Magnetic Field in the Charged Subatomic Swirl

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Quark Matter 2019

Based on the work: arXiv:1904.04704

Outline

Motivation

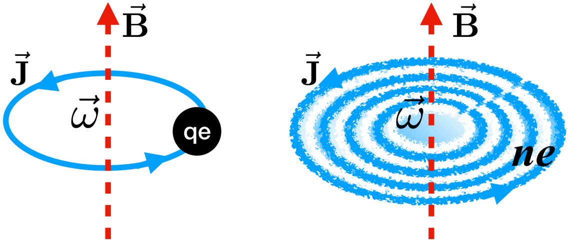
What does the title mean?

In heavy ion collisions:

- Subatomic swirl? $\surd$

- Charged? $\surd$
- Magnetic field? $\surd$ but . . .

Motivation



Charged rotating fluid should produce magnetic fields.

- Relation between B and ω and n?
- Beam energy / centrality dependence?
- Implications?

Single Particle: Classical

For a single classical charged particle with charge qe and mass m, moving with angular velocity ω_0 on a circle of radius R_0

B field:

$$
\begin{aligned}
B_z(\rho) &= B_0 \left[\frac{E\left(\frac{4\tilde{\rho}}{(1+\tilde{\rho})^2}\right)}{\pi(1-\tilde{\rho})} + \frac{K\left(\frac{4\tilde{\rho}}{(1+\tilde{\rho})^2}\right)}{\pi\,(1+\tilde{\rho})} \right] \\
B_0 &= B_z(\rho = 0) = \frac{qe\omega_0}{4\pi R_0}.
\end{aligned}
$$

Where $\tilde{\rho} = \frac{\rho}{R_0}$, $K(x)$ and $E(x)$ are complete elliptic integrals of first and second kind.

$$
\begin{aligned}
B &\propto qe\omega_0 \\
\Phi_B &\sim B_0 \pi R_0^2 \propto L \sim m R_0^2 \omega_0.
\end{aligned}
$$

Single Particle: Quantum Mechanics

For a quantum mechanical particle constrained on a 1D circle with angular momentum $L = k\hbar = mR_0^2\omega_0$, its wave function $\phi = \frac{e^{ik\phi}}{\sqrt{2\pi}}$

$$
\begin{aligned}
B_0 &= \frac{qek\hbar}{2\pi R_0^2} \propto qe\omega_0 \\
\Phi_B &\propto L.
\end{aligned}
$$

Charged Current

For an ideal fluid with charge density n,

$$J^\mu = neu^\mu .$$

Maxwell's equation:

$$\partial_\mu F^{\mu\nu} = J^\nu .$$

Assuming n to be homogeneous or changing slowly,

$$\begin{aligned} ne\omega^\mu &= ne\epsilon^{\mu\nu\rho\sigma} u_\nu \partial_{[\rho} u_{\sigma]} \\ &= \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_{[\rho} J_{\sigma]} \\ &= \epsilon^{\mu\nu\rho\sigma} u_\nu \Box F_{\rho\sigma} . \end{aligned}$$

Static case, local rest frame:

$$ne\omega = \nabla^2 \mathbf{B}.$$

Charged Current: Classical

For a general azimuthal velocity profile

$$\mathbf{v} = v_0 F(\frac{\rho}{R_0})\hat{\phi}, \rho \leq R_0, \int_0^1 \mathrm{d}x F(x) x = \frac{1}{2},$$

,

$$\mathbf{L} \quad = \quad 2\pi \int_0^{R_0} \mathrm{d}\rho \rho n \rho v = 2\pi n v_0 R_0^3 \int_0^1 \mathrm{d}x F(x) x^2 \hat{z}.$$

Define 'average vorticity' by:

$$\begin{aligned} \frac{\pi}{2} n R_0^4 \bar{\omega} &= \mathbf{L} \\ \bar{\omega} &= \frac{4 v_0}{R_0} \int_0^1 \mathrm{d}x F(x) x^2 \hat{z}. \end{aligned}$$

Charged Current: Classical

$$
\begin{aligned}
\mathbf{B} &= nev_0 R_0 \int_{\frac{\rho}{R_0}}^{1} \mathrm{d}x F(x) \hat{z} \\
\bar{\mathbf{B}} &= \frac{\int_0^{R_0} \mathrm{d}\rho \rho \mathbf{B}(\rho)}{\int_0^{R_0} \mathrm{d}\rho \rho} \hat{z} \\
&= nev_0 R_0 \int_0^1 \mathrm{d}x F(x) x^2 \hat{z} \\
e\bar{\mathbf{B}} &= \frac{e^2}{4\pi} nA\bar{\omega}.
\end{aligned}
$$

A is the transverse area of the vortex.

$$
A = \pi R_0^2,
$$

The ratio is independent of $F(x)$!

Magnetic Field in Heavy Ion Collisions: Parameters

$$e\bar{\mathbf{B}} = \frac{e^2}{4\pi} n A \bar{\omega}$$

Take $(20-50)\%$ centrality of Au-Au collision in $(10-200)$GeV at RHIC.

- AMPT simulations give the evolution of vorticity.
- Charge density at freeze-out can be extracted either from AMPT or using thermal freeze-out models. At early time it is about one order of magnetic larger.
- We take $A = \pi R_0^2$, $R_0 = 4fm$.

Estimated Magnetic Field

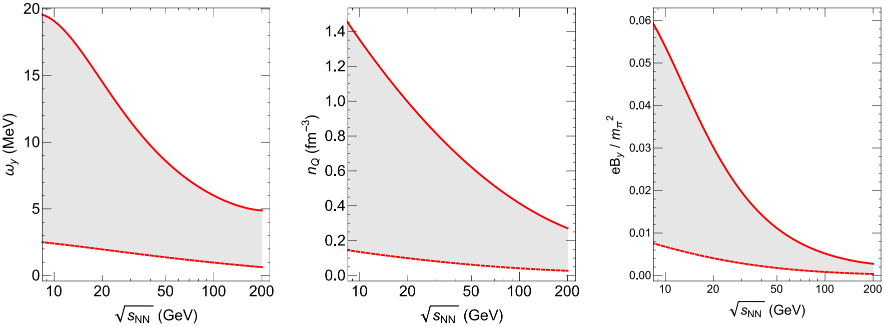


Figure: The vorticity ω_y (left), charge density n_Q (middle) and magnetic field $e\bar{B}$ (right) as functions of collisional beam energy $\sqrt{s_{NN}}$. The upper limits are for $\tau = 0.5\mathrm{fm}$ and lower limits for $\tau = 5\mathrm{fm}$.

Λ Polarization Splitting

With such an magnetic field, one expects the splitting of Λ and $\bar{\Lambda}$ polarization

$$\Delta P \equiv P_{\bar{\Lambda}} - P_{\Lambda} \simeq \frac{2|\mu_\Lambda|\bar{B}}{T_{fo}}$$

we use $|\mu_\Lambda| = 0.613\mu_N = \frac{0.613e}{2M_N}$, $M_N = 938 MeV$, $T_{fo} = 155 MeV$

Λ Polarization Splitting

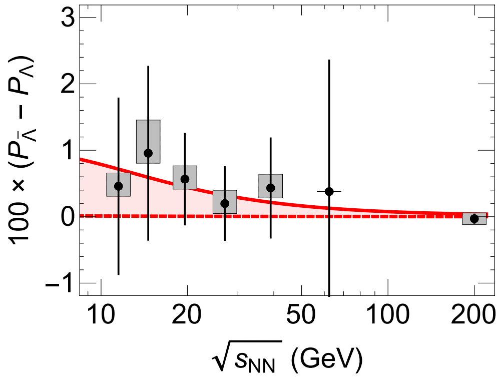


Figure: The induced polarization difference between hyperons and anti-hyperons, $\Delta P = P_{\bar{\Lambda}} - P_{\Lambda}$ as a function of collisional beam energy $\sqrt{s_{NN}}$, in comparison with STAR data[L. Adamczyk *et al.* [STAR Collaboration],Nature **548**, 62 (2017)].

Summary

- We demonstrated the relation between the average vorticity and the average magnetic field of a certain fluid.
- Using this relation, we estimated this magnetic field produced in HICs and its effect on Λ polarization.

- Outlook
  - More quantitative modeling.
  - More application, eg. CME: $eB\tau \simeq (5 \sim 60)\mathrm{MeV}$.