8n@eagle.sophia.ac.jp

Abstract

In the RHIC Beam Energy Scan (BES) experiment, an attempt has been made to investigate properties of quark/hadronic matter at finite density. To search for the critical point (CP) phenomenologically, relativistic hydrodynamics with an appropriate equation of state (EoS) plays an essential role. In this study, adopting several models in their relevant region in T-μ_B plane, we construct a parametrized EoS with the CP and the first order phase transition.

1. Introduction

QCD phase diagram (expected)



A: High temperature and low density region reachable at top RHIC and LHC energies

- First principle calculations are available.
- The equation of state (EoS) has already been obtained from lattice QCD simulations. S. Borsanyi, et al., Phys. Lett. B730 (2014) 99-104

B: Reachable at RHIC-BES, FAIR, NICA, or J-PARC-HI energies(?)

- First principle calculations not available due to the sign problem
- Existence of the critical point (CP) and the first order phase transition? M. Asakawa and K. Yazaki, Nucl. Phys. A504, 668 (1989)
- No consensus on location of the CP

Purpose of this study

Assuming the location of the CP and the boundary between the QGP phase and the hadron phase, we obtain the EoS combining several models according to their relevant region.

2. Model

In order to obtain entropy density in a wide range of QCD phase diagram, we calculate entropy density for near the CP, high temperature and low density region, high temperature and high density region, and low temperature and low density region.

1. Near the CP

Critical phenomena in the QCD phase diagram are known to have the same universality as those in the 3D Ising model. We employ **a parametrized EoS for 3D Ising model**. Magnetization M as a function of external magnetic field h and reduced temperature r can be obtained. Using thermodynamic relations we calculate entropy density.

F. Wilczek. Int. J. Mod. Phys. A7, 3911 (1992) R. Guida and J. Zinn-Justin, Nucl. Phys. B489, 626 (1997)

$$M = M_0 R^\beta \theta,\quad h = h_0 R^{\beta\delta}\tilde{h}(\theta),\quad r = R(1-\theta^2) \;\Rightarrow\; M(h,r) \;\Rightarrow\; s(h,r)$$

where $\beta = 0.326$ and $\delta = 4.80$ are critical exponents, $\tilde{h}(\theta) = \theta + a\theta^3 + b\theta^5$, with $a = -0.76201$, $b = 0.00804$, and M_0 and h_0 are normalization constants.

C. Nonaka and M. Asakawa, Phys. Rev. C71, 044904 (2005)

To obtain entropy density near the CP as a function of T and μ_B, we make **a change of variables from (h, r) to (T, μ_B)** as follows.

$$h = \cos\alpha \frac{T - T_c}{\Delta T_c} + \sin\alpha \frac{\mu_B - \mu_{Bc}}{\Delta T_c}, \qquad r = \sin\alpha \frac{T - T_c}{\Delta \mu_{Bc}} - \cos\alpha \frac{\mu_B - \mu_{Bc}}{\Delta \mu_{Bc}},$$

where T_c and μ_{Bc} are locations of the CP in T-μ_B plane, ΔT_c and $\Delta\mu_{Bc}$ are sizes of critical region, and α is an angle between r and μ_B. In this way we obtain entropy density near the CP, $s_{\mathrm{critical}}(T, \mu_B)$.

2. High temperature and low density region

S. Borsanyi, et al., Phys. Lett. B730 (2014) 99-104

We use a fitting function for the result of **first principle calculations from lattice QCD** with three flavors and physical quark mass.

$$s_{\mathrm{lattice}}(T, \mu_B = 0) = 4T^3 \int_0^T \frac{\mathrm{d}T'}{T'} \frac{I(T')}{T'^4} + \frac{I(T)}{T},$$

where

$$\frac{I(T)}{T^4} = \exp\left(-\frac{h_1}{t} - \frac{h_2}{t^2}\right) \cdot \left(h_0 + \frac{f_0[\tanh(f_1 t + f_2) + 1]}{1 + g_1 t + g_2 t^2}\right),$$

$t = \frac{T}{200\ \mathrm{MeV}}$, $h_0 = 0.1396$, $h_1 = -0.1800$, $h_2 = 0.0350$, $f_0 = 2.76$, $f_1 = 6.79$, $f_2 = -5.29$, $g_1 = -0.47$, $g_2 = 1.04$.

3. Low temperature and low density region

J. Sollfrank et al., Phys. Rev. C55, 392 (1997) Y. Nara et al., Phys. Rev. C61, 024901 (2000)

We employ **the hadron resonance gas (HRG) model** with a particle list from an event generator, JAM, and incorporate **mean field potential into the model**. The repulsive potential is

$$V(n_B) = \frac{1}{2} K n_B^2,$$

where n_B is baryon number density and $K = 450\ \mathrm{fm}^3 \cdot \mathrm{MeV}$.

4. High temperature and high density region

We employ the **bag model**. In this study quarks are massive and have three flavors.

☆**Phase boundary**

We fix a bag constant as a function of μ_B so that a first order transition line becomes $T_c(\mu_B) = T_0\sqrt{1 - \mu_B^2/\mu_0^2}$, where $T_0 = 155\ \mathrm{MeV}$ is pseudo-critical temperature at zero baryon chemical potential and $\mu_0 = 1500\ \mathrm{MeV}$.

Connecting four models

We construct entropy density in the whole region.

$$s(T, \mu_B) = (1 - w)s_{\mathrm{lattice}} + w\left[\frac{1}{2}(1 - \tanh[s_{\mathrm{critical}}(T, \mu_B)])s_{\mathrm{HRG}}(T, \mu_B) + \frac{1}{2}(1 + \tanh[s_{\mathrm{critical}}(T, \mu_B)])s_{\mathrm{Bag}}(T, \mu_B)\right],$$

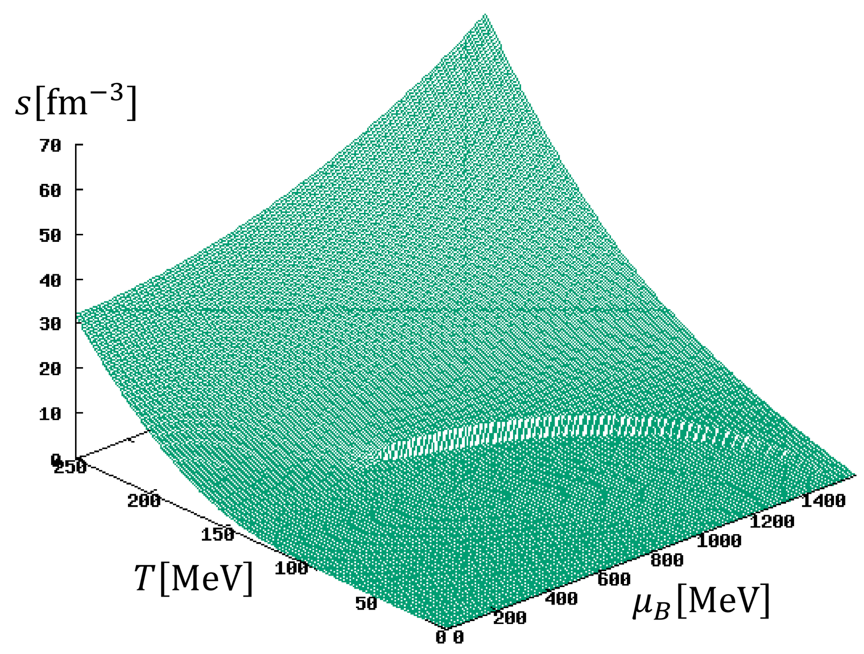
C. Nonaka and M. Asakawa, Phys. Rev. C71, 044904 (2005)

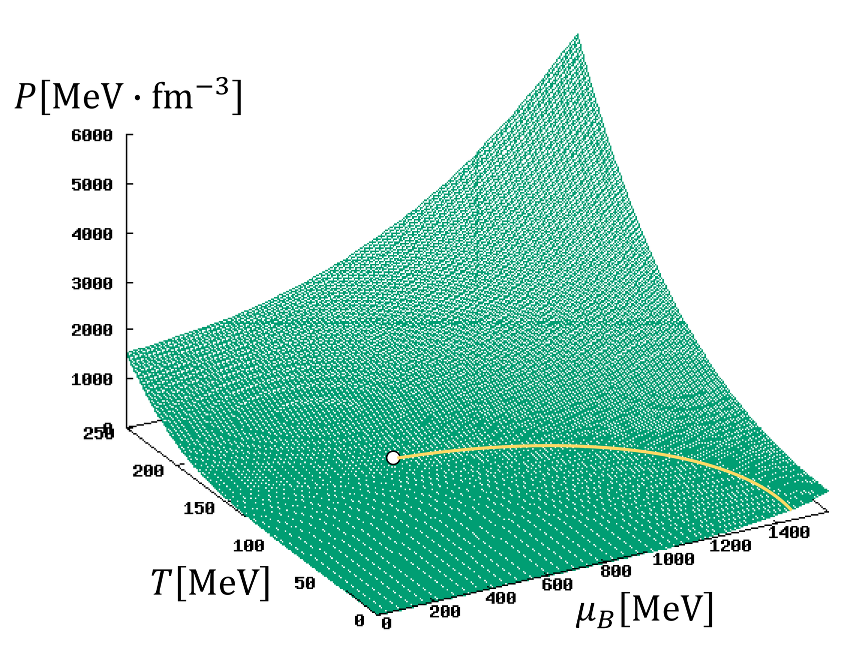
where $w(T, \mu_B) = \frac{1}{\exp\left(\frac{2T - \mu_B}{500\ [\mathrm{MeV}]}\right) + 1}$.

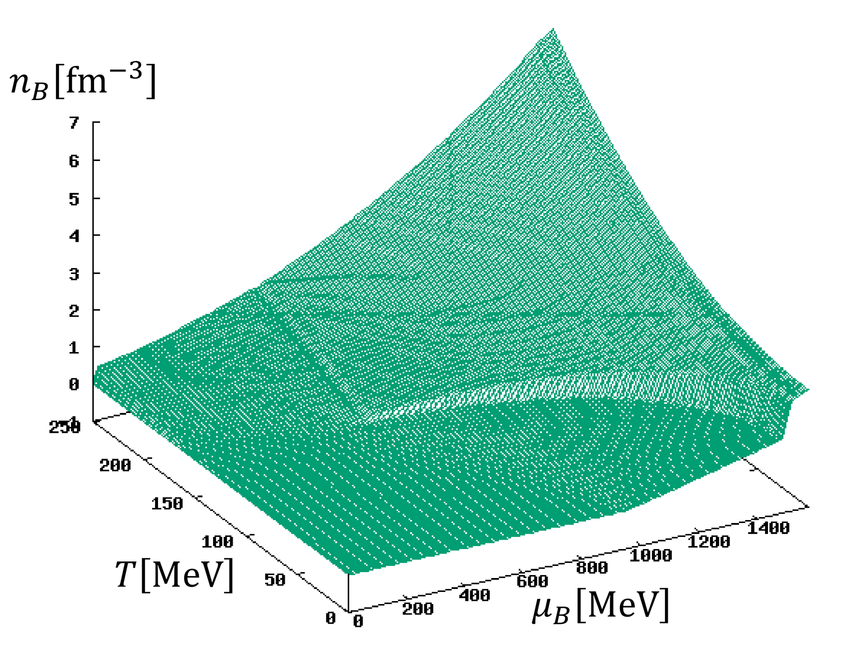
The weight factor, w, is so designed that the lattice QCD results manifest in $\mu_B/T < 2$

3. Results

We calculated **entropy density**, **pressure** and **baryon number density** (from left to right) as a function of T and μ_B.
Pressure and baryon number density are obtained by using thermodynamic relations. The location of the CP is $(T_c, \mu_{Bc}) = (143\ \mathrm{MeV}, 570\ \mathrm{MeV})$.







- The thermodynamic variables **smoothly change below μ_{Bc}**.
- **Near the CP**, the EoS obtained in this way exhibits **the same critical behavior as the 3D Ising model**.
- In the extremely **high density region, the first order phase transition happens** between the QGP phase and the hadron phase.

Summary

- We constructed the EoS **including the CP and the first order phase transition** from several models in their relevant regions.
- We confirmed that the **thermodynamic quantity described the critical behavior near the CP**.

outlook

- We **draw adiabatic curves $n_B/s = \mathrm{const}$.**
- We **calculate pressure as a function of baryon number density and energy density**.
- We **apply to hydrodynamical model and compare hydrodynamic results with heavy ion collision data**.