Hydro+ in action: understanding the off-equilibrium dynamics near the QCD critical point

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Based on Rajagopal-Ridgway-Weller-YY, 1908.08539

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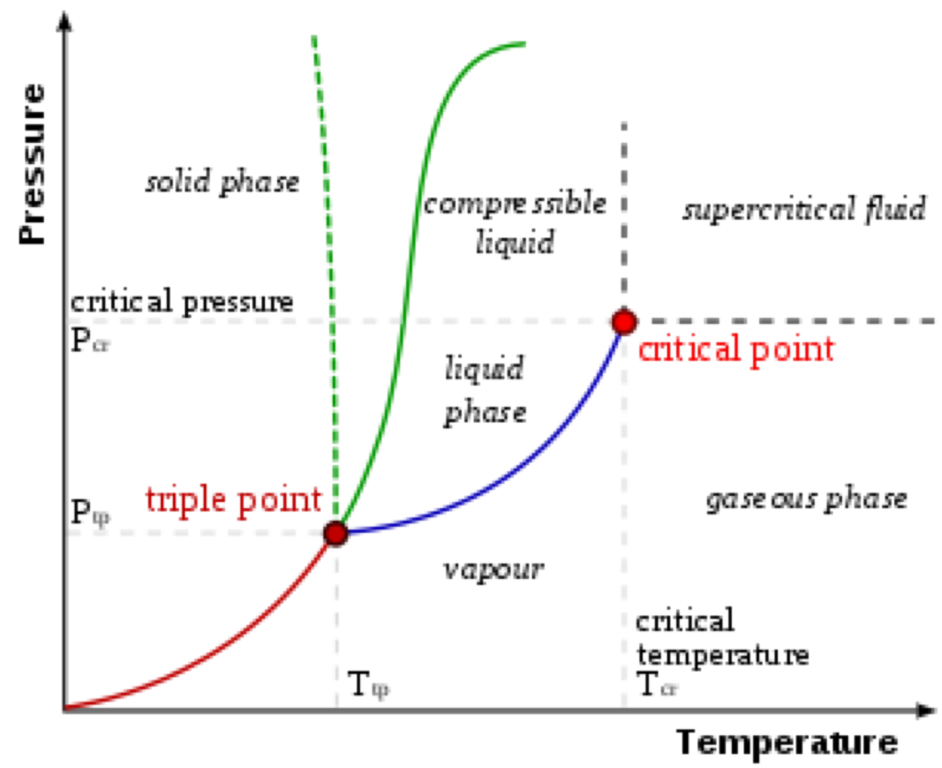
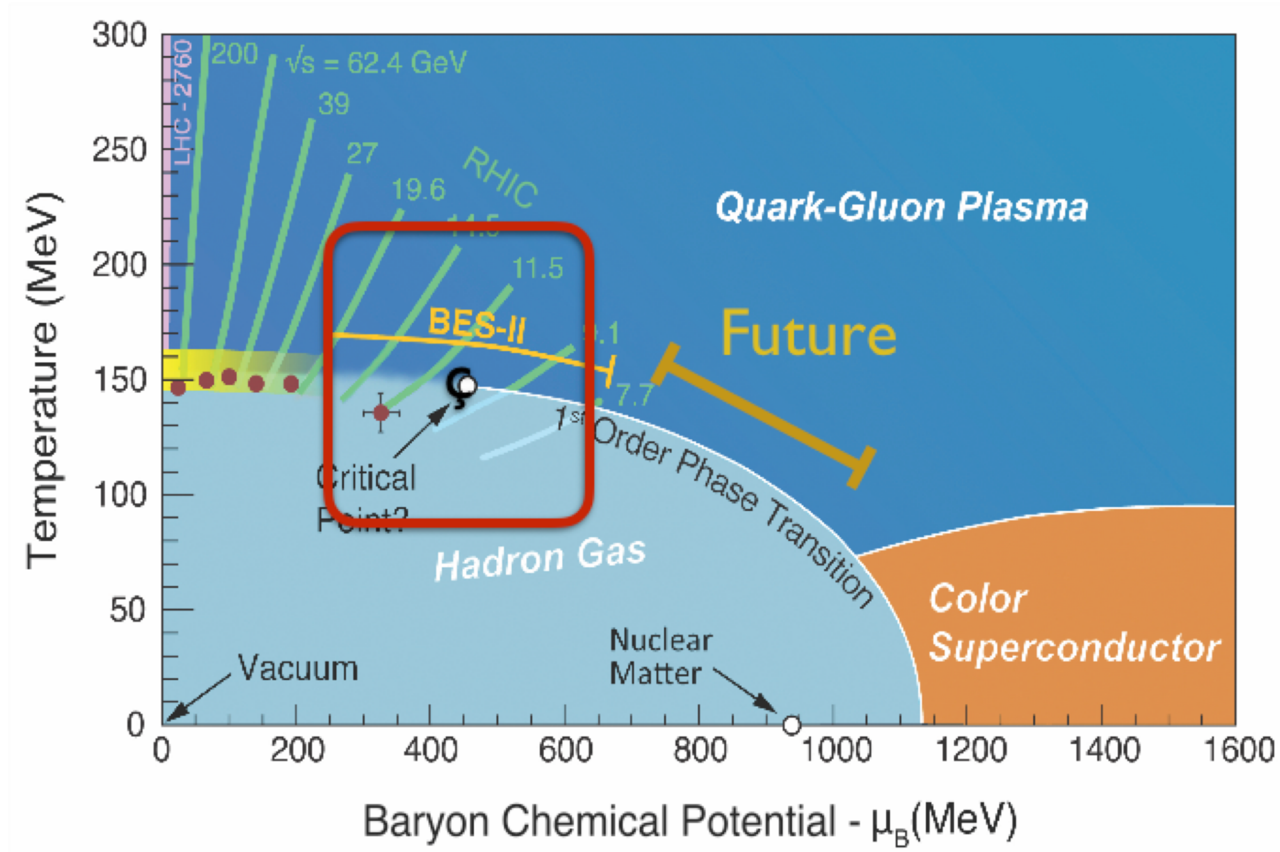
BEST COLLABORATION



Phase diagram of water (from wiki)

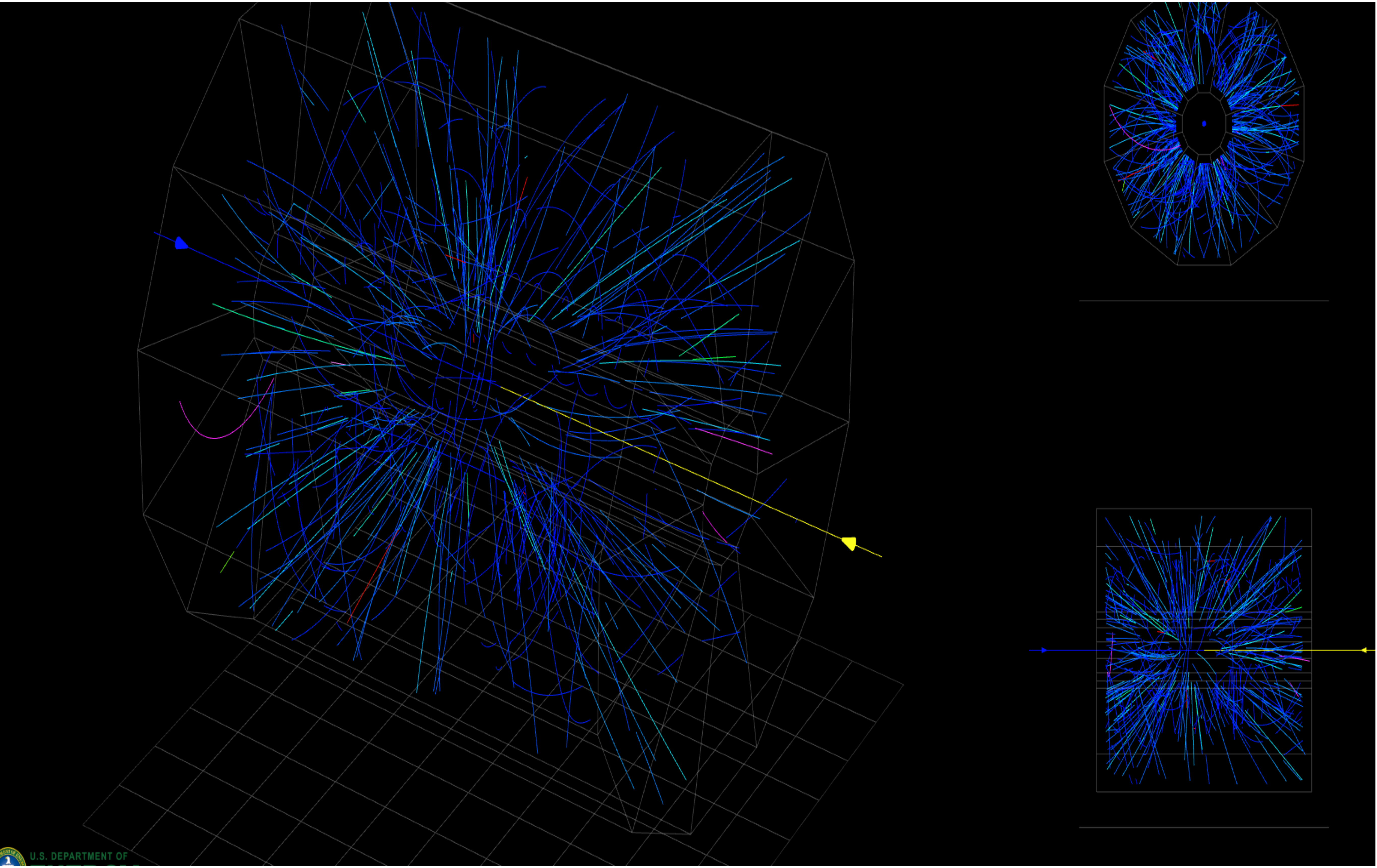


The critical point is a ubiquitous phenomenon.

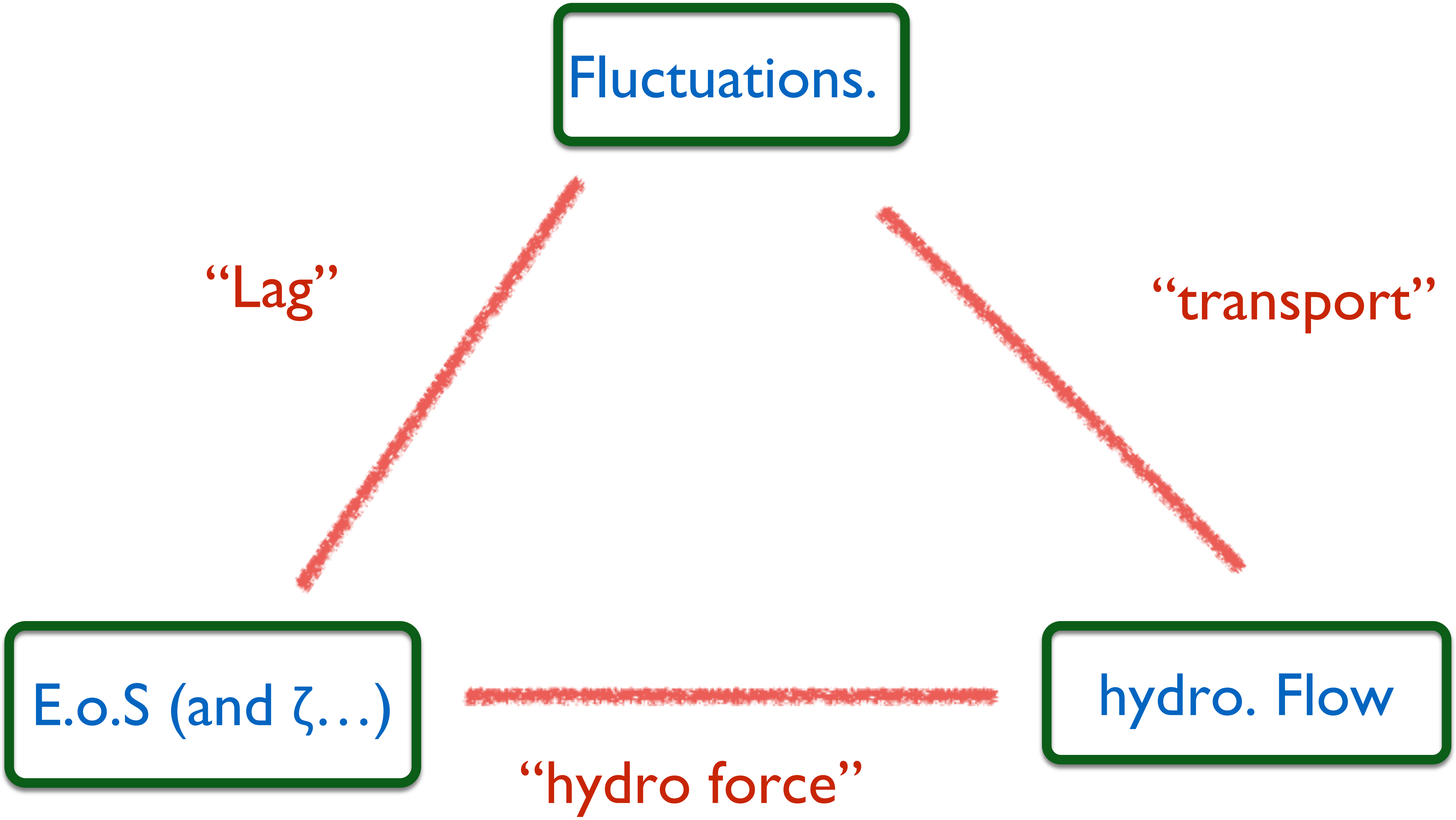
The QCD critical point: the landmark point on the phase diagram.

The discovery of the QCD critical point would in turn raise many new questions for the field!

High precision data is coming!



Interwined dynamics near the critical point

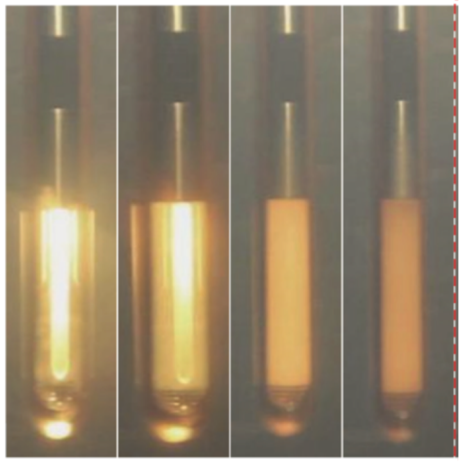


Because of the expansion of the fireball and the critical slowing down, critical fluctuation would *inescapably fall off-equilibrium* and *back-react* on bulk evolution.

This talk: demonstrating the intertwined dynamics among fluctuations and bulk evolution by simulating the “hydro+” in a simplified set-up.

Fluctuations, correlations and criticality

"As the density fluctuations become of a size comparable to the wavelength of light, the light is scattered and causes the normally transparent liquid to appear cloudy." -- wiki



Hou et al, Journal of Chemistry 16'.

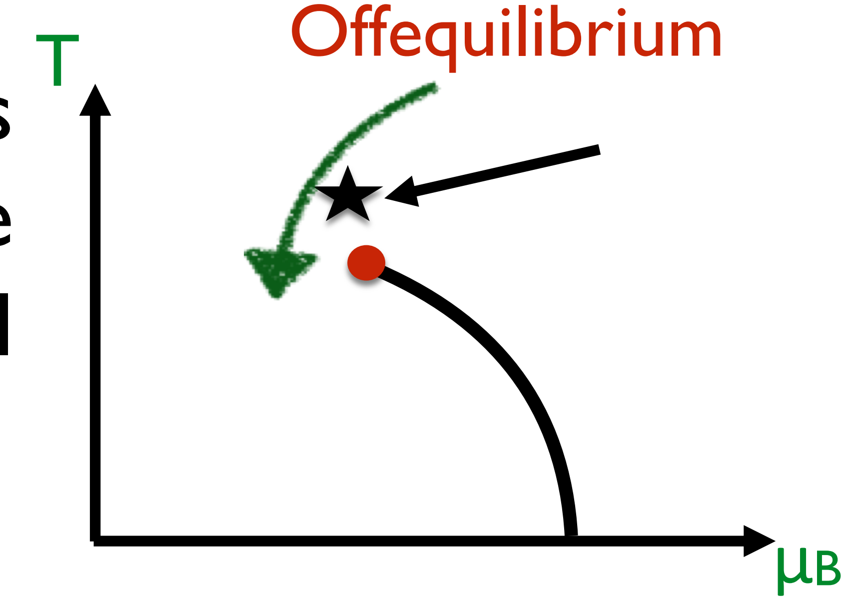
Consider the correlation function of the order parameter field δM (Fourier momentum Q can be considered as the inverse of the size of the fluctuation)

$$\phi_{eq}(Q) \sim \langle \delta M \delta M \rangle_{eq} \sim \frac{1}{Q^2 + \xi^{-2}}$$

The scattering amplitude between the incoming light of momentum Q and the liquid $\propto \phi_{eq}(Q)$ and will *be enhanced* near the critical point for $Q \sim 1/\xi \Rightarrow$ *phenomenon of the critical opalescence*.

Near QCD critical point, the behavior of $\phi_{eq}(Q)$ also leads to the growth of non-Gaussian fluctuations of proton multiplicties and the critical scaling behavior of E.o.S.

The long wavelength critical fluctuation is inescapably fall out of equilibrium near the critical point. ("Critical slowing down": critical fluctuation relaxes very slowly!)



Critical fluctuation can be different from the equilibrium expectation both quantitatively and qualitatively !

e.g. S. Mukherjee, R. Venugopalan and YY, PRC15' ; see YY, 1811.06519 for a brief review on related development.

Further, off-equilibrium effects back-react on bulk evolution.

gradient of p (?) $\simeq$ acceleration of flow

How to describe the interplay among fluctuations and bulk evolutions?

A new dynamical framework: hydro+

Stephanov-YY, 1712.10305, PRD '18;

Conventional approach: adding noise into hydro. (stochastic hydro).
i. Difficult to implement for expanding system and numerically demanding. ii Potentially dependence on cut-off.

Instead "Hydro+" is a hydro-like theory which couples hydro. d.o.f with long wavelength critical fluctuations using deterministic equations.

Eqns for long wavelength fluctuations.

$$\partial_\mu T^{\mu\nu} = 0, \qquad \partial_\mu J^\mu = 0 \text{ with } p \to p_{(+)}$$

Generalizing pressure, $p_{(+)}$: depends on off-equilibrium fluct.

Similar for transport coefficients.

The construction of hydro+

The "+" of "hydro+" is nothing but (Wigner transform of) the two point function of the fluctuating order parameter field δM (For QCD critical point and for description of the dynamics of ϕ, M ~ s/n):

$$\phi(t,x;Q) = \int d\Delta x e^{-i\Delta x Q}\langle \delta M(t,x+\Delta x/2)\delta M(t,x-\Delta x/2)\rangle$$

The evolution of ϕ is modelled by relaxation rate equation.

$$u^{\mu}\partial_{\mu}\phi(\mathrm{t,\ x;Q}) = -\ \Gamma(Q)(\phi-\phi_{eq})$$

The generalized entropy $s_{(+)}$ and generalized pressure $p_{(+)}$ can be derived, e.g.,

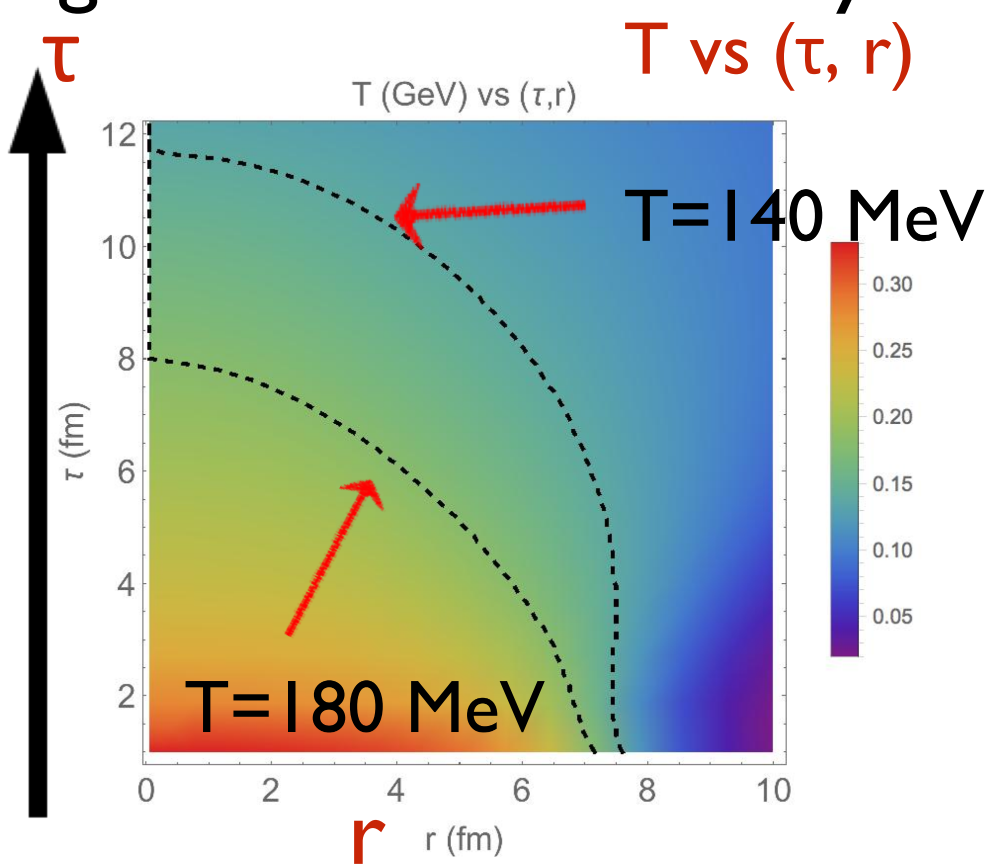
$$s_{(+)} = s(e,n) + \Delta s\,, \qquad \Delta s[\phi] = \frac{1}{2}\int_Q \left[\log(\frac{\phi}{\phi_{eq}}) - \frac{\phi}{\phi_{eq}} + 1\right] + \dots$$

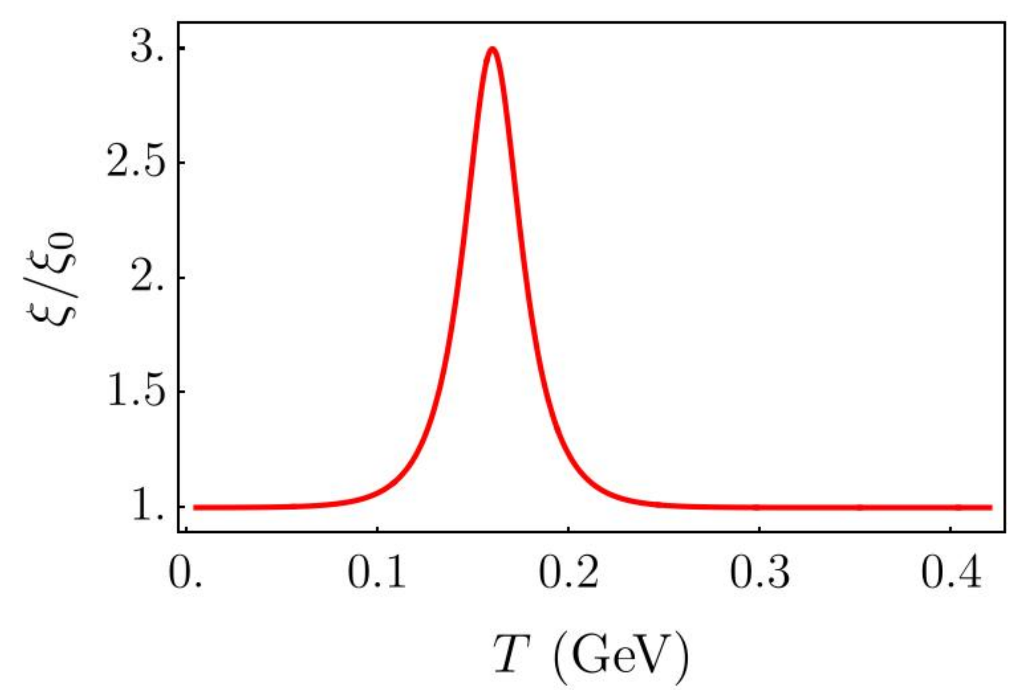
Our (minimum) model for the simulation of hydro+

Solving full hydro+ equation for energy density $\varepsilon(\tau,r)$, radial flow $v_r(\tau,r)$ and wavelength-dependent fluctuations $\phi(\tau,r,Q)$, and showcase the intertwined dynamics among them.

Placing a hypothetical C.P. near $\mu=0$ (no eqn for baryon density.) The critical fluctuation reaches its maximum around Tc=160 MeV.

Considering a Bjorken and radial expanding ($v_r \neq 0$) and inhomogeneous fluid made by a central collision.

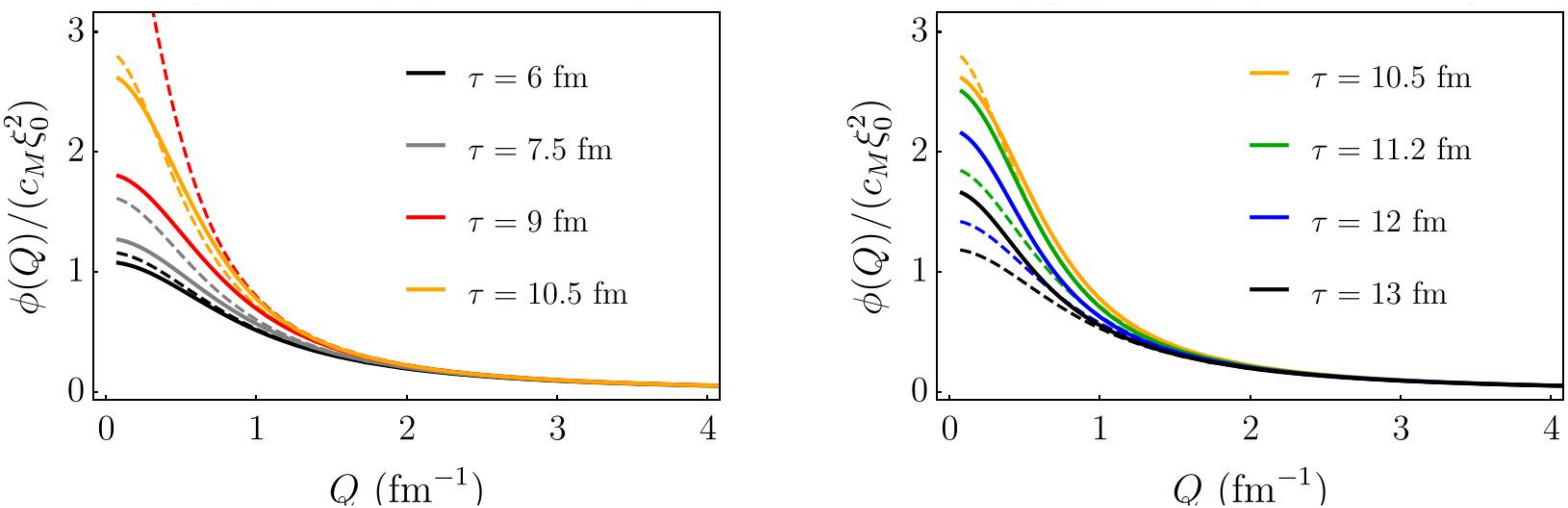




Momentum-depdent off-equilibrium fluctuation at r=1fm.

See also: Berdnikov-Rajagopal 99'; Akamatsu-Teaney-Yan-Yin, PRC 19'

Off-quilibrium : solid ; Equilibrium: dashed.



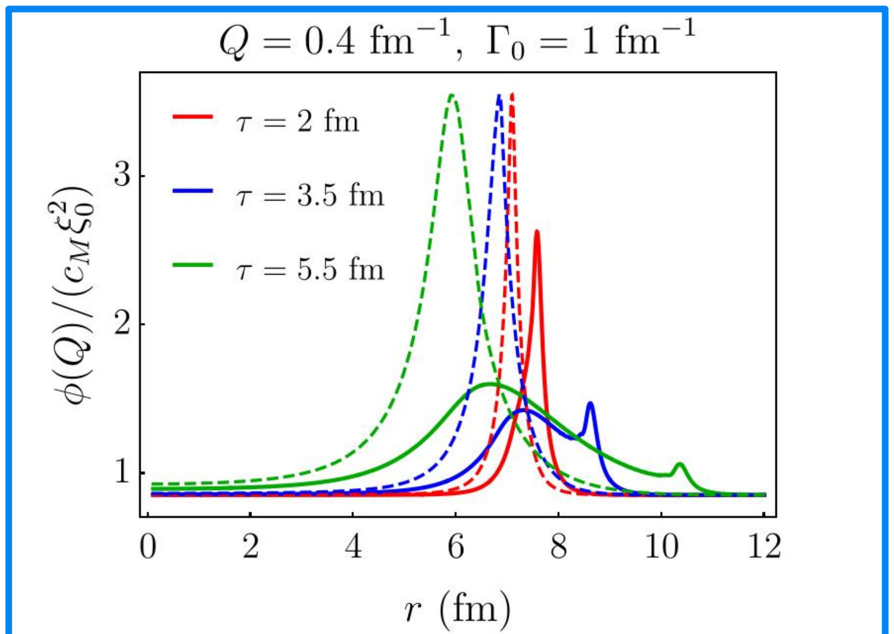
Q-dependence: large Q (shortwave length) modes are in equilibrium while small Q (long wavelength) modes are not.

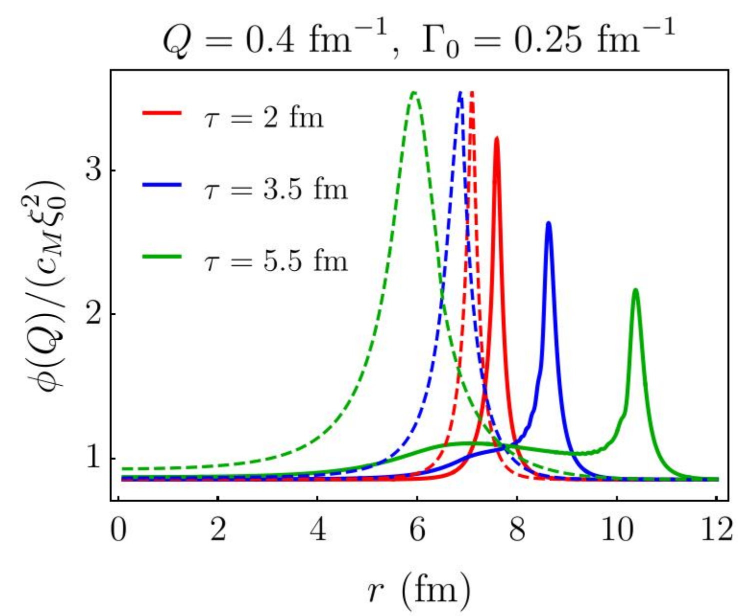
Critical slowing down leads to the *jet-lag* of critical fluctuations: the information of the criticality is encoded in offequilibrium effect!

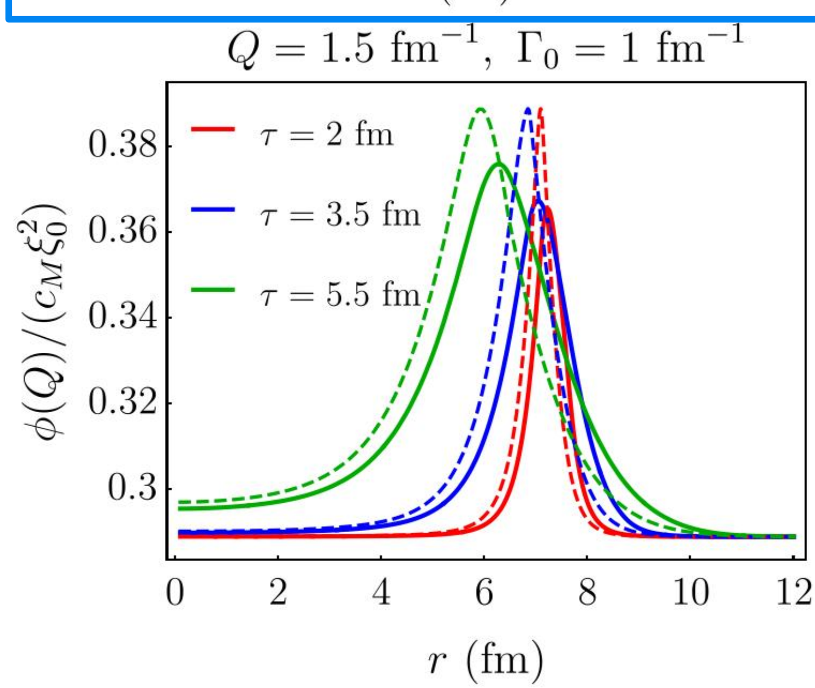
Previously, the "jet-lag" effect has been studied by many others, but mostly for homogeneous expanding fireball.

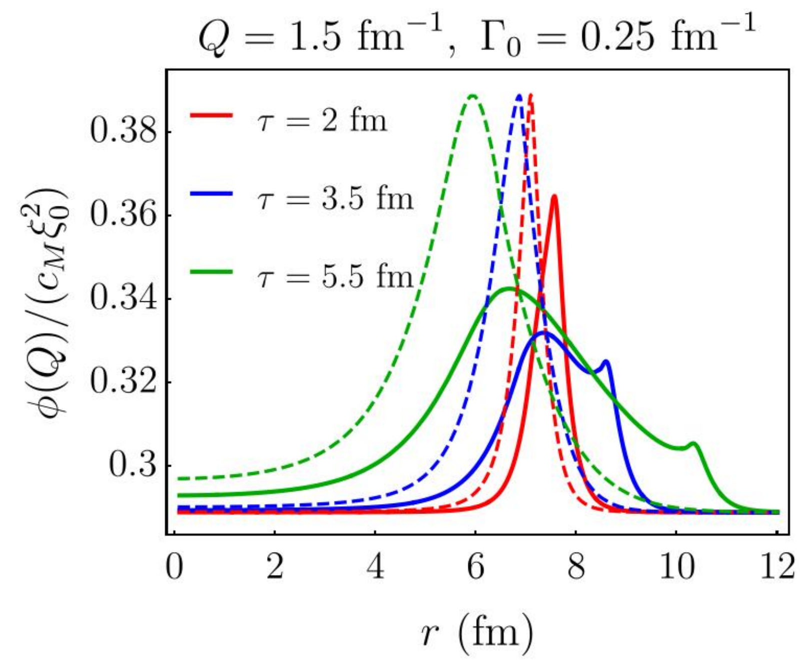
see Teaney's talk

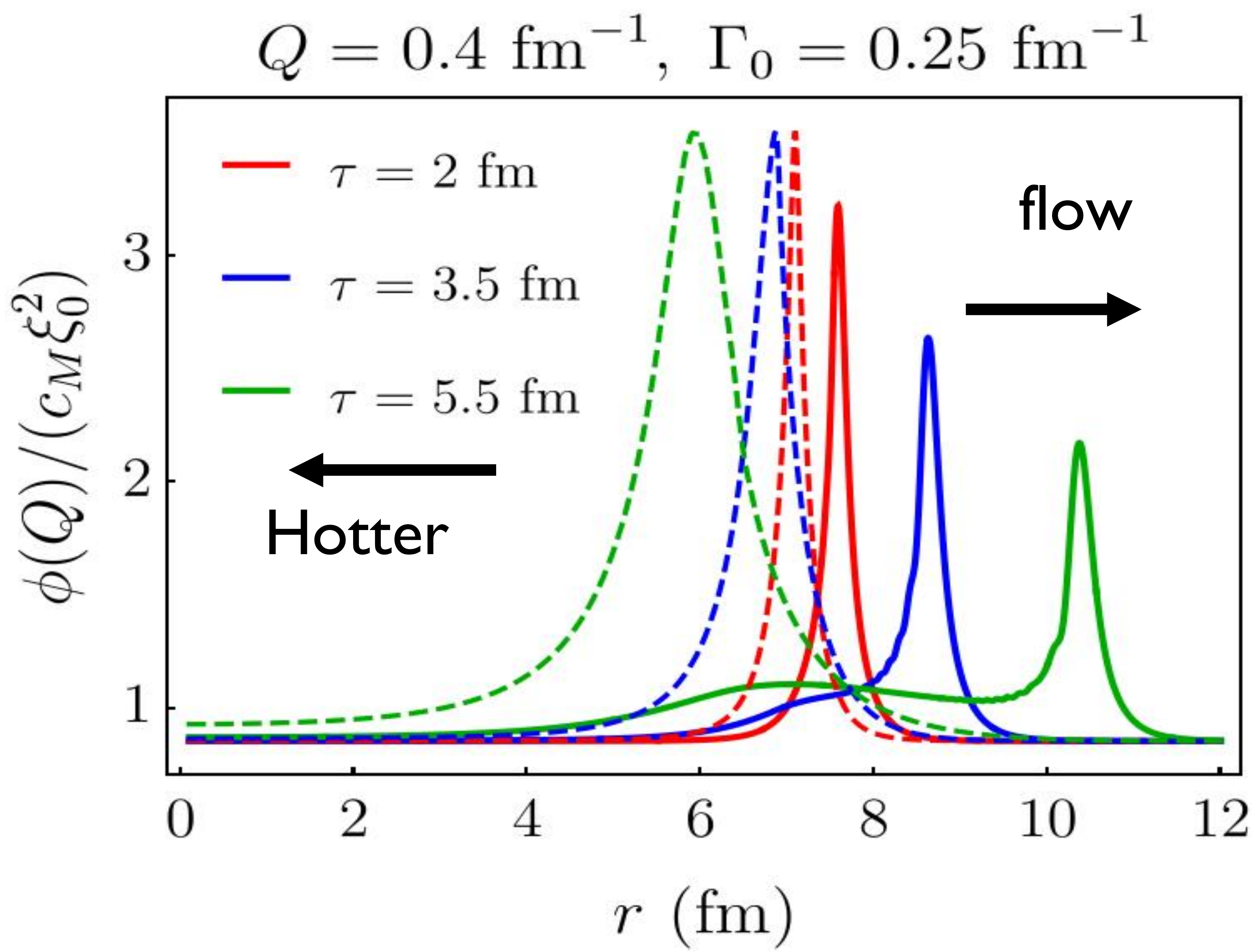
We further watch the nontrivial *spatial distribution* of long wavelength fluctuations.







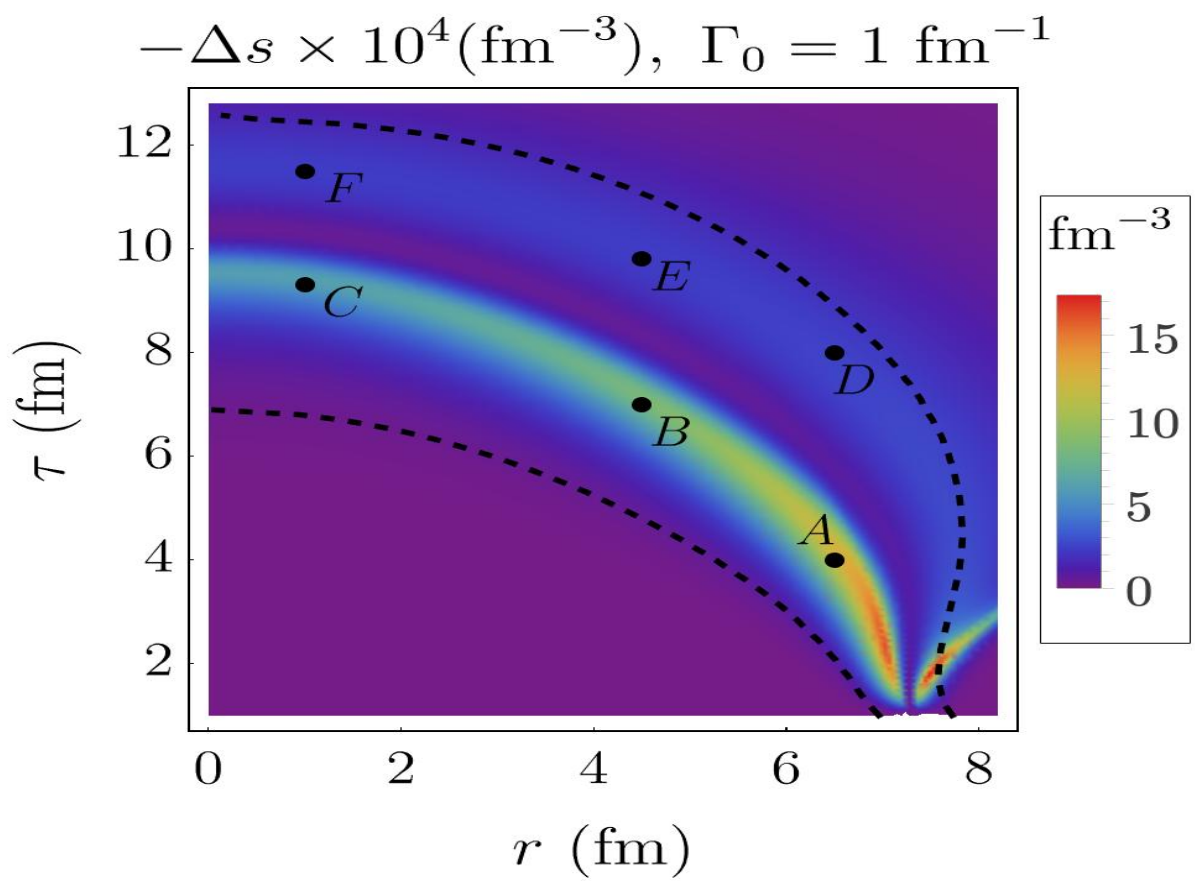


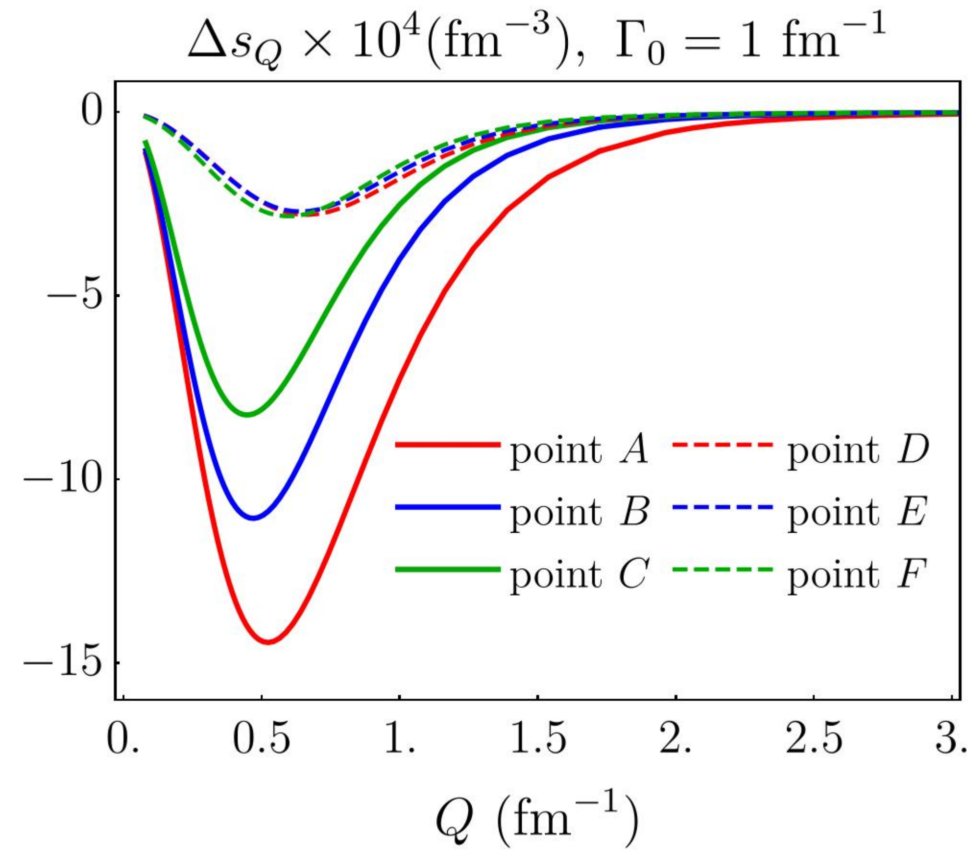


The spatial dependence of the off-equilibrium fluctuations is driven by the critical slowing down and by the advection of the flow.

The modification of E.o.S due to off-equilibrium effects.

Difference between equil. entropy and non-equil. tentropy

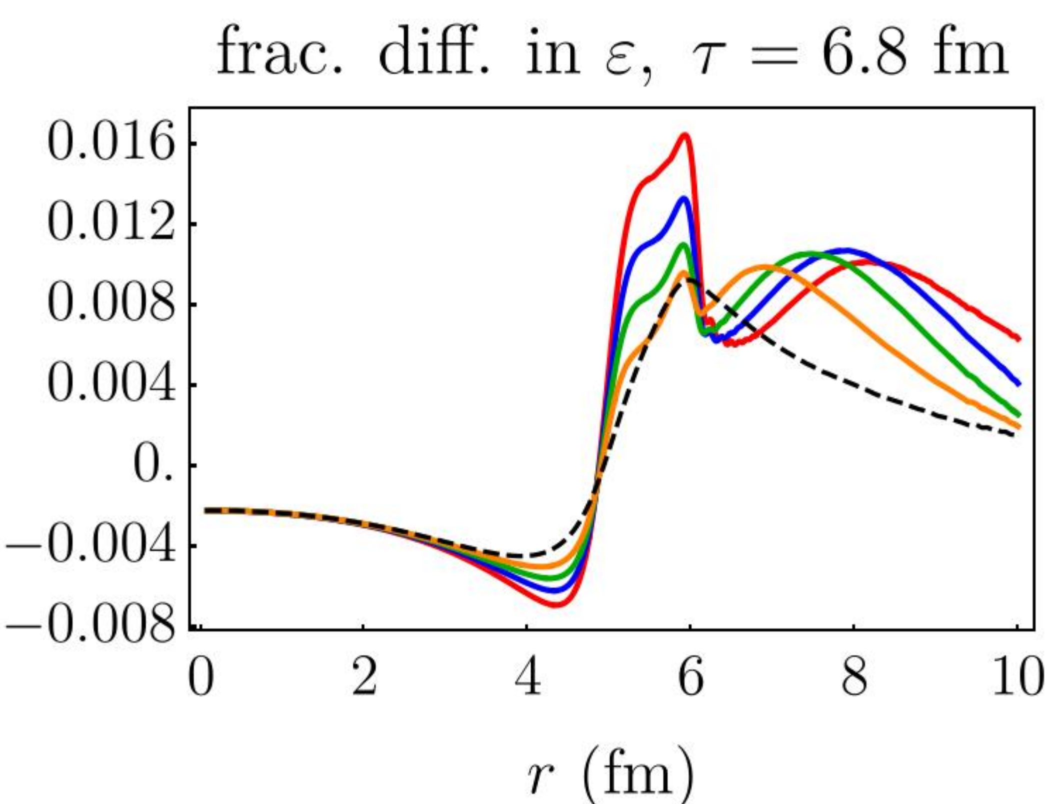


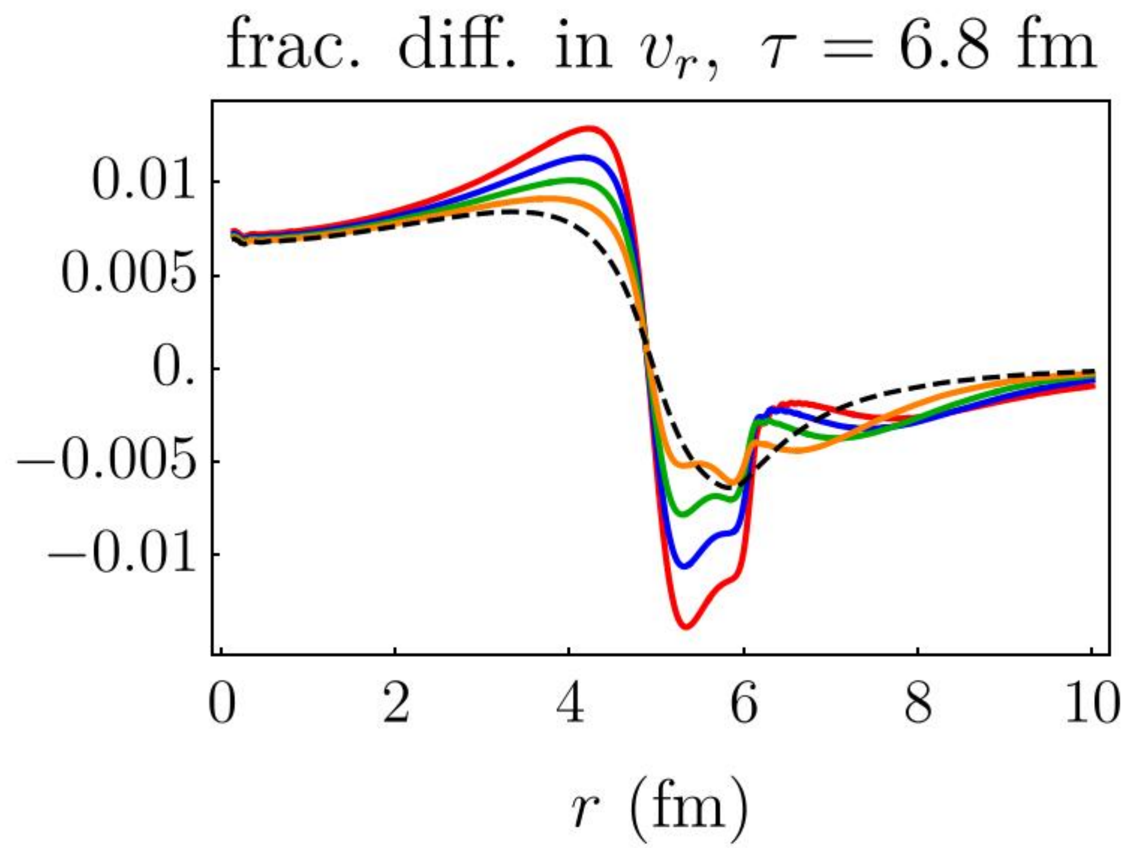


Stronger the offequilibrium effect is, the greater their back-reaction on E.o.S. (e.g. back-reaction is prominent at early times when the fireball expands faster.)

A particular momentum Q_{peak} plays the most important role in back-reaction: *an emergent length scale* induced by dynamical effects.

The snapshot of energy density and radial flow vs r





Dashed, no back-reaction.

From: red, blue, green and orange, results including back-reaction with decreasing relaxation rate.

The slower long wavelength fluctuations are relaxed, the stronger their influence on the energy density and flow.

Within our model, the back-reaction effects are small (within 1 percent).

Summary and outlook I:

The newly-developed hydro+ theory has been successfully exercised in a concrete, albeit simplified settings. We have seen that interplay among flow and fluctuations manifest explicitly.

New lesson learned: advection of the flow will induce the non-trivial spatial distribution of fluctuations.

Discussion: if the back-reaction is indeed small as seen in the present simulation, hydro modes decouples from fluctuation modes (i.e. "+") => significant simplifications for numerical simulations.

NB: the importance of back-reaction also depends on the parameters which maps QCD critical point to those of Ising model.

Airong Liu (Undergrad. of CCNU) and YY, in progress, see also Martinez-Schafer-Skokov et al, 1906.11306 .

Summary and outlook II:

Encouraging progress has been reported from the two different groups (Ohio state U., Wayne state U.) on full numerically simulation of hydro+ . In addition, there are many other progresses on the Beam energy scan theory.

See Lipei Du's poster

The era of quantitative studies of critical dynamics has just begun!

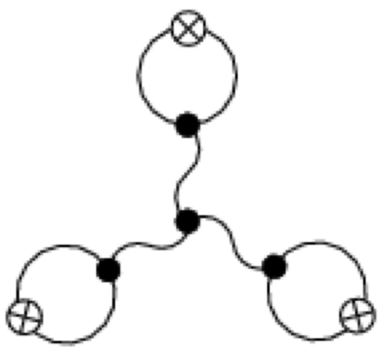
Challenges: extension to non-Gaussian fluctuations? Dynamics near first-order transitions?

Crucial steps: translating density fluctuations into hadron multiplicities fluctuations (particlization)?

Back-up

The critical behavior of correlation function drives the critical phenomenon

As a consequence of the enhanced long-wave length fluctuations, higher cumulants become more sensitive to the growth of the critical fluctuations.

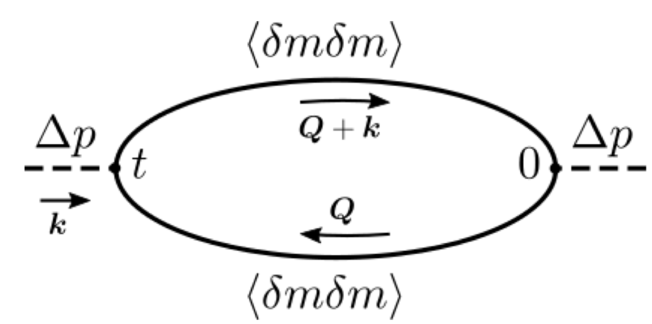


$K_3 \sim \lambda_3\, [\phi(\mathrm{Q}=1/\xi)]^3 \sim \xi^{9/2}$

(Similarly, $K_4 \sim \xi^7$)

Furthermore, E.o.S exhibits the critical scaling behavior. For example, specify heat will also grow and hence sound velocity will be suppressed near the critical point.

$C_V \sim \xi^{\alpha}$, $(cs)^2 = \mathrm{s}/\mathrm{C_V}$



Consequently, one needs to look for observables sensitive to the growth of the fluctuations as well as to the change of the E.o.S.

Qualitative discussion on the importance of back-reaction

The influence of critical fluctuations on E.o.S is enhanced by the growth of the correlation length, but is suppressed by $1/(N_c)^2$

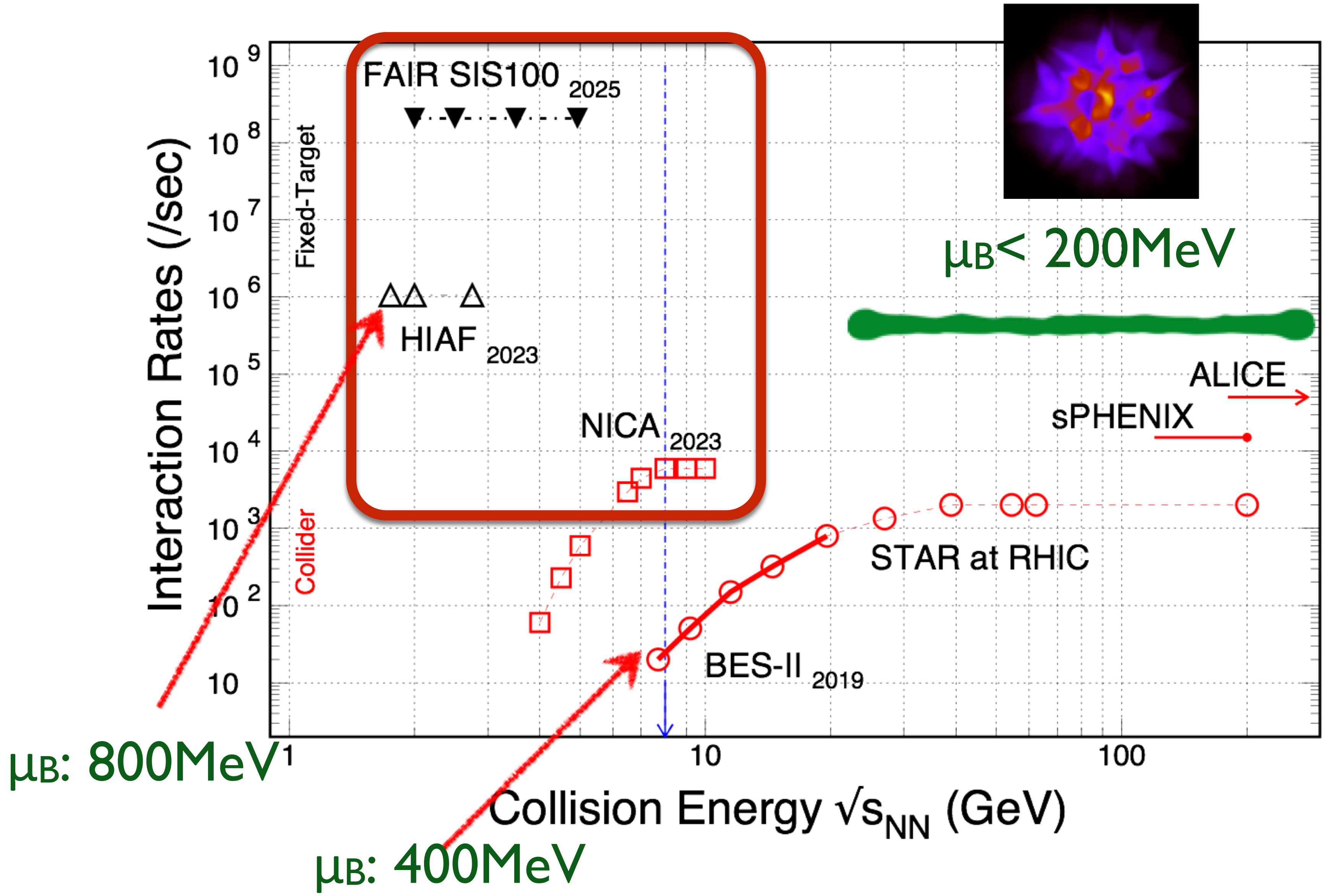
e.g., in our model, ξ grows three times at most. But the critical mode is only one mode. There are 16 gluonic d.o.f. and 36 fermionic d.o.f.

In general, the importance of back-reaction also depends on the parameters which maps QCD critical point to that of Ising model.

Airong Liu (Undergrad. of CCNU) and YY, in progress.

If the back-reaction is indeed small, Hydro decouples from fluctuation(i.e. “+”) for practical purpose. The advection of flow on the fluctuation is much more important than the feedback of fluctuations on the flow.

The voyage of discovery on the QCD phase diagram: from now to future



Future: facilities worldwide (LHC,FAIR/CBM, NICA,HIAF, …) will open new observational frontier in the next decade (μ_B up to 800MeV~M_N). Opportunities around !

Russian	NICA	2023
Germany	FAIR	2025
China	HIAF	2025

Main questions of the field

How does strongly coupled fluid emerge from the asymptotic free QCD as resolution scale decreases?

How does the fluid-like behavior emerges from highly an-isotropic and non-equilibrium quark-gluon matter at early time?

How does the properties of QGP liquid change as baryon density increases? (this talk).