Universality driven analytic structure of QCD crossover: radius of convergence and QCD critical point

Vladimir Skokov in collaboration with **Swagato Mukherjee**
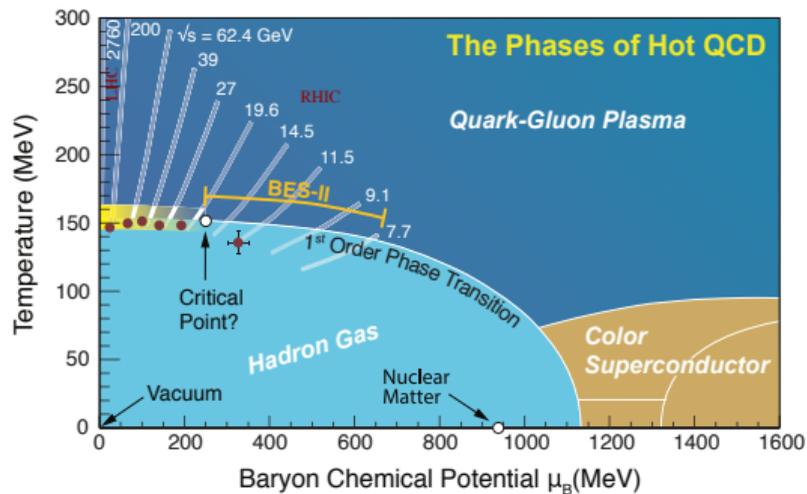**NC State University**   **Brookhaven Natl Lab**

RBRC
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BEST
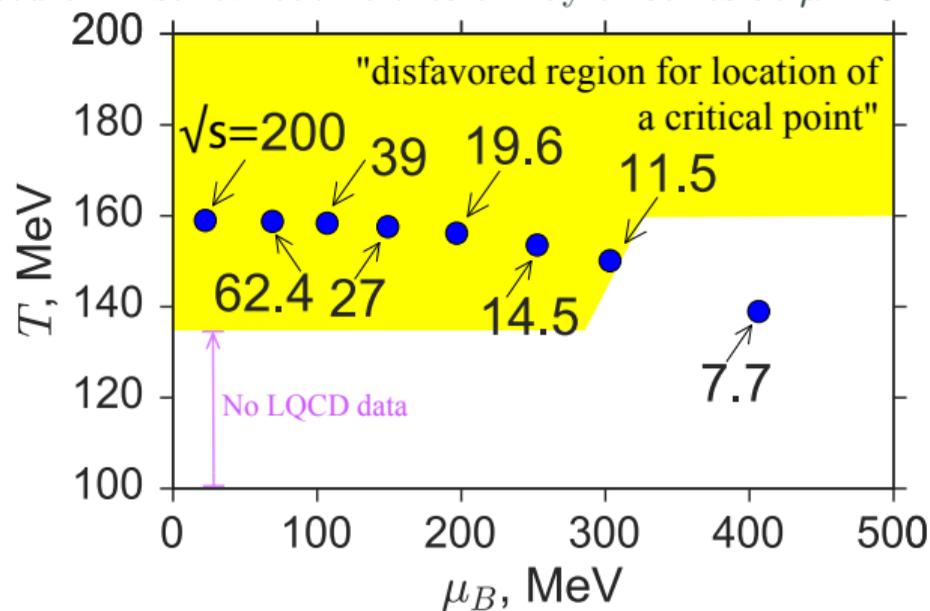COLLABORATION

NC STATE
UNIVERSITY

- Search for a critical point in finite Temperature and Density Landscape

- LQCD + universality argument $\rightsquigarrow$ new input on location of critical point

◆ Radius of convergence of Taylor expansion is defined by closest singularity

◆ Singularities of thermodynamic functions may signal critical point

◆ Analysis is based on first few coefficients of Taylor series at $\mu = 0$:

Reploted using data from BNL-Bi-CCNU Collaboration, arXiv:1701.04325

Radius of convergence II

◆ Precise mathematical statement about radius of convergence

$\rightsquigarrow$ asymptotically high order coefficients

◆ At least model estimates based on 4-th or 6-th order coefficients demonstrate that extracted radius of convergence is not reliable

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V. S., K. Morita, & B. Friman, arXiv:1008.4549
M. Pradeep & M. Stephanov, arXiv:1905.13247

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  – w/o input from experiment
  – based on first principal calculations

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Universality

- Thermodynamics in vicinity of QCD transition is controlled by universal scaling function

$$p(T, \mu_B)/T^4 = -h^{(2-\alpha)/\beta\delta} f_f(z) - f_{\text{regular}}(T, \mu_B)$$

$$z = z_0 t h^{-1/\beta\delta} = z_0 \left[\frac{T - T_c}{T_c} + \kappa_B \left(\frac{\mu_B}{T} \right)^2 \right] \left(\frac{m_{u,d}}{m_s} \right)^{-1/\beta\delta}$$
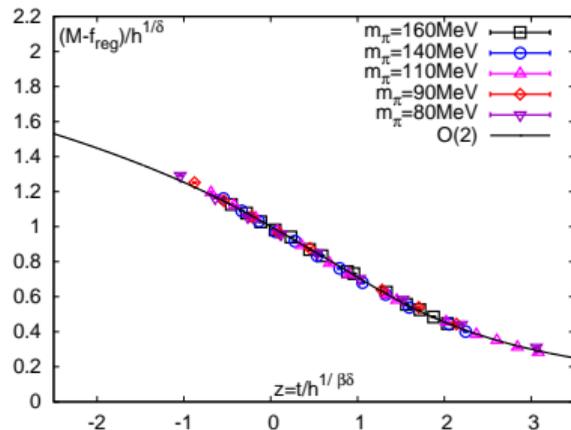
$$t = \frac{T - T_c^0}{T_c^0} + \kappa_B \left(\frac{\mu_B}{T} \right)^2 \; ; \quad h = \frac{m_{u,d}}{m_s} \; .$$

O(4) critical exponents: $\alpha = -0.21$, $\beta = 0.38$, $\delta = 4.82$

F. Karsch, EMMI Workshop 2019

- Recent progress from LQCD established:
  $T_c = 132^{+3}_{-6}$ MeV; $\kappa_B = 0.012(2)$; $z_0 = 1 - 2$

H. T. Ding et al. (2019), 1903.04801

- Additionally, there is a strong indication that QCD is in O(4) scaling regime

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Sheng-Tai Li, Heng-Tong Ding, 1702.01294

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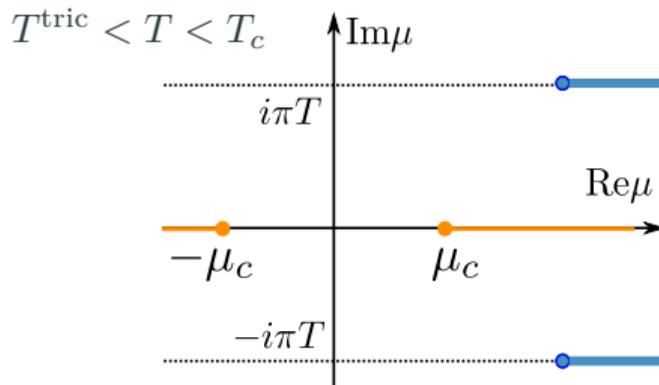$$M/h^{1/\delta} = g(z) \equiv \frac{z}{\beta\delta} f_f'(z) - \left(1 + \frac{1}{\delta}\right) f_f(z)$$

Lattice parametrization
FRG

Mean field
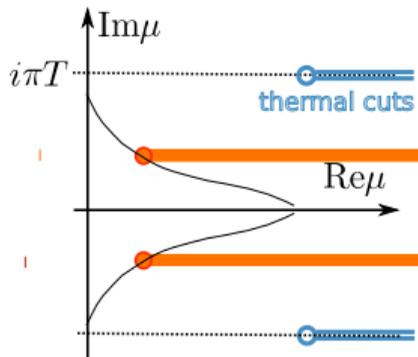
Large N

asymptotics

Chiral limit $(m_{u,d} = 0)$:

$T^{\text{tric}} < T < T_c$
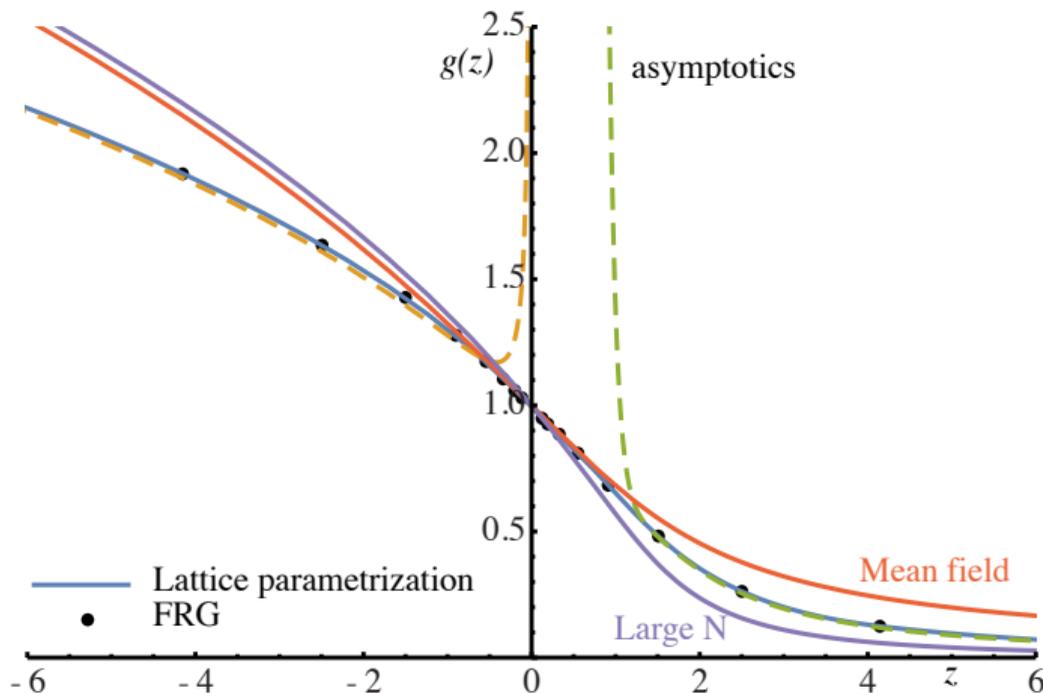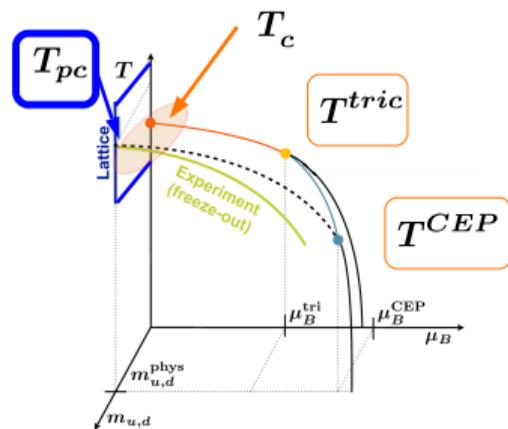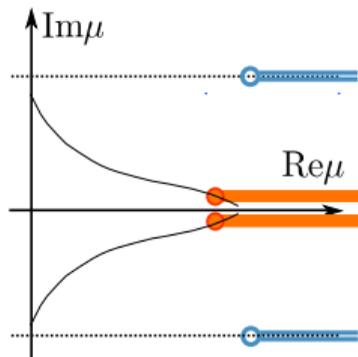
M. Stephanov, hep-lat/0603014; C. Itzykson, et al Nucl.Phys. B220 (1983) 415
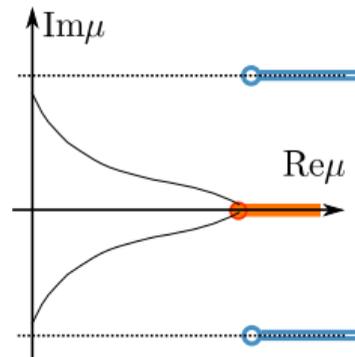
$m_{u,d} \neq 0$:
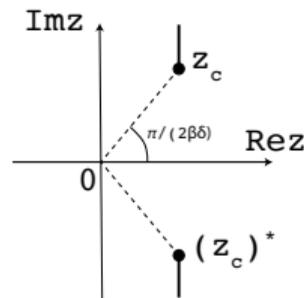
crossover: above but close to CEP: at CEP:

Yang-Lee edge singularity

◆ Function $f_f(z)$ is universal for any given class

  Location of the singularity in complex z plane is also universal
  It is called Yang-Lee edge singularity (M. E. Fisher, 1978)

◆ What is known about z_c?

  ◆ $z_c = |z_c|\, e^{\pm i \frac{\pi}{2\beta\delta}}$

     for QCD, critical exponents are those of O(4) universality class

  ◆ YL edge singularity has its own critical exponent $f_f \sim (z - z_c)^{\sigma+1}$ with $\sigma \approx 0.1$

  ◆ σ is independent of underlying symmetry class
     with only exception $\sigma_{N\to\infty} = 1/2$ as in mean-field approximation

  ◆ Field-theoretically, near z_c: ϕ^3 theory with imaginary coupling

Analytical results for magnetic equation of state

$$g(z) \equiv \frac{z}{\beta\delta} f'_f(z) - \left(1 + \frac{1}{\delta}\right) f_f(z)$$

◆ Large N limit

$$g(z) \left[z + g^2(z)\right]^2 = 1, \quad z_c = \frac{5}{2^{8/5}} \; e^{\pm i \frac{\pi}{5}} \qquad |z_c| \approx 1.649$$

◆ Mean-field approximation

$$g(z) \left[z + g^2(z)\right] = 1, \quad z_c = \frac{3}{2^{2/3}} \; e^{\pm i \frac{\pi}{3}} \qquad |z_c| \approx 1.89$$

Skipping technical details

◆ Consider a theory in the same universality class (e.g. Quark Meson model)

◆ Use your favorite numerical method to solve corresponding FRG

◆ Extract critical exponent to x-check if they coincide with known results for O(4)

◆ Find non-universal parameters (T_c, z_0, ...)

◆ Extract $g(z)$ for real z

◆ By introducing imaginary part to baryon chemical potential or to symmetry breaking field (our choice), extend $g(z)$ to full complex plane z

Main difficulty: this doubles the number of FRG equations to be solved

A. Connelly, G. Johnson and V.S., 2019

◆ Find z_c

◆ Check $z_c = |z_c|\, e^{\pm i \frac{\pi}{2\beta\delta}}$: Arg has to be consistent with critical exponents ✓

◆ Universal location $|z_c| \approx 1.68$ for O(4) scaling function

A. Connelly, G. Johnson and V.S., 2019

◆ Having found z_c and using non-universal parameters from LQCD,
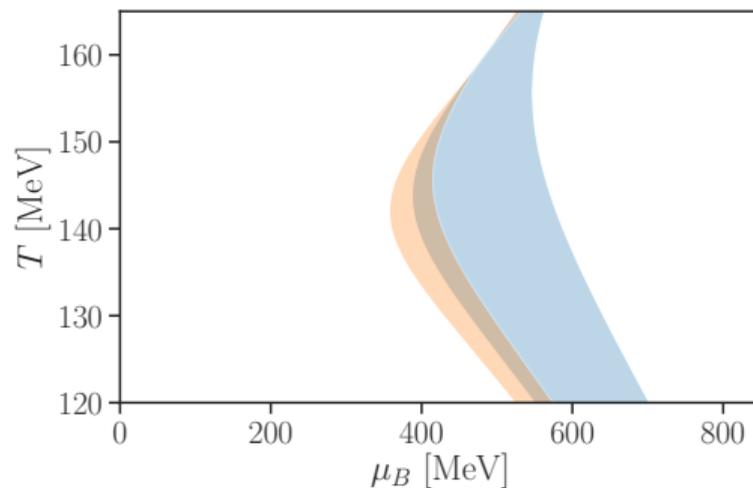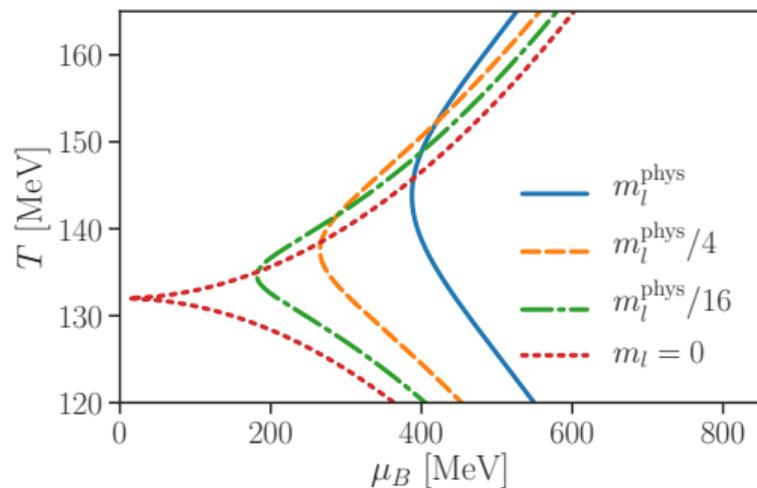one can estimate the radius of convergence

◆ Recall

$$p(T, \mu_B)/T^4 = -h^{(2-\alpha)/\beta\delta} f_f(z) - f_{\text{regular}}(T, \mu)$$

$$z = z_0 \left[\frac{T - T_c}{T_c} + \kappa_B \left(\frac{\mu_B}{T} \right)^2 \right] \left(\frac{m_{u,d}}{m_s} \right)^{-1/\beta\delta}$$

◆ Both singular and regular parts contribute to Taylor series expansion

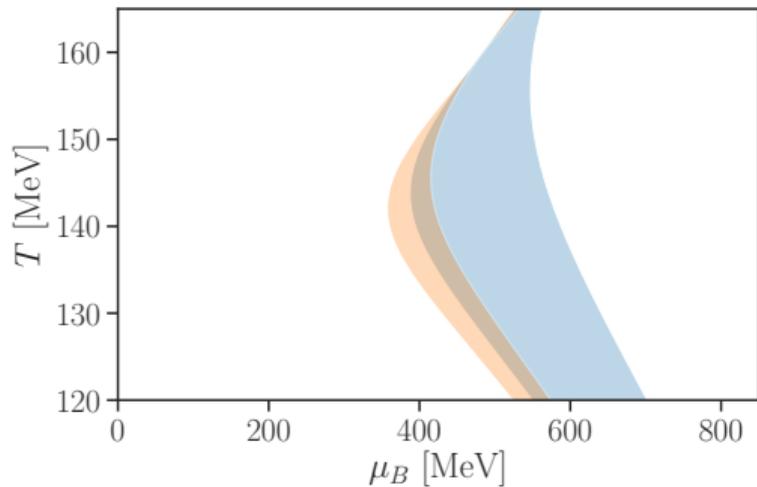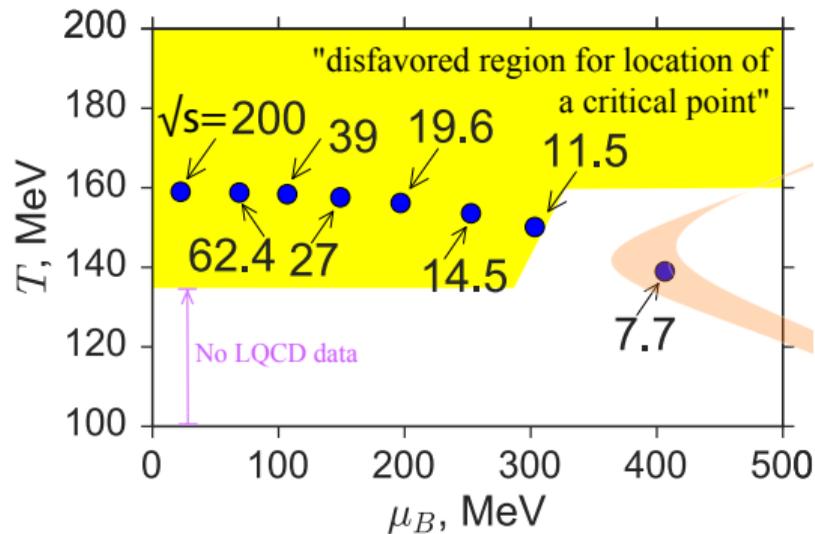◆ Solve $z = z_c$ to find μ_B^c as a function of T and/or $m_{u,d}/m_s$

Orange band is for $z_0 = 2$ and incorporates a 15% uncertainty on the value of $|z_c|$.

Blue band depicts variation of $z_0 = 1 - -2$.

S. Mukherjee & V.S., arXiv:1909.04639

Radius of convergence and BES-II

◆ Consistent with Taylor series analysis

◆ Location of Yang-Lee singularity is universal, but yet unknown for many universality classes in 3d . In this talk, $|z_c|$ for O(4) universality class from Functional Renormalization Group

◆ Input on universal properties and non-universal parameters from lattice QCD $\rightsquigarrow$ radius of convergence in O(4) scaling region

◆ Implication on location of CEP