QCD equation of state at finite densities for nuclear collisions

Akihiko Monnai (KEK)

with Björn Schenke (BNL) and Chun Shen (Wayne/RBRC)

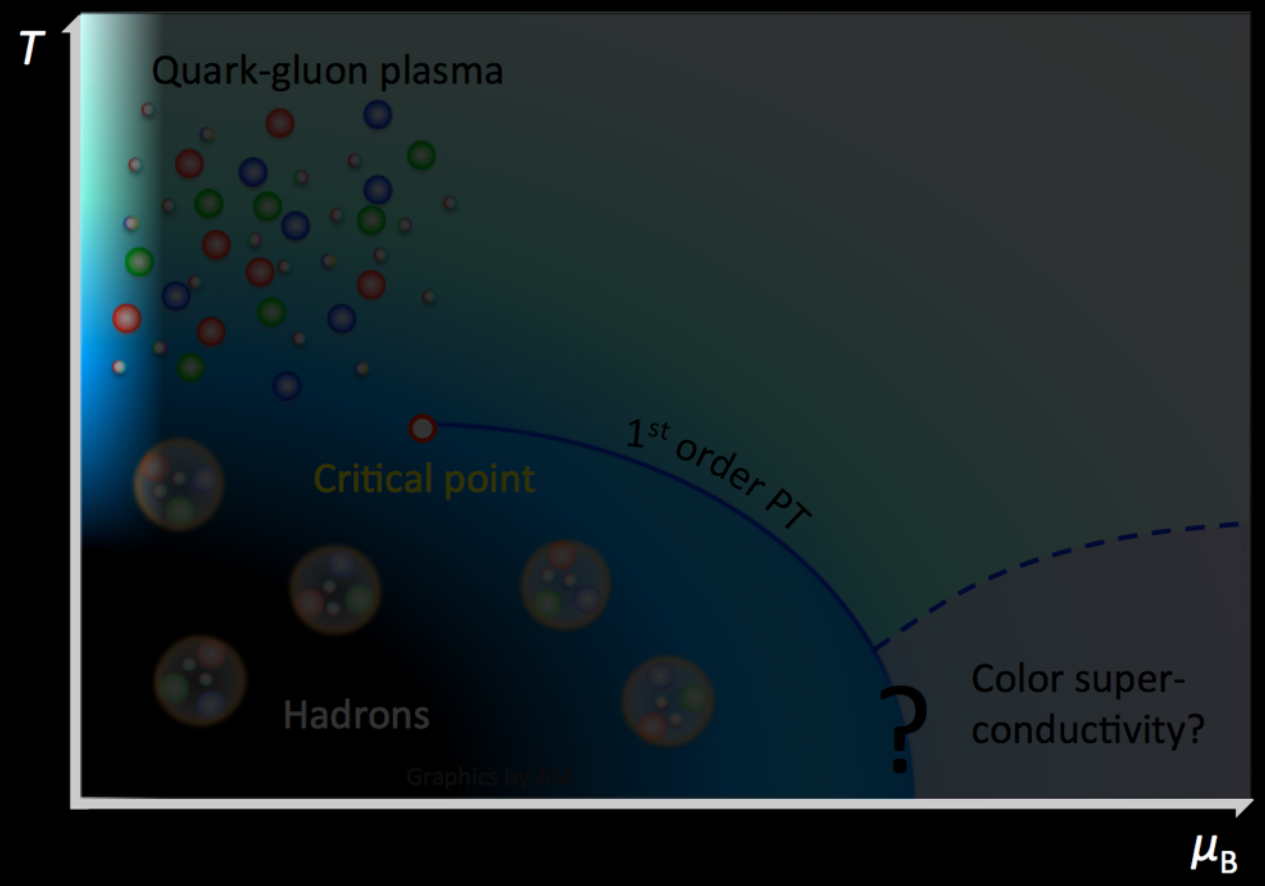
AM, B. Schenke, C. Shen, Phys. Rev. C **100**, 024907 (2019)

Quark Matter 2019

6th November 2019, Wuhan, China

Introduction

- Exploration of the QCD phase diagram



Little is known at finite density
(sign problem of lattice QCD)

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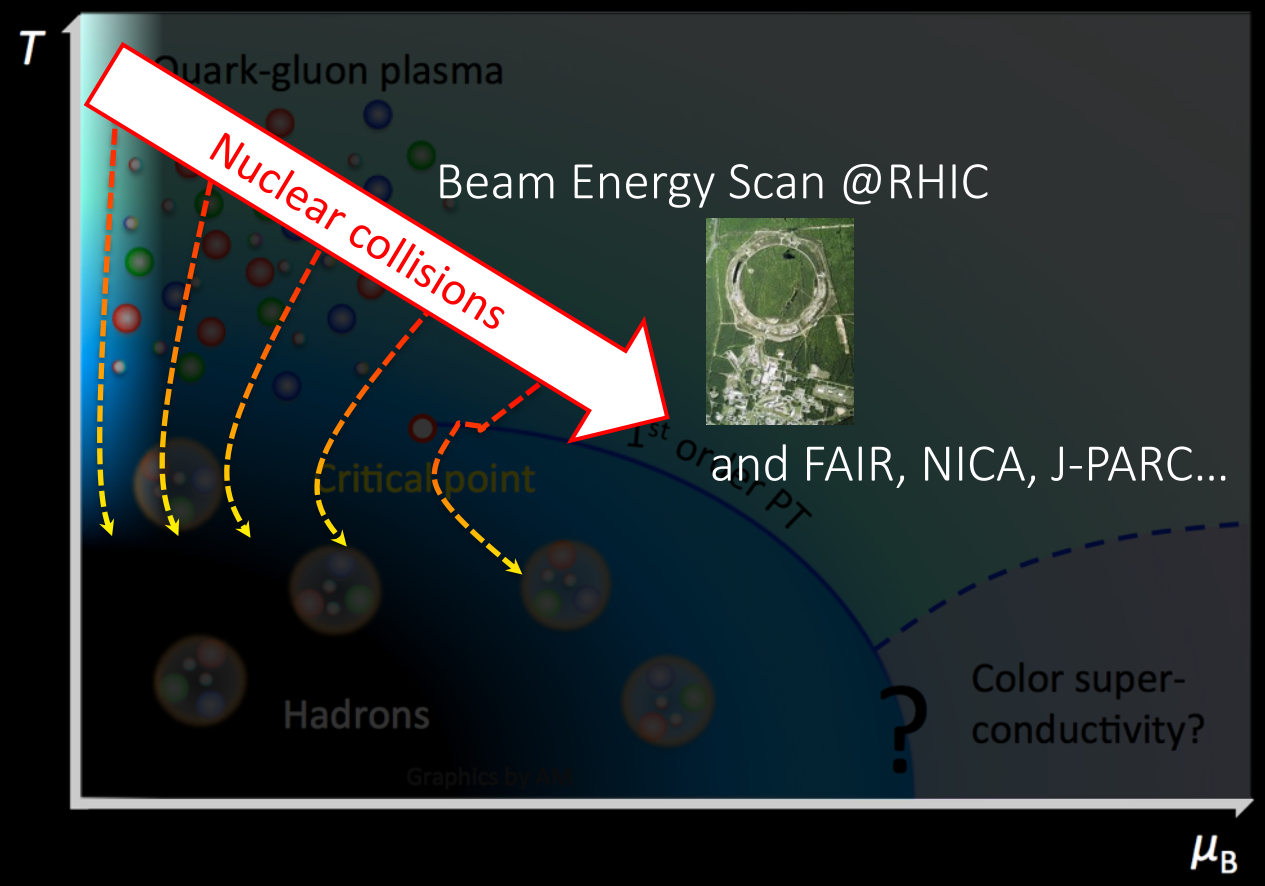

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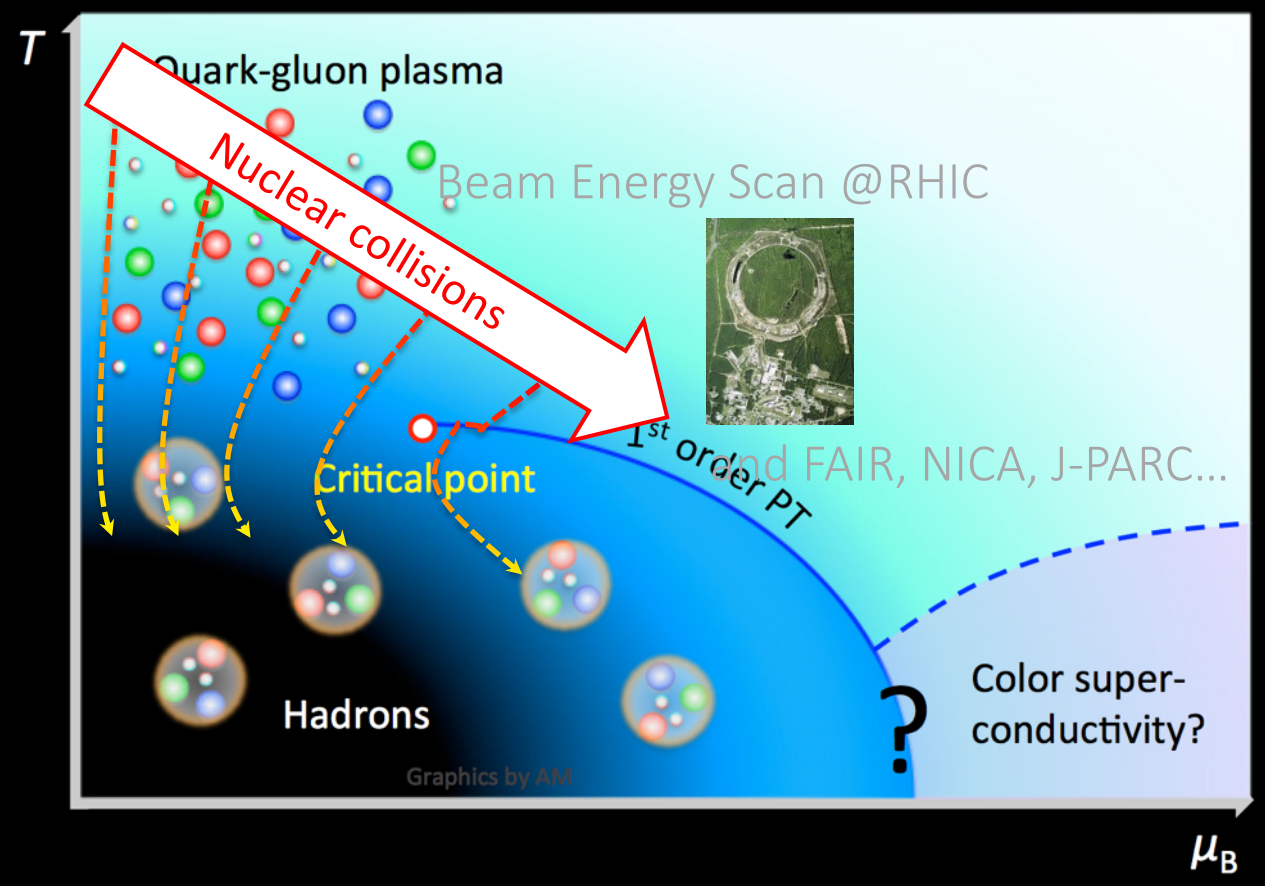


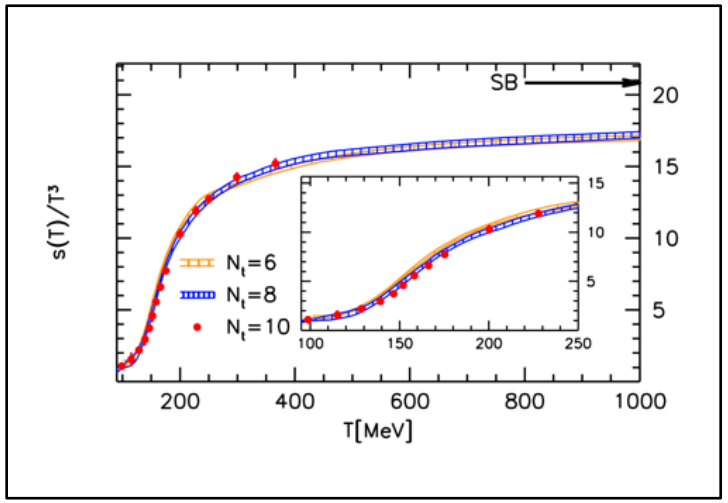
Little is known at finite density (sign problem of lattice QCD)

Use nuclear collisions to study QCD properties at finite T and μ_B

Introduction

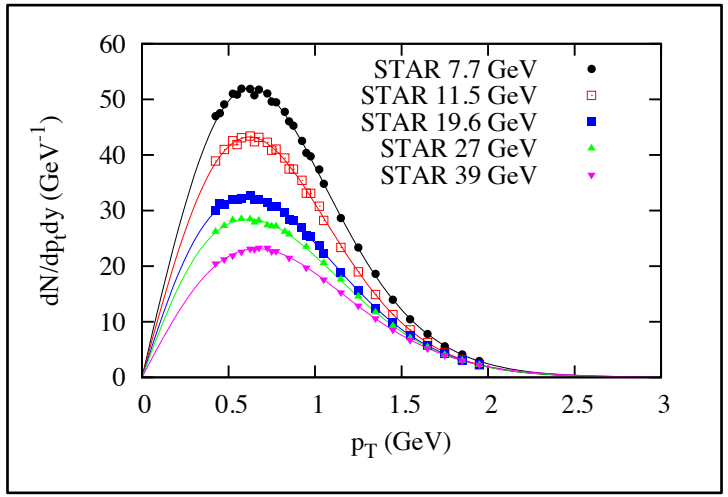
- Modeling nuclear collisions

QCD properties



?

Experimental data



- We need a “link” between experimental data of nuclear collisions and QCD properties ⇨ Relativistic hydrodynamic model

Better Quantitative Studies

- in relativistic nuclear collisions

Baryon number (B)
(> 0 in total)

p n
+1 +1

Electric charge (Q)
(> 0 in total)

p n
+1 0

Strangeness (S)
(= 0 in total)

p n
0 0

Multiple charges are essential in understanding particle-antiparticle ratios

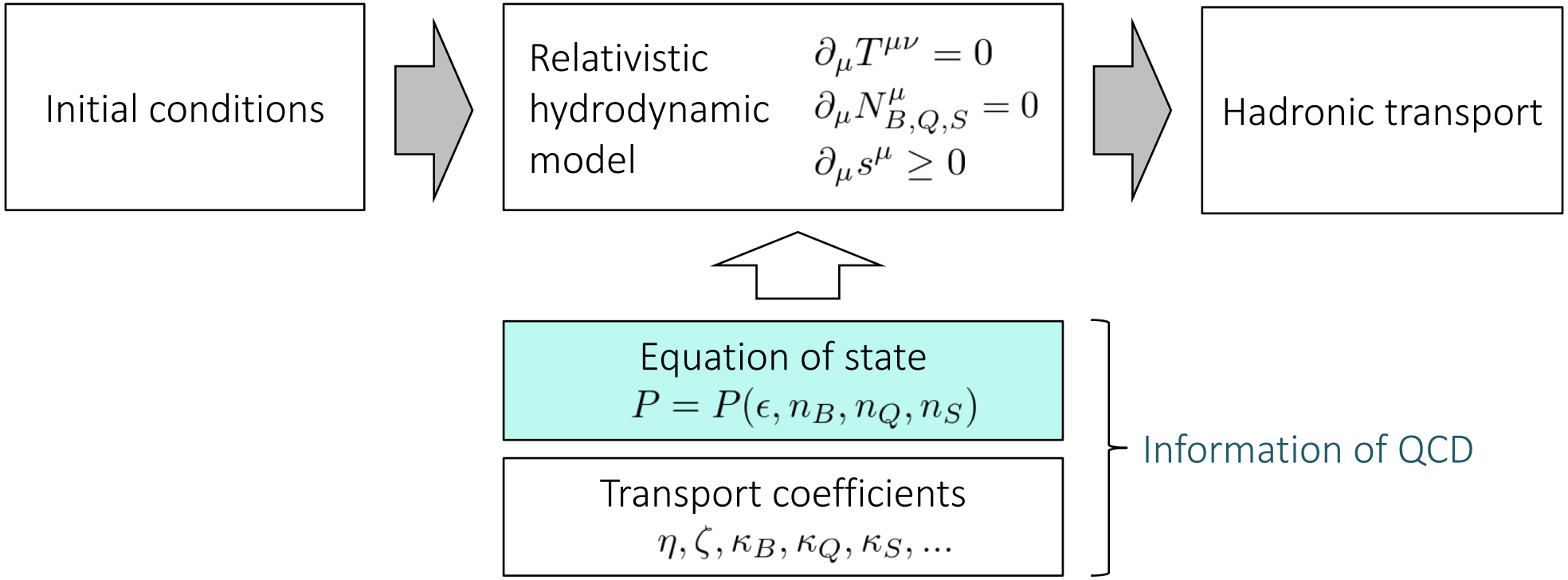
Dunlop et.al., PRC 84 044914

hadron	yield	u	d	s	$\bar{u}$	$\bar{d}$	$\bar{s}$
π^+	72.9	72.9				72.9	
π^-	84.8		84.8		84.8		
π^0 (*)	78.85	39.43	39.43		39.43	39.43	
ϕ	1.17			1.17			1.17
K^+	16.4	16.4					16.4
K^-	5.58			5.58	5.58		
K^0 (*)	10.99		10.99				13.84
$\bar{K}^0$ (*)	10.99			10.99		10.99	
p	46.1	92.2	46.1				
n (*)	70.84	70.84	141.68				
Λ	13.4	13.4	13.4	13.4			
Ξ^-	0.93		0.93	1.86			
$\bar{p}$	0.06				0.12	0.06	
$\bar{n}$ (*)	0.04				0.04	0.08	
$\bar{\Lambda}$	0.1				0.1	0.1	0.1
Sum		305.17	337.33	33	130.07	123.56	28.66

TABLE I: Left two columns: midrapidity yields of common particles from central Pb+Pb collisions measured by the NA49/SPS Collaboration [26–33] at $\sqrt{s_{NN}} = 6.41$ GeV. Starred hadrons are not measured, but estimated from other hadrons.

Overview of hydrodynamic model

- with multiple charges



- We construct the QCD equation of state (EOS) at finite densities (B, Q, S)

Equation of state

- Construction
  - Lattice QCD: Taylor expansion up to the 4th order

$$\frac{P}{T^4} = \frac{P_0}{T^4} + \sum_{l,m,n} \frac{\chi_{l,m,n}^{B,Q,S}}{l!m!n!} \left(\frac{\mu_B}{T}\right)^l \left(\frac{\mu_Q}{T}\right)^m \left(\frac{\mu_S}{T}\right)^n$$

HotQCD Collaboration, PRD 86, 034509 (2012); PRD 90, 094503 (2014); PRD 92, 074043 (2015); PRD 95, 054504 (2017)

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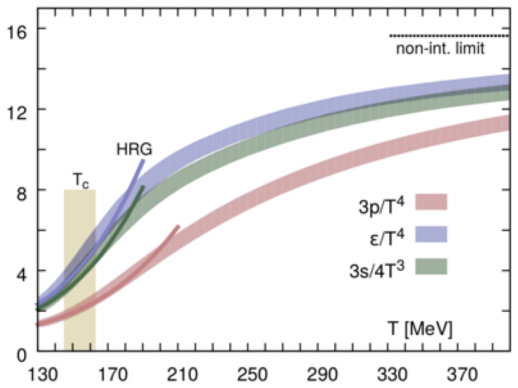
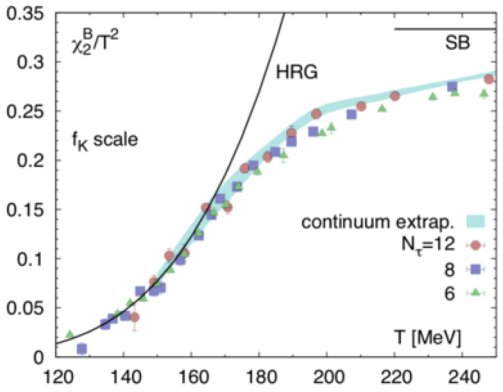
HotQCD Collaboration, PRD 86, 034509 (2012); PRD 90, 094503 (2014); PRD 92, 074043 (2015); PRD 95, 054504 (2017)

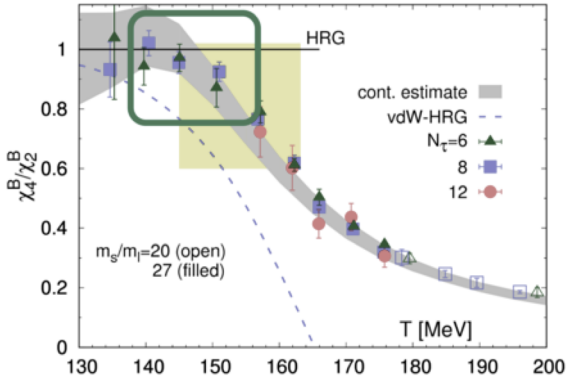
  - Match to hadron resonance gas (HRG) at lower T:

Particle Data Group: PRD 98, 030001 (2018)

1. Taylor expansion is not reliable when the fugacity (μ/T) is large
2. Agreement between lattice QCD and HRG is good in hadronic phase



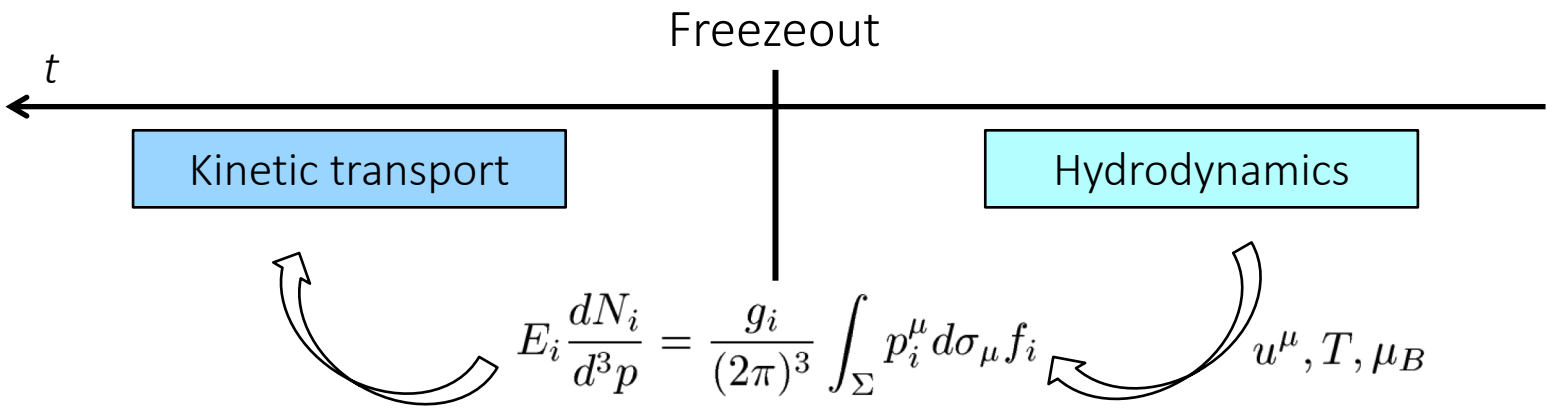




Equation of state

- Construction
  - Match to hadron resonance gas (HRG) at lower T

    3. The EOS of hydro model should match the EOS of kinetic theory at freeze-out *(otherwise EM/charge conservation is lost)*



  - The Stefan-Boltzmann limits are used as anchors at very high T where lattice QCD data are scarce

Equation of state

- Construction

  - Connect to HRG at low T

$$\frac{P}{T^4} = \frac{1}{2}[1 - f(T,\mu_J)]\frac{P_{\rm had}(T,\mu_J)}{T^4} + \frac{1}{2}[1 + f(T,\mu_J)]\frac{P_{\rm lat}(T,\mu_J)}{T^4} \qquad J = \{B, Q, S\}$$

where $f(T,\mu_J) = \tanh[(T - T_c(\mu_B))/\Delta T_c]$

$T_c = 0.16 - 0.4 \times (0.139\mu_B^2 + 0.053\mu_B^4)(\mathrm{GeV}) \quad \Delta T_c = 0.1 T_c(0)$

Crossover-type EOS

Motivated by chemical freeze-out curve J. Cleymans et. al., PRC 73, 034905 (2006)

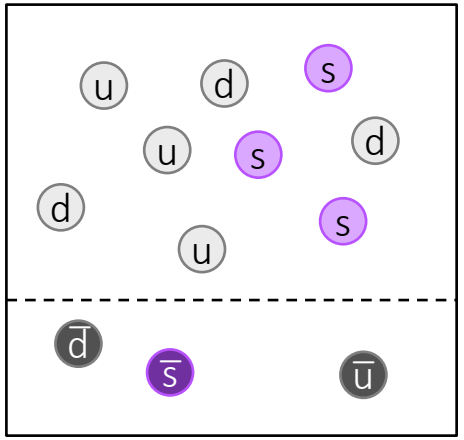
The dependences on sub-leading μ's are approximated to be small

  - Effective 6th order susceptibilities are introduced to satisfy thermodynamic conditions:

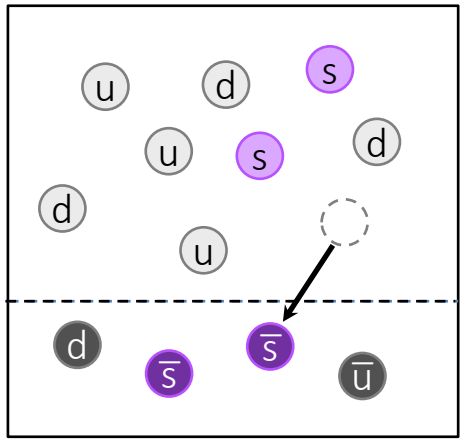
$$\frac{\partial^2 P}{\partial T^2} = \frac{\partial s}{\partial T} > 0\,, \quad \frac{\partial^2 P}{\partial \mu_J^2} = \frac{\partial n_J}{\partial \mu_J} > 0$$

Strangeness and charge densities

- Strangeness neutrality condition ($n_S = 0$)
  - $\mu_S > 0$ when $\mu_B > 0$



$\mu_B > 0$ and $\mu_S = 0$
leads to $n_S \neq 0$



$\mu_S > 0$ is needed for
$\mu_B > 0$ and $n_S = 0$

s quark chemical potential: $\mu_s = \frac{1}{3}\mu_B + \frac{1}{3}\mu_Q - \mu_S = 0$

The condition can be modified by initial fluctuations and diffusion

Strangeness and charge densities

- Charge-to-baryon ratio ($n_Q = c\, n_B$)
  - $\mu_Q < 0$ at $\mu_B > 0$ for neutron rich nuclei ($Z/A < 1/2$)

d quark abundance: $\mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q > \mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q$

Nucleus	${}^{1}_{1}$H	${}^{2}_{1}$H	${}^{3}_{2}$He	${}^{27}_{13}$Al	${}^{63}_{29}$Cu	${}^{96}_{40}$Zr	${}^{96}_{44}$Ru	${}^{127}_{54}$Xe	${}^{197}_{79}$Au	${}^{208}_{82}$Pb	${}^{238}_{92}$U
Z/A	1.000	0.500	0.667	0.481	0.460	0.417	0.458	0.425	0.401	0.394	0.387

proton rich/neutral nuclei
$\mu_Q \geq 0$ for $\mu_B > 0$

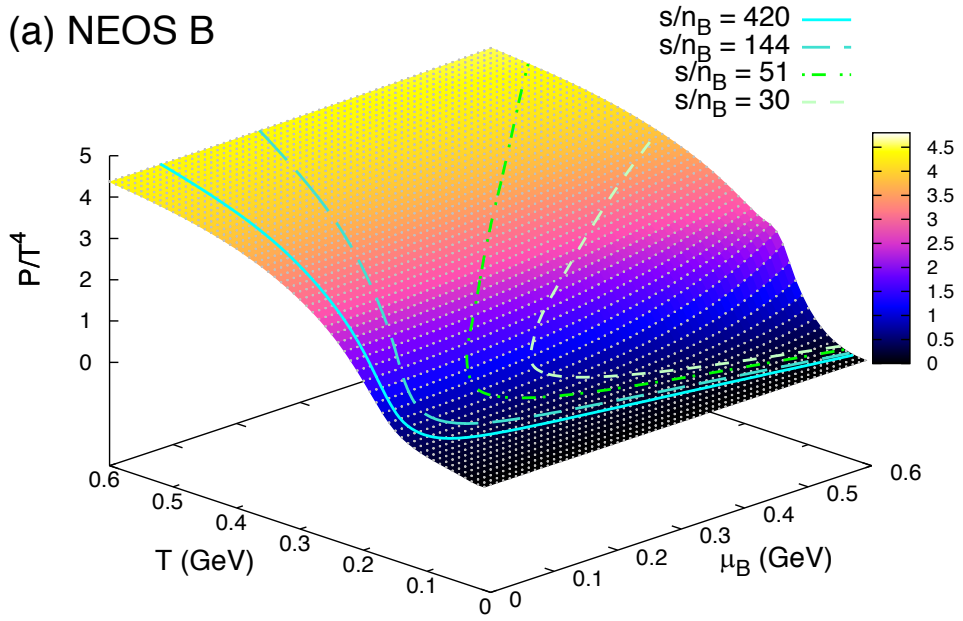
relevant for BG
analyses of isobars

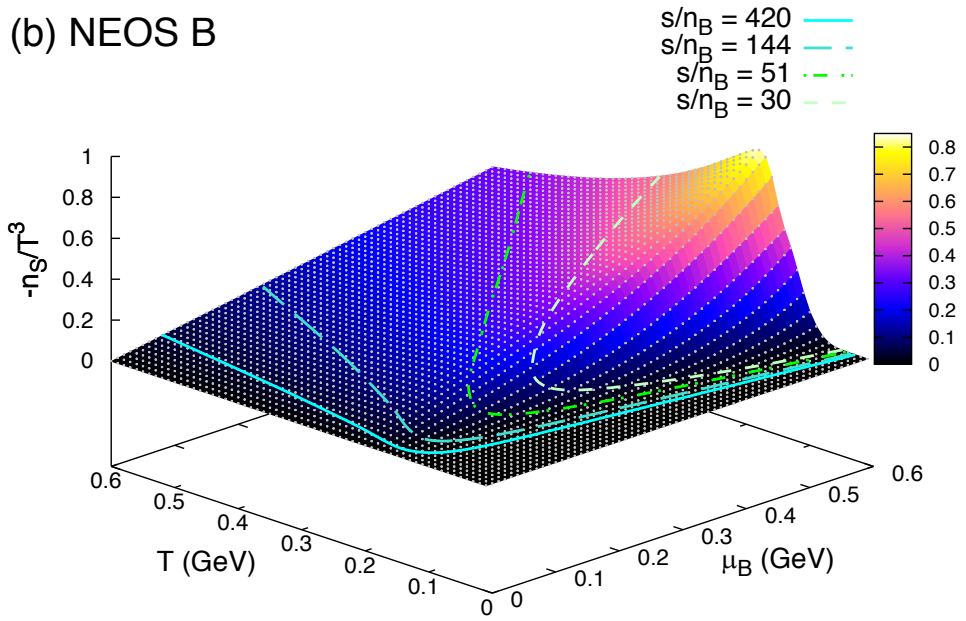
$c \simeq 0.4$ for
Au and Pb nuclei

  - We consider 3 cases: (I) $\mu_S = \mu_Q = 0$, (II) $n_S = 0$, $\mu_Q = 0$ and (III) $n_S = 0$, $n_Q = 0.4 n_B$

Equation of state

- $\mu_S = \mu_Q = 0$ (conventional; denoted as NEOS B)

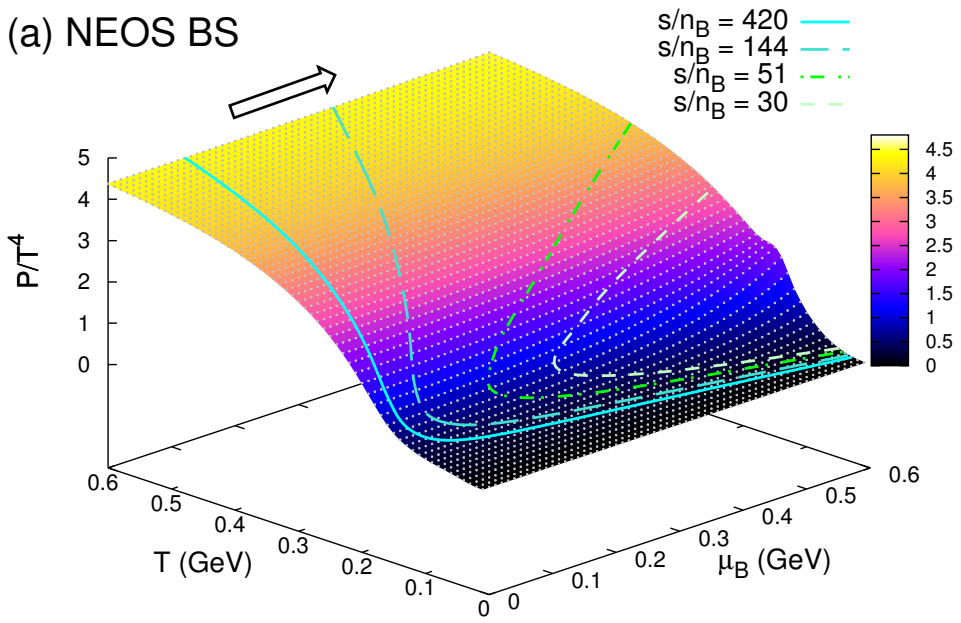


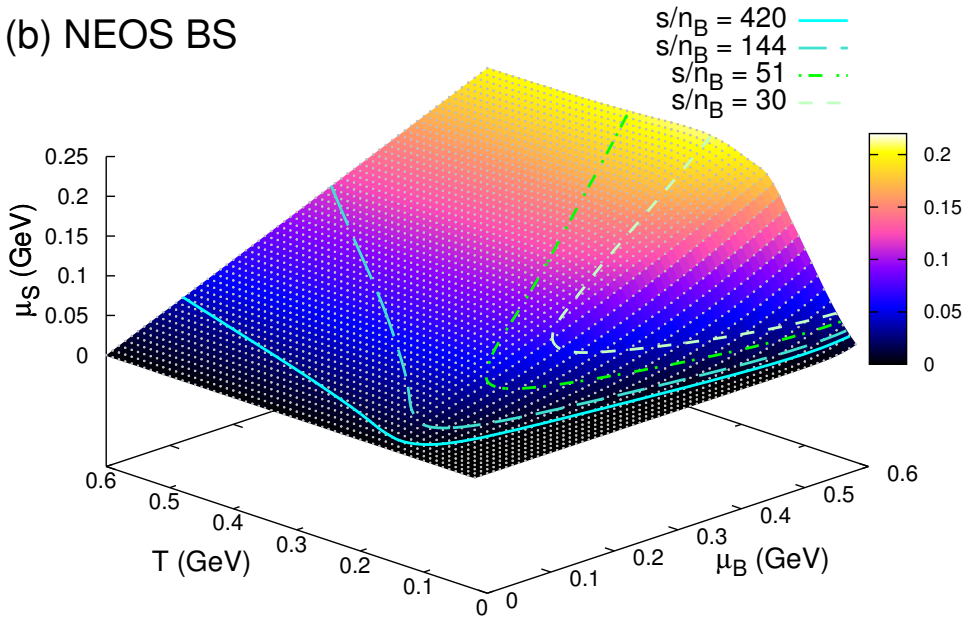


- Thermodynamically consistent smooth EoS is obtained
- The strangeness neutrality condition is violated ($n_S < 0$)

Equation of state

- $n_S = 0$, $\mu_Q = 0$ (strangeness neutral; denoted as NEOS BS)

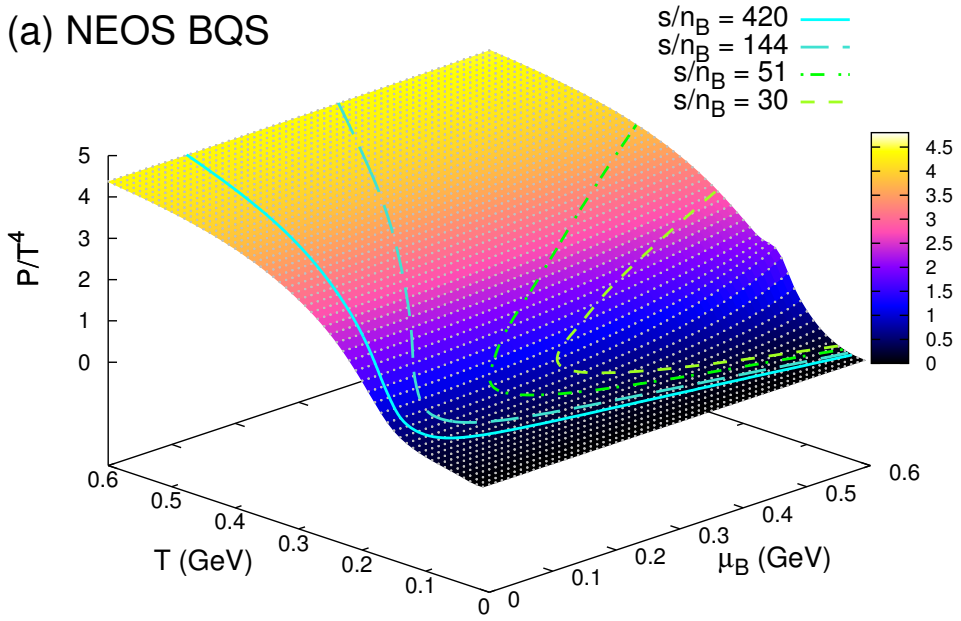


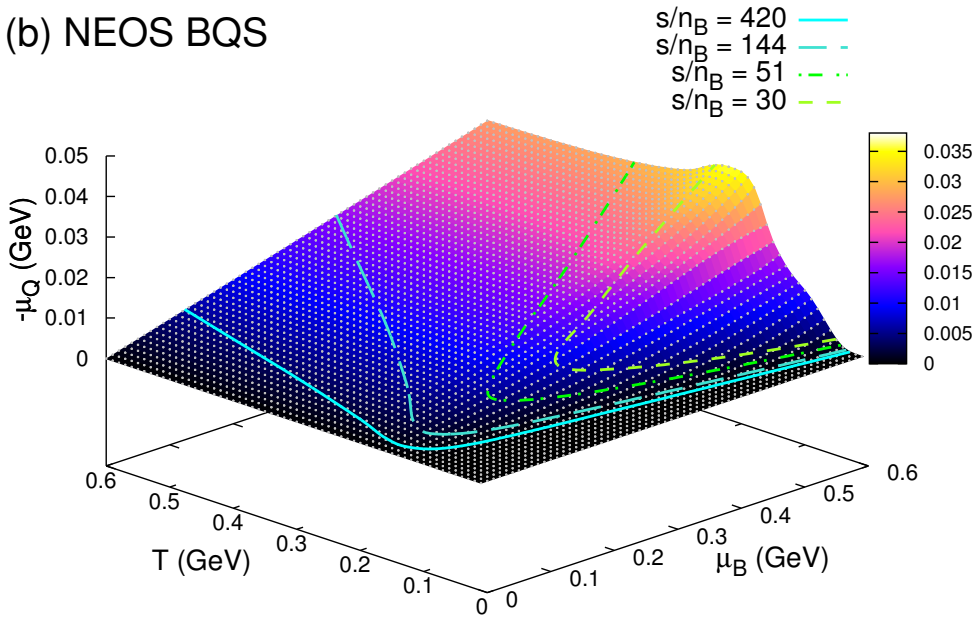


- A visible modification is observed at larger μ_B/T
- Finite positive μ_S is seen owing to the neutrality condition
- μ_B is becomes larger at large T for a given n_B

Equation of state

- $n_S = 0$, $n_Q = 0.4n_B$ (realistic in HIC; denoted as NEOS BQS)

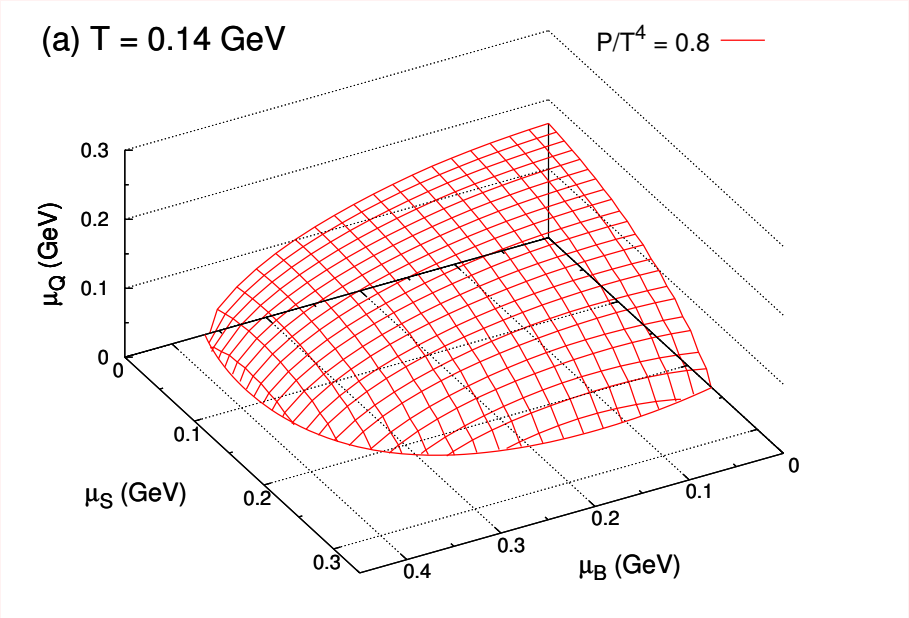




- Finite negative μ_Q owing to the condition $n_Q = 0.4n_B$
- Pressure similar in NEOS BS and BQS because $\mu_Q = 0$ implies $n_Q \sim 0.5n_B$

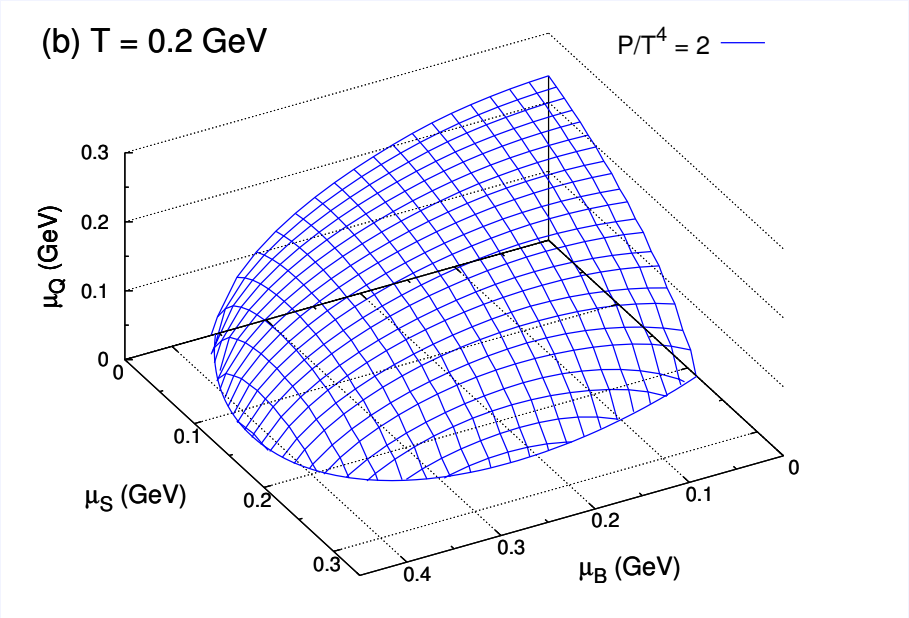
μ_B-μ_Q-μ_S space

- Constant pressure plane at fixed T



$\mu_B^{\rm int} > \mu_S^{\rm int} > \mu_Q^{\rm int}$ in the hadronic phase

Lightest hadrons to carry the charges are ordered in mass as $m_p > m_K > m_\pi$



$\mu_B^{\rm int} > \mu_Q^{\rm int} > \mu_S^{\rm int}$ in the QGP phase

$\chi_2^B = 1/3$, $\chi_2^Q = 2/3$, $\chi_2^S = 1$ in the parton gas limit implies $\mu_B^{\rm int}/3 \sim 2\mu_Q^{\rm int}/3 \sim \mu_S^{\rm int}$

μ_B-μ_Q-μ_S space

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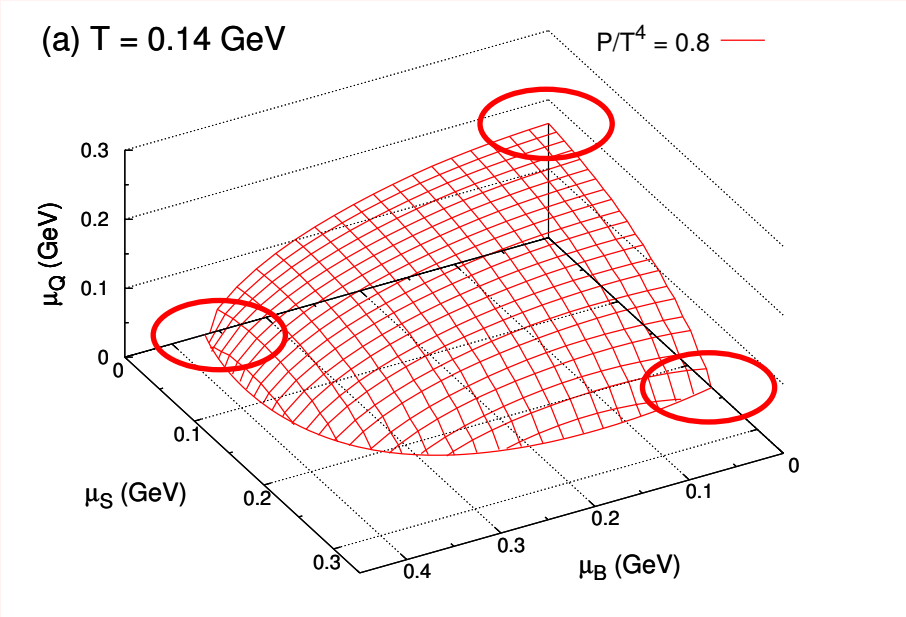

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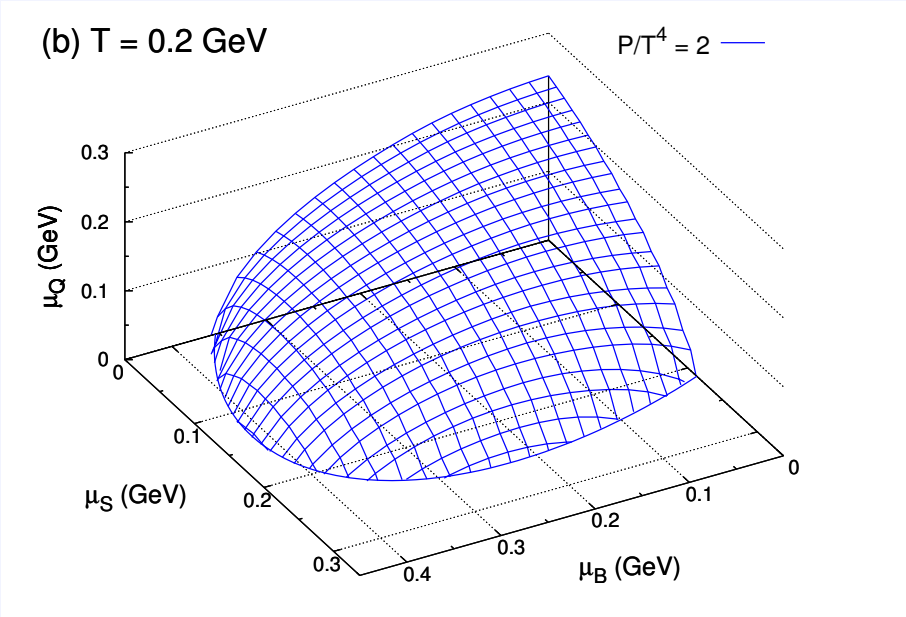

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μ_B-μ_Q-μ_S space

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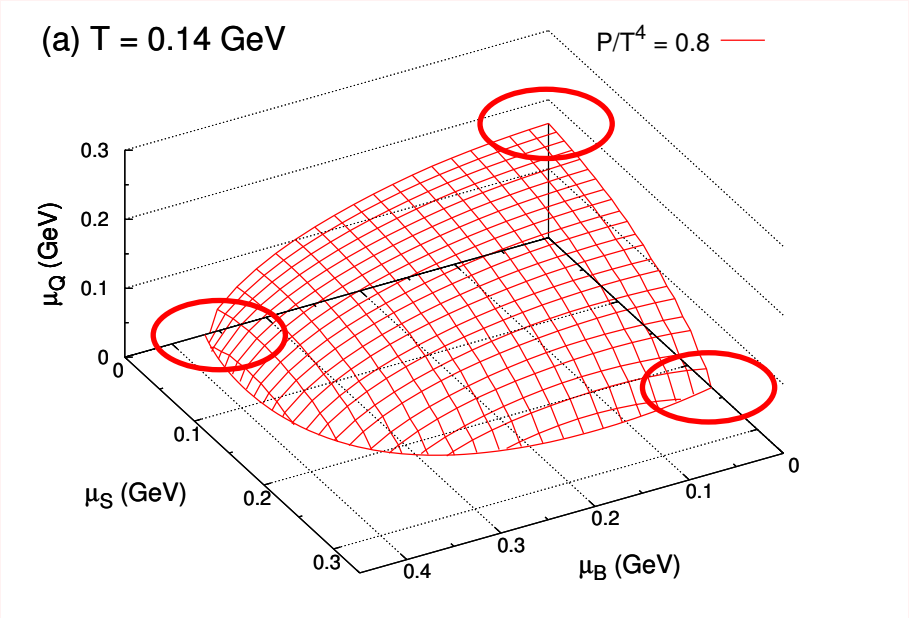

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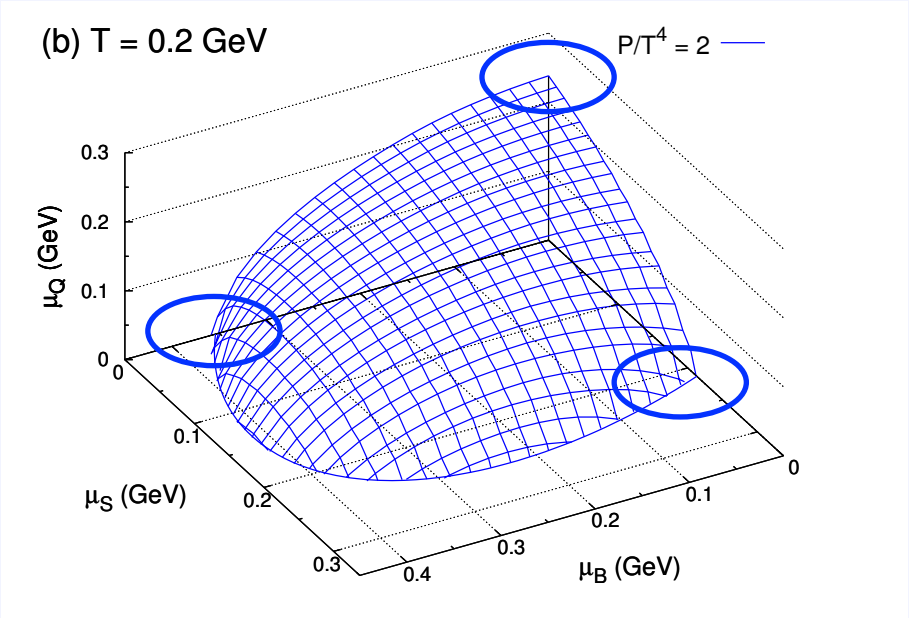

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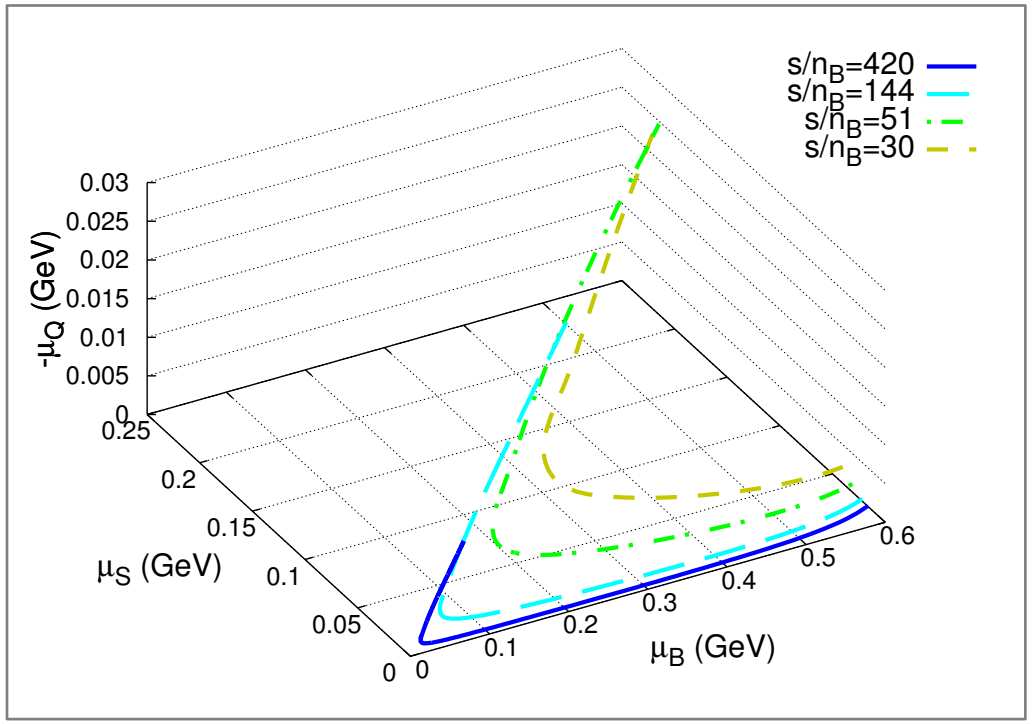
μ_B-μ_Q-μ_S space

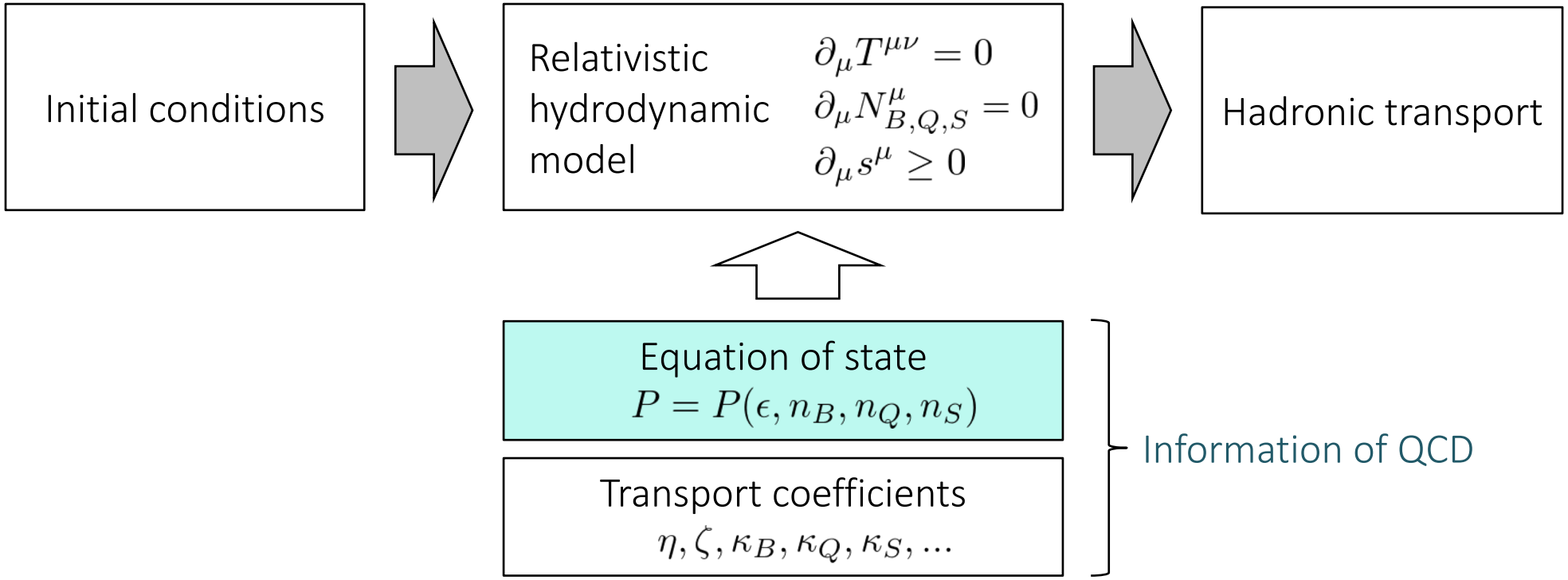
- Exploration of the QCD phase diagram



- The BES experiments do not explore the μ_B-T plane, but a slice in the μ_B-μ_Q-μ_S-T hyperplane
- This may well affect the search for the QCD critical point

Hydrodynamic model

- with multiple charges



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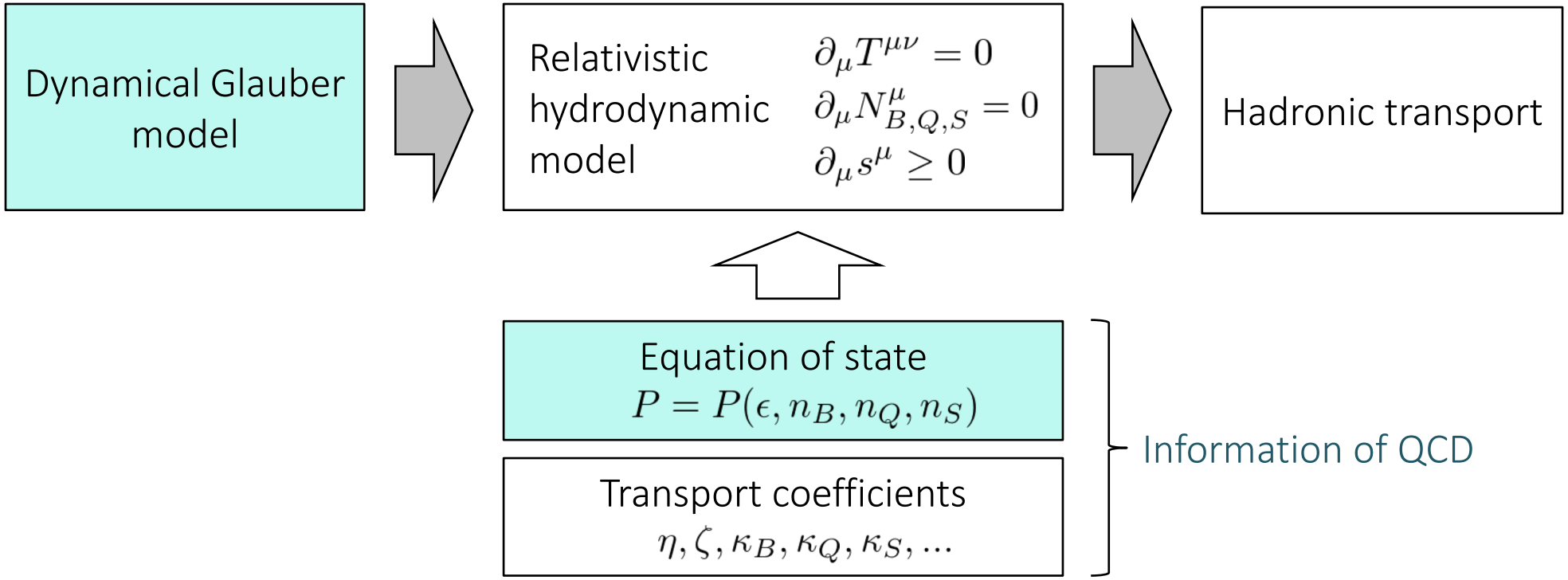

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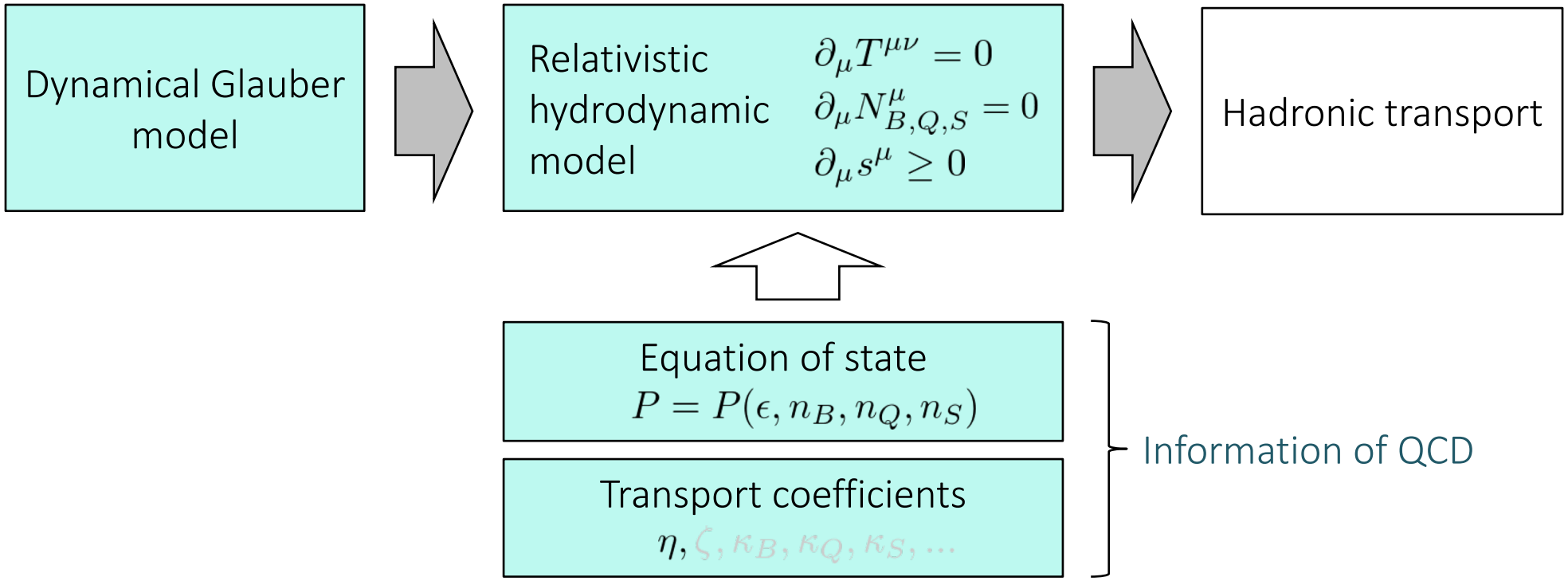

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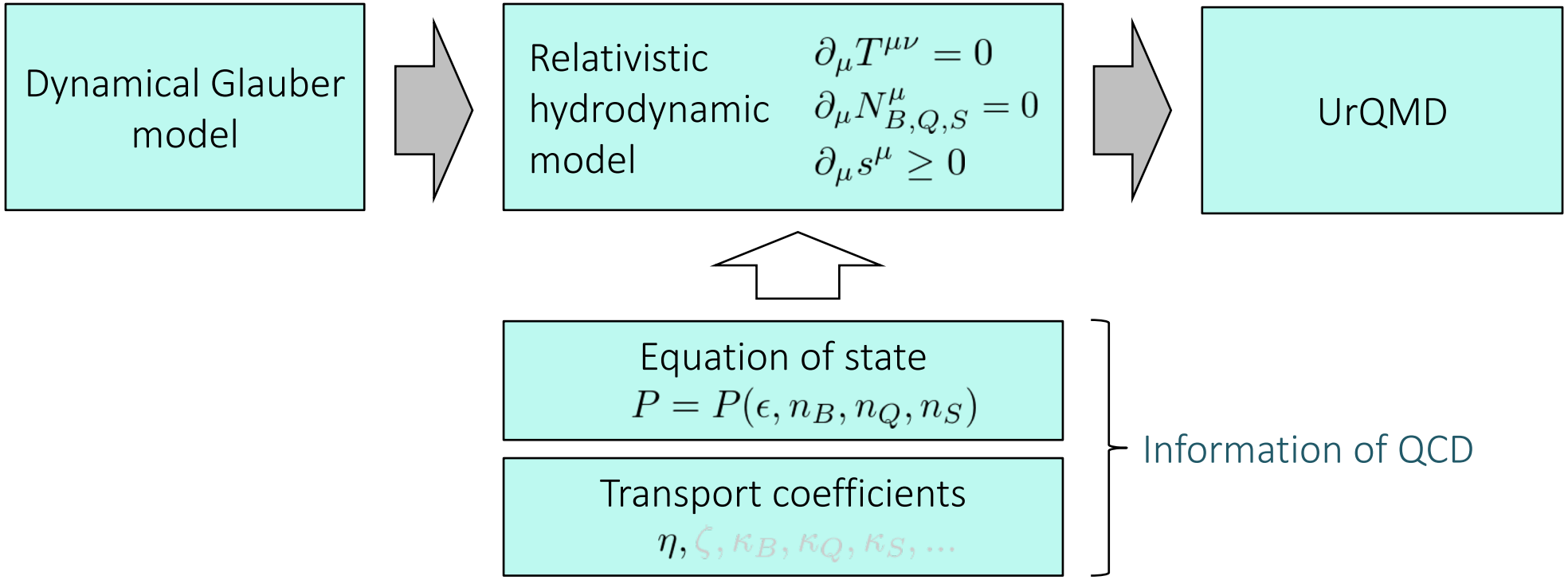
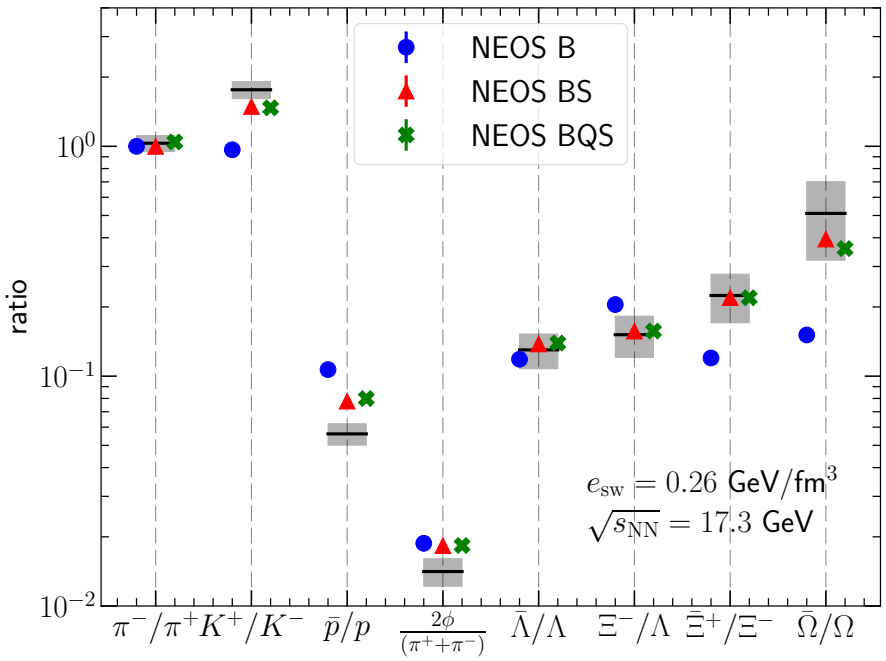
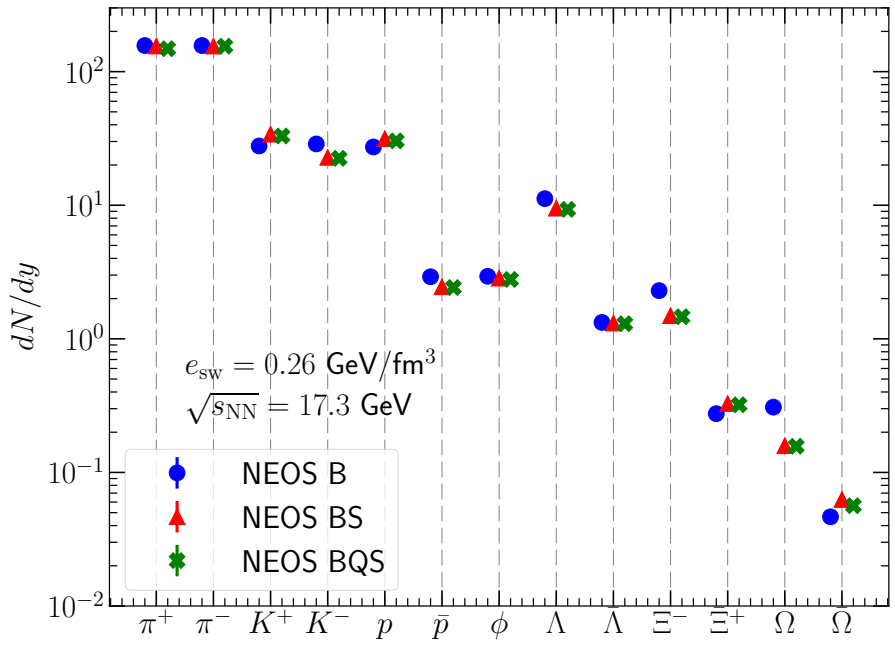


Hydrodynamic results

- 3+1 D viscous hydro + UrQMD for Pb-Pb 17.3 GeV in SPS

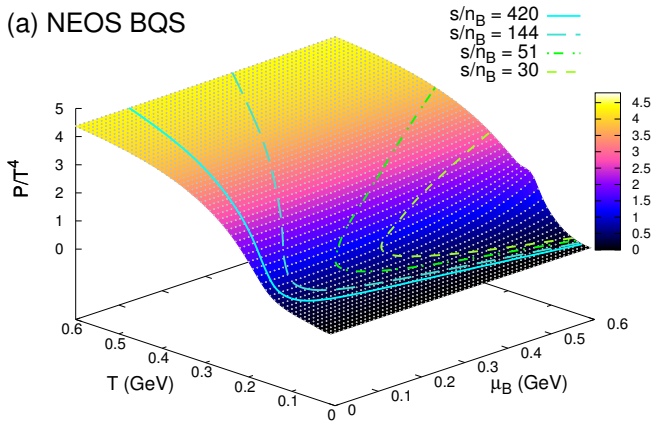




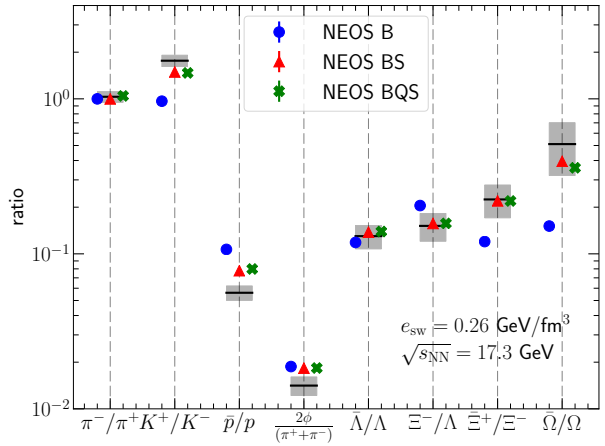
- Strange neutrality improves description of strange hadrons
- Charge-to-baryon ratio fixing has small effects; π-/π+ ratio (> 1) is improved

Summary and outlook

- The QCD matter at finite density poses us challenges:
  - Interplay of multiple charges (B, Q, S) are important
    - Equation of state is constructed
    - Strangeness neutrality condition leads to finite positive μ_S, realistic charge-to-baryon ratio for Au/Pb to finite negative μ_Q
    - Particle-to-antiparticle ratios are described better in hydro model



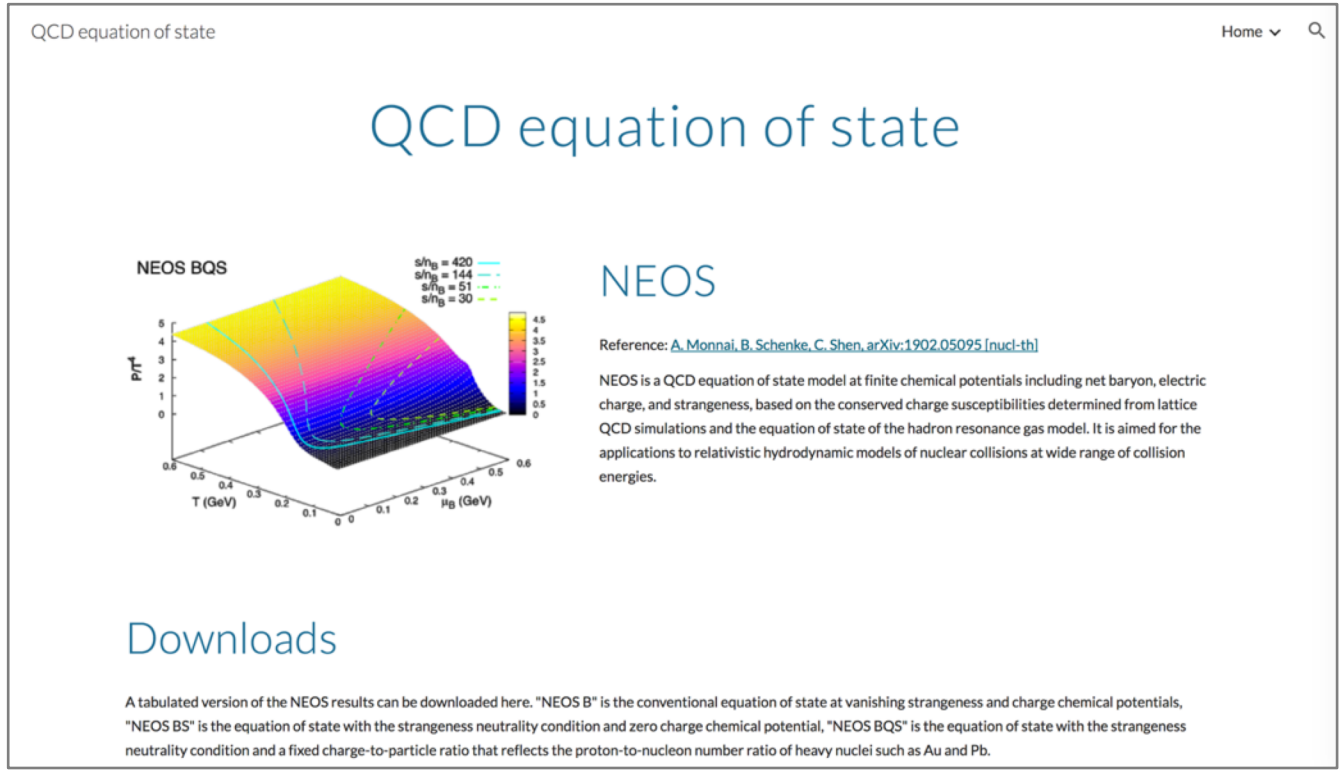
- Future prospects:
  - Estimation of baryon, strangeness and charge diffusion including cross-coupling currents
  - p_T spectra, flow harmonics and rapidity distribution
  - Realistic EoS for small systems as well as isobar experiments



Summary and outlook

- Our QCD equation of state model NEOS is publicly available:

  https://sites.google.com/view/qcdneos/home



谢谢

Thank you for listening!

Backup slides

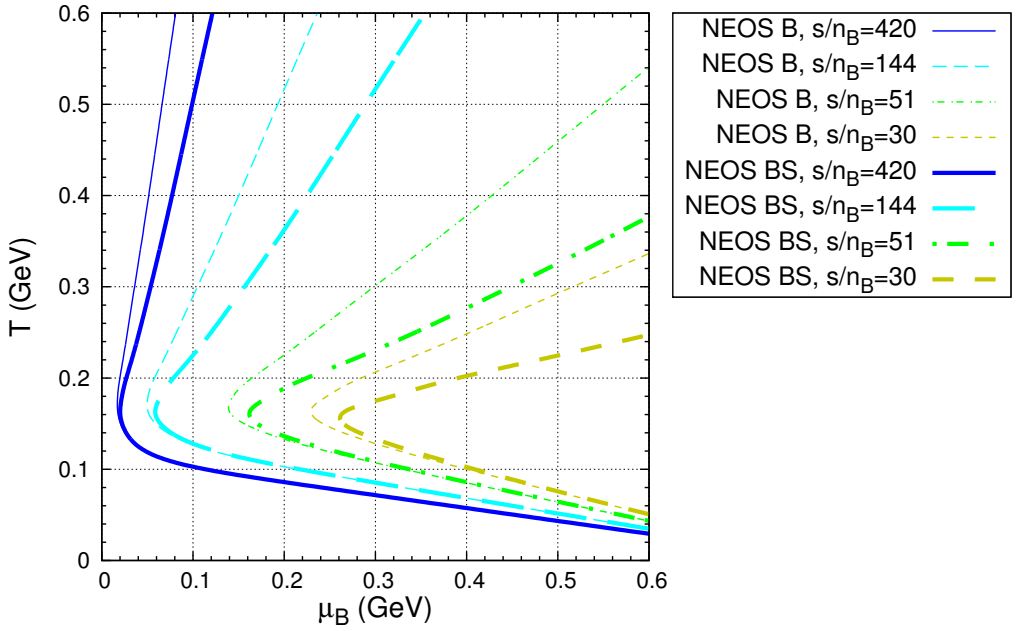
Equation of state

- Where you can probe on the μ_B-T plane

  - s/n_B is constant in a collision when entropy and net baryon number are conserved

$s/n_B = 420$	$\sqrt{s_{NN}} = 200$ GeV
$s/n_B = 144$	$\sqrt{s_{NN}} = 62.4$ GeV
$s/n_B = 51$	$\sqrt{s_{NN}} = 19.6$ GeV
$s/n_B = 30$	$\sqrt{s_{NN}} = 14.5$ GeV

J. Gunther et. al., Nucl. Phys. A 967, 720 (2017)

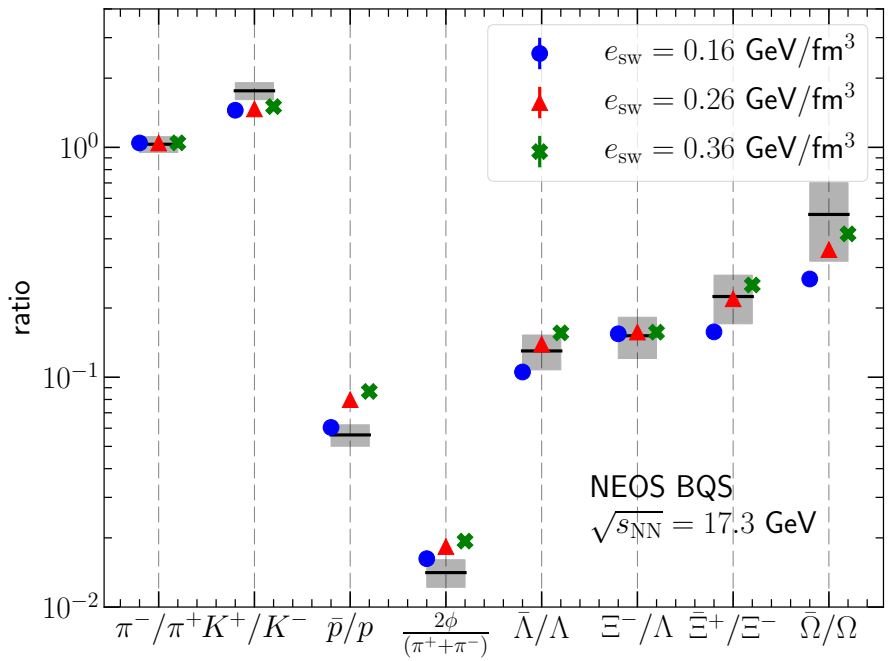


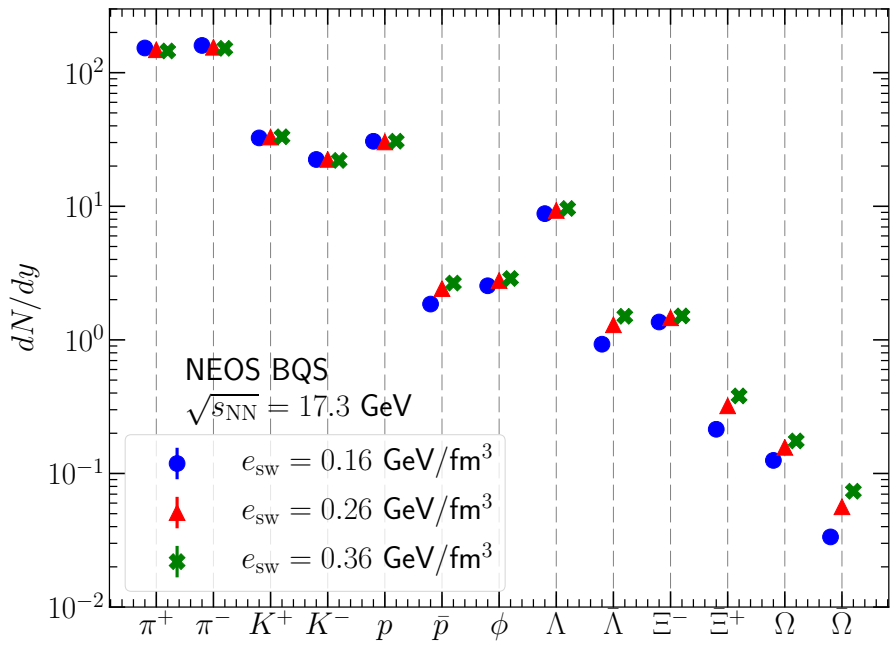
  - QGP phase: s/n_B is a straight line because $s \sim T^3$ and $n_B \sim \mu_B T^2$

    Hadronic phase: s/n_B bends to large μ_B because pions dominate against protons

Hydrodynamic results

- Switching energy density dependence





- The preferred switching energy density to UrQMD is ~ 0.26 GeV/fm^3
- Effects of chemical potential becomes larger for lower e_{sw}

Stefan-Boltzmann limit

- Parton gas pressure

$$\frac{P}{T^4} = \frac{8\pi^2}{45} + \frac{7\pi^2}{60} N_f + \frac{1}{2} \sum_{f=u,d,s} \left(\frac{\mu_f}{T}\right)^2 + \frac{1}{4\pi^2} \sum_{f=u,d,s} \left(\frac{\mu_f}{T}\right)^4$$

where $\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q$, $\mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q$, $\mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S$

- The diagonal and off-diagonal susceptibilities are

$$\chi_2^B = \frac{1}{3}, \quad \chi_2^Q = \frac{2}{3}, \quad \chi_2^S = 1, \quad \chi_{1,1}^{B,Q} = 0, \quad \chi_{1,1}^{B,S} = -\frac{1}{3}, \quad \chi_{1,1}^{Q,S} = \frac{1}{3},$$

$$\chi_4^B = \frac{2}{9\pi^2}, \ \chi_4^Q = \frac{4}{3\pi^2}, \ \chi_4^S = \frac{6}{\pi^2}, \ \chi_{3,1}^{B,S} = -\frac{2}{9\pi^2}, \ \chi_{2,2}^{B,S} = \frac{2}{3\pi^2}, \ \chi_{1,3}^{B,S} = -\frac{2}{\pi^2},$$

$$\chi_{3,1}^{B,Q} = 0, \ \chi_{2,2}^{B,Q} = \frac{4}{9\pi^2}, \ \chi_{1,3}^{B,Q} = \frac{4}{9\pi^2}, \ \chi_{3,1}^{Q,S} = \frac{2}{9\pi^2}, \ \chi_{2,2}^{Q,S} = \frac{2}{3\pi^2}, \ \chi_{1,3}^{Q,S} = \frac{2}{\pi^2},$$

$$\chi_{2,1,1}^{B,Q,S} = \frac{2}{9\pi^2}, \ \chi_{1,2,1}^{B,Q,S} = -\frac{2}{9\pi^2}, \ \chi_{1,1,2}^{B,Q,S} = -\frac{2}{3\pi^2}$$

Stefan-Boltzmann limit

- The chemical potential ratio

$$\begin{pmatrix} n_B \\ n_Q \\ n_S \end{pmatrix} = T^2 \begin{pmatrix} \chi_2^B & \chi_{1,1}^{B,Q} & \chi_{1,1}^{B,S} \\ \chi_{1,1}^{B,Q} & \chi_2^Q & \chi_{1,1}^{Q,S} \\ \chi_{1,1}^{B,S} & \chi_{1,1}^{Q,S} & \chi_2^S \end{pmatrix} \begin{pmatrix} \mu_B \\ \mu_Q \\ \mu_S \end{pmatrix} + \mathcal{O}(\mu^3) \qquad n_J = \left.\frac{\partial P}{\partial \mu_J}\right|_{T,\mu_K}$$

NEOS B ($\mu_S = 0$ and $\mu_Q = 0$)

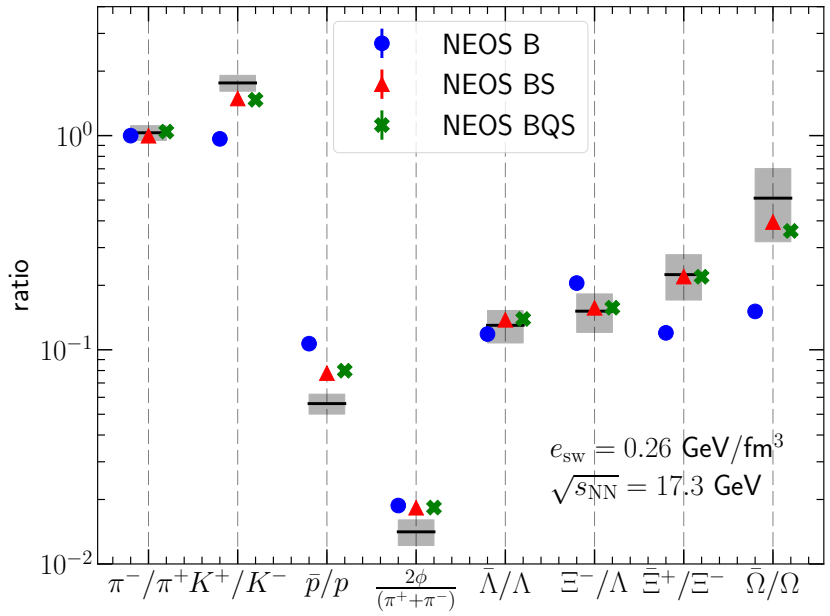
$$\mu_B \simeq 3n_B/T^2, \qquad \mu_S = 0 \qquad , \qquad \mu_Q = 0$$

NEOS BS ($n_S = 0$ and $\mu_Q = 0$)

$$\mu_B \simeq 4.5n_B/T^2, \; \mu_S \simeq 1.5n_B/T^2, \qquad \mu_Q = 0$$

NEOS BQS ($n_S = 0$ and $n_Q = 0.4n_B$)

$$\mu_B \simeq 4.6n_B/T^2, \; \mu_S \simeq 1.6n_B/T^2, \; \mu_Q \simeq -0.2n_B/T^2$$



Sound velocity c_s

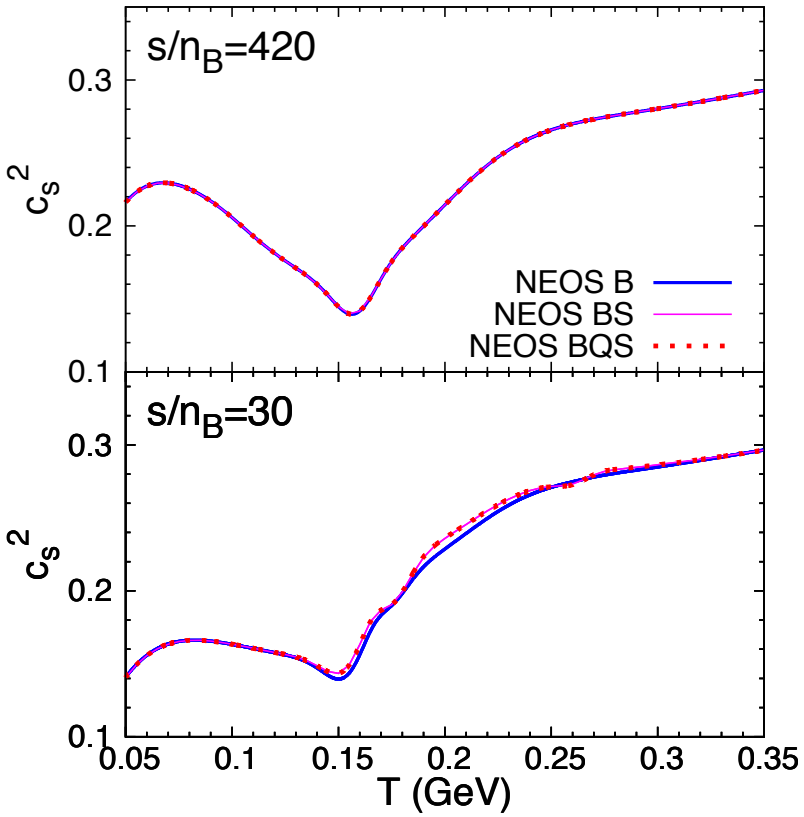
- s/n_B dependence

$$c_s^2 = \left.\frac{\partial P}{\partial e}\right|_{n_K} + \sum_J \frac{n_J}{e+P} \left.\frac{\partial P}{\partial n_J}\right|_{e,n_K} \quad J \neq K$$

- In dense systems, c_s is suppressed at lower T
- The effect of strangeness neutrality becomes more apparent
- Finite n_s is relevant to c_s^2 in NEOS B

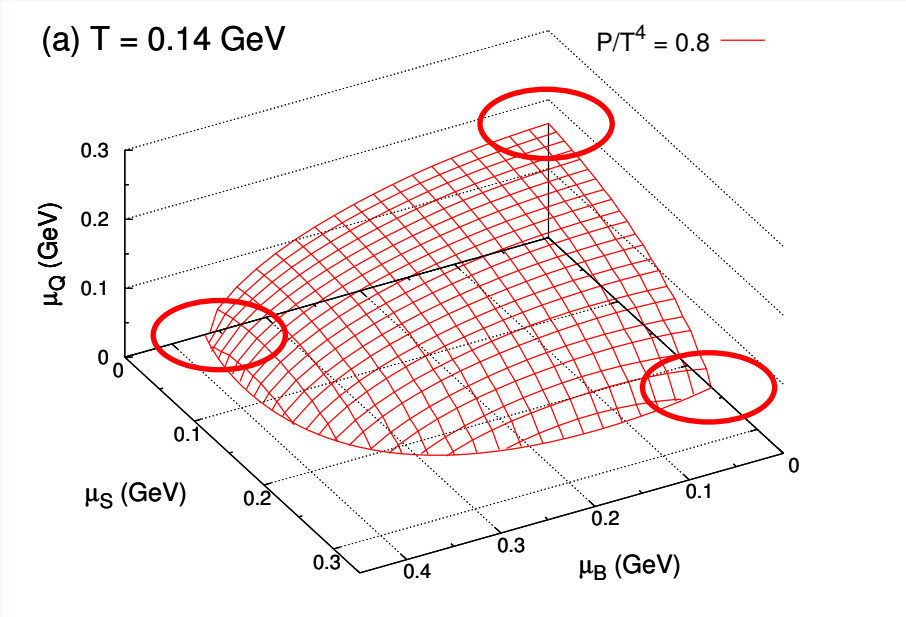
$$\left.\frac{\partial P}{\partial e}\right|_{n_B} + \frac{n_B}{e+P} \left.\frac{\partial P}{\partial n_B}\right|_{e} \neq c_s^2$$

The "conventional definition" w/o Q and S leads to underestimation



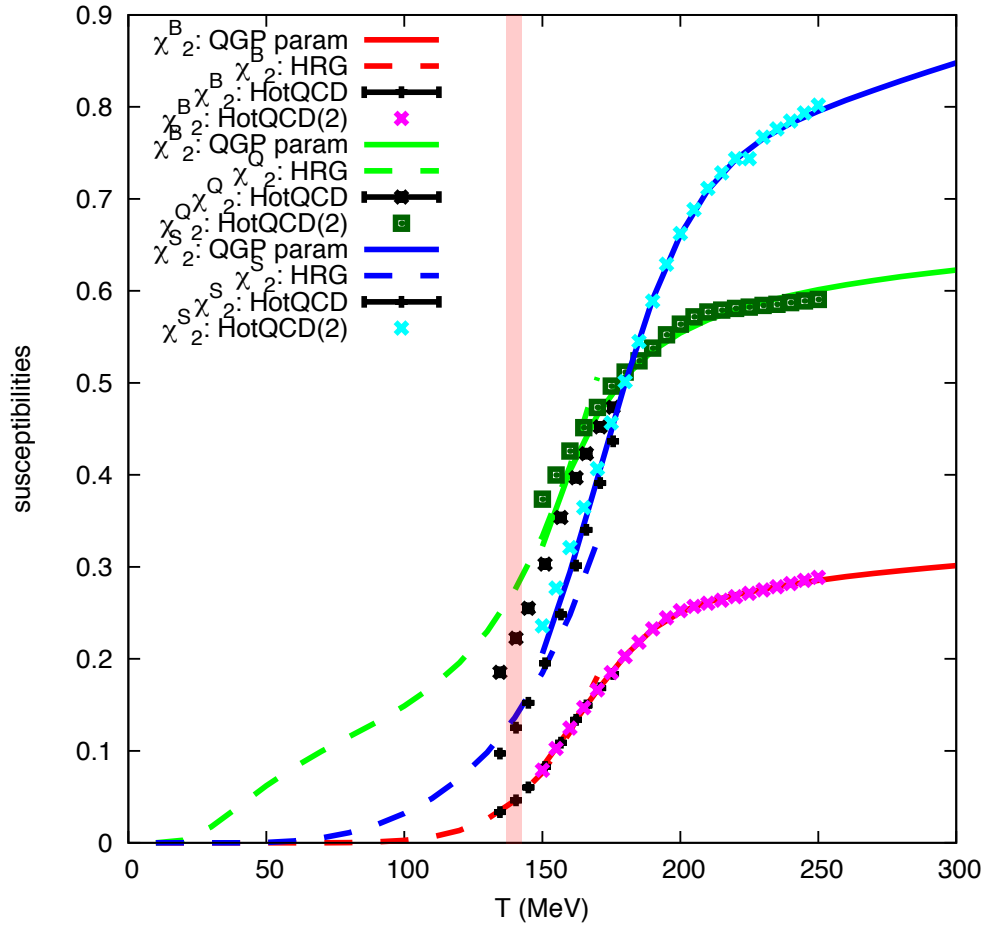
μ_B-μ_Q-μ_S space

- Constant pressure plane at fixed T



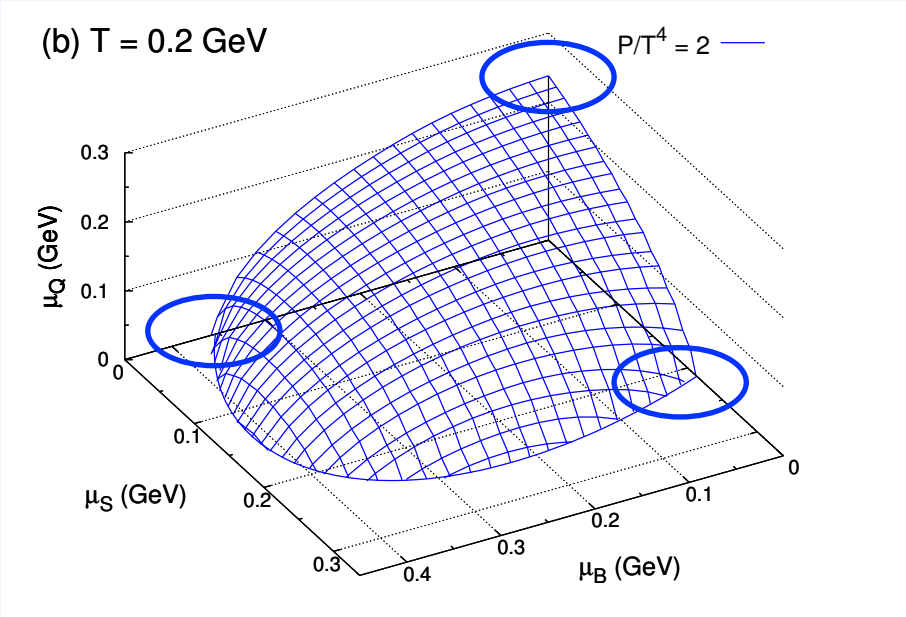
$\mu_B^{\rm int} > \mu_S^{\rm int} > \mu_Q^{\rm int}$ in the hadronic phase

Lightest hadrons to carry the charges are ordered in mass as $m_p > m_K > m_\pi$



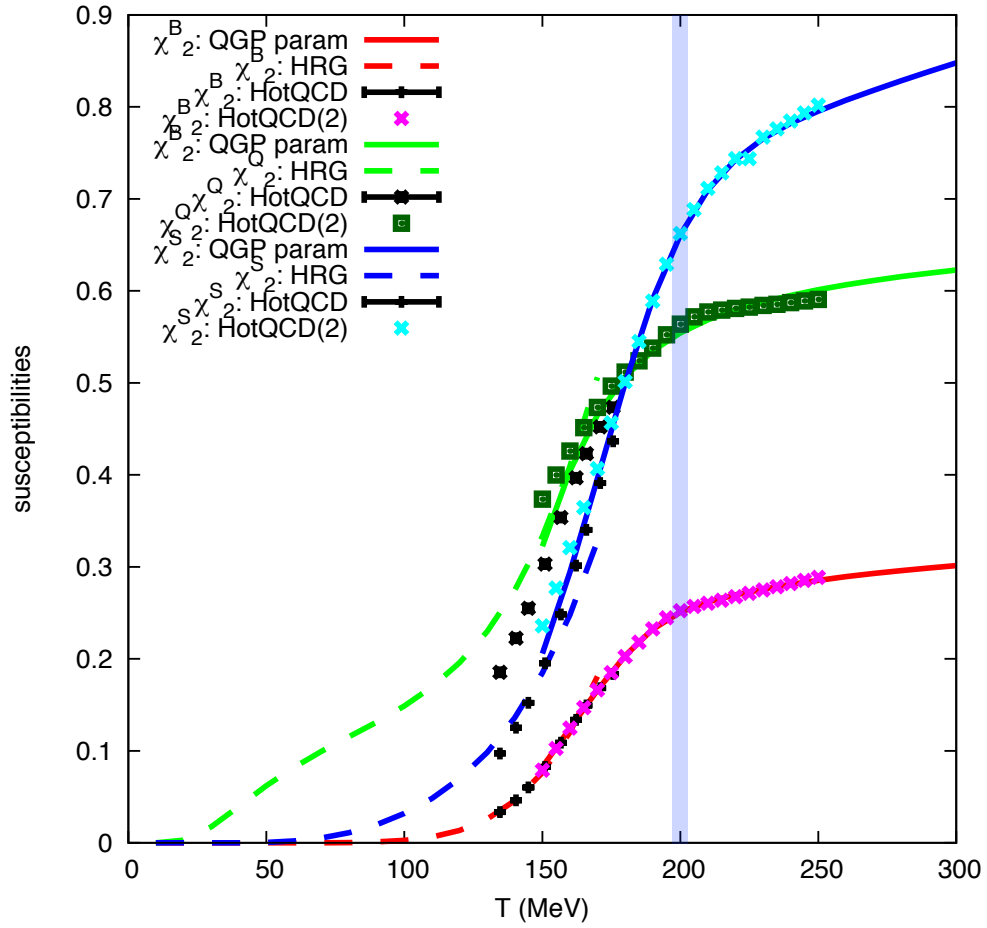
μ_B-μ_Q-μ_S space

- Constant pressure plane at fixed T



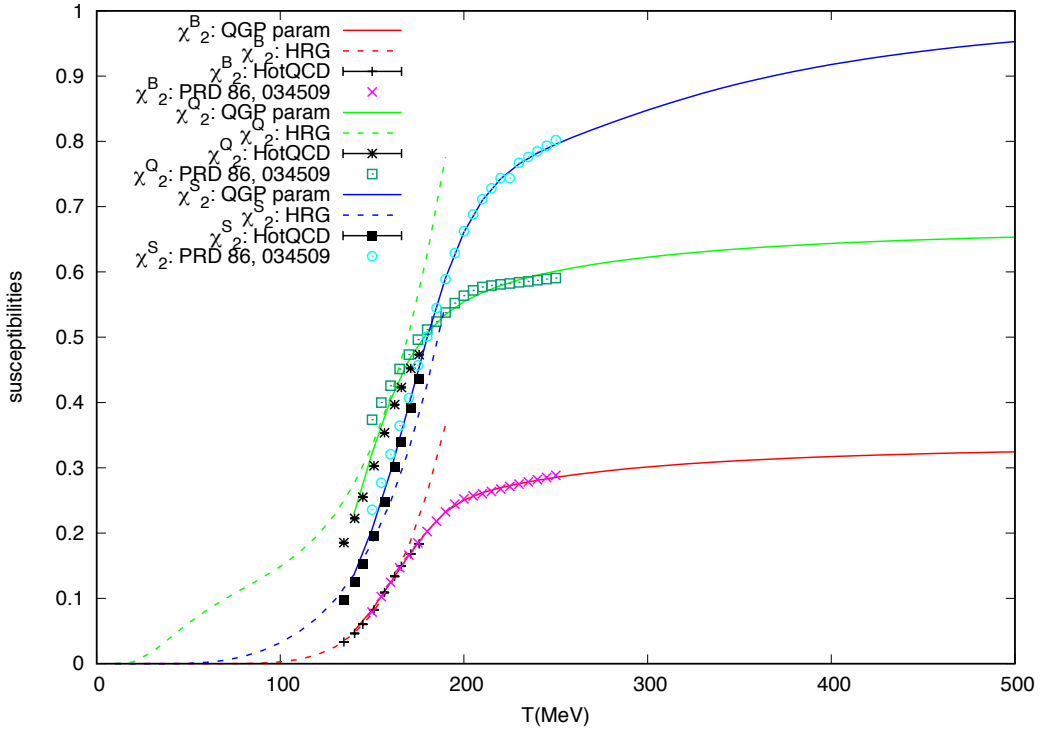
$\mu_B^{\rm int} > \mu_Q^{\rm int} > \mu_S^{\rm int}$ in the QGP phase

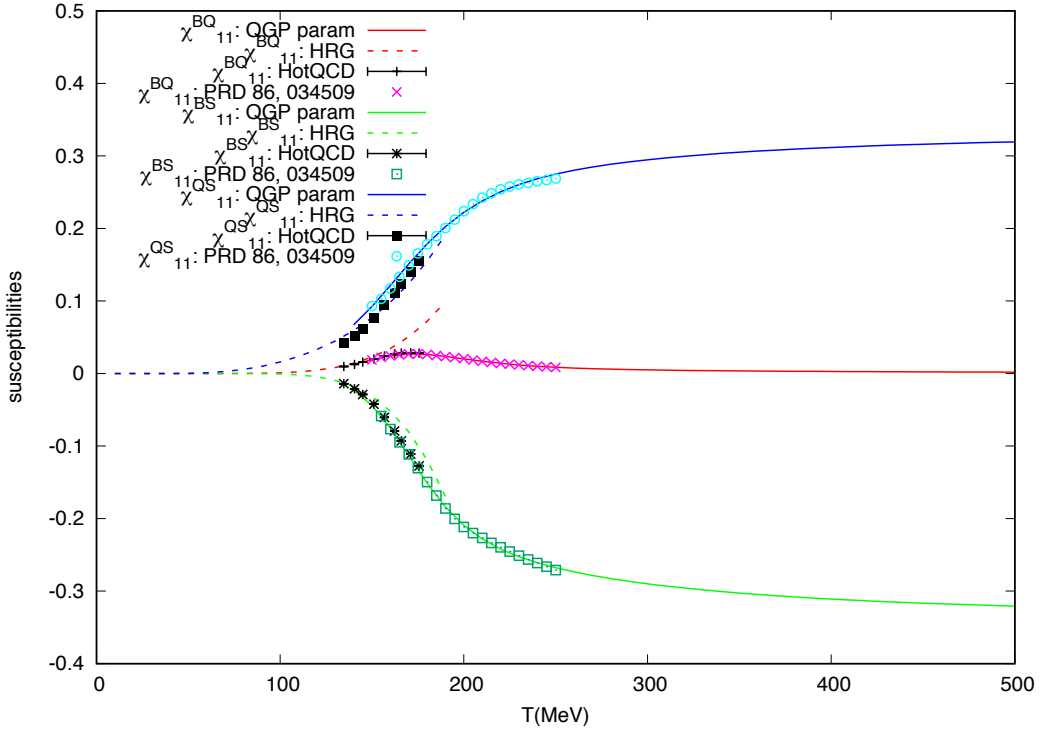
$\chi_2^B = 1/3$, $\chi_2^Q = 2/3$, $\chi_2^S = 1$ in the parton gas limit implies $\mu_B^{\rm int}/3 \sim 2\mu_Q^{\rm int}/3 \sim \mu_S^{\rm int}$



Susceptibilities

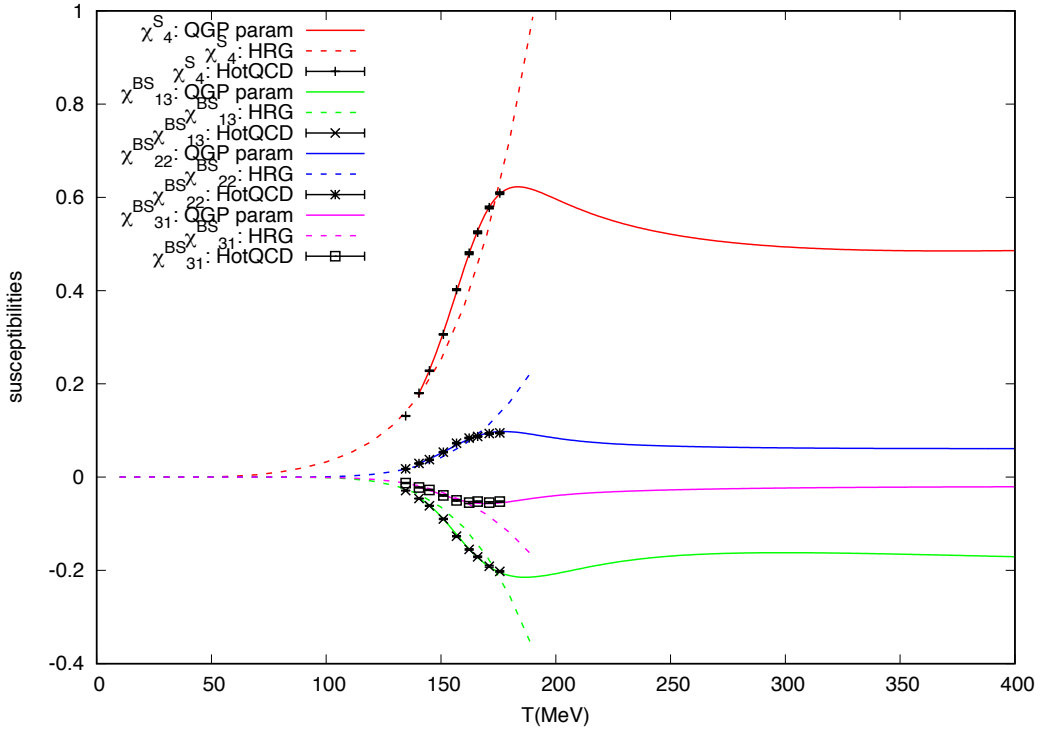
- Lattice QCD vs. Hadron resonance gas: 2nd order

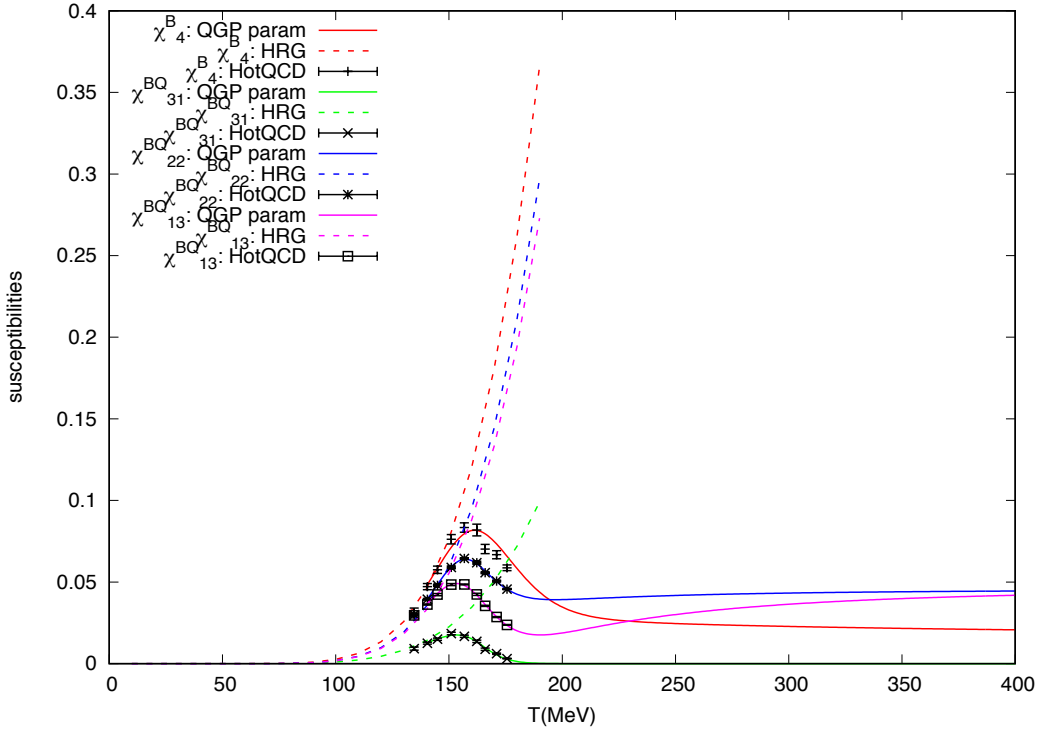




Susceptibilities

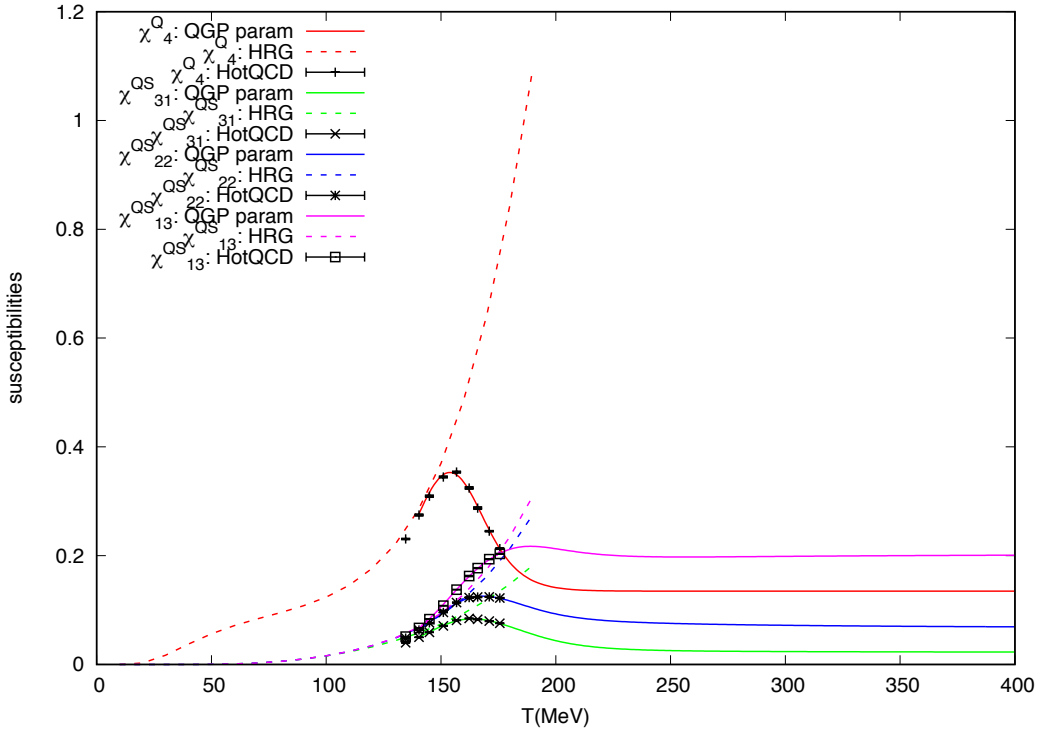
- Lattice QCD vs. Hadron resonance gas: 4th order

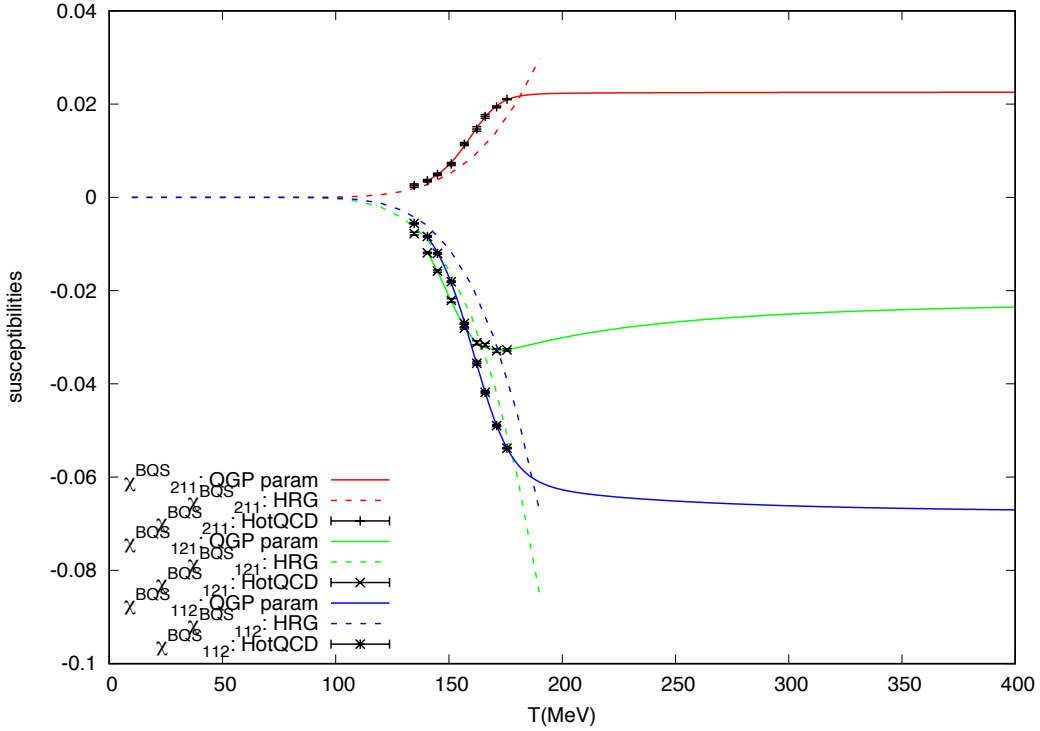




Susceptibilities

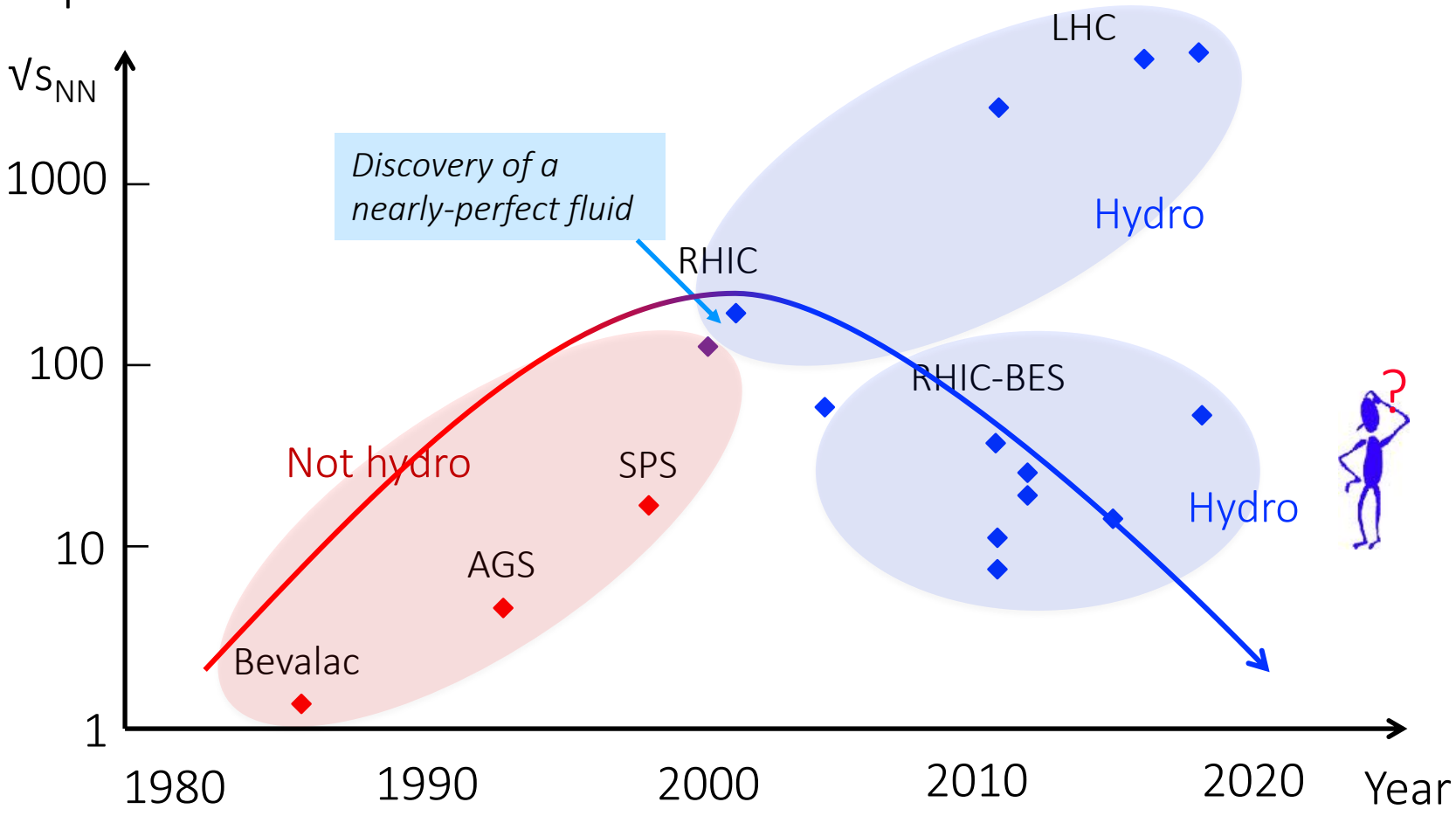
- Lattice QCD vs. Hadron resonance gas





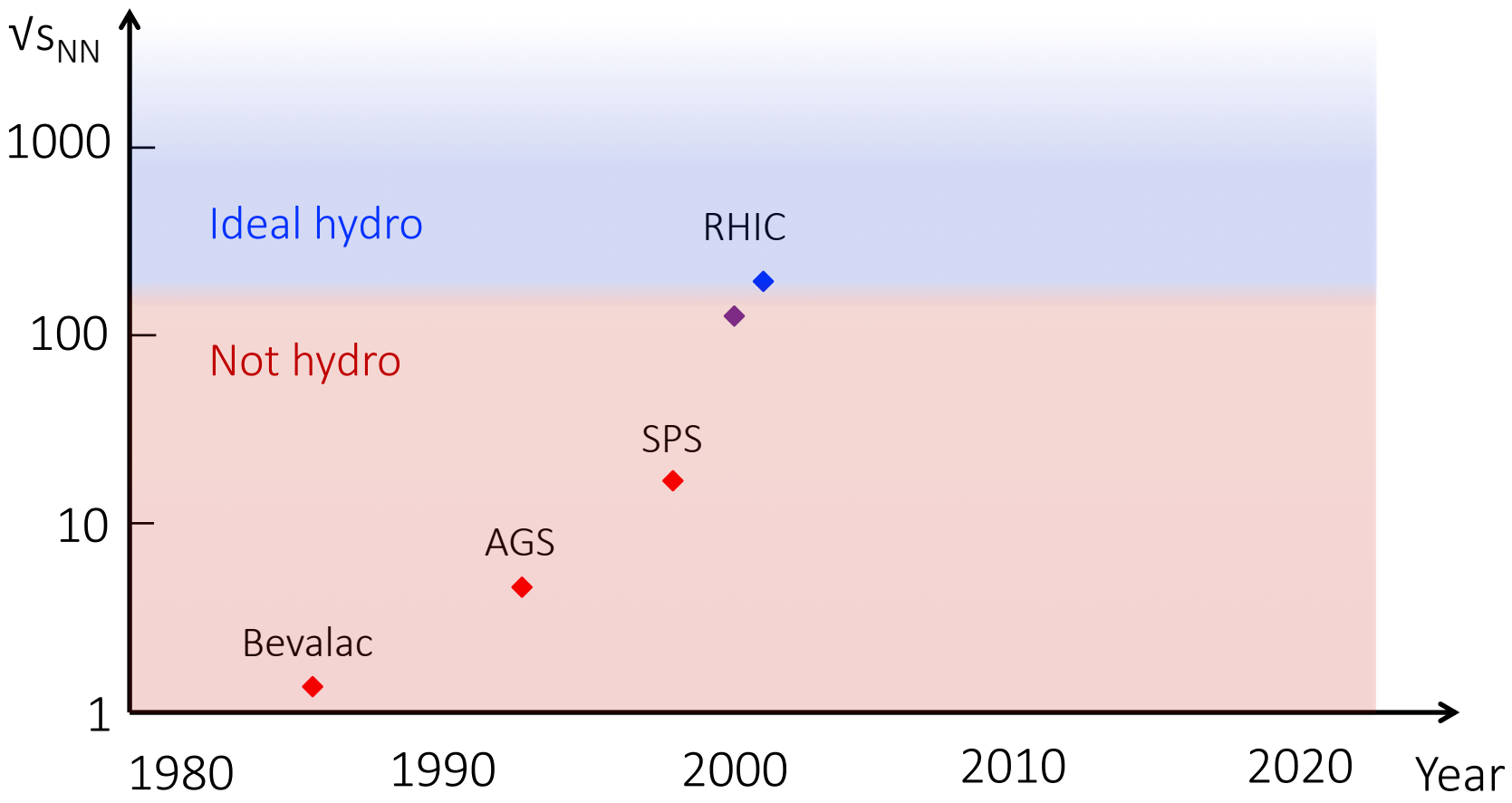
Is hydro good at BES energies?

- A historical point of view



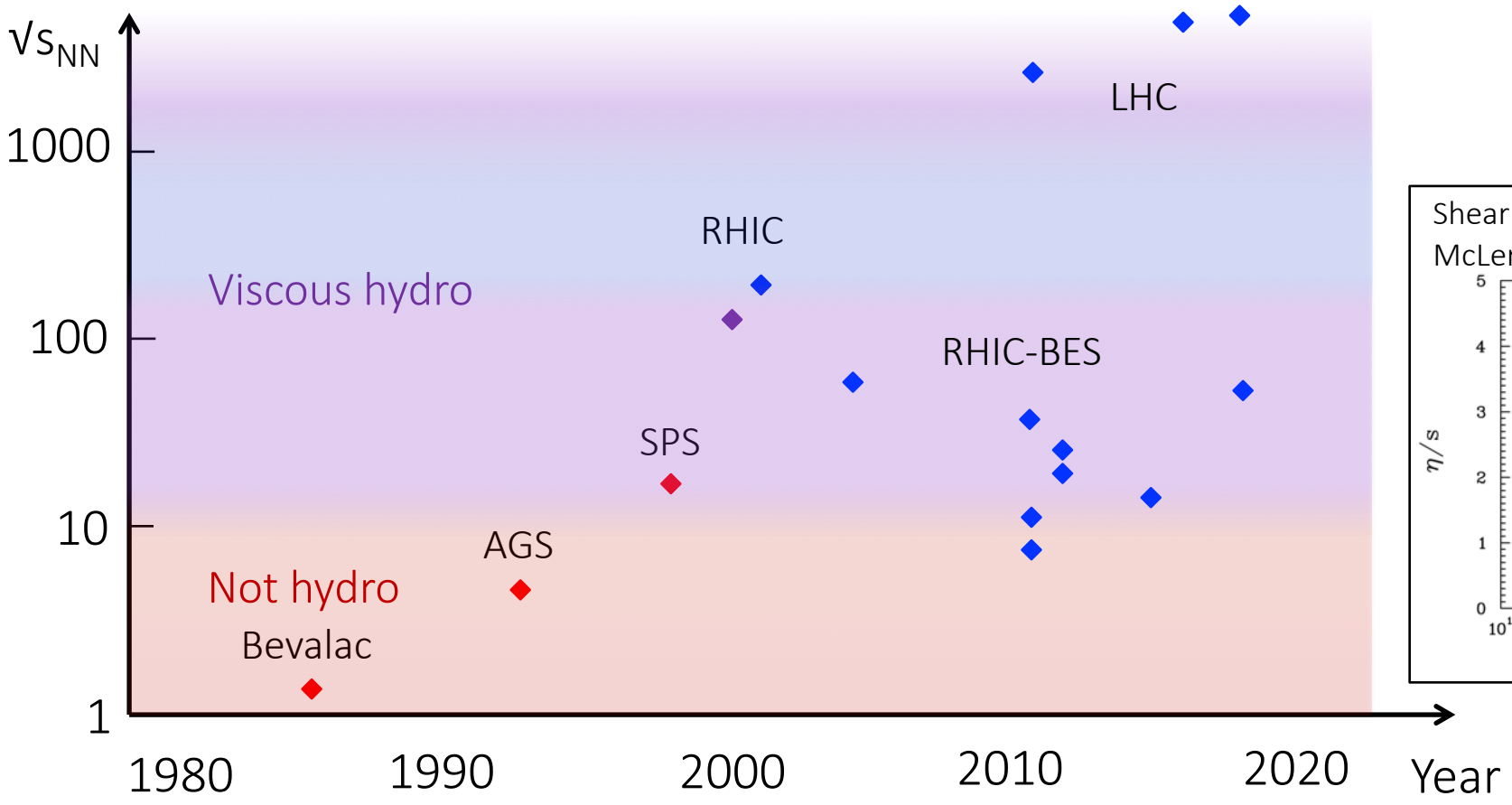
Is hydro good at BES energies?

- A historical point of view (around 2000)



Is hydro good at BES energies?

- A historical point of view (around 2019)



Shear viscosity: Csernai, Kapusta & McLerran, PRL 97, 152303 (2006)