The QCD phase diagram and statistics friendly distributions

A. Bzdak, VK, N. Strodthoff: arXiv:1607.07375

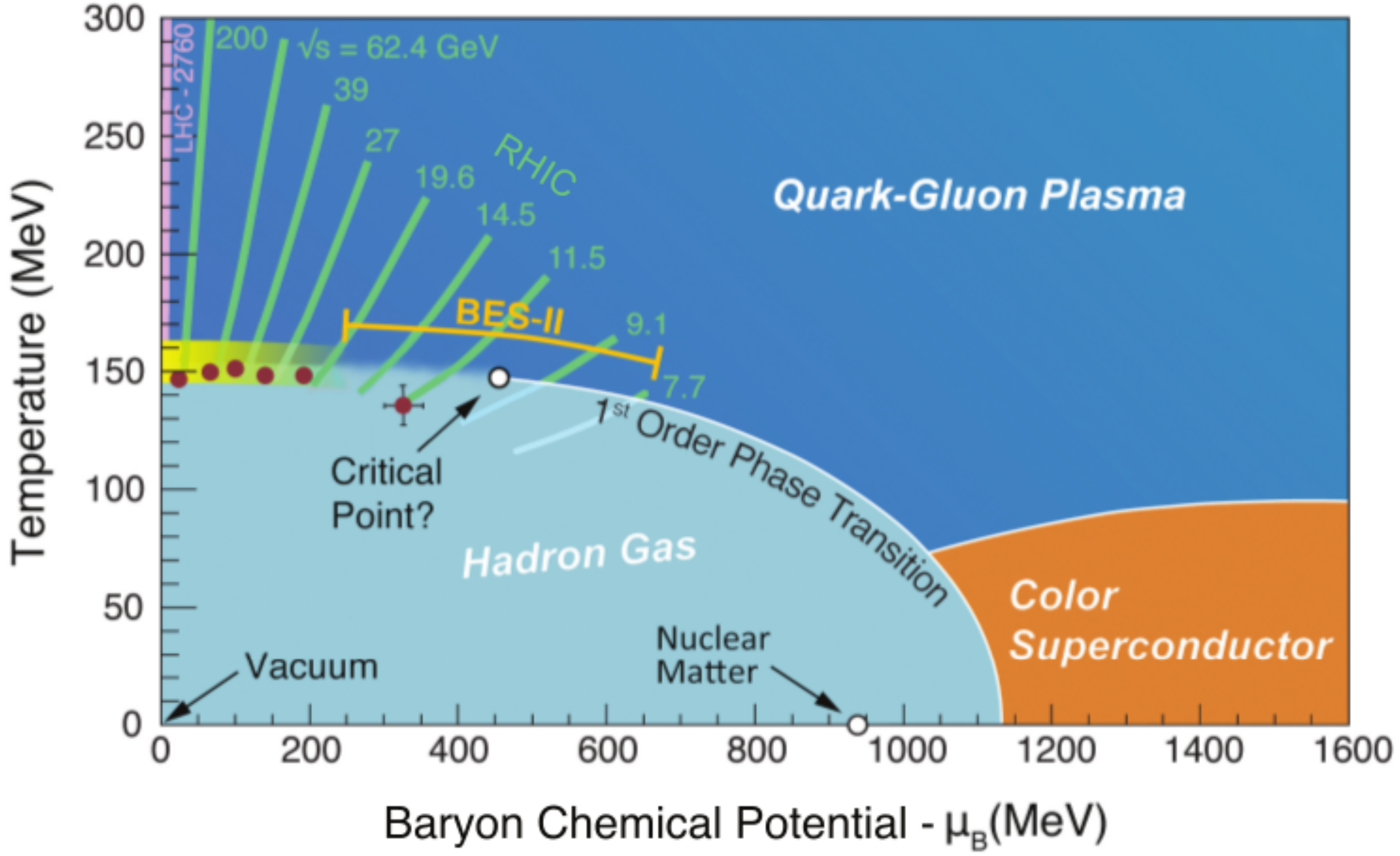
A. Bzdak, VK, V. Skokov: arXiv:1612.05128

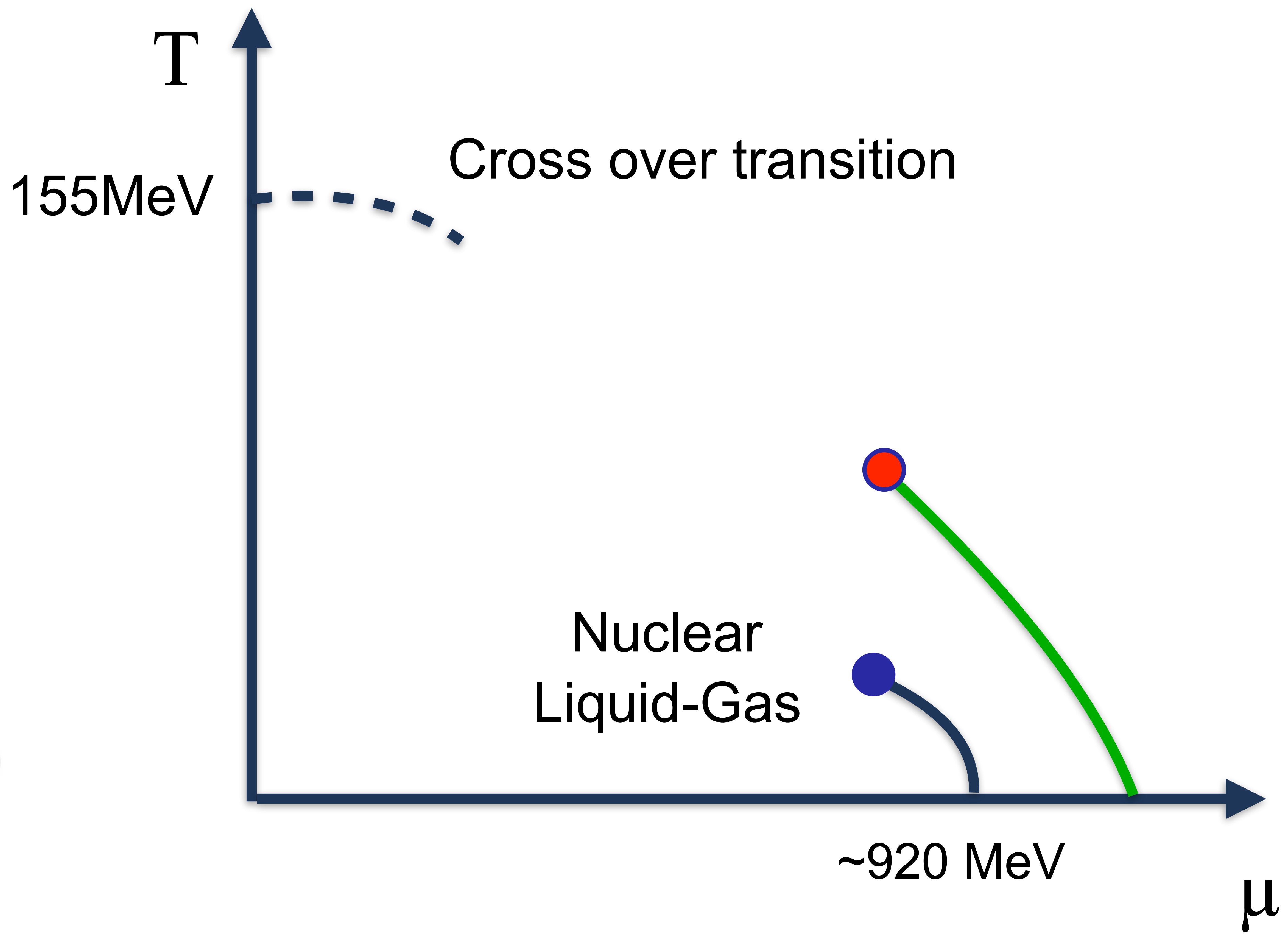
A. Bzdak, D. Oliinychenko, J. Steinheimer, VK: arXiv:1804.04463

A. Bzdak and V.K: arXiv:1811.04456

BEST COLLABORATION

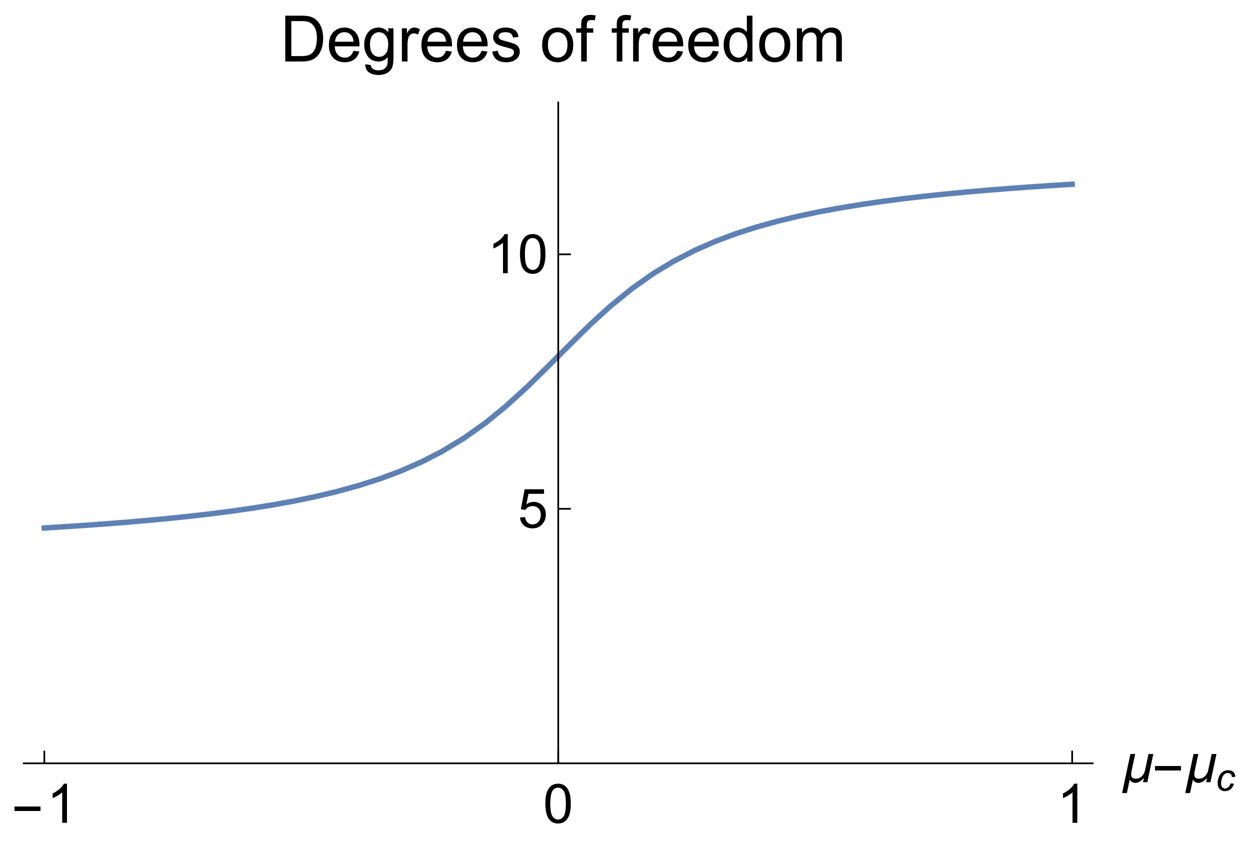
The phase diagram





Cumulants of (Baryon) Number

$$K_n = \frac{\partial^n}{\partial(\mu/T)^n}\ln Z = \frac{\partial^{n-1}}{\partial(\mu/T)^{n-1}}\langle N\rangle$$

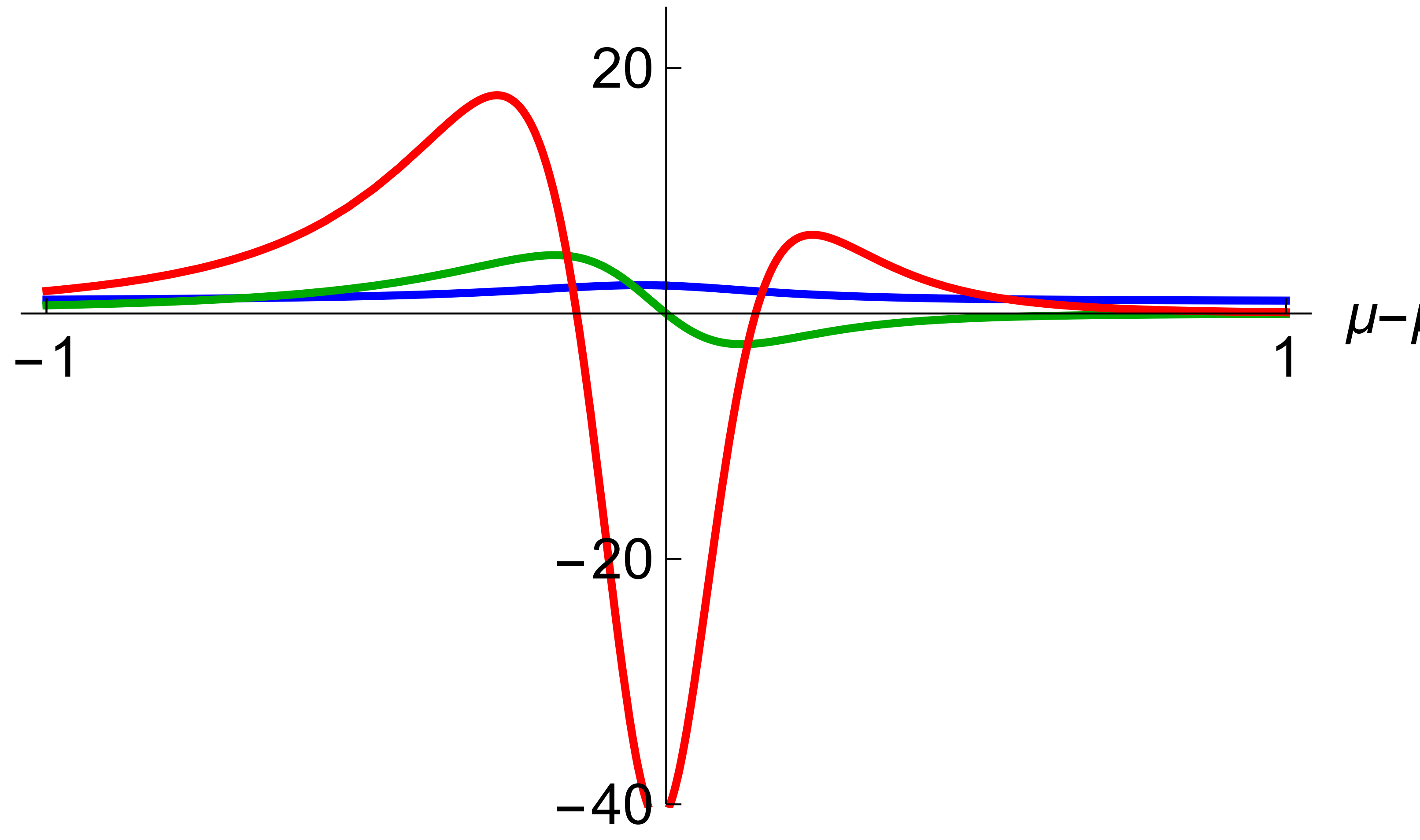


Cumulants measure derivatives of free energy w.r.t chemical potential

$$K_1 = \langle N\rangle\,,\;\; K_2 = \langle N-\langle N\rangle\rangle^2\,,\;\; K_3 = \langle N-\langle N\rangle\rangle^3$$

Cumulants scale with volume (extensive): $K_n \sim V$

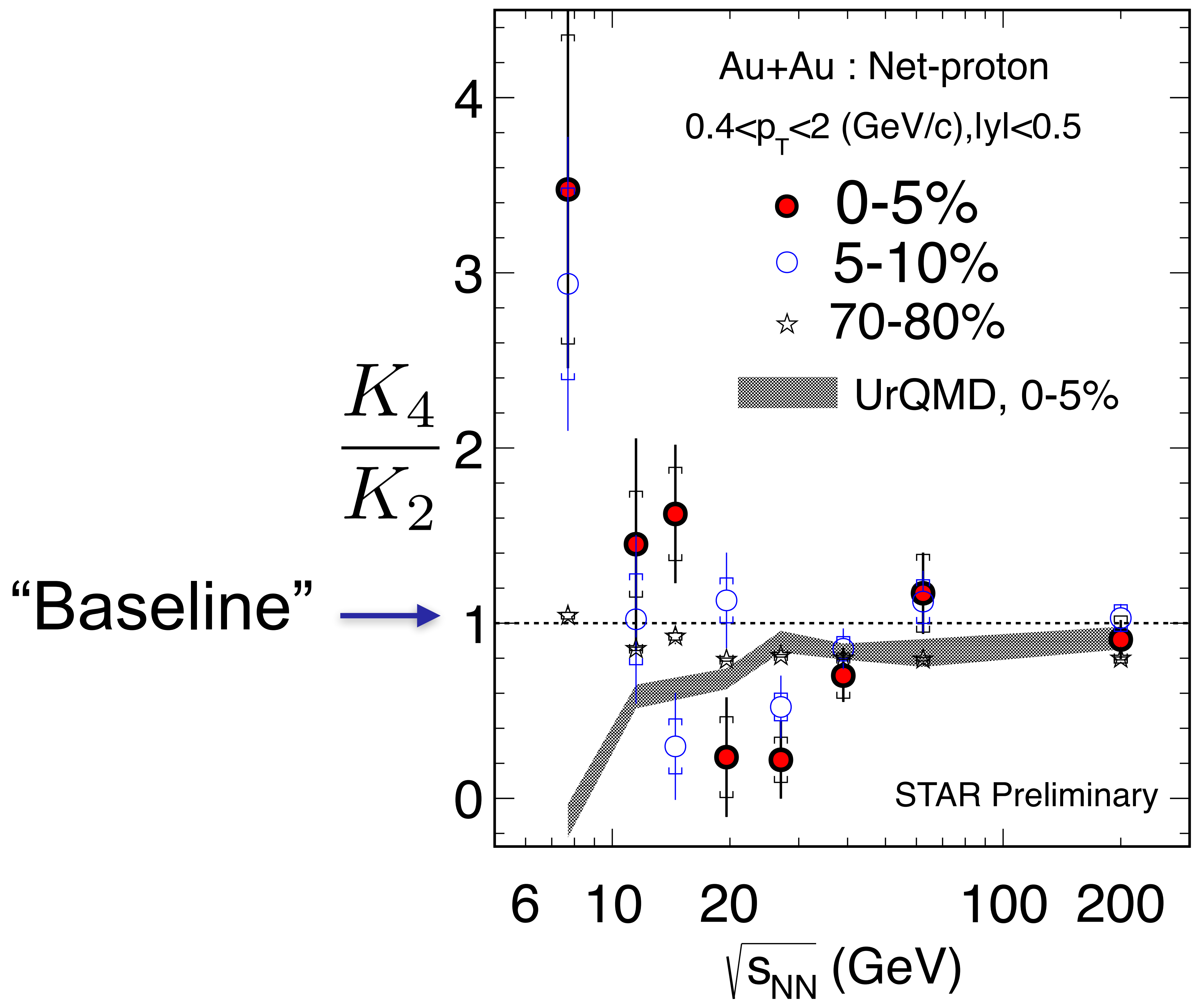
Volume not well controlled in heavy ion collisions

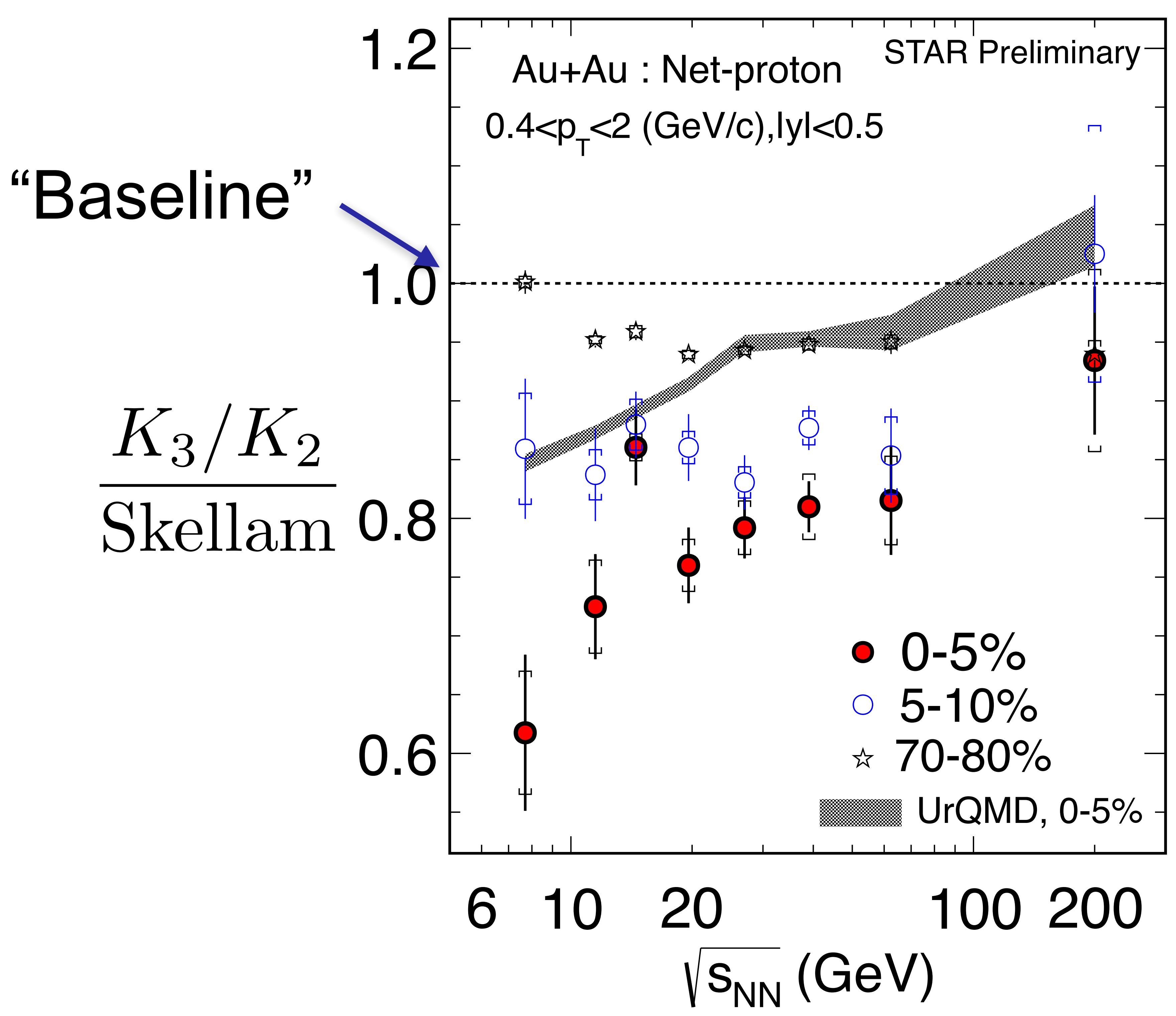


Cumulant Ratios: $$\frac{K_2}{\langle N\rangle},\;\frac{K_3}{K_2},\;\frac{K_4}{K_2}$$

Latest STAR result on net-proton cumulants

X. Luo, NPA 956 (2016) 75





K_4/K_2 follows expectation for CP , K_3/K_2 no so much…..
URQMD totally fails to get trend for K_4/K_2 !

Further insights: Correlations

Cumulants $$K_n = \frac{\partial^n}{\partial \hat{\mu}^n} P/T^4$$

$$K_2 = \langle N - \langle N \rangle \rangle^2 = \left\langle (\delta N)^2 \right\rangle$$

$$\rho_2(p_1, p_2) = \rho_1(p_1)\rho_1(p_2) + C_2(p_1, p_2),$$

C_2: Correlation Function

$$K_3 = \left\langle (\delta N)^3 \right\rangle$$

$$\begin{aligned}\rho_3(p_1, p_2, p_3) = \rho_1(p_1)\rho_1(p_2)\rho_1(p_3) + \rho_1(p_1)\underline{C_2(p_2, p_3)} + \rho_1(p_2)\underline{C_2(p_1, p_3)} \\ + \rho_1(p_3)\underline{C_2(p_1, p_2)} + \underline{C_3(p_1, p_2, p_3)}\end{aligned}$$

From Cumulants to Correlations

Defining integrated correlations function a.k.a factorial cumulants

$$C_n = \int dp_1 \ldots dp_n C_n(p_1, \ldots, p_n)$$

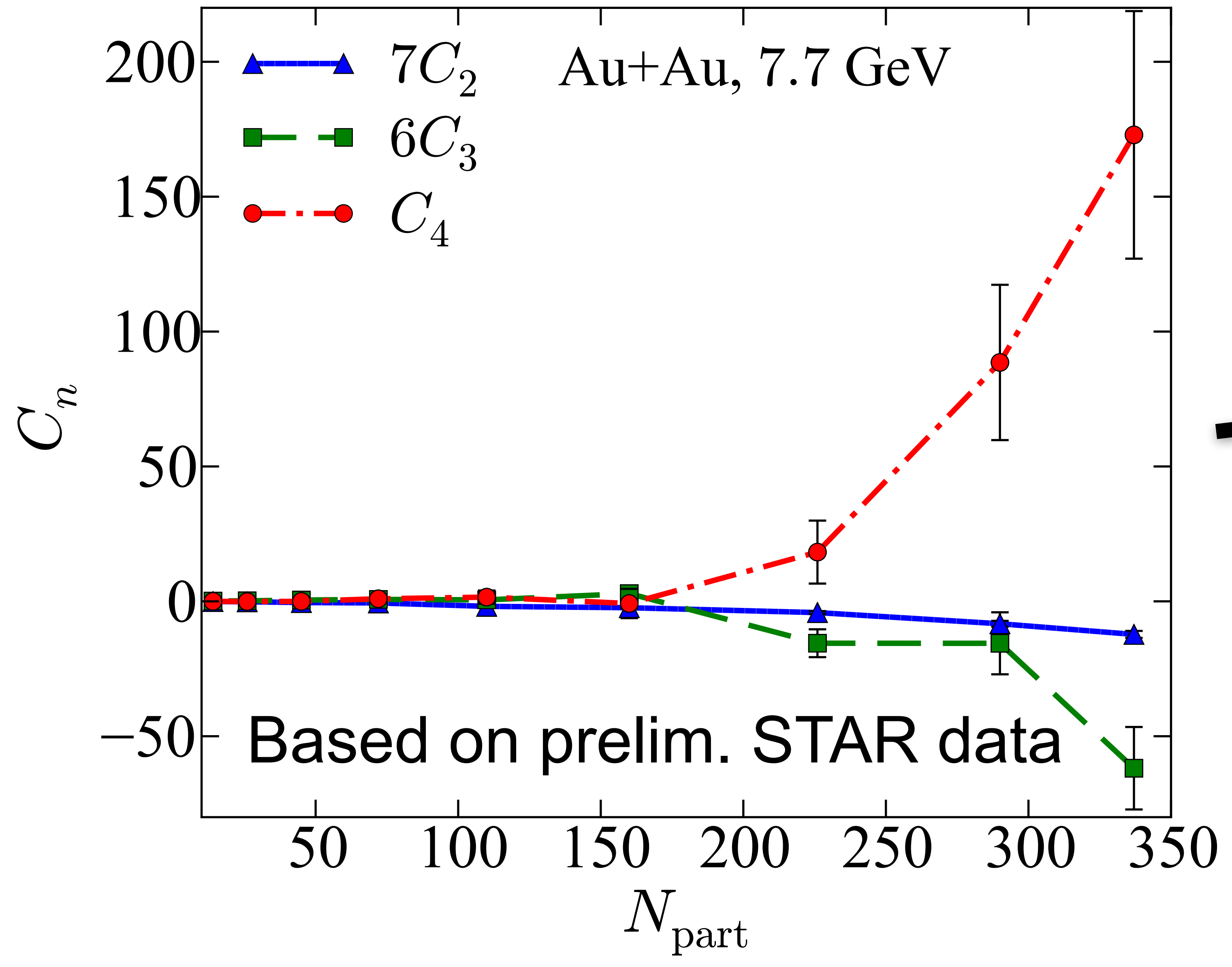
Simple Algebra leads to relation between correlations C_n and K_n

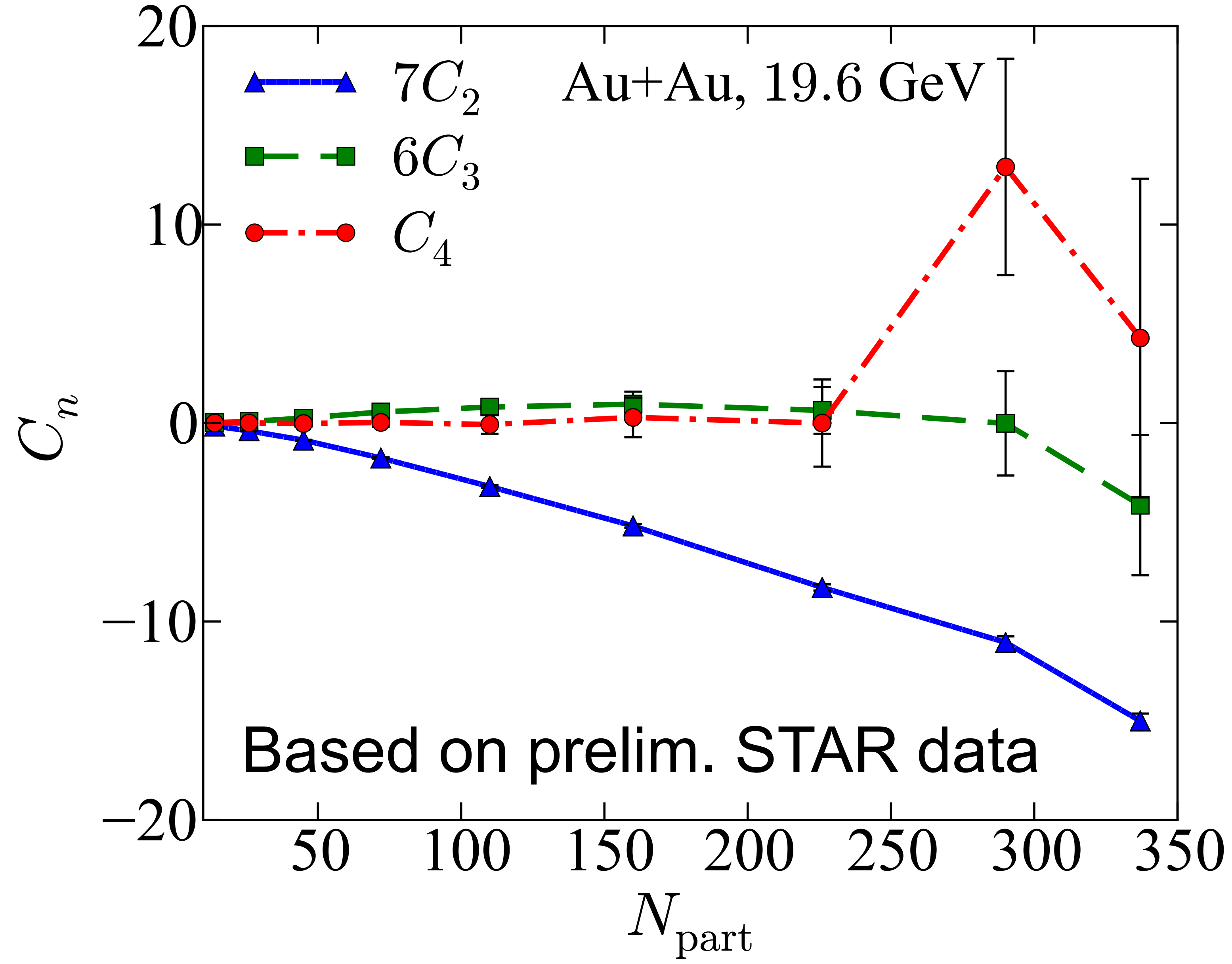
$$C_2 = -K_1 + K_2,$$
$$C_3 = 2K_1 - 3K_2 + K_3,$$
$$C_4 = -6K_1 + 11K_2 - 6K_3 + K_4, .$$

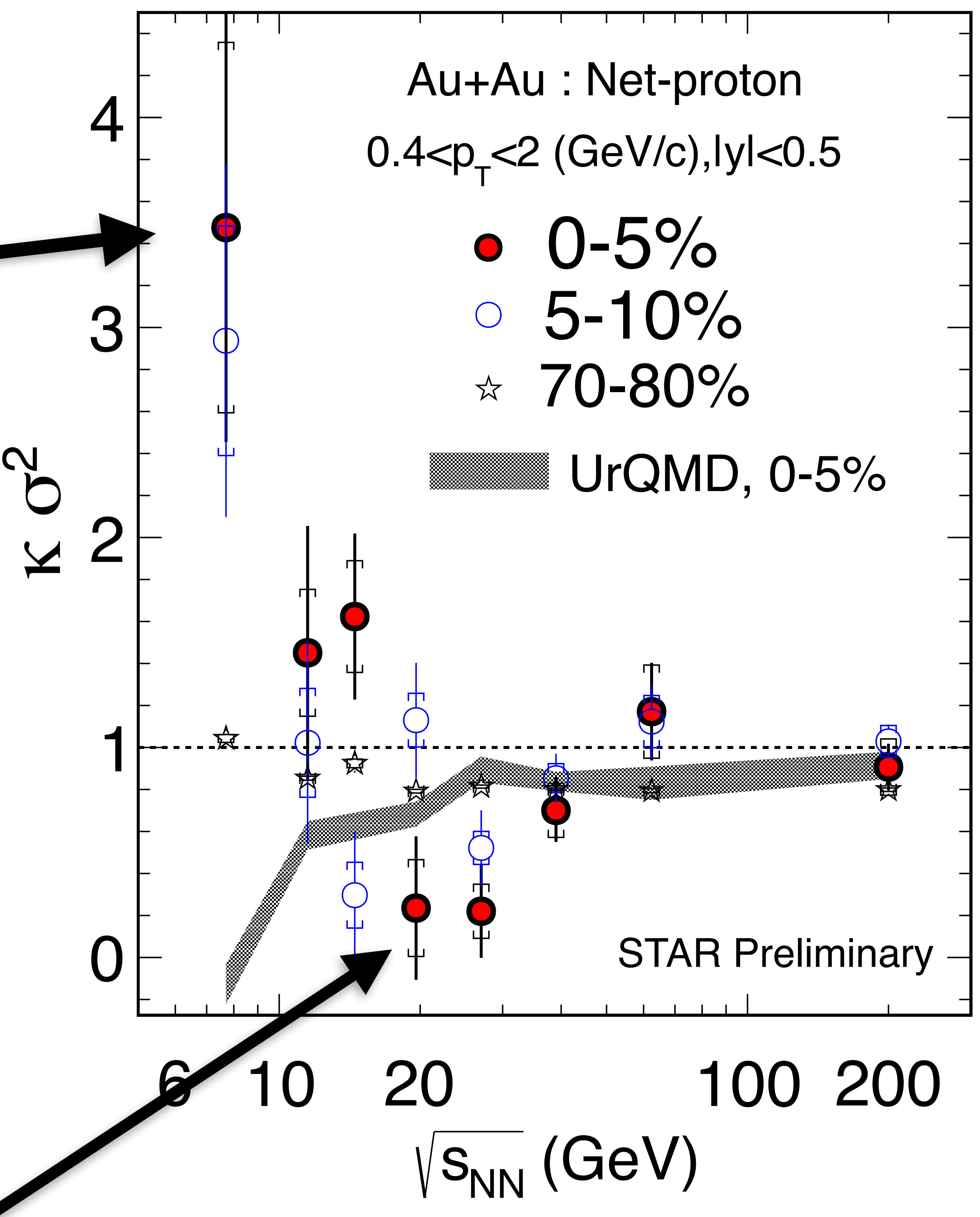
or vice versa

$$K_2 = \langle N \rangle + C_2$$
$$K_3 = \langle N \rangle + 3C_2 + C_3$$
$$K_4 = \langle N \rangle + 7C_2 + 6C_3 + C_4$$

Correlations







Dip at 19.6 GeV from NEGATIVE C_2 !

Correlations

- Four particles correlations kick in for E< 15 GeV

- Data are consistent with long range correlations in rapidity and transverse momentum $\Rightarrow |C_n| \sim \langle N \rangle^n$

- fourth order correlations are large
  - effects from participant fluctuations $\sim 10^{-3}$
  - Cluster model:
    - magnitude requires that 28 out of 30 protons result from 4-proton clusters

- Cluster model is not able to describe both C_4 and C_3

Latest STAR result on net-proton cumulants

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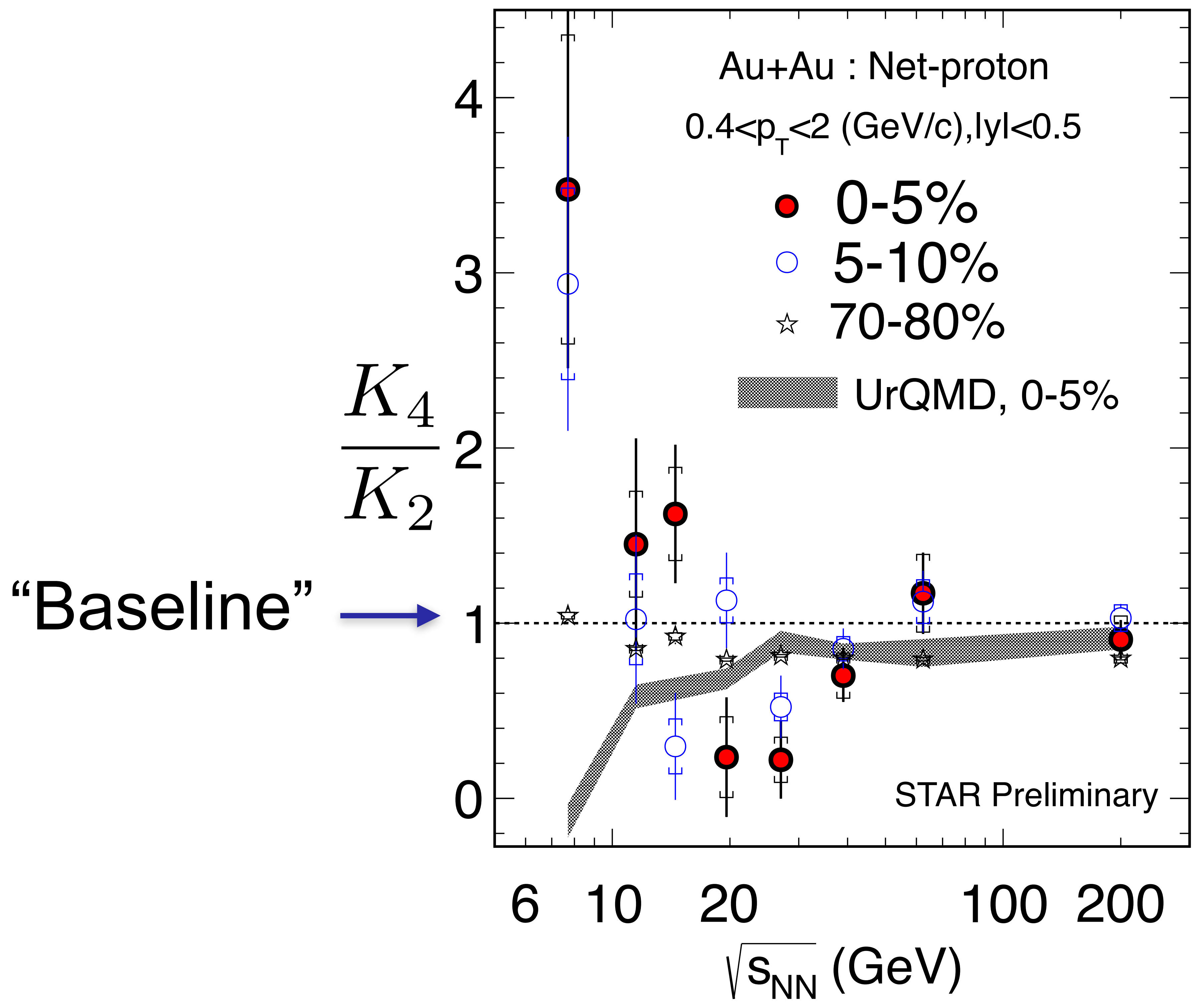
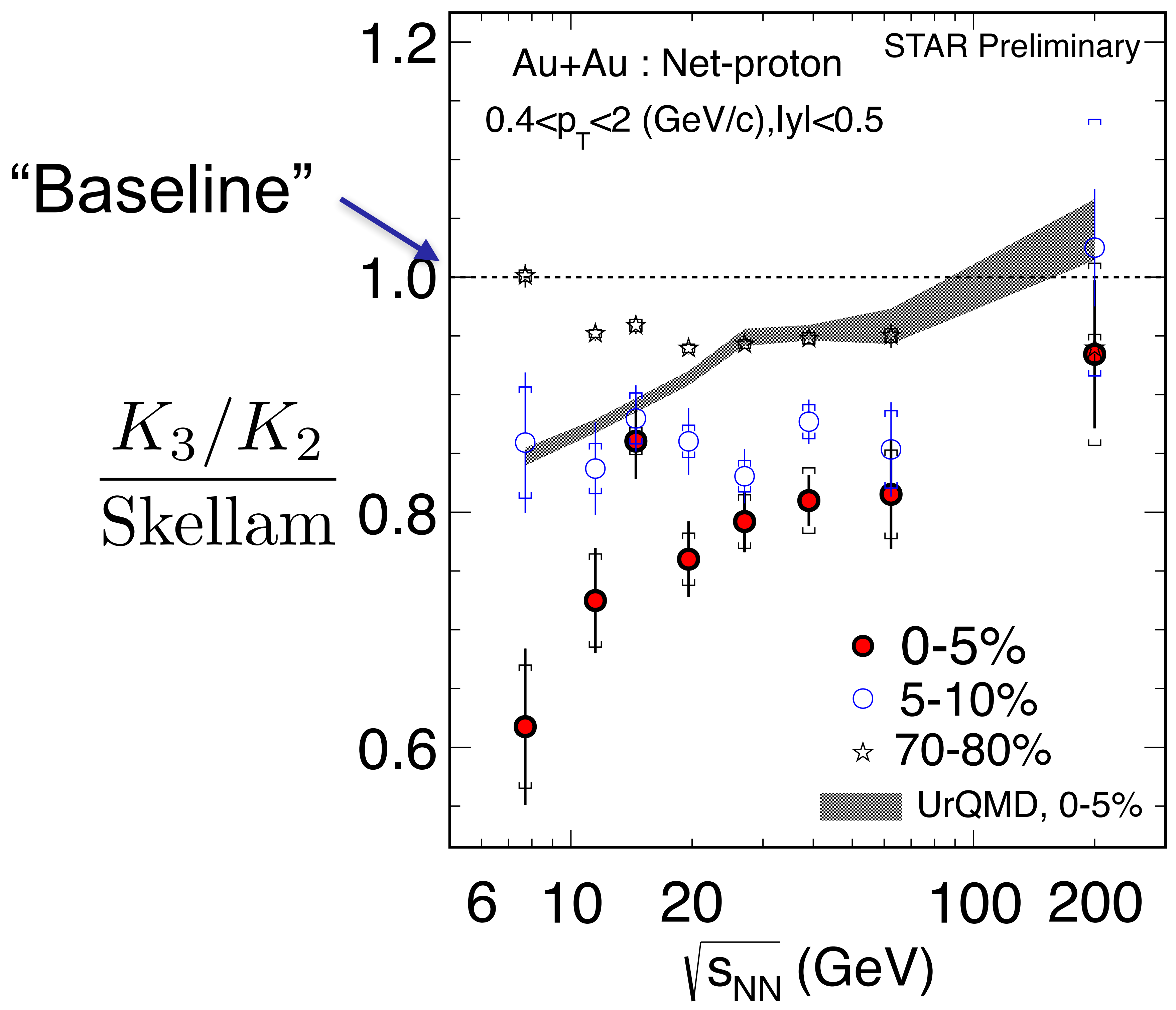


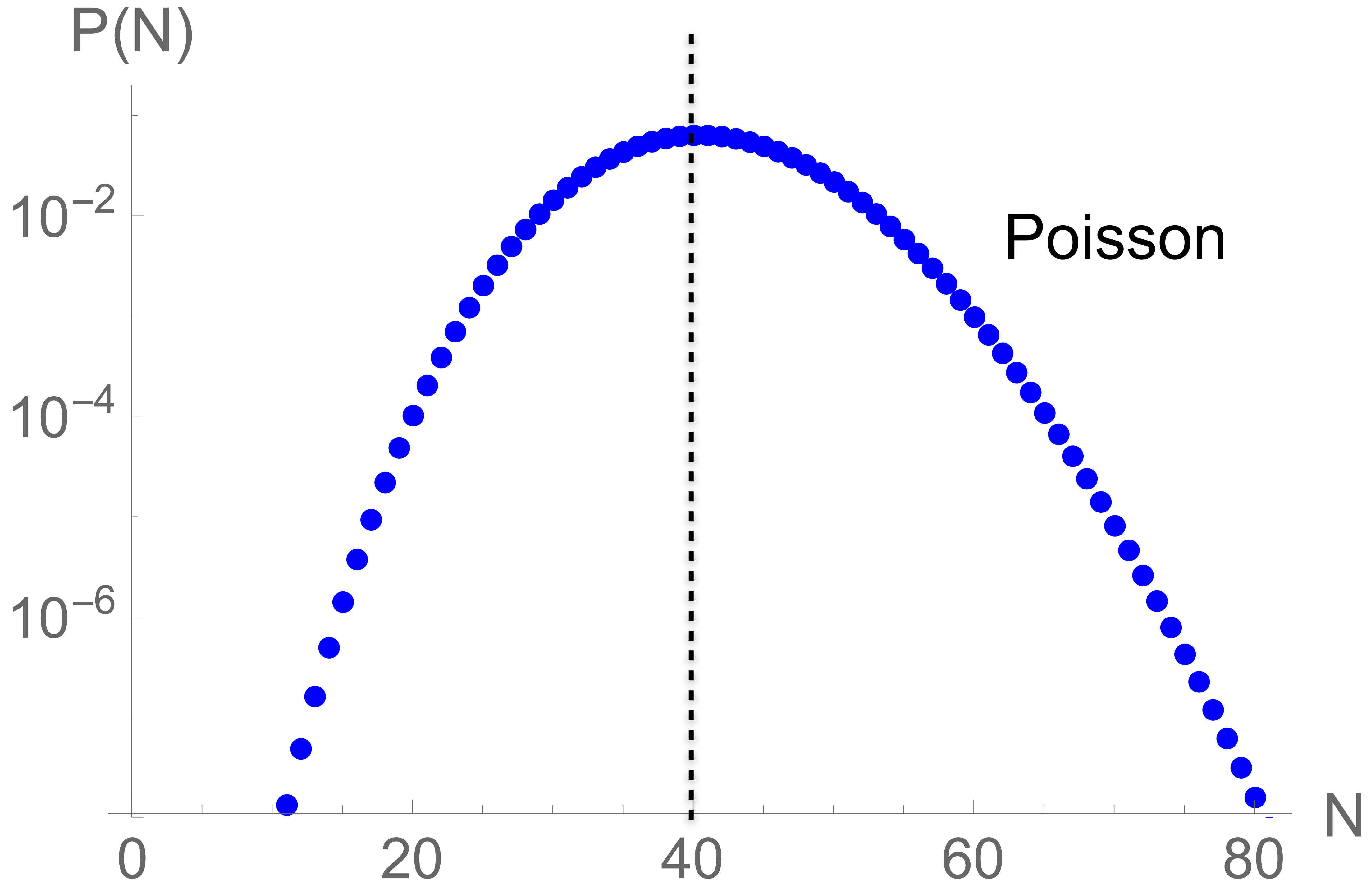


K_4/K_2 above baseline K_3/K_2 below baseline

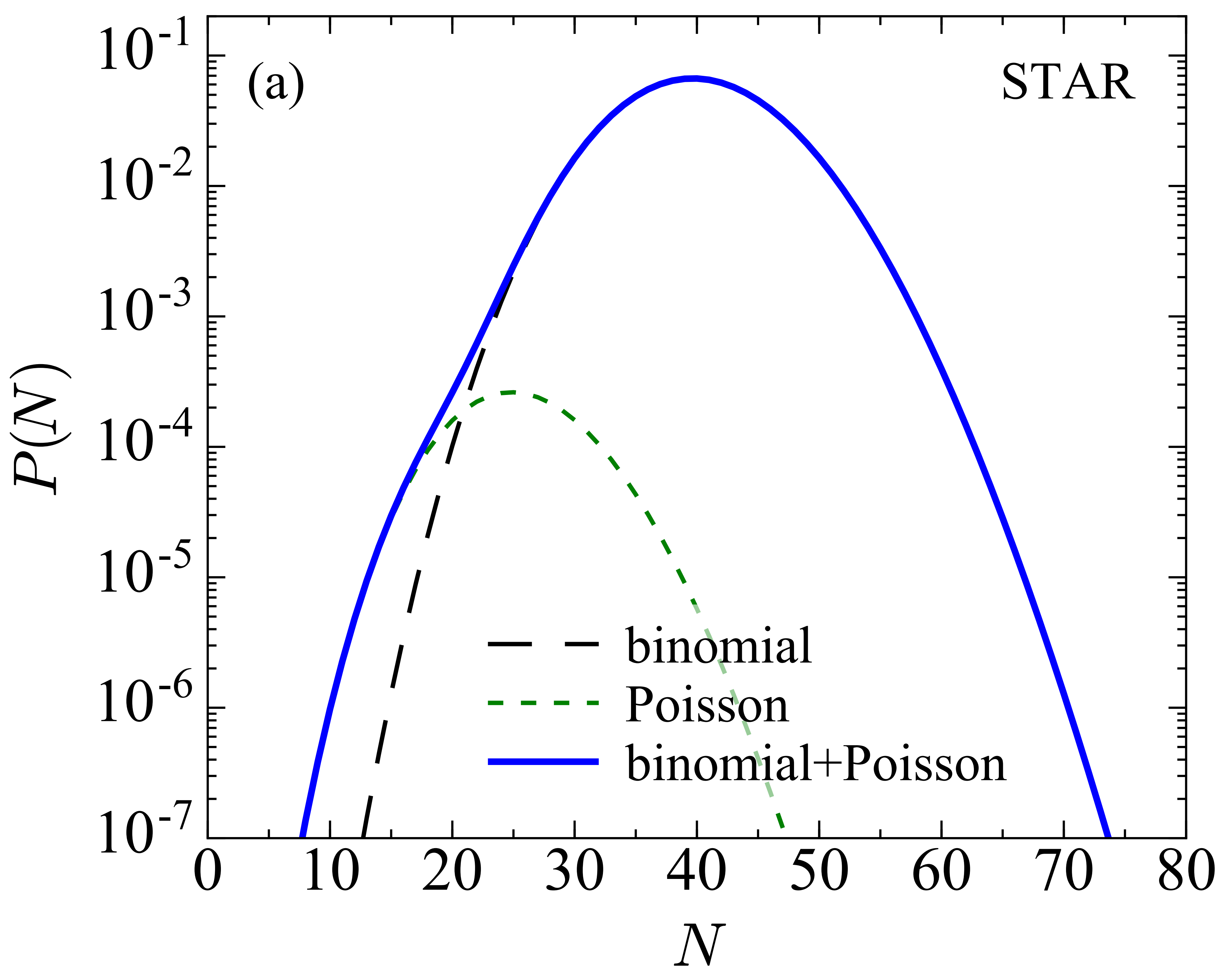
Shape of probability distribution

$$K_3 < \langle N \rangle \qquad K_3 = \langle N - \langle N \rangle \rangle^3$$

$$K_4 > \langle N \rangle \qquad K_4 = \langle N - \langle N \rangle \rangle^4 - 3 \langle N - \langle N \rangle \rangle^2$$



Simple two component model



Weight of small component: ~0.3%

Left bump not visible raw (efficiency) uncorrected distribution

Two component model

$$P(N) = (1-\alpha)P_{(a)}(N) + \alpha P_{(b)}(N)$$

$$\overline{N} = \langle N_{(a)} \rangle - \langle N_{(b)} \rangle$$

$$\begin{aligned}
C_2 =& C_2^{(a)} - \alpha \left\{ \overline{C}_2 - (1-\alpha)\overline{N}^2 \right\} \\
C_3 =& C_3^{(a)} - \alpha \left\{ \overline{C}_3 + (1-\alpha) \left[(1-2\alpha)\overline{N}^3 - 3\overline{N}\,\overline{C}_2 \right] \right\} \\
C_4 =& C_4^{(a)} - \alpha \left\{ \overline{C}_4 - (1-\alpha) \left[\left(1 - 6\alpha + 6\alpha^2\right) \overline{N}^4 - 6(1-2\alpha)\overline{N}^2\overline{C}_2 + 4\overline{N}\,\overline{C}_3 + 3(\overline{C}_2)^2 \right] \right\}
\end{aligned}$$

$$\overline{C}_n = C_n^{(a)} - C_n^{(b)},$$

For Poisson, $C_{(a)}$, $C_{(b)}=0$

Fit to STAR data: $\langle N_{(a)} \rangle \simeq 40, \ \langle N_{(b)} \rangle \simeq 25, \ \alpha \simeq 0.003$

Two component model

$$P(N) = (1-\alpha)P_{(a)}(N) + \alpha P_{(b)}(N)$$

$$\bar{N} = \langle N_{(a)} \rangle - \langle N_{(b)} \rangle > 0$$

For $P_{(a)}$, $P_{(b)}$ Poisson, or (to good approximation) Binomial

$$C_n = (-1)^n K_n^B \bar{N}^n \quad n \geq 2$$

$$K_n^B : \text{ Cumulant of Bernoulli distribution}$$

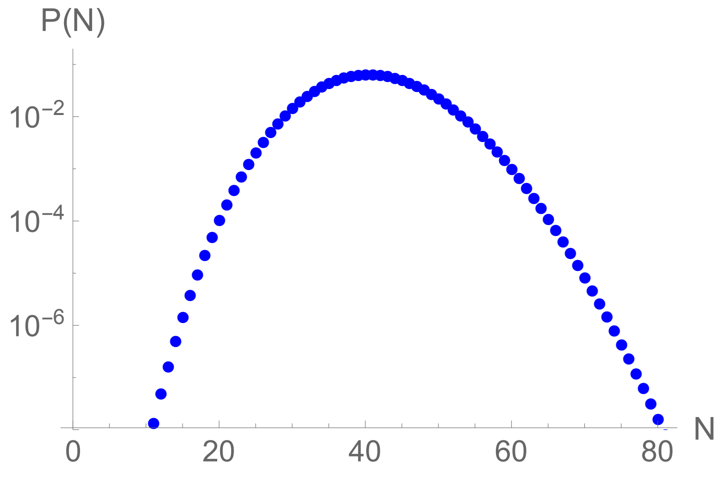
$$\alpha \ll 1, K_n^B = \alpha \;\Rightarrow\; C_n \simeq \alpha(-1)^n \bar{N}^n$$

$\Rightarrow |C_n| \sim \langle N \rangle^n$ as seen by STAR (i.e. “infinite” correlation length)

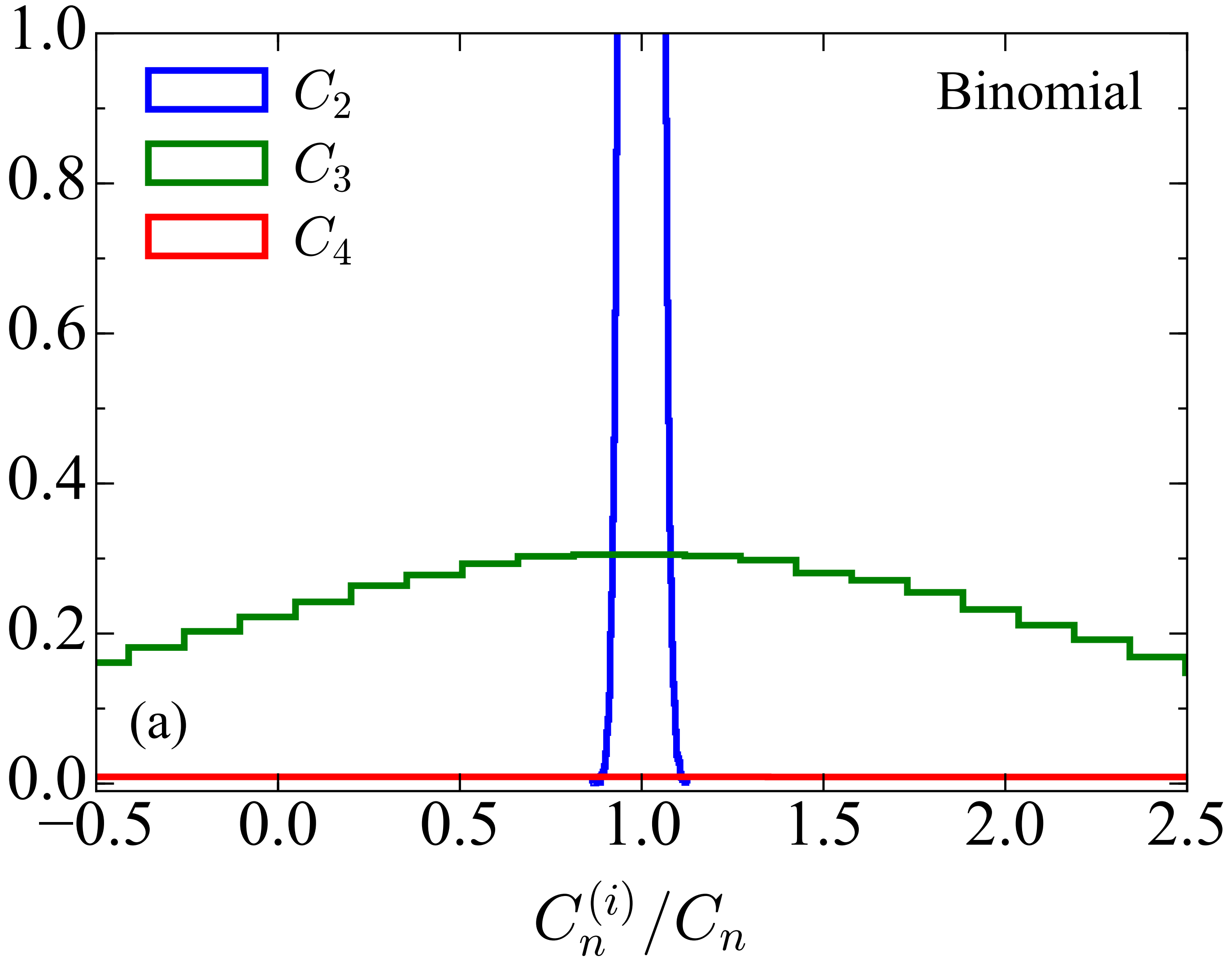
predict: $\boxed{\dfrac{C_4}{C_3} = \dfrac{C_5}{C_4} = \dfrac{C_{n+1}}{C_n} = -\bar{N}}$ $\quad \bar{N} \simeq 15$

Clear and falsifiable prediction: $C_5 \approx -2650 \quad C_6 \approx 41000$

Two component model is “statistics friendly”

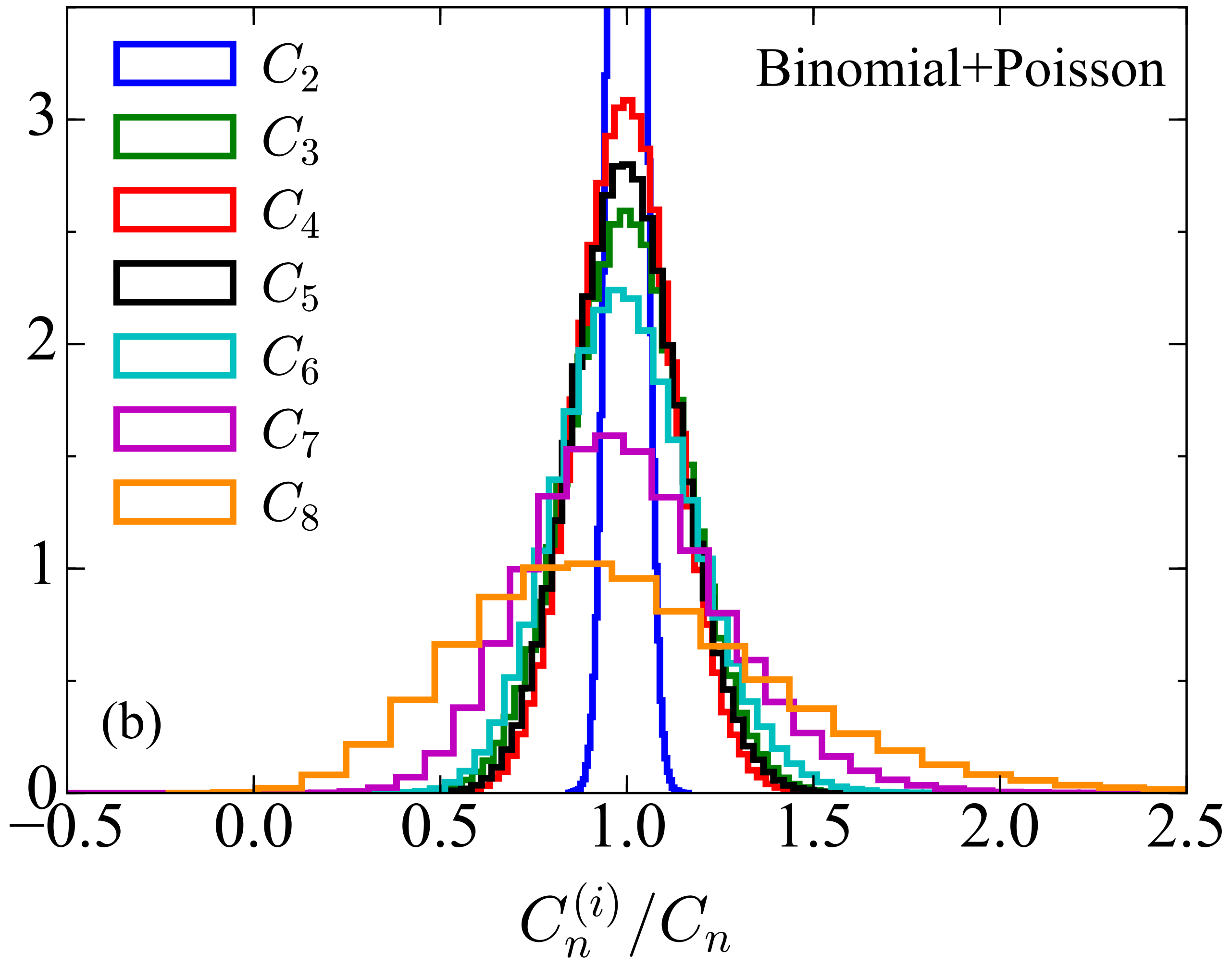


One component

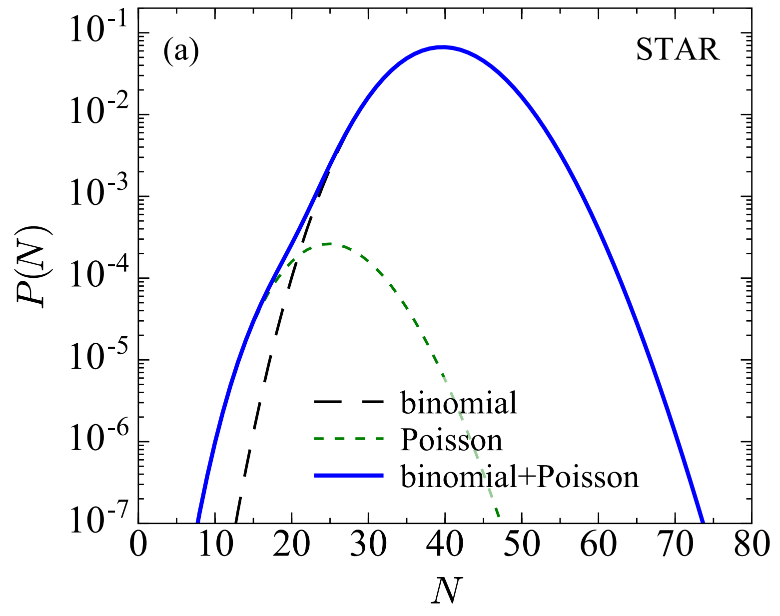


Single component (binomial)

Two component

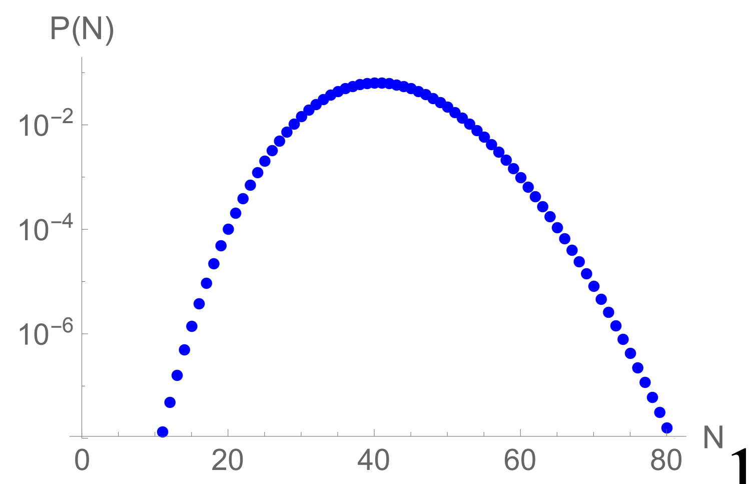


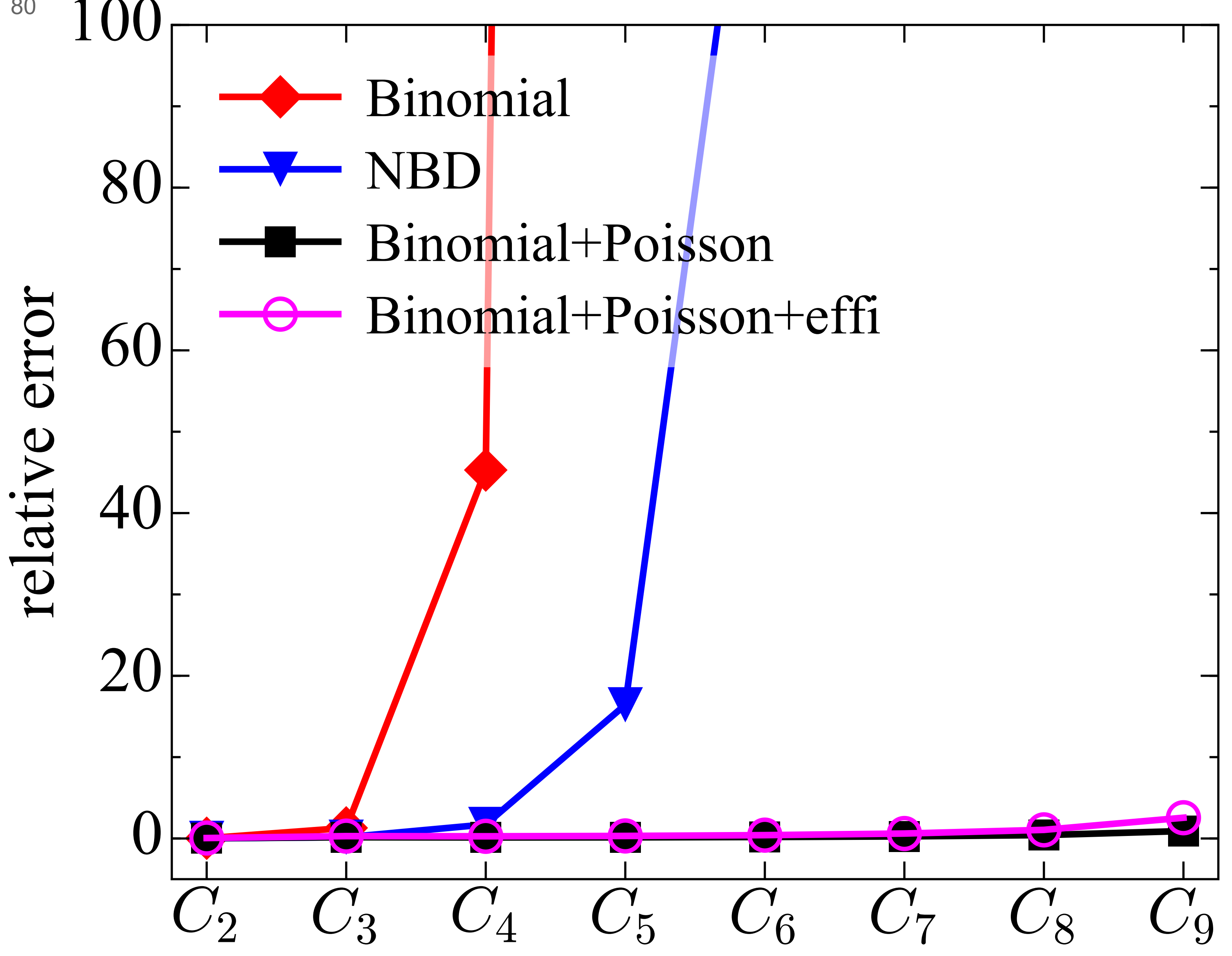
Two component (binomial + Poisson)

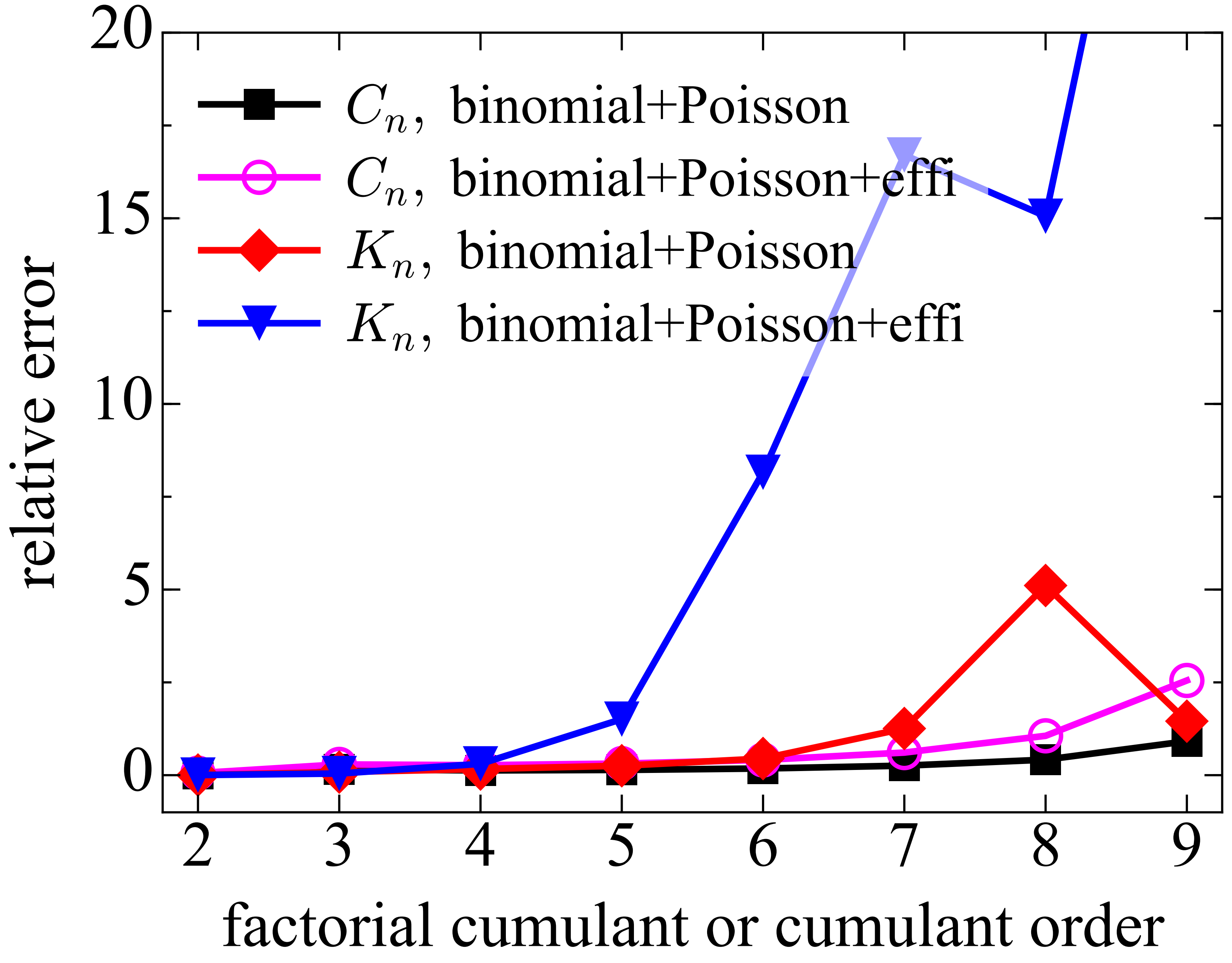


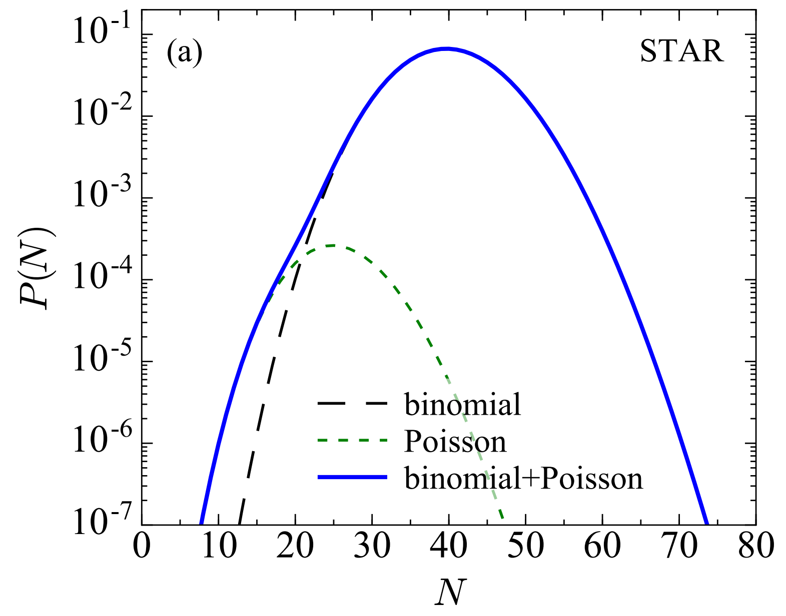
Based on 144393 events (same as STAR 0-5% at 7.7 GEV)

Two component model is "statistics friendly"









Cumulants are less "statistics friendly"

Based on 144393 events (same as STAR 0-5% at 7.7 GEV)

This model can be tested RIGHT NOW!

Model prediction:

$$C_5 = -2645\,(1 \pm 0.14), \quad C_6 = 40900\,(1 \pm 0.18),$$
$$C_7 = -615135\,(1 \pm 0.26), \quad C_8 = 8520220\,(1 \pm 0.42)$$

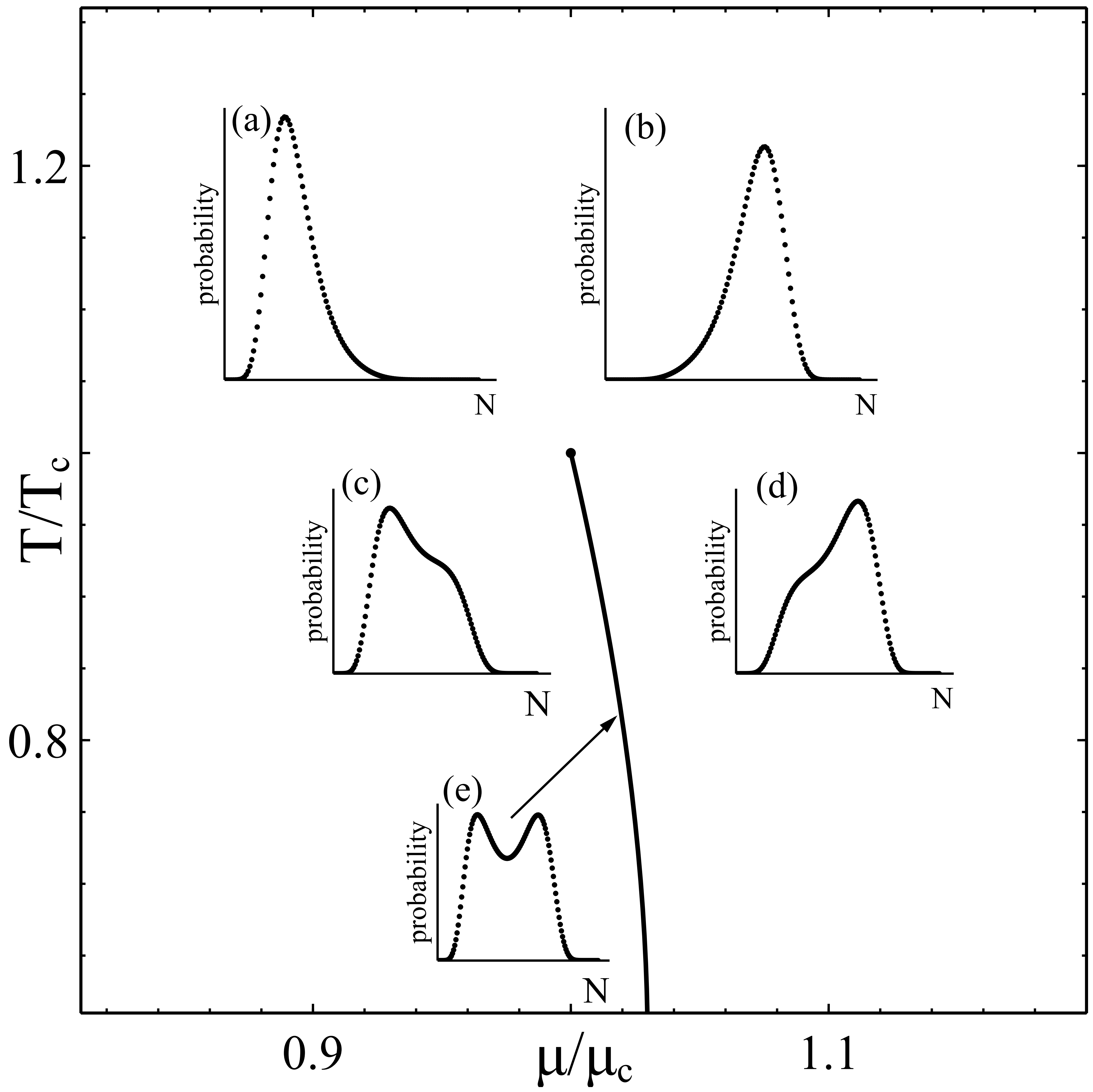
Efficiency corrected

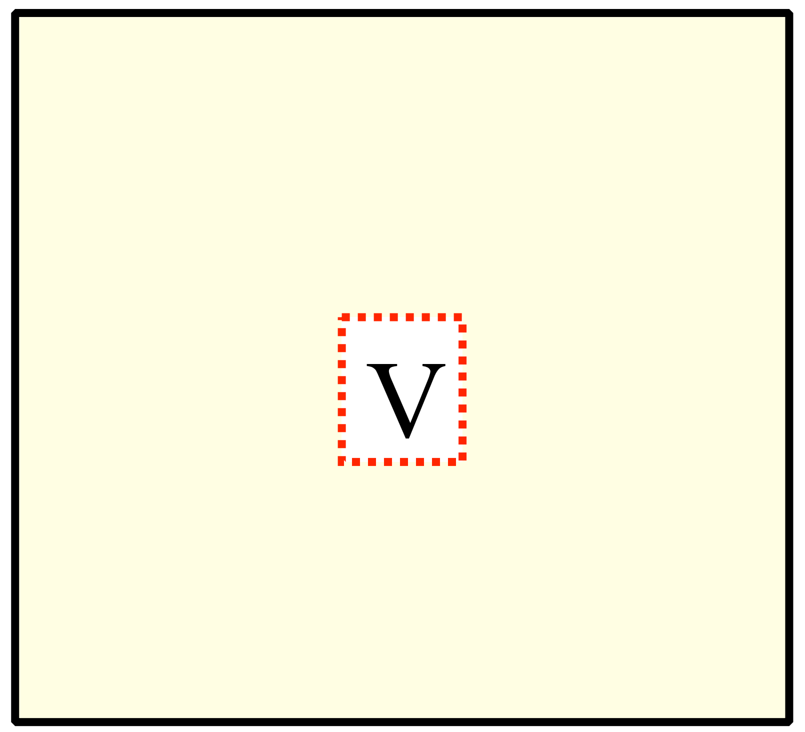
$$C_5 = -307\,(1 \pm 0.31), \quad C_6 = 3085\,(1 \pm 0.41),$$
$$C_7 = -30155\,(1 \pm 0.61), \quad C_8 = 271492\,(1 \pm 1.06),$$

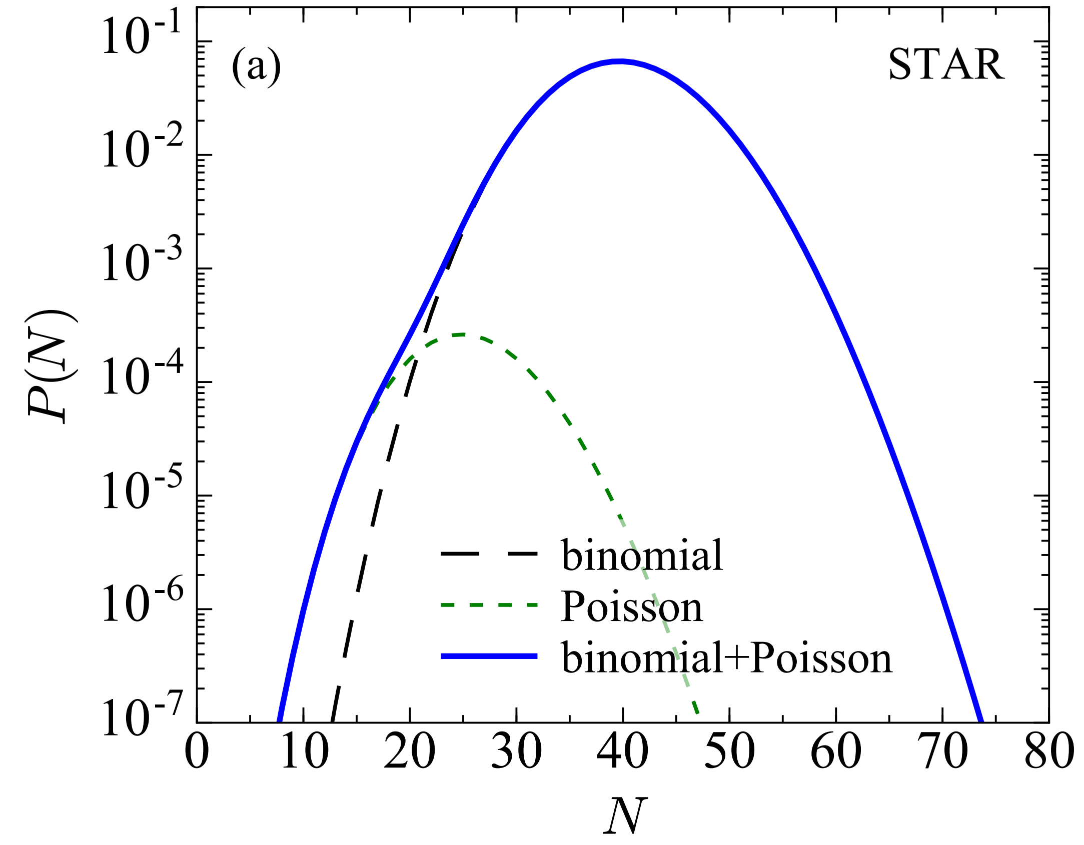
Efficiency UN-corrected

Based on 144393 events (same as STAR 0-5% at 7.7 GEV)

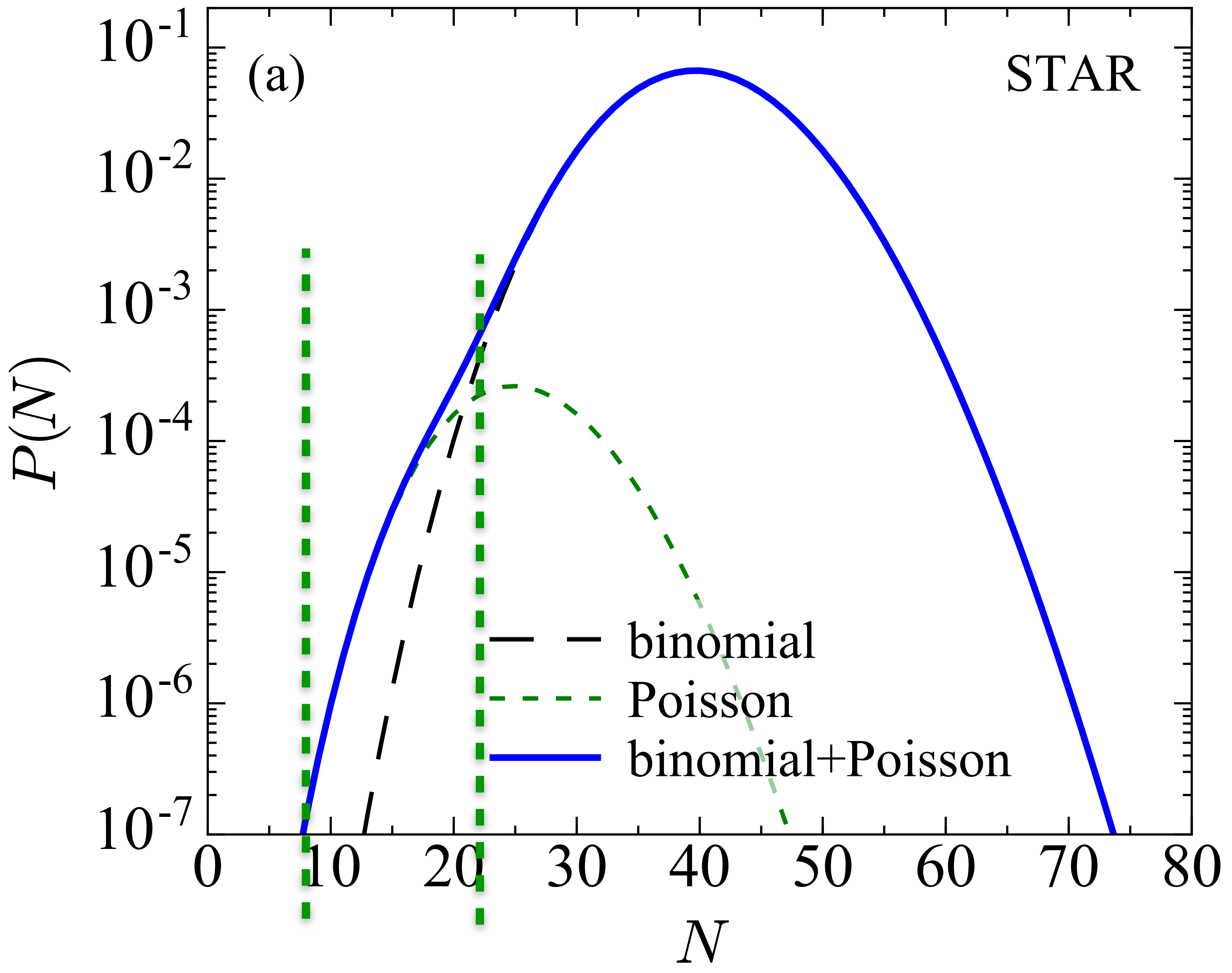
Speculation







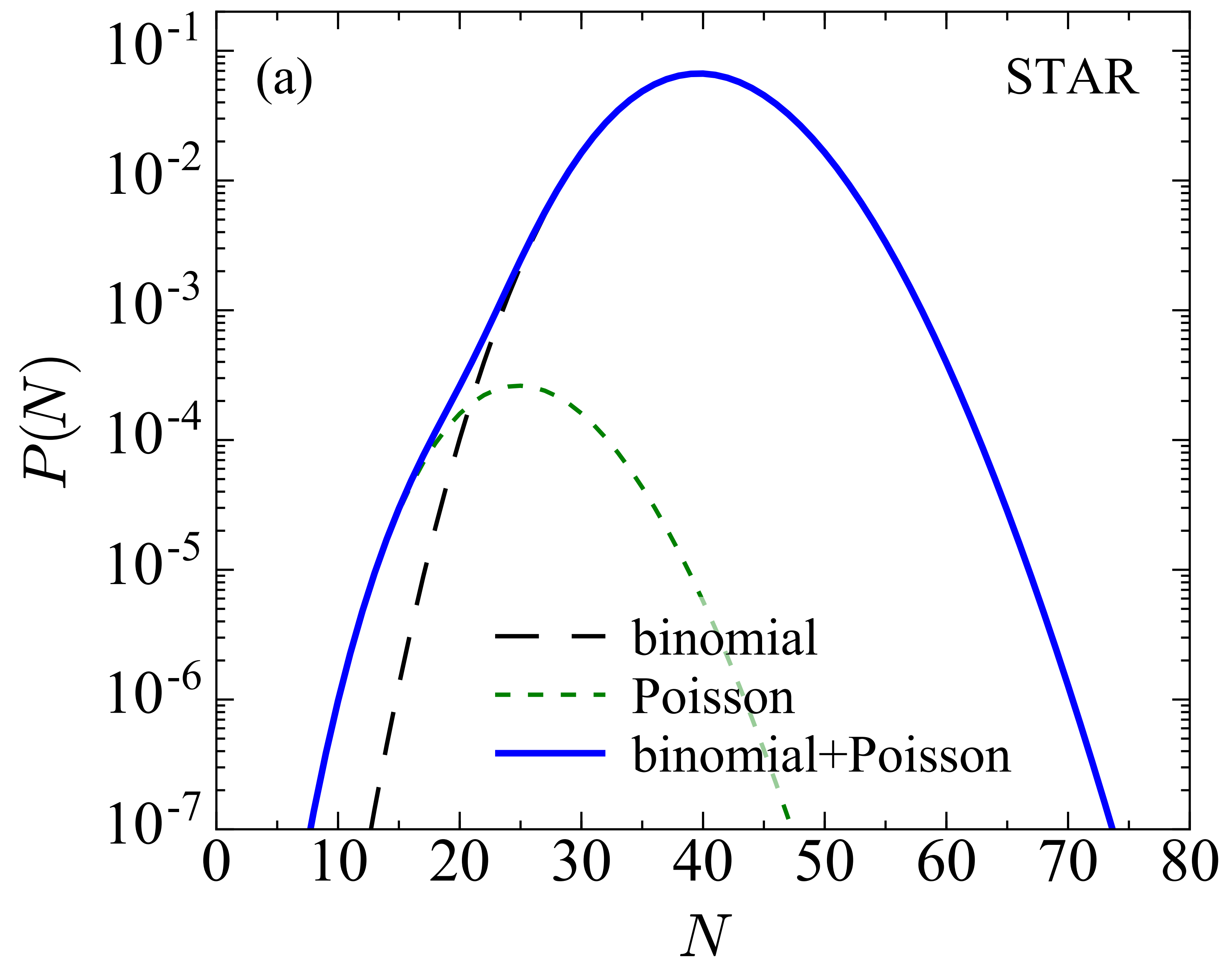
Simple two component model



Analyse data for N_p <20

- Is flow etc different?
- “Inspect by eye (<1% of all events)

Summary



Prediction:

$$C_5 = -2645\,(1 \pm 0.14), \quad C_6 = 40900\,(1 \pm 0.18),$$
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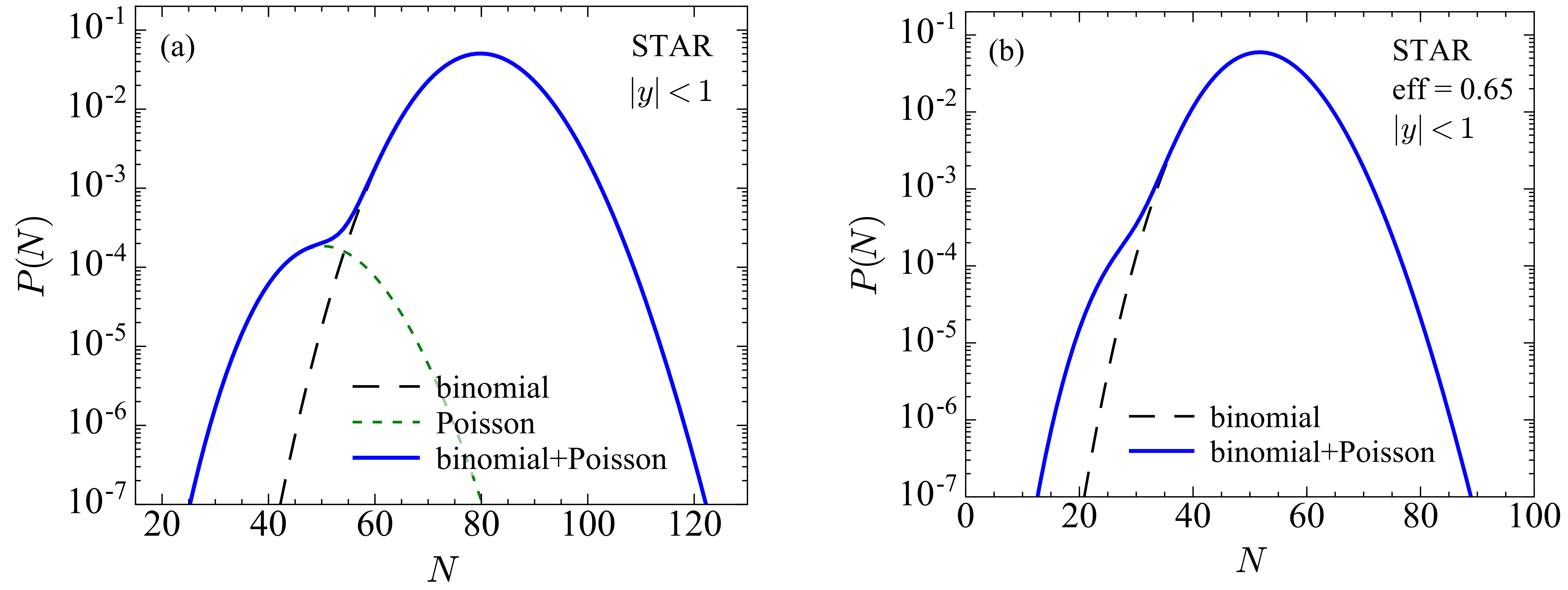
Thank You

Double the acceptance



Should be visible in raw (unfolded) data

Latest STAR result on net-proton cumulants

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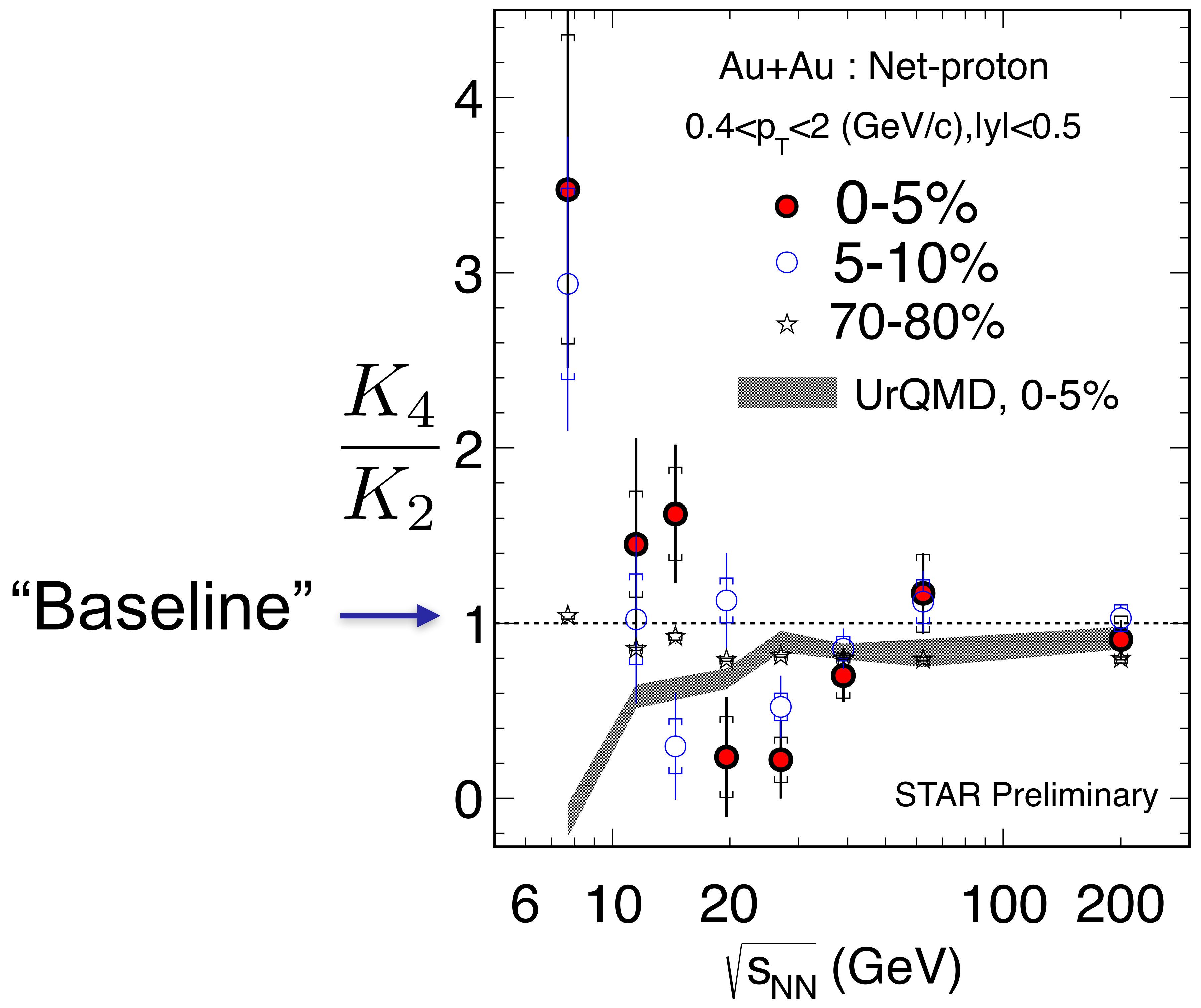
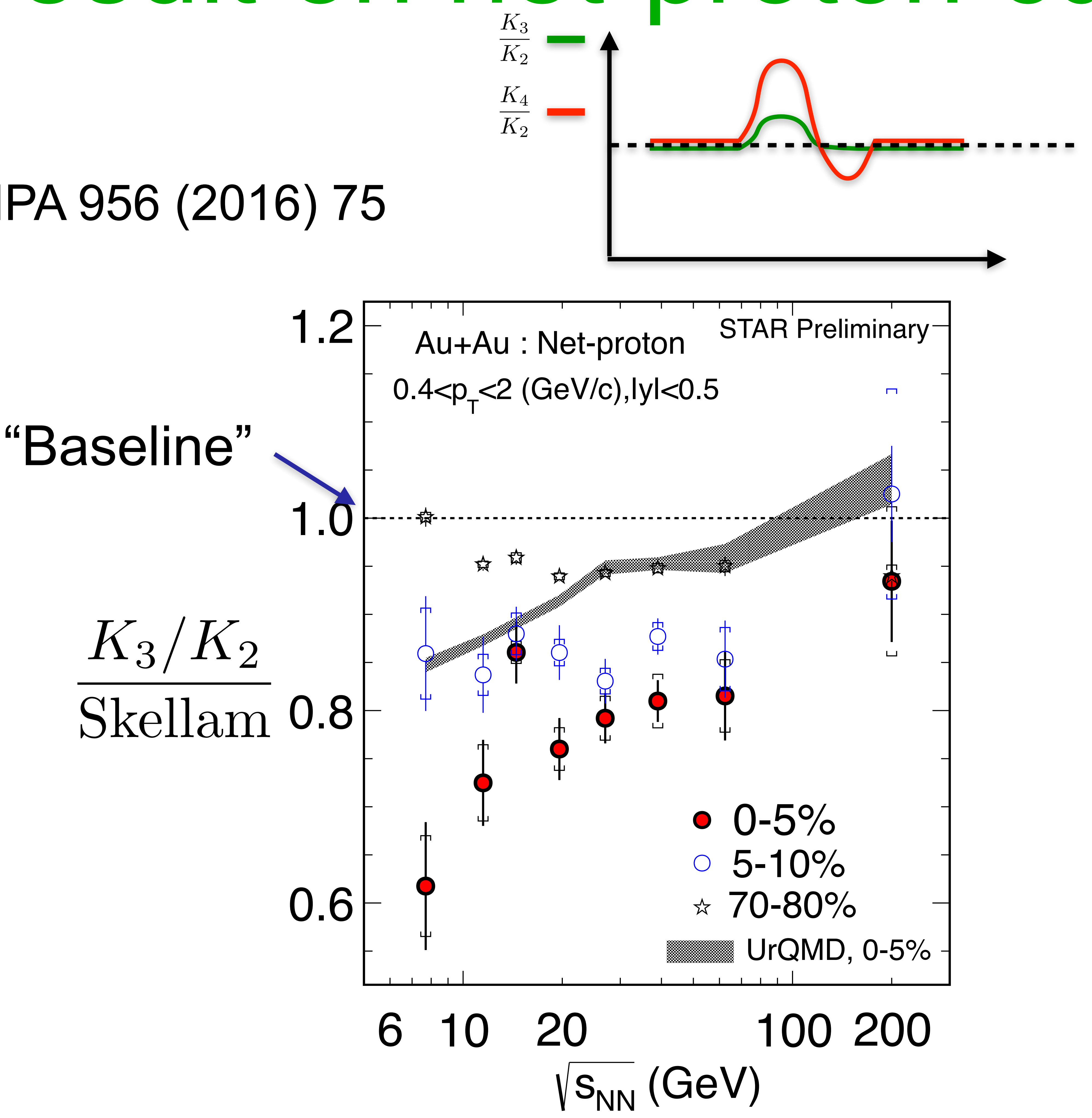
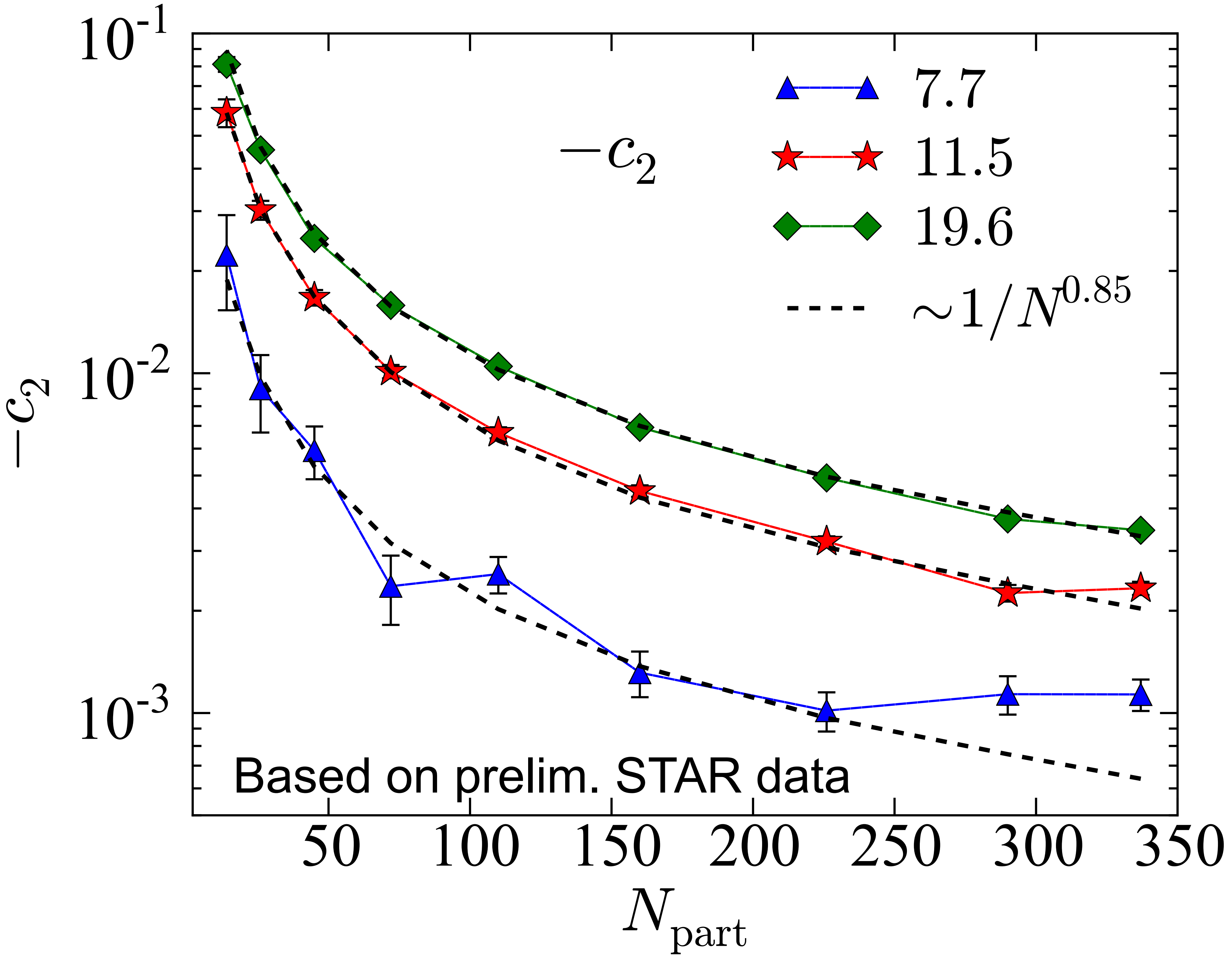




K_4/K_2 follows expectation, K_3/K_2 no so much…..
URQMD totally fails to get trend for K_4/K_2 !

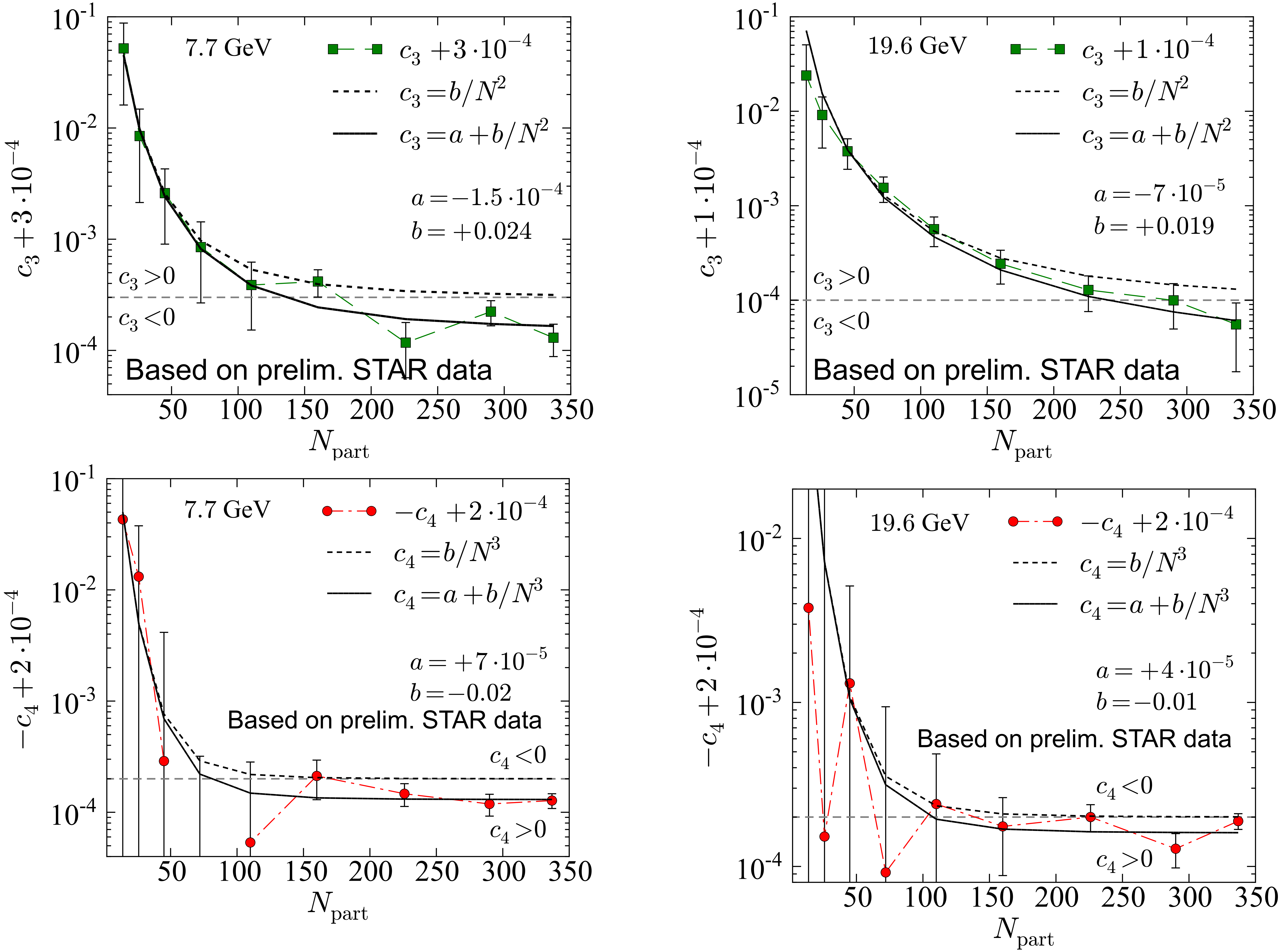
Centrality dependence



Centrality dependence

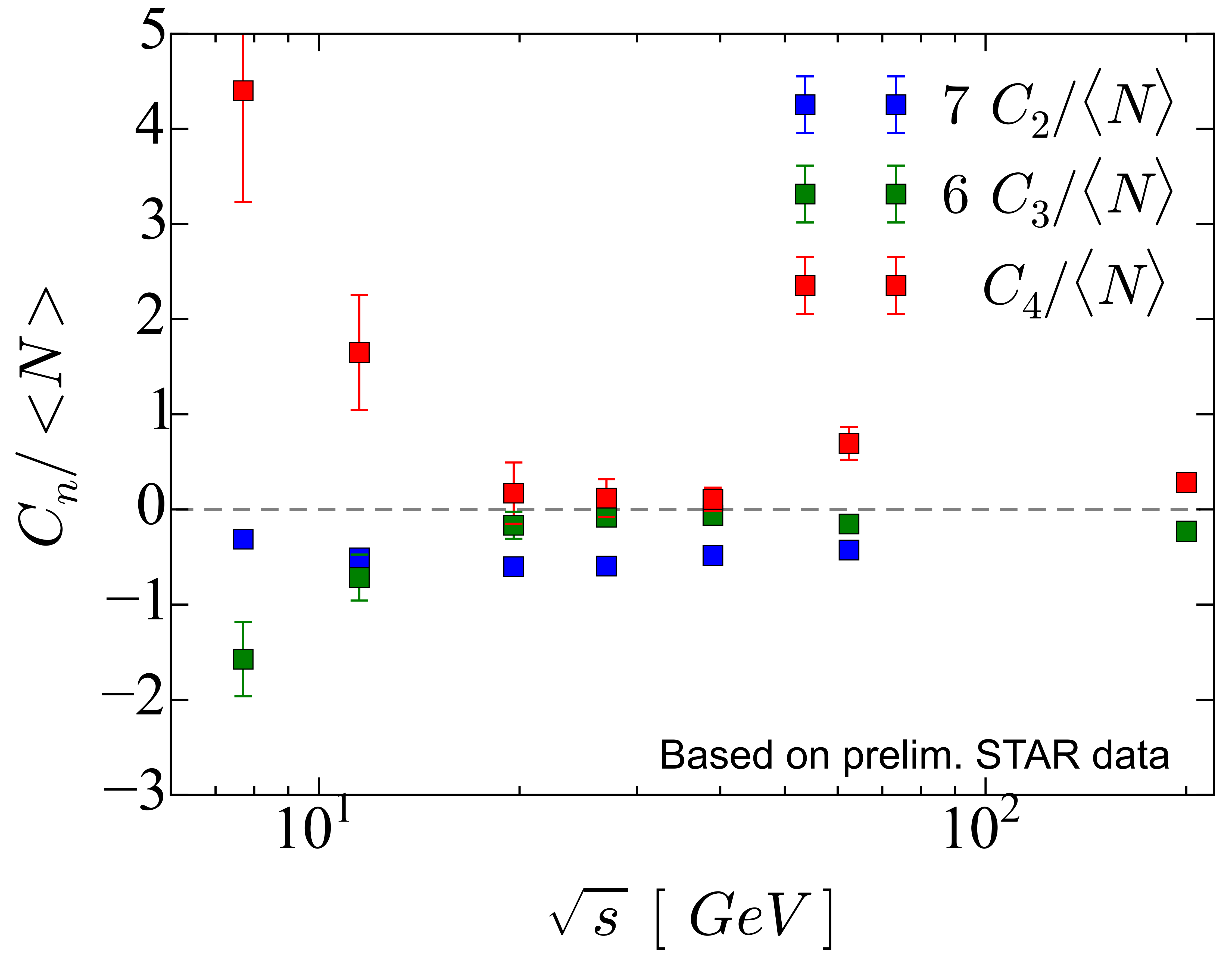
C_3

C_4



7.7 GeV 19.6 GeV

Energy dependence

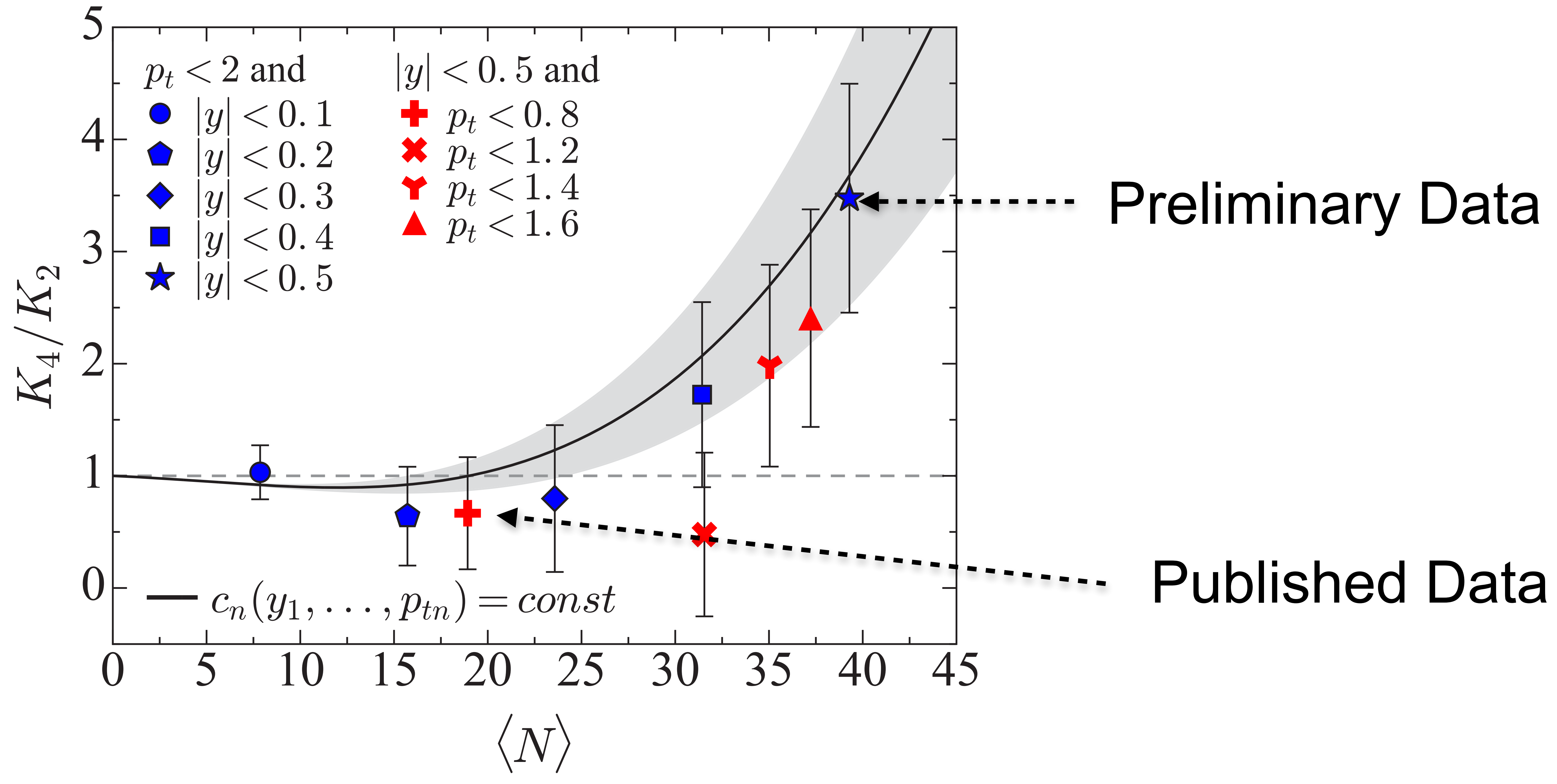


Note: anti-protons are non- negligible above 19.6 GeV
Data are protons only

Long range correlations

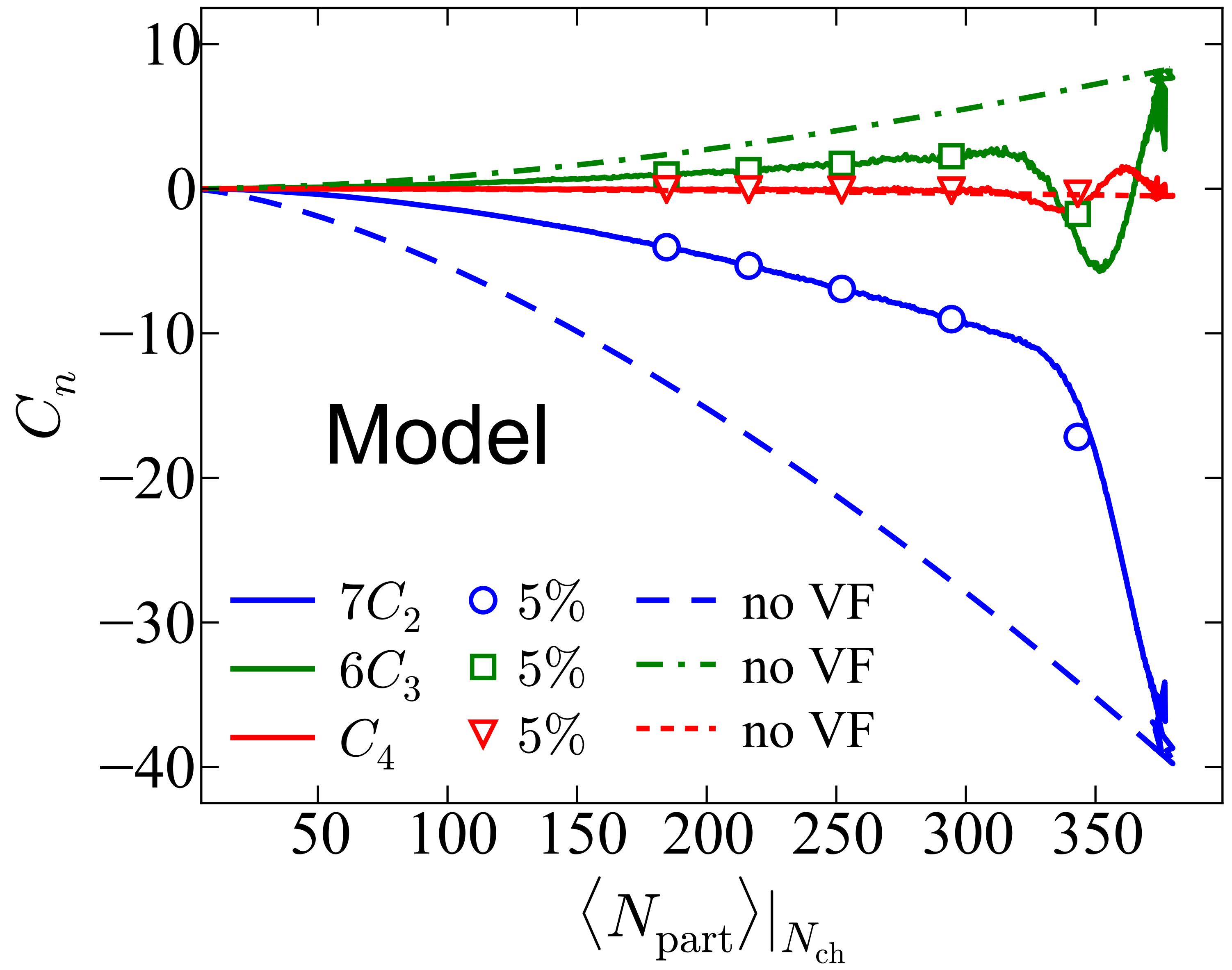
$$C_k = \langle N \rangle^k c_k$$

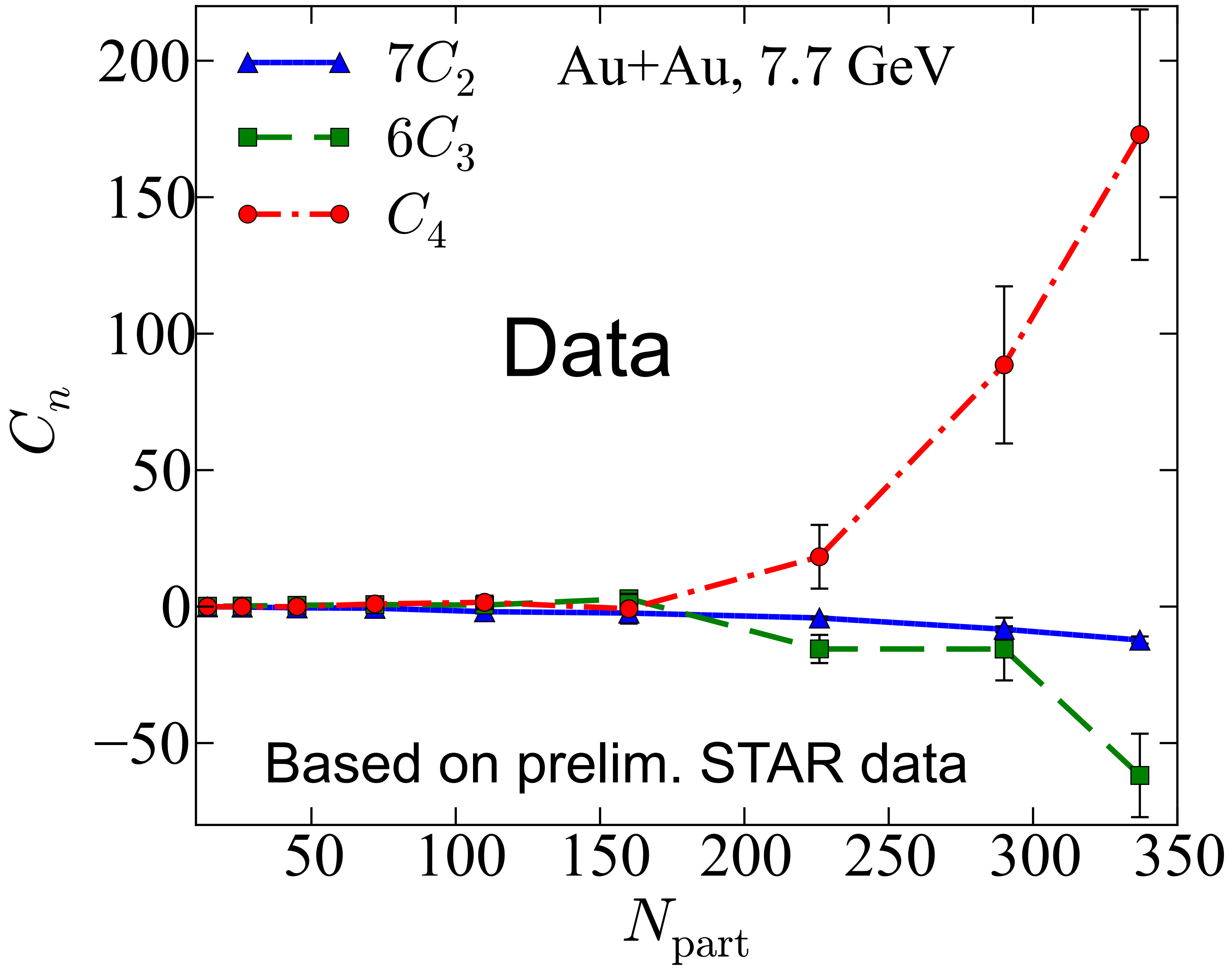
$$c_k = const. \Rightarrow K_n = K_n(\langle N \rangle)$$



Can we understand these correlations?

- Two particle correlations can be understood by simple Glauber model + Baryon number conservation



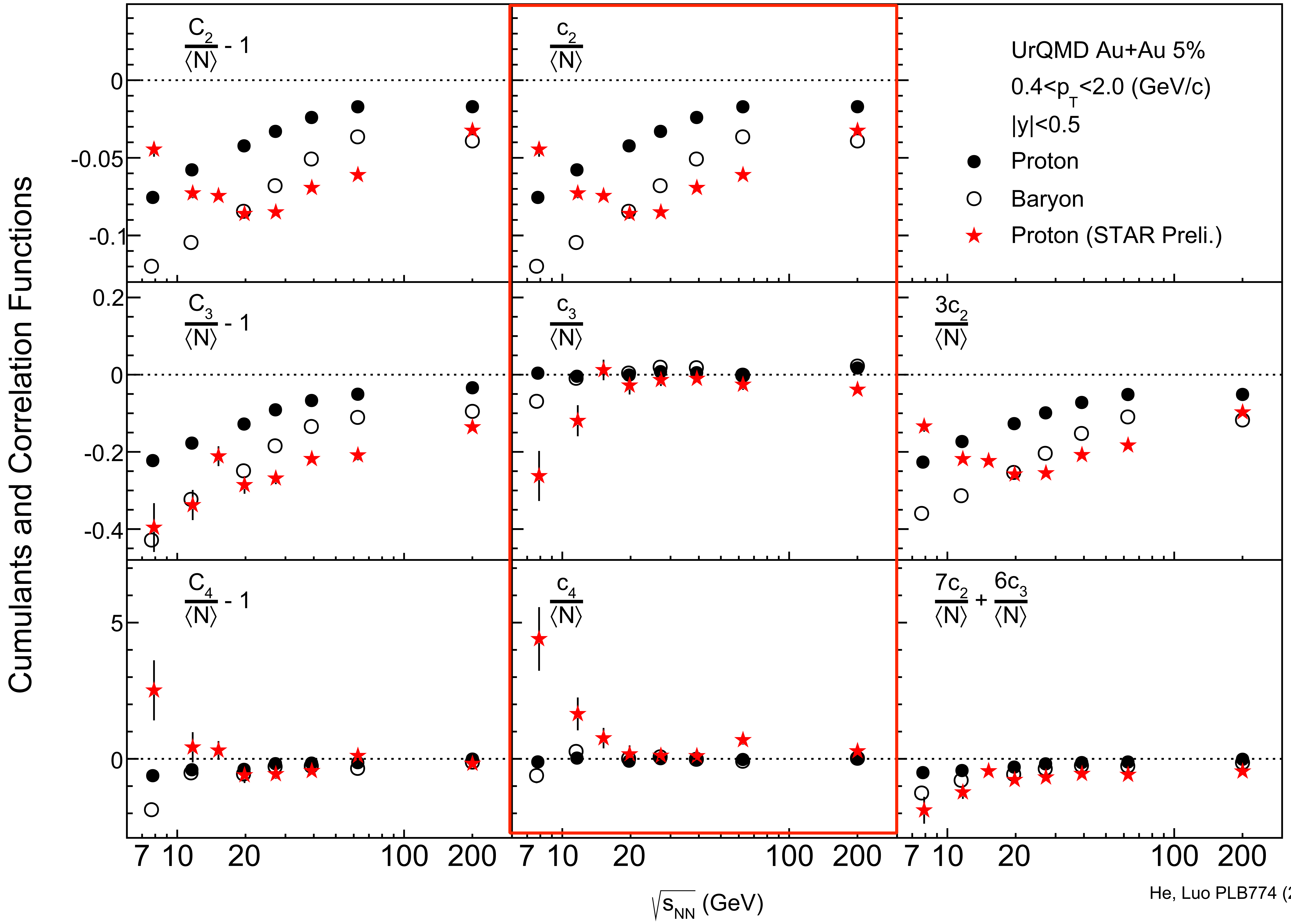


Four particle correlations are orders of magnitudes larger in the data

Also seen in URQMD calculations by He et al. PLB774 (2017) 623

Need to assume the ~40% of protons come from 8-nucleon cluster in order to get magnitude right!

URQMD



He, Luo PLB774 (2017) 623