Initial Condition for Matter in the Fragmentation Region

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Including work with Soren Schlichting and Srimoyee Sen, and
Mawande Lushozi, Gong Ming Yu and Michal Praszalowicz

\[ \gamma t_{\text{formation}} \gg R \quad E/M_T^2 \gg R \]
\[ \delta y > \ln A^{1/3} \quad E_{CM} > 30 \, \text{GeV} \]

This is the conventional region of CGC studies where the nuclei can be treated as two Lorentz contracted sheets with particles produced by the color fields that connect them.

How do we formulate the computation of intial condition in the central region?
Want to:
Review computation of energy density in modern saturation context
(see McLerran 2016)
Develop a theory of the classical fields and baryon density produced in the collision
(in spirit similar to the recent work of Schenke and Shen)

I will consider the fragmentation region at extremely high energies, where there is a fully developed central region. In principle such a description might also be applied at lower energies where the nuclear fragments do not separate, but where a high energy description is valid, but that problem is more complicated.

In the fragmentation region there is an asymmetry between the saturation momentum of the target and projectile.

Target saturation momentum is evaluated at $\Delta y$

Projectile saturation momentum is evaluated at $y_{target} - \Delta y$

$Q_{sat}^{projectile} \gg Q_{sat}^{target}$
Saturation momenta are
\[
Q_{\text{target}}^{\text{sat}} \sim A^{1/3} \Lambda_{QCD}^2 e^{\kappa \Delta y}
\]
\[
Q_{\text{projectile}}^{\text{sat}} \sim A^{1/3} \Lambda_{QCD}^2 e^{\kappa (y_{\text{projectile}} - y_{\text{target}} + \Delta y)}
\]
\[
\kappa \sim 0.2 - 0.3
\]

At LHC energies, the target saturation momenta is of the order of a GeV but the projectile is of order 5-10 GeV. This means that the projectile is “black” to the partons in the target up to a scale of momentum which is the projectile saturation momentum. The dominant particle production occurs in the region of momentum between these two saturation momenta. This is a region where the color sources produce a weak field \( A \ll 1/g \) and there is not much interaction of produced particles in this kinematic region, at least when the degrees of freedom correspond to classical fields. At momenta scales less than the saturation momenta of the target, there are strong fields and classical time evolution of classical fields. This latter region is that of the Glasma.
The multiplicity at low $p_T$ is not much changed due to very high energy

\[
\frac{dN}{dyd^2p_T} \sim \text{cons}, \quad p_T < Q_{\text{target}}^{\text{sat}}
\]

But the dominant contribution comes from intermediate momenta

\[
\frac{dN}{dyd^2p_T} \sim \frac{Q_{\text{target}}^{\text{sat}}}{p_T^2} \cdot \frac{Q_{\text{proj}}^{\text{sat}}}{p_T^2}, \quad Q_{\text{target}}^{\text{sat}} < p_T < Q_{\text{sat}}^{\text{proj}}
\]

And at very high momenta the distribution smoothly goes to a perturbative dependence

\[
\frac{dN}{dy} \sim Q_{\text{target}}^2
\]

\[
< p_T^2 > \sim Q_{\text{proj}}^2
\]

Does not change up to logarithms

Is about 100 times bigger at LHC than at RHIC since

\[
x_{\text{rhic}}^{\text{proj}} \sim 10^{-2} \quad \quad \quad x_{\text{lhc}}^{\text{proj}} \sim 10^{-9}
\]
In addition there is baryon number compression

Why is high energy fragmentation regions somewhat simple?

Anishetty, Koehler and McLerran, Ming and Kapusta

\[ \Delta z \sim 1 - \nu \quad \text{In boosted fame of struck nucleon, compression} \]

\[ \Delta z_{\text{comoving}} \sim \frac{1}{\gamma_{\text{nucleon}}} \]
The compression gamma factor should be of the order of the gamma factor for produced particles

\[
\gamma/M_T \sim R
\]

\[
\gamma \sim Q_{proj} R
\]

So the initial baryon density is of order

\[
N_B/V \sim Q_{targ}^2 Q_{proj}
\]

The number multiplicity of produced particles per unit area scales as

\[
\frac{1}{\pi R^2} \frac{dN}{dy} \sim Q_{targ}^2
\]

The initial longitudinal size scale is set by the typical transverse momenta of produced gluons

\[
Q_{proj}
\]

\[
N_{gluon}/V \sim Q_{targ}^2 Q_{proj}
\]
\[ N_B / N_g \sim \text{cons} \]

However, gluons are not in thermal equilibrium

\[ E / S \sim Q_{proj} \]

but

\[ s \sim Q_{targ}^2 Q_{proj} \]

How might expansion change this?

Interactions and thermalization?

Can one set up CGC-Glasma initial conditions in the fragmentation region including the effects of baryon number density?
Another argument for the rapidity shift of low momentum quark as it passes through the thins sheet of the projectile nucleus

\[ p^2 = 2p^+ p^- - P_T^2 = m^2 \]
\[ p^+ = (p_T^2 + m^2)/2p^- \]
\[ p_i^+ = m^2/2p^- \]
\[ p_f^+ = (Q_{proj}^2 + m^2)/2p^- \sim Q_{proj}^2/2p^- \]

\[ \Delta y = \frac{1}{2} \ln \left( \frac{p_f^+}{p^-} \right) - \frac{1}{2} \ln \left( \frac{p_i^+}{p^-} \right) = \ln \left( \frac{Q_{proj}}{m} \right) \]
Consider a collision of a single “quark” with a classical color charge with a nucleus

Will work in the gauge where the nuclear gauge vector field is entirely transverse and nonzero only before the collisions

Can solve the classical equations:

$$\frac{dp^\mu}{d\tau} = gT \cdot F^{\mu\nu} u^\nu$$

A “quark” at rest is given a transverse momentum kick of

$$\vec{p}_T = -gT \cdot \vec{A}_T$$
The gluon radiation is solved to first order in the strength of the “quark field and all order in the nuclear field.

There are three contributions: Bremstrahlung emission from the quarks, Bertsch Gunion like emission for the gluonic field and interference between the two.

The classical computation must be corrected for recoil at large momentum of emitted gluons. We have checked our classical computation by an explicit fully quantum treatment to lowest order in the nuclear field strength (with Gong Ming Yu, Mawande Lushozi and M. Praszalowicz). The contributions are not universal at large transverse momentum the fragmentation region and depend upon “quark” spin.
The result is

\[ \frac{dN}{dyd^2k_T} = \frac{C_F \alpha_S}{2\pi^2} \int \frac{d^2q_T}{(2\pi)^2} S(k_T - q_T) \left\{ \frac{q^i}{q_T^2 + x^2M^2} - \frac{k^i_T - xp_T^i}{(k_T - xp_T)^2 + x^2M^2} \right\}^2 \]

where

\[ S(x - y) \equiv \frac{1}{N^2 - 1} \langle \text{Tr}(U(x)U^\dagger(y)) \rangle_\rho, \]

The 1 term in S generates the bremsstrahlung and this must be subtracted out and treated properly to get the high momentum limit.
In progress:

We know the gluon fields and distributions of quarks at early times in the collisions on a field by field basis. This allows for a computation of the stress energy tensor for event by event initial conditions for Glasma and for hydrodynamic computations. Should be able to compute for nucleus-nucleus collisions due to asymmetry in saturation momentum of target and projectile at very high energies.