Longitudinal dynamics of multiple conserved charges

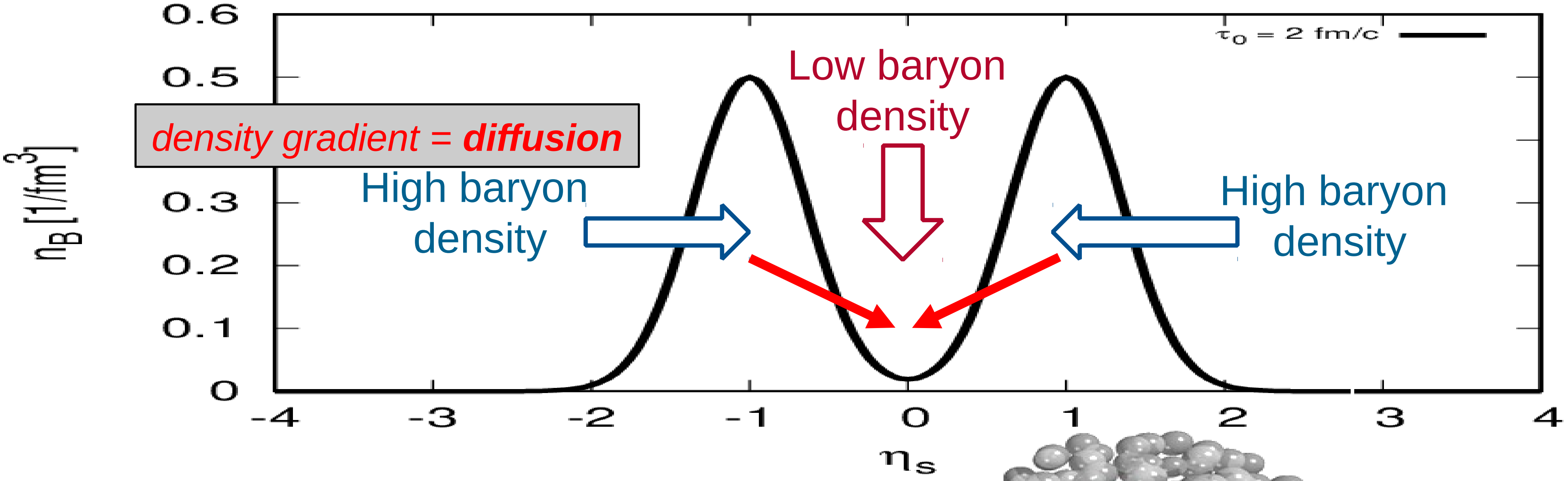
Jan Fotakis

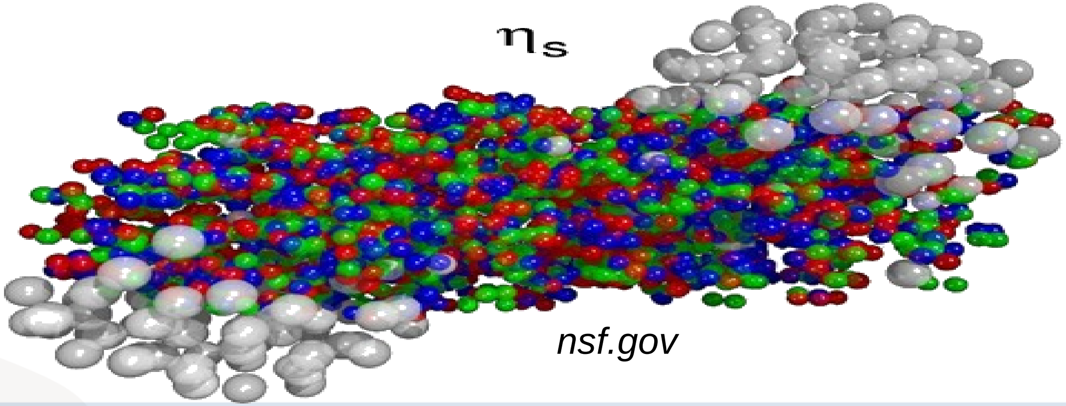
Harri Niemi, Moritz Greif, Gabriel Denicol, Carsten Greiner

*M. Greif et al., Phys. Rev. Lett. **120**, 242301 (2018)*
Coming soon: J.A. Fotakis et al., arXiv:1911.xxxxx

Quark Matter 2019

Motivation





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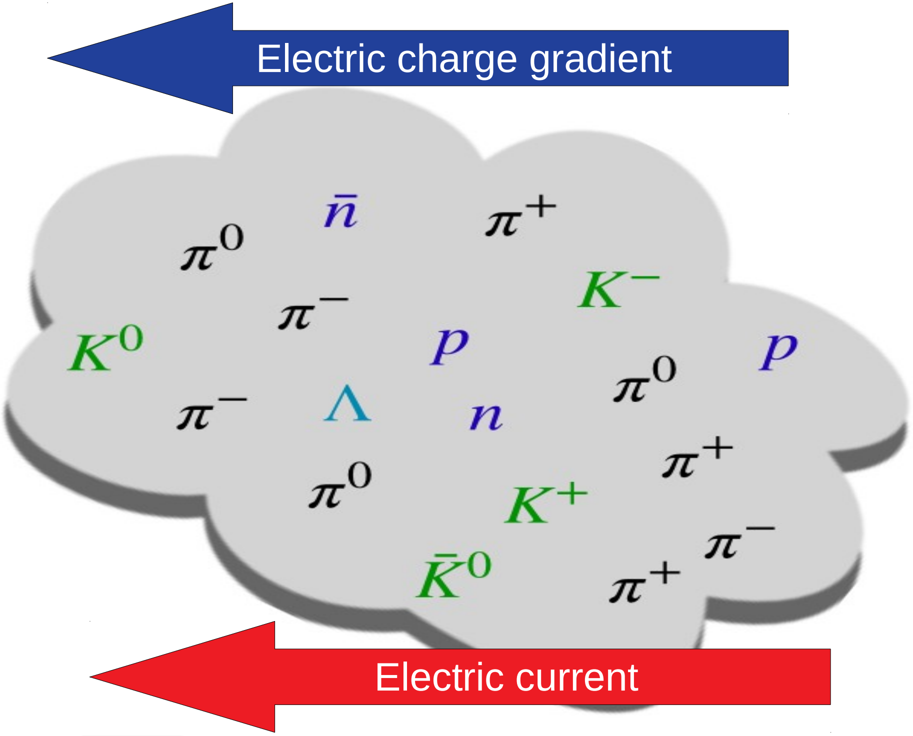
Motivation



Particles carry a multitude of quantum numbers (“*mixed chemistry*”)

= **currents are correlated/coupled!**

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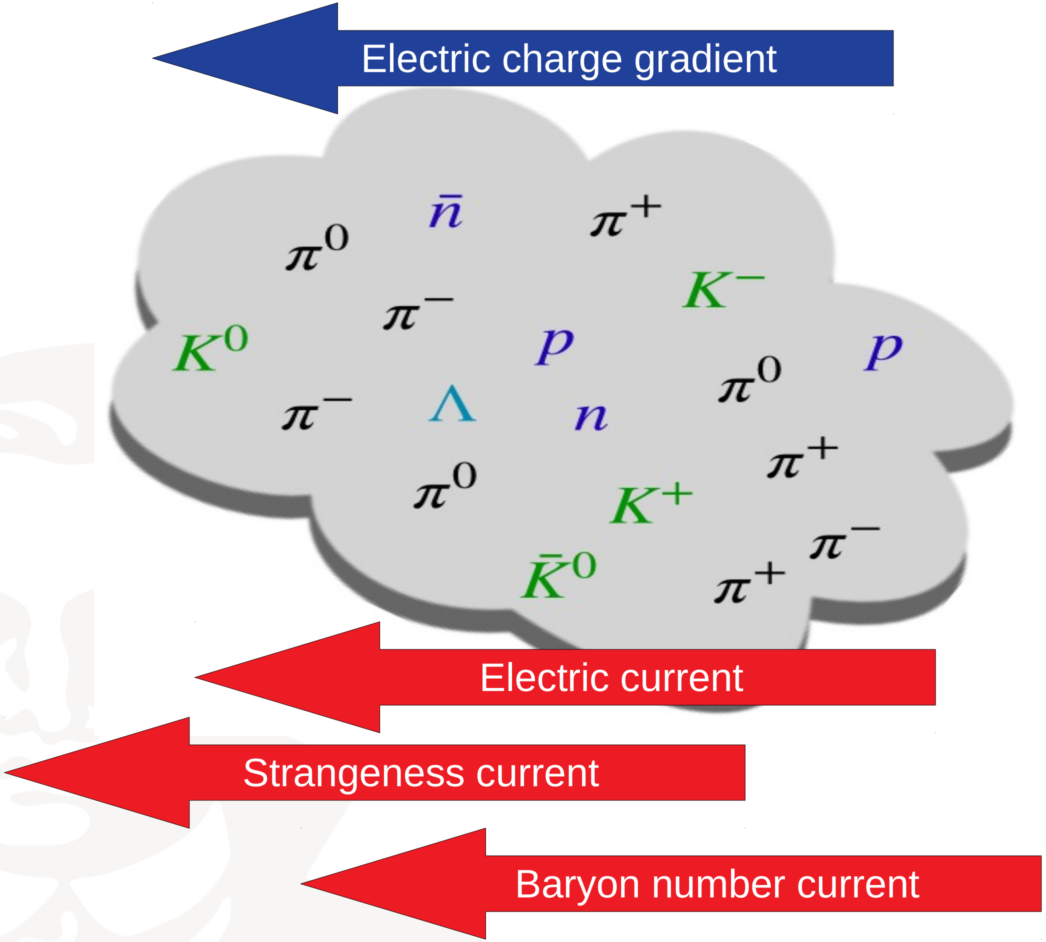

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= **currents are correlated/coupled!**

Also relevant for external electro-magnetic fields!

**J.-B. Rose et al.
(EM1 session)**

Coupled charges in DNMR

- System with **one conserved quantum number** *q* only

$$\tau_q \, \dot{V}_q^{\langle\mu\rangle} + V_q^\mu = \boldsymbol{\kappa_q \nabla^\mu (\mu_q / T)} + \mathcal{O}(2)(\theta, \pi^{\mu\nu}, ...)$$

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- For system with **multiple conserved quantum numbers**:
  mixed chemistry introduces ***coupling of charges through diffusion coefficient matrix***!

$$V_B^\mu \sim \kappa_B \nabla^\mu(\mu_B/T) \;\rightarrow\; \begin{pmatrix} V_B^\mu \\ V_S^\mu \end{pmatrix} \sim \begin{pmatrix} \kappa_{BB} & \kappa_{BS} \\ \kappa_{SB} & \kappa_{SS} \end{pmatrix} \begin{pmatrix} \nabla^\mu(\mu_B/T) \\ \nabla^\mu(\mu_S/T) \end{pmatrix}$$

$$\tau_q \, \dot{V}_q^{\langle\mu\rangle} + V_q^\mu = \boldsymbol{\sum_{q'} \kappa_{qq'} \nabla^\mu(\mu_{q'}/T)} + \mathcal{O}(2)$$

M. Greif, J.A. Fotakis, G.S. Denicol, C. Greiner, Phys. Rev. Lett. ***120****, 242301 (2018)*

The setting

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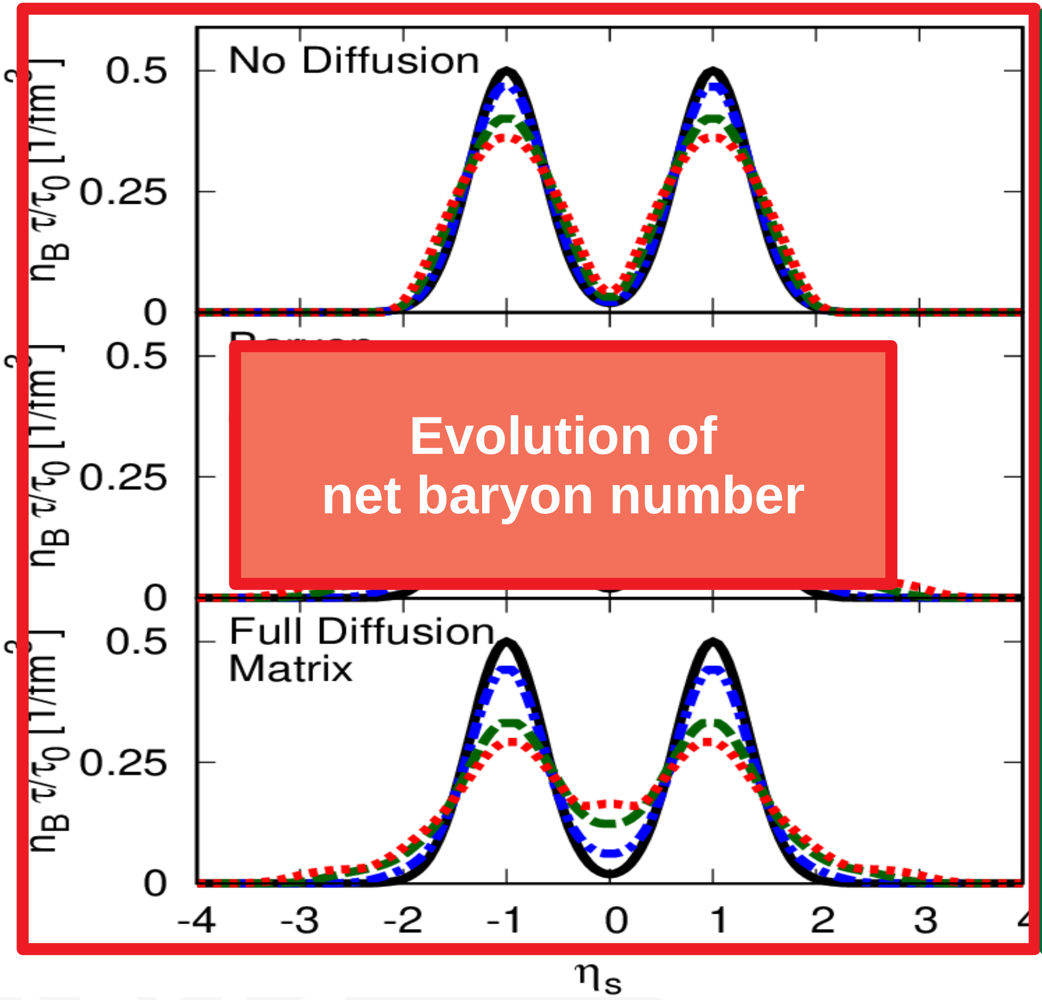
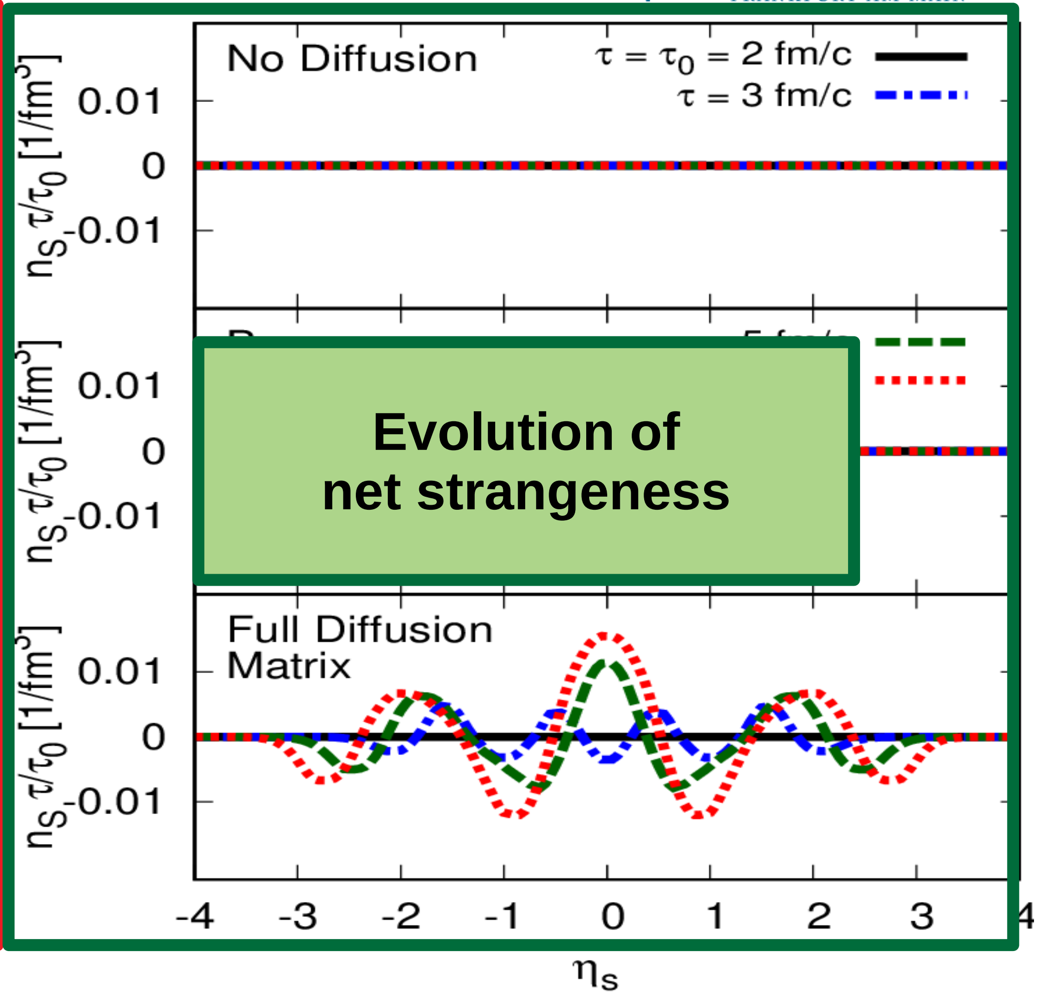
M. Greif, J.A. Fotakis, G.S. Denicol, C. Greiner, Phys. Rev. Lett. ***120****, 242301 (2018)*

- ***Simple initial state:*** T = 160 MeV, no initial net strangeness, longitudinal double-gaussian profile in net baryon number, no initital dissipative currents

Results (Orientation)





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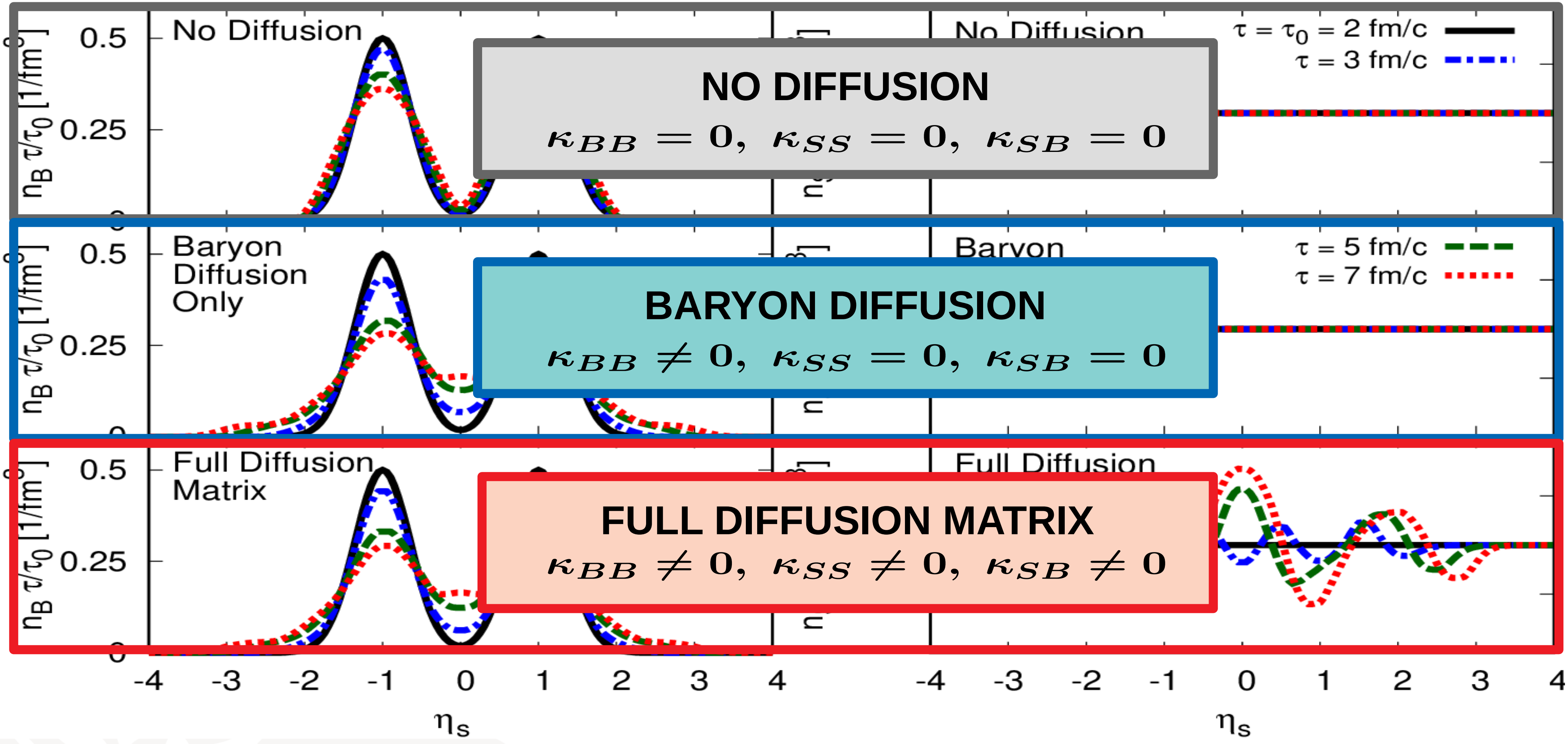

Results

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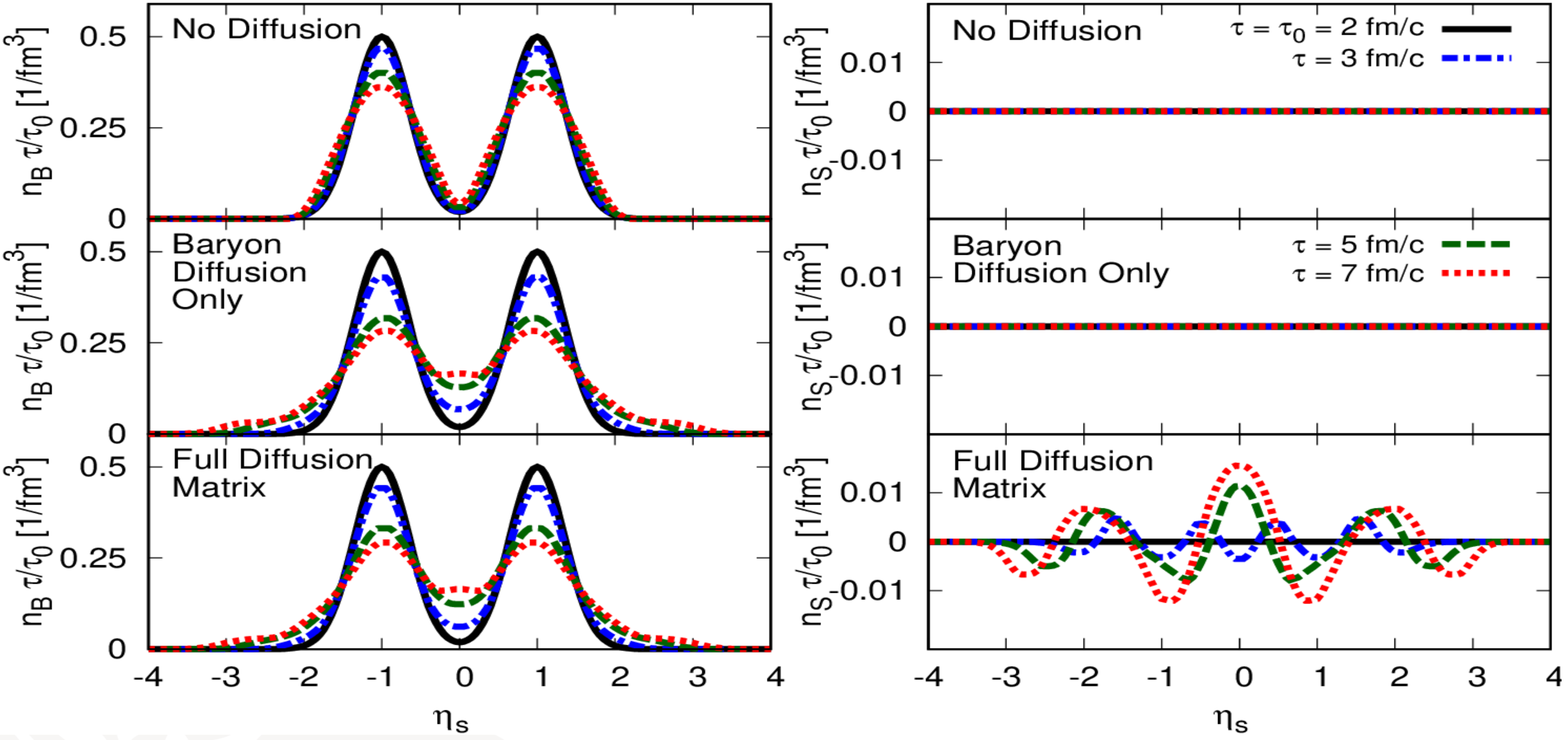


Conclusion

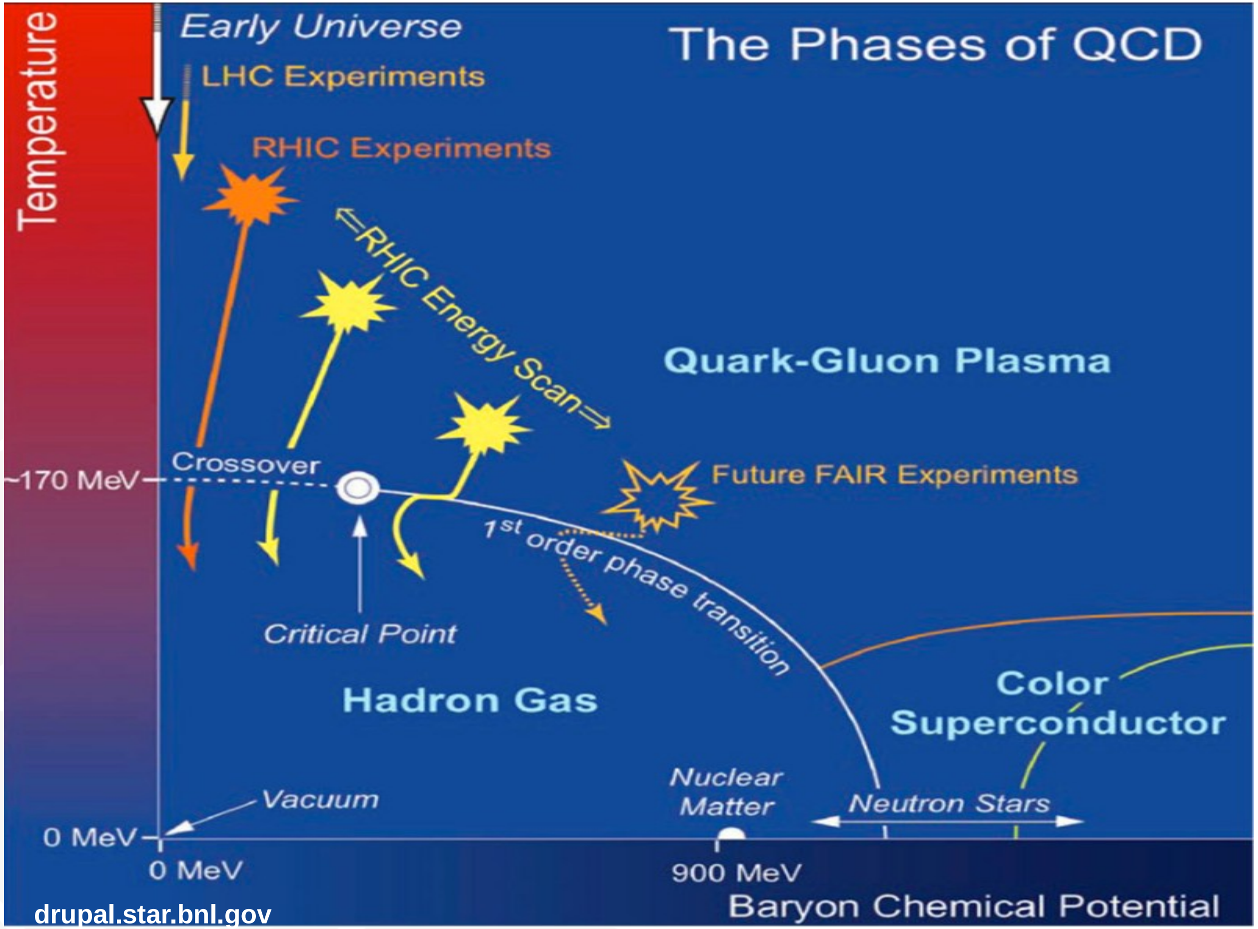
- Introduced multiple conserved charges to (3+1)D-hydro (code)
- Computed diffusion coefficient matrix from the linearized Boltzmann equation for a hadronic system with realistic elastic, isotropic, binary cross sections
- Investigated the *longitudinal* evolution of net baryon number and net strangeness for simple initial conditions
- Found **baryon-strangeness correlation** introduced by EoS and coupled diffusion currents; up-building of non-trivial strangeness profile
- Investigated second-order terms: shear-stress could have a significant impact on diffusive evolution

Outlook

- Investigate transverse and full 3D-evolution
- Use more realistic (fluctuating) initial conditions and equation of state
- Proceed with **particilization (Cooper-Frye), freeze-out and compare to experimental data**
- Use temperature parametrizations for shear viscosity and use hadronic second-order transport coefficients
- Derive ***multi-component fluid dynamics***: coupling second-order terms?
- Compare to transport models (e.g. BAMPS, SMASH)
  and other fluid dynamic approaches

BACKUP

Motivation



LHC:

$\mu_B \approx 0$ and vanishingly small gradients

RHIC BES:

$\mu_B \approx 200 - 400\,\mathrm{MeV}$ and large gradients possible

Fluid dynamics

Bulk matter ***close to local equilibrium*** is characterized by ***macroscopic*** quantities:

- Thermal densities (energy, quantum number)
- Equation of state (isotropic pressure, temperature, chemical potentials)
- Velocity field
- Dissipative currents (bulk viscousity, **diffusion**, shear viscosity)

Energy-momentum current

$$\mathrm{T}^{\mu\nu} = \epsilon\, u^{\mu} u^{\nu} - (P(\epsilon, \{n_q\}) + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

Current of conserved quantum number *q*

(net electric charge, net baryon number, net strangeness, ...)

$$\mathrm{N}_q^{\mu} = n_q u^{\mu} + V_q^{\mu}$$

Fluid dynamics: equations of motion

Dynamics determined by ...

… energy-momentum conservation: $\partial_\nu T^{\mu\nu} = 0$

… conservation of quantum numbers *q*: $\partial_\mu N_q^\mu = 0$

- 4 + N equations, but ...
- … 10 ($T^{\mu\nu}$) + 4N ($\{N_q^\mu\}$) degrees of freedom (d.o.f.)
- 1 d.o.f. is determined by the equation of state
- Additional 5 + 3N equations needed (**dissipation**)

Denicol-Niemi-Molnar-Rischke theory (DNMR)

- Here: neglecting bulk viscosity ($\Pi \equiv 0$)
- Shear-stress:

Second order terms

Relaxation time

$$\boxed{\tau_\pi}\,\dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = \boxed{2\eta\sigma^{\mu\nu}}\ \boxed{- \frac{4}{3}\pi^{\mu\nu}\theta - 2\pi^{\langle\mu}_{\ \ \lambda}\omega^{\nu\rangle\lambda} - \frac{10}{7}\pi^{\lambda\langle\mu}\sigma^{\nu\rangle}_{\ \ \lambda}}$$

Navier-Stokes term

Notation:

$$\text{Expansion parameter:} \quad \theta \equiv \partial_\mu u^\mu$$

$$\text{Shear tensor:} \quad \sigma^{\mu\nu} = \frac{1}{2}(\nabla^\mu u^\nu + \nabla^\nu u^\mu) - \frac{1}{3}\theta\Delta^{\mu\nu}$$

$$\text{Vorticity:} \quad \omega^{\mu\nu} = \frac{1}{2}(\nabla^\mu u^\nu - \nabla^\nu u^\mu)$$

*G. Denicol, H. Niemi, E. Molnar, D. Rischke, Phys. Rev. D **85**, 114047 (2012)*
*K. Gallmeister, H. Niemi, C. Greiner, D. Rischke, Phys. Rev. C **98**, 024912 (2018)*

Equation of state

- Hadronic system including lightest 19 species

$$\pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0, p, \bar{p}, n, \bar{n}, \Lambda^0, \bar{\Lambda}^0, \Sigma^0, \bar{\Sigma}^0, \Sigma^\pm, \bar{\Sigma}^\pm$$

- Assume classical statistics and non-interacting limit

$$P_0(T, \{\mu_q\}) \equiv \frac{1}{3} \sum_{i=1}^{N_{\text{species}}} \int \frac{\mathrm{d}p^3}{(2\pi)^3 E_{i,p}} \left(E_{i,p}^2 - m_i^2\right) g_i \exp(-E_{i,p}/T + \sum_q q_i \alpha_q)$$

- Only assume baryon number and strangeness, neglect electric charge

- Tabulate state variables over energy density and net charge densities

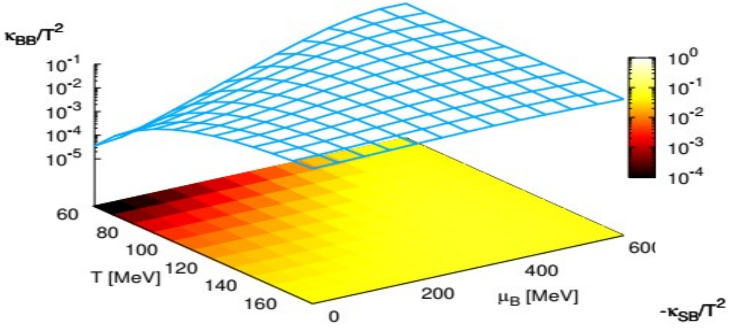
$$T \equiv T(\epsilon, \{n_q\}), \quad \mu_q \equiv \mu_q(\epsilon, \{n_q\}), \quad P_0 \equiv P_0(\epsilon, \{n_q\})$$

Diffusion coefficient matrix

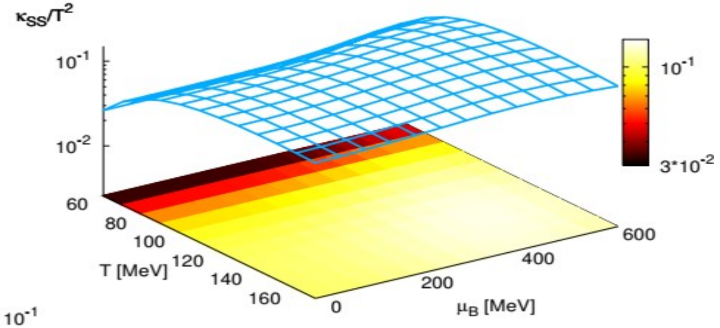
$$\begin{pmatrix} V_B^\mu \\ V_S^\mu \end{pmatrix} \sim \begin{pmatrix} \kappa_{BB} & \kappa_{BS} \\ \kappa_{SB} & \kappa_{SS} \end{pmatrix} \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_S \end{pmatrix}$$

- Matrix is symmetric

L. Onsager, Phys. Rev. ***37****, 405 (1931) & Phys. Rev.* ***38****, 2265 (1931)*

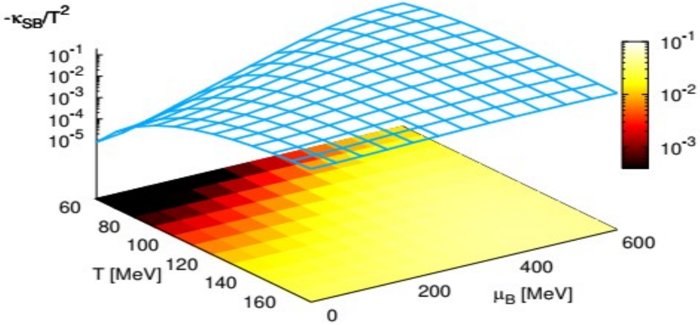


- Elastic isotropic cross sections from PDG, SMASH, GiBUU, UrQMD



κ_{SB} is negative and has similar magnitude as κ_{BB}

$\Longrightarrow$ significant coupling?

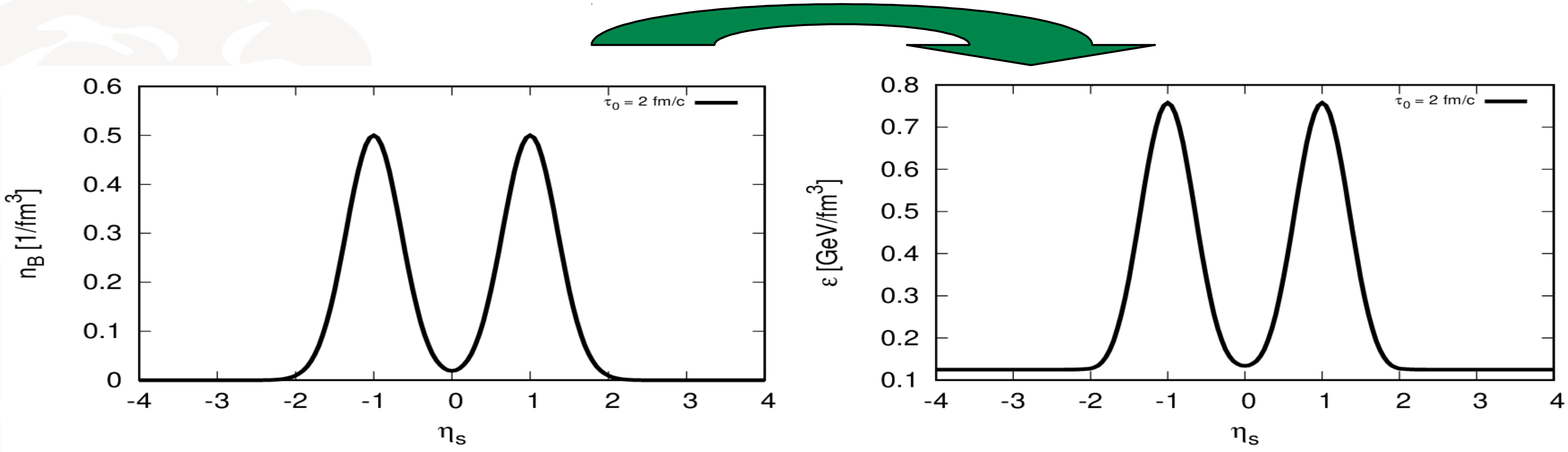


- Tabulate coefficient matrix over $T,\ \mu_B,\ \mu_S$
- $\mu_Q = 0$

M. Greif, J.A. Fotakis, G.S. Denicol, C. Greiner, Phys. Rev. Lett. ***120****, 242301 (2018)*

Initial conditions

- $\tau_0 = 2\ \mathrm{fm/c}$
- Initially: no dissipation and only **Bjorken scaling flow**
- Temperature = 160 MeV
- Double-gaussian profile in net baryon number
- From **EoS**: get energy density



Results (no second-order terms, no shear)

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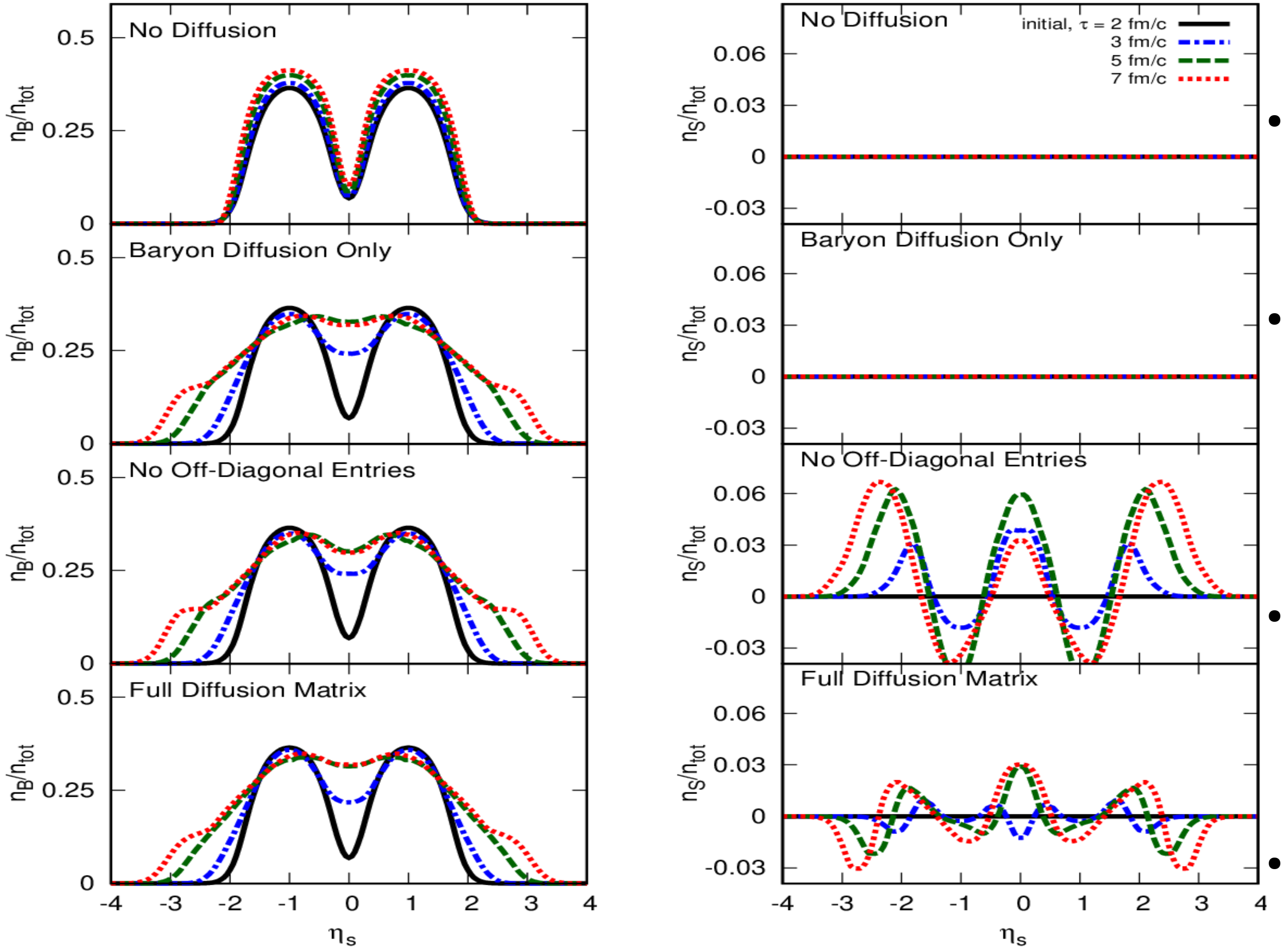


- Chemistry causes baryon-strangeness correlation through ...
- … the EoS which affects the gradients

$$\mu_S \equiv \mu_S(\epsilon, n_B, n_S)$$

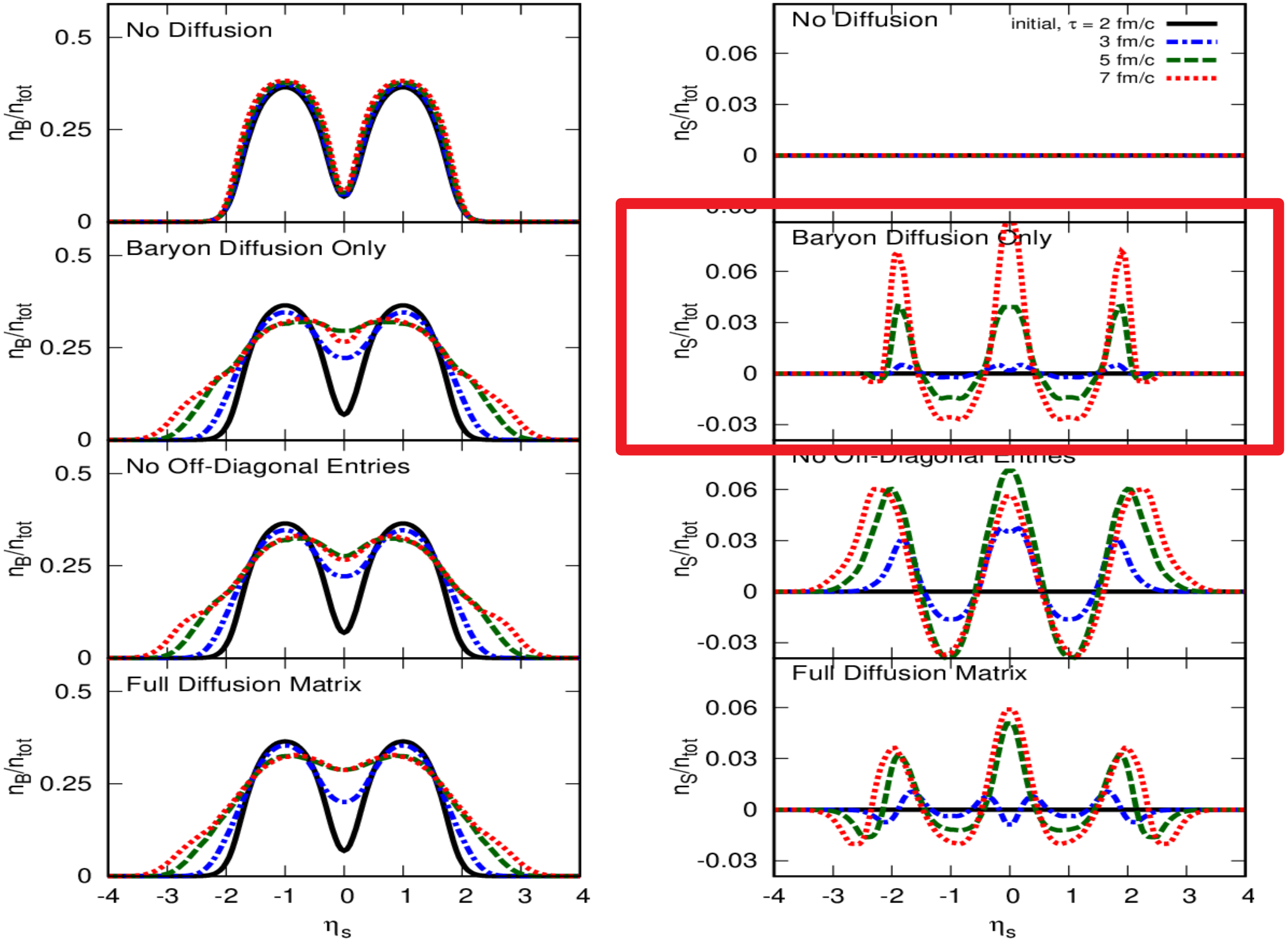
$$\nabla^\mu \alpha_S \sim \nabla^\mu n_B$$

- … the Navier-Stokes terms give diffusive correlation

$$\max\left(\frac{n_S}{n_{\text{tot}}}\right) \approx 3\%$$

- Magnitude of effect in ‘full’ case:

Results (all terms + shear)



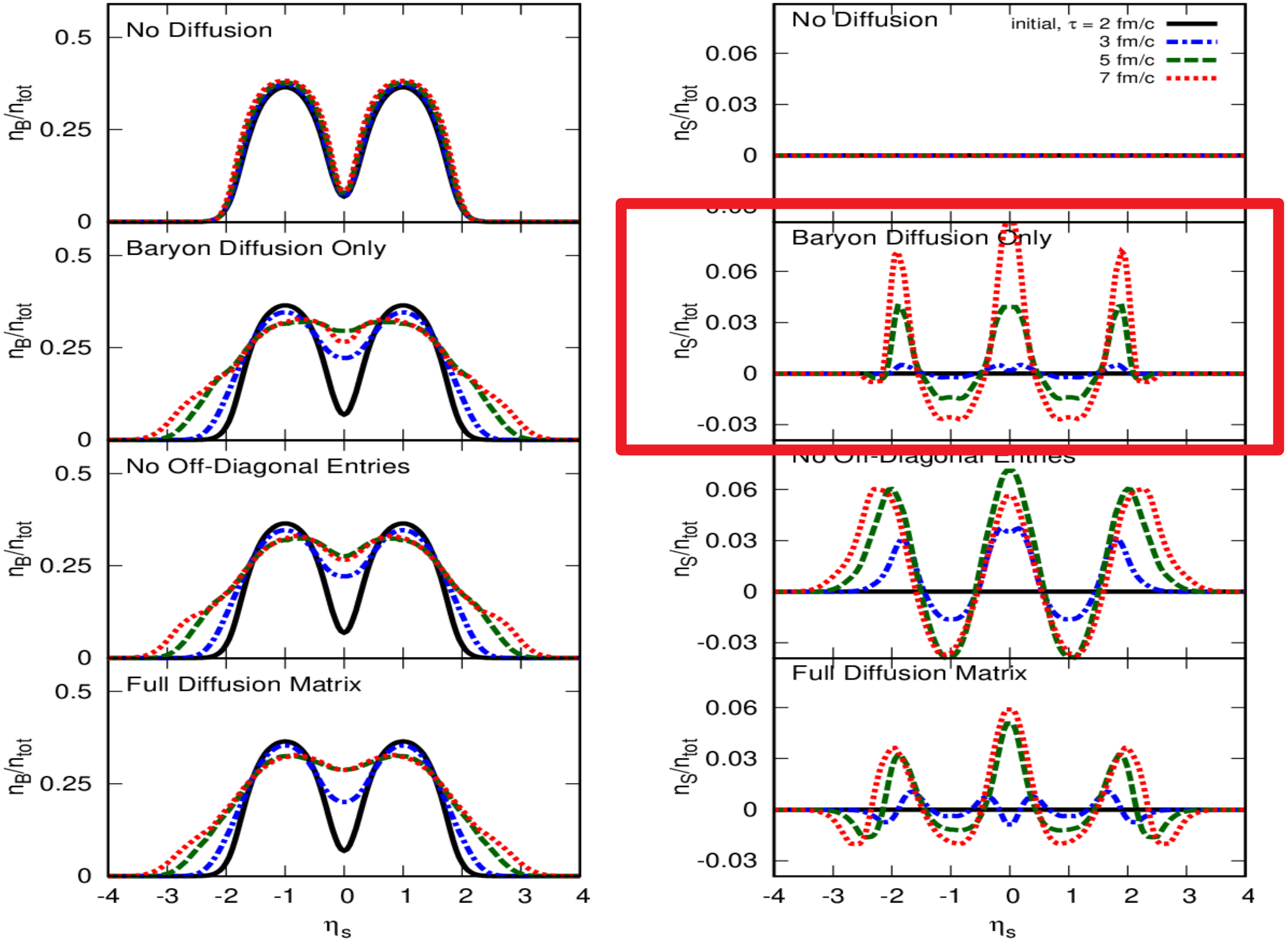
$$\frac{\eta}{s} = \frac{1}{4\pi}$$

- Shear seems to enhance diffusive effects
- Magnitude of effect in 'full' case:

$$\max\left(\frac{n_S}{n_{\mathrm{tot}}}\right) \approx 6\%$$

- At least for 'baryon diffusion only'-case this is problematic!
- Strangeness diffusion should not occur here

Results (all terms + shear)



$$\frac{\eta}{s} = \frac{1}{4\pi}$$

- Reason:

  for second-order terms we assumed transport coefficients from **ultrarelativistic**, single-component case!