Heavy quark momentum diffusion coefficient in 3D gluon plasma

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Introduction: Heavy-ion collisions, non-equilibrium evolution

- After URHIC the IS dominated by nonperturbatively large gluon fields ($f \sim 1/g^2$) out of equilibrium.
- IS gluon fields can be effectively described as classical fields!
- The big questions: Transport properties of this matter? Evolution of heavy quarks in this matter?

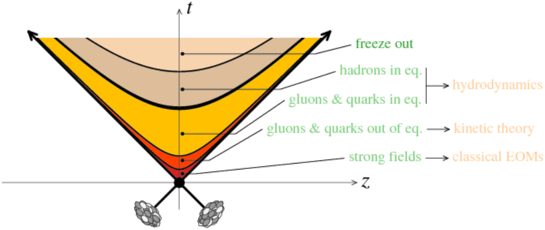


Figure: E. Iancu 1105.0751 [hep-ph]

Initial nonequilibrium evolution in classical gluodynamics

$$f(p,t) = (Qt)^{\alpha} f_s\left((Qt)^{\beta} p\right)$$

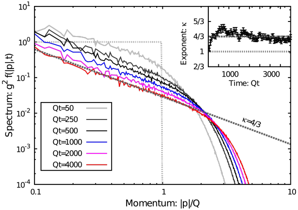


Figure: PRD 89 (2014) 114007

- Self-similar attractor, dynamics determined by powerlaws.
- Dynamical scale separation analogous to HTL PT.

The analogous scales:

- Ultrasoft scale $g^2T \iff$ magnetic scale $m_{\mathrm{mag}} \sim t^{-1/3}$
- Soft scale $gT \iff$ mass scale $\omega_{\mathrm{pl}} \sim t^{-1/7}$
- Temperature $T \iff$ scale of hard excitations $\Lambda \sim t^{1/7}$

Heavy quark diffusion - Introduction

- HQ momentum diffusion coefficient gauge invariantly and nonperturbatively (JHEP 0904 (2009) 053, $A^0 = 0$):

$$\kappa(t, \Delta t) = \frac{1}{3N_c} \int_t^{t+\Delta t} \mathrm{d}t' \frac{1}{V} \int \mathrm{d}^3x \sum_{i=1}^{3} \left\langle g E_i^a(\boldsymbol{x}, t)\, g E_i^a\left(\boldsymbol{x}, t'\right)\right\rangle$$
$$= \int_t^{t+\Delta t} \mathrm{d}t' \; \kappa\left(t, t'\right). \tag{1}$$

- Main ingredients: Kubo relation, HQ effective theory, $M \to \infty$ limit.
- Contains only gauge fields, no need for a fermion simulation!
- Usually $\kappa = \kappa(\Delta t \to \infty)$, corresponds to $\kappa(\omega = 0)$.

Evaluating the correlation function, 3 methods

1. Real time lattice: extract the correlation function on the lattice and compute the time-integral as in (1).
2. HTL: Use PT to evaluate the correlation function. Supplement with our previous measurements (PRD 98 (2018) no.1, 014006) in terms of quasiparticle damping and dispersion relation.
3. Kinetic theory: Extract $f(k)$ from lattice, use kinetic theory to compute κ, use and generalize Phys.Rev. C71 (2005) 064904 (Moore & Teaney).

2. HTL method

- Strategy: Express $\langle E(x,t)E(x,t')\rangle$ (Eq. (1)) using HTL.

2 correlation functions:

- Statistical correlation function, easy to measure classically.

$$\ddot{F}^{jk}_{ab}(x,x') = \frac{1}{2}\left\langle\left\{\hat{E}^j_a(x),\hat{E}^k_b(x')\right\}\right\rangle \tag{2}$$

- Spectral function, measure using linear response (PRD 98 (2018) no.1, 014006)

$$\dot{\rho}^{ab}_{jk}(x,x') = i\left\langle\left[\hat{E}^j_a(x),\hat{A}^b_k(x')\right]\right\rangle. \tag{3}$$

ρ can be evaluated using HTL PT. How to connect F to ρ?
Answer: generalized fluct. dis. relation (agrees with numerics)

$$\frac{\ddot{F}_{T/L}(t,\omega,p)}{\ddot{F}_{T/L}(t,\Delta t=0,p)} = \frac{\dot{\rho}_{T/L}(t,\omega,p)}{\dot{\rho}_{T/L}(t,\Delta t=0,p)}. \tag{4}$$

2. HTL method: main ingredients

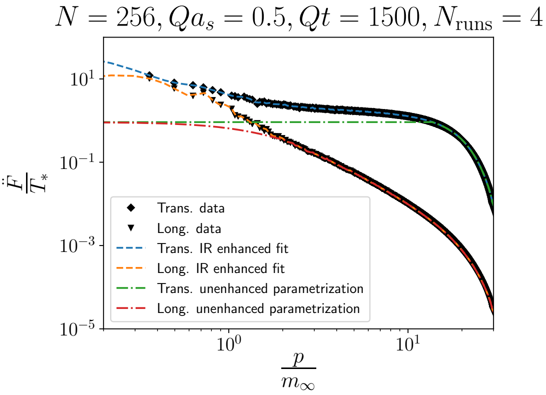
$$\kappa = \frac{d_A}{6N_c} \int \frac{d^3p}{(2\pi)^3} \int \frac{d\omega}{(2\pi)} \frac{2\sin(\Delta t\omega)}{\omega} \left[2g^2 \frac{\ddot{F}_T(t,\Delta t=0,p)}{\dot{\rho}_T(t,\Delta t=0,p)} \dot{\rho}_T(t,\omega,p) + g^2 \frac{\ddot{F}_L(t,\Delta t=0,p)}{\dot{\rho}_L(t,\Delta t=0,p)} \dot{\rho}_L(t,\omega,p) \right]. \quad (5)$$

Spectral function: decompose into quasiparticle and Landau damping parts (transverse and longitudinal):

$$\begin{aligned} \rho_T(\omega,p) = &\quad (2\pi)\beta_T(\omega,p) \\ &+ (2\pi) Z_T(p) \left[h_T(\omega-\omega_T(p),p) - h_T(\omega+\omega_T(p),p) \right] \end{aligned} \quad (6)$$

$$\begin{aligned} \rho_L(\omega,p) = &\frac{p^2}{\omega^2}(2\pi)\beta_L(\omega,p) \\ &+ \frac{p^2}{\omega^2}(2\pi) Z_L(p) \left[h_L(\omega-\omega_L,p(p)) - h_L(\omega+\omega_L(p),p) \right]. \end{aligned} \quad (7)$$

IR enhancement: Equal time statistical correlation function (PRD 98 (2018) no.1, 014006)



- Black points: numerical extraction
- Dashed lines: interpolation
- Dash dotted lines: HTL expectation matched to data in the UV.
- IR enhancement: Deviation of non equilibrium correlation function from the HTL expectation in the infrared.

3. Kinetic theory

In the kinetic theory framework κ is given by ($gq \to gq$, t-channel gluon exchange. Compton suppressed, generalize GDM & Teaney, PRC 71 (2005) 064904)

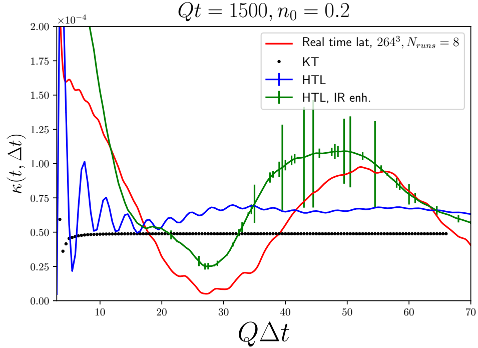
$$|\mathscr{M}|^2_{\text{gluon}}(\omega) = \left[N_c C_H g^4\right] \frac{4M^2(k_0+k_0')^2\left(1+\cos^2\theta_{\boldsymbol{k}\boldsymbol{k}'}\right)}{(q^2-\omega^2+m_D^2)^2}, \qquad (8)$$

with $k_0' = k_0 + \omega$.

$$\kappa(\omega) = \frac{1}{6M}\int \frac{\mathrm{d}^3\boldsymbol{k}\,\mathrm{d}^3\boldsymbol{q}}{(2\pi)^6\, 8|\boldsymbol{k}||\boldsymbol{k}+\boldsymbol{q}|M} 2\pi\delta(|\boldsymbol{k}+\boldsymbol{q}|-|\boldsymbol{k}|-\omega)$$
$$\times\, \boldsymbol{q}^2\, |\mathscr{M}|^2_{\text{gluon}}(\omega) f(k) f(|\boldsymbol{k}+\boldsymbol{q}|) \qquad (9)$$

The dependence on time Δt is obtained by Fourier transforming this expression to time domain.

Dependence on upper integration limit Δt



- With IR enhancement, similar oscillations as in the data.
- KT does not reproduce the oscillations.
- From this plot, $\kappa(t)$ is extracted as an average.
- Frequency ≈ plasmon frequency.

Heavy quark diffusion - time-evolution

Expected scaling in time: $\kappa \sim m_D^2 g^2 T_* \log\left(\frac{\Lambda}{m_D}\right) \sim t^{-5/7}$

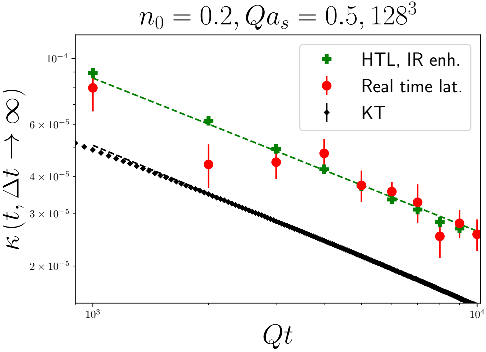


Figure: Extracted heavy quark diffusion coefficient vs. time

- Find $\kappa \sim t^{-B}$ with $B = 0.5 - 0.55, \rightarrow$ log also important.

Conclusions

We have

- Measured heavy quark momentum diffusion coefficient κ. Time evolutions consistent with $\kappa \sim t^{-1/2}$.
- Observed IR enhancement in equal time statistical correlation functions. The effect produces oscillations in $\kappa(t, \Delta t)$ with plasmon frequency. Not reproduced by KT.

Future plans

- Measure also other transport coefficients ($\hat{q}$ etc.) out of equilibrium.
- Spectral functions in 2D gluon plasma (in preparation, extending: arXiv:1907.05892 [hep-ph], to appear in PRD)

Heavy quark diffusion - Dependence on lattice cutoffs

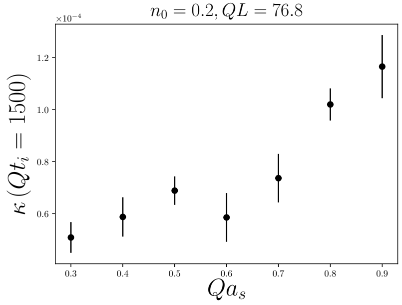


Figure: UV cutoff dependence

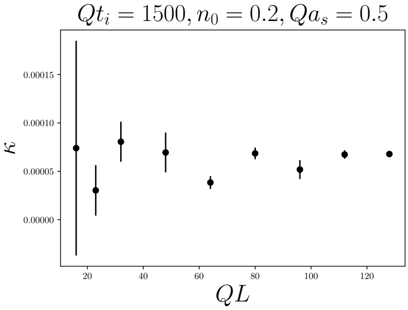


Figure: Volume dependence

Heavy quark diffusion - dependence on occupation number

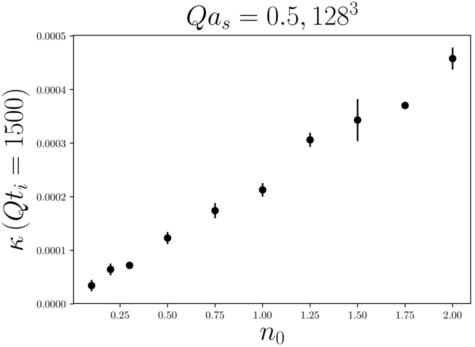


Figure: Extracted heavy quark diffusion coefficient vs. n_0

Separation of scales, HTL vs. classical simulations

On top of these on the lattice we have

- Q the scale of typical hard excitations at $t = 0$.
- n_0 determines the occupation number of the system
- T_* effective temperature

$$T_*(t) \approx \frac{\int \mathrm{d}^3 p f^2(t,p)}{\int \mathrm{d}^3 p f(t,p) / \sqrt{p^2}}. \tag{10}$$

Scales as

$$g^2 T_*(t)/Q \sim (Qt)^{-3/7}. \tag{11}$$

Heavy quark diffusion - Lattice formula

(1) in general gauge (and in Euclidean formulation) is given by:

$$\kappa(\omega) = {}^1\!/_3 \int \mathrm{d}t e^{i\omega t} \sum_{i=1}^{3} \times \mathrm{Tr}\left\langle U(-\infty - i\beta, t)\, gE_i\left(t, \vec{0}\right) U(t,0)\, gE_i\left(0,\vec{0}\right) U(0,-\infty)\right\rangle \times \left(\mathrm{Tr}\left\langle U(-\infty - i\beta, t)\, U(t,0)\, U(0,-\infty)\right\rangle\right)^{-1}, \qquad (12)$$

2. HTL method

Fourier transform to momentum space. Expand statistical functions in Fourier modes

$$\bar{\kappa} = \frac{d_A}{3}\frac{1}{2N_c}\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\int \frac{\mathrm{d}\omega}{(2\pi)}\frac{2\sin(\Delta t\omega)}{\omega}\left[2g^2\ddot{F}_T(t,\omega,p)+g^2\ddot{F}_L(t,\omega,p)\right] \tag{13}$$

Use fluctuation dissipation relation

$$\kappa = \frac{d_A}{6N_c}\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\int \frac{\mathrm{d}\omega}{(2\pi)}\frac{2\sin(\Delta t\omega)}{\omega}\Bigg[2g^2\frac{\ddot{F}_T(t,\Delta t=0,p)}{\dot{\rho}_T(t,\Delta t=0,p)}\,\dot{\rho}_T(t,\omega,p) + g^2\frac{\ddot{F}_L(t,\Delta t=0,p)}{\dot{\rho}_L(t,\Delta t=0,p)}\,\dot{\rho}_L(t,\omega,p)\Bigg]. \tag{14}$$

2. HTL method: main ingredients

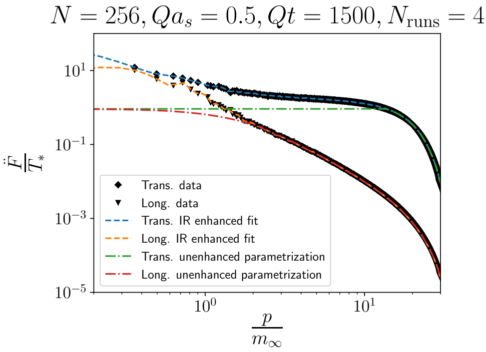
$$\kappa = \frac{d_A}{6N_c} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \int \frac{\mathrm{d}\omega}{(2\pi)} \frac{2\sin(T\omega)}{\omega} \Bigg[2g^2 \frac{\ddot{F}_T(t, \Delta t = 0, p)}{\dot{\rho}_T(t, \Delta t = 0, p)} \dot{\rho}_T(t, \omega, p)$$
$$+ g^2 \frac{\ddot{F}_L(t, \Delta t = 0, p)}{\dot{\rho}_L(t, \Delta t = 0, p)} \dot{\rho}_L(t, \omega, p) \Bigg]. \tag{15}$$

HTL expectation:

$$\ddot{F}_T^{HTL}(\bar{t}, \Delta t = 0, p) = T_*(\bar{t}) \tag{16}$$
$$\ddot{F}_L^{HTL}(\bar{t}, \Delta t = 0, p) = T_*(\bar{t}) \frac{2m^2}{2m^2 + p^2}. \tag{17}$$

2. HTL method: main ingredients

$$\kappa = \frac{d_A}{6N_c} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \int \frac{\mathrm{d}\omega}{(2\pi)} \frac{2\sin(T\omega)}{\omega} \left[2g^2 \frac{\ddot{F}_T(t, \Delta t = 0, p)}{\dot{\rho}_T(t, \Delta t = 0, p)} \dot{\rho}_T(t, \omega, p) + g^2 \frac{\ddot{F}_L(t, \Delta t = 0, p)}{\dot{\rho}_L(t, \Delta t = 0, p)} \dot{\rho}_L(t, \omega, p) \right]. \tag{18}$$



2. HTL method: main ingredients

$$\kappa = \frac{d_A}{6N_c}\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\int \frac{\mathrm{d}\omega}{(2\pi)}\frac{2\sin(\Delta t\,\omega)}{\omega}\Bigg[2g^2\frac{\ddot{F}_T(t,\Delta t=0,p)}{\dot{\rho}_T(t,\Delta t=0,p)}\,\dot{\rho}_T(t,\omega,p) + g^2\frac{\ddot{F}_L(t,\Delta t=0,p)}{\dot{\rho}_L(t,\Delta t=0,p)}\,\dot{\rho}_L(t,\omega,p)\Bigg]. \tag{19}$$

Equal time spectral functions:

$$\dot{\rho}_T^{HTL}(\Delta t=0,p) = 2\int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\dot{\rho}_T^{HTL}(\omega,p) = 1 \tag{20}$$

$$\dot{\rho}_L^{HTL}(\Delta t=0,p) = 2\int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\dot{\rho}_L^{HTL}(\omega,p) = \frac{2m^2}{2m^2+p^2}. \tag{21}$$

2. HTL method: main ingredients

$$\begin{aligned}\rho_T(\omega,p) = & \quad (2\pi)\beta_T(\omega,p) \\ & + (2\pi) Z_T(p)\left[h_T(\omega-\omega_T(p),p) - h_T(\omega+\omega_T(p),p)\right]\end{aligned} \tag{22}$$

$$\begin{aligned}\rho_L(\omega,p) = & \frac{p^2}{\omega^2}(2\pi)\beta_L(\omega,p) \\ & + \frac{p^2}{\omega^2}(2\pi) Z_L(p)\left[h_L(\omega-\omega_L,p(p)) - h_L(\omega+\omega_L(p),p)\right].\end{aligned} \tag{23}$$

Quasiparticle peaks:

$$h_{T/L}\left(\omega \mp \omega_{T/L}(p),p\right) = \frac{1}{\pi}\frac{\gamma_{T/L}(p)}{\left(\omega \mp \omega_{T/L}(p)\right)^2 + \gamma^2_{T/L}(p)} . \tag{24}$$

$\omega_{T,L}$ Quasiparticle dispersion relation, $\gamma_{T,L}$ quasiparticle damping rate (inverse lifetime). Use our data, LO PT predicts $\gamma = 0$.

2. HTL method: main ingredients

$$\rho_T(\omega,p) = (2\pi)\,\beta_T(\omega,p) + (2\pi)\,Z_T(p)\left[h_T(\omega-\omega_T(p),p) - h_T(\omega+\omega_T(p),p)\right] \tag{25}$$

$$\rho_L(\omega,p) = \frac{p^2}{\omega^2}(2\pi)\,\beta_L(\omega,p) + \frac{p^2}{\omega^2}(2\pi)\,Z_L(p)\left[h_L(\omega-\omega_L,p(p)) - h_L(\omega+\omega_L(p),p)\right]. \tag{26}$$

Residues of the quasiparticle peaks:

$$Z_T(p) = \frac{\omega_T(p)\left(\omega_T^2(p)-p^2\right)}{3\omega_{\mathrm{pl}}^2\omega_T^2(p) - \left(\omega_T^2(p)-p^2\right)^2}\,, \tag{27}$$

$$Z_L(p) = \frac{\omega_L(p)\left(\omega_L^2(p)-p^2\right)}{p^2\left(p^2+2m^2-\omega_L^2(p)\right)}\,. \tag{28}$$

2. HTL method: main ingredients

$$\rho_T(\omega,p) = (2\pi)\,\beta_T(\omega,p) + (2\pi)\,Z_T(p)\left[h_T(\omega-\omega_T(p),p) - h_T(\omega+\omega_T(p),p)\right] \tag{29}$$

The Landau damping terms according to HTL at LO are given by:

$$\beta_T(\omega,p) = \left(\frac{m^2}{2}x(1-x^2)\,\theta\left(1-x^2\right)\right)\left(\left[\left[p^2\left(1-x^2\right) + m^2\left(x^2+\frac{x\left(1-x^2\right)}{2}\ln\left|\frac{x+1}{x-1}\right|\right)\right]^2 + \frac{\pi^2}{4}m^4x^2\left(1-x^2\right)^2\right)^{-1} \tag{30}$$

where $x = \dfrac{\omega}{p}$

2. HTL method: main ingredients

$$\rho_L(\omega,p)=\frac{p^2}{\omega^2}(2\pi)\,\beta_L(\omega,p) + \frac{p^2}{\omega^2}(2\pi)\,Z_L(p)\left[h_L(\omega-\omega_L,p(p))-h_L(\omega+\omega_L(p),p)\right]. \tag{31}$$

The Landau damping terms according to HTL at LO are given by:

$$\beta_L(\omega,p)=\frac{m^2x\,\theta\left(1-x^2\right)}{\left[p^2+2m^2\left(1-\frac{x}{2}\ln\left|\frac{x+1}{x-1}\right|\right)\right]^2+\pi^2m^4x^2}\,, \tag{32}$$

where $x=\frac{\omega}{p}$

3. Kinetic theory, $\Delta t \to \infty$

In the kinetic theory framework κ is given by ($gq \to gq$, t-channel gluon exchange. Compton amplitude is suppressed)

$$\kappa = \frac{\langle \Delta k^2 \rangle}{\Delta t} = \frac{1}{6M} \int \frac{\mathrm{d}^3\boldsymbol{k}\,\mathrm{d}^3\boldsymbol{k'}\,\mathrm{d}^3\boldsymbol{p'}}{(2\pi)^9\, 8k^0 k'^0 M} (2\pi)^3\, \delta^3\left(\boldsymbol{p}+\boldsymbol{k'}-\boldsymbol{p'}-\boldsymbol{k}\right)$$
$$\times 2\pi\delta\left(k'-k\right)\boldsymbol{q}^2\, |\mathscr{M}|^2_{\text{gluon}}\, f\left(k\right) f\left(k'\right). \tag{33}$$

k and k' gluon momenta, $q = k - k'$, p and p' incoming and outgoing heavy quark momenta.

$$|\mathscr{M}|^2_{\text{gluon}} = N_c C_H g^4 16 M^2 k_0^2 \left(1+\cos^2\left(\theta_{kk'}\right)\right) \frac{1}{\left(q^2+m_D^2\right)^2} \tag{34}$$

Damping rate, our previous measurements

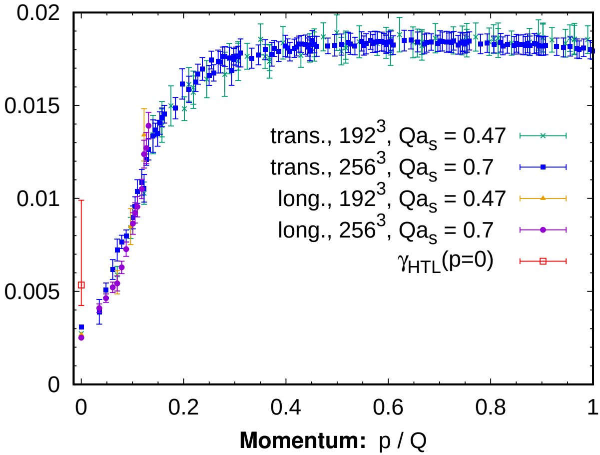


Figure: Measured damping rate

Oscillation frequency in $\kappa(\Delta t)$ vs. plasmon frequency

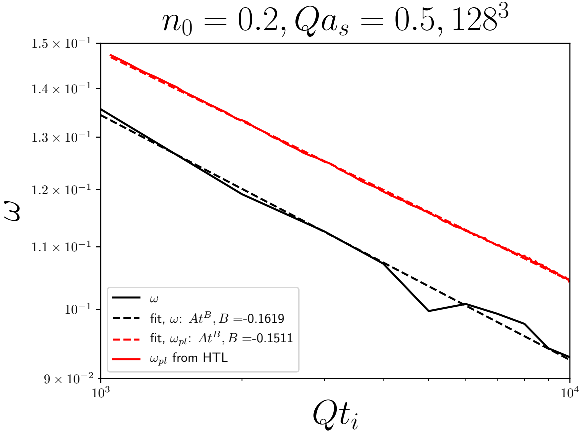


Figure: Extracted frequency of $\kappa(\Delta t)$ from the data together with the extracted plasmon frequency.

1. Direct numerical evaluation of $\langle E(t)E(0)\rangle$

Example of an observed signal using real time lattice:

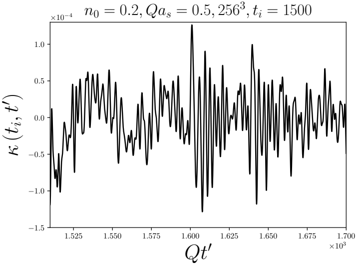


Figure: The extracted signal from a simulation, κ obtained as time integral over this signal.