Non-thermal behavior in the Lifshitz regime

Lifshitz regime ~ Overhauser-Migdal condensate (OM = chiral spirals):

RDP, V. Skokov & A. Tsvelik, 1801.08156

RDP, F. Rennecke, V. Skokov, S. Valgushev, & A. Tsvelik, 1912.....

Critical region *tiny*, Lifshitz regime is *large:*

Wei-Jie Fu, J. M. Pawlowski, & Fabian Rennecke, 1909.02991

Here: *cartoons* of theory, concentrate on a possible experimental signal

Flucuations at *non*-zero momentum can be large *before* OM condensates emerge

At (relatively) high T, low μ!

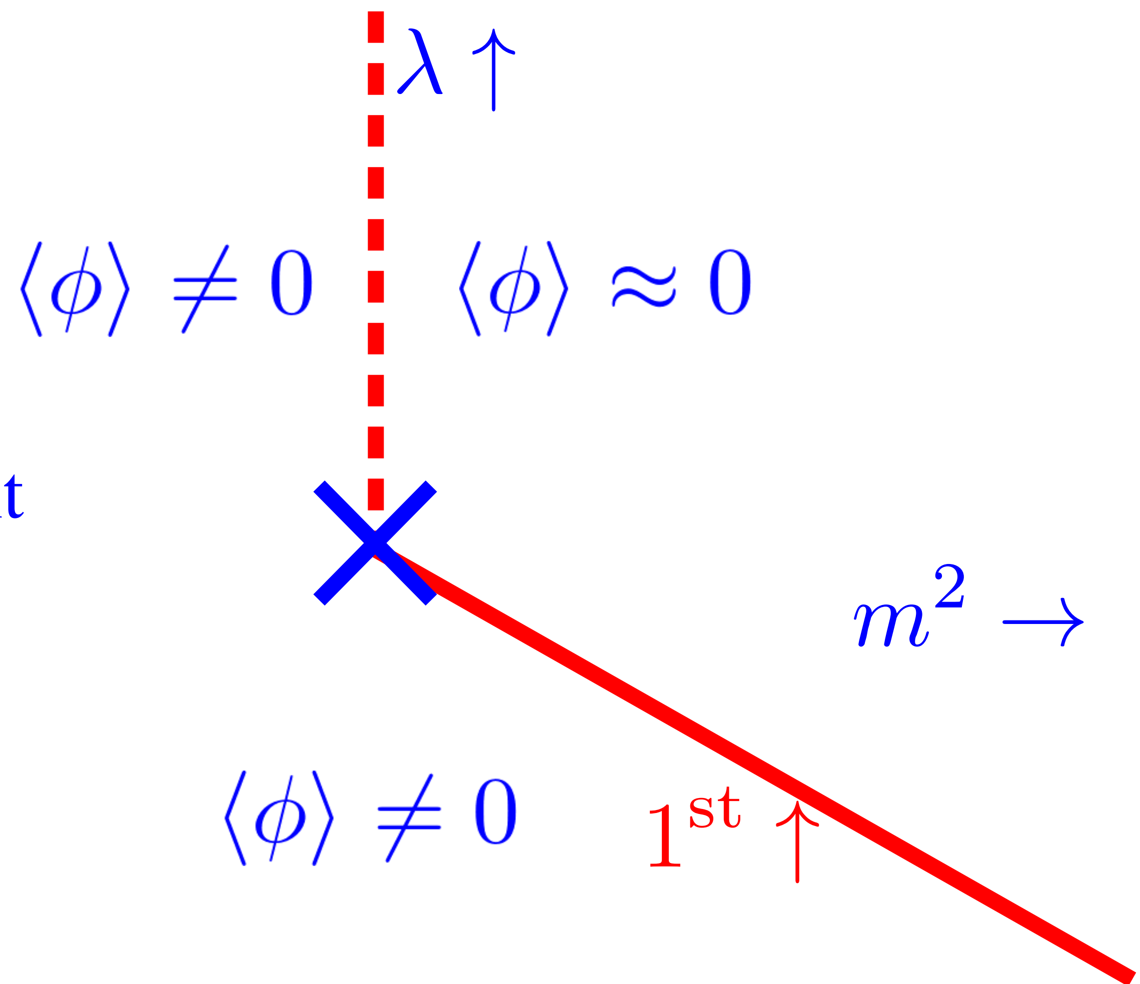
Usual mean field theory

$$\mathcal{L} = (\partial_\mu \phi)^2 - h\phi + m^2\phi^2 + \lambda\phi^4 + \kappa\phi^6$$

$m^2 \approx 0$, $\lambda > 0$: crossover

$m^2 \approx \lambda \approx 0$: critical endpoint

$m^2 > 0$, $\lambda < 0$: 1st order



Plot out phase diagram by varying m^2 & λ. Ok in mean field theory

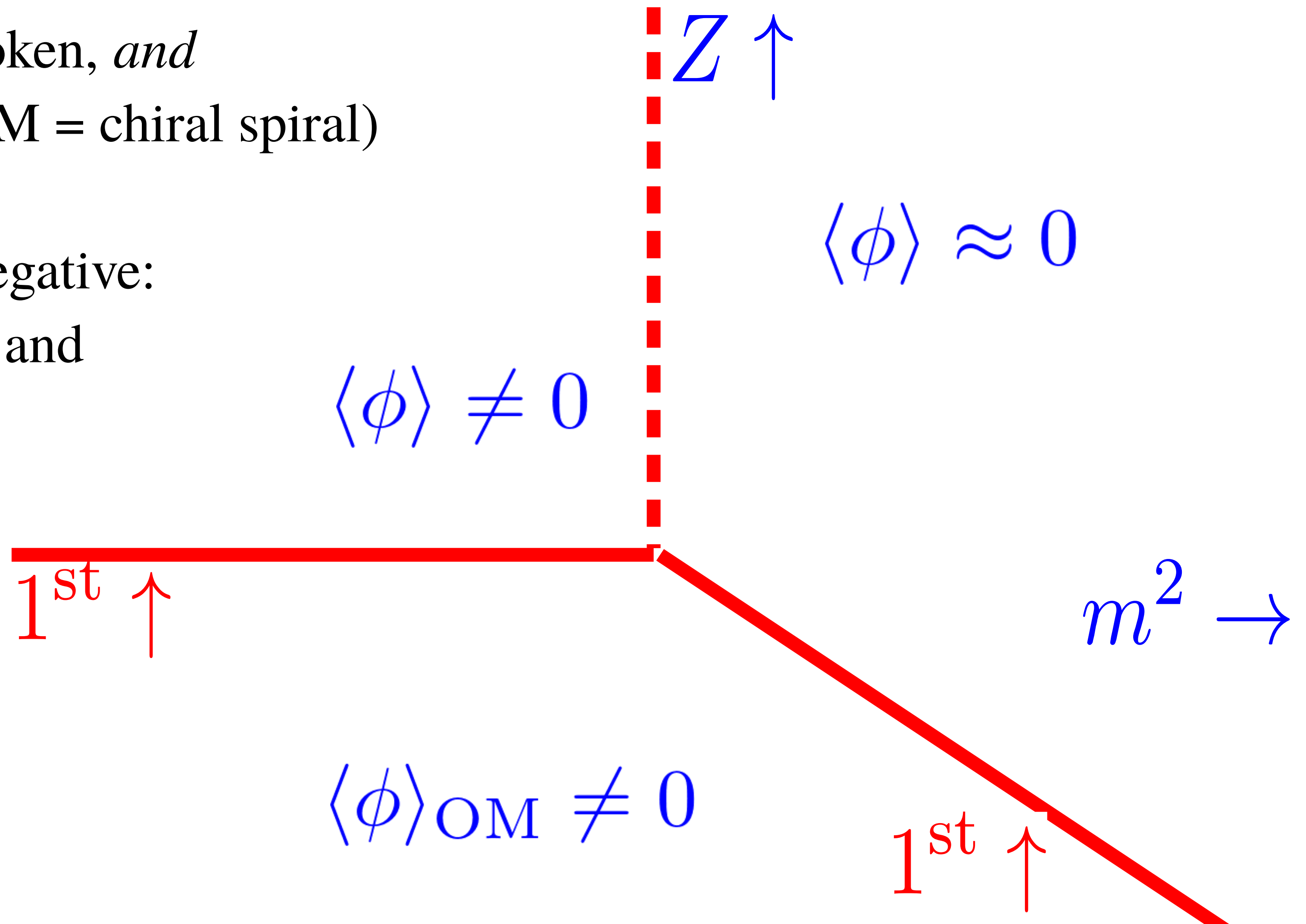
Lifshitz mean field theory

Time derivatives must be positive (causality), spatial derivatives *not*:

$$\mathcal{L} = (\partial_0 \phi)^2 + Z(\partial_i \phi)^2 + \frac{1}{M^2}(\partial_i^2 \phi)^2 - h\phi + m^2\phi^2 + \lambda\phi^4 + \kappa\phi^6$$

Phases: symmetric, broken, *and*
Overhauser-Migdal (OM = chiral spiral)

Both Z and λ can go negative:
Lifshitz regime, Z < 0, and
critical endpoint



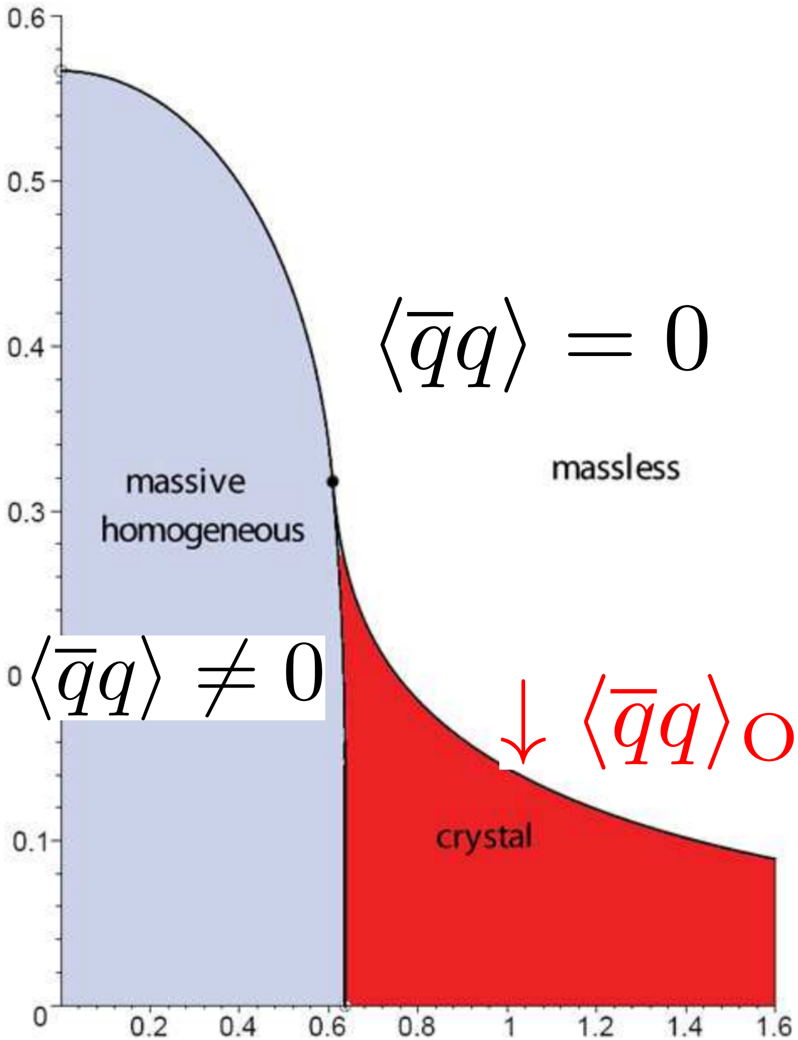
OM condensates are *everywhere* in 1+1 dim.'s

In 1+1 dim's, $\mu \neq 0$ can turn constant condensate into OM.

Gross-Neveu type models soluble for large number of flavors, N_f:

Basar, Dunne & Thies, 0903.1868; Dunne & Thies 1309.2443+ ...

$T \uparrow$



$\langle \overline{q} q \rangle = 0$

$\langle \overline{q} q \rangle \neq 0$

$\downarrow \langle \overline{q} q \rangle_{\mathrm{OM}} \neq 0$

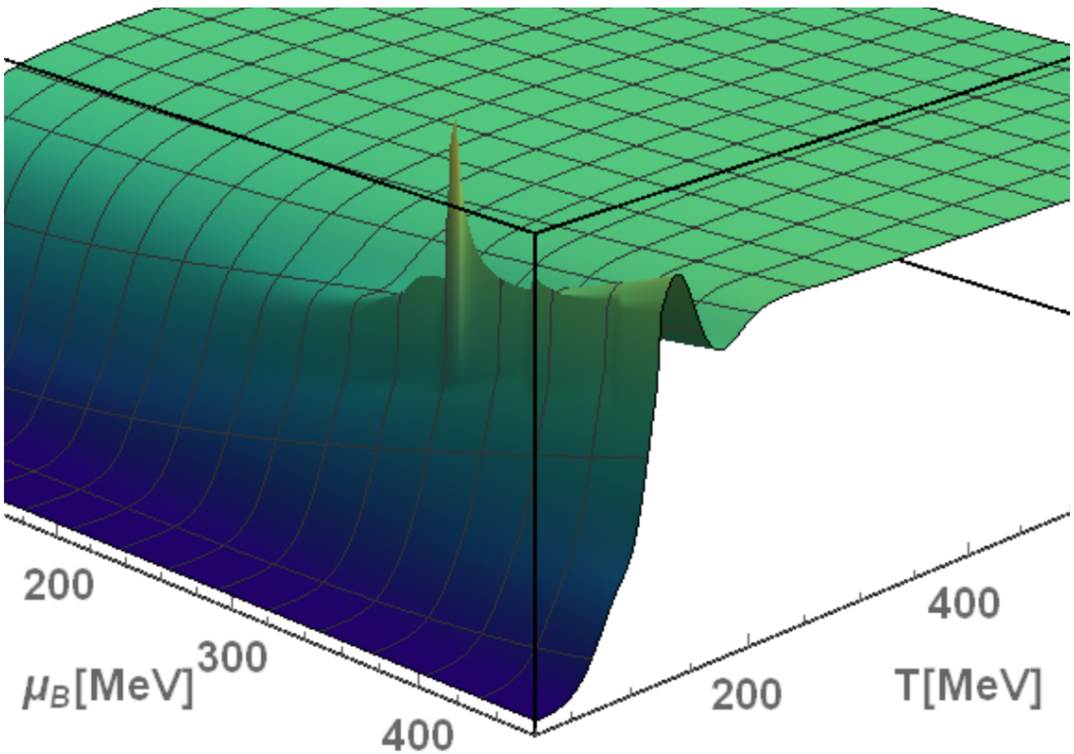
$\mu \rightarrow$

exact solutions,

$N_f = \infty$:

Width of critical region?

Parotto, Bluhm, Mroczek, Nahrgang, Noronha-Hostler, Rajagopal, Ratti, Schaefer, Stephanov, 1805.05249



width of critical region:
w=1 in Eqs. 3.1 & 3.2
w=1?

Wide critical region consistent with lattice about μ=0.
But is it wide?

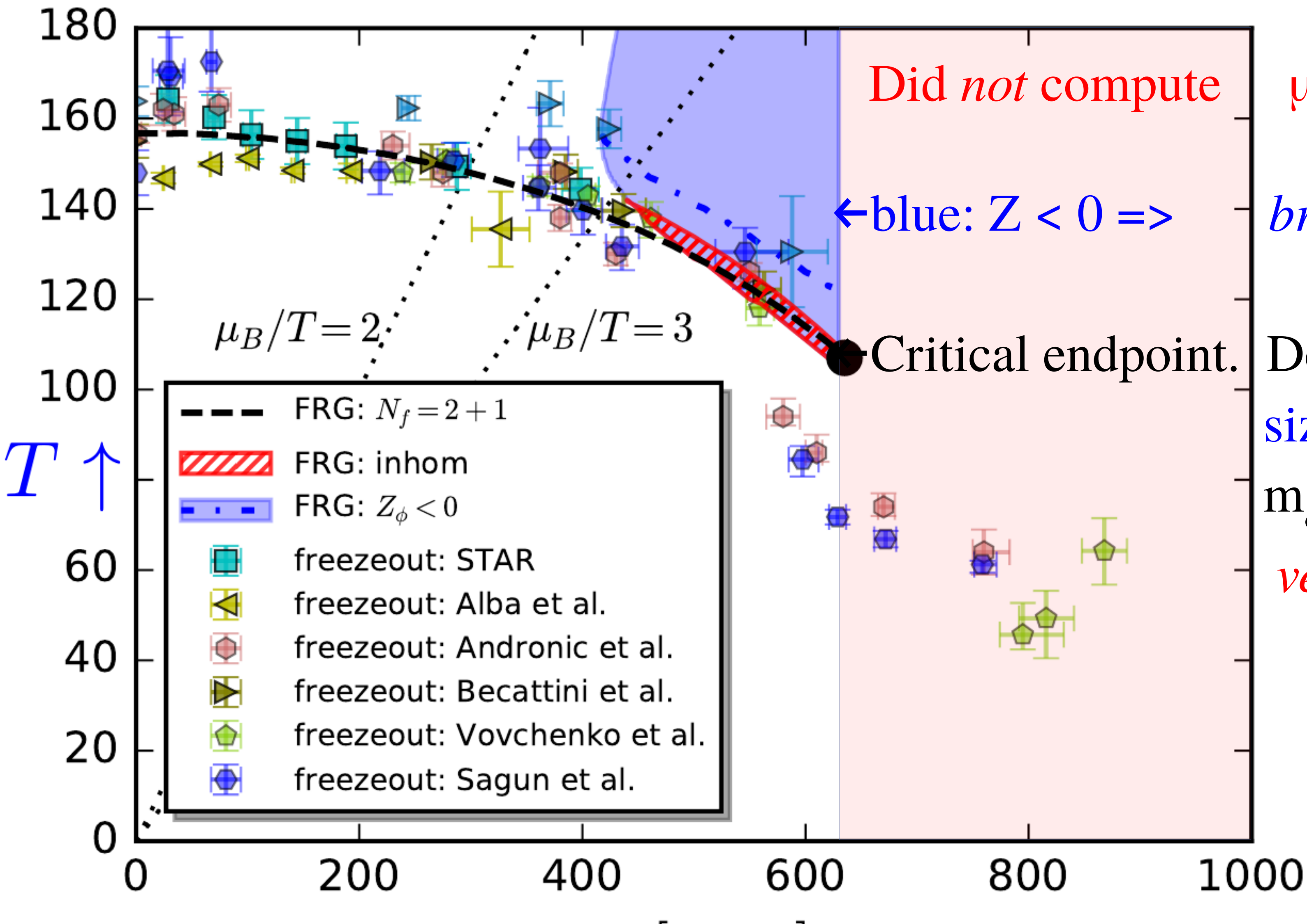
Vovchenko, Steinheimer, Philipsen, Stoecker, 1711.01261:
No critical endpoint up to $\mu_B > \pi T$.

Phase diagram from FRG

Fu, Pawlowski, & Rennecke, 1909.02991

Functional Renormalization Group applied to QCD @ T & μ_B.

Lifshitz regime broad, critical region tiny.



$\mu > \mu_{crit} \sim 620$ MeV

broad Lifshitz region

Dot ≈

size of critical region

$m_\sigma < m_\pi$:

very narrow!

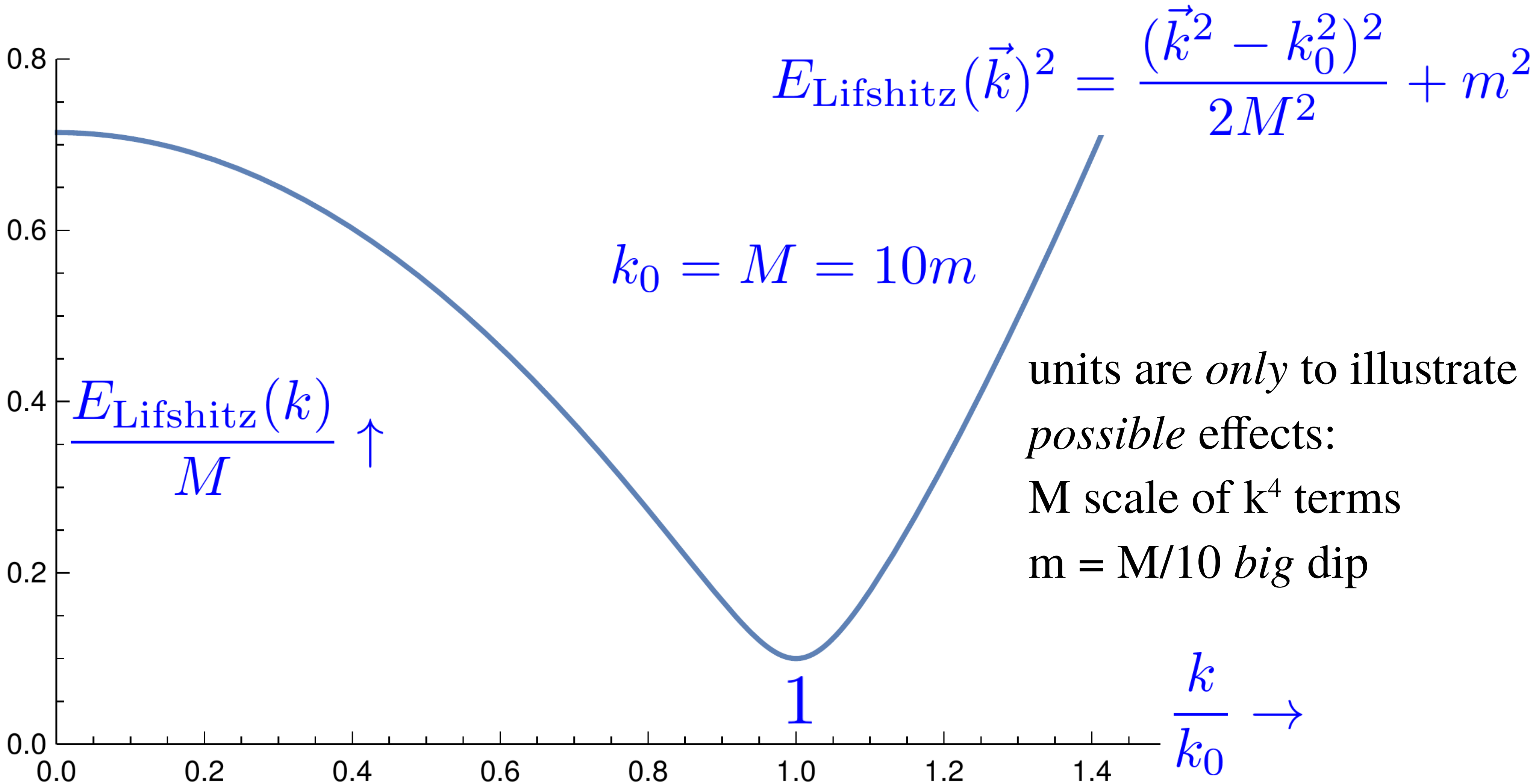
fig. 21, pg. 34.

Cartoon of dispersion relation in the Lifshitz regime

Lifshitz regime: Z < 0 => minimum of energy at *non*-zero momentum, k_0.

Overhauser-Migdal condensate when curve hits zero, $E_{\text{Lifshitz}}(k_0) = 0$.

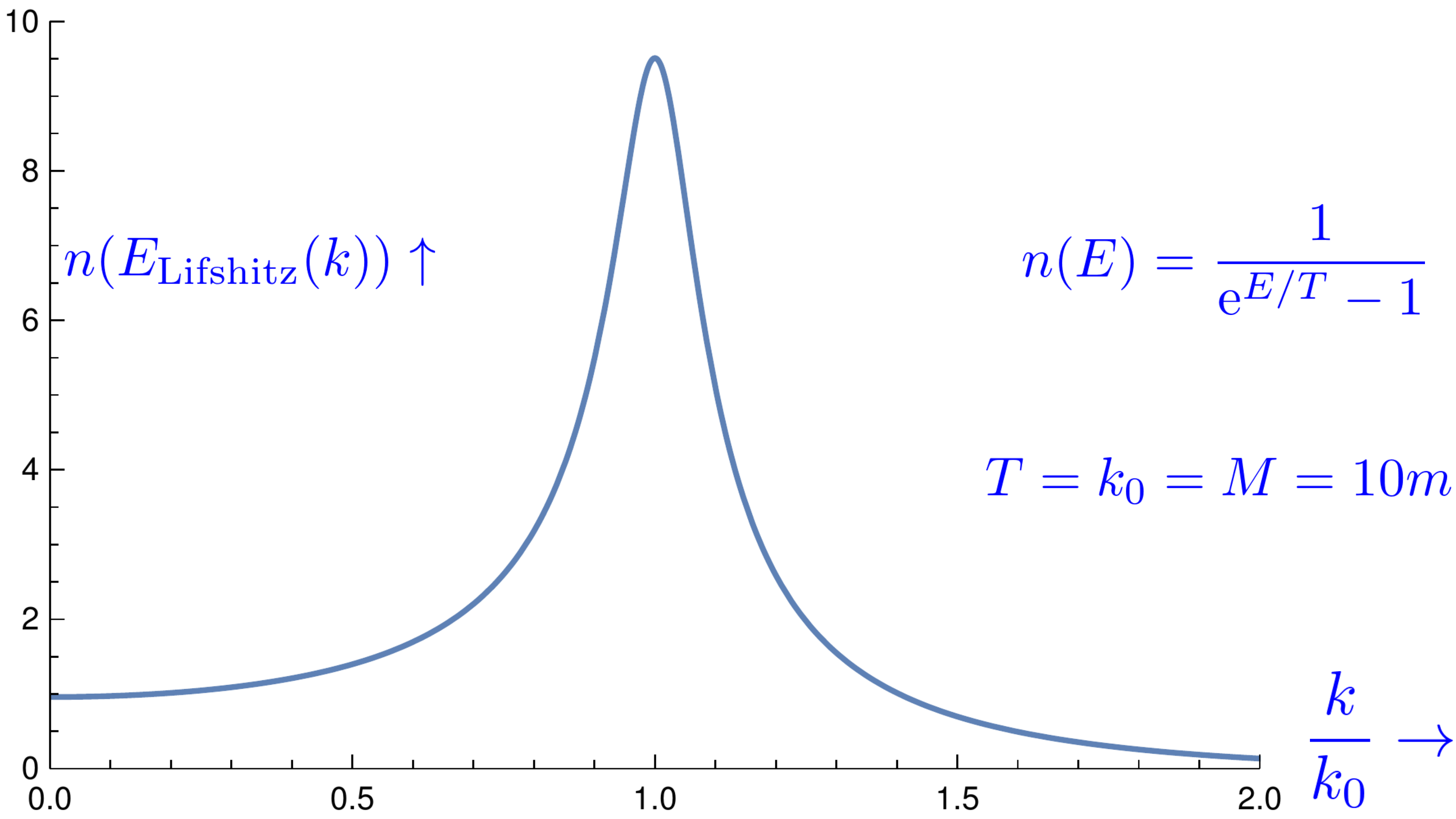
But interesting even when $E_{\text{Lifshitz}}(k_0) > 0$!



Cartoon of Lifshitz statistical distributution function

Lifshitz regime: Z < 0 => minimum of energy at *non*-zero momentum, k_0.

Hence peak of $n(E_{Lifshitz}(k))$ at k_0: *possibly*, non-thermal behavior can be *large*.

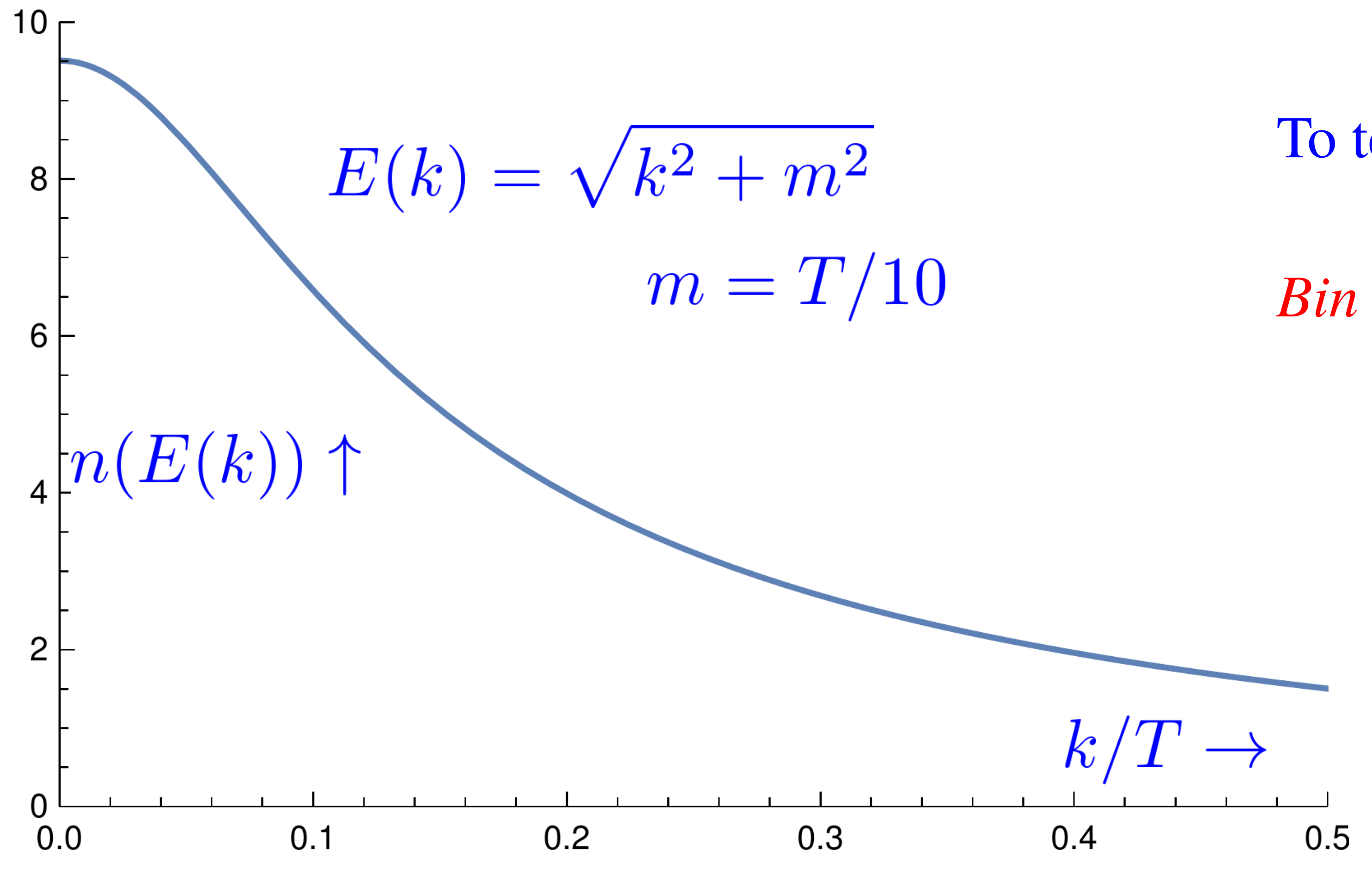


Cartoon of Lifshitz statistical distribututation function

Fluctuations in Lifshitz regime greatest at *non*-zero momentum.

Fluctuations near critical endpoint greatest at *zero* momentum.

Usual χ_2, χ_4, χ_6... integral over momenta.

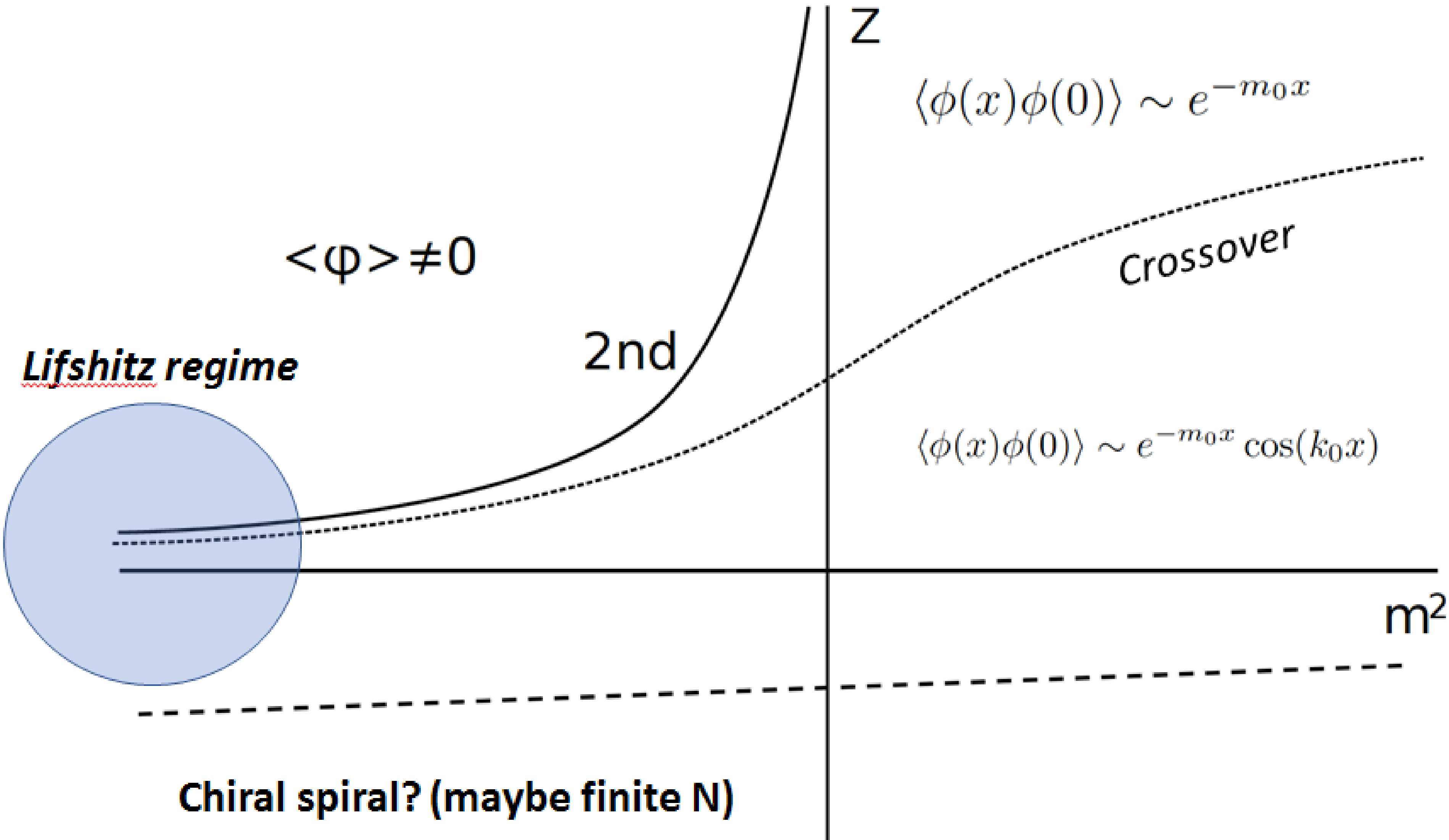


To test Lifshitz regime:

Bin in momenta!

Soluble model at infinite N

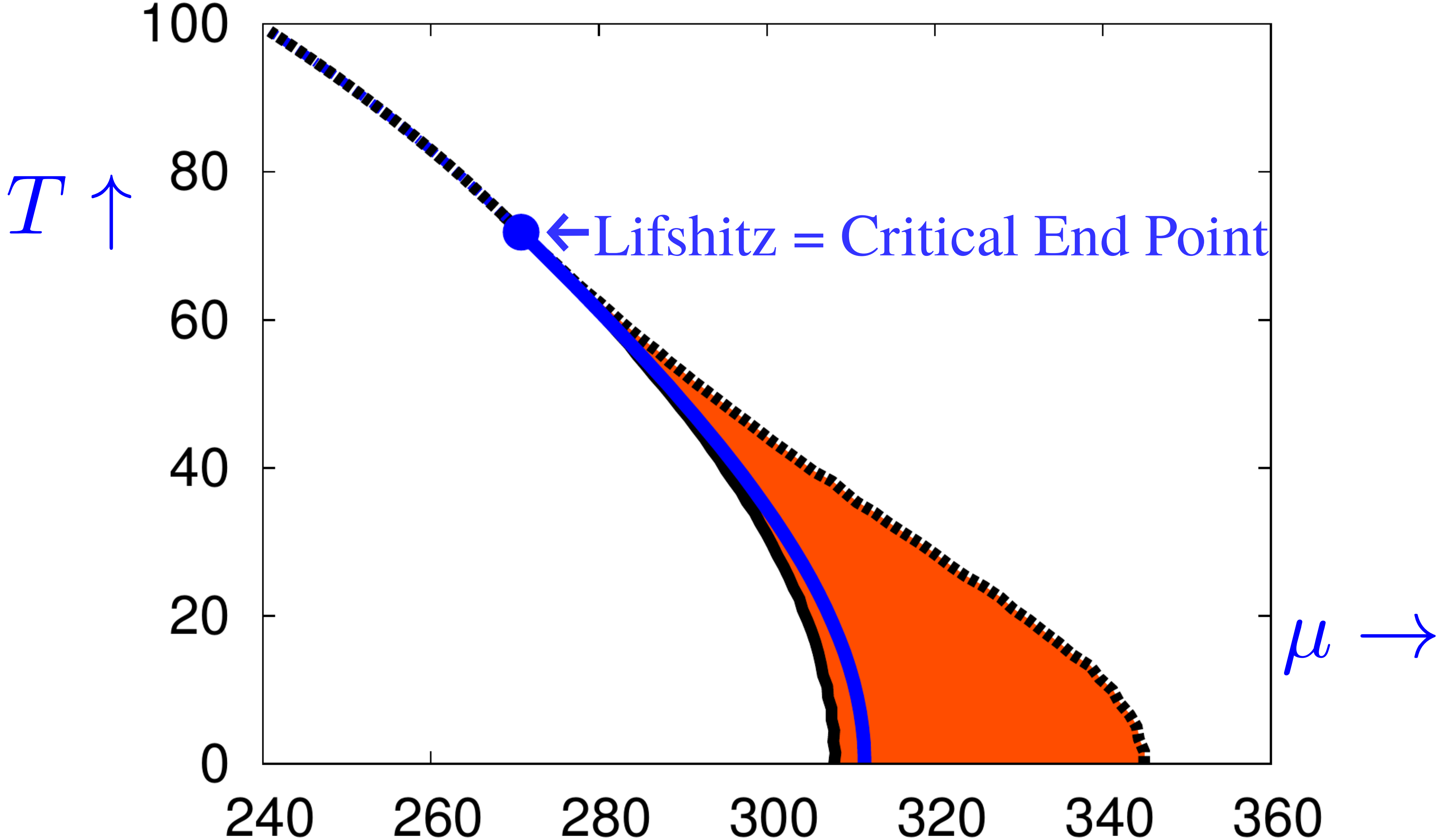
O(N) scalar model soluble at infinite N. Interesting phase structure in Z & m2.
Numerical simulations at small N by Monte Carlo.
Clear signal for Z < 0: oscillatory behavior in 2-point function.



Chiral Spirals in 3+1 dimensions

In 3+1, *common* in NJL models: Nickel, 0902.1778 + ...Buballa & Carignano 1406.1367 + ...

In reduction to 1-dim, $\Gamma_5^{\text{1-dim}} = \gamma_0\gamma_z$, so chiral spiral between $\overline{q}q$ & $\overline{q}\gamma_0\gamma_z\gamma_5 q$



Chiral spirals in 1+1 dimensions

In 1+1 dim., can eliminate μ by chiral rotation:

$$q' = \mathrm{e}^{i\mu z \Gamma_5} q \ , \ \overline{q}(\not{D} + i\mu\Gamma_0)q = \overline{q}' \ \not{D} q' \ , \ \Gamma_5 \Gamma_z = \Gamma_0$$

Thus a constant chiral condensate automatically becomes a chiral spiral:

$$\overline{q}' q' = \cos(2\mu z)\overline{q}q + i \sin(2\mu z)\overline{q}\gamma_5 q$$

Argument is only suggestive.

N.B.: anomaly ok, gives quark number: $\langle \overline{q} \Gamma_0 q \rangle = \mu/\pi$

Pairing is between quark & quark-hole,both at edge of Fermi sea.
Thus chiral condensate varies in z as ~ 2 μ.

Fluctuations at 7 GeV

Beam Energy Scan, down to 7 GeV.

Fluctuations *MUCH* larger when up to 2 GeV than to 0.8 GeV

Trivial multiplicity scaling? ... or Chiral Spiral?

But fluctuations in nucleons, not pions.

X. Luo & N. Xu, 1701.02105, fig. 37; Jowazee, 1708.03364

$\frac{c_4}{c_2}\uparrow$