Hydrodynamics of particles with spin $^1/_2$

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Francesco Becattini, Bengt Friman, Amaresh Jaiswal, Avdhesh Kumar, Radosław Ryblewski, Rajeev Singh, Enrico Speranza

PRC97 (2018) 041901, Phys.Rev. D97 (2018) 116017, Phys.Rev. C98 (2018) 044906
Phys.Rev. C99 (2019) 011901, Phys.Lett. B789 (2019) 419,
Prog.Part.Nucl.Phys. 108 (2019) 103709 – review

QUARK MATTER 2019, Wuhan, China, Nov. 6, 2019

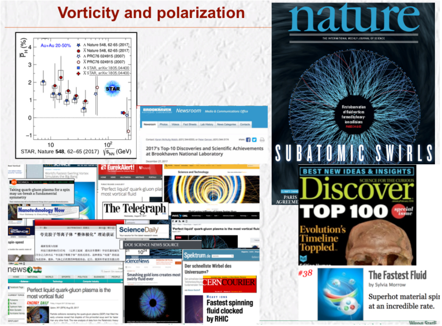
PHYSICS MOTIVATION

Everything started when ...

... the STAR experiment at Brookhaven National Laboratory (USA, Long Island) made the first positive measurements of spin polarization of Λ hyperons produced in relativistic heavy-ion collisions

www.sciencenews.org/article/smashing-gold-ions-creates-most-swirly-fluid-ever

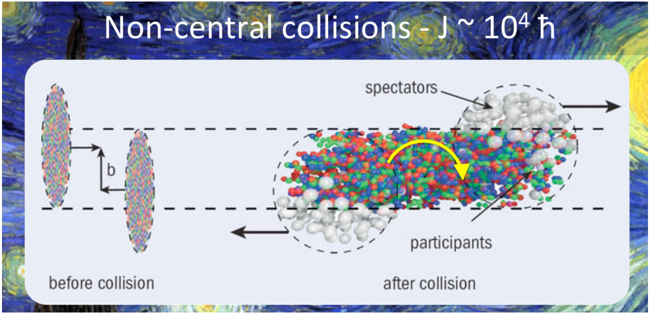
L. Adamczyk et al. (**STAR**), (2017), **Nature 548 (2017) 62-65**



(from Sergiei Voloshin's talk at „Hirschegg 2019 Workshop")

Non-central heavy-ion collisions create fireballs with large global angular momenta, some part of the angular momentum can be transferred from the orbital to the spin part

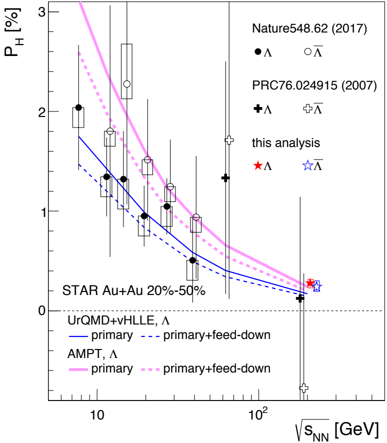
$$\boldsymbol{J}_{\rm init} = \boldsymbol{L}_{\rm init} = \boldsymbol{L}_{\rm final} + \boldsymbol{S}_{\rm final}$$



e. g. $\pi^+ + \pi^- \to \rho^0$
(Michael Lisa, talk „Strangeness in Quark Matter 2016")

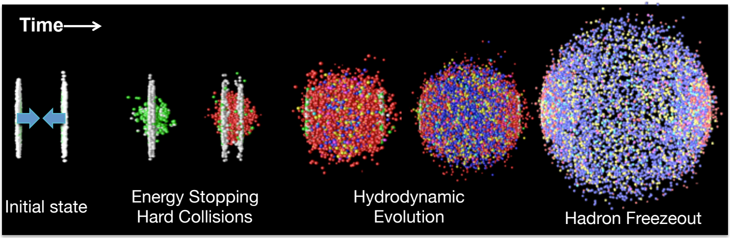
Experimantal results from STAR

- polarization grows with decreasing beam energy, non-zero even for the highest RHIC energies
- within the exp. errors, the spin polarization is the same for particles and antiparticles — most likely, the observed effect has no connection to magnetic fields



(Takafumi Niida, arXiv:1808.10482, talk at "Quark Matter 2016")

RELATIVISTIC HYDRODYNAMICS FORMS THE BASIC INGREDIENT OF THE STANDARD MODEL OF HEAVY-ION COLLISIONS



T. K. Nayak, Lepton-Photon 2011 Conference

HOW CAN WE INCLUDE SPIN POLARIZATION IN A HYDRODYNAMIC FRAMEWORK?

PERFECT-FLUID HYDRODYNAMICS = local equilibrium + conservation laws

one usually includes energy, linear momentum, baryon number, ...

T (temperature), u^μ (three independent components of flow), $\mu = T\xi$ (chemical potential)

$$T^{\mu\nu} = [\varepsilon(T,\mu) + P(T,\mu)]\, u^\mu u^\nu - P(T,\mu) g^{\mu\nu} \qquad \partial_\mu T^{\mu\nu} = 0, \qquad \partial_\mu j^\mu = 0$$

five equations for five unknown functions, **dissipation does not appear**

$$\partial_\mu (s u^\mu) = 0$$

entropy conservation follows from the energy-momentum conservation and the form of the energy-momentum tensor

FOR PARTICLES WITH SPIN, THE CONSERVATION OF ANGULAR MOMENTUM IS NOT TRIVIAL!

new hydrodynamic variables should be introduced

$\Omega_{\mu\nu}$ = T $\omega_{\mu\nu}$ (**spin potential** = temperature × **spin polarization tensor**)

Canonical energy-momentum $\widehat{T}^{\mu\nu}_{\rm can}$ and angular-momentum $\widehat{J}^{\mu,\lambda\nu}_{\rm can}$ tensors from the Noether Theorem:

$$\partial_\mu \widehat{T}^{\mu\nu}_{\rm can} = 0, \quad \partial_\mu \widehat{J}^{\mu,\lambda\nu}_{\rm can} = 0.$$

In general, the energy-momentum tensor is not symmetric

$$T^{\mu\nu}_{\rm can} \neq T^{\nu\mu}_{\rm can}$$

although classical $T^{\mu\nu}$ is always symmetric

$$T^{\mu\nu}_{\rm class} = \frac{\Delta p^\mu}{\Delta \Sigma_\nu} = \frac{\Delta p^\nu}{\Delta \Sigma_\mu} = T^{\nu\mu}_{\rm class} \quad \text{if} \quad \boldsymbol{v} = \frac{\boldsymbol{p}}{E} \quad \text{(1906 Planck)}$$

here $p^\mu = (E, \boldsymbol{p})$ is the four-momentum, while $\Delta\Sigma_\nu$ is a space-time volume element, so, for exmple

$$T^{10} = \frac{\Delta p^x}{\Delta x\, \Delta y\, \Delta z} = \frac{\Delta E\, \Delta x}{\Delta t\, \Delta x\, \Delta y\, \Delta z} = \frac{\Delta E}{\Delta t\, \Delta y\, \Delta z} = T^{01}$$

Total angular momentum $\widehat{J}^{\mu,\lambda\nu}_{\rm can}$ has orbital $\widehat{L}^{\mu,\lambda\nu}_{\rm can}$ and spin $\widehat{S}^{\mu,\lambda\nu}_{\rm can}$ parts:

$$
\begin{aligned}
\widehat{J}^{\mu,\lambda\nu}_{\rm can} &= x^\lambda \widehat{T}^{\mu\nu}_{\rm can} - x^\nu \widehat{T}^{\mu\lambda}_{\rm can} + \widehat{S}^{\mu,\lambda\nu}_{\rm can} \equiv \widehat{L}^{\mu,\lambda\nu}_{\rm can} + \widehat{S}^{\mu,\lambda\nu}_{\rm can} \\
\partial_\mu \widehat{J}^{\mu,\lambda\nu}_{\rm can} &= \widehat{T}^{\lambda\nu}_{\rm can} - \widehat{T}^{\nu\lambda}_{\rm can} + \partial_\mu \widehat{S}^{\mu,\lambda\nu}_{\rm can} = 0, \qquad \partial_\mu \widehat{S}^{\mu,\lambda\nu}_{\rm can} = \widehat{T}^{\nu\lambda}_{\rm can} - \widehat{T}^{\lambda\nu}_{\rm can}
\end{aligned}
$$

Pseudo-gauge transformation

(different localization of energy density and angular momentum, global charges not changed)

$$\begin{aligned} \widehat{T}'^{\mu\nu} &= \widehat{T}^{\mu\nu} + \frac{1}{2}\partial_\lambda\left(\widehat{\Phi}^{\lambda,\mu\nu} - \widehat{\Phi}^{\mu,\lambda\nu} - \widehat{\Phi}^{\nu,\lambda\mu}\right), \\ \widehat{S}'^{\lambda,\mu\nu} &= \widehat{S}^{\lambda,\mu\nu} - \widehat{\Phi}^{\lambda,\mu\nu} \end{aligned}$$

Belinfante's construction (Rosenfeld): superpotential defined as $\widehat{\Phi} = \widehat{S}^{\lambda,\mu\nu}_{\rm can}$

$$\widehat{T}^{\mu\nu}_{\rm Bel} = \widehat{T}^{\mu\nu}_{\rm can} + \frac{1}{2}\partial_\lambda\left(\widehat{S}^{\lambda,\mu\nu}_{\rm can} - \widehat{S}^{\mu,\lambda\nu}_{\rm can} - \widehat{S}^{\nu,\lambda\mu}_{\rm can}\right), \quad \widehat{S}^{\lambda,\mu\nu}_{\rm Bel} = 0$$

in this talk the canonical tensors are considered

physical system under consideration: hadronic gas (Λ hyperons + ...)

Dirac's equation treated as an effective description of baryons with spin 1/2

no EM fields included → see Nora Weickgenannt 's talk

SPIN TENSOR IS KEPT AND USED
ALSO BY THE COMMUNITY THAT STUDIES PROTON'S SPIN

Present phenomenology prescription used to describe the data

Local equilibrium

works by F. Becattini and collaborators
in local equilibrium the spin polarization is determined by thermal vorticity $\omega_{\lambda\nu}(x) = \varpi_{\lambda\nu}(x)$

1) Run any type of hydro, perfect or viscous, or transport, or whatsoever, without spin

2) Find $\beta_\mu(x) = u_\mu(x)/T(x)$ on the freeze-out hypersurface
(defined often by the condition T=const)

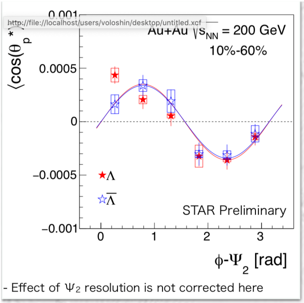
3) Calculate thermal vorticity $\varpi_{\alpha\beta}(x) \neq$ const

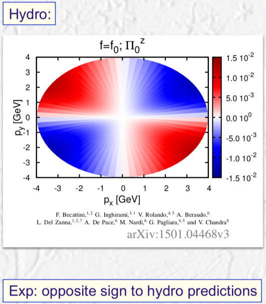
4) Identify thermal vorticity with the spin polarization tensor $\omega_{\mu\nu}$

5) Make predictions about spin polarization

**SUCH A METHOD WORKS WELL,
DESCRIBES MOST OF THE DATA, BUT...**

...CAN WE TAKE IT FOR GRANTED?
THERE ARE PROBLEMS WITH MORE DETAILED DESCRIPTION





Local equilibrium → local spin-thermodynamic equilibrium

in local equilibrium $\widehat{\rho}_{\rm LEQ}$ is approximately constant (with dissipation effects neglected)

$$T^{\mu\nu} = {\rm tr}\left(\widehat{\rho}_{\rm LEQ}\,\widehat{T}^{\mu\nu}\right), \quad S^{\mu,\lambda\nu} = {\rm tr}\left(\widehat{\rho}_{\rm LEQ}\,\widehat{S}^{\mu,\lambda\nu}\right), \quad j^{\mu} = {\rm tr}\left(\widehat{\rho}_{\rm LEQ}\,\widehat{j}^{\mu}\right)$$

these tensors are all functions of the hydrodynamic variables β_μ, $\omega_{\mu\nu}$, and ξ

$$T^{\mu\nu} = T^{\mu\nu}[\beta,\omega,\xi], \quad S^{\mu,\lambda\nu} = S^{\mu,\lambda\nu}[\beta,\omega,\xi], \quad j^{\mu} = j^{\mu}[\beta,\omega,\xi]$$

and satisfy the conservation laws

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\lambda S^{\lambda,\mu\nu} = T^{\nu\mu} - T^{\mu\nu}, \quad \partial_\mu j^{\mu} = 0$$

Spin dependent phase-space distribution functions

standard scalar functions $f(x,p)$ are generalized to 2×2 Hermitean matrices in spin space for each value of the space-time position x and four-momentum p

F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, Annals Phys. 338 (2013) 32

$$
\begin{aligned}
f^+_{rs}(x,p) &= \bar{u}_r(p) X^+ u_s(p) \\
f^-_{rs}(x,p) &= -\bar{v}_s(p) X^- v_r(p)
\end{aligned}
$$

$$
X^\pm = \exp\left[\pm\xi(x) - \beta_\mu(x) p^\mu\right] M^\pm, \quad M^\pm = \exp\left[\pm\frac{1}{2}\omega_{\mu\nu}(x)\Sigma^{\mu\nu}\right]
$$

here $\Sigma^{\mu\nu}$ is the Dirac spin operator, for small polarization

$$
M^\pm = 1 \pm \frac{1}{2}\,\omega_{\mu\nu}\Sigma^{\mu\nu}
$$

Equilibrium Wigner functions

x - space-time position, k - off-shell momentum, p on-shell momentum

De Groot, van Leeuwen, van Weert: *Relativistic Kinetic Theory. Principles and Applications*

GLW framework (approximate, particles on the mass shell)

$$\mathcal{W}^{+}_{\rm eq}(x,k) = \frac{1}{2}\sum_{r,s=1}^{2}\int dP\,\delta^{(4)}(k-p)u^r(p)\bar{u}^s(p)f^{+}_{rs}(x,p)$$

$$\mathcal{W}^{-}_{\rm eq}(x,k) = -\frac{1}{2}\sum_{r,s=1}^{2}\int dP\,\delta^{(4)}(k+p)v^s(p)\bar{v}^r(p)f^{-}_{rs}(x,p)$$

Clifford-algebra expansion

(used in many early works on QED and QGP plasma, e.g., H.T. Elze, M. Gyulassy, D. Vasak, Phys.Lett. B177 (1986) 402)

$$\mathcal{W}^{\pm}(x,k) \quad = \quad \frac{1}{4}\left[\mathcal{F}^{\pm}(x,k) + i\gamma_5\mathcal{P}^{\pm}(x,k) + \gamma^\mu\mathcal{V}^{\pm}_{\mu}(x,k) + \gamma_5\gamma^\mu\mathcal{A}^{\pm}_{\mu}(x,k) + \Sigma^{\mu\nu}\mathcal{S}^{\pm}_{\mu\nu}(x,k)\right]$$

WF, A. Kumar, R. Ryblewski, Phys. Rev. C98 (2018) 044906

$$\big(\gamma_\mu K^\mu - m\big)\mathcal{W}(x,k) = C[\mathcal{W}(x,k)]$$

Here K^μ is the operator defined by the expression

$$K^\mu = k^\mu + \frac{i\hbar}{2}\,\partial^\mu$$

In the case of global equilibrium, with the vanishing collision term, the Wigner function $\mathcal{W}(x,k)$ exactly satisfies the equation

$$\big(\gamma_\mu K^\mu - m\big)\mathcal{W}(x,k) = 0$$

the leading order terms in $\hbar$ can be taken from Becattini's $\mathcal{W}^{\pm}_{\mathrm{eq}}(x,k)$

in the NLO in $\hbar$ we get the kinetic equation (well known in the literature)

$$k^\mu \partial_\mu \mathcal{F}_{\rm eq}(x,k) = 0$$

$$k^\mu \partial_\mu \, \mathcal{A}^\nu_{\rm eq}(x,k) = 0, \quad k_\nu \, \mathcal{A}^\nu_{\rm eq}(x,k) = 0$$

Global equilibrium — these equations are exactly fulfilled, what about local equilibrium

Local equilibrium — only moments of the kinetic equations are satisfied, standard method for going from the kinetic-theory description to hydrodynamics

$$\partial_\alpha N^\alpha_{\rm eq}(x) = 0, \quad \partial_\alpha T^{\alpha\beta}_{\rm GLW}(x) = 0, \quad \partial_\lambda S^{\lambda,\mu\nu}_{\rm GLW}(x) = 0$$

GLW — forms proposed by de Groot, van Leeuwen, van Weert

EQUATIONS DEFINED ABOVE ARE EXACTLY THE HYDRODYNAMIC EQUATIONS WE HAVE BEEN LOOKING FOR
11 equations for 11 unknown functions of space and time

NLO corrections in $\hbar$ again

LO generates corrections in the NLO

$$\mathcal{P}^{(1)} = -\frac{1}{2m}\,\partial^\mu \mathcal{A}_{\mathrm{eq},\mu}$$

$$\mathcal{V}^{(1)}_\mu = -\frac{1}{2m}\partial^\nu \mathcal{S}_{\mathrm{eq},\nu\mu}$$

$$\mathcal{S}^{(1)}_{\mu\nu} = \frac{1}{2m}\left(\partial_\mu \mathcal{V}_{\mathrm{eq},\nu} - \partial_\nu \mathcal{V}_{\mathrm{eq},\mu}\right)$$

IMPORTANT IF the canonical formalism is used

$$T^{\mu\nu}_{\mathrm{GLW}}(x) = \frac{1}{m}\mathrm{tr}_4 \int d^4k\, k^\mu\, k^\nu \mathcal{W}(x,k) = \frac{1}{m}\int d^4k\, k^\mu\, k^\nu \mathcal{F}(x,k)$$

$$T^{\mu\nu}_{\mathrm{can}}(x) = \int d^4k\, k^\nu \mathcal{V}^\mu(x,k)$$

quantum corrections induce asymmetry $T^{\mu\nu}_{\mathrm{can}}(x) \neq T^{\nu\mu}_{\mathrm{can}}(x)$

From canonical to GLW case

Including the components of $\mathcal{V}^\mu(x,k)$ up to the first order in the equilibrium case we obtain

$$T^{\mu\nu}_{\rm can}(x) = T^{\mu\nu}_{\rm GLW}(x) + \delta T^{\mu\nu}_{\rm can}(x)$$

where

$$\delta T^{\mu\nu}_{\rm can}(x) = -\frac{\hbar}{2m}\int d^4k k^\nu \partial_\lambda \mathcal{S}^{\lambda\mu}_{\rm eq}(x,k) = -\partial_\lambda S^{\nu,\lambda\mu}_{\rm GLW}(x)$$

The canonical energy-momentum tensor is conserved

$$\partial_\alpha T^{\alpha\beta}_{\rm can}(x) = 0$$

$$\begin{aligned} S^{\lambda,\mu\nu}_{\rm can} &= \hbar\cosh(\xi)\int dP\, e^{-\beta\cdot p}\left(\omega^{\mu\nu}p^\lambda + \omega^{\nu\lambda}p^\mu + \omega^{\lambda\mu}p^\nu\right) \\ &= S^{\lambda,\mu\nu}_{\rm GLW} + S^{\mu,\nu\lambda}_{\rm GLW} + S^{\nu,\lambda\mu}_{\rm GLW} \end{aligned}$$

The canonical spin tensor is not conserved!

$$\partial_\lambda S^{\lambda,\mu\nu}_{\rm can}(x) = T^{\nu\mu}_{\rm can} - T^{\mu\nu}_{\rm can} = -\partial_\lambda S^{\mu,\lambda\nu}_{\rm GLW}(x) + \partial_\lambda S^{\nu,\lambda\mu}_{\rm GLW}(x)$$

if we introduce the tensor $\Phi_{\rm can}^{\lambda,\mu\nu}$ defined by the relation

$$\Phi_{\rm can}^{\lambda,\mu\nu} \equiv S_{\rm GLW}^{\mu,\lambda\nu} - S_{\rm GLW}^{\nu,\lambda\mu}$$

we can write

$$S_{\rm can}^{\lambda,\mu\nu} = S_{\rm GLW}^{\lambda,\mu\nu} - \Phi_{\rm can}^{\lambda,\mu\nu}$$

and

$$T_{\rm can}^{\mu\nu} = T_{\rm GLW}^{\mu\nu} + \frac{1}{2}\partial_\lambda \left(\Phi_{\rm can}^{\lambda,\mu\nu} + \Phi_{\rm can}^{\mu,\nu\lambda} + \Phi_{\rm can}^{\nu,\mu\lambda}\right)$$

The canonical and GLW frameworks are connected by a pseudo-gauge transformation. Similarly to Belinfante, it leads to a symmetric energy-momentum tensor, however, does not eliminate the spin degrees of freedom.

1. Foundations for the theoretical framework of perfect-fluid hydrodynamics with spin have been laid.
2. Relations between different definitions of the energy-momentum tensors have been clarified in this context.
3. Intensive work is going on now aiming at the numerical implementation of our ideas and making more detailed comparisons with the experimental data.
4. More conceptual work is needed to include dissipation.

more in Prog.Part.Nucl.Phys. 108 (2019) 103709

BACK-UP SLIDES

Back-up slides 1: Jan Weyssenhoff's circle

CC

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TPJU 94-10
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CRACOW INVESTIGATIONS IN THE THEORY OF RELATIVISTIC SPIN PARTICLES

Bronisław Średniawa

Institute of Physics, Jagellonian University, Cracow

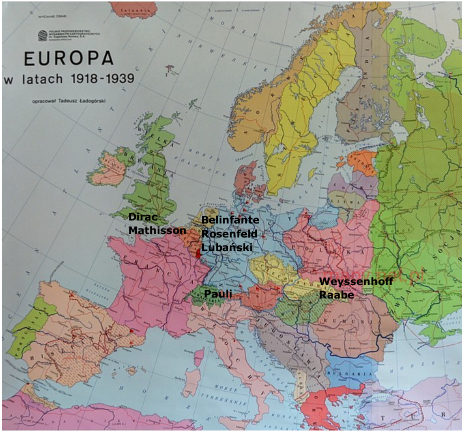
Abstract

The first papers of M. Mathisson on the theory of relativistic spin particles, written in early thirties in Warsaw are presented. Then the beginnings of the work on this theory in Cracow, performed by J. Weyssenhoff, M. Mathisson and their collaborators in the last years before the outbreak of the II World War is described. Further the development of this theory by Weyssenhoff and A. Raabe during the years of the war is discussed. Their work demonstrated the similarity of the properties of relativistic spin particles with those of Dirac's electron. After the end of the war Weyssenhoff and his collaborators, B. Średniawa, Z. Borelowski and A. Białas studied the properties of spin particles, working especially on the canonical formalism of this theory and on the radiation and motion of magnetic dipoles. The research in these problems lasted in Cracow until the late sixties.

TPJU 10/94

April 1994

Institute of Physics, Reymonta 4, 30-059 Kraków, POLAND, tlx 322723 ifuj pl



Jan Weyssenhoff
1889-1972

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J. K. LUBAŃSKI

Obituary notice by L. ROSENFELD, Manchester.

The circle of Lubański's friends was not large. During the wartime which was so difficult for him he lived very retired and he was rather shy and taciturn. But those who were in closer contact with him have been able to discover his sensitive and refined personality. After the liberation a striking change came over him, that made us only then understand how much he had suffered during the war. He surprised us by an unknown alacrity and optimism. His new task in Delft gave him great satisfaction and he fulfilled it with an enthusiasm and energy that authorised the greatest expectations for his scientific and personal future. This made the shock the more violent for his friends when the news reached them of his unexpected death on the 8-th December 1946, after only a very short illness.

Jozeph Kazimir Lubański was born in 1914 in Rumania from Polish parents. He spent his youth in Russia; only in 1926 did he come to Poland where he studied Physics at the Universities of Wilno and Kraków. In Kraków he worked under the direction of the very original theoretician Mathisson (who died in England during the war); his first paper in 1937 is based on Mathisson's theories. Until the autumn of 1938 Lubański was assistant at the Institute of Theoretical Physics at Wilno. In December 1938 he came to Leiden with a stipend from the Polish Government to work under the direction of Professor Kramers. Since that time he lived in Holland, where he was greatly helped during the war by the Lorentz Fund. As a Polish citizen he was forced already in 1940 to leave the coastal region and after a short stay in the country-side he settled in Utrecht, where he stayed until his appointment at Delft in October 1945.

In Leiden he worked with Kramers and Belinfante on the Theory of Particles with arbitrary spin. His investigations on this subject are set-out in three papers, published in the Dutch journal Physica. These papers witness his profound knowledge of the abstract theories of modern algebra and his mastery in applying them to fundamental problems of theoretical physics. The same qualities characterize his further publications which contained the results of the work he did in Utrecht.

Back-up slides 2: classical treatment of spin

internal angular momentum tensor $s^{\alpha\beta}$

M. Matthison, Neue mechanik materieller systemes, Acta Phys. Polon. 6 (1937) 163.

$$s^{\alpha\beta} = \frac{1}{m}\epsilon^{\alpha\beta\gamma\delta}p_\gamma s_\delta. \tag{1}$$

$$s \cdot p = 0, \quad s^\alpha = \frac{1}{2m}\epsilon^{\alpha\beta\gamma\delta}p_\beta s_{\gamma\delta} \tag{2}$$

A straightforward generalization of the phase-space distribution function $f(x, \boldsymbol{p})$ is a spin dependent distribution $f(x, \boldsymbol{p}, s)$

$$\int dS \ldots = \frac{m}{\pi\,\mathfrak{s}} \int d^4s\,\delta(s \cdot s + \mathfrak{s}^2)\,\delta(p \cdot s) \ldots \tag{3}$$

$$\mathfrak{s}^2 = \frac{1}{2}\left(1 + \frac{1}{2}\right) = \frac{3}{4} \tag{4}$$

$$\int dS = \frac{m}{\pi\,\mathfrak{s}} \int d^4s\,\delta(s \cdot s + \mathfrak{s}^2)\,\delta(p \cdot s) = 2 \tag{5}$$

equilibrium distribution functions for particles and antiparticles in the form

$$f^{\pm}_{\rm eq}(x,p,s) = \exp\left(-p\cdot\beta(x) \pm \xi(x) + \frac{1}{2}\omega_{\alpha\beta}(x) s^{\alpha\beta}\right). \tag{6}$$

conserved “currents”

$$N^{\mu}_{\rm eq} = \int dP \int dS\; p^{\mu} \left[f^{+}_{\rm eq}(x,p,s) - f^{-}_{\rm eq}(x,p,s)\right], \tag{7}$$

$$T^{\mu\nu}_{\rm eq} \quad = \quad \int dP \int dS\; p^{\mu} p^{\nu} \left[f^{+}_{\rm eq}(x,p,s) + f^{-}_{\rm eq}(x,p,s)\right] \tag{8}$$

$$S^{\lambda,\mu\nu}_{\rm eq} \quad = \quad \int dP \int dS\; p^{\lambda}\, s^{\mu\nu} \left[f^{+}_{\rm eq}(x,p,s) + f^{-}_{\rm eq}(x,p,s)\right] \tag{9}$$

For $|\omega_{\mu\nu}| < 1$ we obtain the formalism that agrees with that based on the quantum description of spin (in the GLW version)

PL vector can be expressed by the simple expression

$$\pi_\mu = -\mathfrak{s}\,\frac{\tilde{\omega}_{\mu\beta}}{P}\,\frac{p^\beta}{m}\,L(P\,\mathfrak{s}), \tag{10}$$

where $L(x)$ is the Langevin function defined by the formula

$$L(x) = \coth(x) - \frac{1}{x}. \tag{11}$$

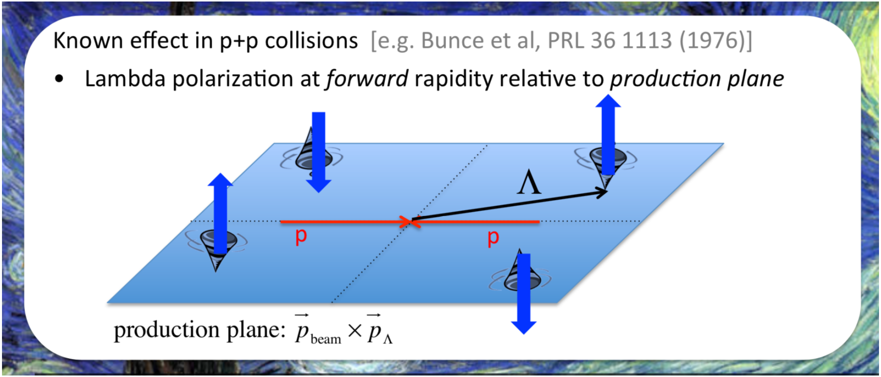
in PRF the direction of the PL vector agrees with that of the polarization vector $\boldsymbol{P}$. For small and large P we obtain two important results:

$$\boldsymbol{\pi}_* = -\mathfrak{s}\,\frac{\boldsymbol{P}}{P}, \quad |\boldsymbol{\pi}_*| = \mathfrak{s} = \sqrt{\frac{3}{4}}, \quad \text{if} \quad P \gg 1 \tag{12}$$

and

$$\boldsymbol{\pi}_* = -\mathfrak{s}^2\,\frac{\boldsymbol{P}}{3}, \quad |\boldsymbol{\pi}_*| = \mathfrak{s}^2\,\frac{P}{3} = \frac{P}{4}, \quad \text{if} \quad P \ll 1. \tag{13}$$

Back-up slides 3: *pp* collisions


Known effect in p+p collisions [e.g. Bunce et al, PRL 36 1113 (1976)]
• Lambda polarization at *forward* rapidity relative to *production plane*
Λ
p
p
production plane: $\vec{p}_{\text{beam}} \times \vec{p}_{\Lambda}$

Back-up slides 4: GLW expressions

charge current

$$N^{\alpha}_{\rm GLW} = n u^{\alpha}, \qquad n = 4\,\sinh(\xi)\, n_{(0)}(T)$$

energy-momentum tensor (with a perfect-fluid form)

$$T^{\mu\nu}_{\rm GLW}(x) \quad = \quad (\varepsilon + P) u^{\mu} u^{\nu} - P g^{\mu\nu}$$

$$\varepsilon = 4\,\cosh(\xi)\, \varepsilon_{(0)}(T)\,, \quad P = 4\,\cosh(\xi)\, P_{(0)}(T)$$

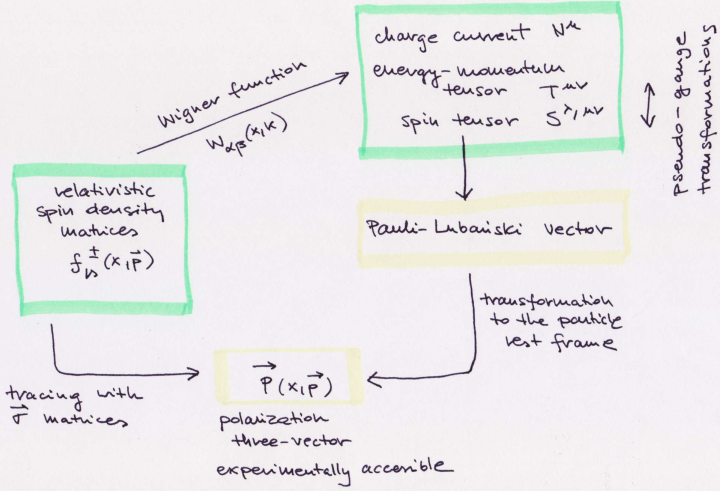
$n_{(0)}(T)$, $\varepsilon_{(0)}(T)$, $P_{(0)}(T)$ — particle density, energy density, and pressure of classical particles at the temperature T

spin tensor

$$S^{\lambda,\mu\nu}_{\rm GLW} \quad = \quad \frac{\hbar \cosh(\xi)}{m^2} \int dP\, e^{-\beta\cdot p} p^{\lambda} \left(m^2 \omega^{\mu\nu} + 2 p^{\alpha} p^{[\mu} \omega^{\nu]}{}_{\alpha} \right) = S^{\lambda,\mu\nu}_{\rm ph} + S^{\lambda,\mu\nu}_{\Delta}.$$

only $S^{\lambda,\mu\nu}_{\rm ph}$ was used in WF, B. Friman, A. Jaiswal, E.Speranza, Phys.Rev. C97 (2018) 041901

Back-up slides 5: consistency checks



charge current N^μ
energy-momentum tensor $T^{\mu\nu}$
spin tensor $S^{\lambda,\mu\nu}$
Wigner function
$W_{\alpha\beta}(x,k)$
pseudo-gauge transformations
relativistic spin density matrices
$f_{rs}^{\pm}(x,\vec{p})$
Pauli-Lubanski vector
transformation to the particle rest frame
$\vec{P}(x,\vec{p})$
tracing with $\vec{\sigma}$ matrices
polarization three-vector
experimentally accessible

relativistic spin density matrices
$f^{\pm}_{rs}(x,\vec{p})$

$\xrightarrow[\text{for the Wigner function coefficients}]{\text{kinetic equations}}$

$$k^\mu \partial_\mu \mathcal{F}_{eq}(x,k)=0$$
$$k^\mu \partial_\mu \mathcal{A}^\nu_{eq}(x,k)=0$$
$$k_\nu \mathcal{A}^\nu_{eq}(x,k)=0$$

$\downarrow$ moments (integrals over k)

$$\partial_\mu N^\mu=0$$
$$\partial_\mu T^{\mu\nu}_{GLW}=0$$
$$\partial_\lambda S^{\lambda,\mu\nu}_{GLW}=0$$

hydrodynamic equations

classical spin
$f(x,\vec{p},s)$

$\xrightarrow[|\omega_{\mu\nu}|<1]{s^2=\frac{1}{2}(1+\frac{1}{2})}$ hydrodynamic equations

$\downarrow$ arbitrary $\omega_{\mu\nu}$

anisotropic hydrodynamics with spin