The crossover line in the (T, μ)-phase diagram of QCD

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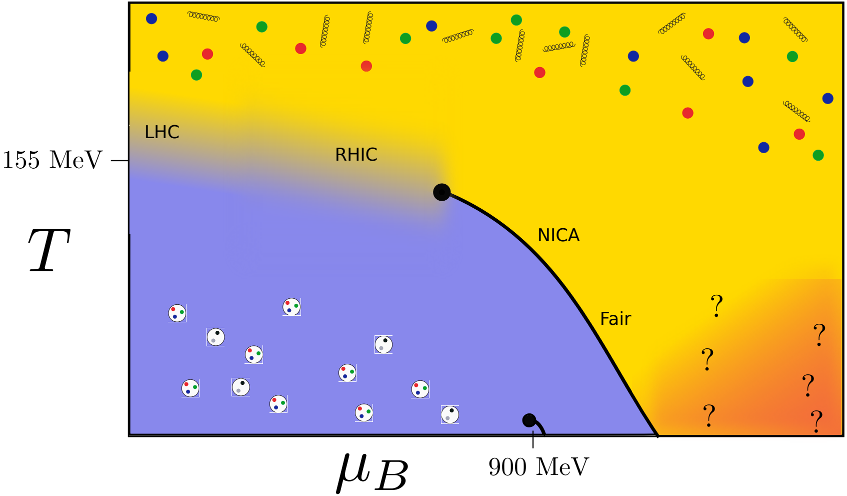
SFB TR55

WB collaboration

The (T, μ_B)-phase diagram of QCD



Our observables: T_c, Equation of state, Fluctuations ($\rightarrow$ poster by Paolo Parotto)

The sign problem

The QCD partition function:

$$Z(V,T,\mu) = \int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}\; e^{-S_F(U,\psi,\bar{\psi})-\beta S_G(U)}$$
$$= \int \mathcal{D}U \;\det M(U) e^{-\beta S_G(U)}$$

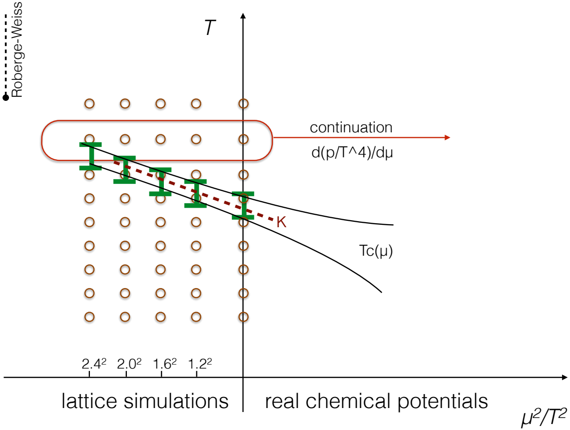
- For Monte Carlo simulations $\det M(U) e^{-\beta S_G(U)}$ is interpreted as Boltzmann weight
- If there is particle-antiparticle-symmetry $\det M(U)$ is real
- If $\mu^2 > 0$ $\det M(U)$ is complex

Dealing with the sign problem

- Reweighting techniques
- Canonical ensemble
- Complex Langevin
- Lefshetz Thimble
- Density of state methods
- Dual variables
- ...

- Taylor expansion $\longrightarrow$ [Bazavov et al., Bazavov:2017dus], [Bonati et al., Bonati:2018nut]
- Imaginary μ

Analytic continuation



Common technique:

- [deForcrand:2002hgr]
- [Bonati:2015bha]
- [Cea:2015cya]
- [DElia:2016jqh]
- [Bonati:2018nut]
- ...

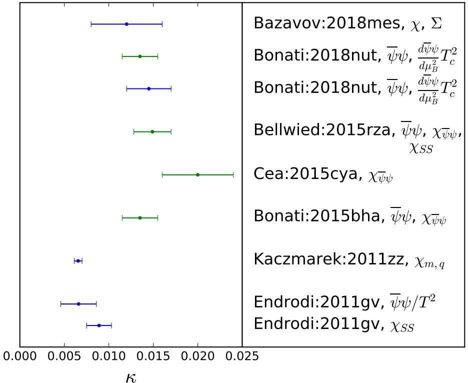
Current status

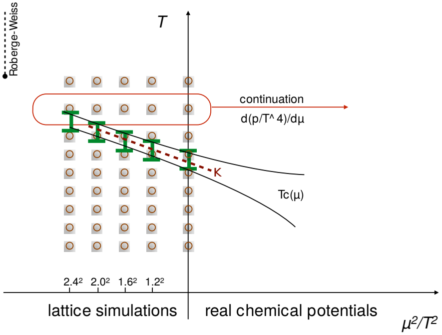
Curvature function:

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T_c}\right)^2 + \mathcal{O}(\mu_B^4)$$

$$C_1(\hat\mu) = 1 + a\hat\mu^2 + b\hat\mu^4, \quad C_2(\hat\mu) = \frac{1 + a\hat\mu^2}{1 + b\hat\mu^4}$$

$$C_3(\hat\mu) = \frac{1}{1 + a\hat\mu^2 + b\hat\mu^4}, \quad \hat\mu = \frac{\mu_B}{T}$$



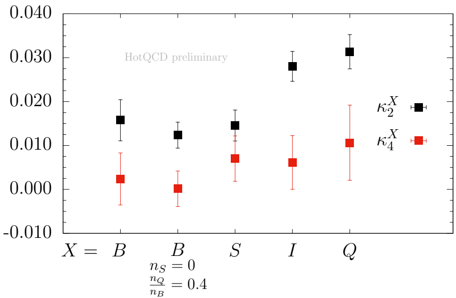


Current status

Curvature function:

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_c}\right)^4 + \mathcal{O}(\mu_B^6)$$

[Steinbrecher:2018phh]



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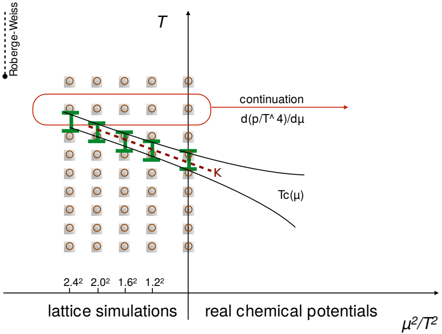

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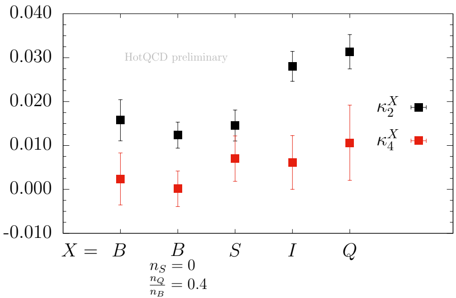

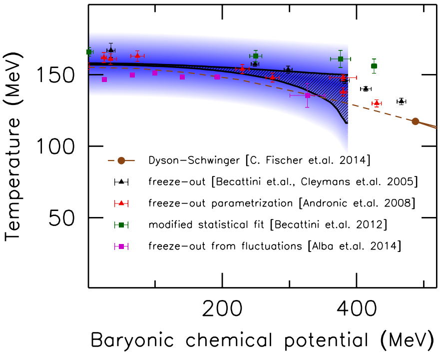

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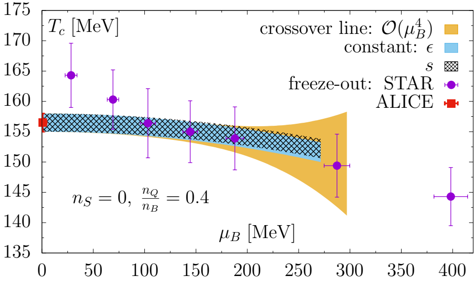
[Bazavov:2018mes]



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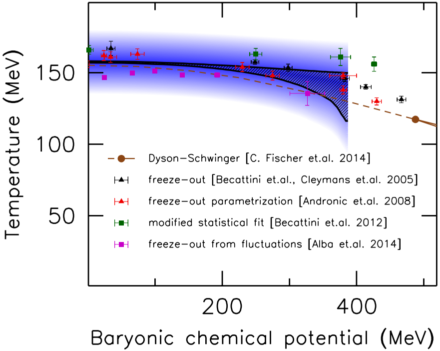
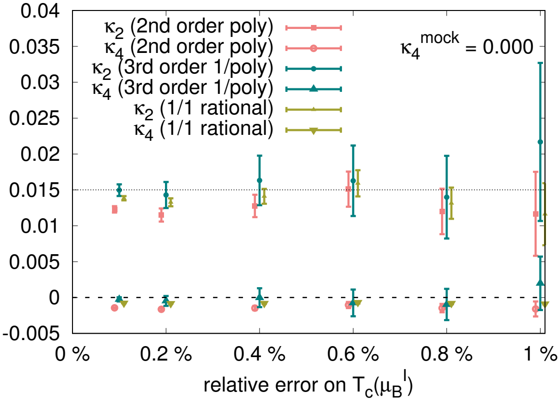


Mock-Analysis



$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_c}\right)^4$$

When reducing the error on T_c we need higher order fit functions for reliable results.

Choice: $C_1(\hat{\mu}) = 1 + a\hat{\mu}^2 + b\hat{\mu}^4 + c\hat{\mu}^6$
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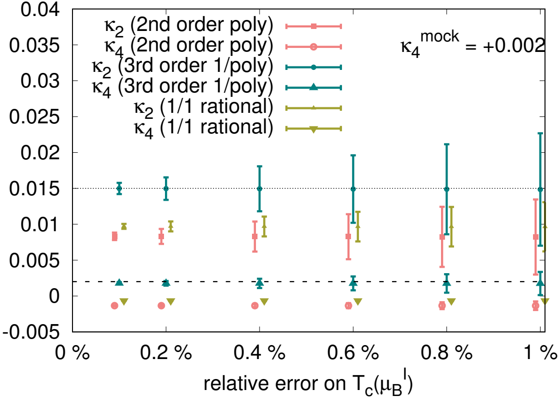

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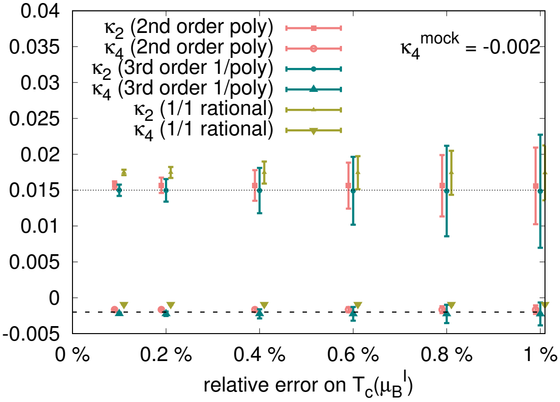

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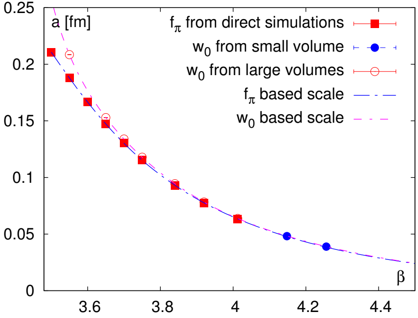
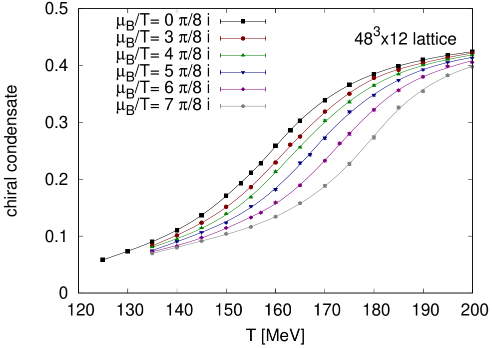
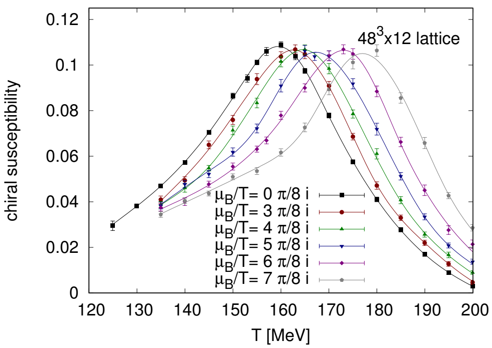
Actual-Analysis



- Action: tree-level Symanzik improved gauge action, with four times stout smeared staggered fermions
- 2+1+1 flavour, on LCP with pion and kaon mass
- Simulation at $\langle n_S \rangle = 0$ and $\langle n_Q \rangle = 0.4 \langle n_B \rangle$
- Continuum extrapolation from lattice sizes: $40^3 \times 10$, $48^3 \times 12$ and $64^3 \times 16$
- $\frac{\mu_B}{T} = \mathrm{i}\frac{j\pi}{8}$ with $j = 0, 3, 4, 5, 6, 6.5$ and 7
- Two methods of scale setting: f_π and w_0, $Lm_\pi > 4$

Observables





Chiral condensate:

$$\langle\bar{\psi}\psi\rangle = \frac{T}{V}\frac{\partial \ln Z}{\partial m_q}$$

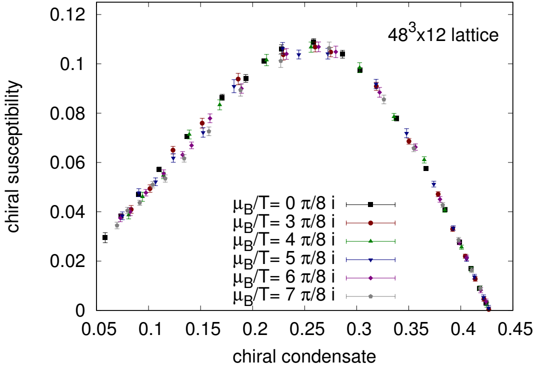
$$\langle\bar{\psi}\psi\rangle^r = \left(\langle\bar{\psi}\psi\rangle(0,\beta) - \langle\bar{\psi}\psi\rangle(T,\beta)\right)\frac{m_l}{m_\pi^4}$$

Chiral susceptibility:

$$\chi_{\bar{\psi}\psi} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial (m_q)^2}$$

$$\chi^r_{\bar{\psi}\psi} = \left(\chi_{\bar{\psi}\psi}(T,\beta) - \chi_{\bar{\psi}\psi}(0,\beta)\right)\frac{m_l^2}{m_\pi^4}$$

$\chi_{\bar\psi\psi}(\langle\bar\psi\psi\rangle)$

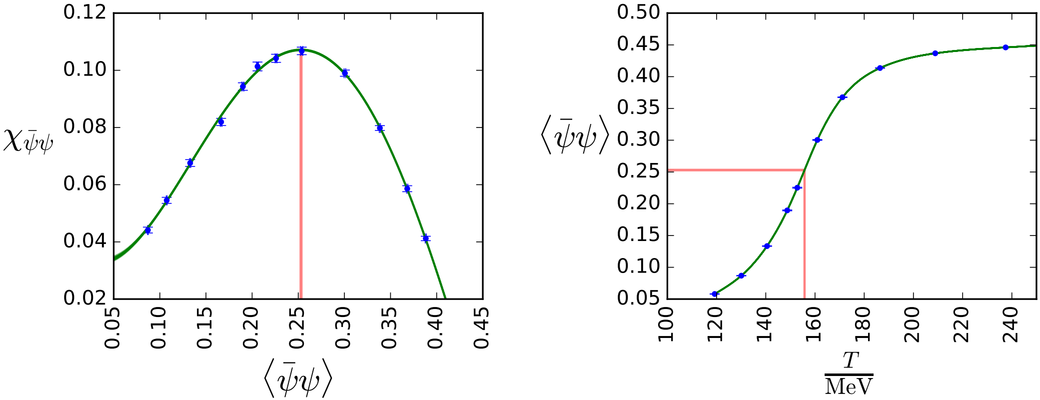


$$\chi_{\bar\psi\psi}(\langle\bar\psi\psi\rangle) = \sum_{i=0}^{n} \alpha_i \left(1 + \beta_i \left(\frac{\mu_B}{T}\right)^2\right) \langle\bar\psi\psi\rangle^i,$$

$$n \in \{2, 3, 4\}$$

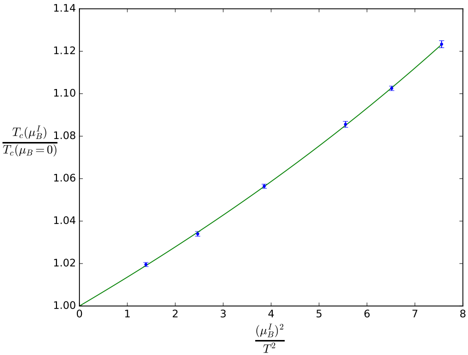
Fitting $\chi_{\bar\psi\psi}(\langle\bar\psi\psi\rangle)$ removes most of the μ_B dependence and allows for a price determination of the transition value of $\langle\bar\psi\psi\rangle$. In a next step this has to be translated into temperature.

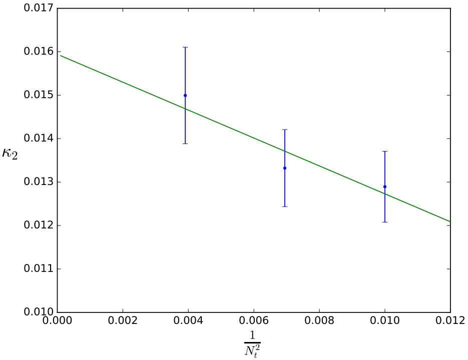
$\langle\bar{\psi}\psi\rangle(T)$



To determine the temperature from the the $\langle\bar{\psi}\psi\rangle$ value we use a spline.

Extrapolation

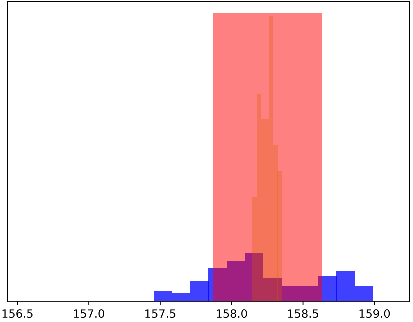




We perform a combined extrapolation in $\frac{\mu_B^2}{T^2}$ and to the continuum, which uses a fully correlated fit.

Systematic Errors

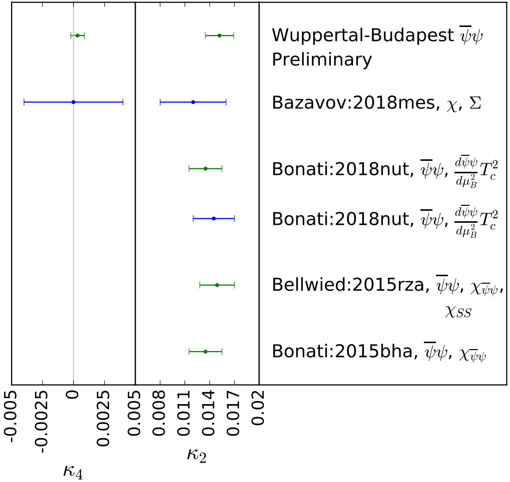
- 2 renormalization fits for $\langle\bar{\psi}\psi\rangle$
- 2 renormalization fits for $\chi_{\bar{\psi}\psi}$
- 4 cuts in $\chi_{\bar{\psi}\psi}$
- 3 fits for $\chi_{\bar{\psi}\psi}(\langle\bar{\psi}\psi\rangle)$
- 2 two choices of nodepoints for the spline
- 2 function for the extrapolation
- including or discarding the highest μ_B^I
- scale setting either with w_0 or f_π



histogram for $T_c(\mu_B = 0)$

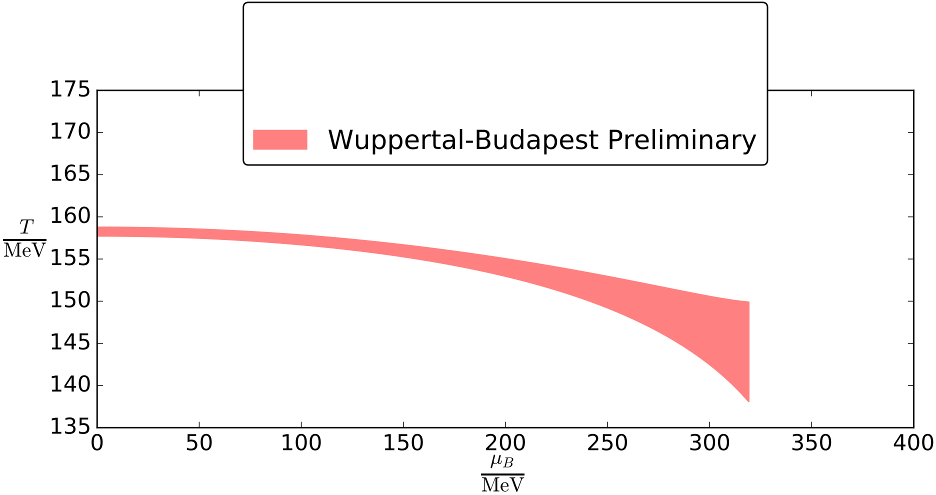
In total we perform 764 analysis. We weight every result with a $Q > 0.1$ uniformly

Result

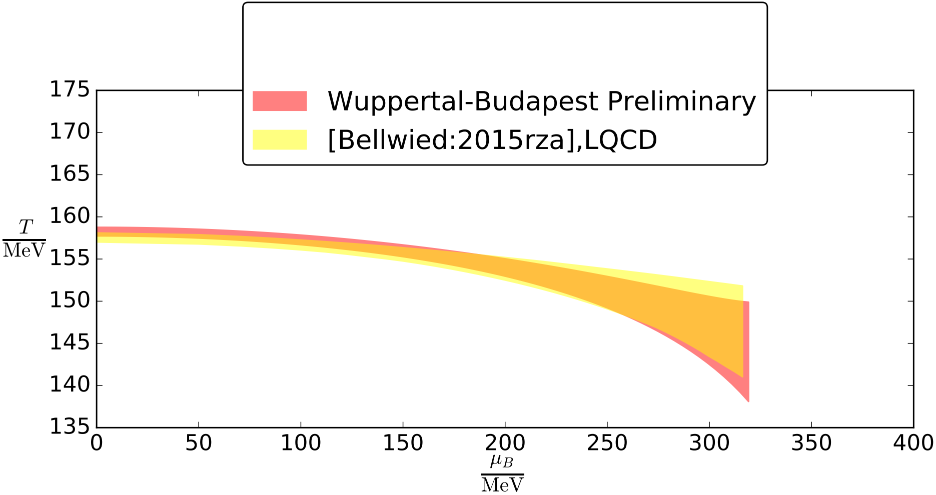


$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_c}\right)^4 + \mathcal{O}(\mu_B^6)$$

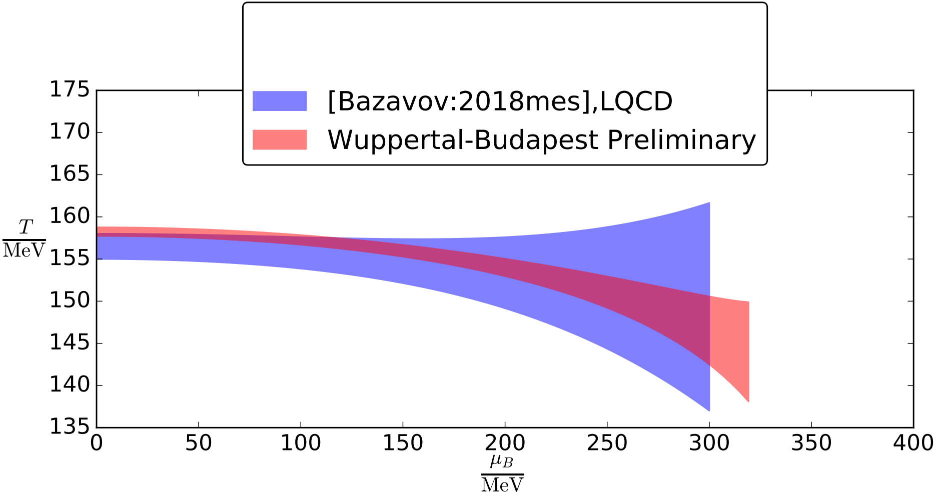
Extrapolation



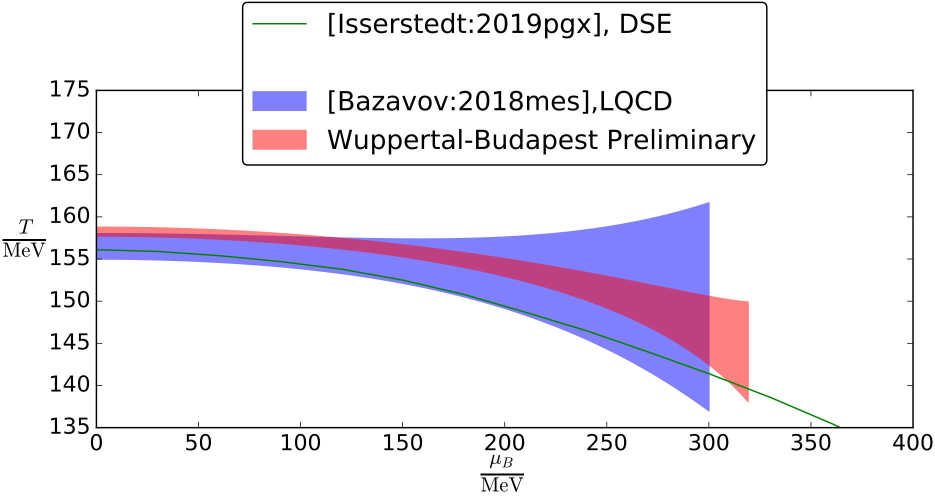
Extrapolation



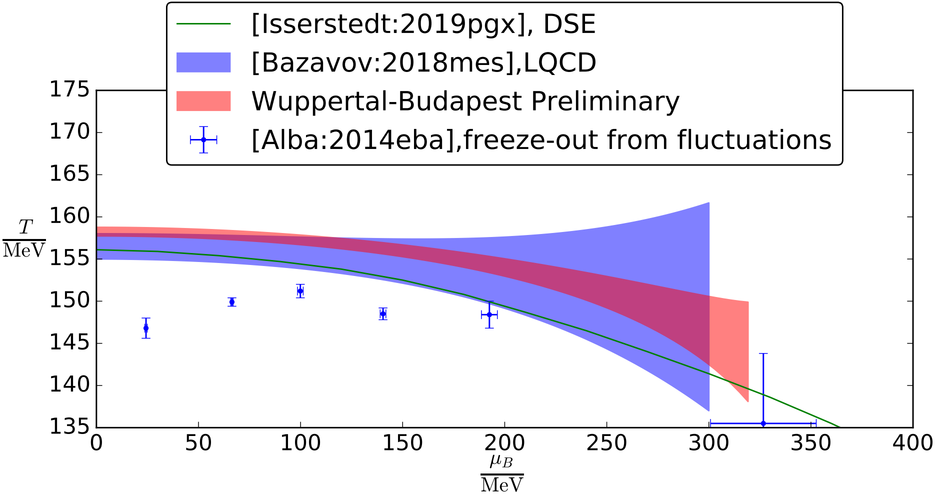
Extrapolation



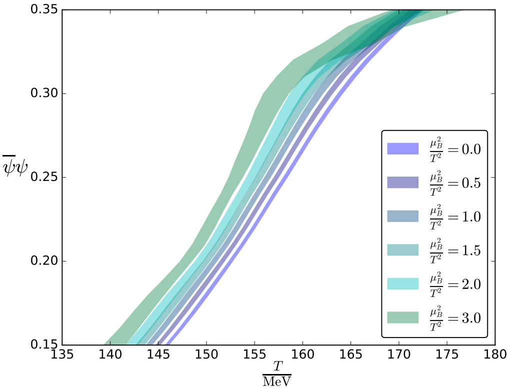
Extrapolation

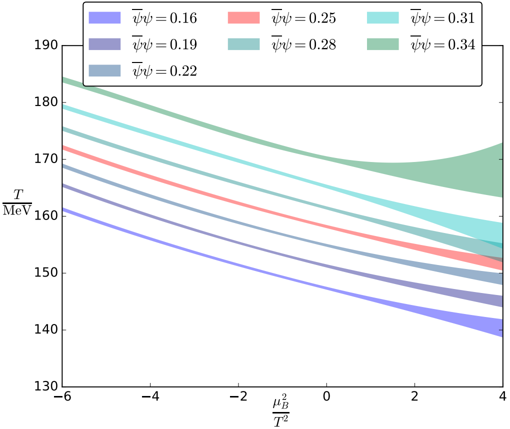


Extrapolation



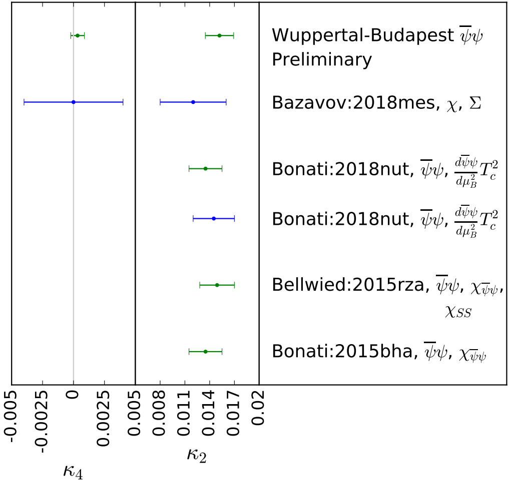
Outlook



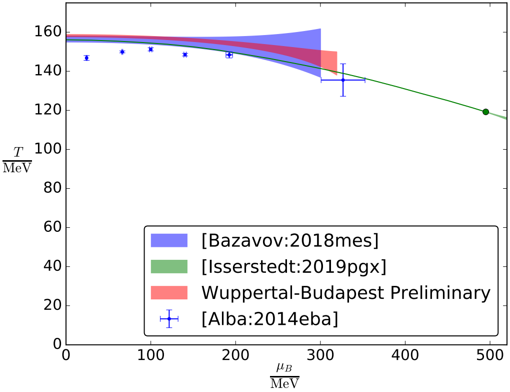


For more information please visit the poster by Ruben Kara.

Summary



Wuppertal-Budapest $\overline{\psi}\psi$
Preliminary
Bazavov:2018mes, χ, Σ
Bonati:2018nut, $\overline{\psi}\psi$, $\frac{d\overline{\psi}\psi}{d\mu_B^2}T_c^2$
Bonati:2018nut, $\overline{\psi}\psi$, $\frac{d\overline{\psi}\psi}{d\mu_B^2}T_c^2$
Bellwied:2015rza, $\overline{\psi}\psi$, $\chi_{\overline{\psi}\psi}$, χ_{SS}
Bonati:2015bha, $\overline{\psi}\psi$, $\chi_{\overline{\psi}\psi}$
-0.005
-0.0025
0
0.0025
0.005
κ_4
0.008
0.011
0.014
0.017
0.02
κ_2



$\frac{T}{\mathrm{MeV}}$
160
140
120
100
80
60
40
20
0
100
200
300
400
500
$\frac{\mu_B}{\mathrm{MeV}}$
[Bazavov:2018mes]
[Isserstedt:2019pgx]
Wuppertal-Budapest Preliminary
[Alba:2014eba]