

# Momentum-dependent flow fluctuations as a hydrodynamic response to initial geometry

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In collaboration with D.D. Chinellato, M. Luzum, J. Noronha, T. Nunes da Silva, J. Takahashi

*(The ExTrEM Collaboration)*

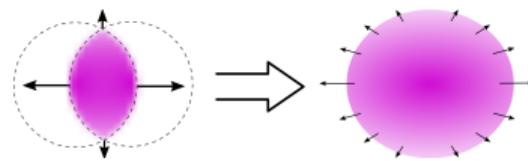
Instituto de Física “Gleb Wataghin” - Universidade Estadual de Campinas

November 6, 2019



# Motivation

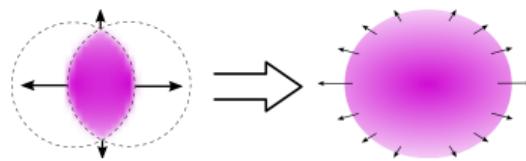
- Conversion of **initial geometry** to **momentum anisotropy**.



- How does **small-scale structure** affect the flow pattern?
- How to measure momentum-dependent **flow fluctuations**?

# Motivation

- Conversion of **initial geometry** to **momentum anisotropy**.



- How does **small-scale structure** affect the flow pattern?
- How to measure momentum-dependent **flow fluctuations**?

*Principal Component Analysis and connection to properties of the initial state*

# Principal Component Analysis (PCA)

- General statistical method.
- Here, applied to complex **flow vectors**:

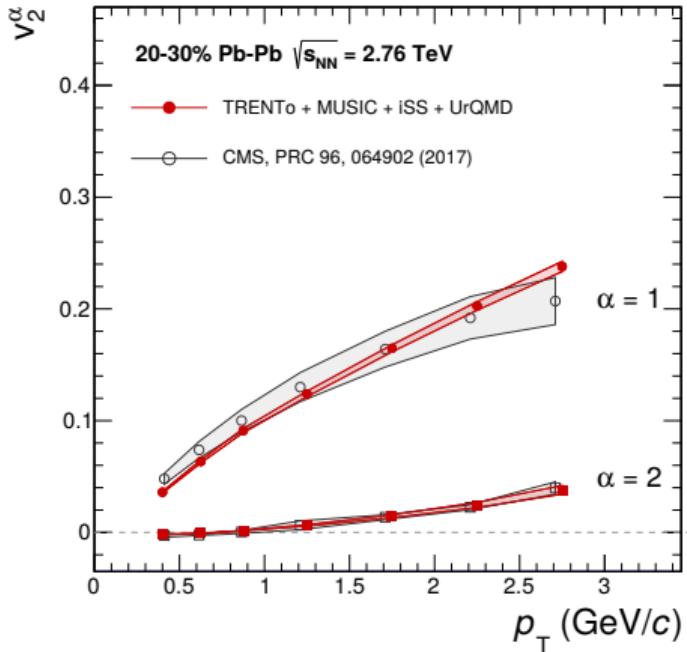
$$\frac{dN}{p_T dp_T d\varphi} = \frac{1}{2\pi} N(p_T) \sum_{n=-\infty}^{\infty} \mathbf{V}_n(\mathbf{p}_T) e^{-in\varphi}.$$

- Diagonalizing flow **covariance matrix**, one finds linearly **uncorrelated combinations** of variables  $V_n^{(\alpha)}(p_T)$ , such that:

$$V_{n\Delta}(p_{T1}, p_{T2}) = \langle V_n(p_{T1}) V_n^*(p_{T2}) \rangle \simeq \sum_{\alpha=1}^{\alpha_{\max}} V_n^{(\alpha)}(p_{T1}) V_n^{(\alpha)}(p_{T2}).$$

R. S. Bhalerao, J. Y. Ollitrault, S. Pal and D. Teaney, PRL **114** (2015).

# Principal Component Analysis



- Eigenvalues are **strongly ordered**  $\Rightarrow$  truncation.
- Characterization of **two-particle correlations** from few  $V_n^{(\alpha)}(p_T)$ .
- Leading mode  $V_n^{(1)}(p_T) \sim v_2\{2\}(p_T)$ .

A. M. Sirunyan *et al.* [CMS Collaboration], PRC **96** (2017),

T. Nunes da Silva, D.D. Chinellato, R. Derradi De Souza, M.H.,  
M. Luzum, J. Noronha and J. Takahashi, arXiv:1811.05048.

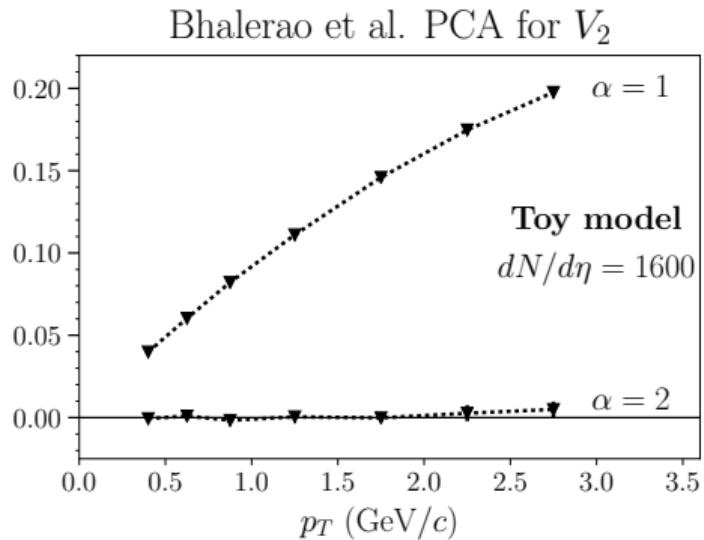
# PCA on a toy model

- What is the **information** contained in the PCA observables?

A. Mazeliauskas and D. Teaney,  
Phys. Rev. C **91** (2015) and **93** (2016).

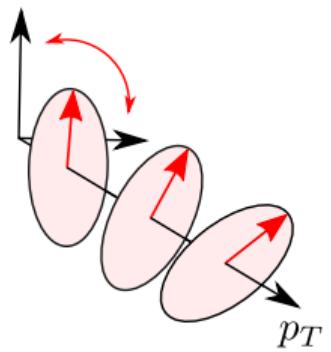
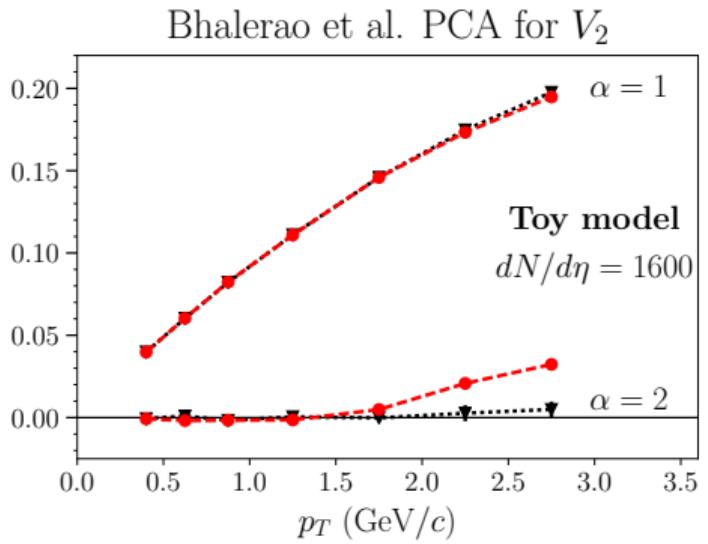
- **Test:** Events generated in **toy model**.
- Sample **particle distribution** in  $p_T$ , with:
  - ① Azimuthal distribution with constant  $|V_2(p_T)|$  and global fluctuations of the charged multiplicity.
  - ② **Fluctuations** of the **event plane**, with some  $p_T$  profile.
  - ③ **Fluctuations** of the **spectrum** (mean  $p_T$ ).

## Proof of Concept: Toy model results I



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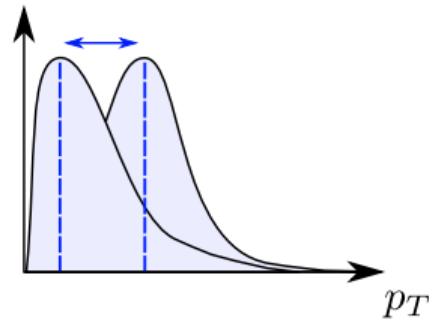
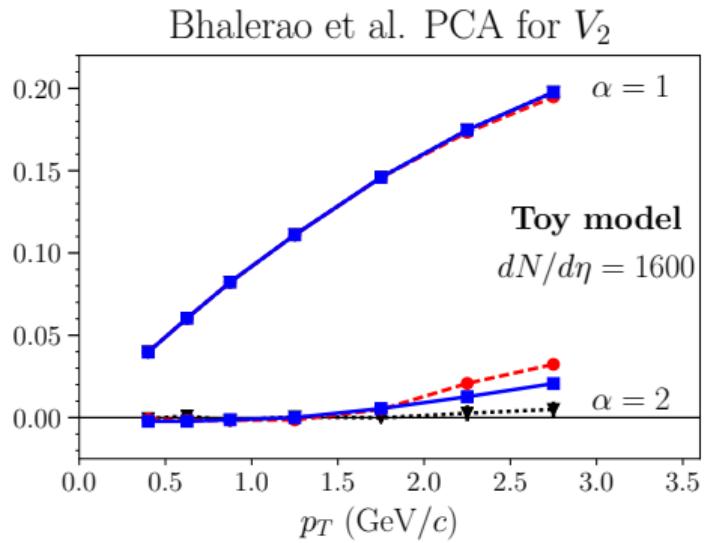
Global Fluctuations  
 $\Psi_{EP}$  Fluctuations



- **PCA** observables are sensitive to **anisotropic flow** fluctuations.

# Proof of Concept: Toy model results I

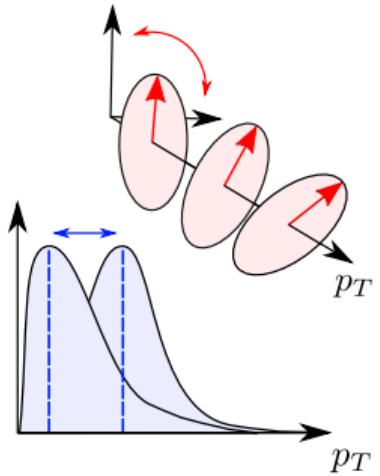
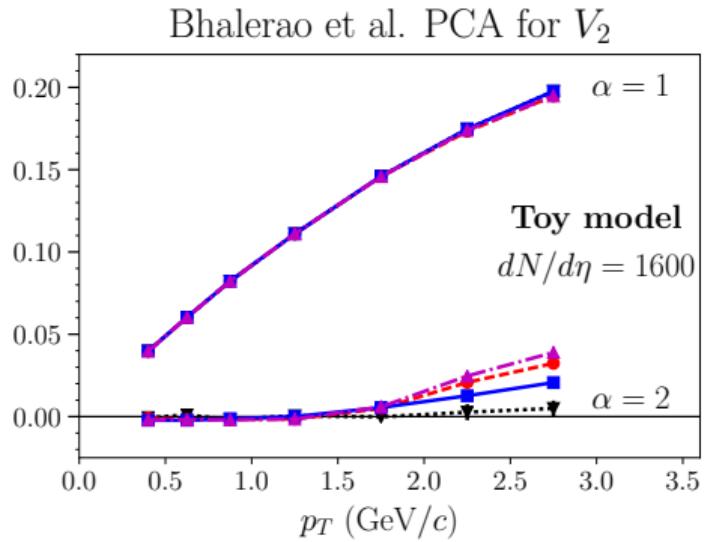
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- **PCA** observables are sensitive to **anisotropic flow** fluctuations.
- Also sensitive to fluctuations of the **spectrum**.

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- +---  $\overline{p_T}$  and  $\Psi_{EP}$  Fluctuations



- **PCA** observables are sensitive to **anisotropic flow** fluctuations.
- Also sensitive to fluctuations of the **spectrum**.
- The latter can **obscure** the former.

# Principal Component Analysis (ExTrEM Collaboration)

- Originally measured PCA from

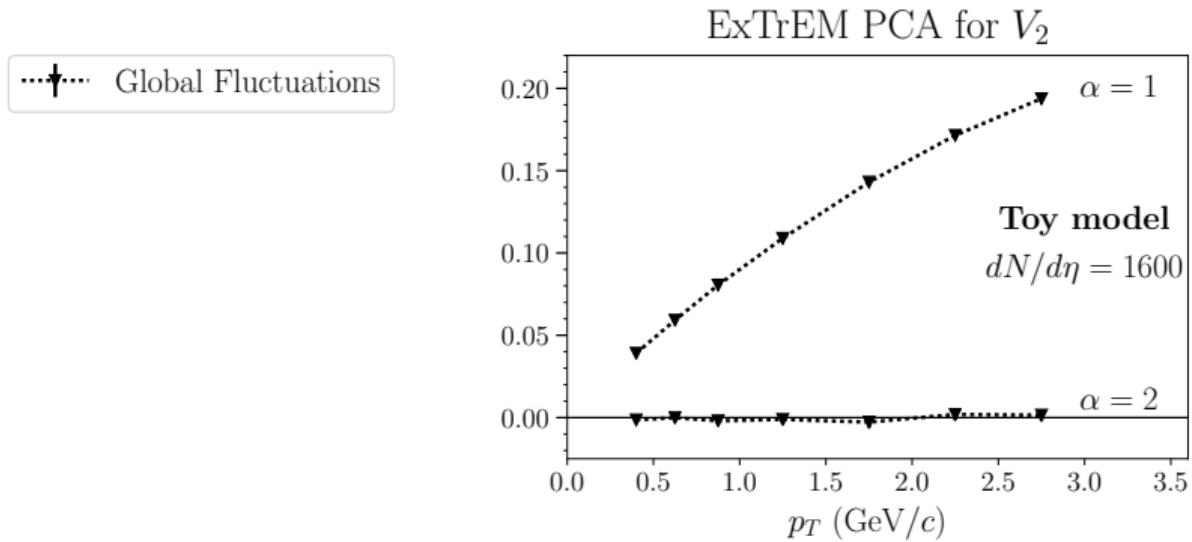
$$V_{n\Delta}^N(p_{T1}, p_{T2}) = \langle N(p_{T1}) N(p_{T2}) V_n(p_{T1}) V_n^*(p_{T2}) \rangle.$$

- Anisotropic flow fluctuations obscured by  $\langle \Delta N(p_{T1}) \Delta N(p_{T2}) \rangle$ .
- Issue fixed by using **redefined** covariance matrix:

$$V_{n\Delta}^R \equiv \frac{\langle N(p_{T1}) N(p_{T2}) V_n(p_{T1}) V_n^*(p_{T2}) \rangle}{\langle N(p_{T1}) N(p_{T2}) \rangle} \simeq \langle V_n(p_{T1}) V_n^*(p_{T2}) \rangle. \quad (1)$$

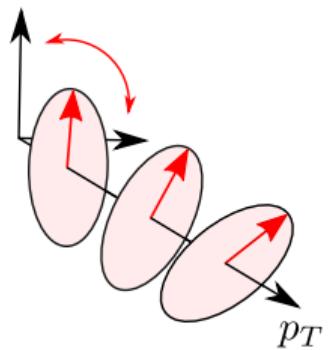
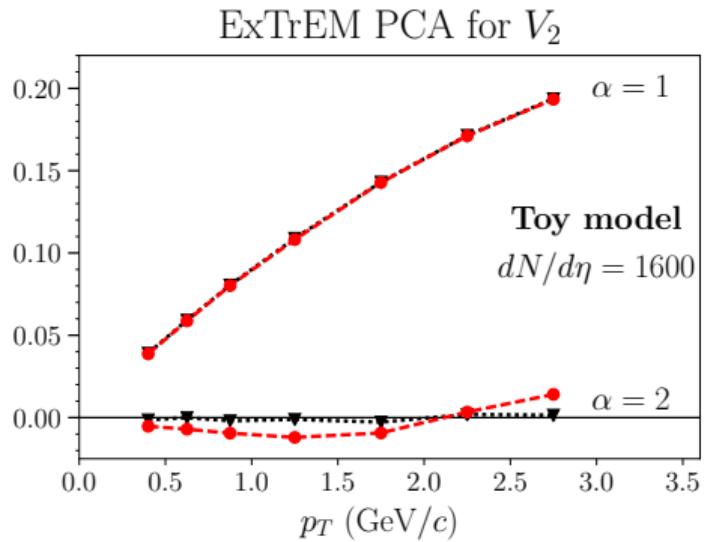
MH, D.D. Chinellato, M. Luzum, J. Noronha,  
T. Nunes da Silva and J. Takahashi, arXiv:1906.08915

# Proof of Concept: Toy model results II



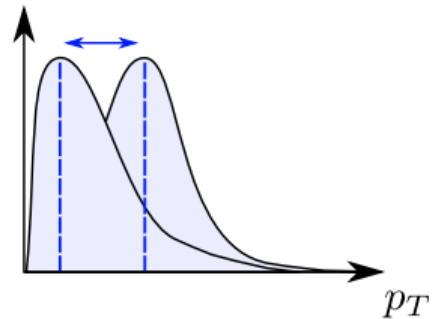
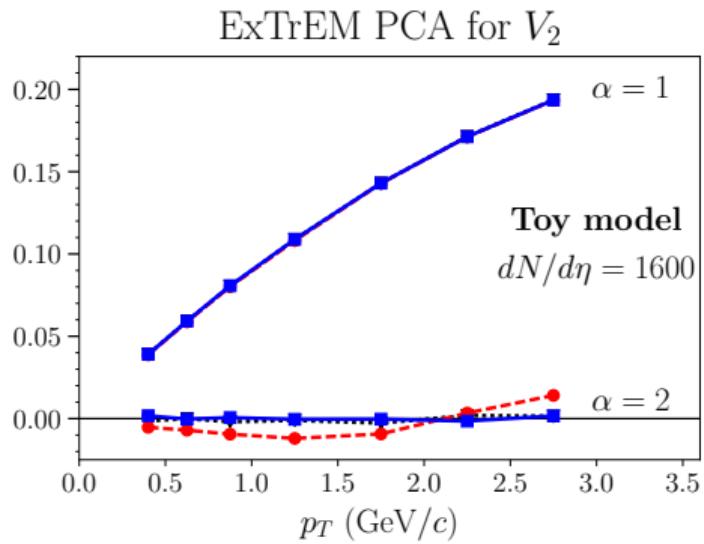
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Global Fluctuations  
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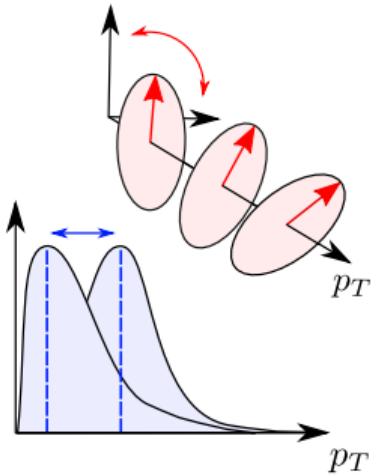
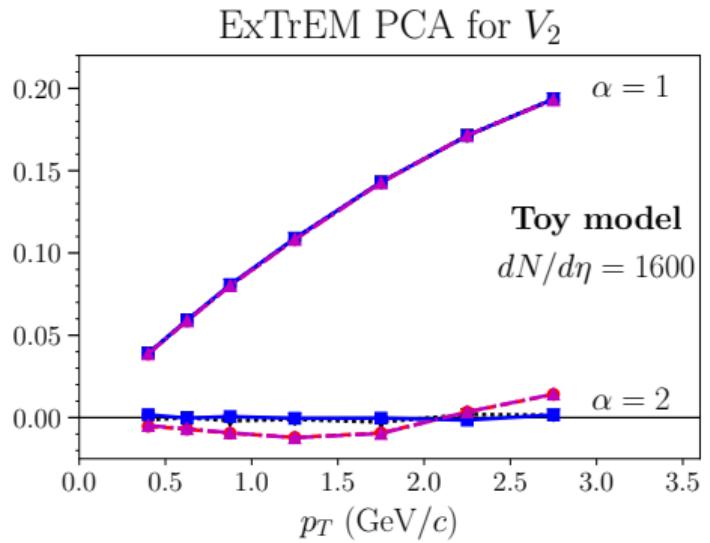
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- Redefined PCA observables **not affected** by fluctuations of the **spectrum**.

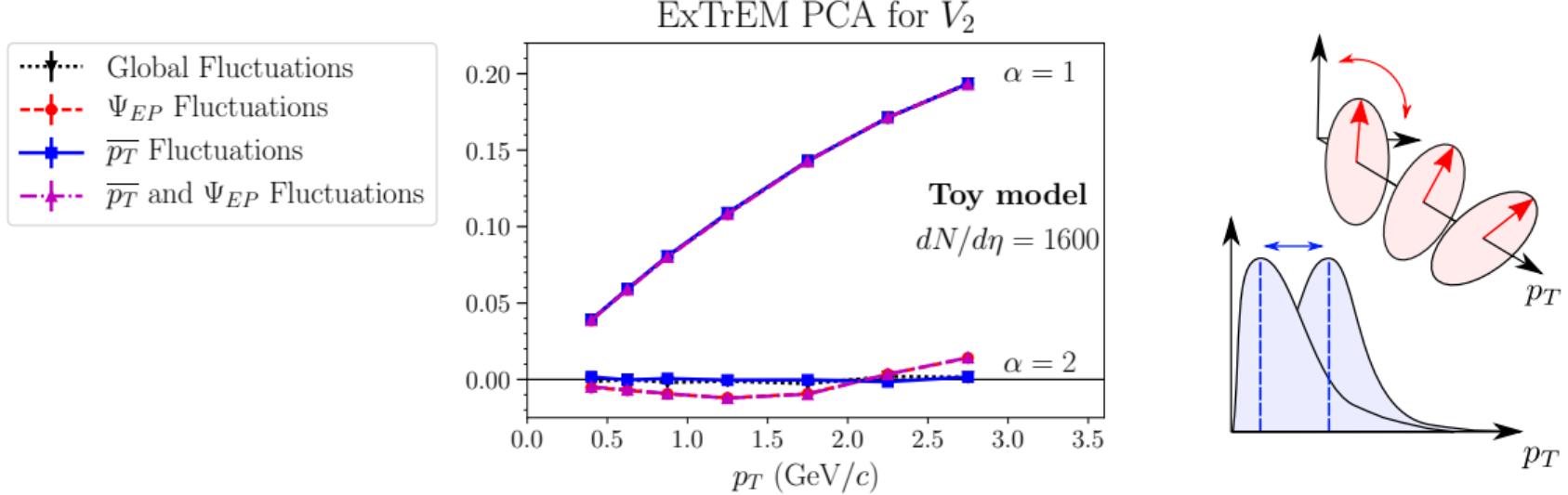
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- **Redefined PCA** observables **not affected** by fluctuations of the **spectrum**.
- Redefined PCA observables characterize **anisotropic flow only**.

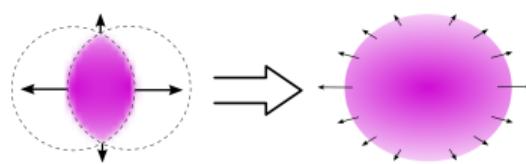
# Proof of Concept: Toy model results II



- **Redefined PCA** observables **not affected** by fluctuations of the **spectrum**.
- Redefined PCA observables characterize **anisotropic flow only**.
- Can we understand them from the **initial state**?

# Mapping hydrodynamic response

- How to connect **PCA** observables to the **initial state**?
- Conversion of **initial geometry** to **momentum anisotropy**:



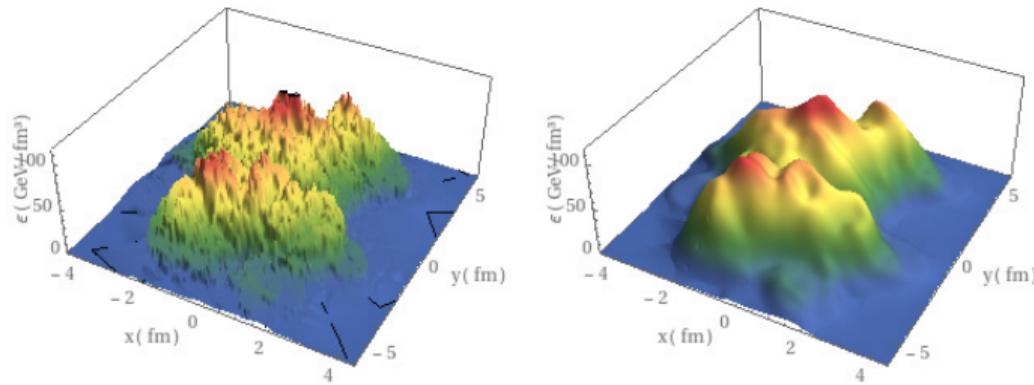
- Captured by **systematic expansion**  $V_n \simeq \kappa \epsilon_n + \dots$

# Assumptions

- ① Posterior evolution and one-body distribution determined by initial transverse profile  $\rho(\tau_0, \mathbf{x})$

$$E \frac{dN}{d^3 p} = \mathcal{F} \left[ \rho(\tau_0, \mathbf{x}) \right]$$

- ② Hierarchy of scales: large scales dominate



[arXiv:1807.05213]

# Mapping hydrodynamic response

- Eccentricities  $\epsilon_{n,m}$  from cumulants of transverse density:

$$\rho(\mathbf{x}) \rightarrow \epsilon_{\textcircled{n,m}}^{\text{lengthscale, angular dependence}}$$

- Flow  $V_n$  can be predicted by the set of all  $\epsilon_{n,m}$ :

$$V_n = \mathcal{F}_n[\rho(\tau_0, \mathbf{x})] = f_n[\{\epsilon_{n',m}\}]$$

- Large scales (small  $m$ ) dominate  $\Rightarrow$  **truncated expansion**

See talk by Matthew Luzum.

D. Teaney and L. Yan, PRC **83** (2011).  
 F. G. Gardim, F. Grassi, M. Luzum and J. Y. Ollitrault, PRC **85** (2012).

# Estimators

- The cumulant expansion allows us to estimate the flow harmonics  $V_n$ .
- Map hydrodynamic response with **linear and nonlinear** terms:

$$V_2 \simeq \kappa \cdot \epsilon_2$$

- Extension to **differential flow**  $V_n(p_T)$  via  $\kappa_i \rightarrow \kappa_i(p_T)$ .
- Coefficients  $\kappa_i$  **for each value of p<sub>T</sub> independently** determined.

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$$V_2 \simeq \kappa_0 \epsilon_{2,2} + \kappa_1 \epsilon_{2,4} + \kappa_2 \epsilon_{2,6} + \kappa_3 \epsilon_{2,8} + \mathcal{O}(m=10)$$

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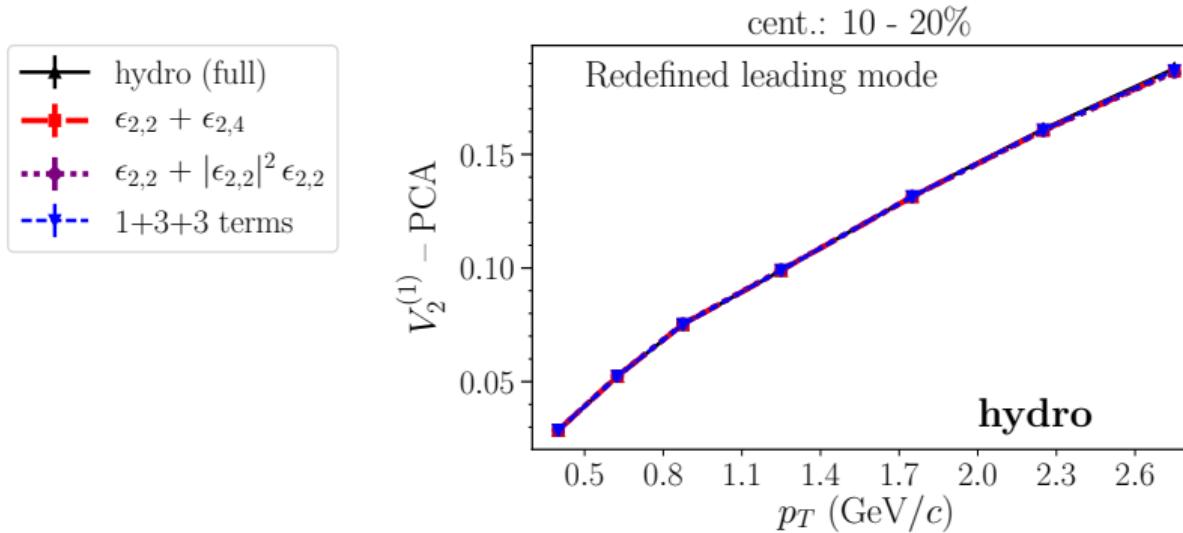
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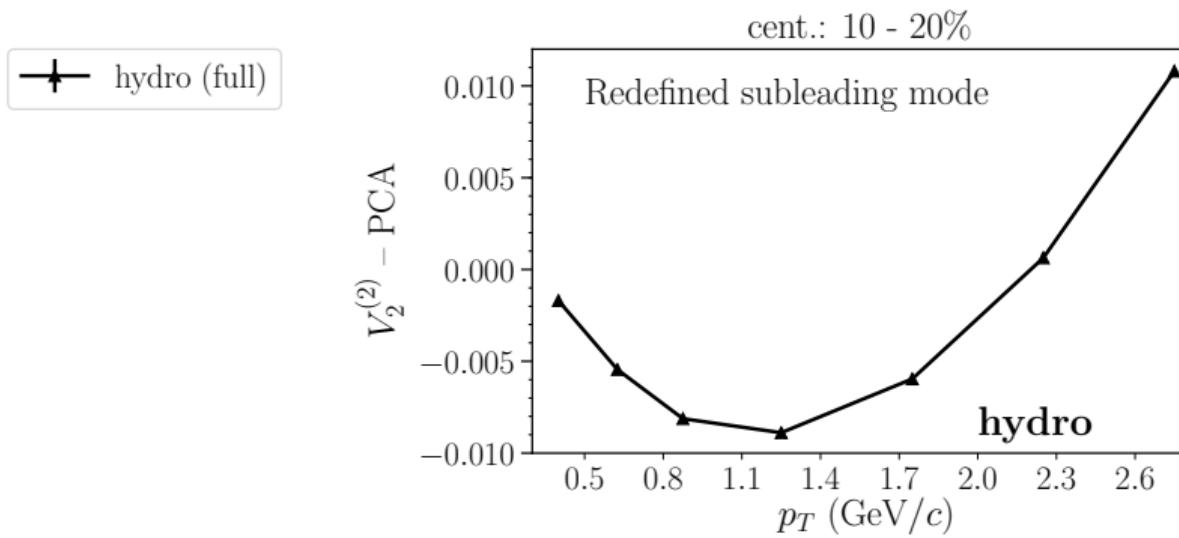
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- Do estimators reproduce PCA observables?

# Mapping PCA observables: Leading flow

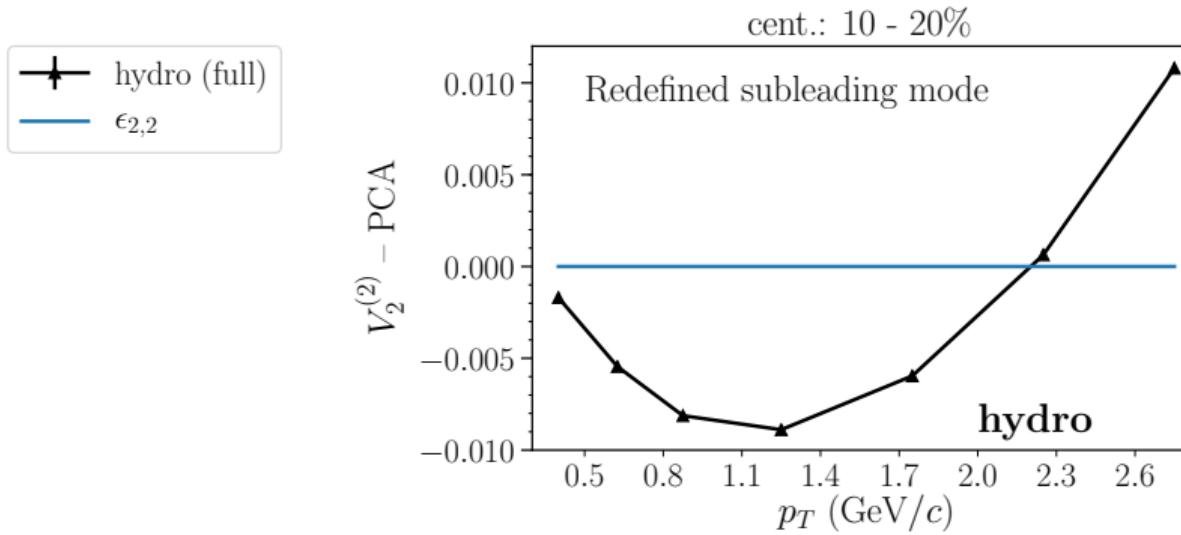


- Leading PCA mode  $\Rightarrow$  leading term in eccentricity expansion.
- Are there effects from subleading terms?

# Mapping PCA observables: Subleading flow

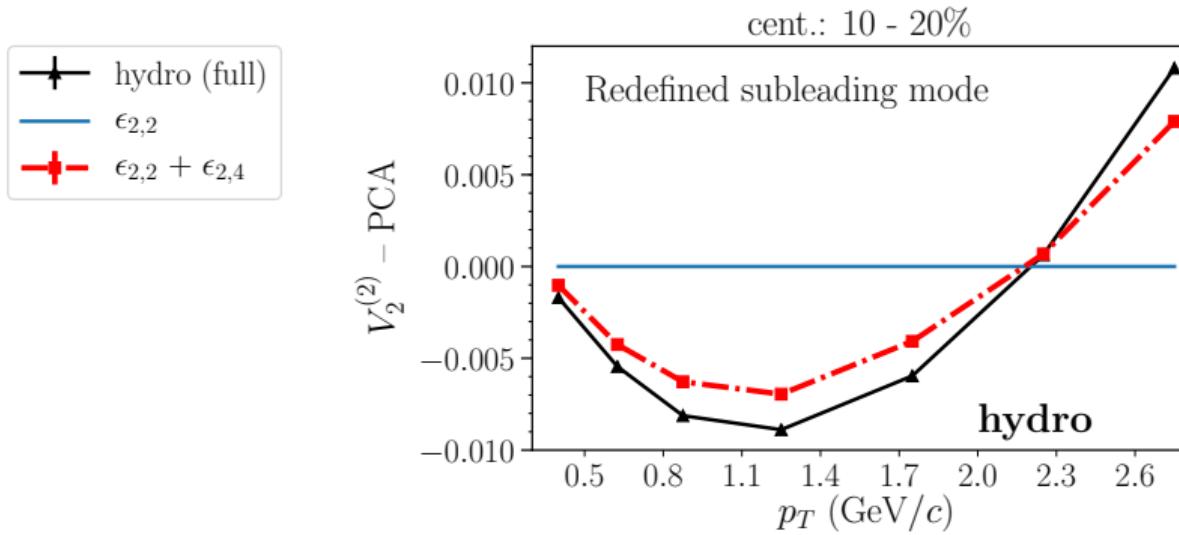


# Mapping PCA observables: Subleading flow



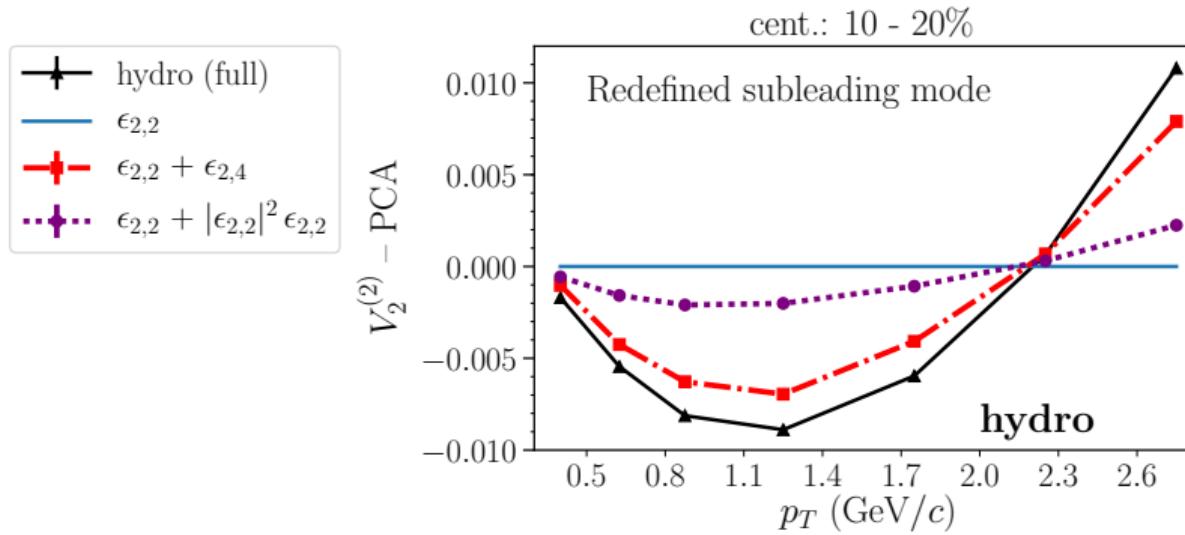
- At least two terms required for subleading mode.

# Mapping PCA observables: Subleading flow



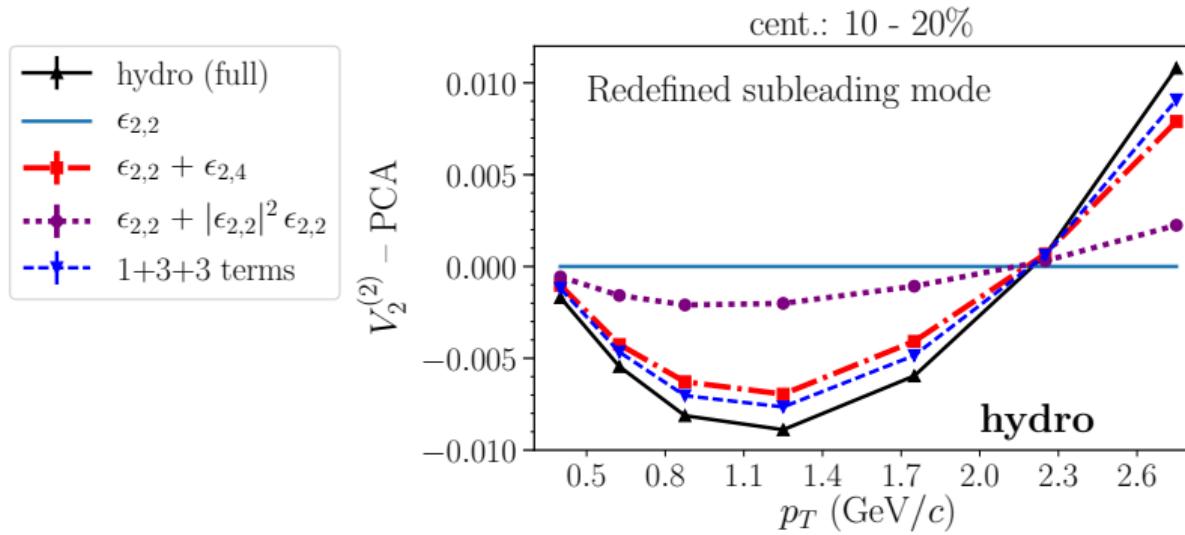
- At least two terms required for subleading mode.
- **Subleading  $V_2$  mode is sensitive to smaller-scale structure.**

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- Not as sensitive to nonlinear terms in central collisions.

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- Not as sensitive to nonlinear terms in central collisions.
- Estimators approach full result  $\Rightarrow$  **validation** of the framework.

# Conclusions

- New PCA observables **isolate** anisotropic flow **fluctuations**.
- Flow **fluctuations** at **differential level** captured by **cumulant expansion**.
- **Subleading mode** of  $V_2$  probes initial-state fluctuations at **smaller scales**.

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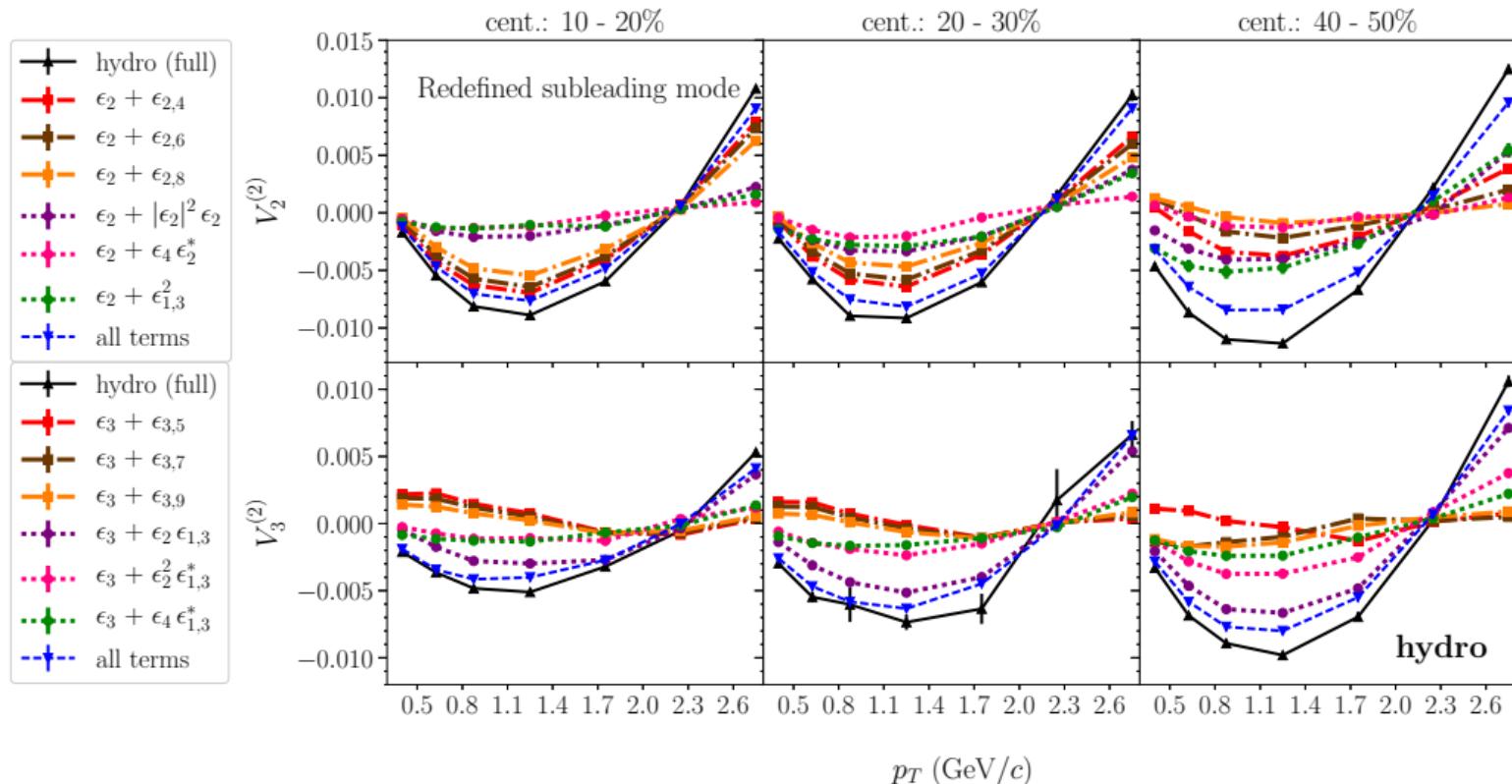


Grants:  
17/05685-2, 18/07833-1

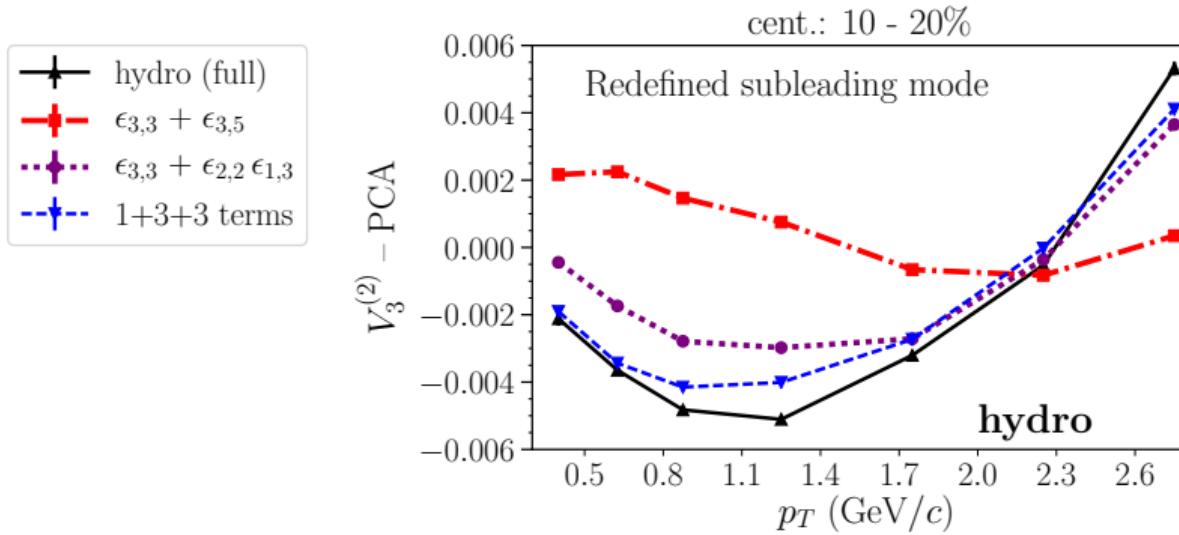
# Backup Slides

Backup slides...

# Subleading modes

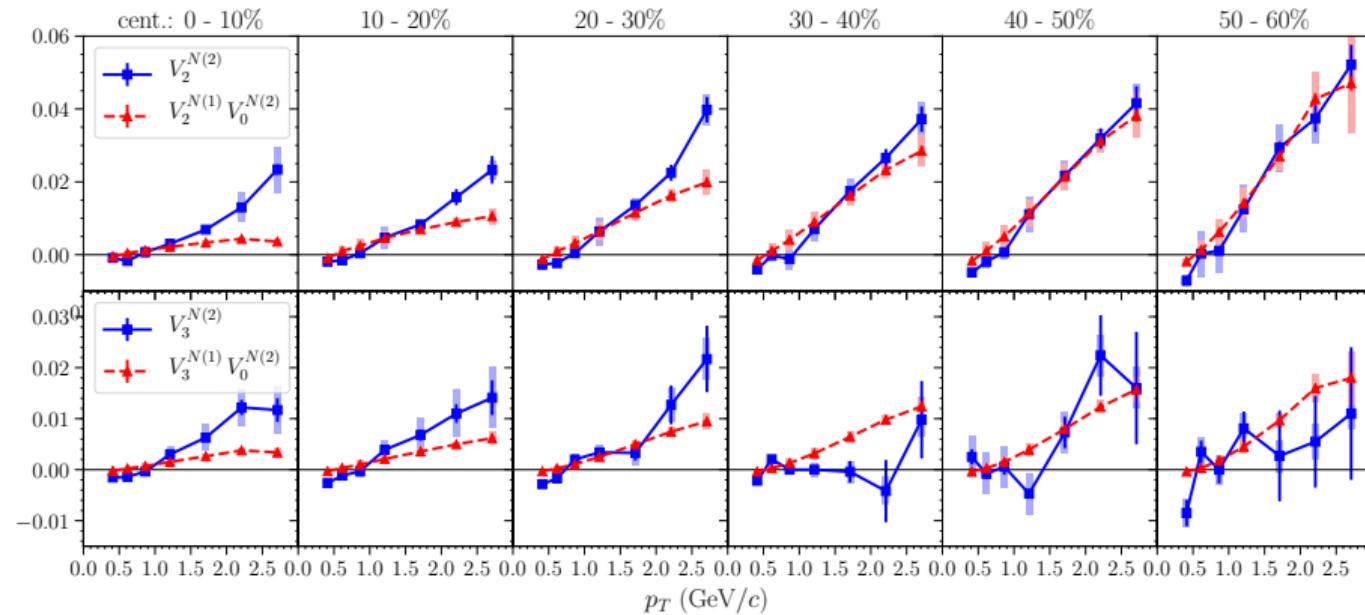


# Mapping PCA observables: Triangular flow



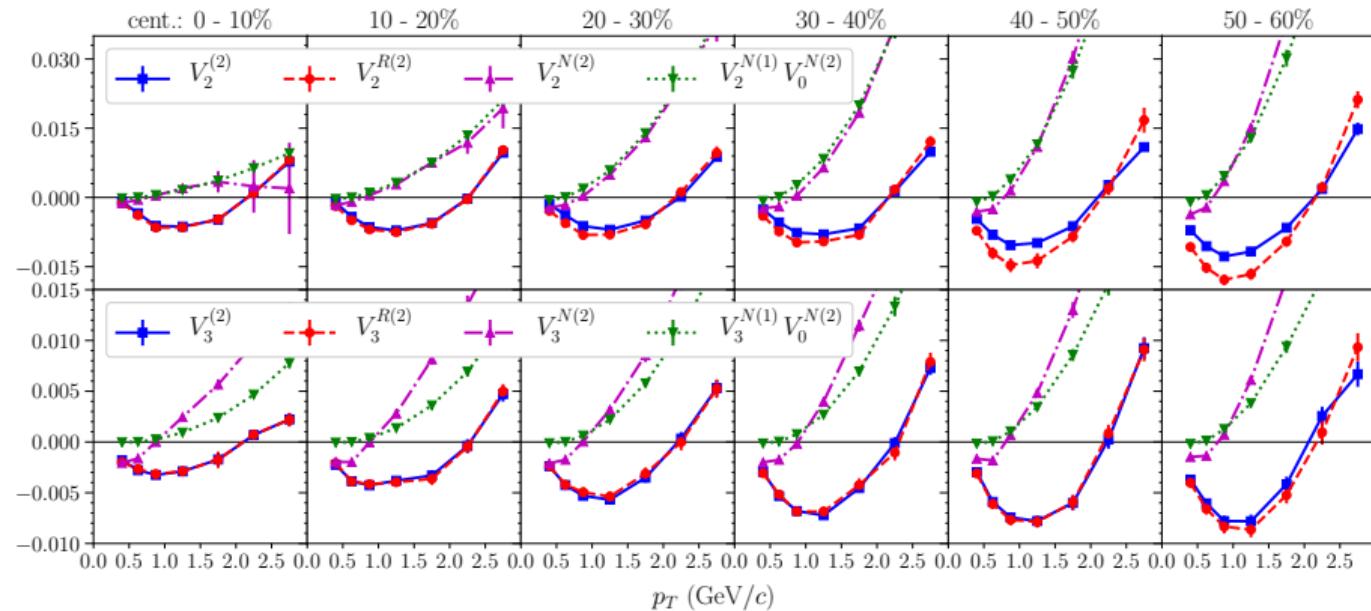
- **Subleading  $V_3$**  mode exhibits **nonlinear response** already at semicentral collisions.
- Estimators approach full result  $\Rightarrow$  **validation** of the framework.

# Multiplicity Fluctuations



MH, D.D. Chinellato, M. Luzum, J. Noronha,  
T. Nunes da Silva and J. Takahashi, arXiv:1906.08915

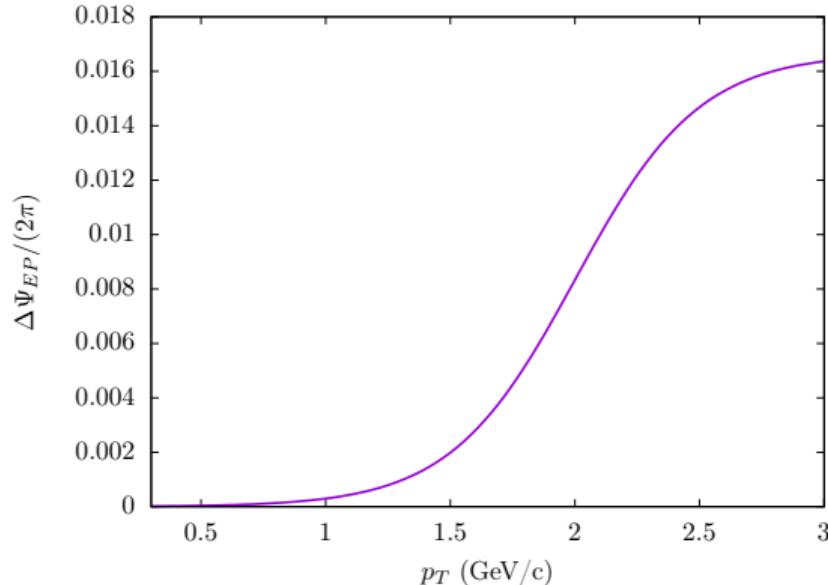
# Redefined PCA



MH, D.D. Chinellato, M. Luzum, J. Noronha,  
T. Nunes da Silva and J. Takahashi, arXiv:1906.08915

# Toy model

- Anisotropic flow via arbitrary azimuthal distribution.
- Multiplicity fluctuations: 10%-width fluctuations of  $\bar{p}_T$  and 20%-width fluctuations of the total multiplicity.
- Flow fluctuations via event-plane fluctuations with the profile below.



# Toy model

- Spectrum  $N(p_T) = N_0 p_T e^{-p_T/\overline{p_T}}$ , with  $\overline{p_T} = 0.5 \text{ GeV}/c$ .
- Angular distribution

$$\frac{dN}{d\varphi} = \frac{N(p_T)}{2\pi} \left[ 1 + 2 \sum_{n=2}^3 v_n(p_T) \cos(n(\varphi - \Psi_{EP})) \right], \quad (2)$$

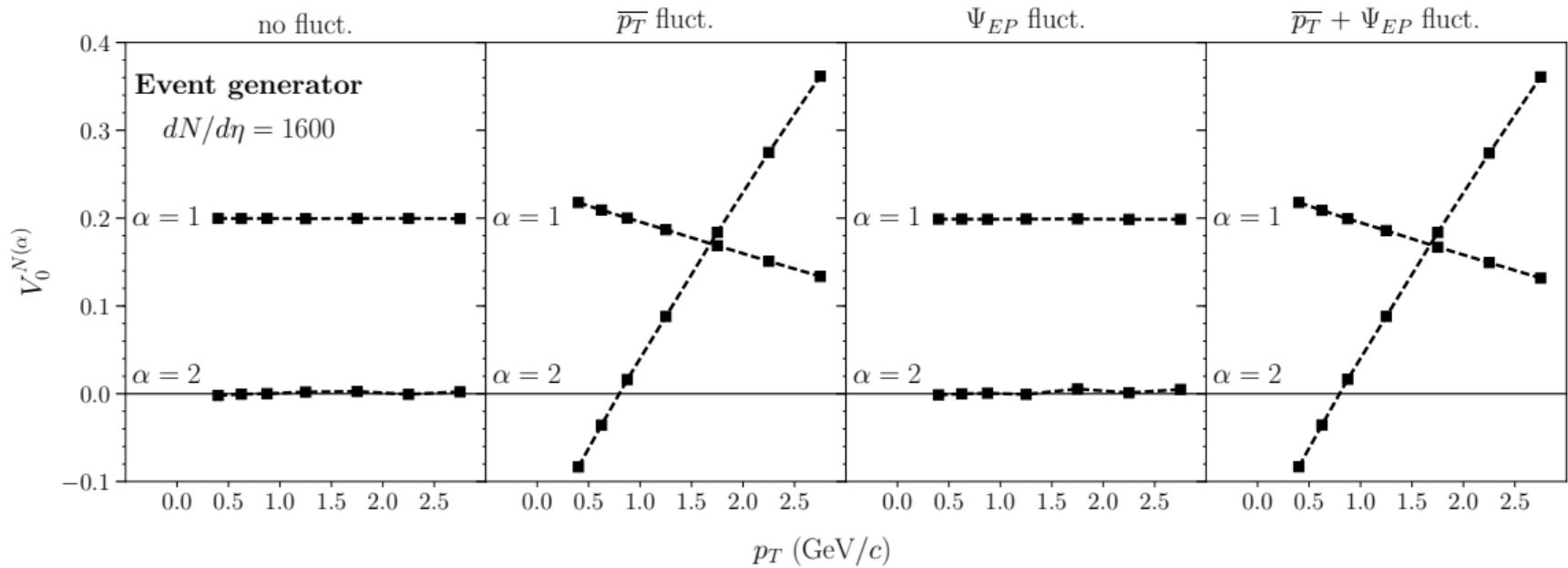
with

$$v_2(p_T) = N_2 \frac{p_T}{1 + (p_T/p_2^{\text{sat}}) (1 + p_T/p_2^{\text{dec}})}. \quad (3)$$

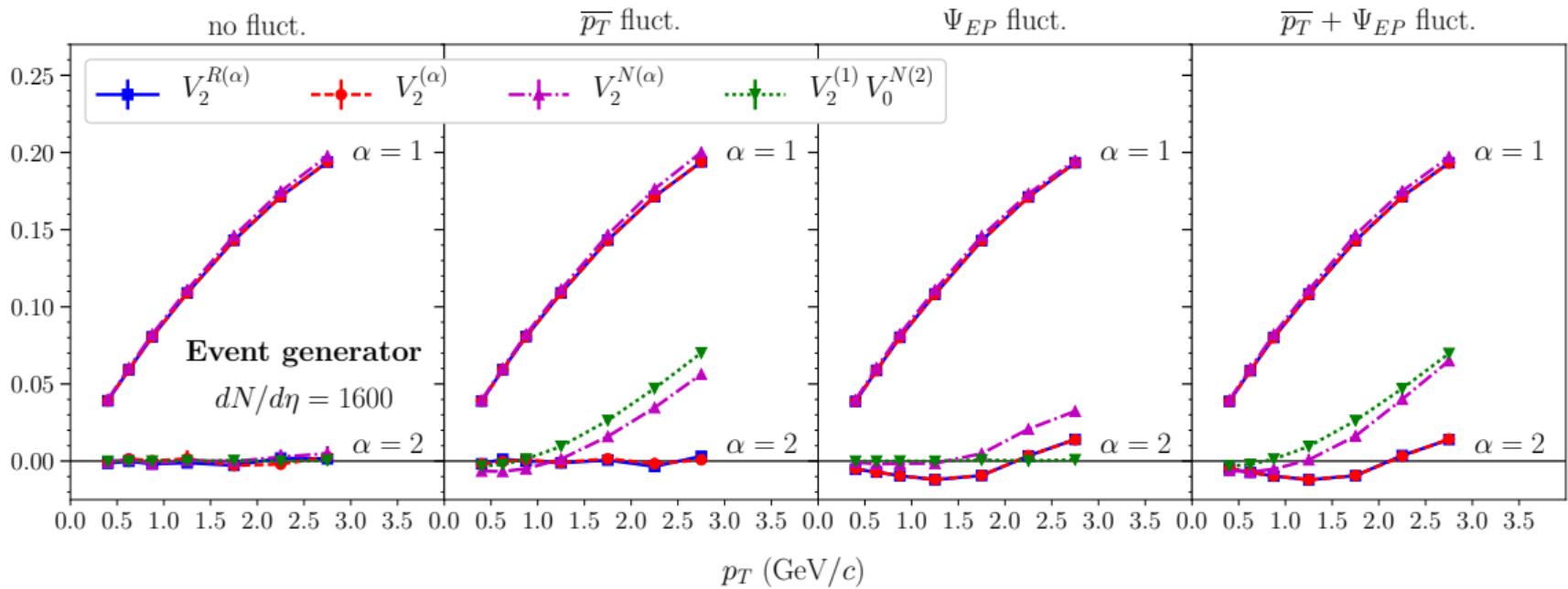
- Event-plane fluctuations with standard deviation

$$\Delta\Psi_{EP}(p_T) = N_{EP} \tanh\left(-\frac{p_T - p_{EP}}{\Delta p_{EP}}\right). \quad (4)$$

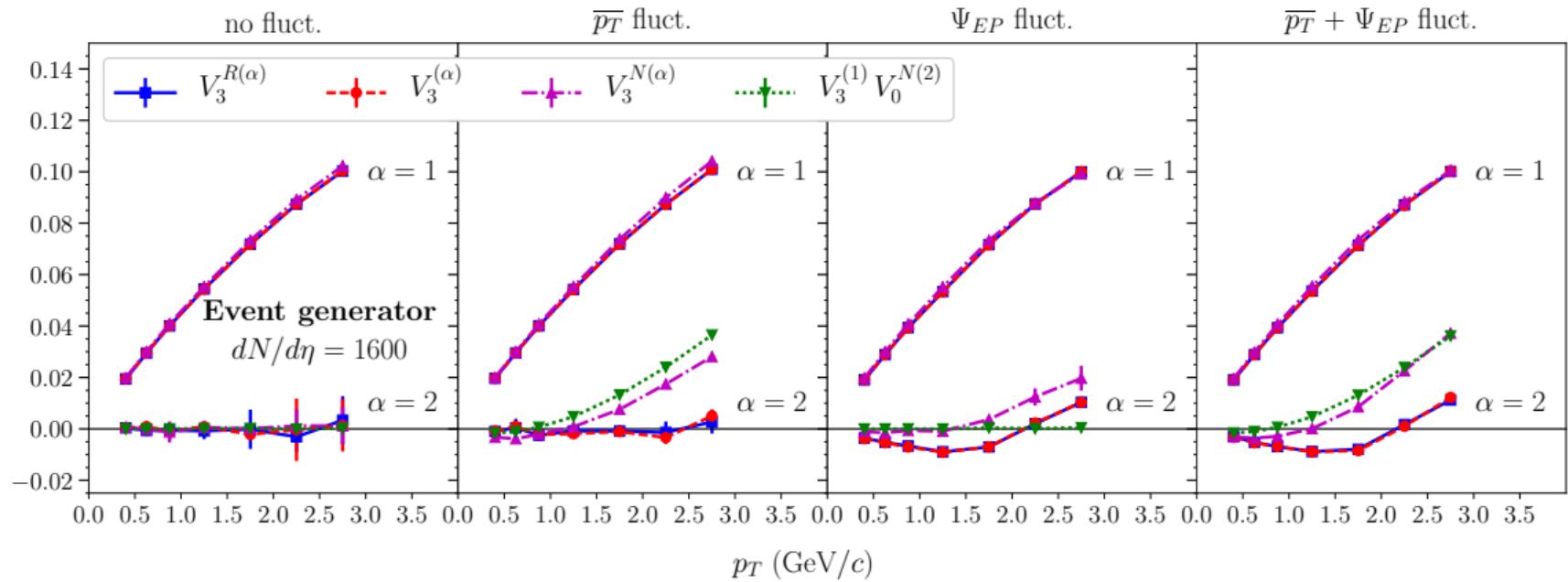
# Toy model results – Multiplicity Fluctuations



# Toy model results – Elliptic Flow



# Toy model results – Triangular flow



# Mapping hydrodynamic response

- **Eccentricities**  $\epsilon_{n,m}$  from cumulants of transverse density:  $\rho(\mathbf{x})$

$$W(\vec{k}) \equiv \ln \left( \int d^2x \rho(\mathbf{x}) e^{i\mathbf{k}\cdot\mathbf{x}} \right) \equiv \sum_{\substack{m=0 \\ n=-\infty}}^{\infty} W_{n,m} k^m e^{-in\phi_{\vec{k}}}$$

- Flow  $V_n$  can be predicted by the set of all  $\epsilon_{n,m}$ :

$$V_n = \mathcal{F}_n[\rho(\mathbf{x})] = f_n[\{\epsilon_{n',m}\}]$$

- Large scales (**small  $m$** ) dominate  $\Rightarrow$  **truncated expansion**

D. Teaney and L. Yan, PRC **83** (2011).

F. G. Gardim, F. Grassi, M. Luzum and J. Y. Ollitrault, PRC **85** (2012).

# Generalized eccentricities

Anisotropy and orientation of the initial density profile  $\rho(\vec{x})$  given by

$$W(\vec{k}) := \log \left( \int d^2x \rho(\vec{x}) e^{i\vec{k}\cdot\vec{x}} \right) := \sum_{m=0}^{\infty} \sum_{n=-\infty}^{\infty} W_{n,m} k^m e^{-i n \phi_{\vec{k}}} . \quad (5)$$

Dimensionless eccentricities from cumulants:

$$\epsilon_{n,m} := W_{n,m} / (W_{0,2})^{m/2} . \quad (6)$$

# General hydrodynamic estimators

$$\begin{aligned} V_n^{\text{est}}(p_T, y) = & \sum_{m=\max[n,2]}^{m_{\max}} \kappa_{n,m}(p_T, y) \epsilon_{n,m} + \\ & + \sum_{p=2}^{p_{\max}} \sum_{\substack{\{n_i, m_i\} \\ \sum n_i = n \\ m < m_{\max}}} \kappa_{\{n_i, m_i\}}^{(p)}(p_T, y) \prod_{i=1}^p \epsilon_{n_i, m_i}, \end{aligned} \tag{7}$$

# Cumulant examples

$$W_{0,2} = \frac{i^2}{2!} \frac{1}{2} \left[ \{r^2\} - \{r e^{-i\phi}\} \{r e^{-i\phi}\} \right], \quad (8)$$

$$W_{2,2} = \frac{i^2}{2!} \frac{1}{4} \left[ \{r^2 e^{2i\phi}\} - \{r e^{i\phi}\}^2 \right], \quad (9)$$

$$W_{1,3} = \frac{i^3}{3!} \frac{3}{8} \left[ \{r^3 e^{i\phi}\} - \{r^2 e^{2i\phi}\} \{r e^{-i\phi}\} \right], \quad (10)$$

where

$$\{\dots\} := \frac{\int d^2x \rho(\vec{x}) (\dots)}{\int d^2x \rho(\vec{x})} \quad (11)$$

# The framework

- Flow harmonics  $V_n$  can be predicted by  $\epsilon_{n,m}$ .
- The  $\kappa$ 's are found by minimizing

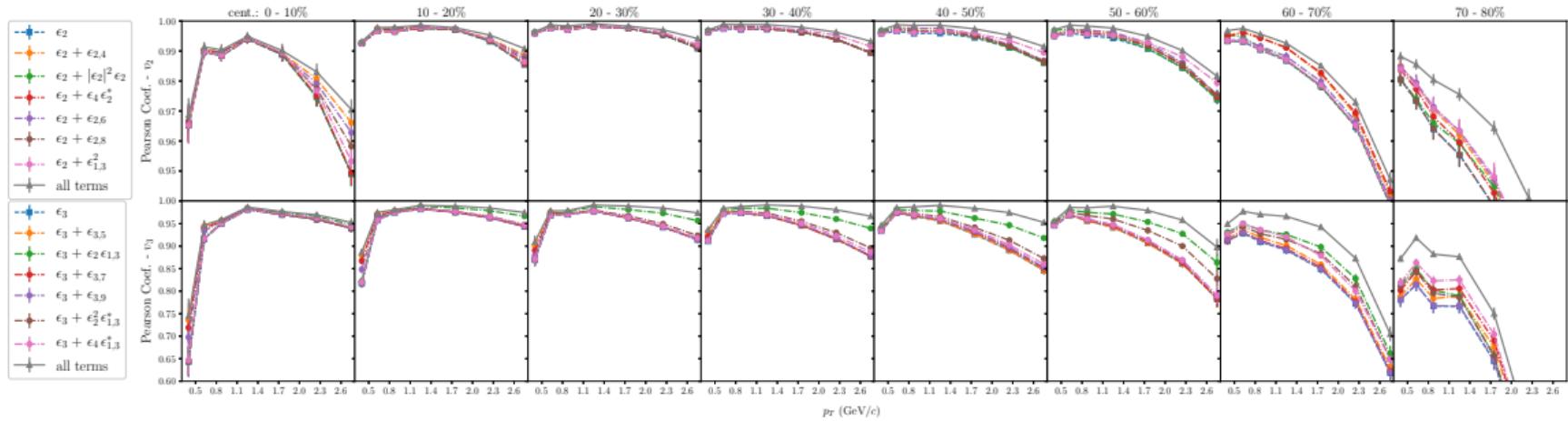
$$\langle |\delta_{\text{residual}}| \rangle^2 := \langle |V_n^{\text{est.}} - V_n|^2 \rangle. \quad (12)$$

- The quality of the predictor can be measured by the Pearson coefficient

$$\frac{\langle V_n^* V_n^{\text{est.}} \rangle}{\sqrt{\langle |V_n|^2 \rangle \langle |V_n^{\text{est.}}|^2 \rangle}} \leq 1. \quad (13)$$

- To predict differential flow,  $V_n(p_T)$ , find  $\kappa(p_T)$ .

# Pearson coefficients



## Bin-independent orthonormality condition

Naive diagonalization of  $V_{n\Delta}^{ab}$  with conventional algorithms will result in

$$\sum_a \psi_n^{(\alpha)a} \psi_n^{(\beta)a} = \delta_{\alpha\beta}, \quad (14)$$

not compatible with

$$\int dp_T \psi_n^{(\alpha)}(p_T) \psi^{(\beta)}(p_T) = \delta_{\alpha\beta}. \quad (15)$$

We instead diagonalize

$$\sqrt{\Delta p_T^a \Delta p_T^b} V_{n\Delta}^{ab} = \sum_i \lambda^{(\alpha)} \tilde{\psi}_n^{(\alpha)a} \tilde{\psi}_n^{(\alpha)b} \quad (16)$$

$$\sum_a \tilde{\psi}_n^{(\alpha)a} \tilde{\psi}_n^{(\beta)a} = \delta_{\alpha\beta}. \quad (17)$$

Replacing  $\tilde{\psi}^{(\alpha)a} \rightarrow \sqrt{\Delta p_T^a} \psi^{(\alpha)a}$ :

$$V_{n\Delta}^{ab} = \sum \lambda^{(\alpha)} \psi_n^{(\alpha)a} \psi_n^{(\alpha)b}, \quad \sum \Delta p_T^a \psi_n^{(\alpha)a} \psi_n^{(\beta)a} = \delta_{\alpha\beta}. \quad (18)$$