Chirality production, chiral magnetic effect and Schwinger mechanism

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Outline

1. Chirality production, Schwinger mechanism and recent development (Non-perturbative method for dynamical quantities under strong EB fields)

2. Solving Boltzmann equations on GPUs (New numerical framework with high performance on GPU)
1. Chirality production and Schwinger Mechanism

Ref: Patrick Copinger, Kenji Fukushima, SP, PRL 2018; in preparation
Axial Ward identity

- **Axial Ward identity**

\[
\partial_\mu j_5^\mu = 2im\bar{\psi}\gamma^5\psi - \frac{e^2}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}
\]

Chiral current  Pseudo-scalar  \sim E.B, Chiral anomaly

- **Volume integral**

\[
\frac{d}{dt}N_5 = \int d^3x \left(2im\bar{\psi}\gamma^5\psi - \frac{e^2}{2\pi^2}E \cdot B\right)
\]
Pseudo-scalar

\[ \partial_\mu j_5^\mu = 2i m \overline{\psi} \gamma^5 \psi - \frac{e^2}{16\pi^2} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} \]

- **Massless limit:** Pseudo-scalar term -> 0
- **Method:**
  - Perturbative:
    - Weickgenannt, Sheng, Speranza, Q. Wang, 1902.06513;
    - Hattori, Hidaka, Yang, 1903.01653;
    - Wang, Guo, Shi, Zhuang, 1903.03461

Also see Prof. Jianhua Gao’s talk
  - Non-perturbative: World-line formulism
World-line Formulism: IN-OUT Propagator

- **Spinor Feynman propagator at background fields:**

  \[ S_A(x, y) = \underbrace{\Delta(x, y)}_{\text{Path integral: (Homogenous, Constant E,B)}} \]

  \[ = \left( i \mathcal{D}_x + m \right) \Delta(x, y) \]

- **Path integral:**

  \[ \Delta(x, y) = \int_0^\infty ds \exp\left[ -im^2 s + \frac{e^2}{(4\pi)^2} \frac{E B \exp\left[ -\frac{i}{2} e F \sigma s + \frac{i}{2} x e E F y - \frac{i}{4} z \coth(e F s) e F z \right]}{\sinh(e E s) \sin(e B s)} \right] \]

  \[ s: \text{Schwinger proper time} \]

*References*

**M. D. Schwartz, Quantum Field theory and the standard model;**

**Christian Schubert: lecture note on the Worldline Formalism**

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Puzzle: chirality production vanishes?

- **Homogenous Constant E,B at z direction**

- **Using world-line formulism (or original Schwinger’s methods):**

\[
\langle \partial_{\mu} j_5^{\mu} \rangle = -2im \langle \overline{\psi} \gamma^5 \psi \rangle - \frac{e^2}{2\pi^2} \mathbf{E} \cdot \mathbf{B} = 0
\]

\[
\langle \overline{\psi} \gamma^5 \psi \rangle = i \frac{1}{4\pi^2} \frac{EB}{m}
\]

*J. Schwinger, Phys. Rev. 82,5 (1951); M. D. Schwartz, Quantum Field theory and the standard model;*
Puzzle: All vanishing?

\[ < \partial_\mu j^\mu_5 > = -2im < \overline{\psi} \gamma^5 \psi > - \frac{e^2}{2\pi^2} E \cdot B = 0 \]

- **Taking** \( m \rightarrow 0 \) **at the very beginning**: Weyl fermions

\[ \partial_\mu j^\mu_5 = - \frac{e^2}{2\pi^2} E \cdot B \]

- **After all the calculations, taking** \( m \rightarrow 0 \).

\[ < \partial_\mu j^\mu_5 > = -2im < \overline{\psi} \gamma^5 \psi > - \frac{e^2}{2\pi^2} E \cdot B = 0 \]

- **We can take the smooth massless limits** \((m \rightarrow 0)\) **in world-line formulism.**
IN and OUT States

- **Homogenous Constant \( E_z, B_z \) field:** (Schwinger Fock gauge)
  \[
  A^z(t) = eE_z t, \quad H = H(A(t)),
  \]

- **Vacuum at In states is different with it at OUT states:**

  \[ |\text{IN}\rangle \rightarrow |\text{OUT}\rangle \]

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Unstable vacuum

- $|0, \text{IN}\rangle$ is NOT equal to $|0, \text{OUT}\rangle$

\[ |<0, \text{out} | 0, \text{in} > |^2 \neq 1 \]

- **Schwinger Pair Production Rate:**

\[ P_0 = 1 - |<0, \text{out} | 0, \text{in} > |^2 = \frac{e^2 E_z B_z}{4\pi^2} \coth \left( \frac{B_z}{E_z} \pi \right) \exp \left( -\frac{m^2 \pi}{|eE_z|} \right) \]

(n=1 world-line instanton)
Expectation Value: IN-IN states

• **Transition amplitude:** IN-OUT

\[ \langle 0, \text{out} | \partial_\mu j_5^\mu | 0, \text{in} \rangle \]

• **Expectation value:** IN-IN

\[ \langle 0, \text{in} | \partial_\mu j_5^\mu | 0, \text{in} \rangle \]

*Review: F. Gelis, N. Tanji 2015; N. Tanji 2009*

To compute a quantity at IN-IN states, we can use all the expression in ordinary world-line formulism but with new integral paths (red one in figures).

**IN-OUT Propagator:** Path in Blue

\[
S_A(x, y) = \left( i \mathcal{D}_x + m \right) \Delta(x, y)
\]

\[
\Delta(x, y) = \left[ \theta(x_3 - y_3) \int_{\Gamma_c} + \theta(y_3 - x_2) \int_{\Gamma_c} \right] ds \times e^{-im^2s} g(x, y, s),
\]

**IN-IN Propagator:** Path in Red

\[
S_{in}(x, y) = \left( i \mathcal{D}_x + m \right) \Delta_{in}(x, y)
\]

\[
\Delta_{in}(x, y) = \left[ \theta(x_3 - y_3) \int_{\Gamma_>} + \theta(y_3 - x_2) \int_{\Gamma_<} \right] ds \times e^{-im^2s} g(x, y, s),
\]


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Chirality Production

- **We obtained the chirality production rate:**

  \[ \partial_\mu j_5^\mu = \frac{e^2 E B}{2\pi^2} \exp \left( -\frac{\pi m^2}{eE} \right) \]

  smooth massless limit: \( m \to 0 \), Chiral anomaly

- **Consistent with physical picture**

  \[ \frac{1}{2} \partial_t n_5 = \text{Schwinger Pair Production rate} \]

  \[ K. \text{ Fukushima, D.Kharzeev, H. Warringa PRL 2010} \]
Mass correction to CME

• **Assuming** \( E, B \) at \( z \) direction, \( \text{we obtain the current} \)

\[
\dot{j}_3 = \frac{e^2 E B}{2\pi^2} \coth \left( \frac{B}{E} \pi \right) \exp \left( -\frac{\pi m^2}{eE} \right) t
\]

• **Non-perturbative:** \( \sim 1/(eE) \)

• **Sum over all Landau levels:** \( \text{Coth}(B/E \ \pi) \)
**Connection to closed-time-path formalism**

- **The world-line IN-IN formalism is equivalent to a Schwinger Keldysh formalism.**

![Diagram of IN-IN path in Schwinger proper time]

- **Key:**
  - Integral path: \([-\infty, \infty] \oplus [\infty + i\epsilon, \infty - i\epsilon] \oplus [\infty, -\infty]\)
  - Compute the propagator \(S_{++}\)

\[
S_{in}^{c}(x, y) \approx \frac{1}{2} \int d^{3}x' \int dx'_{0} \left[ \begin{array}{c}
\text{Tr} S^{c}(x, x') \lim_{\epsilon \to \infty} (i\hat{\mathcal{P}}_{x''} - m)S^{c}(x'', y) \\
\times [\theta(z_{3}) - \theta(-z_{3})] \int_{h_{1}} ds \text{Im} g(x, y, s)
\end{array} \right]
\approx \frac{1}{2} (i\hat{\mathcal{P}}_{x} + m) \\
\times \left[ \theta(z_{3}) \int_{\Gamma^{+}} + \theta(-z_{3}) \int_{\Gamma^{-}} \right] g(x, y, s)
\]

- **Using Wick theorem, we make connection between IN-IN and IN-OUT propagators**
- **After a long and complicated calculation, we find the non-trivial small piece in the close-time-path formalism will give essential parts for the path in IN-IN propagator.**
Recent development

Patrick Copinger, Kenji Fukushima, SP, in preparation

- **We have a better and deeper understanding of IN-IN formalism:**
  - We prove that the world-line IN-IN formalism is equivalent to a Schwinger Keldysh formalism.
  - We also obtain the IN-IN formulism through the canonical operator methods.

- **We will show all the techniques to compute quantities under the strong electromagnetic fields.**

- **We will also compute some other interesting physical quantities.**
Summary for world-line formalism

• We introduce a non-perturbative method to compute dynamical quantities in strong EB fields.

  ➢ Axial Ward identity, correct mass correction!
  ➢ Mass correction to CME
  ➢ Dynamical chiral condensate

• We have shown that our IN-IN formalism is equivalent with closed-time-path formulism.
2. Toward a full solution of complete Relativistic Boltzmann equation on GPUs

Ref: Jun-jie Zhang, Hong-zhong Wu, SP, Guang-you Qin, Qun Wang, in preparation

Also see Jun-jie Zhang’s talk in Parallel Session - Chirality II (Ball Room 3) at 12:20 on Nov.6th(Wed)
Relativistic Boltzmann Equations (BE)

- **Boltzmann equations: kinetic theory**
  - microscopic theory for many body-systems

\[
\frac{\partial}{\partial t} f_p + \frac{\partial x}{\partial t} \cdot \nabla_x f_p + \frac{\partial p}{\partial t} \cdot \nabla_p f_p = C[f_p]
\]

- **A semi-classical description of many body theory.**

One can derive it symmetrically from quantum field theory with closed-time-path formulism.

Main difficulties for solving complete relativistic BE

- **6+1 dimensional phase space**
  - 3 coordinates, 3 momentum, 1 time
- **Extremely complicated collisional term**
  - 5 dimensional integrals including many terms
- **Time cost:** “infinite” (years)

Assuming we only have 10 grids in all space and momentum directions, so totally, we have $10^6$ grids. We choose the time step could be 1000. Totally, we need to compute $10^9$ times high dimensional integrals. Assuming that it costs 1 sec to compute one 5 dim integral, then totally it will cost 30 years!

- **Particle number non-conservation:**
  - comes from errors of collisional integrals.
- **A usual way to deal with collision term:**
  - Test particle method;
- **Could we solve the complete BE directly?**
Numerical framework for solving BE on GPU

- **Systems which we simulate:**
  - **Collison:** 2->2 quarks and gluons
  - **Dynamic Debye mass:** Hard Thermal Loop
    (Do not need to worry about soft region)

- **Feature of the framework:**
  - A full solution of complete relativistic BE
  - High performance
  - Particle number is strictly conserved
Collisional term via ZMCintegral

• **5 dimensional integral on each phase space grid:**

Wu, Zhang, Pang, Q. Wang, 2019, accepted by Computer Physics Communications

Test of 9 dim Gaussian integrals

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ZMCintegral 4.2

High performance

[https://github.com/Letianwu/ZMCintegral](https://github.com/Letianwu/ZMCintegral)

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Chirality production and Schwinger mechanism
Symmetrical sampling method on GPUs

- **Symmetrical sampling:**
  - restore the time reversal symmetry
- **We ensure particle number conservation:**

![Graph showing gluon numbers vs. independent evaluations with and without balancing procedure](image)
Time evolution

Gluons + quarks

Phase space box is of size \([-3\text{fm}, 3\text{fm}]^3 \times [-2\text{GeV}, 2\text{GeV}]^3\).

- **Grids:** space: 1 grid; momentum: 30x30x30=27,000
- **Phase space size:** \([-3\text{fm}, 3\text{fm}]^3 \times [-2\text{GeV},2\text{GeV}]^3\)
- **Time step:** \(dt=0.0005\text{fm}\); 100,000 steps
- **Time cost:** around 50 hours on a Nvidia Tesla V100 card

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Chirality production and Schwinger mechanism
Summary for solving BE on GPUs

• We introduce a new **numerical framework** to derive **full solutions** of a **complete relativistic BE** on **GPUs**.

  - **Full collisional term:**
    high dimensional integrals.
  - **High performance:**
    space 10x10x10,  
momentum 30x30x30,  
Time steps: $10^4$-$10^6$,  
on one Nvidia Tesla V100 card costs a few days!
  - **Particle number is strictly conserved.**
  - **Dynamical Debye mass**
Analytic solution for Anomalous magneto-hydrodynamics

- **Siddique, R.-j. Wang, Pu, Q. Wang, PRD 2019**

- **Anomalous MHD:**
  Hydrodynamic eq. + Maxwell’s eq. + Chiral currents

- **We obtained the analytic solutions of anomalous magneto-hydrodynamics in Bjorken flow with transverse EB fields**

\[
n_5(\tau) = n_{5,0} \left( \frac{\tau_0}{\tau} \right) \left\{ 1 + a_2 e^{\sigma \tau_0} [E_1(\sigma \tau_0) - E_1(\sigma \tau)] \right\},
\]

\[
\epsilon(\tau) = \epsilon_0 \left( \frac{\tau_0}{\tau} \right)^{1+\delta_2} \left\{ 1 + \sigma \frac{E_0^2}{\epsilon_0} e^{2\sigma \tau_0} [\tau_0 E_1(2\sigma \tau_0) - \tau \left( \frac{\tau_0}{\tau} \right)^{1/2} E_1(2\sigma \tau')] + \frac{a_3}{\tau_0} e^{\sigma \tau_0} [\tau_0 E_2(2\sigma \tau_0) - \tau \left( \frac{\tau_0}{\tau} \right)^{3/2} E_2(2\sigma \tau)] \right\}.
\]

- **Time evolution of EB fields**
  - In lab frame, B field decays much slower than in the vacuum
  - By decays like \( \sim 1/\tau \)
  - \( Bx \) decays like \( \sim \exp(-\sigma \tau)/\tau \)

- **Time evolution of chirality and energy density, EB fields with quantum corrections from CME, chiral anomaly.**
1. We introduce a non-perturbative method to compute the dynamical quantities in strong electromagnetic fields.

2. We introduce a new numerical framework to solve the complete Boltzmann equation on GPUs.
Thank you for your time!

Hope to see you at HeFe2019! (USTC, Nov. 10-Nov.11, 2019)

A Workshop on Heavy Flavor and Dilepton Production in Relativistic Heavy-Ion Collisions (HeFe2019)

10-11 November 2019
University of Science and Technology of China (USTC), Hefei, Anhui, China
Asia/Shanghai timezone

This is a Quark Matter 2019 Satellite meeting

Starts 10 Nov 2019 08:00
Ends 11 Nov 2019 18:00
Asia/Shanghai

University of Science and Technology of China (USTC), Hefei, Anhui, China
No. 96 Jinzhai Road, Hefei, Anhui Province, China

Prof. Tang, Zebo
Prof. Pu, Shi

Registration Deadline: Oct. 18, 2019

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Chiral Magnetic Effect

- Magnetic fields
- Nonzero axial chemical potential
- Number of Left handed fermions ≠ Number of Right handed fermions

Charge current: charge separation

\[ j = \frac{e^2}{2\pi^2} \mu_5 B, \]

Kharzeev, Fukushima, Warrigna, (08,09), etc. ...
Experiments: signal VS background

Waiting for the results from Isobar

\[ \langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \rangle \]

\[ \times 10^{-3} \]

\% Most central

Red: same charge
Blue: opp charge

\[ \text{STAR PRL 103, 251601(2009); PRC 81, 054908} \]

\[ \text{CMS PRL 118, 122301 (2016); PRC 97, 044912} \]

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Chirality production and Schwinger mechanism
Energy conservation

- Not conserved in HTL (physically)
- Not conserved (from errors): because of discrete grid

But, if we increases the number of grid, the variation of energy is tiny.
Symmetric Sampling

Time reversal symmetry is restored!

\[ C_g(x, p) \]

\[ c_p^g (\text{sample 1}) \]
\[ c_p^g (\text{sample 2}) \]
\[ \vdots \]
\[ c_p^g (\text{sample } s_i) \] \[ g(k_1) + g(k_2) \rightarrow g(k_3) + g(p) \]
\[ \vdots \]
\[ c_p^g (\text{sample } N - 1) \]
\[ c_p^g (\text{sample } N) \]

\[ C_g(x, k_1) \]
\[ \vdots \]
\[ c_k^g (\text{sample } 1) \]
\[ c_k^g = -c_p^g \]
\[ g(k_3) + g(p) \rightarrow g(k_2) + g(k_1) \]
\[ \vdots \]
\[ c_k^g (\text{sample } N) \]

\[ C_g(x, k_2) \]
\[ \vdots \]
\[ c_k^g (\text{sample } 1) \]
\[ c_k^g = -c_p^g \]
\[ g(k_3) + g(p) \rightarrow g(k_1) + g(k_2) \]
\[ \vdots \]
\[ c_k^g (\text{sample } N) \]

\[ C_g(x, k_3) \]
\[ \vdots \]
\[ c_k^g (\text{sample } 1) \]
\[ c_k^g = c_p^g \]
\[ g(k_1) + g(k_2) \rightarrow g(p) + g(k_3) \]
\[ \vdots \]
\[ c_k^g (\text{sample } N) \]
space: 1 grid; momentum: 30x30x30 = 27,000
Phase space box is of size $[-3\text{fm}, 3\text{fm}]^3 \times [-2\text{GeV}, 2\text{GeV}]^3$.
Time step $dt=0.001\text{fm}$; 20,000 steps
on one Nvidia Tesla V100 card: costs around 2 hours
Parameters

• **Physical parameters:**
  ➢ coupling constant; initial conditions.

• **Parameters for simulations:**
  ➢ Size, number of grids.
Collisional term

\[
C_{ab\rightarrow cd} \equiv \int \prod_{i=1}^{3} \frac{d^3 k_i}{(2\pi)^3 2E_{k_i}} \left| \frac{M_{ab\rightarrow cd}}{2E_p} \right|^2 (2\pi)^4 \delta^{(4)}(k_1 + k_2 - k_3 - p) \left[ f_{k_1}^a f_{k_2}^b F_{k_3}^c F_{k_4}^d - F_{k_1}^a f_{k_2}^b f_{k_3}^c f_{k_4}^d \right],
\]

For 2-\(\rightarrow\)2 scatterings including quarks and gluons: e.g.