Relativistic Dynamics of Fluctuations Away and Near the QCD Critical Point

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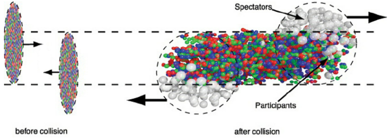
†Parallel talk in Session NTH I on Tuesday.

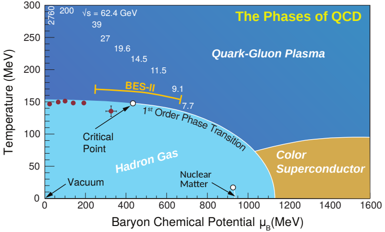
Introduction

- Motivation: Search for the QCD critical point.
- Experimental approach: Beam Energy Scan (BES) program.
- Theory toolkit: Generalized hydrodynamic formalism with

I) fluctuations in arbitrary relativistic flow.
Misha's talk (Tuesday)

II) critical slowing down (Hydro+).
Stephanov-Yin, 2017



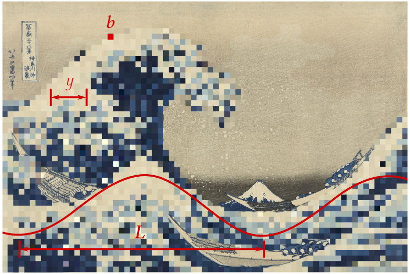


Historical review

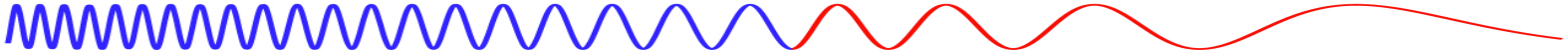
authors	year	relativistic	arbitrary flow	non-conformal	charged	critical
Andreev	1970s	✘	✔	✔	✔	✘
Akamatsu-Mazeliauskas-Teaney	2017	✔	✘	✔	✘	✘
Martinez-Schäfer	2018	✔	✘	✘	✔	✘
XA-Başar-Stephanov-Yee	2019	✔	✔	✔	✔	✔

Separation of hydrodynamic scales

The Great Wave
K. Hokusai, 1830
Left: Original;
Right: Coarse-grained.



	microscopic		hydro cell		fluctuation		background
Length:	ℓ_{mic}	$\ll$	b	$<$	y	$\ll$	L
Wavenumber:	T	$\gg$	Λ	$>$	q	$\gg$	k



In equilibrium $\longleftarrow$ q^* $\longrightarrow$ Out of equilibrium

Relaxation rate $\Gamma_{\text{relax}} = \gamma q^2 \xleftrightarrow{\text{competition}}$ Evolution rate $\omega = c_s k \implies q^* \sim \sqrt{c_s k/\gamma}$.

From stochastic to deterministic approach

- Stochastic hydrodynamic equations:

$$\partial_t \breve{\psi} = -\nabla \cdot \Big(\text{flux}\,[\breve{\psi}] + \text{noise} \Big) \qquad \text{(conservation)}$$

where $\breve{\psi}$ = energy, momentum and *baryon charge* densities. click

- Deterministic hydrodynamic equations:

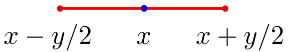
$$\begin{cases} \partial_t \psi = -\nabla \cdot \text{flux}\,[\psi; G]\,, \\ \partial_t G = \text{relaxation}[G - G^{(\text{eq})}; \psi]\,. \end{cases}$$

where

$$\psi(x) = \langle \breve{\psi}(x) \rangle\,, \quad \delta\psi(x) = \breve{\psi} - \langle \breve{\psi} \rangle\,; \quad \text{(1-pt)}$$

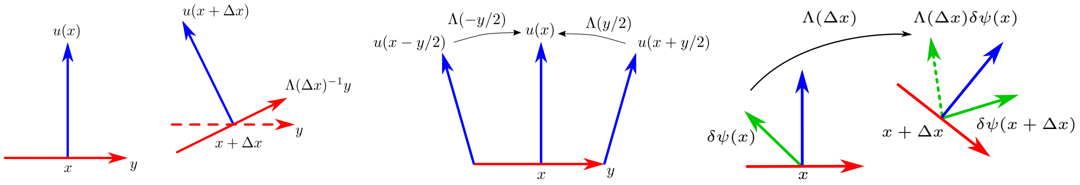
$$G(x,y) = \langle \delta\psi(x + y/2)\delta\psi(x - y/2) \rangle\,; \quad \text{(2-pt)}$$

$$\langle \text{noise}(x + y/2)\text{noise}(x - y/2) \rangle \sim \delta^{(4)}(y)\,. \quad \text{(FDT)}$$



"Confluent" quantities

- "Confluent" quantities: correlators and derivatives adjusted by the fluid. XA, *et al*, 2019



equal time: $u \cdot y = 0$ correlator: $G \xrightarrow{\Lambda} \bar{G}$ derivative: $\partial \xrightarrow{\Lambda} \bar{\partial}$

- The "confluent" Wigner function

$$W(x, q) = \int_y e^{-iq\cdot y}\, \bar{G}(x, y)$$

satisfies a "confluent" evolution equation. click

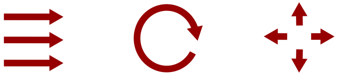
"Confluent" evolution equations

- Upon averaging over fast modes on relevant time scales, seven slow modes survive. The sound mode decouples and matches the kinetic equation for phonons: XA, *et al*, 2019

$$\mathcal{L}[W_{++}] = \left[(u+v)\cdot\bar{\nabla} + \text{force}\cdot\frac{\partial}{\partial q}\right] W_{++} = -\Gamma_L q^2 (W_{++} - T/E)\,,$$

where $v = c_s\hat{q}$, $E = c_s|q|$ and

$$\text{force} \supset$$



Inertial Coriolis "Hubble"

"Finally, after about six months of work off and on, all the pieces suddenly fitted together, producing miraculous cancellation, and I was staring at the amazingly simple final result." (C.N. Yang)

Renormalization and long-time tails

- flux $=$ flux $[\psi; G]$ where

$$G(x) = \int^{\Lambda} d^3q\, W(x,q) \sim \underbrace{\Lambda^3 + \Lambda\,\partial(u,\,\mu/T)}_{\text{divergent}} + \underbrace{\widetilde{G}}_{\text{remaining}} .$$

Divergent part: renormalize flux, i.e., $\psi \to \psi_R$ (EOS), $\gamma \to \gamma_R$ (transp.coeff.).

Remaining part: non-analytic terms (long-time tails)

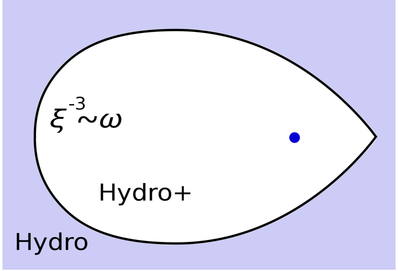
$$\widetilde{G} \sim q^{*3} \sim (c_s k/\gamma)^{3/2} \gg \mathcal{O}(k^2) .$$

- We obtain a closed set of cutoff-independent equations suitable for simulation : XA, *et al*, 2019

$$\begin{cases} \partial_t \psi = -\nabla \cdot \text{flux}\,[\psi; G]\,, \\ \partial_t G = \text{relaxation}[G - G^{(\text{eq})}; \psi]\,. \end{cases} \longrightarrow \begin{cases} \partial_t \psi_R = -\nabla \cdot \text{flux}\,[\psi_R; \widetilde{G}]\,, \\ \bar{\partial}_t \widetilde{G} = \text{relaxation}[\widetilde{G}; \psi_R]\,. \end{cases}$$

Hydro+

- Critical slowing down: $\Gamma_{\text{relax}} \sim \xi^{-z} \to 0$ as the correlation length $\xi \to \infty$ near the critical point.
- Hydrodynamics breaks down when relaxation rate $\lesssim$ evolution rate (i.e., $\Gamma_{\text{relax}} \lesssim \omega$) .
- Hydro+: Extending Hydro by adding one critical slow mode G relaxing with $\Gamma_\lambda \sim \xi^{-3}$. Stephanov-Yin, 2017
- Our formalism merges with and extends Hydro+.
  XA, *et al*, 2019



Dynamics of fluctuations near the critical point

- Sound mode decouples, while remaining slow modes mix :

$$\begin{cases} \mathcal{L}[W_{\sigma\sigma}] = -\,2\Gamma_\lambda \left(W_{\sigma\sigma} - W_{\sigma\sigma}^{(\mathrm{eq})}\right) + L[W_{\sigma i}]\,, \\ \mathcal{L}[W_{\sigma i}] = -\,(\Gamma_\eta + \Gamma_\lambda)\,W_{\sigma i} + L[W_{\sigma\sigma}, W_{\sigma i}, W_{ij}]\,, \\ \mathcal{L}[W_{ij}] = -\,2\Gamma_\eta \left(W_{ij} - W_{ij}^{(\mathrm{eq})}\right) + L[W_{ij}, W_{\sigma i}]\,. \end{cases}$$

- The correlation length ξ increases as the system approaches the critical point, thus

$$\Gamma_\lambda \sim \xi^{-3} \quad \ll \quad \Gamma_\eta \sim \xi^{-2} \quad \ll \quad \xi^{-1}\,.$$

- Don't forget that Γ_{relax} and ω compete !

Dynamics of fluctuations near the critical point

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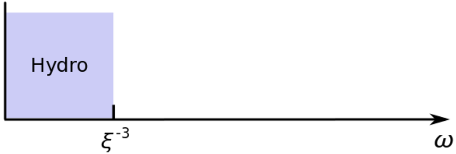
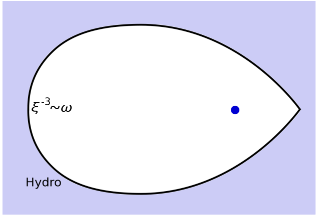
- $\omega \;\ll\; \Gamma_\lambda \;\ll\; \Gamma_\eta$:



toward the smaller system (increasing ω)



approach the critical point (increasing ξ)

Dynamics of fluctuations near the critical point

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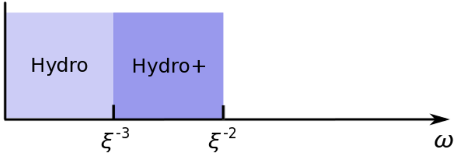
- The correlation length ξ increases as the system approaches the critical point, thus

$$\Gamma_\lambda \sim \xi^{-3} \quad \ll \quad \Gamma_\eta \sim \xi^{-2} \quad \ll \quad \xi^{-1}\,.$$

- $\Gamma_\lambda \;\ll\; \omega \;\ll\; \Gamma_\eta \;\ll\; \xi^{-1}$:



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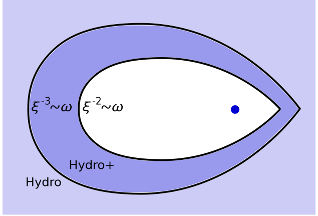


approach the critical point (increasing ξ)

Dynamics of fluctuations near the critical point

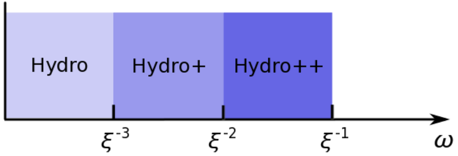
- Sound mode decouples, while remaining slow modes mix :

$$
\begin{cases}
\mathcal{L}[W_{\sigma\sigma}] = -\,2\Gamma_\lambda \left(W_{\sigma\sigma} - W_{\sigma\sigma}^{(\mathrm{eq})}\right) + L[W_{\sigma i}]\,, \\
\mathcal{L}[W_{\sigma i}] = -\,(\Gamma_\eta + \Gamma_\lambda)\, W_{\sigma i} + L[W_{\sigma\sigma}, W_{\sigma i}, W_{ij}]\,, \\
\mathcal{L}[W_{ij}] = -\,2\Gamma_\eta \left(W_{ij} - W_{ij}^{(\mathrm{eq})}\right) + L[W_{ij}, W_{\sigma i}]\,.
\end{cases}
$$

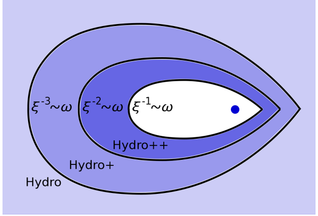
- The correlation length ξ increases as the system approaches the critical point, thus

$$
\Gamma_\lambda \sim \xi^{-3} \quad \ll \quad \Gamma_\eta \sim \xi^{-2} \quad \ll \quad \xi^{-1}\,.
$$

- $\Gamma_\lambda \;\ll\; \Gamma_\eta \;\ll\; \omega \;\ll\; \xi^{-1}$:



toward the smaller system (increasing ω)



approach the critical point (increasing ξ)

Dynamics of fluctuations near the critical point

- Sound mode decouples, while remaining slow modes mix :

Equations need nonlocal modification ! $\longrightarrow$ Model H

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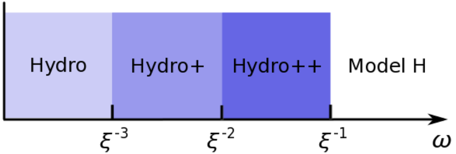
$$\Gamma_\lambda \sim \xi^{-3} \quad \ll \quad \Gamma_\eta \sim \xi^{-2} \quad \ll \quad \xi^{-1}\,.$$

- $\Gamma_\lambda \ \ll \ \Gamma_\eta \ \ll \ \xi^{-1} \ \lesssim \ \omega$:



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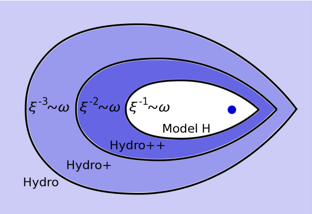
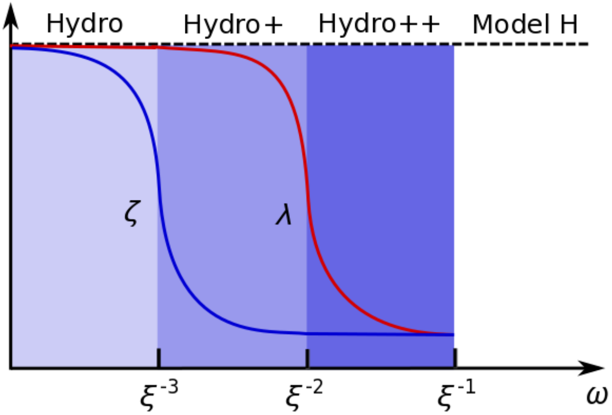


approach the critical point (increasing ξ)

Dynamics of fluctuations near the critical point

- Schematic frequency dependence of the transport coefficients due to fluctuations:



Summary and outlook

- Our formalism is designed to

  1) apply to arbitrary relativistic flow;

  2) extend hydrodynamics to smaller system;

  3) extend Hydro+ closer to the QCD critical point.

- To Do List:

  1) higher-points functions;

  2) first order phase transition;

  3) freeze-out of heavy ion collisions;

  4) hydrodynamics with other DOFs.

  A lot of work could to be done and interesting physics is just ahead!

Backup

Hydrodynamic quantities and equations

- The conserved quantities

$$\psi = (T^{00},\, T^{i0},\, J^0)\,, \quad \text{flux} = (T^{i0},\, T^{ij},\, J^i)\,,$$

where the stress tensor $T^{\mu\nu}$ and charge current J^{μ} are given by back

$$T^{\mu\nu}(\epsilon, n, u) = w(\epsilon, n) u^\mu u^\nu + p(\epsilon, n) g^{\mu\nu} + \Pi^{\mu\nu}, \quad w \equiv \epsilon + p(\epsilon, n)\,,$$
$$J^\mu(\epsilon, n, u) = n u^\mu + \nu^\mu,$$

and

$$\Pi^{\mu\nu} = -2\eta \left(\theta^{\mu\nu} - \frac{1}{3}\Delta^{\mu\nu}\theta\right) - \zeta \Delta^{\mu\nu}\theta\,, \quad \nu^\mu = -\lambda\, \partial_\perp^\mu (\mu/T)\,,$$

$$\theta^{\mu\nu} \equiv \frac{1}{2}\left(\partial^\mu u^\nu + \partial^\nu u^\mu\right), \quad \theta \equiv \partial \cdot u\,, \quad \Delta^{\mu\nu} \equiv g^{\mu\nu} + u^\mu u^\nu, \quad \Delta^{\mu\nu}\partial_\nu \equiv \partial_\perp^\mu.$$

- Conservation equations

$$\partial_\mu \breve{T}^{\mu\nu} = 0\,, \quad \partial_\mu \breve{J}^{\mu} = 0\,.$$

“Confluent” fluctuation evolution equation

- Fluctuation evolution equation: back

$$u\cdot\bar{\nabla}W(x,q) = -\left[i\mathbb{L}^{(q)} + \mathbb{K}^{(a)}, W\right] - \left\{\frac{1}{2}\bar{\mathbb{L}} + \mathbb{D}^{(q)} + \mathbb{K}^{(s)}, W\right\} + \theta W + 2\mathbb{Q}^{(q)}$$
$$+ (\partial_{\perp\lambda}u_\mu)q^\mu \frac{\partial W}{\partial q_\lambda} + \frac{1}{2}\left(a_\lambda + \frac{\partial_{\perp\lambda}c_s}{c_s}\right)\left\{\mathbb{L}^{(q)}, \frac{\partial W}{\partial q_\lambda}\right\}$$
$$+ \frac{\partial}{\partial q_\lambda}\left(\{\Omega^{(s)}_\lambda, W\} + [\Omega^{(a)}_\lambda, W] - \frac{1}{4}[\mathbb{H}_\lambda, [\mathbb{L}^{(q)}, W]]\right),$$

where schematically

$$\mathbb{L}^{(q)} \sim c_s q, \quad \bar{\mathbb{L}} \sim c_s\bar{\nabla}_\perp, \quad \mathbb{D}^{(q)} \sim \mathbb{Q}^{(q)} \sim \gamma q^2,$$
$$\mathbb{K} \sim \partial(u,\alpha), \quad \mathbb{H} \sim c_s\partial u, \quad \Omega \sim c_s^2\omega q.$$