

Prescaling and far-from-equilibrium hydrodynamics in the quark-gluon plasma

Aleksas Mazeliauskas

Theoretical Physics Department, CERN

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AM and Jürgen Berges, *Phys. Rev. Lett.* 122 (2019) (arXiv:1810.10554)



UNIVERSITÄT
HEIDELBERG
ZUKUNFT
SEIT 1386

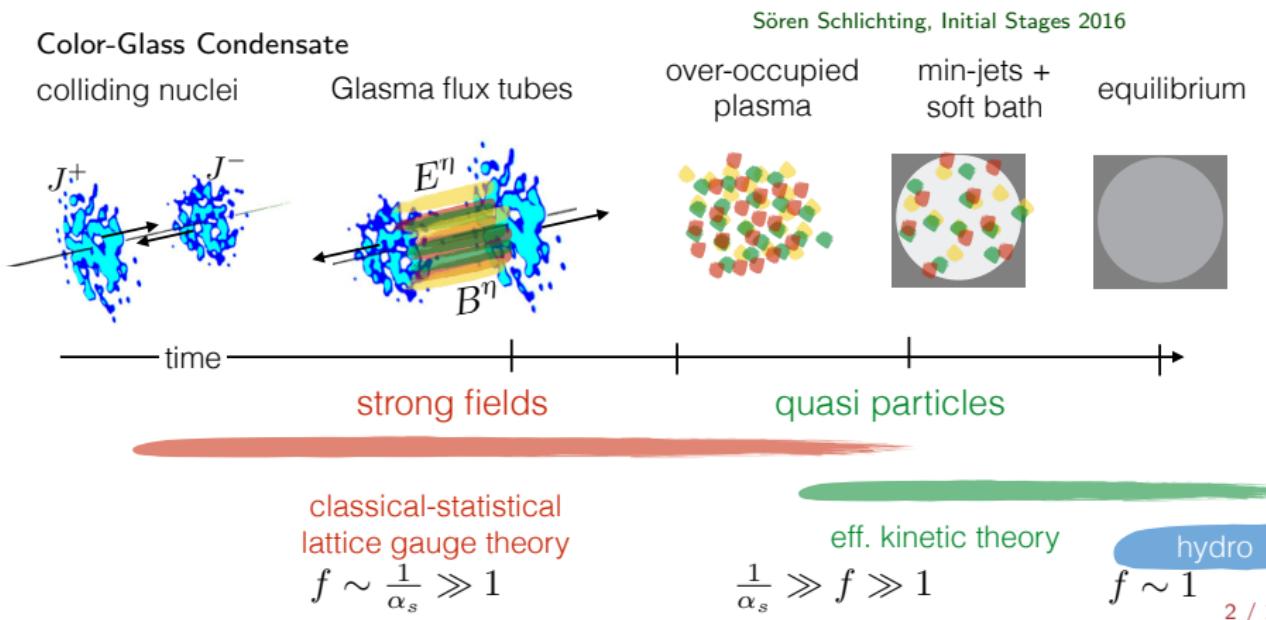


Isolated quantum systems and universality in extreme conditions

Non-equilibrium QCD descriptions at weak coupling $\alpha_s \rightarrow 0$

At high energies mid-rapidity is dominated by small Bjorken- x gluons

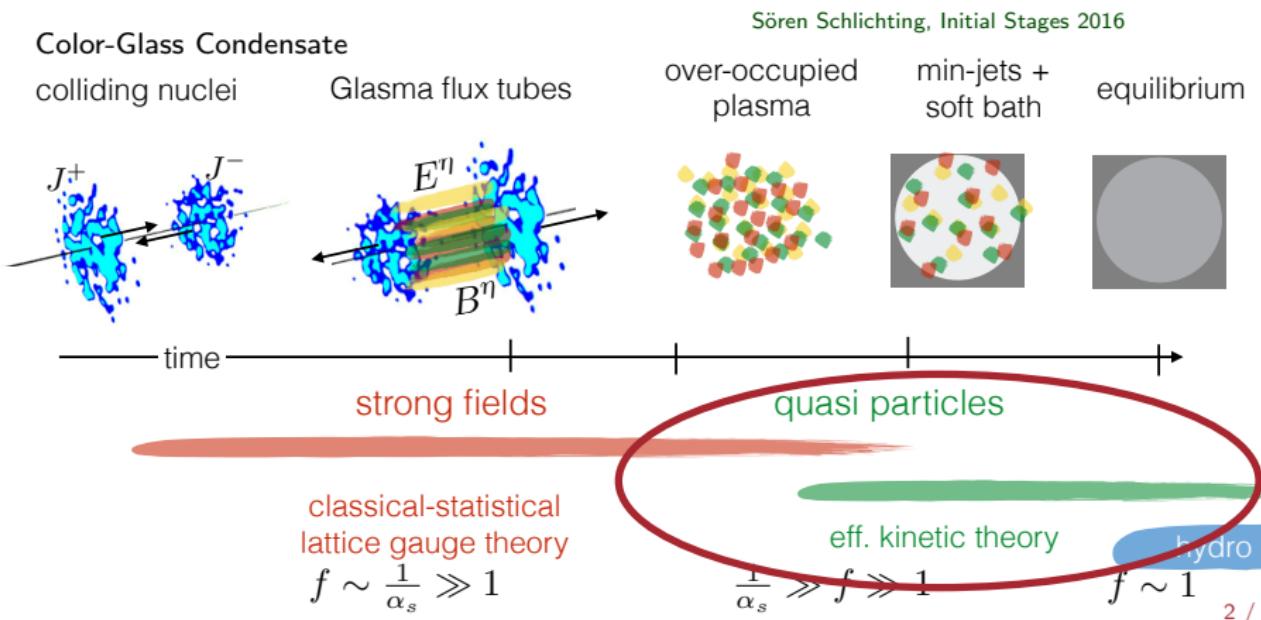
- $p \sim Q_s$ saturation scale $\gg \Lambda_{QCD}$, strong gluon fields $A_\mu \sim \frac{1}{\alpha_s} \gg 1$
 \implies classical-statistical simulations
- decoherence of classical fields at $\tau Q_s \gg 1$
 \implies kinetic evolution of gluon phase space distribution f



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High temperature gauge kinetic theory

Boltzmann equation for distribution f of quark and gluon quasi-particles.

Arnold, Moore, Yaffe (2003)[1]

$$\partial_\tau f_{g,q} - \frac{p_z}{\tau} \partial_{p_z} f_{g,q} = -\mathcal{C}_{2\leftrightarrow 2}[f] - \mathcal{C}_{1\leftrightarrow 2}[f]$$

Leading order processes in the coupling constant $\lambda = 4\pi\alpha_s N_c$:

- 1 2 \leftrightarrow 2 elastic scatterings: $gg \leftrightarrow gg$, $qq \leftrightarrow qq$, $qg \leftrightarrow gq$, $gg \leftrightarrow q\bar{q}$

$$= |\mathcal{M}_{gg}^{gg}|^2 = \lambda^2 16 \frac{d_F C_F}{C_A^2} \left[C_F \left(\frac{u}{t} + \frac{t}{u} \right) - C_A \left(\frac{t^2 + u^2}{s^2} \right) \right]$$

Hard Thermal Loop resumed propagators, screening mass $m_D \sim gT$

Microscopic studies of QGP equilibration with QCD kinetic theory:

- Bottom-up thermalization of gluons.

Kurkela and Zhu (2015) [2]

- Equilibration of perturbations.

Keegan, Kurkela, AM and Teaney (2016) [3]

- Preflow computer code K ϕ MP ϕ ST

Kurkela, AM, Paquet, Schlichting and Teaney (2018)[4, 5]

see Gale's talk on Tuesday 9:20

- Chemical equilibration of quarks and gluons.

Kurkela and AM (2018) [6, 7]

- Pre-scaling phenomena.

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Leading order processes in the coupling constant $\lambda = 4\pi\alpha_s N_c$:

- 2 1 \leftrightarrow 2 medium induced collinear radiation: $g \leftrightarrow gg$, $q \leftrightarrow qg$, $g \leftrightarrow q\bar{q}$

$$= |\mathcal{M}_{qq}^g|^2 = \frac{k'^2 + p'^2}{k'^2 p'^2 p^3} \underbrace{\mathcal{F}_q(k'; -p', p)}_{\text{splitting rate}}$$

Resummed multiple scatterings with the medium (LPM suppression).

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"Bottom-up" thermalization scenario

Baier, Mueller, Schiff, and Son (2001)[10]

Evolution of initially over-occupied hard gluons $p \sim Q_s \gg \Lambda_{\text{QCD}}$

I) over-occupied

$$p_z \sim \frac{Q_s}{(Q_s \tau)^{1/3}}$$

$$1 \ll Q_s \tau \ll \alpha_s^{-3/2}$$

II) under-occupied

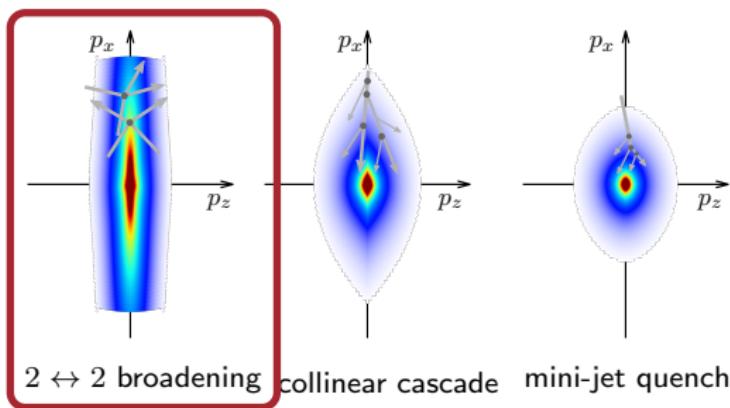
$$p_z \sim \sqrt{\alpha_s} Q_s$$

$$\alpha_s^{-3/2} \ll Q_s \tau \ll \alpha_s^{-5/2}$$

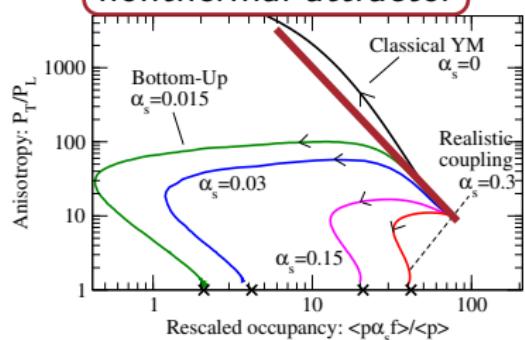
III) mini-jet quenching

$$p_z \sim \alpha_s^{3/2} Q_s (Q_s \tau)$$

$$\alpha_s^{-5/2} \ll Q_s \tau \ll \alpha_s^{-13/5}$$



nonthermal attractor



Berges, Boguslavski, Schlichting, Venugopalan (2014) [9]

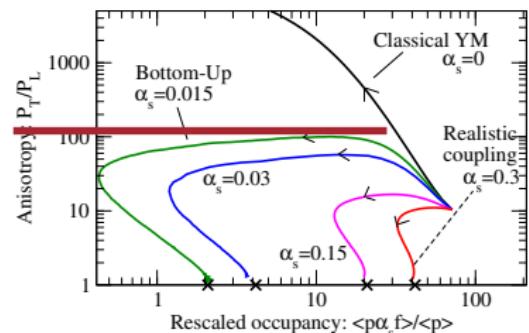
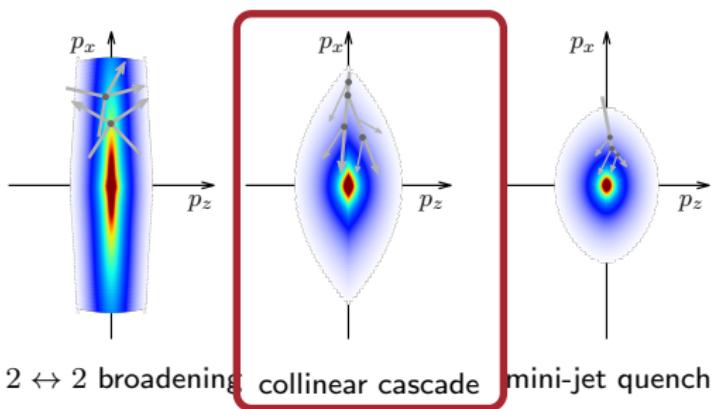
Kurkela and Zhu (2015), Keegan, Kurkela, AM and Teaney (2016), Kurkela, AM, Paquet, Schlichting and Teaney (2018)
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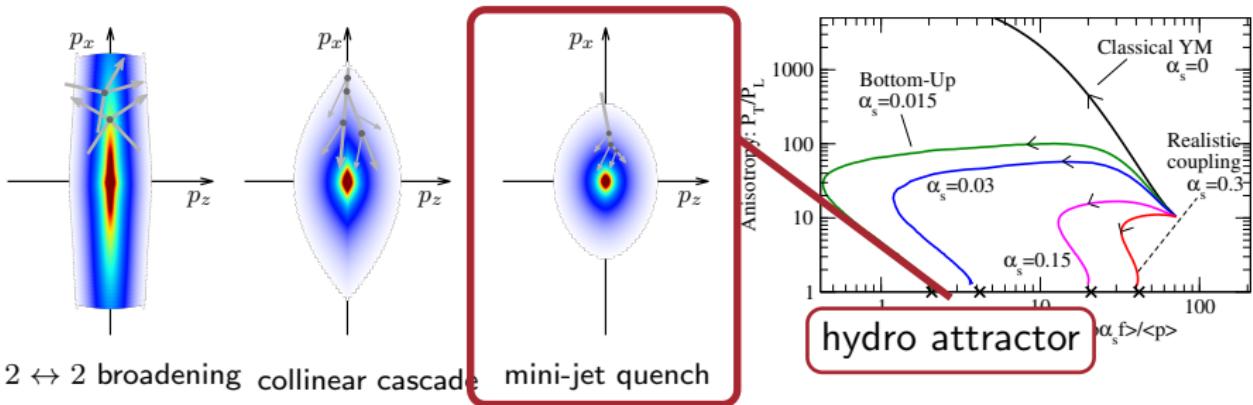
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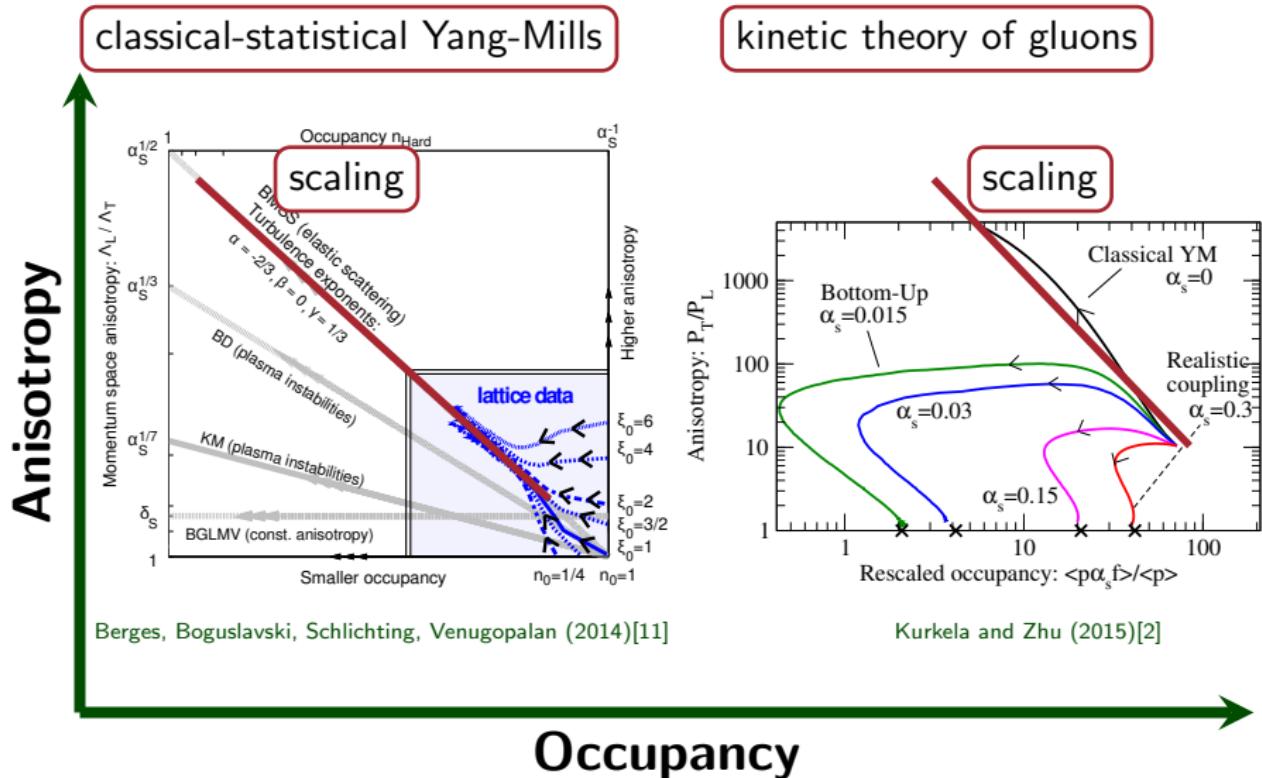
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From classical simulations to kinetic theory



Self-similar evolution of distribution function

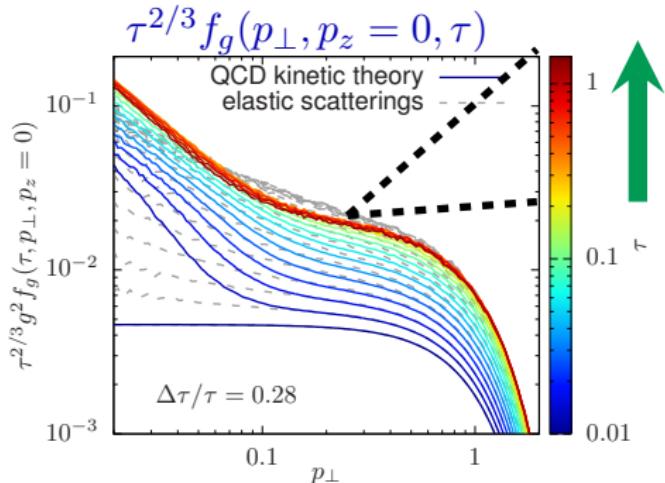
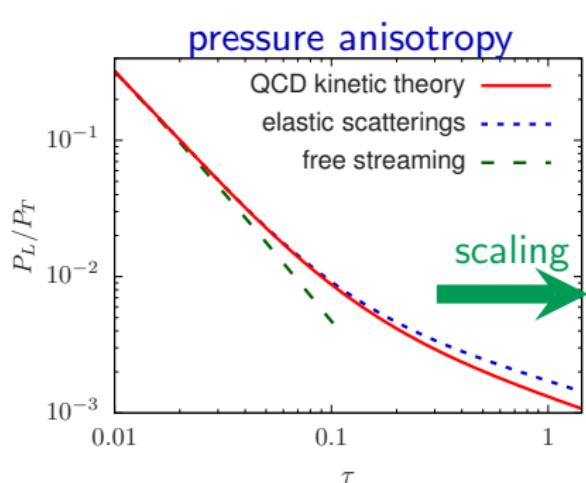
$$f_g(p_\perp, p_z, \tau) = \tau^\alpha f_S(\tau^\beta p_\perp, \tau^\gamma p_z), \quad \alpha \approx -\frac{2}{3}, \quad \beta \approx 0, \quad \gamma \approx \frac{1}{3}$$

Scaling in leading order QCD kinetic theory

Initial conditions $f_g = \frac{\sigma_0}{g^2} e^{-(p_\perp^2 + \xi^2 p_z^2)}$, $\sigma_0 = 0.1$, $g = 10^{-3}$, $\xi = 2$

Scaling regime is reached at late times

$$f_g(p_\perp, p_z, \tau) = \tau^{-2/3} f_S(p_\perp, \tau^{1/3} p_z), \quad \tau \rightarrow \tau/\tau_{\text{ref}}$$



Approach to a non-thermal fixed point in full QCD kinetic evolution.

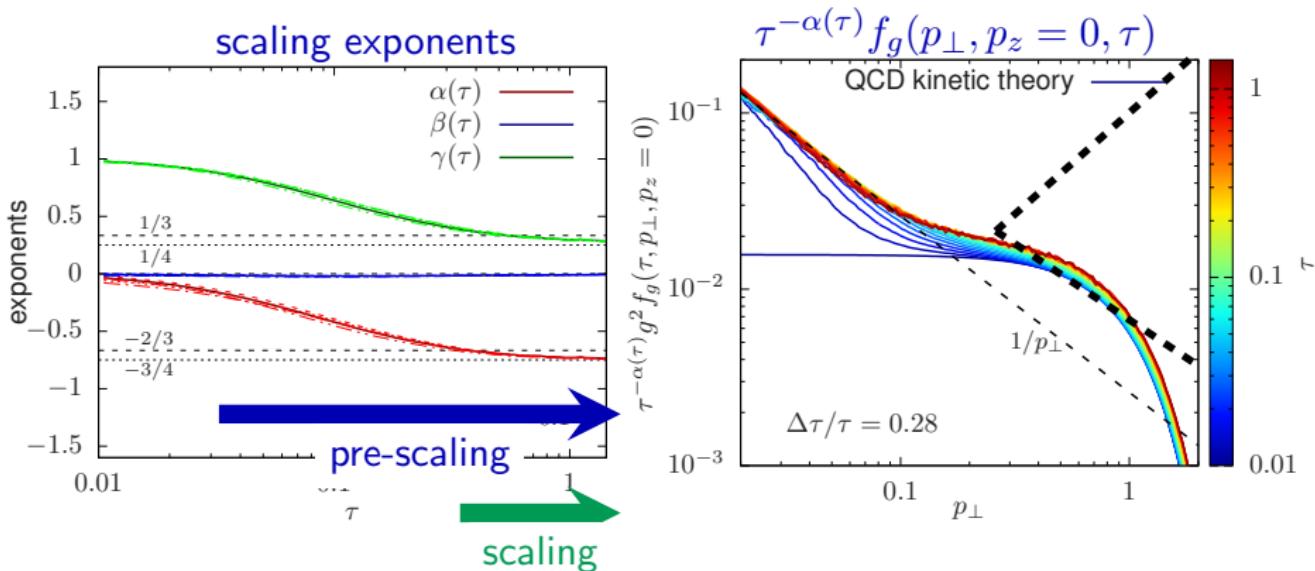
Pre-scaling regime in QCD kinetic theory

Non-equilibrium dynamics undone by self-similar renormalization

$$f_g(p_\perp, p_z, \tau) = \tau^{\alpha(\tau)} f_S(\tau^{\beta(\tau)} p_\perp, \tau^{\gamma(\tau)} p_z)$$

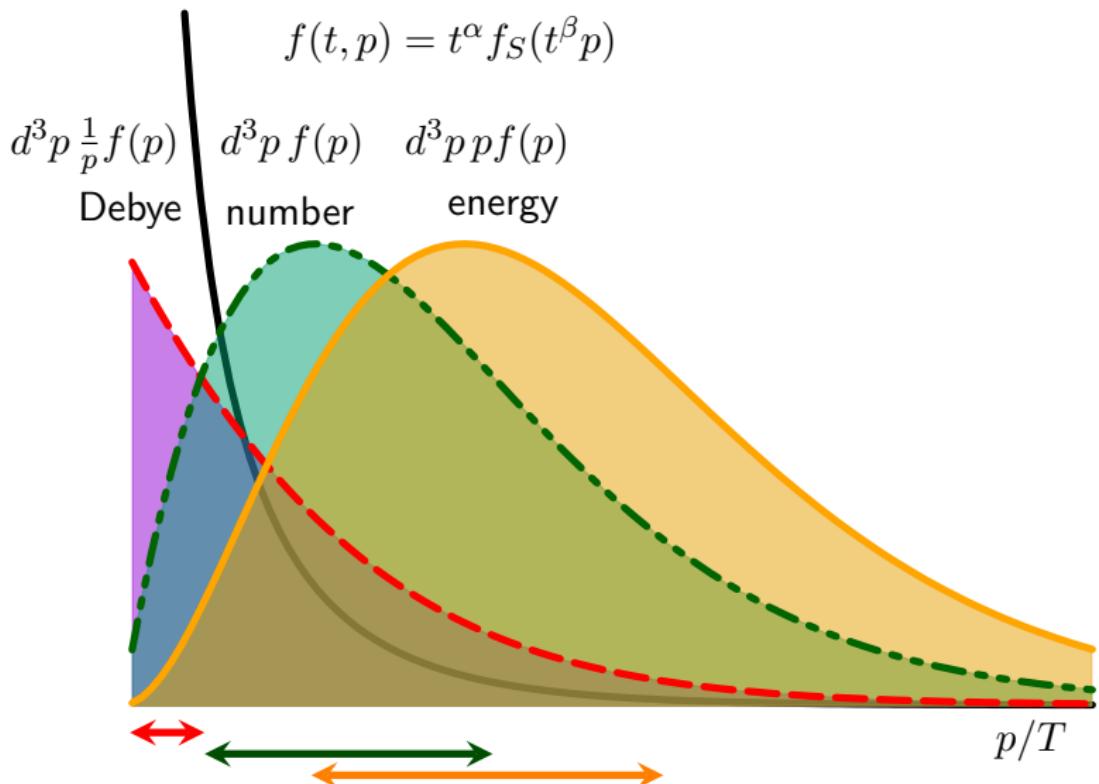
AM and Berges (2018) [8], cf. Micha and Tkachev (2004) [12]

Scaling exponents $\alpha(\tau), \beta(\tau), \gamma(\tau)$ can be time dependent!



Much earlier collapse to scaling solution f_S — pre-scaling regime.

Weighted momentum distribution



Moments of distribution function probe different momentum scales.

Extracting scaling exponents from integral moments

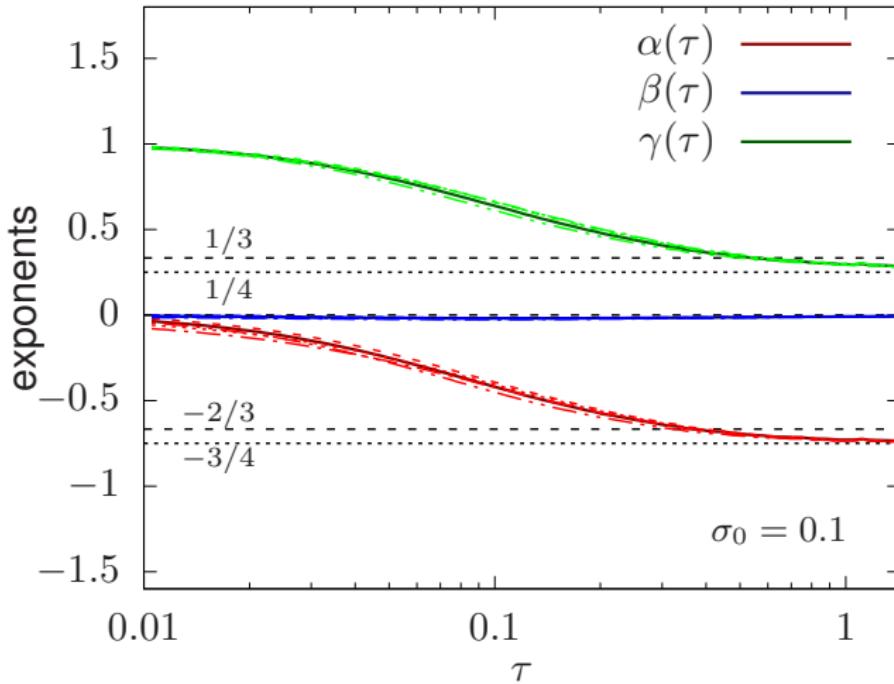
- Pre-scaling evolution $f_g(p_\perp, p_z, \tau) = \tau^{\alpha(\tau)} f_S(\tau^{\beta(\tau)} p_\perp, \tau^{\gamma(\tau)} p_z)$ imposes relations between integral moments

$$n_{m,n}(\tau) \equiv \int_{\mathbf{p}} p_\perp^m |p_z|^n f_g(p_\perp, p_z, \tau) \sim \tau^{\alpha(\tau) - (m+2)\beta(\tau) - (n+1)\gamma(\tau)}$$

- *Momentum range* of scaling \Leftrightarrow number of moments obeying scaling.
- Consider 5 triples of moments: $\{1, p_\perp, |p_z|\}$, $\{1, p_\perp^2, p_z^2\}$, $\{p_\perp, p_\perp^2, p_\perp |p_z|\}$, $\{p_\perp^2, p_\perp^3, p_\perp |p_z|\}$, $\{1, p_\perp^3, |p_z|^3\}$

Time dependent exponents

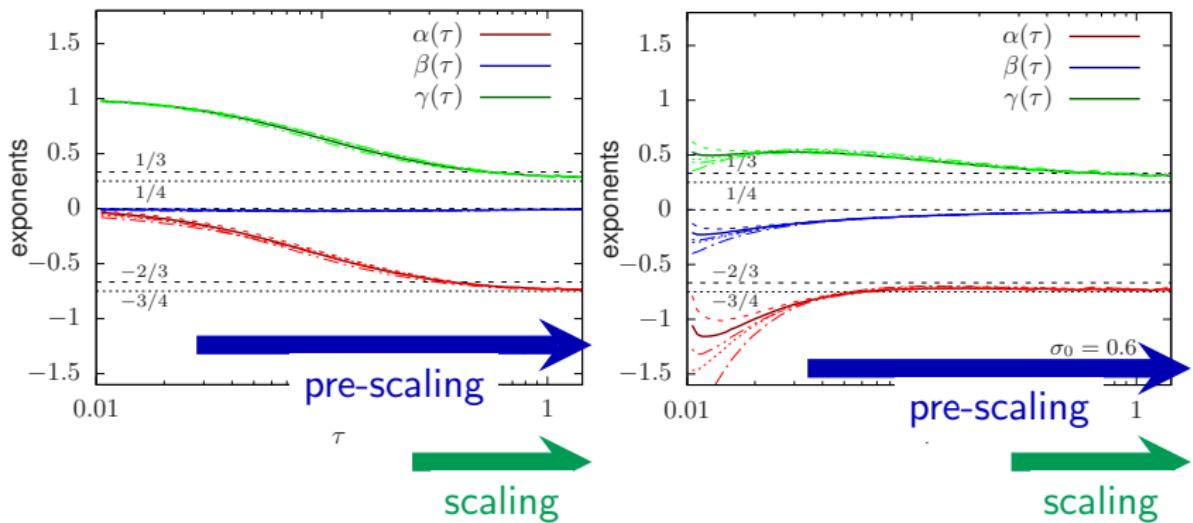
$$f_g(p_\perp, p_\perp, \tau) = \tau^{\alpha(\tau)} f_S(\tau^{\beta(\tau)} p_\perp, \tau^{\gamma(\tau)} p_z)$$



Time evolution encoded into a few hydrodynamic degrees of freedom.

Dependence on initial conditions

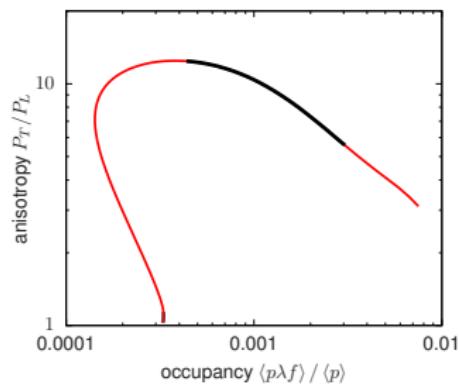
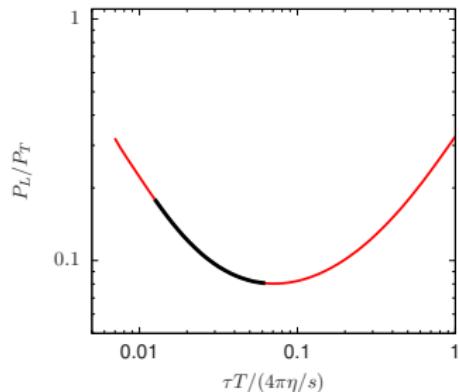
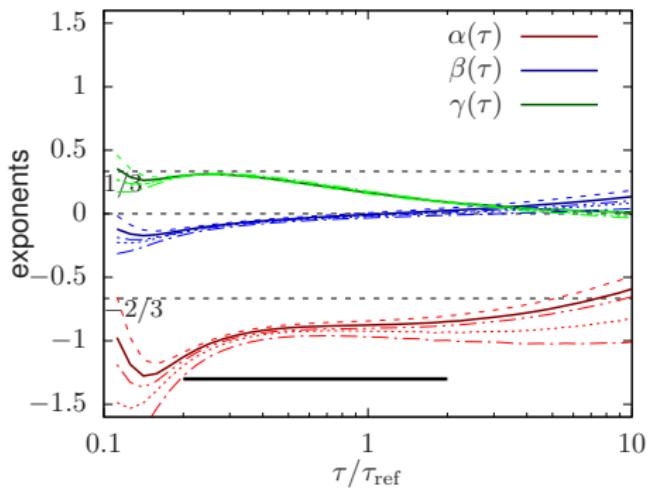
Vary initial gluon occupation $\sigma_0 = 0.1, 0.6$: $f_g = \frac{\sigma_0}{g^2} e^{-(p_\perp^2 + \xi^2 p_z^2)}$



Non-universal pre-scaling evolution of $\alpha(\tau)$, $\beta(\tau)$, $\gamma(\tau)$

The onset of thermalization

Consider larger couplings $g = 0.1$
and evolve until equilibration.

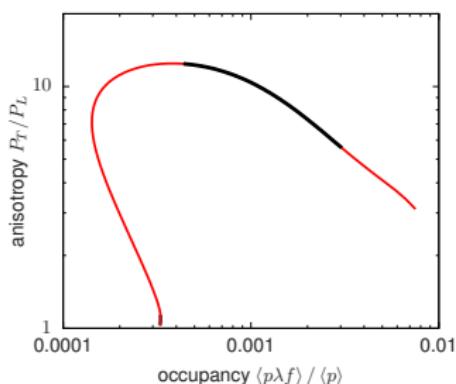
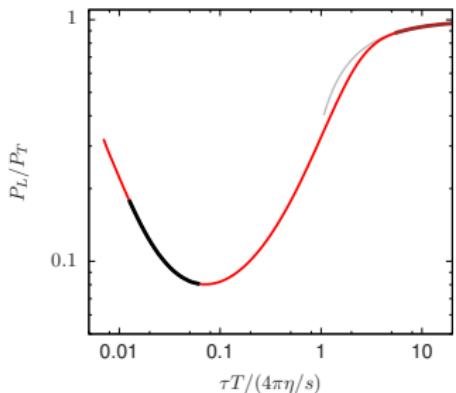
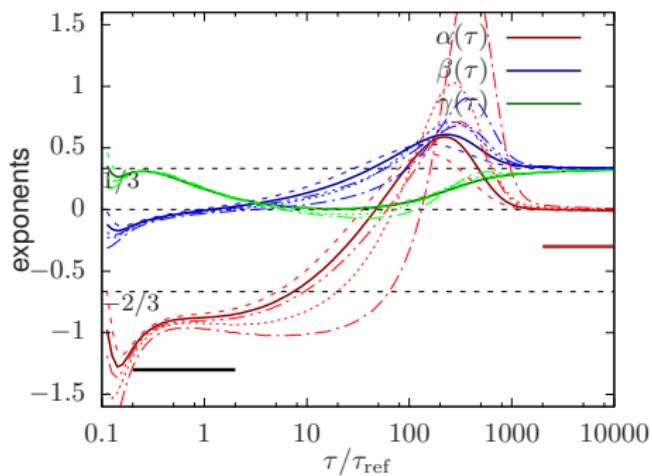


- Pre-scaling before isotropization
- Thermal scaling seen at late times.

Early time pre-scaling disconnected from late time hydrodynamics.

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Early time pre-scaling disconnected from late time hydrodynamics.

Summary and Outlook

Self-similar evolution in weakly coupled QGP

$$f_g(p_\perp, p_z, \tau) = \tau^{\alpha(\tau)} f_S(\tau^{\beta(\tau)} p_\perp, \tau^{\gamma(\tau)} p_z)$$

- Scaling and pre-scaling present in full QCD kinetic theory evolution.

AM and Berges (2019)

- $\alpha(\tau), \beta(\tau), \gamma(\tau)$ —new hydrodynamic-like degrees of freedom for evolution *not around equilibrium*.
- Can Bottom-up thermalization be understood as (pre)-scaling + hydrodynamic attractor?

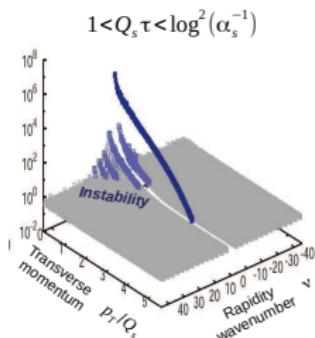
Berges, AM, Mikheev, *work in progress*

Non-thermal fixed point (NTFP) for gauge theories

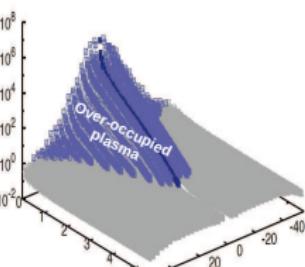
For $f \sim A^2 \gg 1$ classical-statistical Yang-Mills describes gluon evolution

Aarts, Berges (2002), Mueller, Son (2004), Jeon (2005)

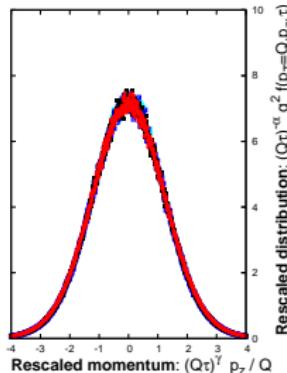
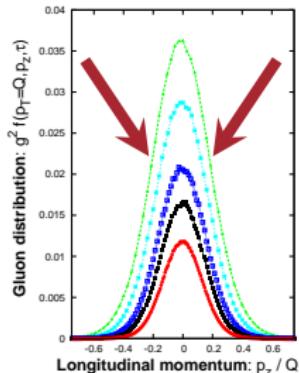
initial plasma instabilities



$Q_s \tau \sim \log^2(\alpha_s^{-1})$



later evolution scaling rescaled



Berges, Schenke, Schlichting, Venugopalan (2014) [13] Berges, Boguslavski, Schlichting, Venugopalan (2014) [9]

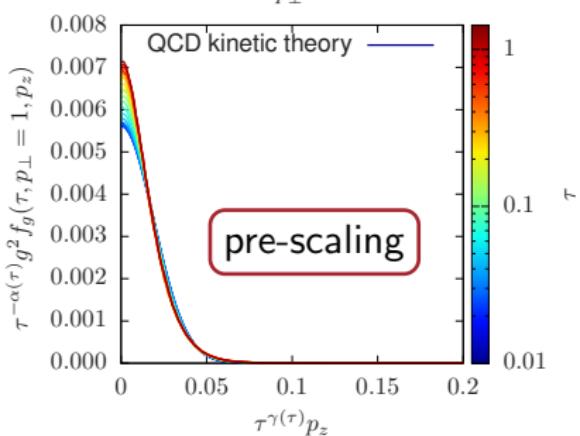
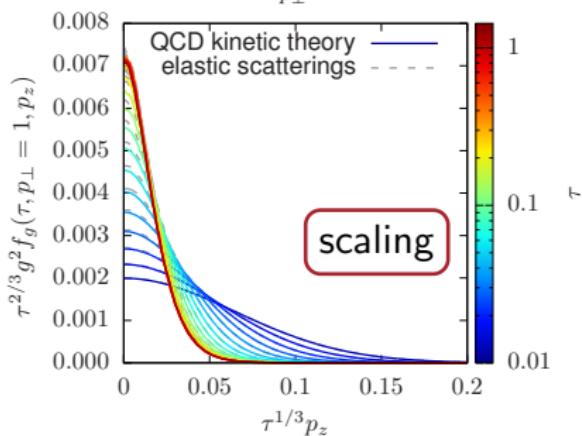
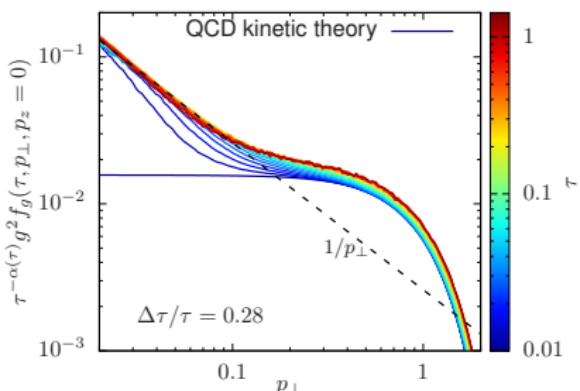
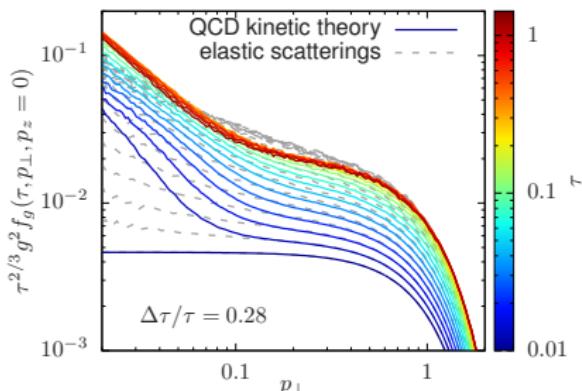
Self-similar scaling \Rightarrow simplification of non-equilibrium physics

$$f_g(p_{\perp}, p_z, \tau) = \tau^{\alpha} f_S(\tau^{\beta} p_{\perp}, \tau^{\gamma} p_z), \quad \tau = \sqrt{t^2 - z^2}$$

Universal exponents: $\alpha \approx -\frac{2}{3}$, $\beta \approx 0$, $\gamma \approx \frac{1}{3}$

scaling in other systems: Orioli et al. (2015) [14], Mikheev et al. (2018) [15], Prüfer et al. (2018) [16], Erne et al. (2018) [17]

Comparison between constant and time dependent exponents



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