Warm pions in real time

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Context

- Lattice constraints on real-time phenomena at finite temperature need some knowledge of spectral densities.
- Knowledge-free methods of using spectral densities have been explored (MEM, etc) with spectacular success in some cases.
- Theoretical constraints on spectral densities largely utilize weak-coupling methods, and have increased our understanding substantially. (AMY et sectatores)
- Most studies to date have concentrated on high-temperature properties, namely $T > T_c$. However, the fireball produced in heavy ion collisions spend more than half its life time in the hadronic phase. So understanding the warm hadronic plasma ($T_c > T > 0$) is important.
- This study utilizes an Effective Field Theory (EFT) approach to address the analytic continuation of lattice data in the warm region of the hadronic phase.

The EFT approach

EFTs are useful for describing physics where there are multiple length scales. We want to explain physics at length scales much longer than $1/T$ for T slightly less than T_c. Since all hadron masses other than m_π are significantly larger than T_c, long distance physics is likely to be dominated by pions.

We tested this by writing a relativistic EFT constrained only by chiral symmetry, spatial rotations and CPT (no boosts: heat bath chooses a special frame). This explains lattice measurements of long-distance pion and axial current correlators in the warm plasma.

Here we address the relation between pion rest mass in Minkowski space-time and the pion screening mass measured on the lattice: simplest problem of analytic continuation.

Lagrangian of EFT

Lagrangian is sum of the three terms

$$\mathcal{L}_3 = d^3 T_0 \overline{\psi}\psi \quad \mathcal{L}_4 = \overline{\psi}\partial\!\!\!/_4\psi + d^4\overline{\psi}\nabla\!\!\!/\psi \quad \mathcal{L}_6 = \sum_j \frac{d_j^6}{T_0^2}\overline{\psi}\Gamma_j\psi\,\overline{\psi}\Gamma_j\psi$$

1. Note that d^i are dimensionless. Dimensions provided by the UV cutoff of the EFT; here chosen to be T_0, the temperature of interest.
2. $\mathcal{L}_4$ and $\mathcal{L}_6$ symmetric under CPT and $SU(2)_L \times SU(2)_R$ chiral symmetry.
3. $\mathcal{L}_3$ breaks the axial part of this symmetry. The combination $T_0 d^3 = m_0$ is the quark mass.
4. At $T = 0$ full Lorentz symmetry would force $d^4 = 1$. When $d^4 \neq 1$ then there is a difference between the mass m_0 (pole mass) and the mass that can be deduced from the decay of static correlation functions (screening mass).

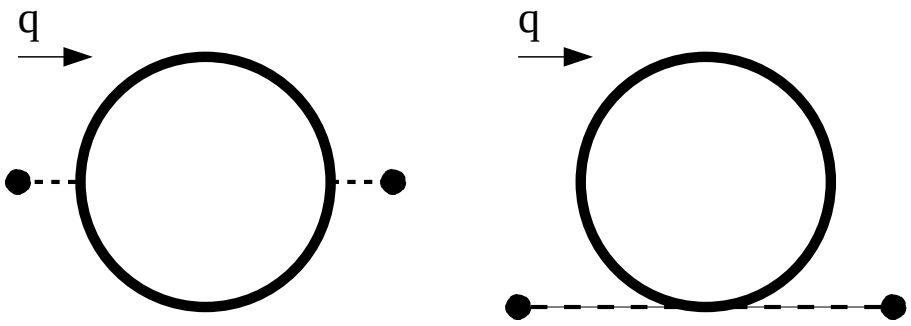
Pion correlators

Mean field theory has expected form, with a single coupling λ. All couplings fixed previously by matching lattice data in Euclidean. Error bands on parameters by error propagation from lattice. Use Minkowski version of the same Lagrangian.

Introduce small fluctuations about the mean field (pions) using a Hubbard-Stratanovich trick

$$\psi \to \exp\left[\frac{i\pi^a\tau^a\gamma^5}{2f}\right]\psi.$$

Expand in EFT Lagrangian to second order in π: gives coupled pion-quark action; overcounts degrees of freedom. Integrate out the quarks to get (retarded) pion correlator.



Matching to a pion theory

The correlator is expanded in powers of q_0 and $\mathbf{q}$. The expression, when only quadratic terms are retained, can be matched to the quadratic Lagrangian,

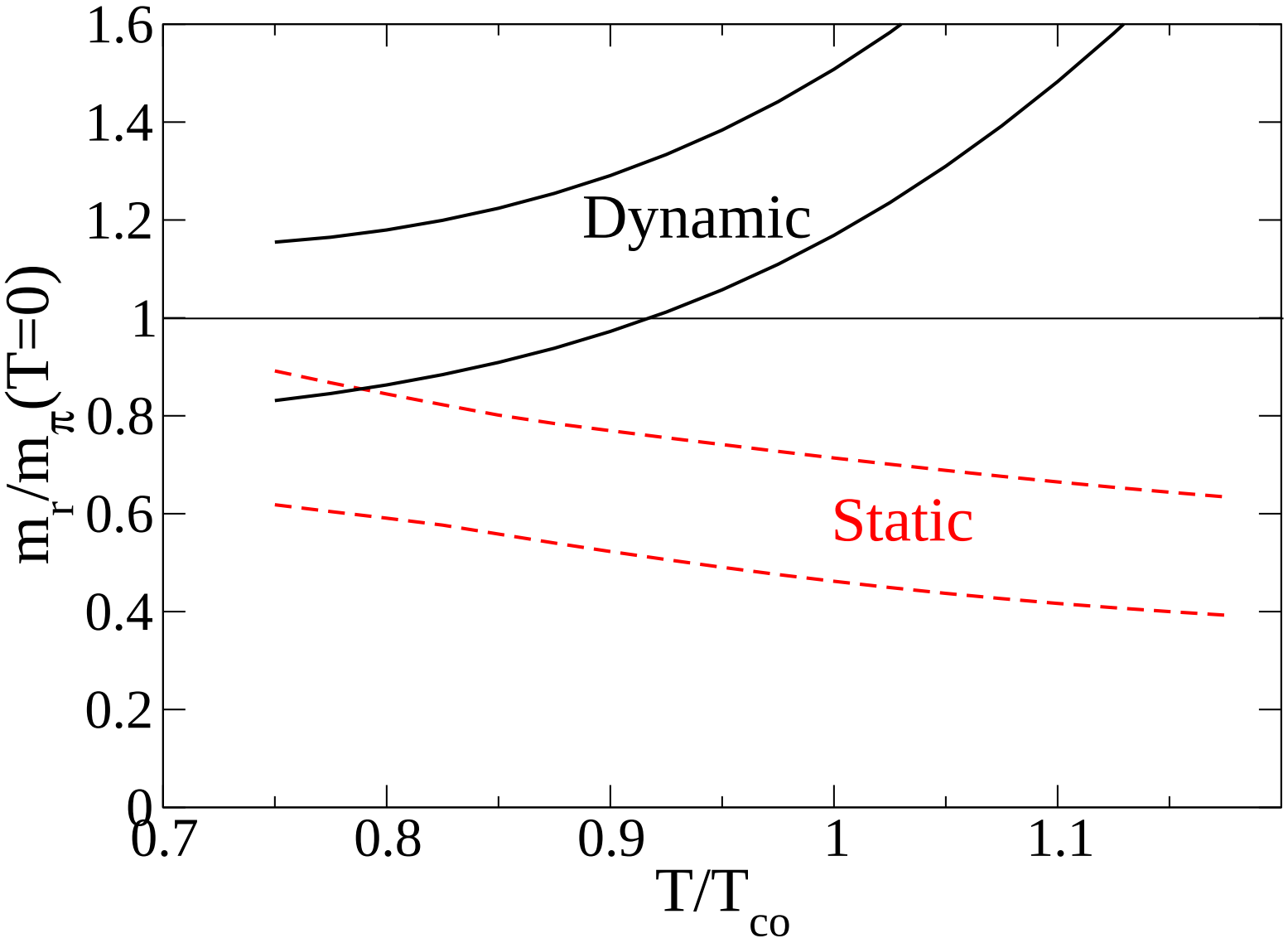
$$\mathcal{L}_\pi = \frac{1}{2} m_\pi^2 \pi^2 - \frac{1}{2}(\partial_0 \pi)^2 + \frac{1}{2}\, u_\pi^2 (\nabla \pi)^2$$

The choice of the coefficient of $(\partial_0 \pi)^2$ defines f_π. The low-momentum limit of loop integrals depends on the order of limits.

The static correlators involve space-like external momenta q: take $q_0 = 0$ first, then $\mathbf{q} \to 0$. The values of m_π, f_π, and u_π are the same as in the Euclidean. Sanity check!

Dynamical phenomena involve time-like external momentum q: take $\mathbf{q} = 0$ first and then the long-time limit $q_0 \to 0$. Then m_π, f_π and u_π turn out to be quite different.

Main results



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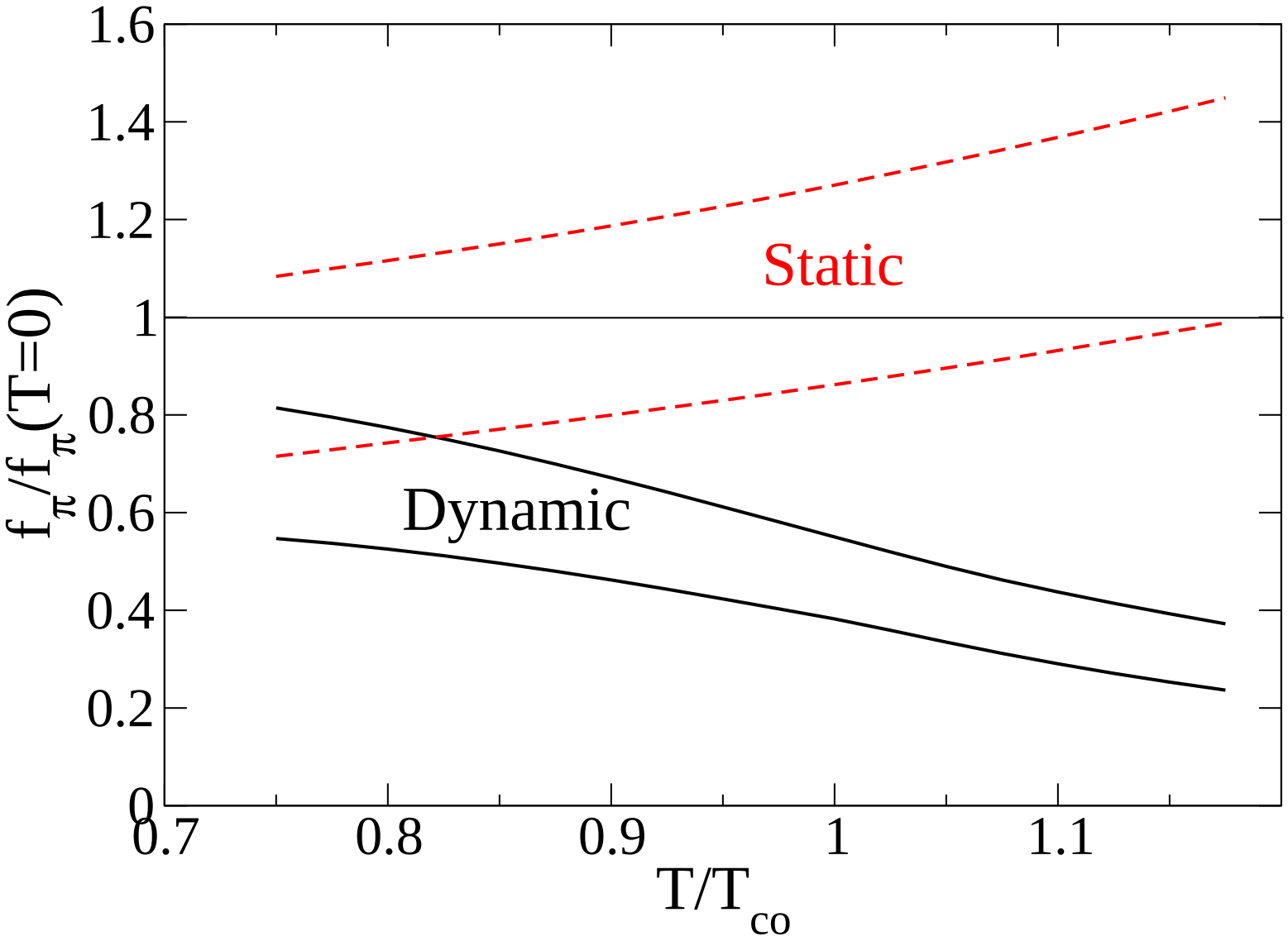

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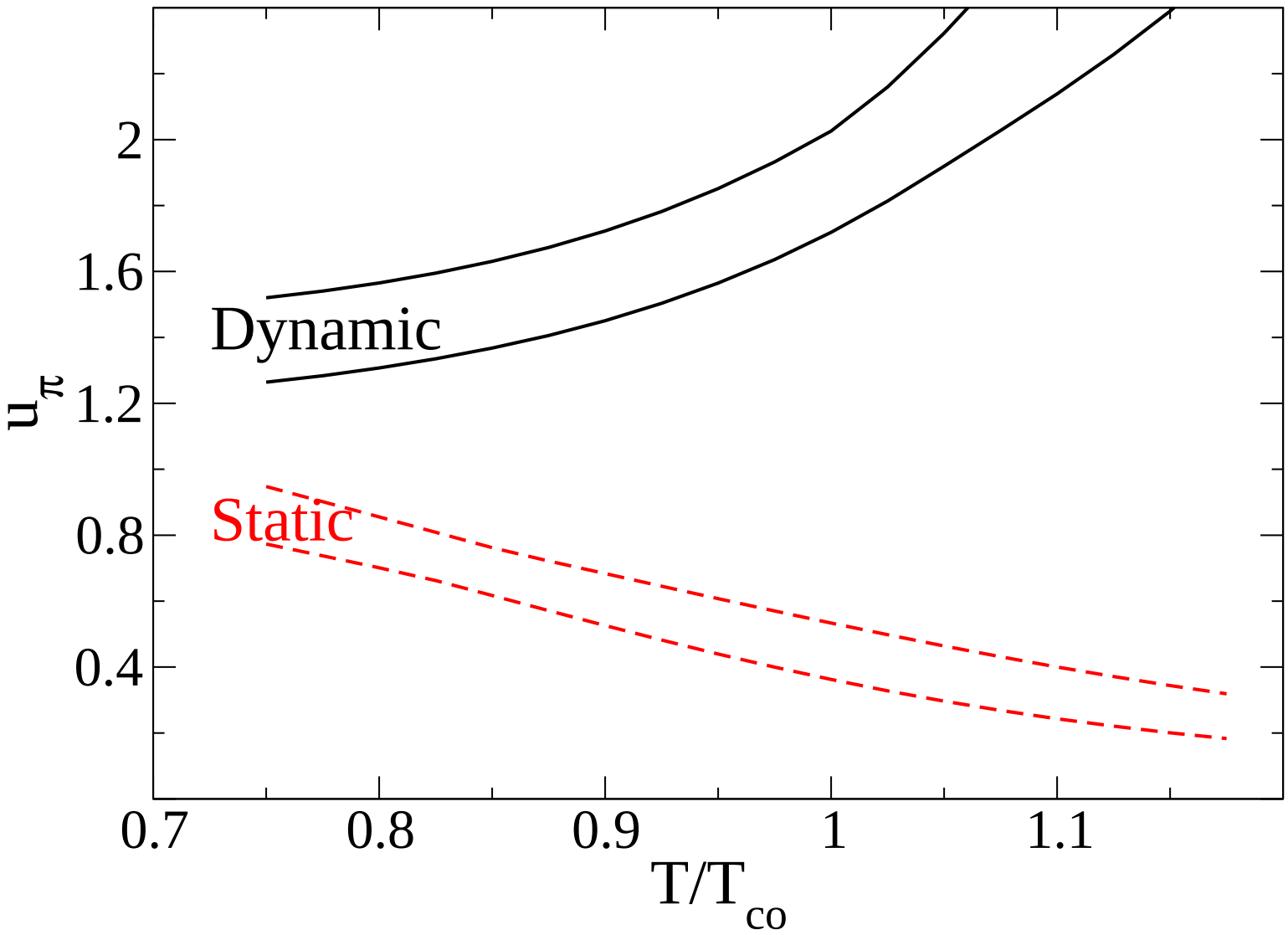


Dispersion relations

Near the dynamic limit, if one considers propagation of spatially inhomogenous fields with slow spatial variation, then the dispersion relation is

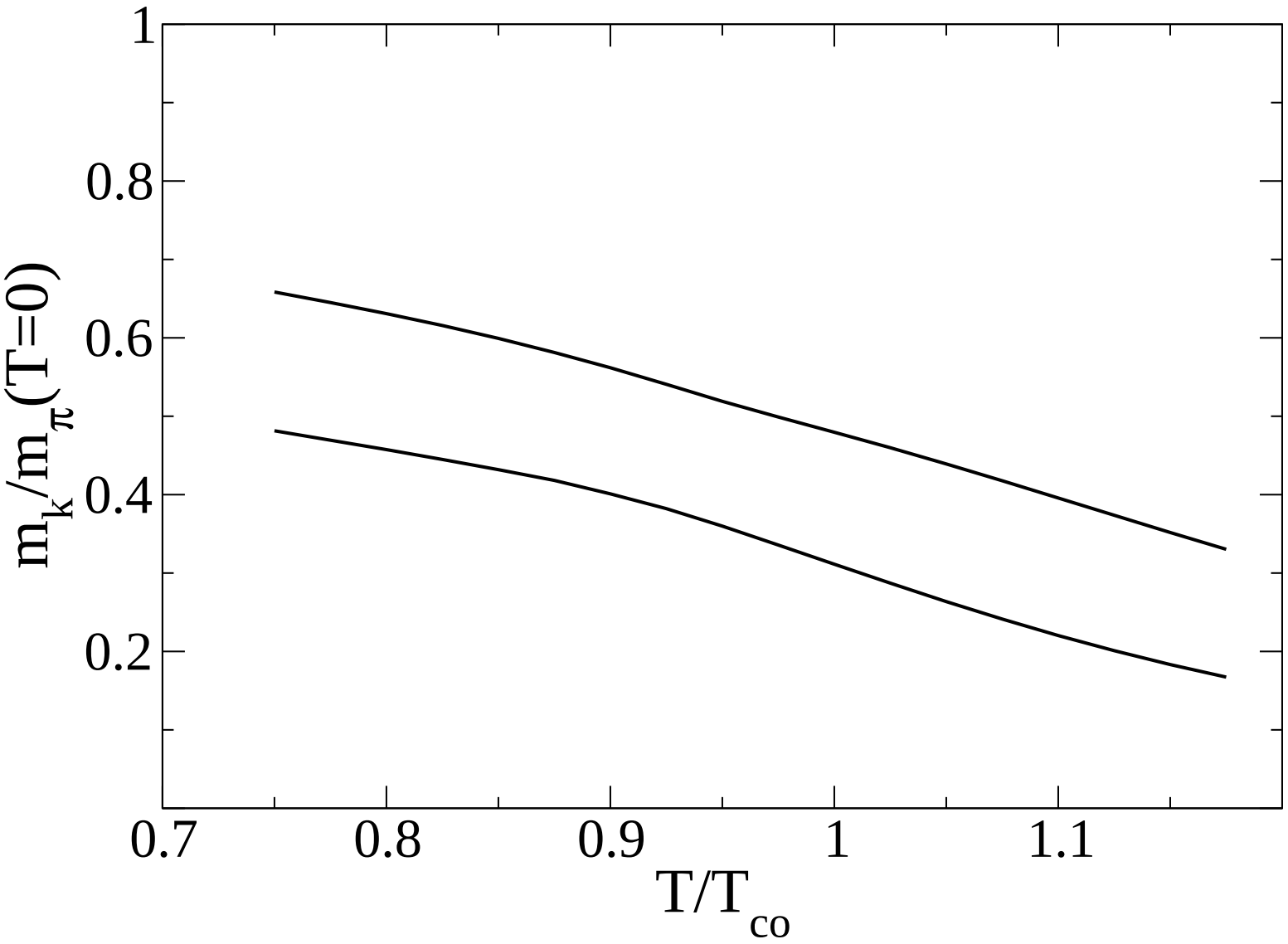
$$E^2 = m_\pi^2 + u_\pi^2 |\mathbf{q}|^2, \qquad E = m_\pi + \frac{|\mathbf{q}|^2}{2m_\pi/u_\pi^2} + \cdots$$

The "rest mass" is m_π, but the kinetic energy is controlled by a different "kinetic mass"

$$m_k = \frac{m_\pi}{u_\pi^2}.$$

A pion gas in a finite box, has single pion states lying at energy m_π above the vacuum, and discrete states of kinetic energy $k^2/(2m_k)$ above it.

Kinetic mass

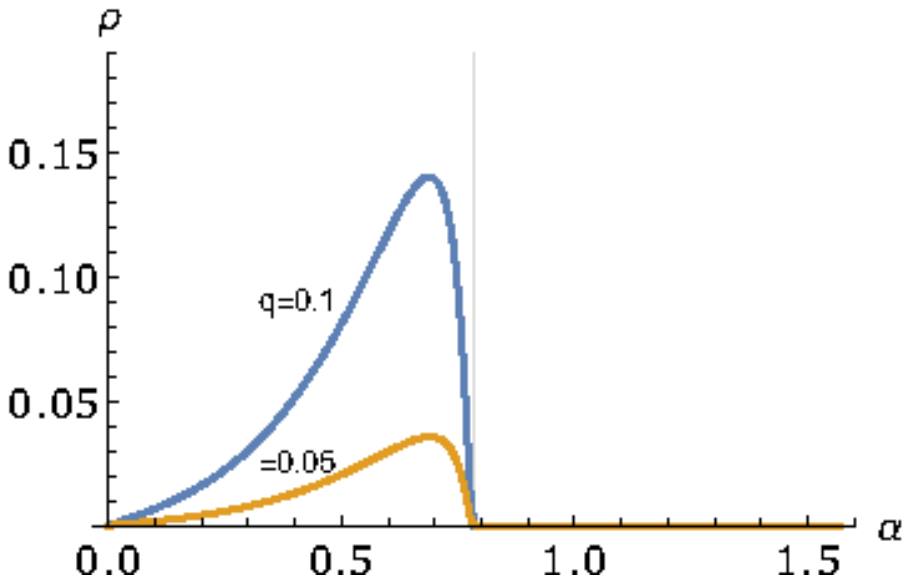


Why so complicated? The spectral function

Spectral function is defined as

$$\rho(q_0, \mathbf{q}) = iD_R(q_0, \mathbf{q}) - iD_A(q_0, \mathbf{q}).$$

Easily calculated to one loop order: not a delta function with support on $q_0^2 = |\mathbf{q}|^2 + m_\pi^2$.



Change of variables: $\tan\alpha = q_0/|\mathbf{q}|$, $q^2 = q_0^2 + |\mathbf{q}|^2$. Spectral function non-zero for $\tan\alpha \leq d^4$: Landau cut.

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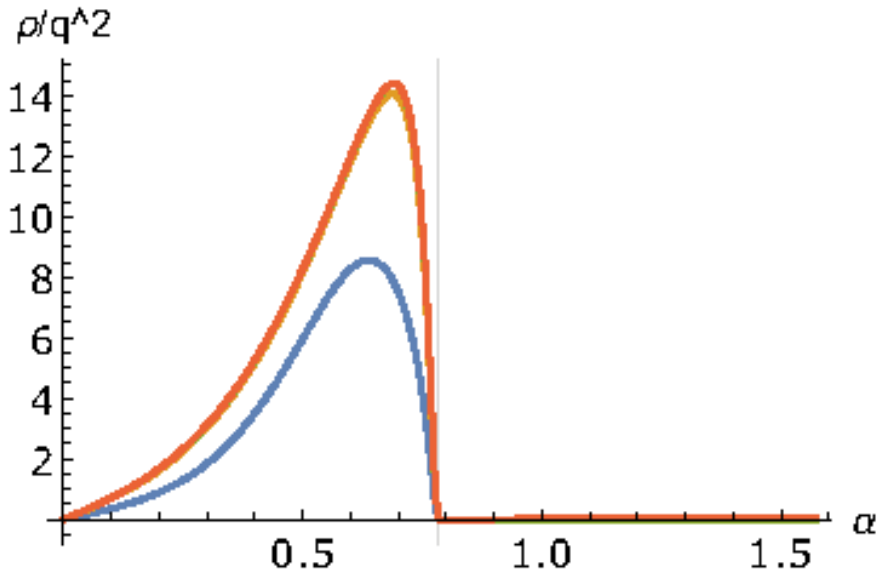

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Additional remarks

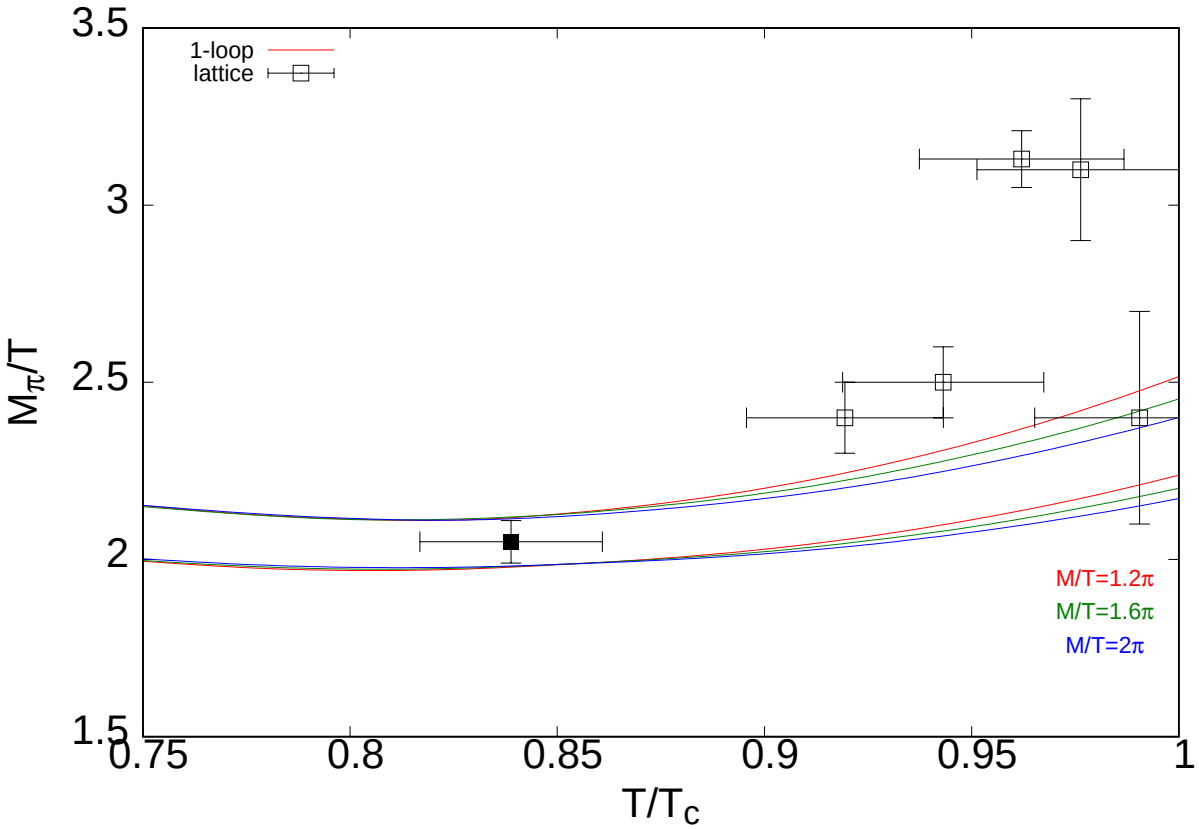
1. Straightforward to prove that the product $f_\pi m_\pi$ does not depend on how the small q_0, $|\mathbf{q}|$ limit is taken.
2. Decrease of Euclidean $m_\pi(T)$ near the cross over temperature T_{co} related to increasing static correlation length near the chiral critical point. Interesting that the dynamic mass instead increases with T.
3. Does the decrease of the pion decay width have observable consequences? Requires further study.
4. Would a full QCD computation give a Landau cut in the pion propagator? Open question near T_{co}. We have been able to develop lattice observables which could give information on this.
5. For the kinetics of a pion gas, the mass and dispersion relations must be taken from the Minkowski theory, and not directly from lattice measuement of screening masses.

Backup: full set of dimension-6 terms

$$
\begin{aligned}
\mathcal{L}_6 \;=\; & \frac{d^{61}}{T_0^2}\left[(\overline{\psi}\psi)^2 + (\overline{\psi} i\gamma_5\tau^a\psi)^2\right] + \frac{d^{62}}{T_0^2}\left[(\overline{\psi}\tau^a\psi)^2 + (\overline{\psi} i\gamma_5\psi)^2\right] \\
& + \frac{d^{63}}{T_0^2}(\overline{\psi}\gamma_4\psi)^2 + \frac{d^{64}}{T_0^2}(\overline{\psi} i\gamma_i\psi)^2 + \frac{d^{65}}{T_0^2}(\overline{\psi}\gamma_5\gamma_4\psi)^2 \\
& + \frac{d^{66}}{T_0^2}(\overline{\psi} i\gamma_5\gamma_i\psi)^2 + \frac{d^{67}}{T_0^2}\left[(\overline{\psi}\gamma_4\tau^a\psi)^2 + (\overline{\psi}\gamma_5\gamma_4\tau^a\psi)^2\right] \\
& + \frac{d^{68}}{T_0^2}\left[(\overline{\psi} i\gamma_i\tau^a\psi)^2 + (\overline{\psi} i\gamma_5\gamma_i\tau^a\psi)^2\right] \\
& + \frac{d^{69}}{T_0^2}\left[(\overline{\psi} iS_{i4}\psi)^2 + (\overline{\psi} S_{ij}\tau^a\psi)^2\right] \\
& + \frac{d^{60}}{T_0^2}\left[(\overline{\psi} iS_{i4}\tau^a\psi)^2 + (\overline{\psi} S_{ij}\psi)^2\right]
\end{aligned}
$$

10 couplings d^{6i}, collapse to one (λ) in mean field theory.

Backup: Euclidean predictions



Numerical approximations in prediction: λ fitted in the chiral limit; T_0 without power corrections; d^3, d^4 and λ assumed to be independent of T. **Lattice measurements taken from Brandt et al, 1406.5602**

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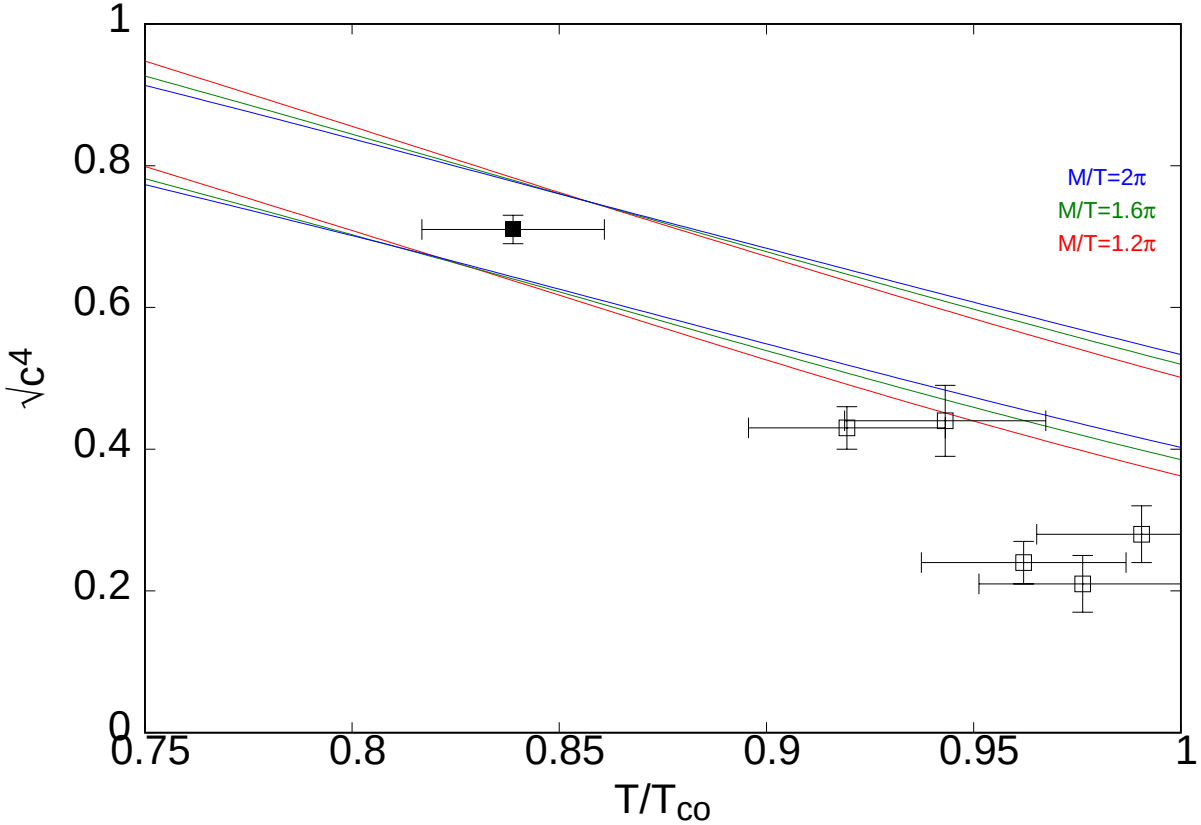

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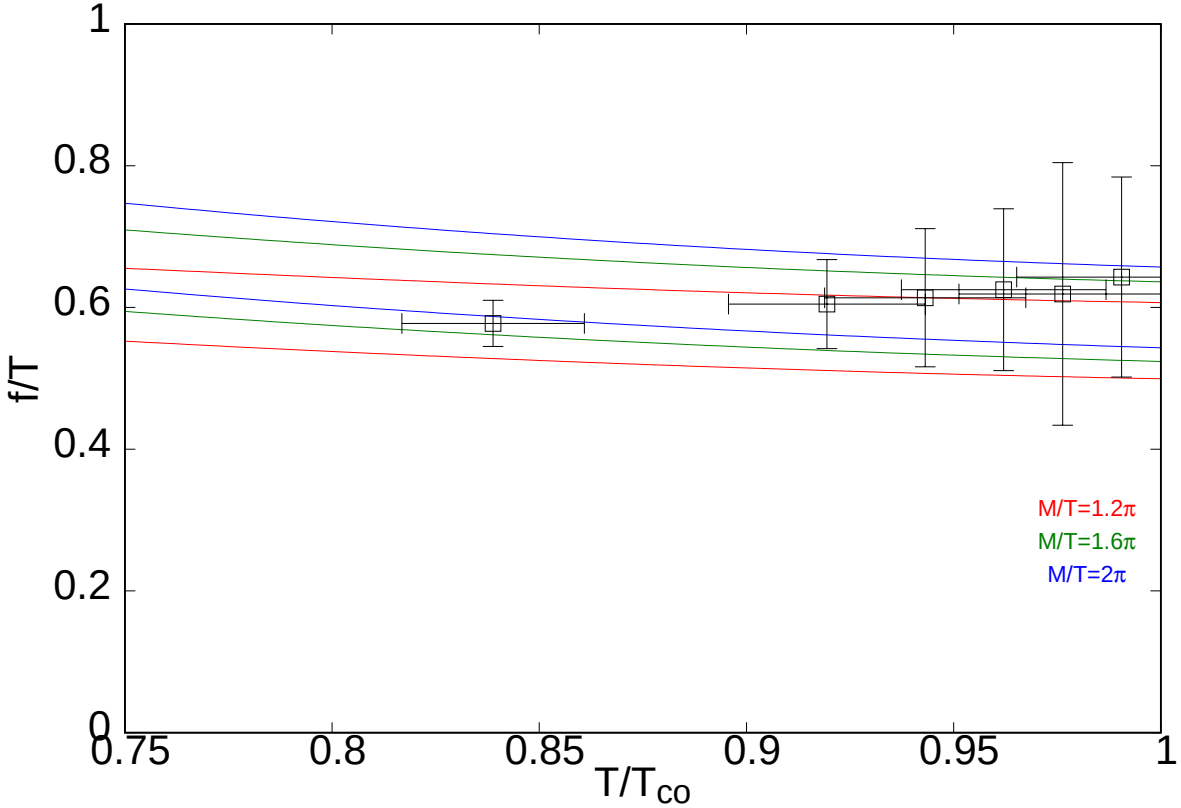

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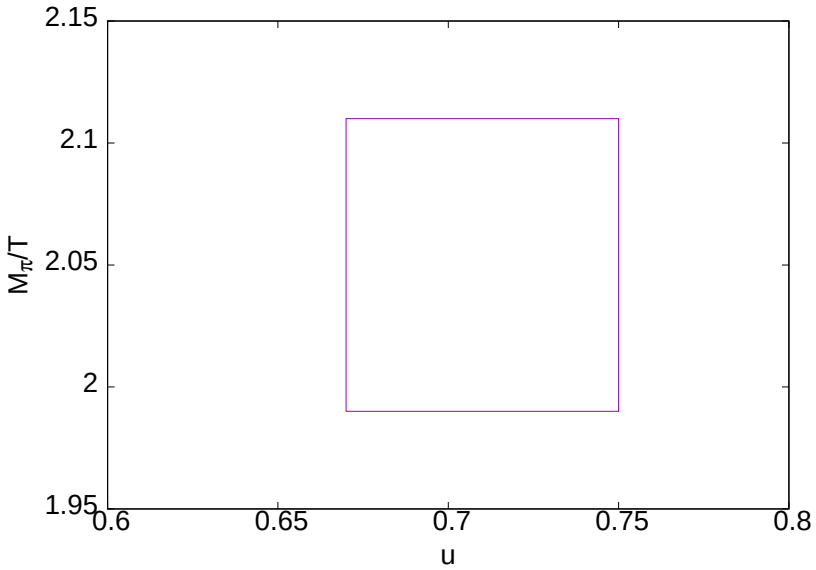
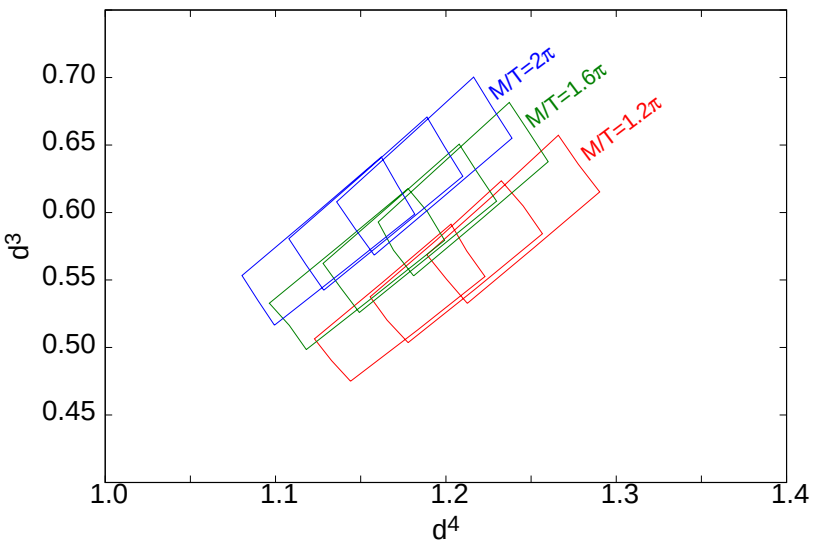
Backup: Error propagation



Input errors: lattice measurements



Output errors: EFT parameters

Covariances of lattice measurements not reported, but can be extracted. In future covariances can be propagated to the EFT parameters.