Critical Fluctuation in a Dynamically Expanding Heavy-Ion Collisions

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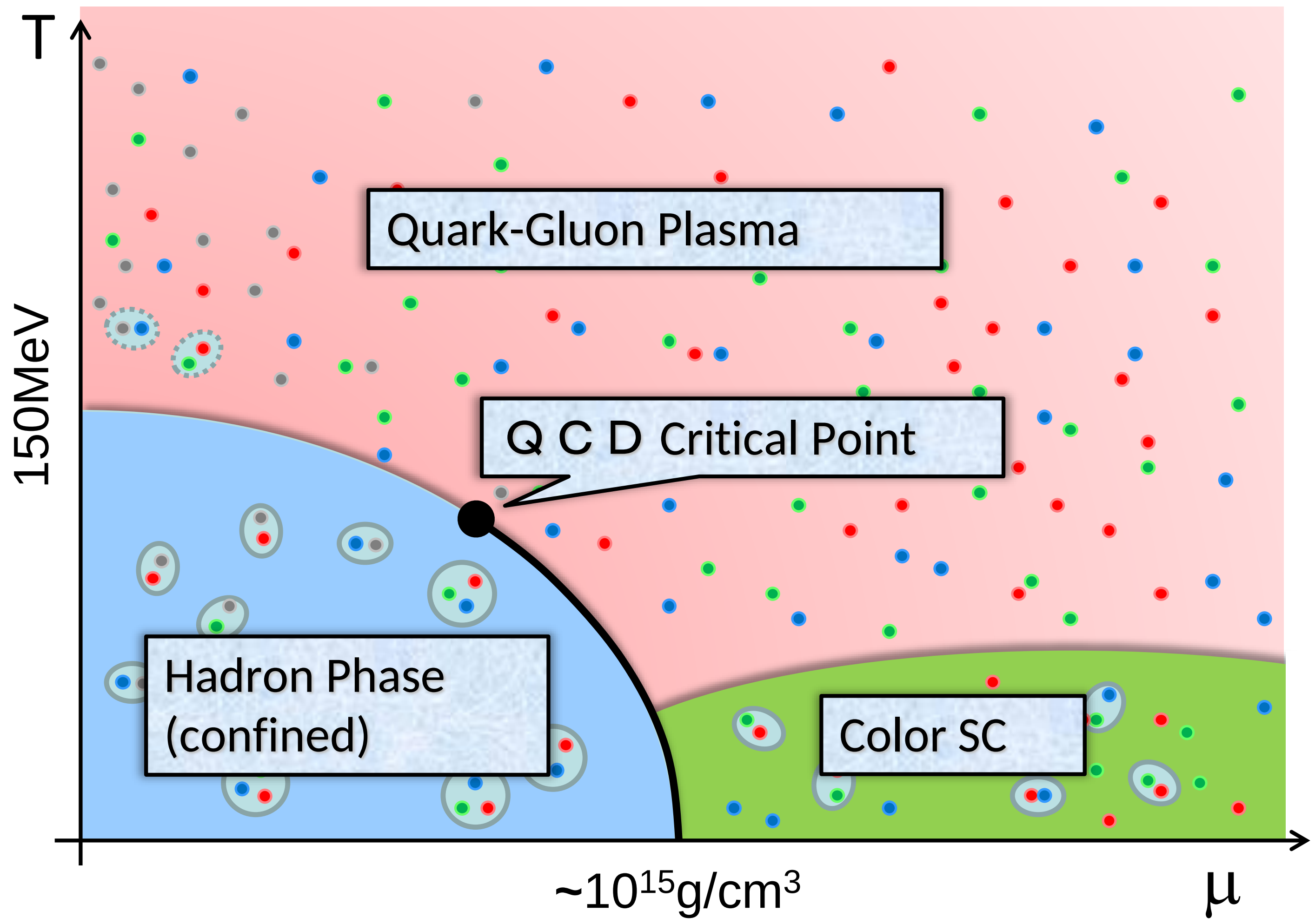
Wanda Reign Hotel, Wuhan, China, 5/Nov./2019

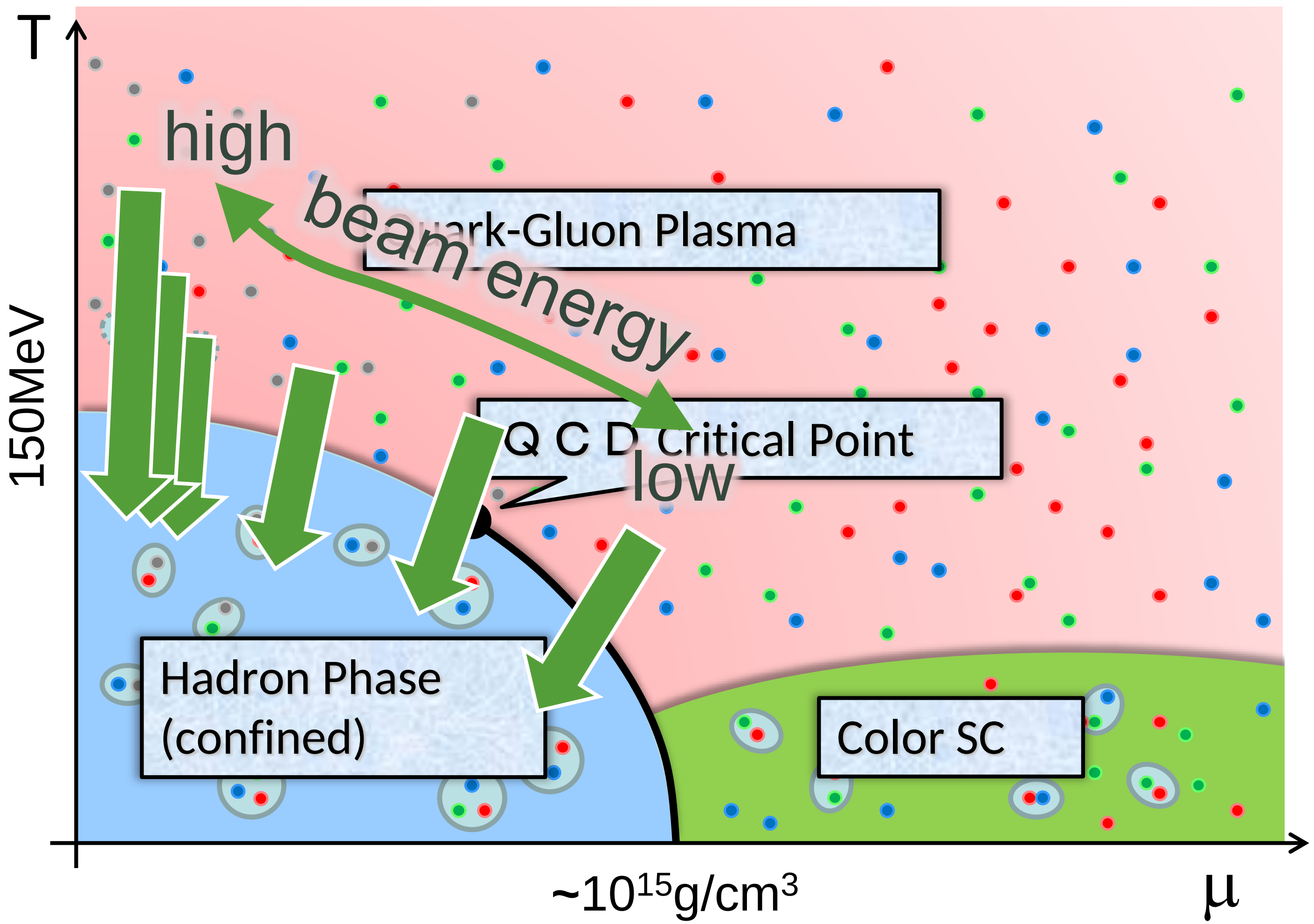
Search for QCD Phase Structure



Possible existence of

- ☐ 1st order transition
- ☐ QCD critical point

Search for QCD Phase Structure



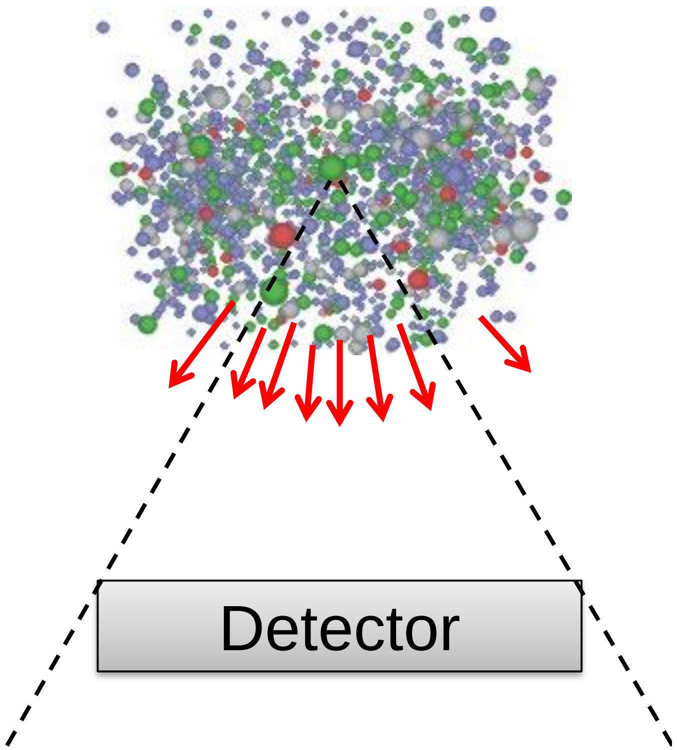
Possible existence of

- ☐ 1st order transition
- ☐ QCD critical point

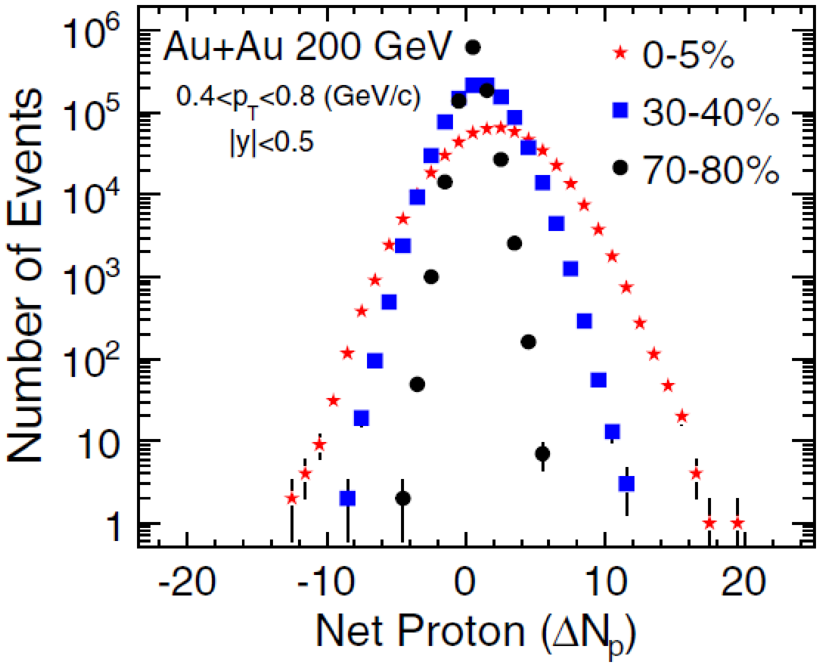
Beam-energy scan

- ☐ RHIC-BES-I 2010～
- ☐ RHIC-BES-II 2019～
- ☐ Future: FAIR, NICA, J-PARC-HI, HADES, …

Event-by-Event Fluctuations



STAR, PRL**105** (2010)



Cumulants

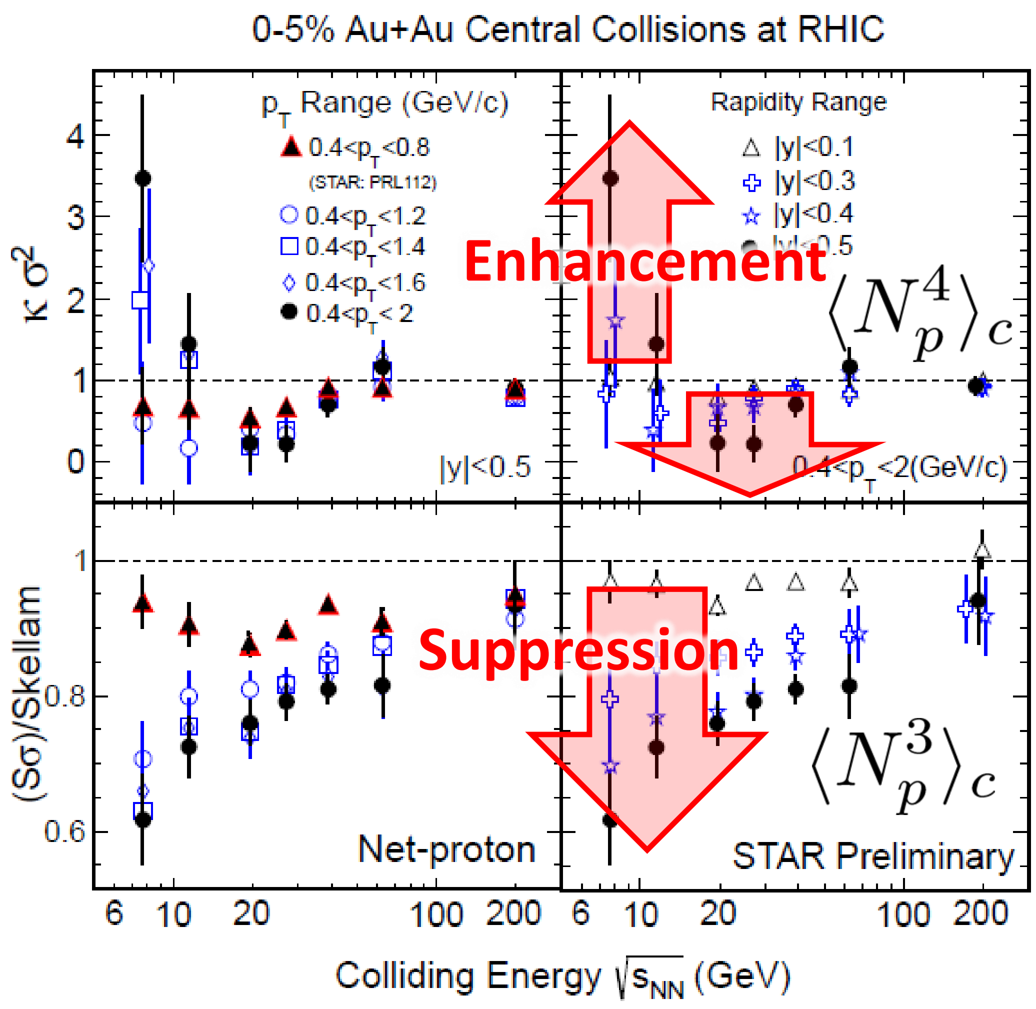
$$\langle N^2 \rangle_c = \langle \delta N^2 \rangle = \sigma^2$$

$$\langle N^3 \rangle_c = S\sigma^3$$

$$\langle N^4 \rangle_c = \kappa\sigma^4$$

General Review:
Asakawa, MK, PPNP (2016)

Event-by-Event Fluctuations

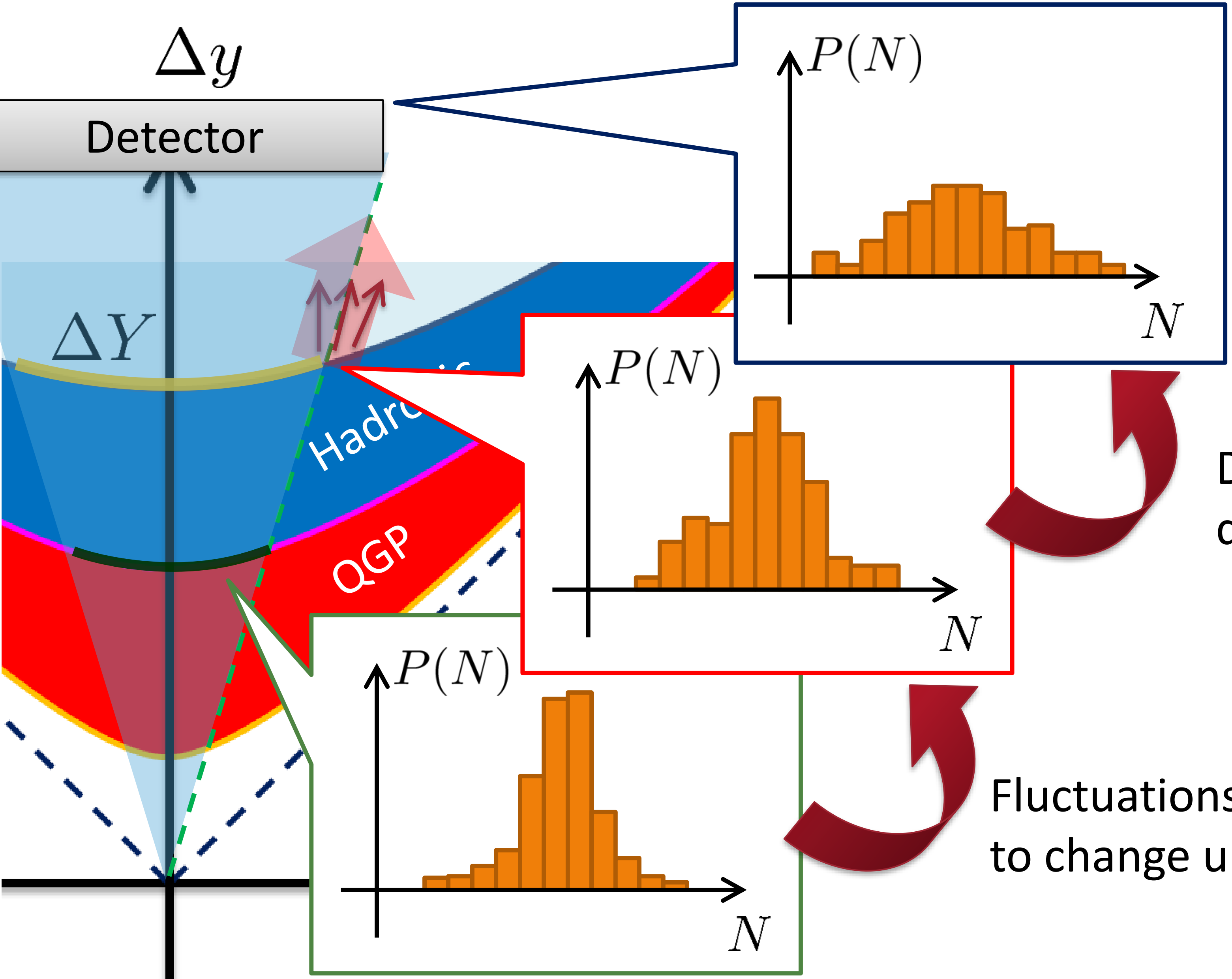


STAR 2010~

Net-proton number cumulants

- **Non**-zero **non**-Gaussian cumulants have been established experimentally!
- **Are they the signal of the QCD-CP?**
- **Note:** Baryon number cumulants are actually needed! MK, Asakawa, 2012;2012

Time Evolution of Fluctuations

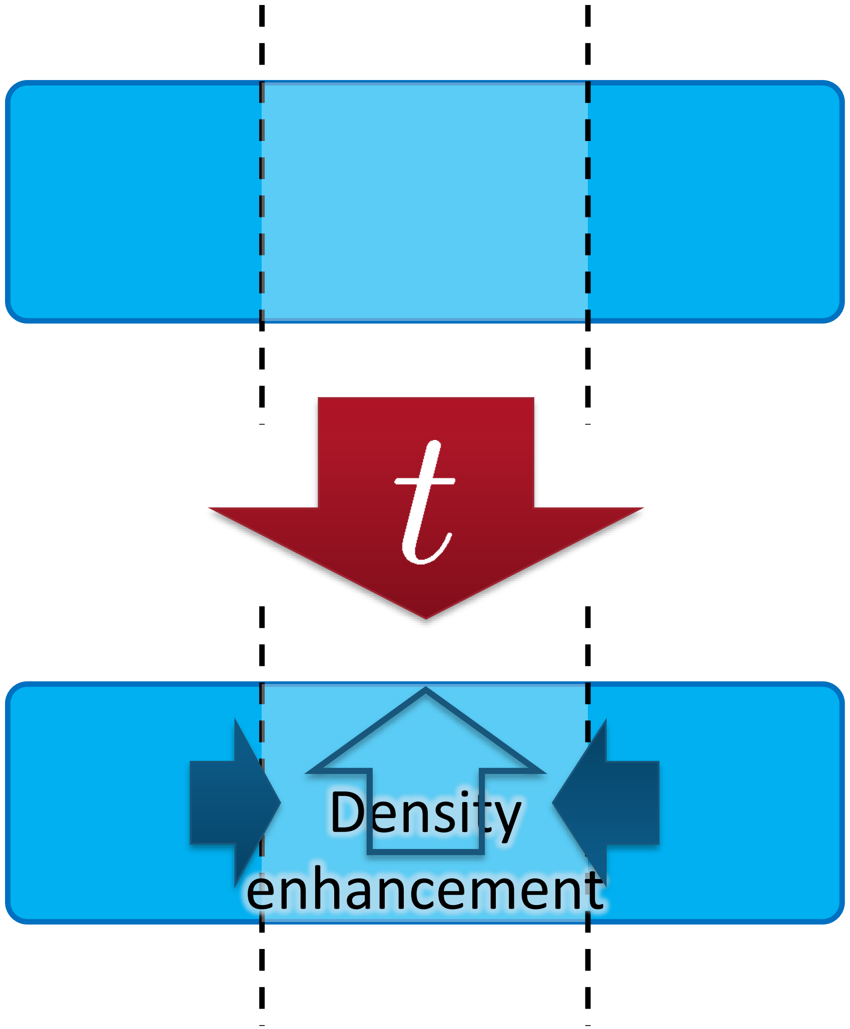


Distributions in ΔY and Δy are different due to “thermal blurring”.

Ohnishi, MK, Asakawa, PRC(2016)

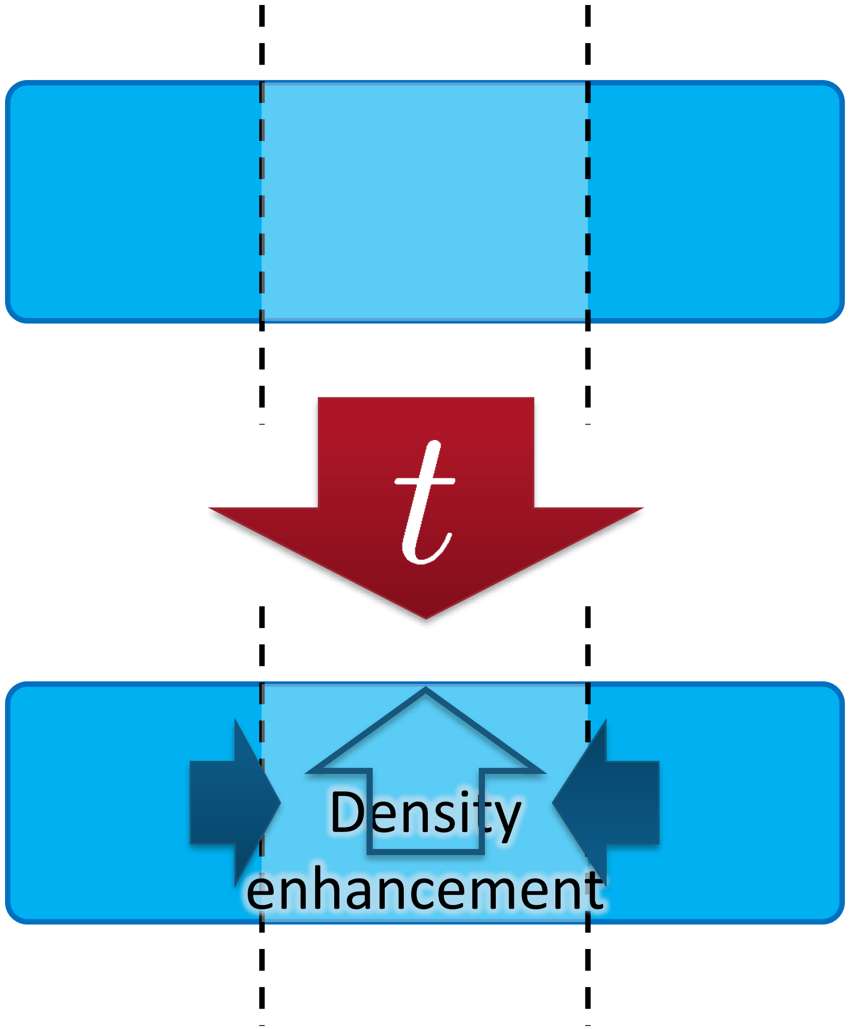
Fluctuations in ΔY continue to change until kinetic f.o.

Evolution of Conserved-Charge Fluct.



Equations describing the transport of *n*:

Evolution of Conserved-Charge Fluct.



Equations describing the transport of *n*:

- Diffusion Equation

$$\frac{\partial n}{\partial t} = D\nabla^2 n$$

- Stochastic Diffusion Equation (SDE)

$$\frac{\partial n}{\partial t} = D\nabla^2 n + \nabla\xi(x,t)$$

$$\langle\xi(1)\xi(2)\rangle = 2D\chi_2\delta(1-2)$$

- SDE with non-linear terms

$$\frac{\partial n}{\partial t} = \kappa\nabla^2\frac{\delta\mathcal{F}}{\delta n} + \frac{\partial}{\partial x}\xi(x,t)$$

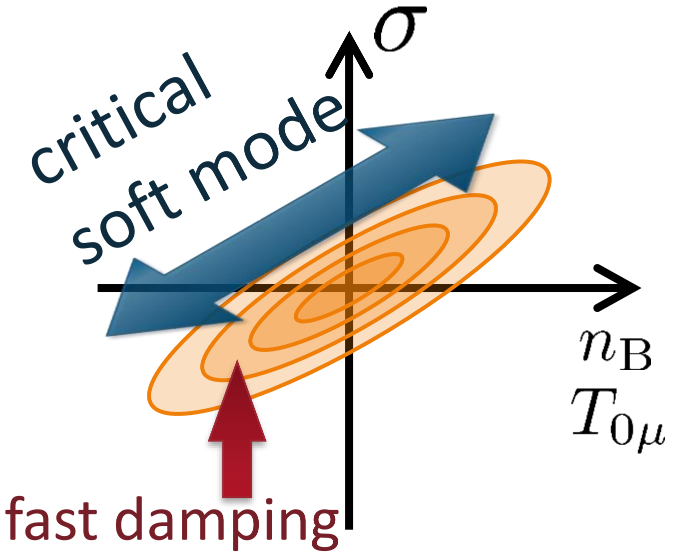
$$\mathcal{F} = \int dx\left(a\Delta n^2 + c(\nabla n)^2 + \lambda_3\Delta n^3 + \cdots\right)$$

Soft Mode of QCD-CP

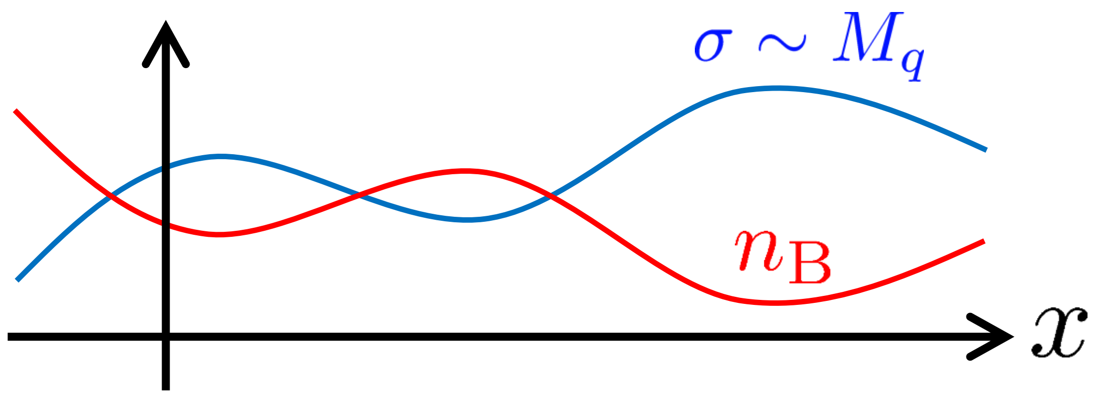
Fujii 2003; Fujii, Ohtani, 2004; Son, Stephanov, 2004

Fluctuations of σ and n_B are coupled around the CP!

$$\delta\sigma \simeq \delta n_B$$



$$F(\sigma, n) = A\sigma^2 + B\sigma n + Cn^2 + \cdots$$



- To a first approximation, SDE describes the soft mode of the CP.
- Coupling to σ & $T_{\mu\nu}$ has to be included for more accurate description.

Contents

1. CrossOver with SDE (Gaussian)
   - 2nd cumulant/correlation func.

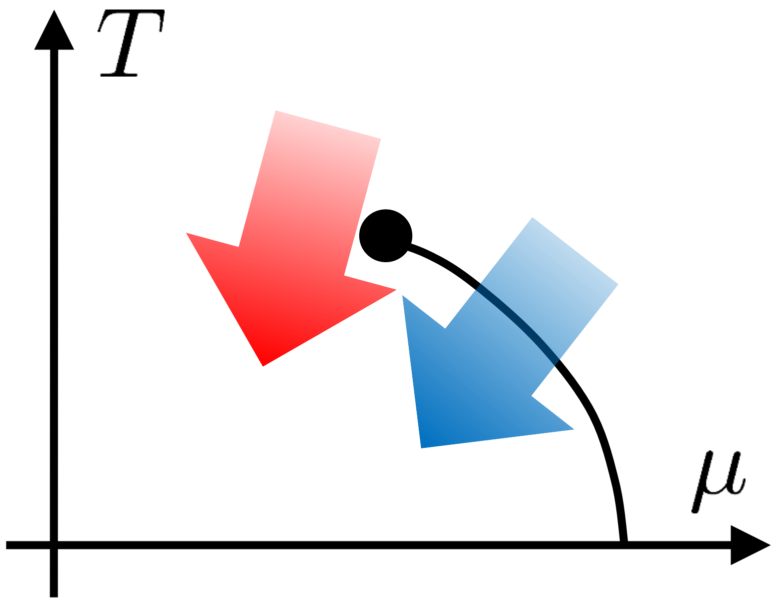
     Sakaida, Asakawa, Fujii, MK, PRC95 (2017)

2. CrossOver with Non-Linear SDE
   - higher-order cumulants

     Nahrgang, Bluhm, Schaefer, Bass PRD99 (2019);
     Pihan, Touroux, Nahrgang, Bluhm, Sami, MK, in prep.

3. 1st-Order with Non-Linear SDE

   Nonaka, MK, et al., in prep.



Stochastic Diffusion Equation (Gaussian)

$$\partial_t n = D(t)\partial_x^2 n + \partial_x \xi$$

$$\langle \xi(x_1,t_1)\xi(x_2,t_2)\rangle = 2D\chi_2\delta^{(2)}(1-2)$$

$D(t),\ \chi_2(t)$:parameters characterizing evolution of the medium

- Analytic solution is obtained.
- Study 2nd order cumulant & correlation function.

Cartesian coordinates

$$\partial_t n = D(t)\partial_x^2 n + \partial_x \xi$$

Milne coordinates

$$\partial_\tau n = \frac{D(t)}{\tau^2}\partial_Y^2 n + \frac{1}{\tau}\partial_Y \xi - \frac{n}{\tau}$$

suppression of diffusion

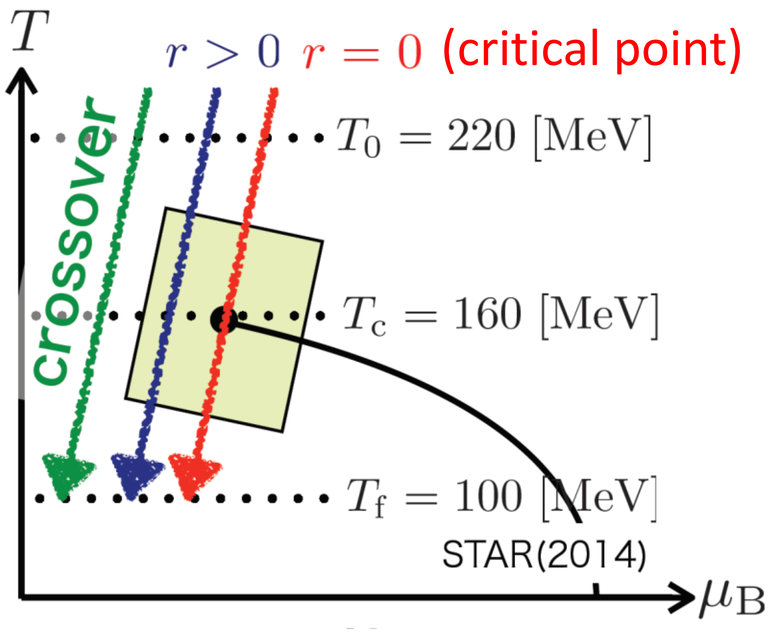
density reduction

Evolution of Parameters

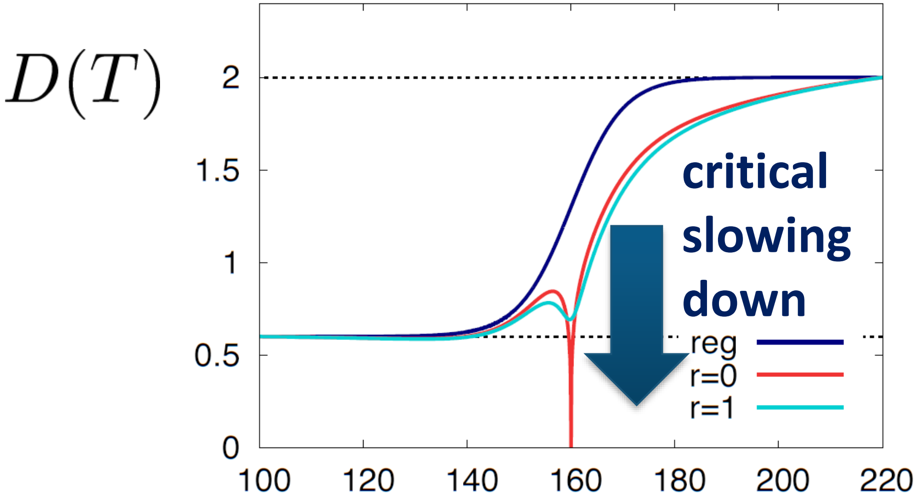
□Critical behavior

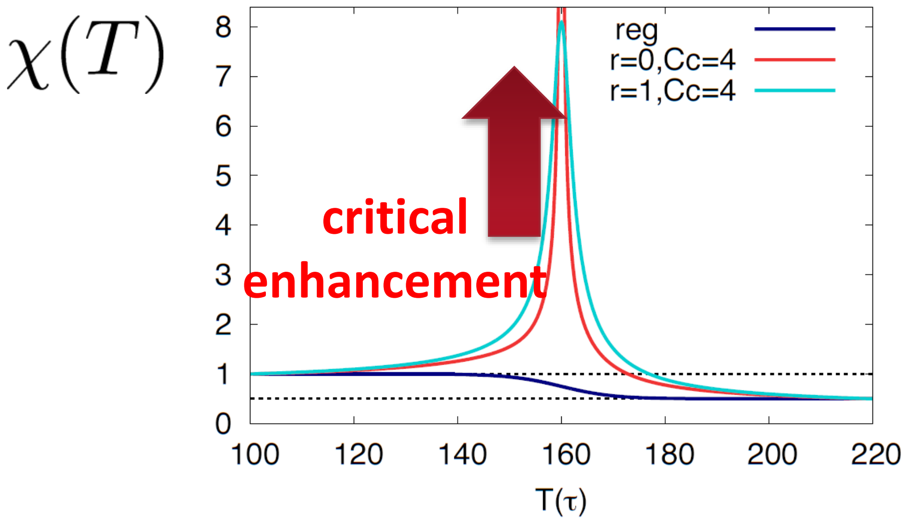
- 3D Ising (r,H)
- model H

Berdnikov, Rajagopal (2000)
Stephanov (2011); Mukherjee+(2015)



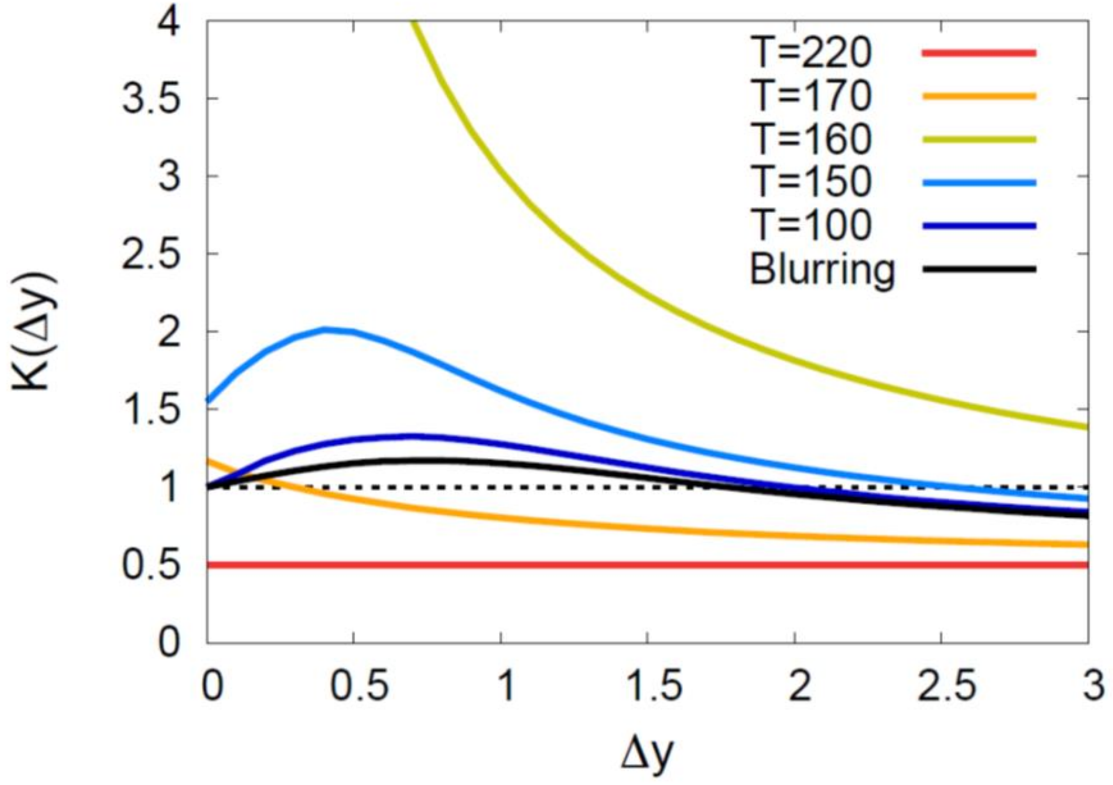
□Temperature dependence



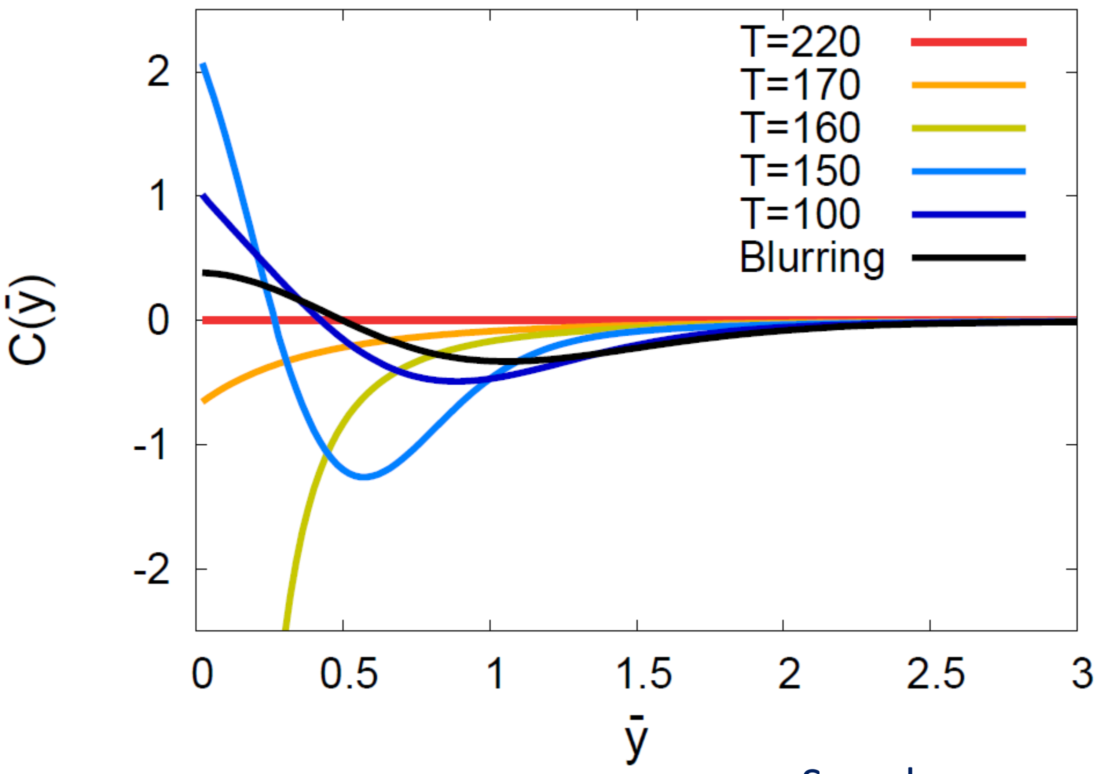


Time Evolution

Cumulant $K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\mathrm{eq.}}$



Corr. Func. $C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\mathrm{H}}$



Non-monotonic rapidity dependence

Analytic property: $K(\Delta y),\ C(\bar{y})$ non-monotonic ⇒ $\chi(\tau)$ non-monotonic

See also,
Kapusta, Torres-Rincon, PRC86 (2012);
Wu, Song, Chin. Phys. C43 (2019)

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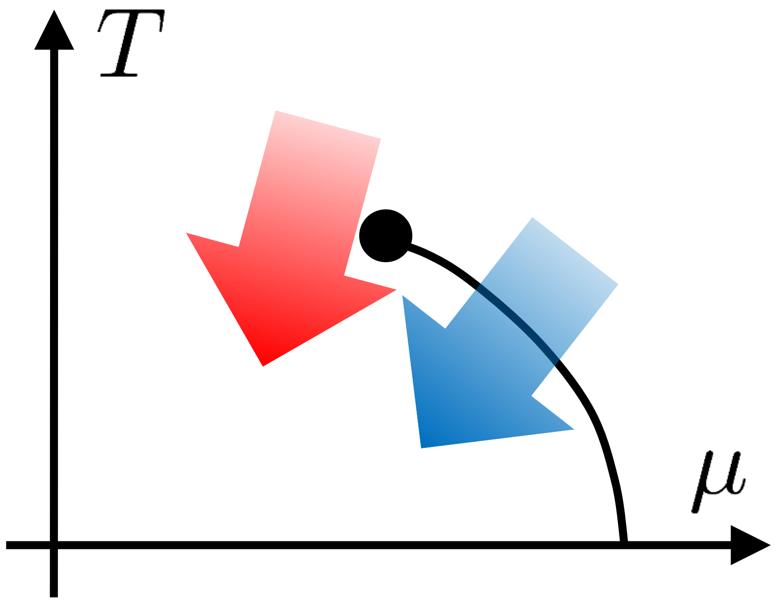
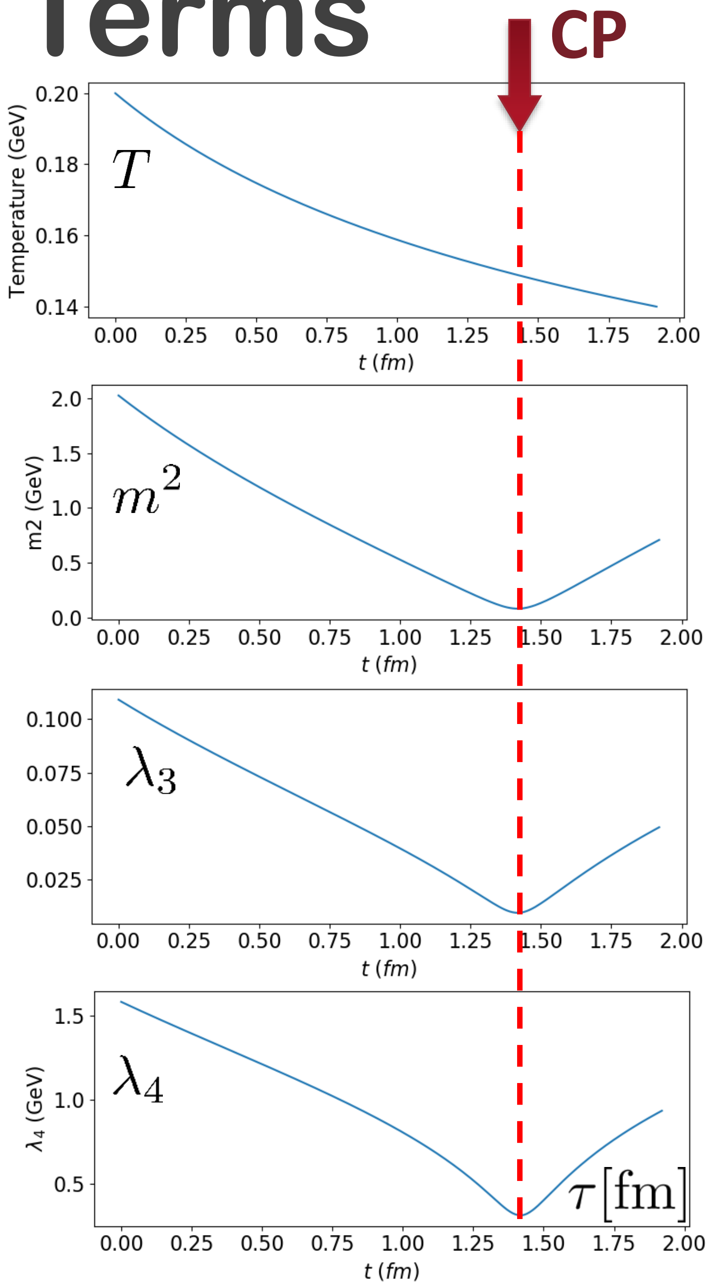


Introducing Non-Linear Terms

$$\frac{\partial n}{\partial t} = \kappa \nabla^2 \frac{\delta \mathcal{F}}{\delta n} + \frac{\partial}{\partial x}\xi(x,t)$$

$$\mathcal{F}[n] = T\int d^3r\Big(\frac{m^2}{2n_c^2}\Delta n^2 + \frac{K}{2n_c^2}(\nabla n)^2 + \frac{\lambda_3}{3n_c^3}\Delta n^3 + \frac{\lambda_4}{4n_c^4}\Delta n^4 + \frac{\lambda_6}{6n_c^6}\Delta n^6\Big)$$

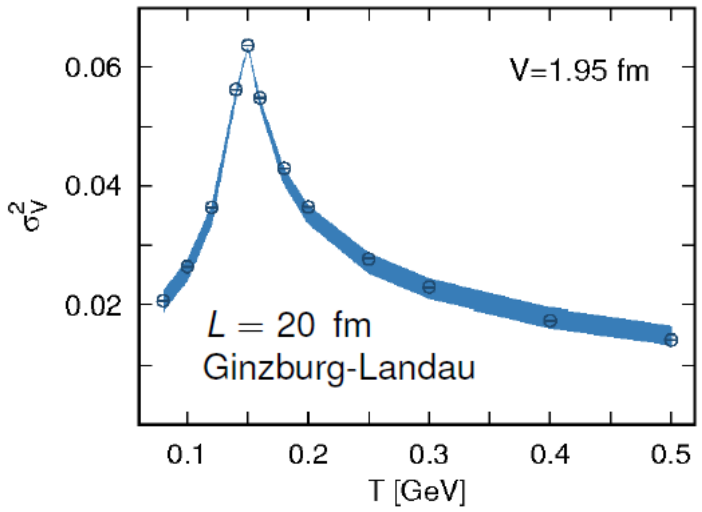
- Diffusive dynamics of **higher order** cumulants can be described.
- No analytic solution. Need numerical analysis.
- Parameters: $\kappa,\ m,\ K,\ \lambda_3,\ \lambda_4,\ \lambda_6$
  ←Hubble expansion, Ising universality



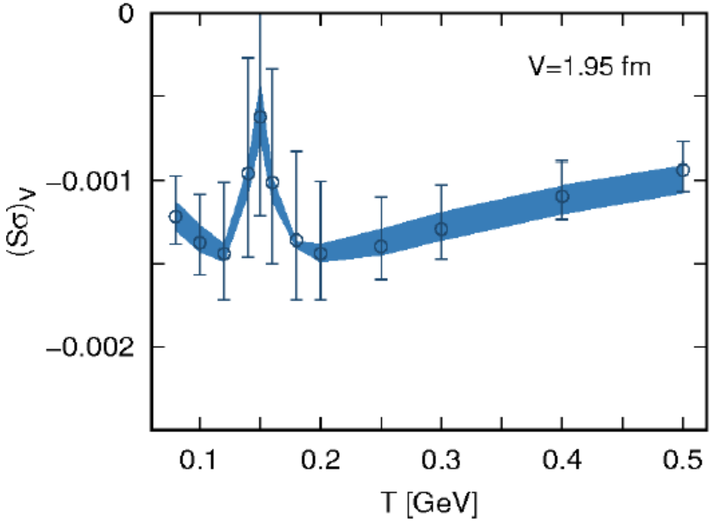
Cumulants in Equilibrium

Nahrgang, Bluhm, Schaefer, Bass PRD99 (2019)

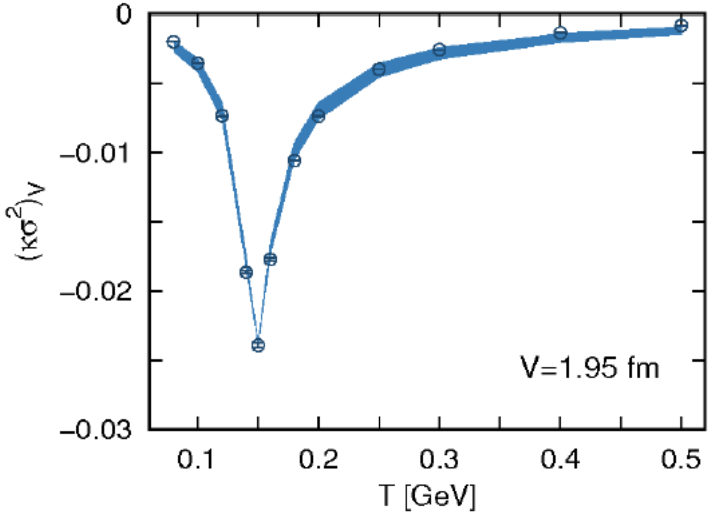
2nd: σ^2



3rd: $S\sigma$



4th: $\kappa\sigma^2$



- ☐ Simulation with fixed *T*.
- ☐ Spatial length L=20fm
- ☐ Weaker criticality due to the finite volume effects
- ☐ Shape of Sσ can be explained by the finite volume effects

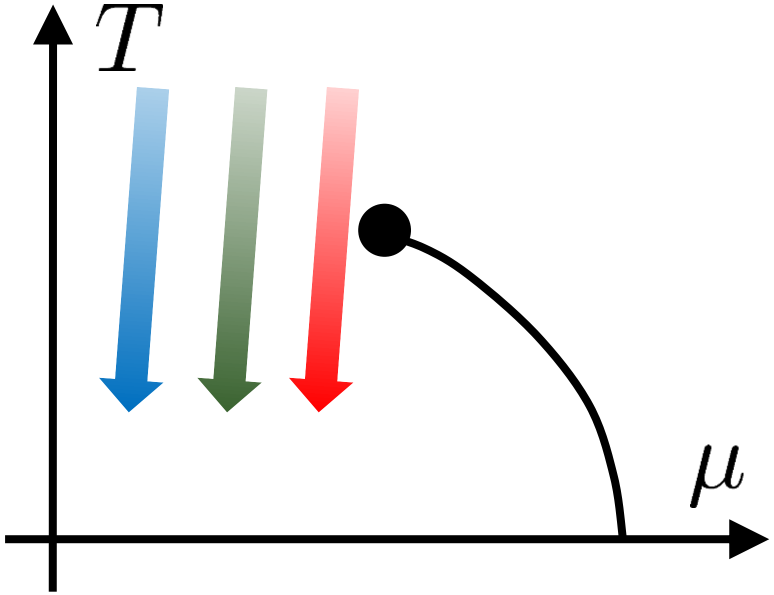
M. Agah Nouhou+, arXiv:1906.02647; Bluhm, SQM2019

Evolution with Bjorken Expansion

Pihan, Touroux, Nahrgang, Bluhm, Sami, MK, in prep.

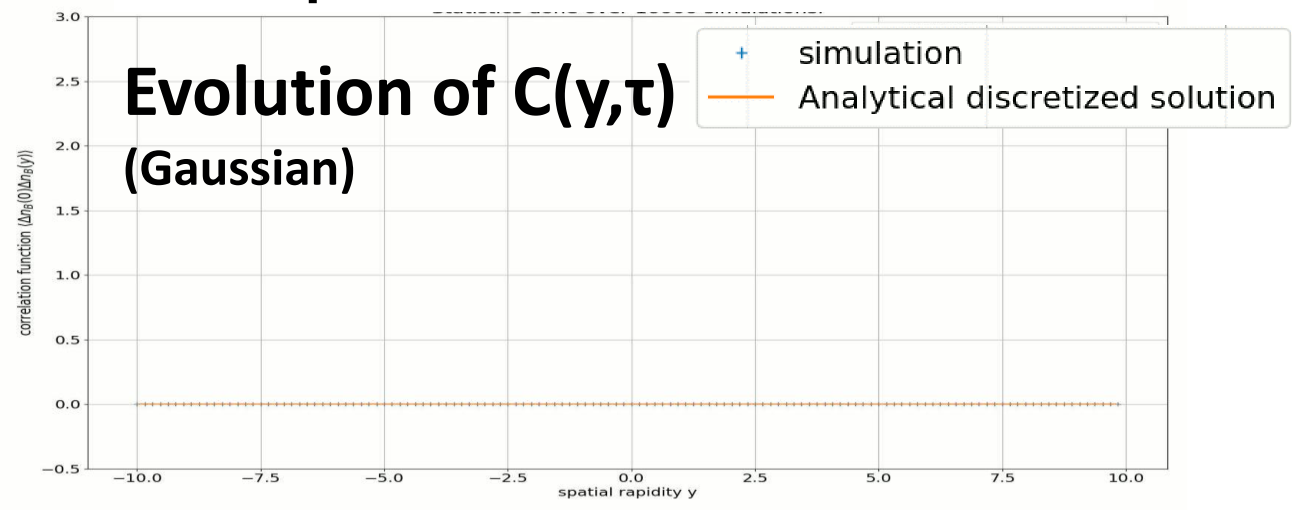
Milne coordinates

$$\partial_\tau n = \frac{\kappa(t)}{\tau^2}\partial_y^2 \frac{\delta \mathcal{F}}{\delta n} + \frac{1}{\tau}\partial_y \xi - \frac{n}{\tau}$$



- Critical Point: T=150 MeV, μ=390 MeV
- Initial temperature: T=200 MeV
- μ=50, 200, 300, 350 MeV
- Cumulants on a single cell
- Compare results with & without NL terms

Evolution of C(y,τ)
(Gaussian)

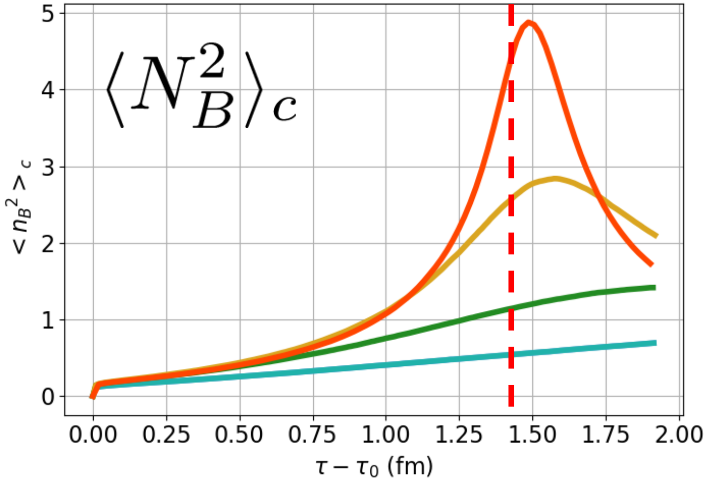


Numerical Result

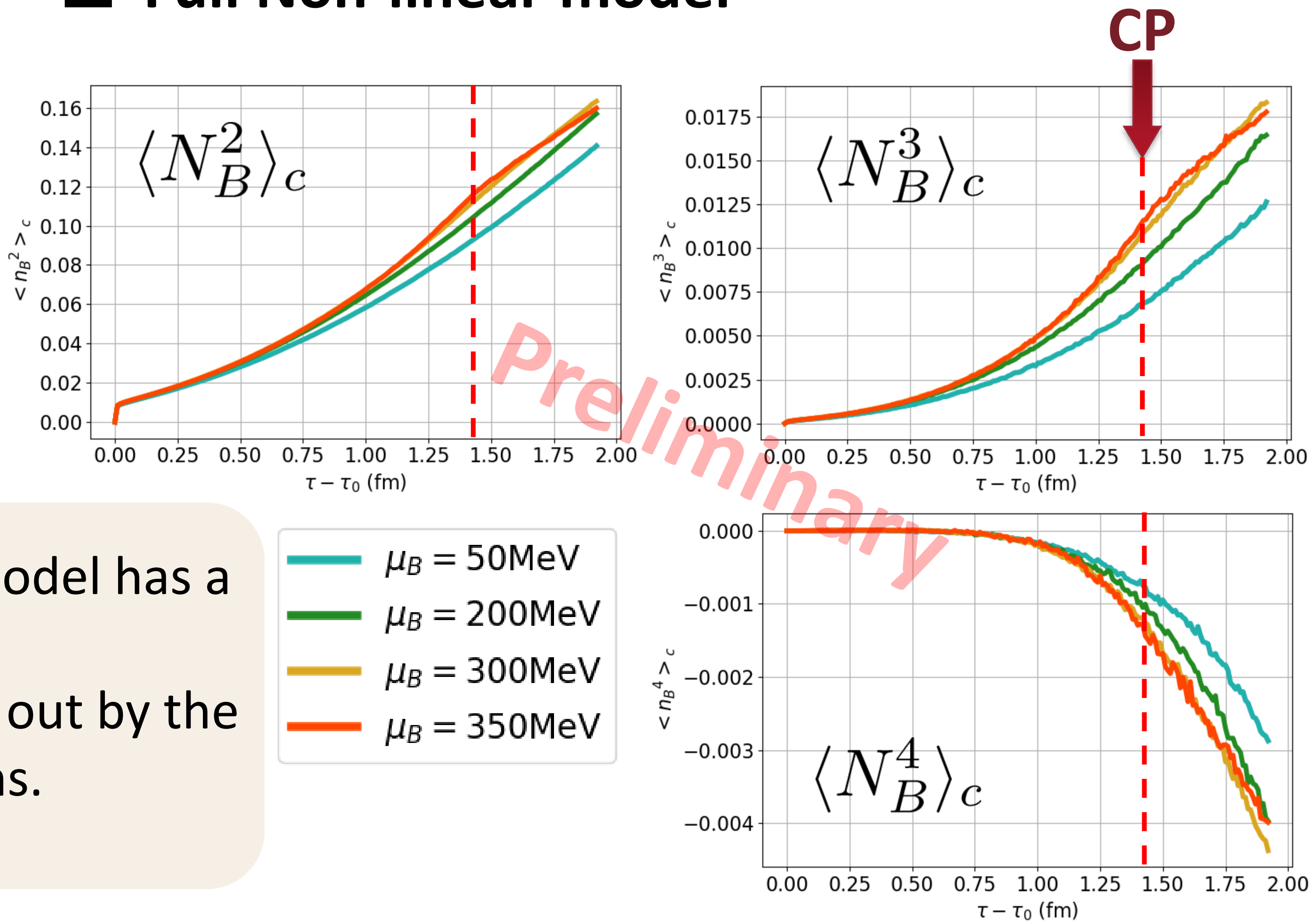
Pihan, Touroux, Nahrgang, Bluhm, Sami, MK, in prep.

☐ Gaussian

(without nonlinear terms)



☐ Full Non-linear model



- ☐ 2nd cumulant in Gaussian model has a peak at the CP.
- ☐ But, this behavior is washed out by the effect of the non-linear terms.
- ☐ Need further investigation.

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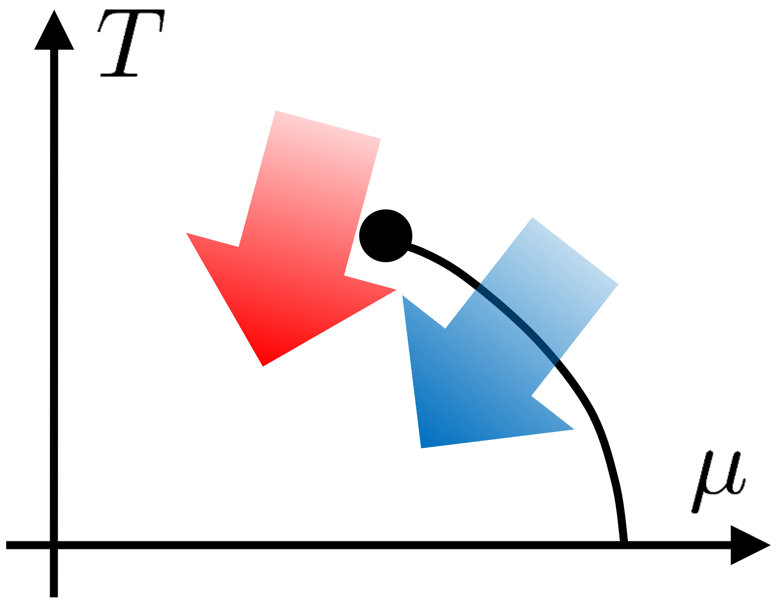
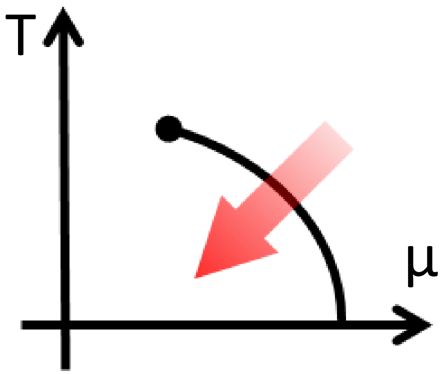
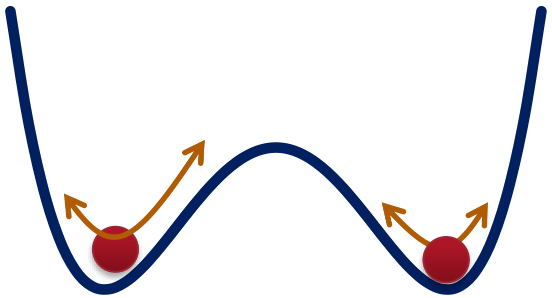


1st-Order Transition



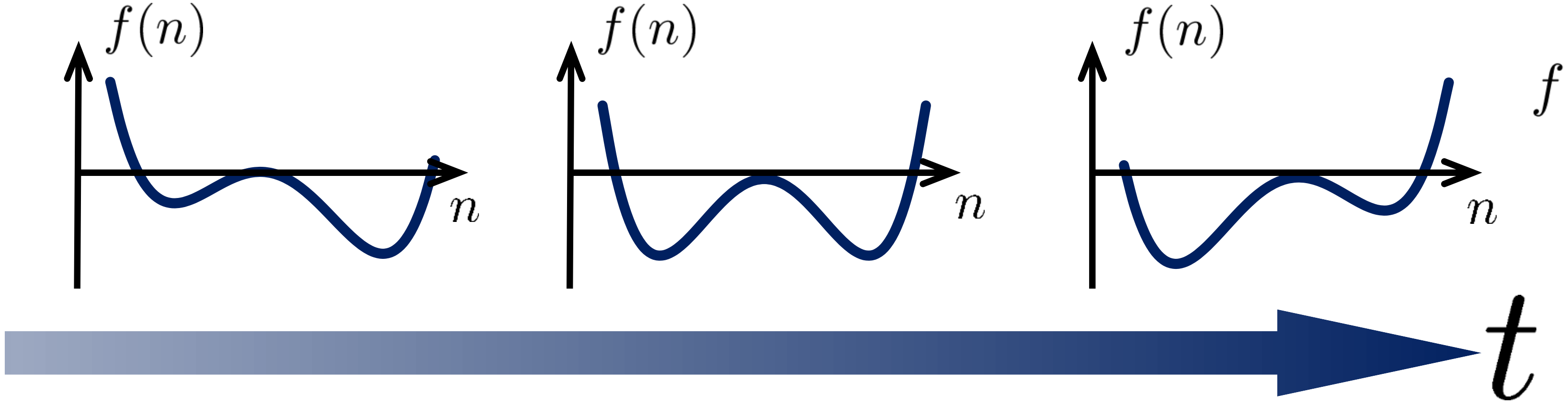
- Domain formation
- Non-uniform system



Herold, Nahrgang, et al. (2011~);
Steinheimer, Randrup (2012; 2013)

Free Energy

- **At 1st transition point**

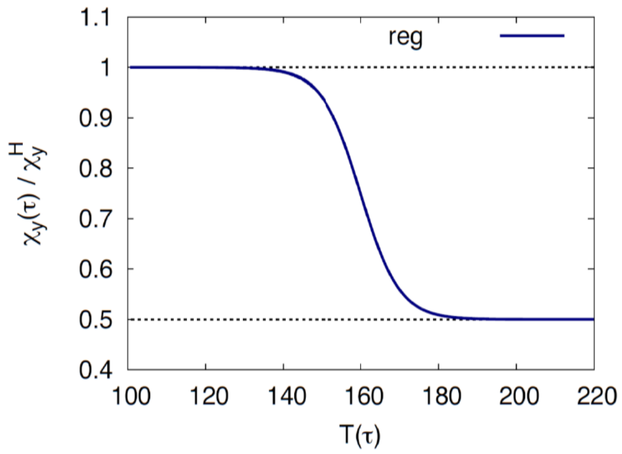


$$f(n) = \frac{1}{2}a(n-n_s)^2 + \frac{1}{4}b(n-n_s)^4 + c(\tau)n + k(\partial_Y n)^2$$

- **Large and small n**

$$\chi(n) = \frac{\partial^2 f}{\partial n^2} \quad \to \chi_{\mathrm{QGP}} \ (n \to \infty)$$
$$\to \chi_{\mathrm{hadron}} \ (n \to 0)$$

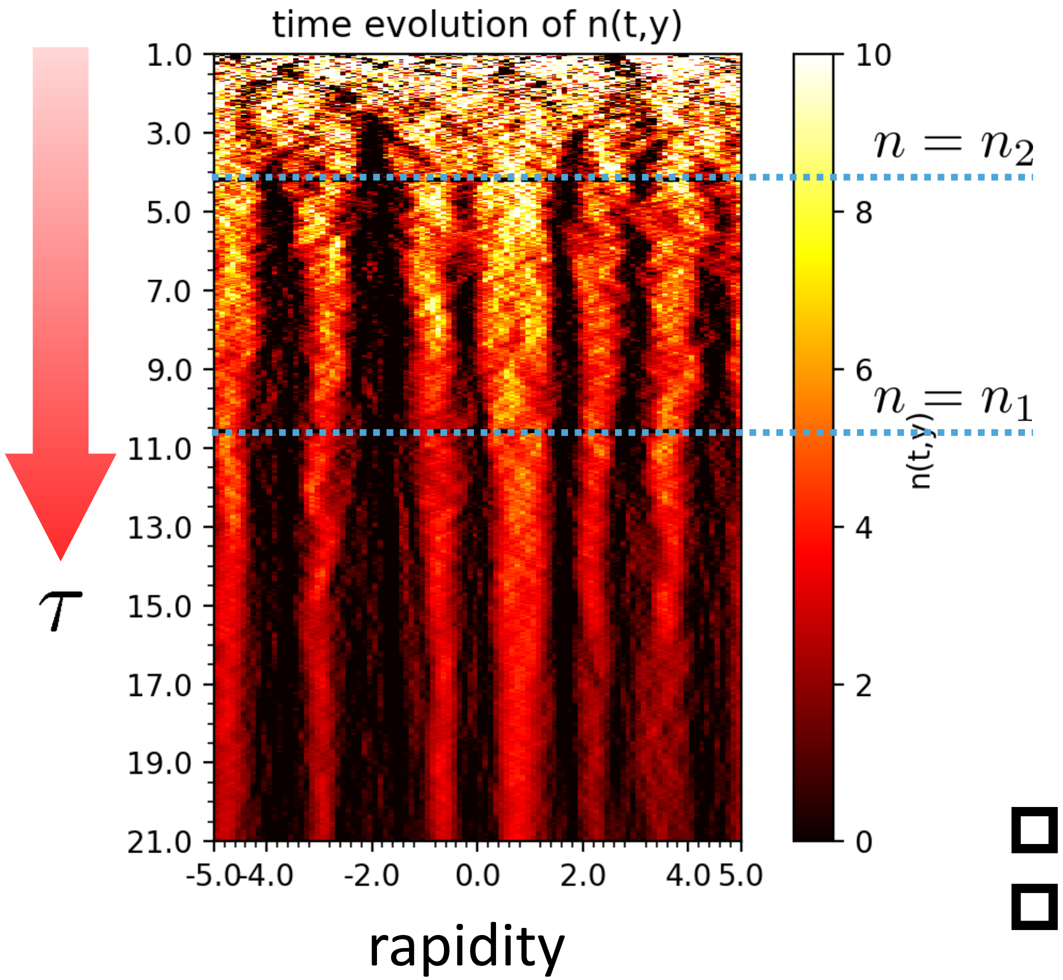
Poisson

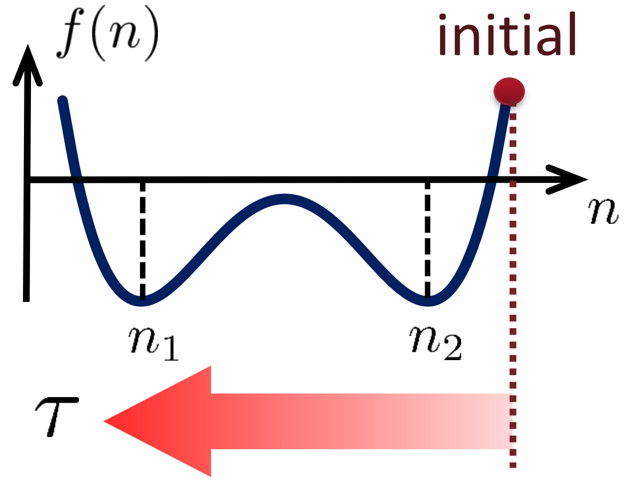


- κ: positive
- adjust κ and A to reproduce the behavior of D at small and large n

$$\tilde{D} = \Gamma\left(\frac{\partial^2 f}{\partial n^2} + X\right) \qquad A = 2D\chi_2$$

Time Evolution



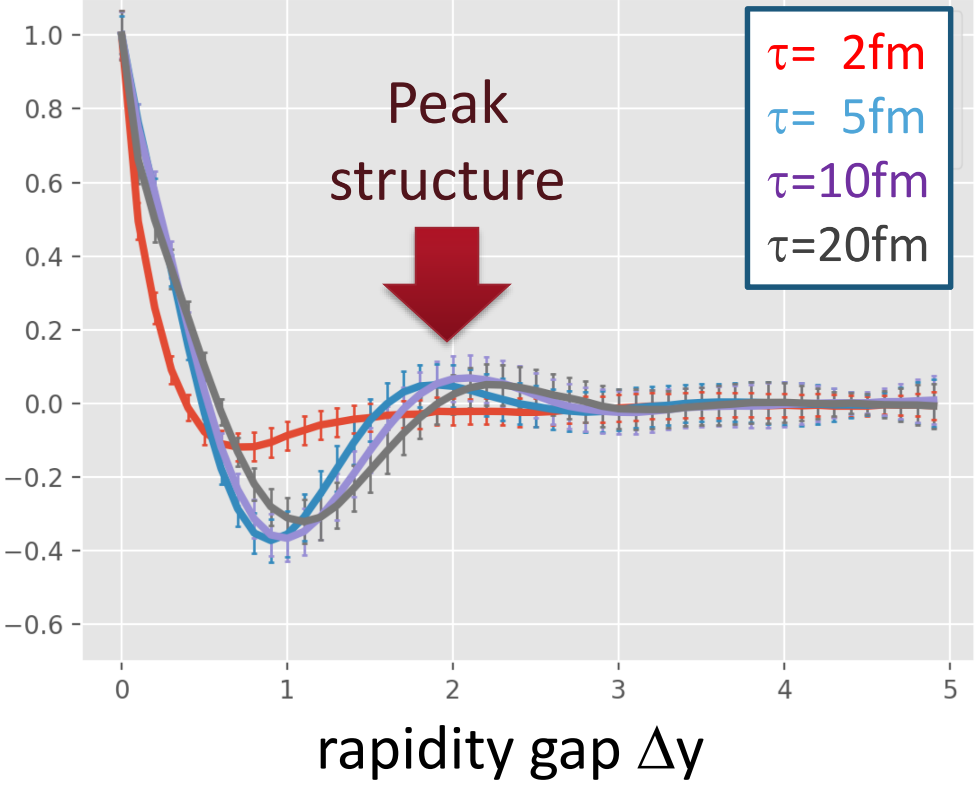


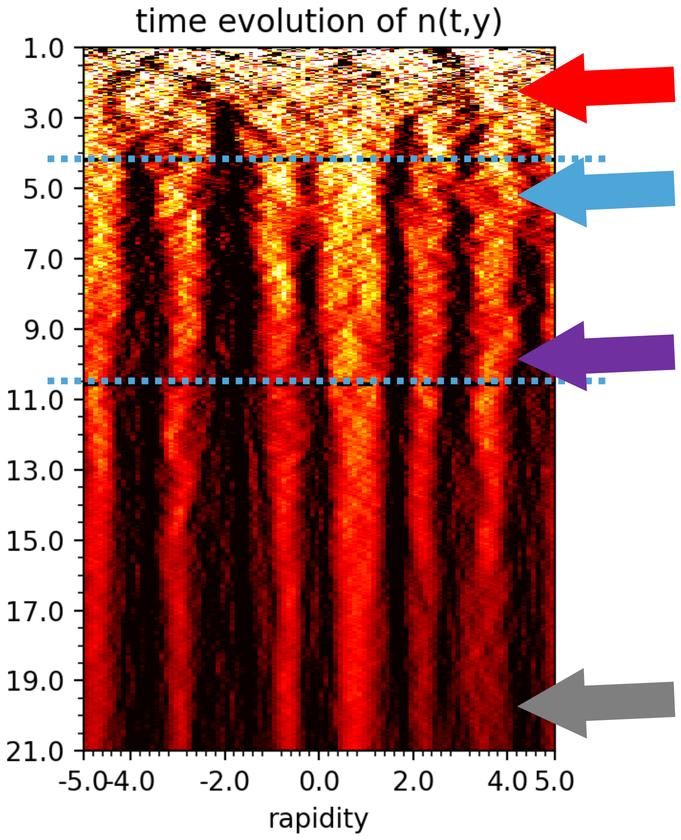
- Dynamical domain formation
- Domains survive even after 1st transition

Correlation Function

Correlation Function

$$C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\text{hadron}}$$





- Domain leads to a peak structure in C(y).
- The peak can survive even in the final state.

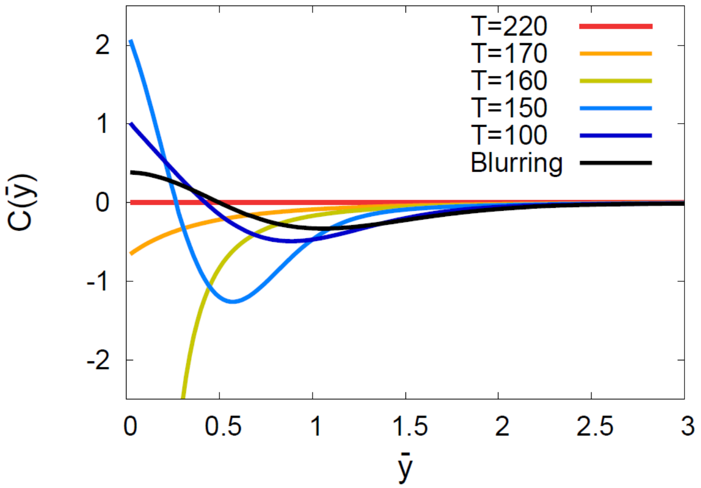
Summary

- Diffusive dynamics is important in describing fluctuations in heavy-ion collisions.
- We studied dynamical evolution near the QCD-CP and at the 1st transition in stochastic diffusion equation with and without non-linear terms.

- Future: coupling with sigma & momentum / more realistic space-time evolution

Correlation function

Critical point/ crossover



1st order transition