Equation of state for hot QCD and compact stars

based on [1905.00866]

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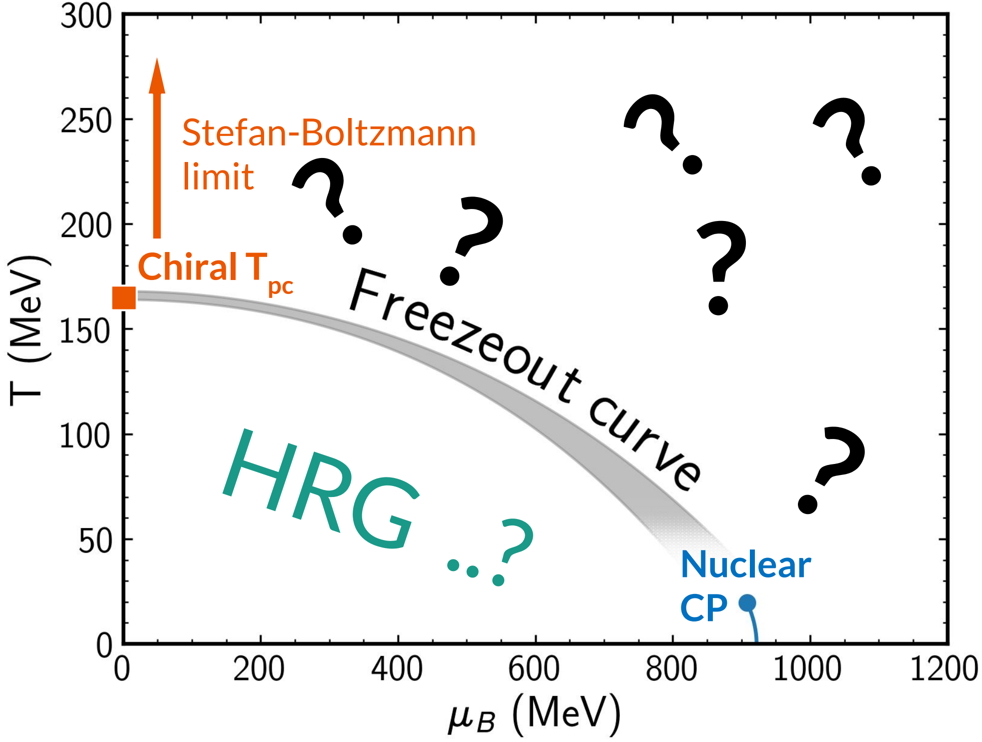
FIAS Frankfurt Institute for Advanced Studies

GSI

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Quark Matter 2019, Wuhan, China **November 6, 2019**

QCD phenomenology for the EoS



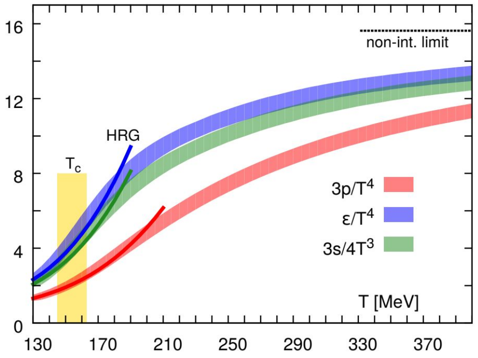
How one can map all known phenomenology to the **QCD phase diagram**?

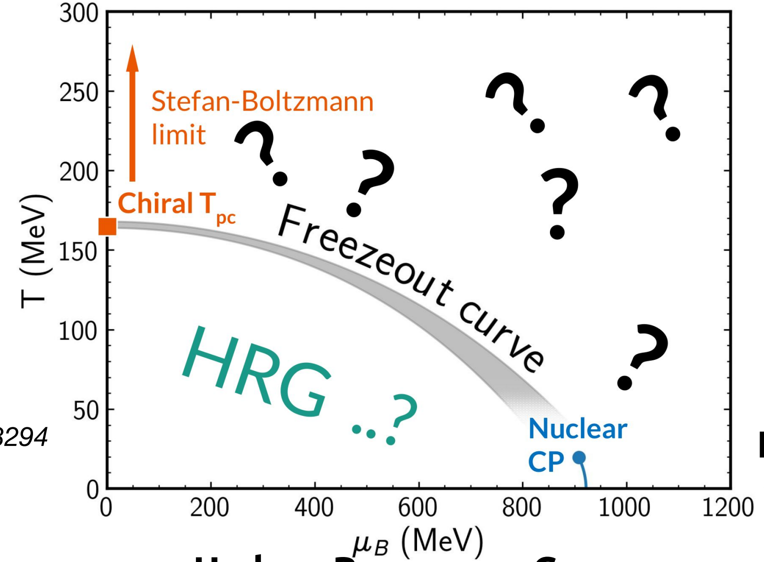
We build a **unified approach to equation of state** that incorporates most features of QCD phenomenology.

based on 1905.00866, **A. Motornenko**, QM2019, Wuhan China, November 2019

QCD phenomenology for the EoS

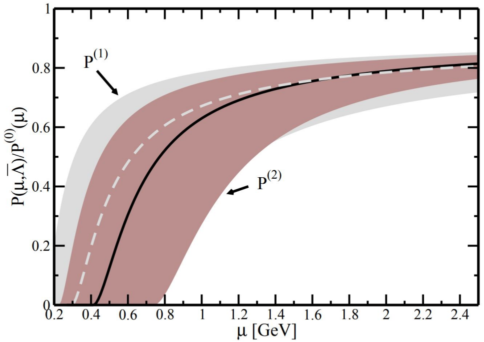
Stefan-Boltzmann limit μ_B=0 *HotQCD, 1407.6387*



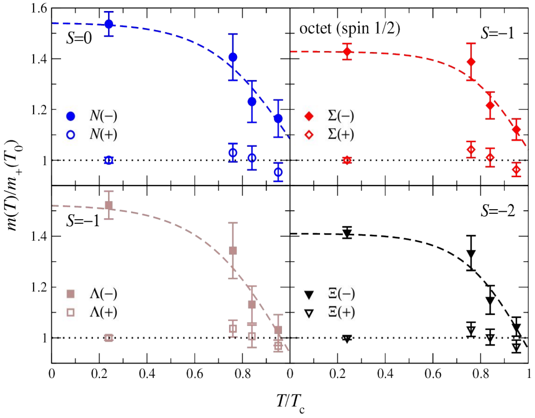


Stefan-Boltzmann limit T=0

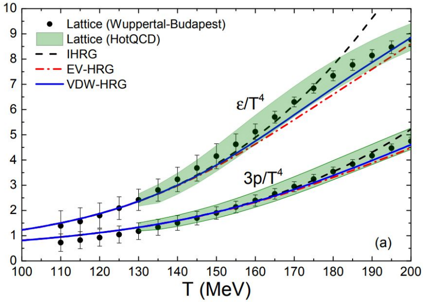
A. Kurkela, P. Romatschke, A. Vuorinen, 0912.1856



Parity doubling *G. Aarts et al., 1710.08294*



Hadron Resonance Gas

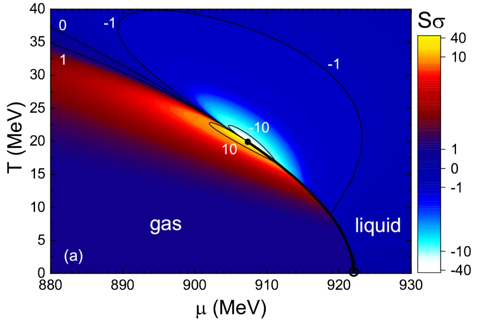


Vovchenko, Gorenstein, Stoecker, 1609.03975

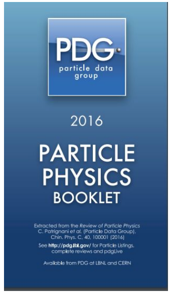
Nuclear matter properties

J. Pochodzalla et al., Phys.Rev.Lett. 75 (1995)

V. Vovchenko et al., 1506.05763

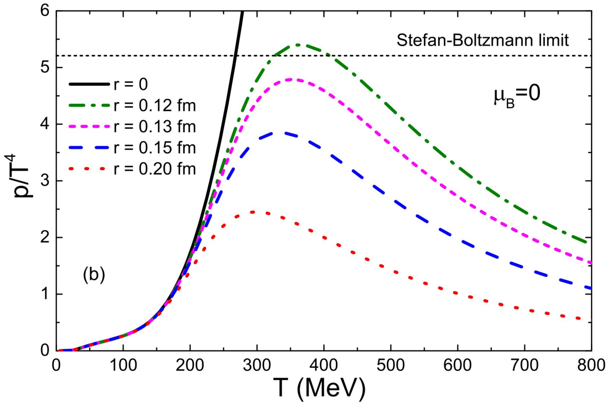


Hadronic part: QCD matter at low densities



$$\rho_i = \frac{\rho_i^{\rm id}(T, \mu_i^* - v_i\, p)}{1 + \sum_j v_j \rho_j^{\rm id}(T, \mu_j^* - v_j\, p)}$$

$$\varepsilon_i = \frac{\varepsilon_i^{\rm id}(T, \mu_i^* - v_i\, p)}{1 + \sum_j v_j \rho_j^{\rm id}(T, \mu_j^* - v_j\, p)}$$



V. Vovchenko, D. Anchishkin, M. Gorenstein, 1412.5478

PDG list of known hadrons is included with Excluded Volume interactions.
EV suppress hadrons at high energy densities.

EV of baryons: 1 fm^3
EV of mesons: 1/8 fm^3

EV triggers the switch between hadron and quark degrees of freedom: **hadron** pressure is **suppressed** as function of T and μ_B — quarks are dominant at high densities.

Including Quarks: Stefan-Boltzmann limit for QCD matter

We include quarks within **PNJL** inspired approach (*Fukushima, hep-ph/0310121*):

$$\Omega_q = -VT \sum_{i \in Q} \frac{d_i}{(2\pi)^3} \int d^3k \frac{1}{N_c} \Big[\ln\left(1 + 3\Phi e^{-\left(E_i^* - \mu_i^*\right)/T} + 3\bar{\Phi} e^{-2\left(E_i^* - \mu_i^*\right)/T} + e^{-3\left(E_i^* - \mu_i^*\right)/T}\right)$$
$$+ \ln\left(1 + 3\bar{\Phi} e^{-\left(E_i^* + \mu_i^*\right)/T} + 3\Phi e^{-2\left(E_i^* + \mu_i^*\right)/T} + e^{-3\left(E_i^* + \mu_i^*\right)/T}\right)\Big]$$

Polyakov loop Φ — is assumed as deconfinement order parameter:
Φ=0 — no quarks, **Φ=1** — free quarks.

Effective masses of the quarks are generated by chiral fields **σ** and **ζ**:

$$E_i^* = \sqrt{k^2 + m_i^{*2}}$$
$$m_q^* = -g_{q\sigma}\sigma + \delta m_q + m_{0q}$$
$$m_s^* = -g_{s\zeta}\zeta + \delta m_s + m_{0q}$$

Polyakov loop **Φ** is controlled by the **potential U(Φ)**:

$$U = -\frac{1}{2}(a_0 T^4 + a_1 T_0 T^3 + a_2 T_0^2 T^2)\Phi\Phi^*$$
$$+ b_3 T_0^4 \log[1 - 6\Phi\Phi^* + 4(\Phi^3 + \Phi^{*3}) - 3(\Phi\Phi^*)^2]$$

Fukushima, hep-ph/0310121
Roessner, Ratti, Weise, hep-ph/0609281

$SU(3)_f$ octet and parity doubling: nuclear matter and lattice

We include all states of the **$SU(3)_f$** baryon octet:

$$\begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -2\frac{\Lambda}{\sqrt{6}} \end{pmatrix}$$

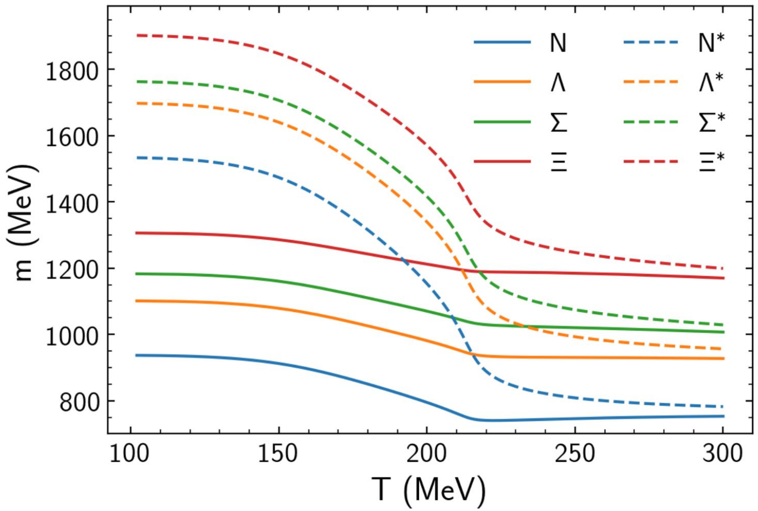
together with their **parity partners** *(G. Aarts et al., 1710.08294)*, i.e. states with the same quantum numbers but **opposite parity**. Those interact within $SU(3)_f$ σ model:

$$\mathcal{L}_B = \sum_i (\bar{B}_i i \partial\!\!\!/ B_i) + \sum_i (\bar{B}_i m_i^* B_i) + \sum_i (\bar{B}_i \gamma_\mu (g_{\omega i}\omega^\mu + g_{\rho i}\rho^\mu + g_{\phi i}\phi^\mu) B_i)$$

with effective masses generated by chiral fields **σ** and **ζ**:

$$m_{i\pm}^* = \sqrt{\left[(g_{\sigma i}^{(1)}\sigma + g_{\zeta i}^{(1)}\zeta)^2 + (m_0 + n_s m_s)^2\right]} \pm g_{\sigma i}^{(2)}\sigma \pm g_{\zeta i}^{(2)}\zeta$$

'+' stands for positive and '–' for negative parity states

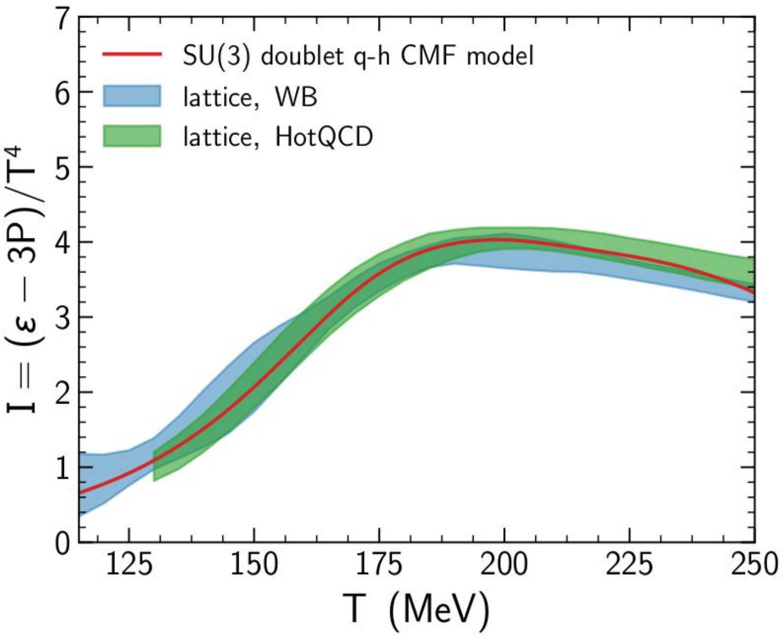


Chiral SU(3)$_f$ parity-doublet Polyakov-loop quark-hadron model

- **Chiral** — chiral symmetry restoration among parity partners and in the quark sector, chiral field is a proxy interaction between quarks and hadrons
- **SU(3)** — 3-flavor (u, d, s) chiral Lagrangian: respective baryon octet interacts through mesonic fields. Realization of σ-ω model.
  P. Papazoglou et al., nucl-th/9706024
- **parity-doublet** — parity doubling among the baryon octet
  C. E. Detar and T. Kunihiro, Phys.Rev. D39 (1989)
  T. Hatsuda and M. Prakash, Phys.Lett. B224 (1989)
  G. Aarts et al., 1703.09246 and 1812.07393
- **quark-hadron** — realization of the deconfinement, PNJL-like
  K. Fukushima, hep-ph/0310121
  C. Ratti, M.A. Thaler, W. Weise, hep-ph/0506234
  J. Steinheimer, S. Schramm, H. Stoecker, 1009.5239

A **single framework** for QCD thermodynamics, **simultaneously** satisfies constraints from **lattice QCD** and known **nuclear matter properties**, as well as **neutron star** observations.

The CMF model and lattice data, μ_B=0



Wuppertal-Budapest collab., 1309.5258
HotQCD collab., 1407.6387

The Interaction measure ***I*** effectively measures rate of increase/decrease of degrees of freedom (N_{dof}~p/T^4) function of temperature *(Fukushima, 0804.3318)*:

$$I = \frac{\varepsilon - 3p}{T^4} = T\frac{\partial}{\partial T}\left(\frac{p}{T}\frac{1}{T^3}\right)$$

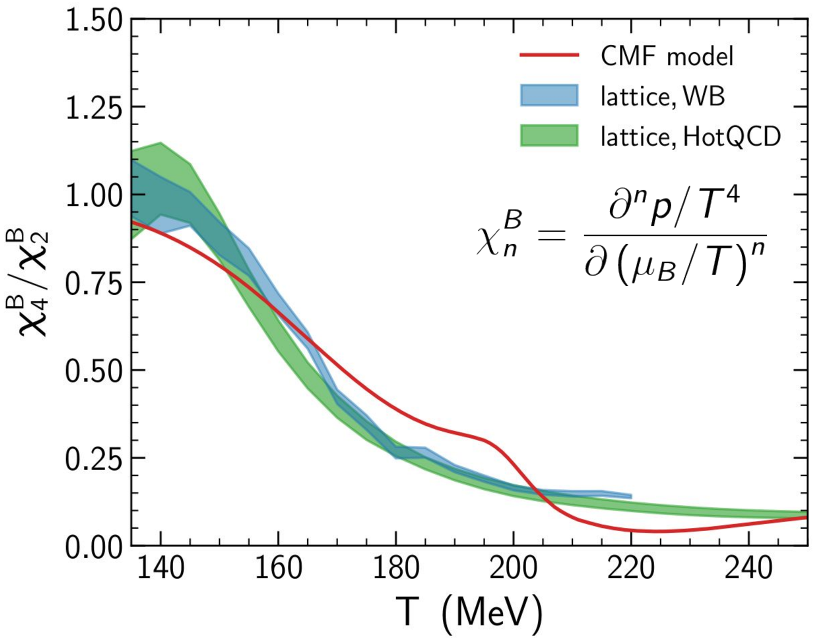
I is an illustrative quantity for quark appearance.

We reproduce ***I*** by fitting coefficients of Polyakov loop potential $U(\Phi, \bar{\Phi}, T)$ and quark couplings $g_{q\sigma}$ and $g_{q\zeta}$ to the chiral fields.

$$m_q^* = -g_{q\sigma}\sigma + \delta m_q + m_{0q},$$
$$m_s^* = -g_{s\zeta}\zeta + \delta m_s + m_{0q}$$

$$U(\Phi, \bar{\Phi}, T) = -\frac{1}{2}a(T)\Phi\bar{\Phi} + b(T)\log[1 - 6\Phi\bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi\bar{\Phi})^2]\,,$$
$$a(T) = a_0 T^4 + a_1 T_0 T^3 + a_2 T_0^2 T^2,\ \ b(T) = b_3 T_0^4$$

The CMF model and lattice data, μ_B=0



Wuppertal-Budapest collab., 1112.4416
HotQCD collab., 1203.0784

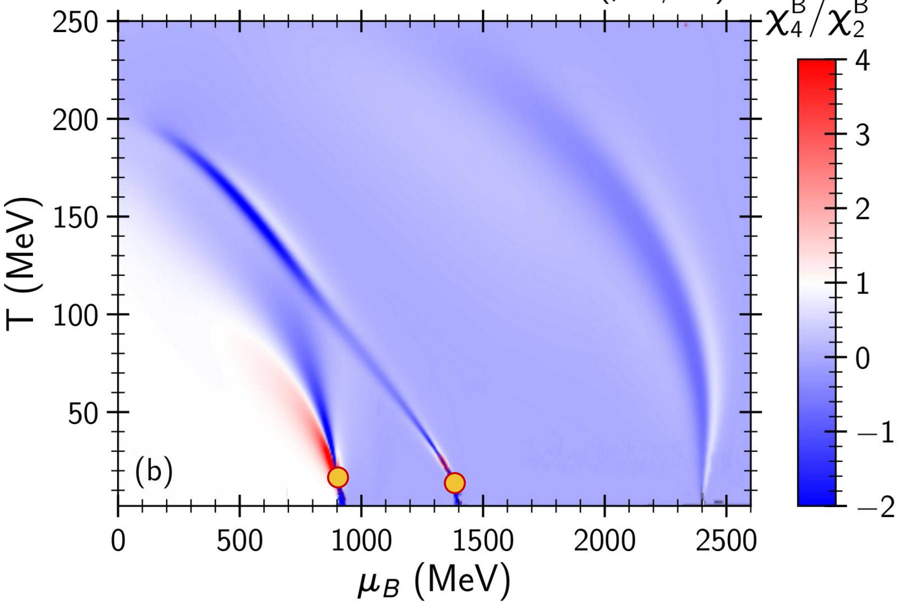
The change between two baselines is in correct region w.r.t. lattice QCD Kurtosis

The CMF model contains two phase transitions, remnants are already observable at μ_B=0:

- ➔ General trend comes from nuclear liquid-vapor phase transition: dilute hadron gas ➜ dense hadronic medium
- ➔ Small bump is produced by the chiral field σ, though the chiral phase transition is at high μ_B and low T

Kurtosis and the phase diagram

Kurtosis : $\chi_4^B/\chi_2^B, \quad \chi_n^B = \frac{\partial^n p/T^4}{\partial(\mu_B/T)^n}$



- Remnants of the well established nuclear liquid-vapor transition are visible at μ_B=0
- Chiral transition do not change the baseline in kurtosis
- Third transition is always a crossover, transition from quark-hadron mixture to **quark matter**

Three transitions:

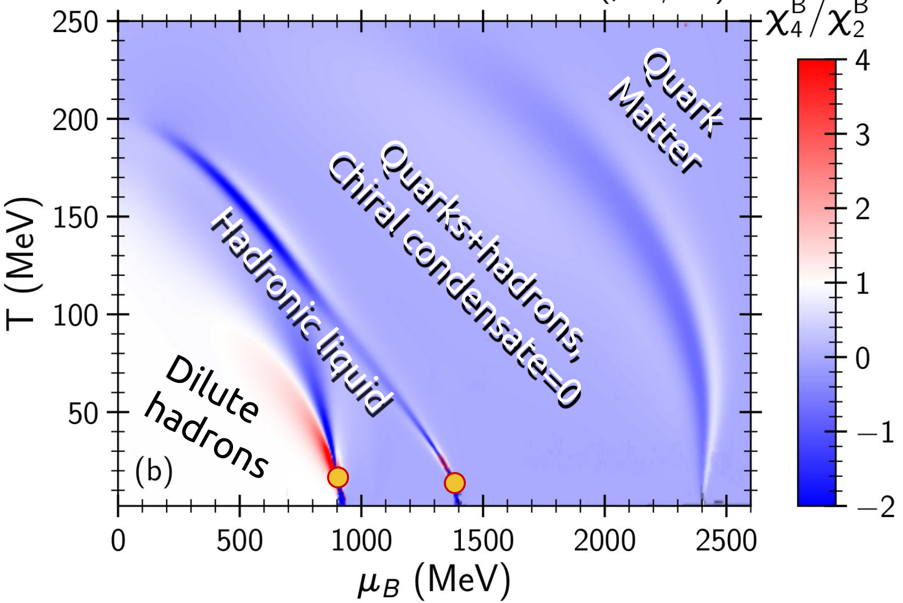
hadron gas ➜ hadronic liquid ➜ chiral symmetry restoration ➜ quark matter

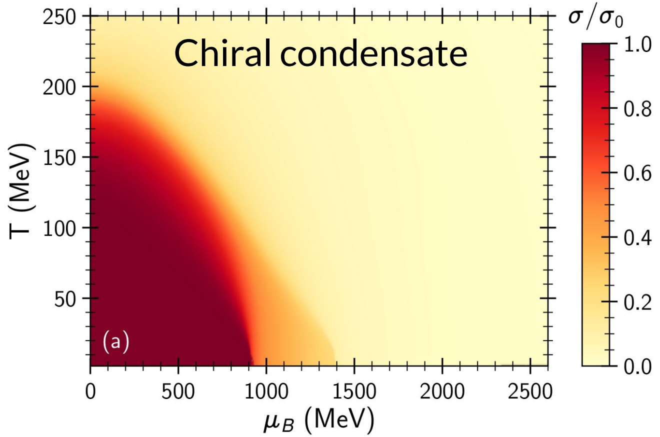
Two critical points: nuclear CP T_{CP}≈ 17 MeV, chiral CP T_{CP}≈ 17 MeV

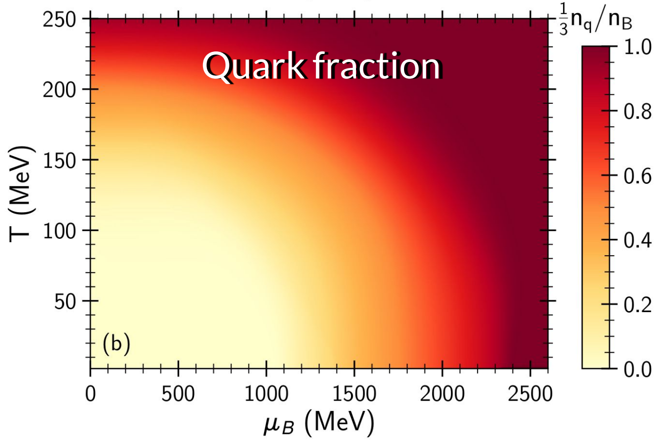
based on 1905.00866, **A. Motornenko**, QM2019, Wuhan China, November 2019

Kurtosis and the phase diagram

Kurtosis : $\chi_4^B/\chi_2^B, \quad \chi_n^B = \dfrac{\partial^n p/T^4}{\partial(\mu_B/T)^n}$







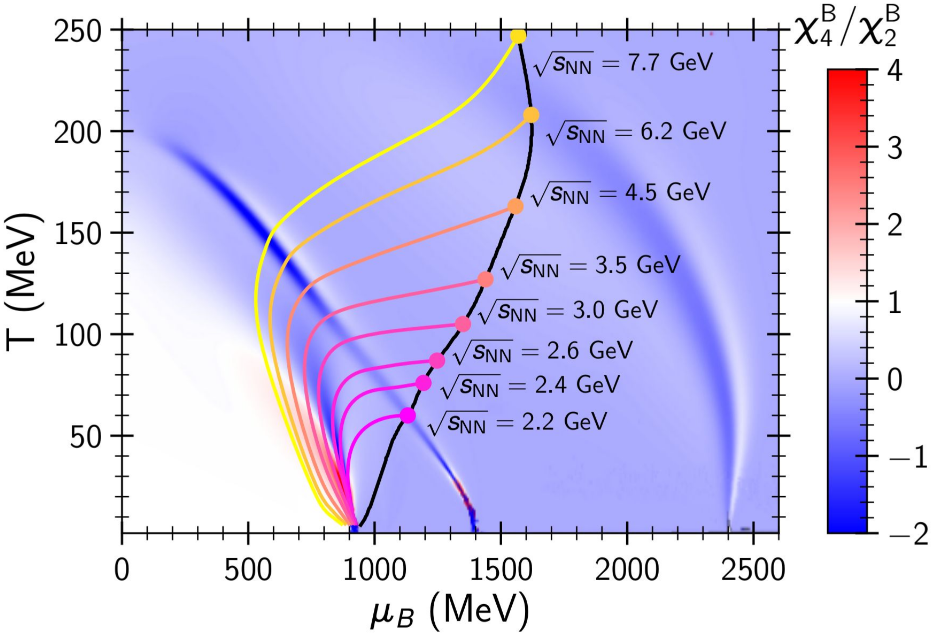
Three transitions:

hadron gas ➜ hadronic liquid ➜ chiral symmetry restoration ➜ quark matter

Two critical points: nuclear CP $T_{CP} \approx 17$ MeV, chiral CP $T_{CP} \approx 17$ MeV

based on 1905.00866, **A. Motornenko**, QM2019, Wuhan China, November 2019

Probing the phase diagram by heavy ions



Hydrodynamical evolution of the **hot central region in heavy ion collision** is done by isentropic expansion (S/A = const, no dissipations).

Initial entropy per baryon S/A is estimated by the relativistic Rankine-Hugoinot-Taub adiaibat (shock wave solution) *A. H. Taub, Phys. Rev. 74, 328 (1948)*

$$(P_0 + \varepsilon_0)(P + \varepsilon_0)\, n^2 = (P_0 + \varepsilon)(P + \varepsilon)\, n_0^2$$

Here the initial conditions:

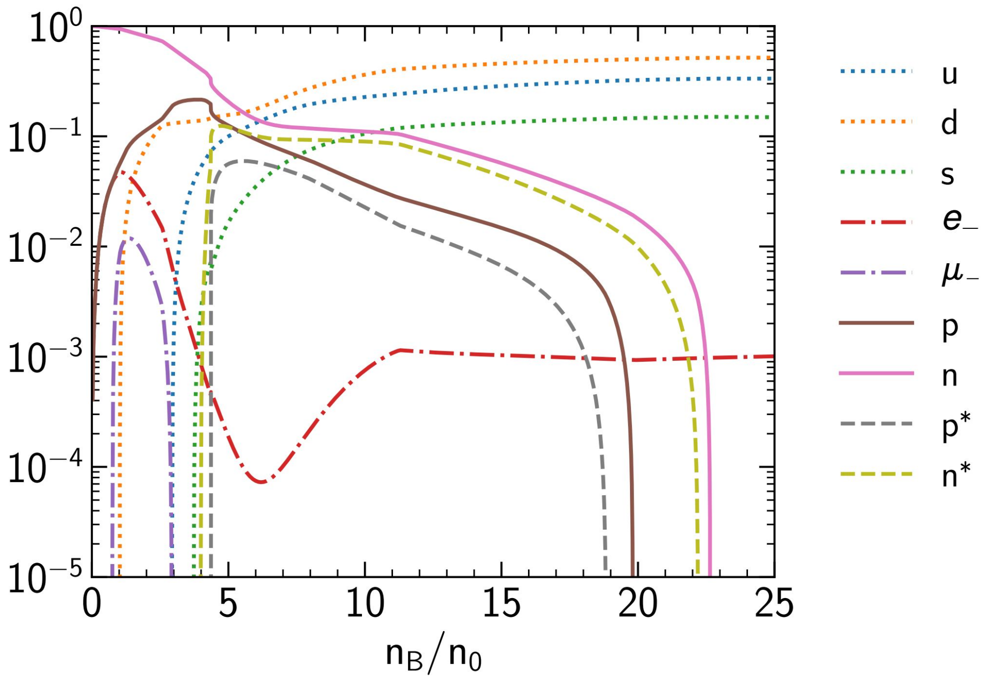
$P_0 = 0,\ \varepsilon_0/n_0 - m_N = -16\,\mathrm{MeV}$ and $n_0 = 0.16\,\mathrm{fm}^{-3}$

correspond to the initial pressure, energy density, and baryon density in the local rest frame of each colliding slab.

based on 1905.00866**, A. Motornenko**, QM2019, Wuhan China, November 2019

Equation of state at T=0: neutron stars

Particle yields normalized to baryon density:



CMF is easily employed for the description of **neutron star matter** in ***β*-equilibrium** with no changes to the parameters.

The EoS is applied to neutron stars by solving Tolman–Oppenheimer–Volkoff equation:

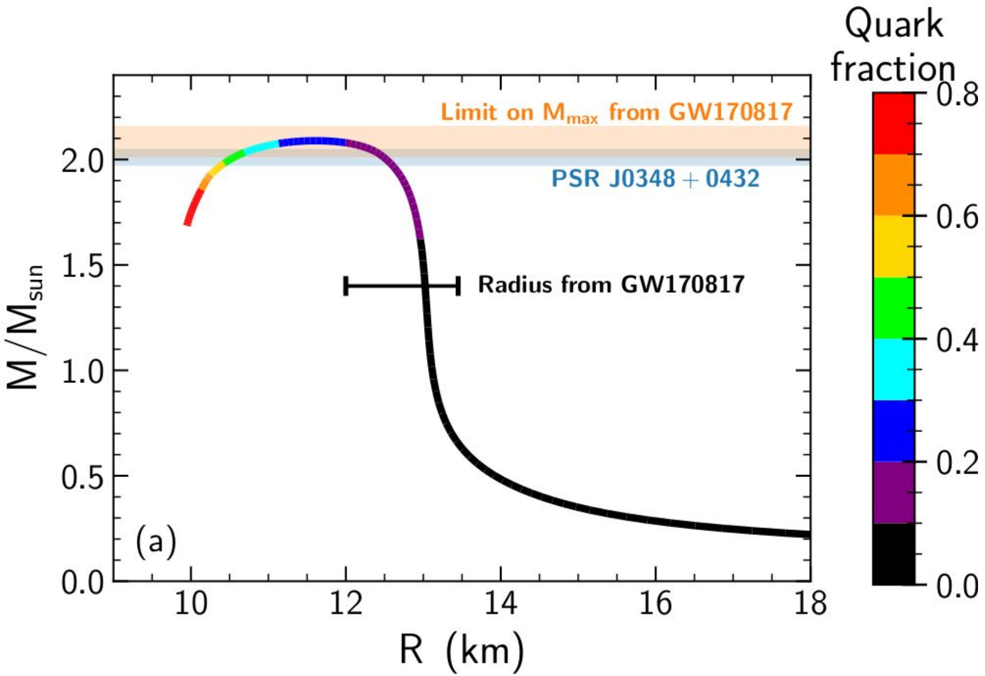
$$\frac{dP}{dr} = -\frac{Gm}{r^2}\rho\left(1+\frac{P}{\rho}\right)\left(1+\frac{4\pi r^3 P}{m}\right)\left(1-\frac{2Gm}{r}\right)^{-1}$$

Equation of state for static **neutron stars**:

- ➔ **T=0**
- ➔ Electric charge is **zero**
- ➔ **Leptons** are included
- ➔ **Strange quarks** are included
- ➔ No nuclear ground state
- ➔ Chiral transition is at $n \approx 4n_0$
- ➔ Quark matter is at $n \approx 23n_0$
- ➔ **Hyperons** are suppressed by the EV repulsion

based on [1905.00866](1905.00866)**, A. Motornenko**, QM2019, Wuhan China, November 2019

Mass-radius relations: another benchmark for QCD EoS

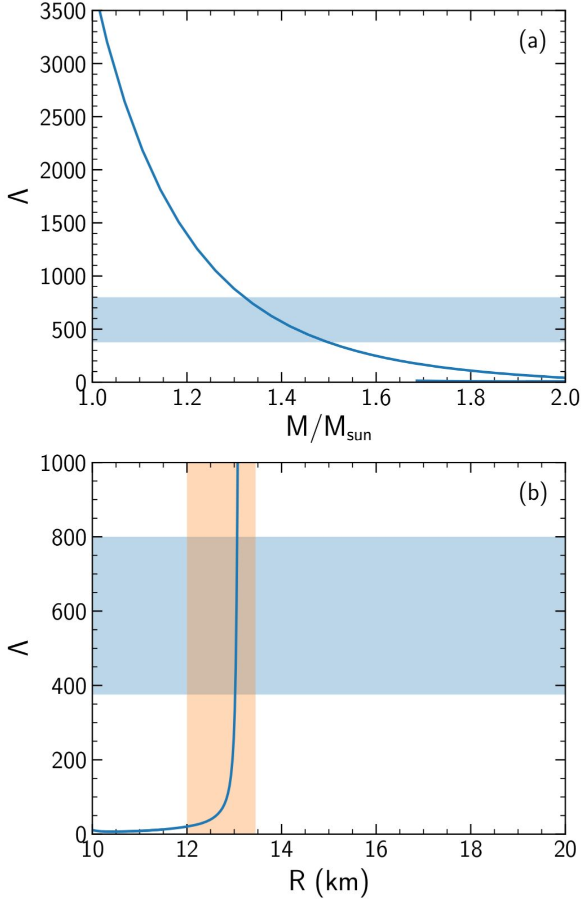


- ➔ The CMF EoS is stiff enough to provide $2M_{sun}$ neutron stars
- ➔ As soon as significant abundance of quarks appears — NS are unstable due to the soft quark EoS
- ➔ No repulsion among quarks— no second family
- ➔ quark fraction < 30% for stable stars
- ➔ Agreement with the analysis of GW170817 (Most, Weih, Rezzolla, Schaffner-Bielich, 1803.00549)
- ➔ Maximal mass of NS is in agreement with constraints form universal relations *(Rezzolla, Most, Weih, 1711.00314)*

$$2.01^{+0.04}_{-0.04} < M_{\rm TOV}/M_{\rm sun} < 2.16^{+0.17}_{-0.15}$$

based on [1905.00866](), **A. Motornenko**, QM2019, Wuhan China, November 2019

NS tidal deformabilities: new benchmark from GW170817



Tidal deformability Λ — EoS dependent property of neutron star. After the GW170817 is extensively studied to constrain EoS of QCD matter.

Λ measures stars' induced quadruple moment Q_{ij} as a response to the external tidal field $\mathcal{E}_{ij}$:

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

important EoS-dependent quantity for inspiral phase of binary neutron star system.

One presents the dimensionless tidal deformability Λ:

$$\Lambda = \frac{\lambda}{M^5}$$

Bands — recent constraints for radius and tidal deformability of $1.4M_{sun}$ star.
Most, Weih, Rezzolla, Schaffner-Bielich., 1803.00549
Line — results on Λ using EoS obtained from the CMF model.

Summary

Chiral SU(3) parity-doublet quark-hadron mean-field model — unified phenomenological approach to model QCD thermodynamics at wide range of scales, contains nuclear liquid-vapor phase transition, chiral symmetry restoration and crossover to quark matter

- Model produces neutron stars in agreement with the modern constraints and with analysis of GW170817
- Model's EoS can be used as an input for both finite T and T=0 neutron star physics
- … as well as for hydro simulations of heavy ions collisions

Thanks for your attention!