Quantum Kinetic Theory of Spin Polarization of Massive Quarks in Perturbative QCD: Leading Log

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Introduction

Experimentally observation of Λ polarization in HIC

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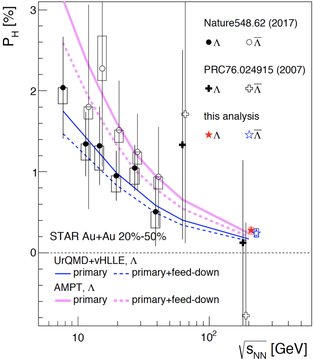
Theoretical progress:

- **Spin-orbital coupling in HIC from global OAM** Liang and Wang 2005
  Becattini et al 2008 Kapusta et al 2019
- **Relativistic Hydrodynamics with spin**
  W. Florkowski et al.2018 K. Hattori et al 2019
- **Free streaming Boltzmann equation for massive spin-$\frac{1}{2}$ fermions**
  Mueller and Venugopalan, arXiv:1901.10492
  K. Hattori et al, arXiv:1903.01653
  N. Weickgenannt et al, arXiv:1902.06513
  Gao and Liang, arXiv:1902.06510



Our objective: Formulate "quantum kinetic equations" for massive spin-$\frac{1}{2}$ quarks with collision term in pQCD

Note: The characteristic relaxation rate for spin polarization of massive quarks $\Gamma_0 \sim \alpha_s^2 \log(1/\alpha_s) T$

Approach

Objective: Derive Lindblad equation for the time evolution of density matrix $\hat{\rho}$ for quarks:

$$\frac{\partial \hat{\rho}}{\partial t} = -\frac{i}{\hbar}[H_{eff}, \hat{\rho}] - \Gamma \cdot \hat{\rho}$$

where:

$$H_{eff} = -\frac{\hbar}{2}\boldsymbol{\sigma} \cdot (\boldsymbol{\omega} + e\mathbf{B})$$

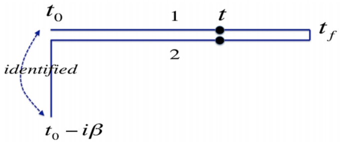
$$\Gamma = \Gamma_0 + \Gamma_1(\boldsymbol{\omega}, \boldsymbol{B}) + O(\boldsymbol{\omega}^2, \boldsymbol{B}^2)$$

We begin by Γ_0, which describes how the spin density matrix, initially polarized relaxes to un-polarized one with vanishing of vorticity and external magnetic field.

Density matrix

- Density matrix $\hat{\rho}$ in phase space$(\mathbf{x}, \mathbf{p})$ in Schwinger-Keldysh formalism

$$\hat{\rho}(\mathbf{p}_1, \mathbf{p}_2) = \hat{\rho}(\mathbf{p})(2\pi)^3 \delta(\mathbf{p}_1 - \mathbf{p}_2)$$



- $\boldsymbol{p}_a = (\boldsymbol{p}_1 - \boldsymbol{p}_2) \sim \boldsymbol{\partial}_{\boldsymbol{x}_r}$

Note: Assuming the background is spatially homogeneous!!!!

$$\hat{\rho}(\boldsymbol{p}) = \sum_{s,s'=\pm} |\boldsymbol{p}, s\rangle \rho_{s,s'}(\boldsymbol{p}) \langle \boldsymbol{p}, s'| = \frac{1}{2} f(\mathbf{p}) + \mathbf{S}(\mathbf{p}) \cdot \boldsymbol{\sigma}$$

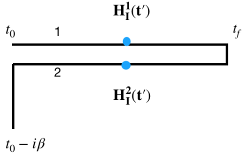
- Particle Number $N = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} f(\boldsymbol{p})$, • Spin polarization $\boldsymbol{S} = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \boldsymbol{S}(\boldsymbol{p})$

Time evolution of density matrix

- $\hat{\rho}(t) = \left\langle U_1(t, t_0)\hat{\rho}(t_0)U_2^\dagger(t, t_0)\right\rangle_A$

- Unitary time evolution operator

$U_{1(2)}(t, t_0) = \mathcal{P}e^{-i\int_{t_0}^{t} dt'\, H_I^{1(2)}(t')}$

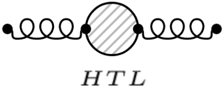


- QCD Interaction Hamiltonian:

$H_I^{1(2)} = g\int d^3\boldsymbol{x}\,\bar{\psi}(\boldsymbol{x})\gamma^\mu t^a\psi(\boldsymbol{x})A_\mu^{a1(2)}(\boldsymbol{x})$

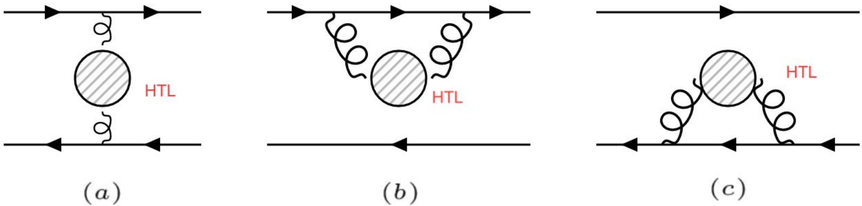
- Single point function vanish $\langle A_\mu^{1(2)}(x)\rangle_A = 0$

- Two point function $\langle A_\mu^i A_\nu^j\rangle_A \quad \sim$



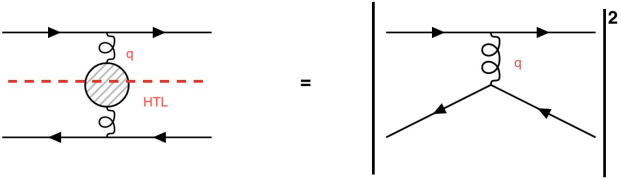
- Expand to second order in perturbation

$$U_I(\Delta t, 0) \approx 1 - i\int_0^{\Delta t} dt H_I^{\mathrm{int}}(t) + (-i)^2\int_0^{\Delta t} dt\int_0^t dt' H_I^{\mathrm{int}}(t)H_I^{\mathrm{int}}(t') + \cdots$$



$$\frac{d}{dt}\rho_{s,s'}(\boldsymbol{p},t) = g^2 C_2(F)(\Gamma_{\rm cross} + \Gamma_{\rm self\ energy})\,, \qquad C_2(F) = \frac{N_c^2 - 1}{2N_c}$$

• *Leading (log) contribution from t channel soft gluon exchange* $gT \ll q \ll T$



• *Similar cutting applies to self-energy diagram.*

• *The log arises from:* $\int_{m_D}^{T} \frac{dq}{q} \sim \log(T/m_D) \sim \log(1/g)$

• $g^2 \log(1/g)$ *contribution cancel, and remaining contribution is proportional to* $g^4 \log(1/g)$

Quantum kinetic equation

•Density Matrix: $\hat{\rho}(\boldsymbol{p}) = \frac{1}{2} f(\boldsymbol{p}) + \boldsymbol{S}(\boldsymbol{p}) \cdot \boldsymbol{\sigma}$

•Distribution function in leading log:

$$\frac{\partial f(\boldsymbol{p}, t)}{\partial t} = C_2(F) \frac{m_D^2 g^2 \log(1/g)}{(4\pi)} \sigma_f$$

$$\sigma_f = \boldsymbol{\nabla}_{p^i} \Big(T(\frac{3}{4} - \frac{E_p^2}{4p^2} + \frac{\eta_p m^4}{4p^3 E_p}) \boldsymbol{\nabla}_{p^i} f(\boldsymbol{p}) + \boldsymbol{p}^i \frac{Tm^2}{4p^3 E_p} (\eta_p + \frac{3E_p}{p} - \frac{3\eta_p E_p^2}{p^2}) \boldsymbol{p} \cdot \boldsymbol{\nabla}_p f(\boldsymbol{p}) + \frac{\boldsymbol{p}^i}{2p^2} (E_p - \frac{\eta_p m^2}{p}) f(\boldsymbol{p}) \Big)$$

m: quark mass $\quad E_p = \sqrt{m^2 + p^2} \quad \eta = \frac{1}{2} ln \frac{E_p + p}{E_p - p}$ *:rapidity.*

•Detailed balance $\sigma_f = 0$ when $f(\boldsymbol{p}) = f^{\mathrm{eq}}(\boldsymbol{p}) = ze^{-E_p/T}$

$$\frac{\partial \boldsymbol{S}(\boldsymbol{p},t)}{\partial t} = C_2(F)\frac{m_D^2 g^2 \log(1/g)}{(4\pi)}\frac{1}{2pE_p}\boldsymbol{\Gamma}_S$$

$$\begin{aligned}
\boldsymbol{\Gamma}_S^i = {} & (2p + \frac{TE_p}{p} - \frac{\eta m^2 T}{p^2})\boldsymbol{S}^i(p) + (pTE_p - \frac{m^2 TE_p}{2p} + \frac{\eta m^4 T}{2p^2})\nabla_p^2 \boldsymbol{S}^i(\boldsymbol{p}) \\
& + [\frac{\eta m^2 T}{2p^2}(1 - \frac{3E_p^2}{p^2}) + \frac{3m^2 TE_p}{2p^3}](\boldsymbol{p}\cdot\boldsymbol{\nabla}_p)^2 \boldsymbol{S}^i(\boldsymbol{p}) \\
& + \frac{1}{p^2}[pE_p^2 - \frac{3m^2 TE_p}{2p} + \eta m^2(-E_p - \frac{T}{2} + \frac{3TE_p^2}{2p^2})](\boldsymbol{p}\cdot\boldsymbol{\nabla}_p)\boldsymbol{S}^i(\boldsymbol{p}) \\
& + 2T[\eta(\frac{1}{2} - \frac{E_p^2}{p^2} + \frac{mE_p}{2p^2} + \frac{E_p^3}{2p^2(E_p+m)}) + (\frac{E_p}{p} - \frac{m}{2p} - \frac{m^2}{2p(E_p+m)})]\boldsymbol{p}^i(\boldsymbol{\nabla}_p\cdot\boldsymbol{S}(\boldsymbol{p})) \\
& - 2T[\eta(\frac{1}{2} - \frac{E_p^2}{p^2} + \frac{mE_p}{2p^2} + \frac{E_p^3}{2p^2(E_p+m)}) + (\frac{E_p}{p} - \frac{m}{2p} - \frac{m^2}{2p(E_p+m)})]\boldsymbol{\nabla}_p^i(\boldsymbol{p}\cdot\boldsymbol{S}(\boldsymbol{p})) \\
& - \frac{T}{p^2}[\frac{E_p(E_p+2m)}{p(E_p+m)} + \frac{\eta m E_p}{E_p+m}(-\frac{3E_p}{p^2} + \frac{1}{E_p+m})]\boldsymbol{p}^i(\boldsymbol{p}\cdot\boldsymbol{S}(\boldsymbol{p}))
\end{aligned}$$

Summary and Outlook

•Go beyond spatial homogeneity limit to include the advective term **x**

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}_p \cdot \frac{\partial}{\partial \boldsymbol{x}}\right) \hat{\rho}(\boldsymbol{x}, \boldsymbol{p}) = \Gamma \cdot \hat{\rho}(\boldsymbol{x}, \boldsymbol{p}), \quad \boldsymbol{v}_p \equiv \frac{\boldsymbol{p}}{E_p}$$

•To include the spin coupling to vorticity $\boldsymbol{\omega}$ and external magnetic field $\boldsymbol{B}$ in both the free streaming and collision terms

$$\Gamma = \Gamma_0 + \Gamma_1(\omega, \mathbf{B}) + O(\omega^2, \mathbf{B^2})$$

•To calculate the transport coefficients in the recent developed spin hydrodynamics K. Hattori et al 2019.

Backup

In massless limit

$$\hat{\rho}(\boldsymbol{p}) = f_+(\boldsymbol{p})\mathcal{P}_+(\boldsymbol{p}) + f_-(\boldsymbol{p})\mathcal{P}_-(\boldsymbol{p}), \quad \mathcal{P}_\pm(\boldsymbol{p}) = \frac{1}{2}(\mathbf{1} \pm \hat{\boldsymbol{p}} \cdot \boldsymbol{\sigma})$$

$$\hat{\rho}(\boldsymbol{p}) = \frac{1}{2}(f_+(\boldsymbol{p}) + f_-(\boldsymbol{p})) + \frac{1}{2}(f_+(\boldsymbol{p}) - f_-(\boldsymbol{p}))\,\hat{\boldsymbol{p}} \cdot \boldsymbol{\sigma}$$

$$f(\boldsymbol{p})_{eq} = f_+(\boldsymbol{p}) + f_-(\boldsymbol{p}) = e^{-p/T}(e^{\mu_+/T} + e^{\mu_-/T}) = C * e^{-p/T}$$

$$\boldsymbol{S}_{eq} = \frac{1}{2}(f_+(\boldsymbol{p}) - f_-(\boldsymbol{p}))\,\hat{\boldsymbol{p}} \equiv \frac{1}{2}e^{-p/T}(e^{\mu_+/T} - e^{\mu_-/T})\hat{\boldsymbol{p}} = C2 * e^{-p/T}\hat{\boldsymbol{p}}$$

•Non trivial check of detailed balance in equilibrium

$$\Gamma_f(\boldsymbol{p}) = \Gamma_{\boldsymbol{S}} = 0$$

Time evolution of density matrix

$$\frac{d}{dt}\rho_{s,s'}(\boldsymbol{p},t) = g^2 C_2(F)(\Gamma_{\text{cross}} + \Gamma_{\text{self energy}})\,, \quad C_2(F) = \frac{N_c^2-1}{2N_c}$$

$$\Gamma_{\text{cross}} = \int \frac{d^3\boldsymbol{p}'}{(2\pi)^3}\frac{1}{4E_pE_{p'}}\sum_{s'',s'''}[\bar{u}(\boldsymbol{p},s)\gamma^\mu u(\boldsymbol{p}',s'')]\rho_{s'',s'''}(\boldsymbol{p}')$$

$$[\bar{u}(\boldsymbol{p}',s''')\gamma^\nu u(\boldsymbol{p},s')]G^{(12)}_{\mu\nu}(E_p-E_{p'},\boldsymbol{p}-\boldsymbol{p}')$$

$$\Gamma_{\text{self energy}} = \int \frac{d^3\boldsymbol{p}'}{(2\pi)^3}\frac{1}{4E_pE_{p'}}\sum_{s''}[\bar{u}(\boldsymbol{p},s)\gamma^\mu u(\boldsymbol{p}',s'')][\bar{u}(\boldsymbol{p}',s'')\gamma^\nu u(\boldsymbol{p},s)]$$

$$\rho_{s,s'}(\boldsymbol{p})G^{(21)}_{\mu\nu}(E_p-E_{p'},\boldsymbol{p}-\boldsymbol{p}')$$

$$G^{(ij)}_{\mu\nu}(q^0,\boldsymbol{q}) = \int d^4x\, e^{i(q^0t-\boldsymbol{q}\cdot\boldsymbol{x})}\langle A^{(i)}_\mu(\boldsymbol{x},t)A^{(j)}_\nu(\boldsymbol{0},0)\rangle_A\,, i,j=1,2\,(\text{SK contour}$$

$$G^{(12)}_{\mu\nu}(q^0,\boldsymbol{q}) = n_B(q^0)\rho_{\mu\nu}(q^0,\boldsymbol{q}) \quad G^{(21)}_{\mu\nu}(q^0,\boldsymbol{q}) = (n_B(q^0)+1)\rho_{\mu\nu}(q^0,\boldsymbol{q})$$