Francesco Becattini
University of Florence

Does the spin tensor play a role in non-gravitational physics?

OUTLINE

- Introduction: QFT and pseudo-gauge transformations
- Local thermodynamic equilibrium
- Polarization in heavy ion collisions

Quark Matter 2019, Wuhan

Introduction

In QFT in flat space-time, from translational and Lorentz invariance one obtains two conserved Noether currents:

$$\widehat{T}^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Psi^a)} \partial^\nu \Psi^a - g^{\mu\nu} \mathcal{L}$$

$$\widehat{\mathcal{S}}^{\lambda,\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\lambda \Psi^a)} D^A (J^{\mu\nu})^a_b \Psi^b$$

$$\partial_\mu \widehat{T}^{\mu\nu} = 0$$

$$\partial_\lambda \widehat{\mathcal{J}}^{\lambda,\mu\nu} = \partial_\lambda \left(\widehat{\mathcal{S}}^{\lambda,\mu\nu} + x^\mu \widehat{T}^{\lambda\nu} - x^\nu \widehat{T}^{\lambda\mu} \right) = \partial_\lambda \widehat{\mathcal{S}}^{\lambda,\mu\nu} + \widehat{T}^{\mu\nu} - \widehat{T}^{\nu\mu} = 0$$

However, the Lagrangian density can be changed and so, are those tensors objectively defined? (well known problem already for the EM stress-energy tensor)

Pseudo-gauge transformations with a superpotential $\widehat{\Phi}$

F. W. Hehl, Rept. Math. Phys. 9 (1976) 55   (see also F. B., L. Tinti Phys. Rev. D 84 (2011) 025013)

$$\widehat{T}'^{\mu\nu} = \widehat{T}^{\mu\nu} + \frac{1}{2}\partial_\alpha \left(\widehat{\Phi}^{\alpha,\mu\nu} - \widehat{\Phi}^{\mu,\alpha\nu} - \widehat{\Phi}^{\nu,\alpha\mu} \right)$$

$$\widehat{S}'^{\lambda,\mu\nu} = \widehat{S}^{\lambda,\mu\nu} - \widehat{\Phi}^{\lambda,\mu\nu}$$

They leave the conservation equations and spacial integrals (=generators, or total energy, momentum and angular momentum operators) invariant

EXAMPLE: Belinfante symmetrization

$$\widehat{T}'^{\mu\nu} = \widehat{T}^{\mu\nu} + \frac{1}{2}\partial_\alpha \left(\widehat{S}^{\alpha,\mu\nu} - \widehat{S}^{\mu,\alpha\nu} - \widehat{S}^{\nu,\alpha\mu} \right)$$

$$\widehat{S}'^{\lambda,\mu\nu} = 0$$

Free Dirac field:

$$\widehat{T}^{\mu\nu} = \frac{1}{2}\overline{\Psi}\gamma^\mu \overset{\leftrightarrow}{\partial}{}^\nu \Psi$$

$$\widehat{S}^{\lambda,\mu\nu} = \frac{1}{2}\overline{\Psi}\{\gamma^\lambda, \Sigma^{\mu\nu}\}\Psi = \frac{i}{8}\overline{\Psi}\{\gamma^\lambda[\gamma^\mu,\gamma^\nu]\}\Psi$$

Canonical pseudo-gauge

$$\widehat{T}'^{\mu\nu} = \frac{i}{4}\left[\overline{\Psi}\gamma^\mu \overset{\leftrightarrow}{\partial}{}^\nu \Psi + \overline{\Psi}\gamma^\nu \overset{\leftrightarrow}{\partial}{}^\mu \Psi \right]$$

$$\widehat{S}'^{\lambda,\mu\nu} = 0$$

Belinfante pseudo-gauge

Consequence of pseudo-gauge invariance

They look very much alike the vector potential in gauge theories and arbitrary up to a quantum correction (spin tensor is proportional to ℏ).

Naively, if one could measure energy or momentum or spin at some place and time, then one would objectively single out $(T, \mathcal{S})$

In Quantum Field Theory, this is thought to be impossible! Only in classical gravity energy can be precisely localized.

Summarized in the adage: *You cannot separate orbital angular momentum and spin in relativistic theories*

Heavy ion Physics: our beloved stress-energy tensor $T^{\mu\nu}(x) = \mathrm{Tr}(\widehat{\rho}\widehat{T}^{\mu\nu}(x))$ *is not objective up to quantum terms. It plays the same role as of a vector potential in electrodynamics. What is objective are just momentum distributions which <u>should</u> be invariant under pseudo-gauge transformations*

Is it really unhackable?

$$|p, \sigma\rangle$$

Quantum free particle physical states depend on the generators P and J
Of the Poincare' group and are independent of the pseudo-gauge, i.e. of

$$(T, \mathcal{S})$$

$$\sum C(p_1, \sigma_1, p_2, \sigma_2, \ldots)|p_1, \sigma_1\rangle|p_2, \sigma_2\rangle \ldots$$

However, the coefficients may be an implicit function of $(T, \mathcal{S})$

$$C(p_1, \sigma_1, p_2, \sigma_2, \ldots)_{(T, \mathcal{S})}$$

More generally, one has a density operator $\widehat{\rho}(\widehat{T}, \widehat{\mathcal{S}})$
which may or may not be pseudo-gauge invariant. In this case, measurements would
break this pseudo-gauge invariance

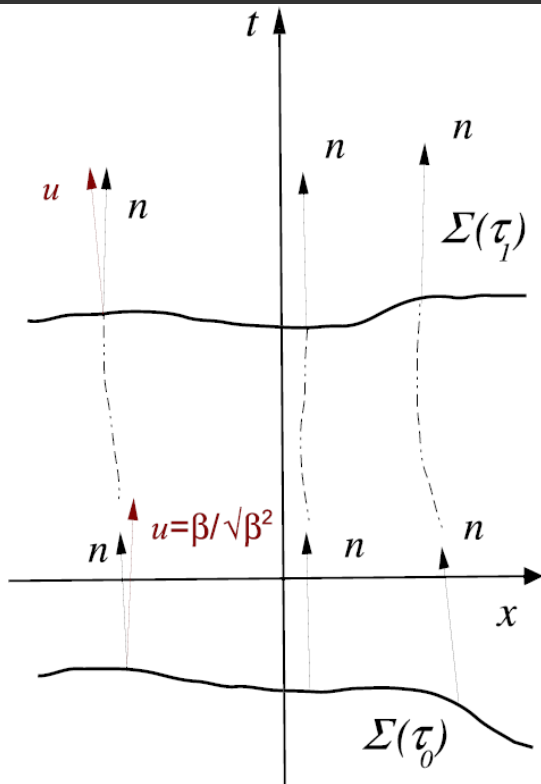*Can we prepare a state which is NOT pseudo-gauge invariant?*

Local thermodynamic equilibrium

$$\beta = \frac{1}{T}u$$

$$\zeta = \frac{\mu}{T}$$

General covariant
Local thermodynamic
Equilibrium density operator

$$\widehat{\rho} = \frac{1}{Z}\exp\left[-\int_{\Sigma}\mathrm{d}\Sigma_{\mu}\left(\widehat{T}^{\mu\nu}\beta_{\nu} - \zeta\widehat{j}^{\mu}\right)\right]$$

The operator is obtained by maximizing the entropy

$$S = -\mathrm{tr}(\widehat{\rho}\log\widehat{\rho})$$

with the constraints of fixed energy-momentum density
on a given 3D hypersurface Σ
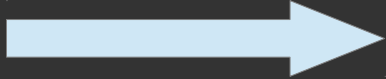
Zubarev 1979, Ch. Van Weert 1982

F. B., L. Bucciantini, E. Grossi, L. Tinti,
Eur. Phys. J. C 75 (2015) 191 (β frame)

T. Hayata, Y. Hidaka, T. Noumi, M. Hongo,
Phys. Rev. D 92 (2015) 065008

Pseudo-gauge dependence analysis

Start from Belinfante pseudo-gauge:

$$\widehat{\rho}_{\text{LE}} = \frac{1}{Z} \exp\left[-\int_\Sigma d\Sigma_\mu \left(\widehat{T}_B^{\mu\nu} \beta_\nu - \zeta \widehat{j}^\mu \right) \right] ,$$

$$\widehat{\rho}_{\text{LE}} = \frac{1}{Z} \exp\left[-\int_\Sigma d\Sigma_\mu \left(\widehat{T}_C^{\mu\nu} \beta_\nu - \frac{1}{2}\varpi_{\lambda\nu} \widehat{\mathcal{S}}_C^{\mu,\lambda\nu} - \frac{1}{2}\xi_{\lambda\nu} \left(\widehat{\mathcal{S}}_C^{\lambda,\mu\nu} + \widehat{\mathcal{S}}_C^{\nu,\mu\lambda} \right) - \zeta \widehat{j}^\mu \right) \right] ,$$

Bel to Can transformation

$$\varpi_{\lambda\nu} = \frac{1}{2}(\nabla_\nu \beta_\lambda - \nabla_\lambda \beta_\nu)$$

$$\xi_{\lambda\nu} = \frac{1}{2}(\nabla_\nu \beta_\lambda + \nabla_\lambda \beta_\nu)$$
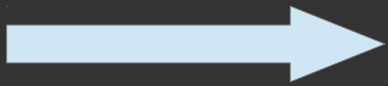
This operator is DIFFERENT from what we would get if we implemented Local Thermodynamic Equilibrium using Canonical or any other couple of tensors

Pseudo-gauge dependence analysis -II

If the spin tensor is non-zero (non-Belinfante) angular momentum constraints must be additionally implemented

$$\widehat{\rho}_{\mathrm{LE}} = \frac{1}{Z} \exp\left[-\int_{\Sigma} \mathrm{d}\Sigma_{\mu} \left(\widehat{T}^{\mu\nu} \beta_{\nu} - \frac{1}{2}\Omega_{\lambda\nu}\widehat{\mathcal{S}}^{\mu,\lambda\nu} - \zeta \widehat{j}^{\mu} \right) \right].$$

$$\Omega_{\lambda\nu} \equiv \mathrm{spin\ potential}$$

Can to Bel transformation

$$\widehat{\rho}_{\mathrm{LE}} = \frac{1}{Z} \exp\left[-\int_{\Sigma} \mathrm{d}\Sigma_{\mu} \left(\widehat{T}_B^{\mu\nu} \beta_{\nu} - \frac{1}{2}(\Omega_{\lambda\nu} - \varpi_{\lambda\nu})\widehat{\mathcal{S}}^{\mu,\lambda\nu} + \frac{1}{2}\xi_{\lambda\nu}\left(\widehat{\mathcal{S}}^{\lambda,\mu\nu} + \widehat{\mathcal{S}}^{\nu,\mu\lambda} \right) - \zeta \widehat{j}^{\mu} \right) \right],$$

which is the same as the previous (in form)

$$\widehat{\rho}_{\mathrm{LE}} = \frac{1}{Z} \exp\left[-\int_{\Sigma} \mathrm{d}\Sigma_{\mu} \left(\widehat{T}_B^{\mu\nu} \beta_{\nu} - \zeta \widehat{j}^{\mu} \right) \right],$$

only if

$$\varpi = \Omega$$

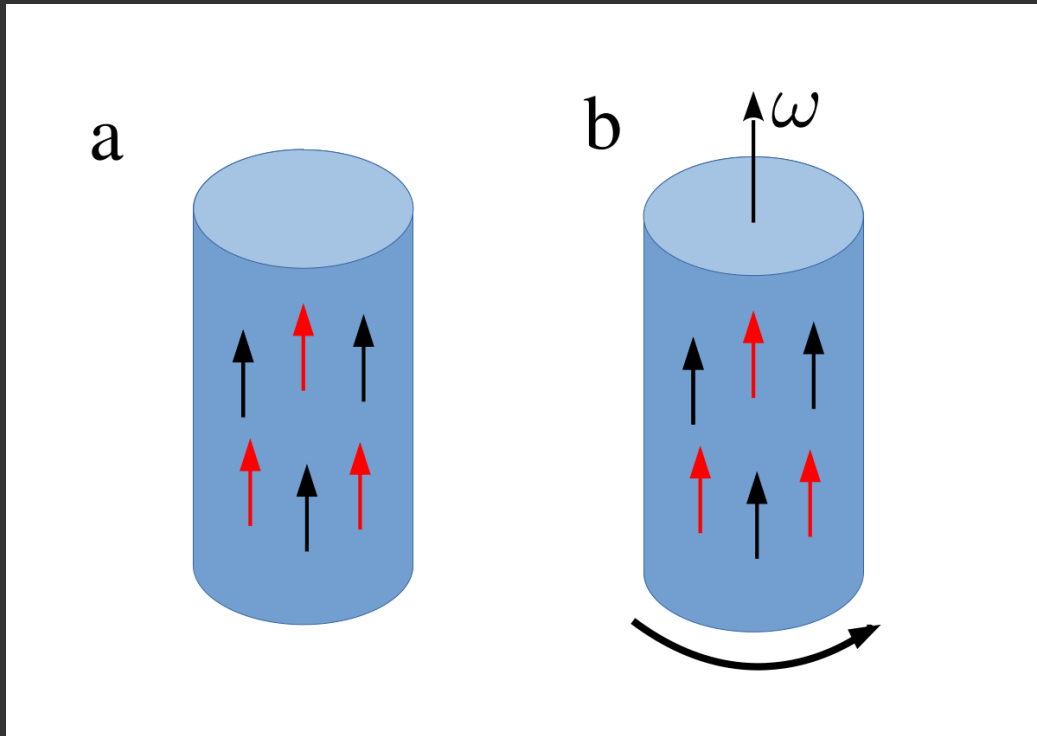$$\xi_{\mu\nu} = \frac{1}{2}(\nabla_{\mu}\beta_{\nu} + \nabla_{\nu}\beta_{\mu}) = 0$$

GLOBAL THERMODYNAMIC EQUILIBRIUM

What is the physical difference?

$$\widehat{\rho}_{\text{LE}} = \widehat{\rho}_{\text{LE}}(\widehat{T}, \widehat{S})$$ is NOT pseudo-gauge invariant, except at global equilibrium

C-invariant relativistic matter

a) with u=(1,0,0,0) T = constant
and particles-antiparticles polarized
in the same direction requires
$\Omega \neq \varpi$ and thus breaks pseudo-gauge
invariance.

b) Belinfante pseudo-gauge:
only thermal vorticity can sustain such a
configuration

Non-Belinfante allows to describe non-rotating polarized C-invariant matter
(NO non-relativistic Limit) as a local equilibrium state.

The question is whether we can prepare such a state.

Polarization in heavy ion collisions

Is a spin tensor needed? NO (it is a momentum-dependent quantity) and
we cannot localize it; YES if the density operator depends on the pseudo-gauge

$$S^\mu(p) = \sum_{i=1}^{3} [p]_i^\mu \, \mathrm{tr}(D^S(\mathsf{J}^i)\Theta(p)),$$

$$\Theta(p)_{\sigma\tau} = \frac{\mathrm{Tr}(\widehat{\rho}\,\widehat{a}_\tau^\dagger(p)\widehat{a}_\sigma(p))}{\sum_\sigma \mathrm{Tr}(\widehat{\rho}\,\widehat{a}_\sigma^\dagger(p)\widehat{a}_\sigma(p))},$$

$$S^\mu(p) = -\frac{1}{4}\epsilon^{\mu\beta\gamma\delta}p_\delta \frac{\int_\Sigma d\Sigma_\mu \mathrm{tr}_4(\{\gamma^\mu, \Sigma_{\beta\gamma}\}W_+(x,p))}{\int_\Sigma d\Sigma_\mu p^\mu \mathrm{tr}_4 W_+(x,p)}$$

Wigner function

$$W(x,k) = \mathrm{Tr}(\widehat{\rho}\widehat{W}(x,k))$$

$$\widehat{W}(x,k)_{AB} = -\frac{1}{(2\pi)^4}\int d^4y \, e^{-ik\cdot y} : \Psi_A(x-y/2)\overline{\Psi}_B(x+y/2) :$$

$$= \frac{1}{(2\pi)^4}\int d^4y \, e^{-ik\cdot y} : \overline{\Psi}_B(x+y/2)\Psi_A(x-y/2) :$$

$$S^\mu(p) = \mathrm{Tr}(\widehat{\rho} \;\; \widehat{\mathrm{Op}}(p))$$

Possibly pseudo-gauge dependent

Pseudo-gauge independent

Leading order

$$S^\mu(p) = \frac{1}{8m} \epsilon^{\mu\nu\rho\sigma} p_\sigma \frac{\int_\Sigma d\Sigma_\tau p^\tau n_F (1 - n_F) \partial_\nu \beta_\rho}{\int_\Sigma d\Sigma_\tau p^\tau n_F}$$
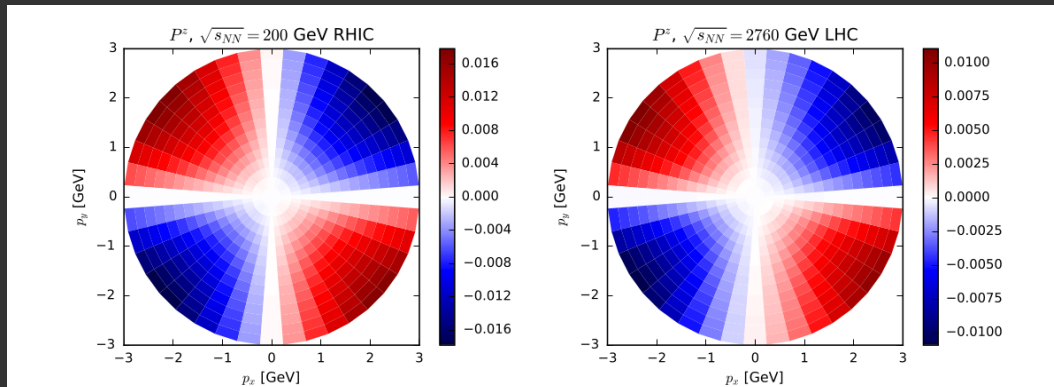
F. B., V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338, 32 (2013)

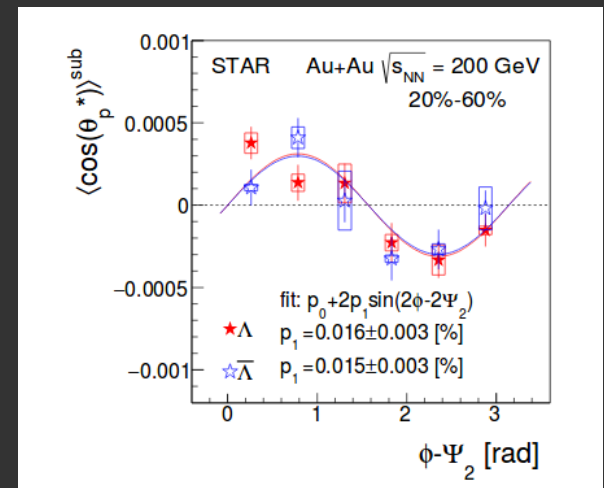For the Belinfante pseudo-gauge, corrections are second-order in derivatives

In any other pseudo-gauge, corrections are expected of the order

$$\Delta S^\mu(p) \propto \epsilon^{\mu\nu\rho\sigma} p_\sigma \int_\Sigma d\Sigma_\tau \ldots (\Omega_{\nu\rho} - \partial_\nu \beta_\rho)$$

A solution of the sign puzzle?

F. B., I. Karpenko, Phys. Rev. Lett. 120, 012302 (2018)

STAR coll. Phys. Rev. Lett. 123, 132301 (2019)

Hydrodynamics with a spin tensor

$$\partial_\mu j^\mu = 0 \qquad\qquad \partial_\mu T^{\mu\nu} = 0 \qquad\qquad \partial_\lambda \mathcal{S}^{\lambda,\mu\nu} = T^{\nu\mu} - T^{\mu\nu}$$

$$j^\mu = j^\mu(\beta,\zeta,\Omega), \qquad T^{\mu\nu} = T^{\mu\nu}(\beta,\zeta,\Omega), \qquad \mathcal{S}^{\lambda,\mu\nu} = \mathcal{S}^{\lambda,\mu\nu}(\beta,\zeta,\Omega).$$

Is it all relevant to heavy ion collisions? Strictly speaking, the prepared initial state are two colliding nuclei, which is not pseudo-gauge dependent. However,

If pseudo-gauge invariance is really broken, what is THE spin tensor?
In the Einstein-Cartan gravity theory, this is the dual of the axial current

$$\mathrm{Ric}_\alpha{}^\beta - \frac{1}{2}\delta_\alpha^\beta \mathrm{Ric}_\gamma{}^\gamma = \kappa\, \mathcal{T}_\alpha{}^\beta,$$

$$T_{\alpha\beta}{}^\gamma - \delta_\alpha^\gamma T_{\mu\beta}{}^\mu + \delta_\beta^\gamma T_{\mu\alpha}{}^\mu = \kappa\, \mathcal{S}_{\alpha\beta}{}^\gamma.$$

$$\mathcal{S}^{\lambda,\mu\nu} = \frac{1}{2}\bar\Psi\{\gamma^\lambda, \Sigma^{\mu\nu}\}\Psi$$

F. W. Hehl and Y. N. Obukhov, *Conservation of energy-momentum of matter as the basis for the gauge theory of gravitation*, arXiv:1909.01791

Summary and questions

• Pseudo-gauge invariance implies that the spin tensor is arbitrary and that the stress-energy tensor is not objective up to quantum corrections

• Physical observables in QFT in flat spacetime are supposedly pseudo-gauge invariant - such as spectra, elliptic flow, polarizations etc.

• Quantum states can be defined which break such an invariance: they can single out a particular stress-energy and spin tensor.

• Can we prepare such states? Is QGP one of those? In this case, what is the "correct" spin tensor?

• Polarization in heavy ion collisions could be the most sensitive observable to probe such an invariance in QFT
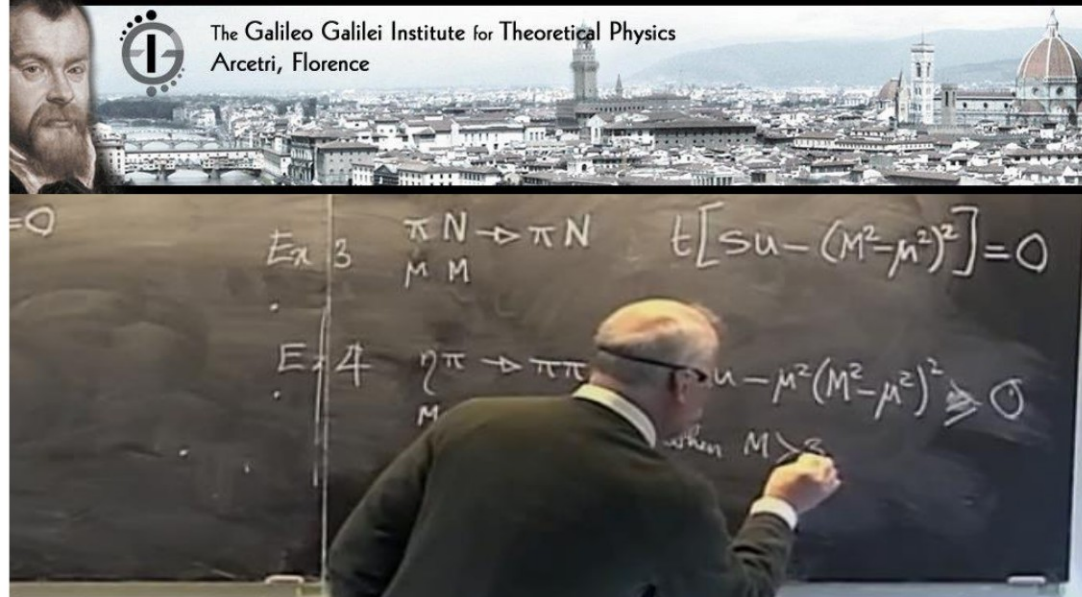
Frontiers in Nuclear and Hadronic Physics

24 February – 6 March 2020

The Galileo Galilei Institute for Theoretical Physics – Arcetri, Florence

Organizers

Francesco BECATTINI
(University and INFN, Firenze)

Ignazio BOMBACI
(University and INFN, Pisa)

Angela BONACCORSO
(INFN, Pisa)

Maria COLONNA
(LNS-INFN, Catania)

Vincenzo GRECO
(University and LNS-INFN, Catania)

Marzia NARDI
(INFN, Torino)

Giovanni SALMÈ
(INFN, Roma)

Elena SANTOPINTO
(INFN, Genova)

Enrico VIGEZZI
(INFN, Milano)

Lecturers and Topics

Gert AARTS (College of Science, Swansea University, UK)
QCD phase diagram and thermodynamics

Jean-Paul BLAIZOT (IPhT, Université Paris Saclay, France)
Introduction to ultra-relativistic collisions and Hard Probes of the Hot QCD matter

Wojciech FLORKOWSKI (Jagiellonian University, Krakow, Poland)
Relativistic hydrodynamics for nuclear collisions and recent advances

Sanjay REDDY (INT – Washington University, USA)
Dense matter inside neutron stars and its implications for multi-messenger astrophysics

Raju VENUGOPALAN (Brookhaven Natl. Lab, USA)
Universal dynamics in high energy QCD: classicalization, scrambling and thermalization

The school is primarily addressed to Ph.D. students in Theoretical Nuclear and Hadron Physics. Participation of experimentalists and post-docs is also encouraged. The 2020 edition will be devoted to the strong interaction at finite temperature and density and it will provide a pedagogical introduction to the basic concepts and tools needed for carrying cutting edge research in Hot QCD matter, high energy QCD, Quark-Gluon Plasma probes and properties, ultra-relativistic nuclear collisions (pA, AA), Relativistic hydrodynamics, QCD phase diagram at finite temperature and chemical potential with applications to Neutron Stars physics and multi-messenger astrophysics.

Deadline for application: 10 January 2020
https://www.ggi.infn.it/showevent.pl?id=341

Revisiting

These results deserve some discussion concerned with the physical meaning of the spin tensor, which has been the crucial ingredient to obtain (52) and the ensuing formulae. In principle, the definition of the polarization vector (50) involves the *total* angular momentum of the particle; hence the formula should be invariant under a change of the stress–energy tensor and spin tensor operators keeping the integral of the total angular momentum density invariant, a so-called *pseudo-gauge* transformation of the stress–energy tensor [22] (see also detailed discussion in Refs. [9,10]). However, in a formula like (52), one defines the local value of polarization and, therefore, a dependence on the particular spin tensor is implied. It could be expected that a space-integrated expression like (58) would be independent of the spin tensor choice; in fact this is not the case because of the explicit time-dependence of the local equilibrium density operator (see Discussion in Section 4). Such a dependence does not enable us to write the mean value of the divergence of an operator as the divergence of its mean value, thus breaking the pseudo-gauge invariance of the total angular momentum. For instance, had we used the Belinfante symmetrized stress–energy tensor, the ensuing value of polarization at local thermodynamical equilibrium (58) would vanish. To summarize, the choice of a specific spin tensor operator is necessary to calculate the polarization of particles and we have chosen the canonical spin tensor (see Eq. (16), which is the same used in Ref. [23]) to calculate the polarization of electrons. Even though it might appear disturbing that polarization at local thermodynamical equilibrium depends on the particular quantum spin tensor (whence the stress–energy tensor) of the theory, it has been recently shown that in thermodynamics this is a general feature [9,10].

This is incorrect because it is not true in general.
Its validity is limited to the approximated expression
of the Wigner Function (De Groot et al.):

Wrong

$$W(x,k) \simeq \frac{1}{2} \sum_{\sigma,\tau} \int \frac{d^3 p}{\varepsilon}\, \delta^4(k-p) u_\sigma(p) f(x,p)_{\sigma\tau} \bar{u}_\tau(p) - \delta^4(k+p) v_\sigma(p) \bar{f}(x,p)_{\tau\sigma} \bar{v}_\tau(p)$$
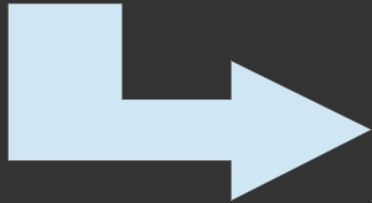
Global thermodynamic equilibrium

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-\int_\Sigma d\Sigma_\mu \left(\widehat{T}^{\mu\nu} \beta_\nu - \zeta \widehat{j}^\mu \right) \right]$$

Independent of the 3D hypersurface Σ if

$$\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0 \qquad\qquad \partial_\mu \zeta = 0$$

$$\beta_\mu = b_\mu + \varpi_{\mu\nu} x^\nu$$

The density operator becomes

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-b_\mu \widehat{P}^\mu + \frac{1}{2} \varpi_{\mu\nu} \widehat{J}^{\mu\nu} + \zeta \widehat{Q} \right]$$

True statistical operator (Zubarev theory)

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-\int_{\Sigma(\tau_0)} d\Sigma_\mu \left(\widehat{T}_B^{\mu\nu}\beta_\nu - \zeta\widehat{j}^\mu\right)\right].$$
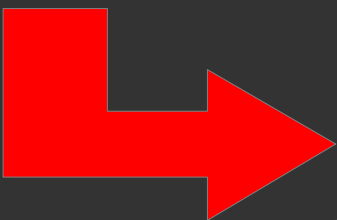
With the Gauss theorem

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-\int_{\Sigma(\tau)} d\Sigma_\mu \left(\widehat{T}_B^{\mu\nu}\beta_\nu - \zeta\widehat{j}^\mu\right) + \int_\Theta d\Theta \left(\widehat{T}_B^{\mu\nu}\nabla_\mu\beta_\nu - \widehat{j}^\mu\nabla_\mu\zeta\right)\right],$$

Local equilibrium, non-dissipative terms

Dissipative terms

$$W(x,k) = \mathrm{Tr}(\widehat{\rho}\widehat{W}(x,k))$$

Local thermodynamic equilibrium: leading order

$$W(x,k) = \frac{1}{Z}\mathrm{Tr}\left(\exp[-\beta(x)\cdot\widehat{P} + \frac{1}{2}\varpi(x):\widehat{J}]\widehat{W}(x,k)\right)$$

The exact solution at global equilibrium is still missing

Educated *ansatz* of the Wigner function of the Dirac field at global equilibrium
F. B., V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338, 32 (2013)

$$W(x,k) \simeq \frac{1}{2}\sum_{\sigma,\tau}\int\frac{\mathrm{d}^3\mathrm{p}}{\varepsilon}\,\delta^4(k-p)u_\sigma(p)f(x,p)_{\sigma\tau}\bar{u}_\tau(p) - \delta^4(k+p)v_\sigma(p)\bar{f}(x,p)_{\tau\sigma}\bar{v}_\tau(p)$$
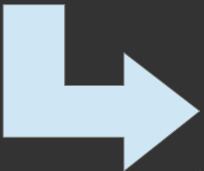
$$f(x,p) = \frac{1}{2m}\bar{U}(p)\left(\exp[\beta(x)\cdot p - \xi(x)]\exp[-\frac{1}{2}\varpi(x):\Sigma] + I\right)^{-1}U(p)$$

See also R. H. Fang, L. G. Pang, Q. Wang, X. N. Wang, Phys. Rev. C 94 (2016) 024904
W. Florkowski, A. Kumar and R. Ryblewski, Phys. Rev. C 98 (2018) 044906
Y. C. Liu, L. L. Gao, K. Mameda and X. G. Huang, Phys. Rev. D 99 (2019) 085014
N. Weickgenannt, X. L. Sheng, E. Speranza, Q. Wang and D. H. Rischke, Phys. Rev. D 100 (2019) 056018

$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\sigma\frac{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau n_F(1-n_F)\partial_\nu\beta_\rho}{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau\, n_F}$$
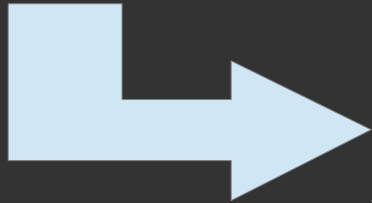
Global thermodynamic equilibrium

$$\widehat{\rho} = \frac{1}{Z} \exp \left[-\int_{\Sigma} d\Sigma_{\mu} \left(\widehat{T}^{\mu\nu} \beta_{\nu} - \zeta \widehat{j}^{\mu} \right) \right]$$

Independent of the 3D hypersurface Σ if

$$\partial_{\mu} \beta_{\nu} + \partial_{\nu} \beta_{\mu} = 0 \qquad\qquad \partial_{\mu} \zeta = 0$$

$$\beta_{\mu} = b_{\mu} + \varpi_{\mu\nu} x^{\nu}$$

The density operator becomes

$$\widehat{\rho} = \frac{1}{Z} \exp \left[-b_{\mu} \widehat{P}^{\mu} + \frac{1}{2} \varpi_{\mu\nu} \widehat{J}^{\mu\nu} + \zeta \widehat{Q} \right]$$