Spin Polarization in high energy heavy-ion collisions

Qun Wang

Department of Modern Physics
Univ of Science & Technology of China (USTC)

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Outline

- **A clue of solution to the sign problem in longitudinal polarization and possible implication [Wu, Huang, Pang, QW, 1906.09385]**
- **What can we learn from the global spin alignment of the phi meson? A new insight. [Sheng, Oliva, QW, 1910.13684]**

Global OAM and Magnetic field in HIC

- **Huge global orbital angular momenta are produced**

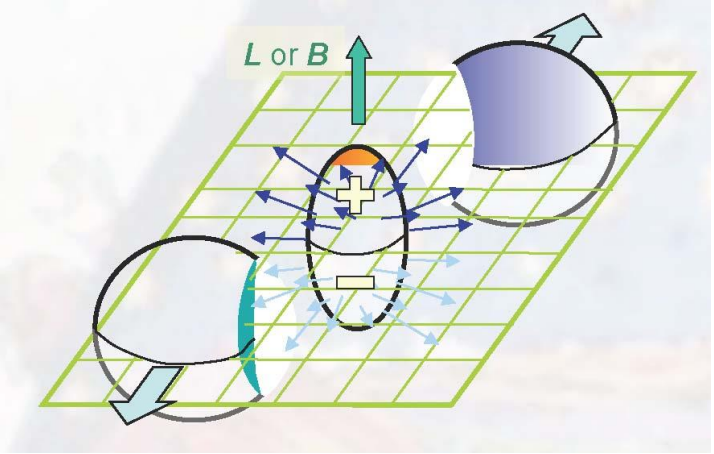
$$\mathbf{L} \sim 10^5 \hbar$$

- **Very strong magnetic fields are produced**

$$\mathbf{B} \sim m_\pi^2 \sim 10^{18}\ \mathrm{Gauss}$$

- **How are orbital angular momenta transferred to the matter created?**

- **How is spin coupled to local vorticity in a fluid?**



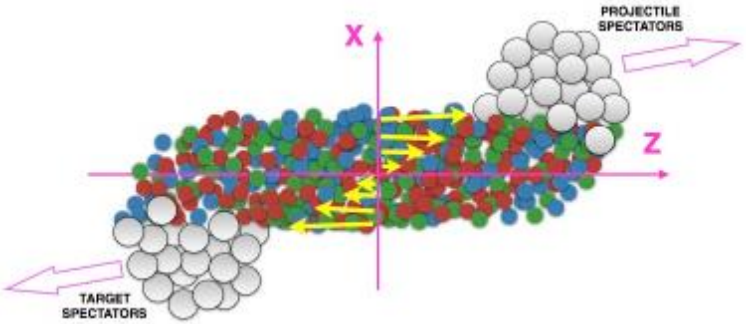


Figure taken from Becattini et al, 1610.02506

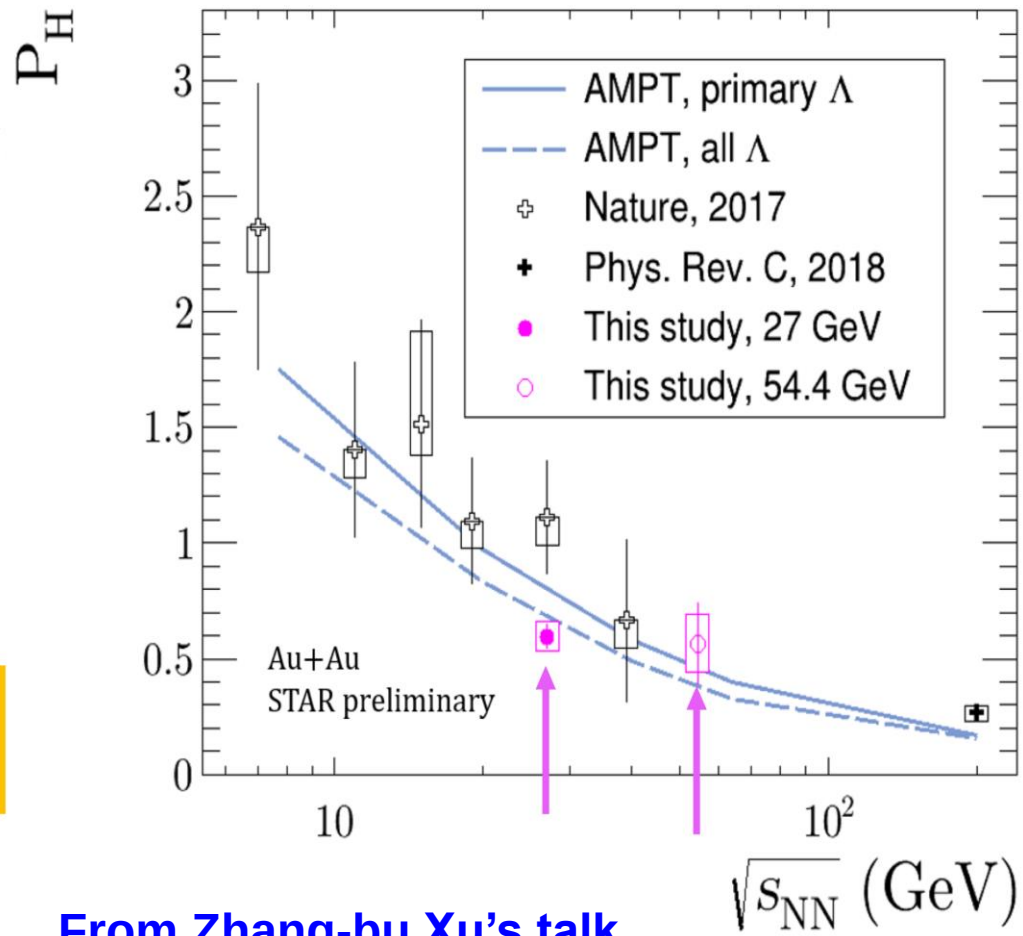
$\sqrt{s_{NN}}$ dependence of global polarization

Joey Adams, Wed 14:00 BR1

- Previous study across broad range of $\sqrt{s_{NN}}$ suggests strong beam energy dependence

- New data at 27 & 54.4 GeV with high statistics for $\Lambda/\bar{\Lambda}$, centrality, rapidity and p_T dependence

New datasets with high statistics follow previous measured trend



[2] The STAR Collaboration, *Global Lambda hyperon polarization in nuclear collisions: evidence for the most vortical fluid.* Nature **548** (2017) 62

From Zhang-bu Xu's talk

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Theoretical models and proposals: early works on global polarization in HIC

With such correlation between rotation and polarization in materials, we expect the same phenomena in heavy ion collisions. Some early works along this line:

- **Polarizations of Λ hyperons and vector mesons through spin-orbital coupling in HIC from global OAM**
- **-- Liang and Wang, PRL 94,102301(2005), PRL 96, 039901(E) (2006) [nucl-th/0410079]**
- **-- Liang and Wang, PLB 629, 20(2005) [nucl-th/0411101]**

- **Polarized secondary particles in un-polarized high energy hadron-hadron collisions**
- **-- Voloshin, nucl-th/0410089**

- **Polarization as probe to vorticity in HIC**
- **-- Betz, Gyulassy, Torrieri, PRC 76, 044901(2007) [0708.0035]**

- **Statistical model for relativistic spinning particles**
- **-- Becattini, Piccinini, Annals Phys. 323, 2452 (2008) [0710.5694]**

Theoretical models for global polarization

- **Spin-orbit coupling or microscopic models**
- **[Liang and Wang, PRL 94,102301(2005); Gao, Chen, Deng, Liang, QW, Wang, PRC 77, 044902(2008); Zhang, Fang, QW, Wang, arXiv:1904.09152.]**

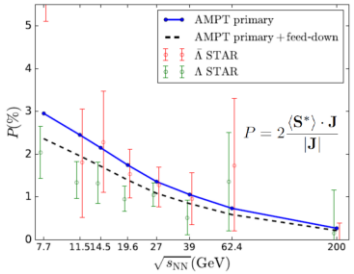
- **Statistical-hydro models**
- **[Zubarev (1979); Weert (1982); Becattini et al. (2012-2015); Hayat, et al. (2015); Floerchinger (2016).]**

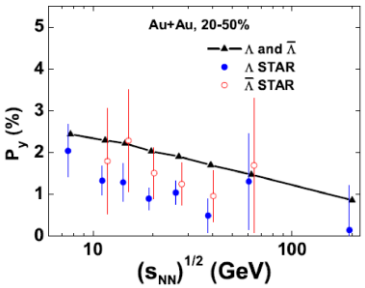
- **Spin hydrodynamic model**
- **[Florkowski,Friman,Jaiswal,Ryblewski,Speranza (2017-2018); Montenegro,Tinti,Torrieri (2017-2019); Hattori,Hongo,Huang,Matsuo,Taya (2019)]**

- **Kinetic theory for massive fermions with Wigner functions**
- **[Early works: Heinz (1983); Vasak, Gyulassy and Elze (1987); Elze, Gyulassy, Vasak (1986); Zhuang, Heinz (1996).]**
- **[Recent developments: Fang, Pang, QW, Wang (2016); Weickgenannt, Sheng, Speranza, QW, Rischke (2019); Gao, Liang (2019); Wang, Guo, Shi, Zhuang (2019); Hattori, Hidaka, Yang (2019).]**

Theoretical results for global polarization in y direction (OAM)

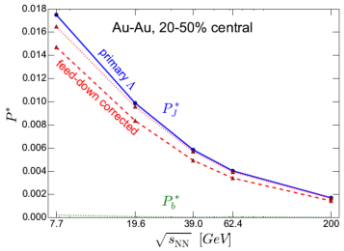
- **AMPT transport model**
- **-- Li, Pang, QW, Xia, PRC96,054908(2017)**

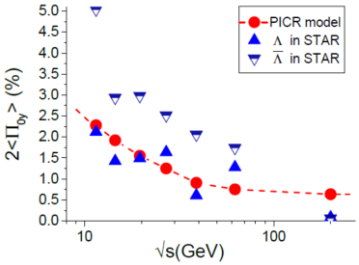




- **UrQMD + vHLLE hydro**
- **-- Karpenko, Becattini, EPJC 77, 213(2017)**

- **PICR hydro**
- **-- Xie, Wang, Csernai, PRC 95,031901(2017)**





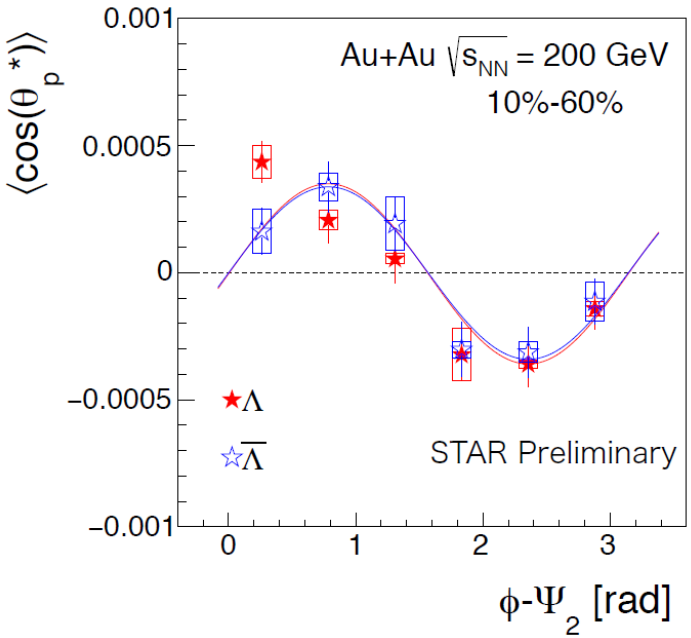
- **Chiral Kinetic Approach + Collisions**
- **-- Sun, Ko, PRC96, 024906(2017)**

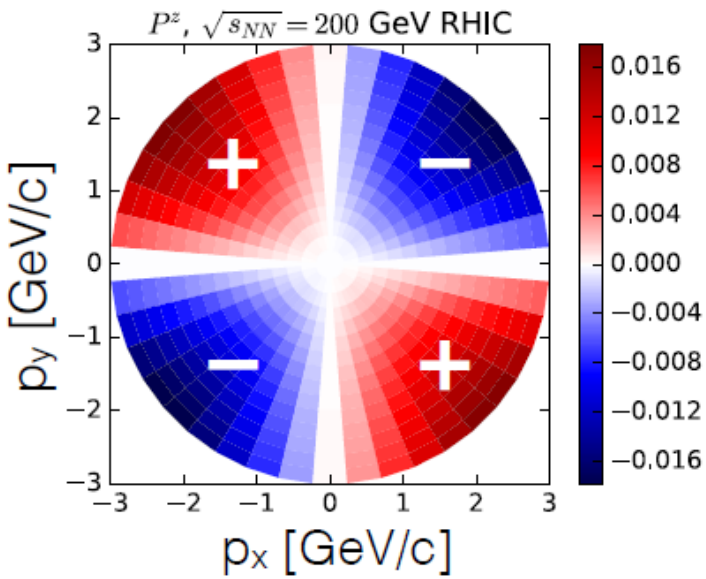
- **Many other works ……**

- **Take-home message I: All models can describe data: the global polarization is a robust and significant effect!**

Sign problem in polarization along the beam direction

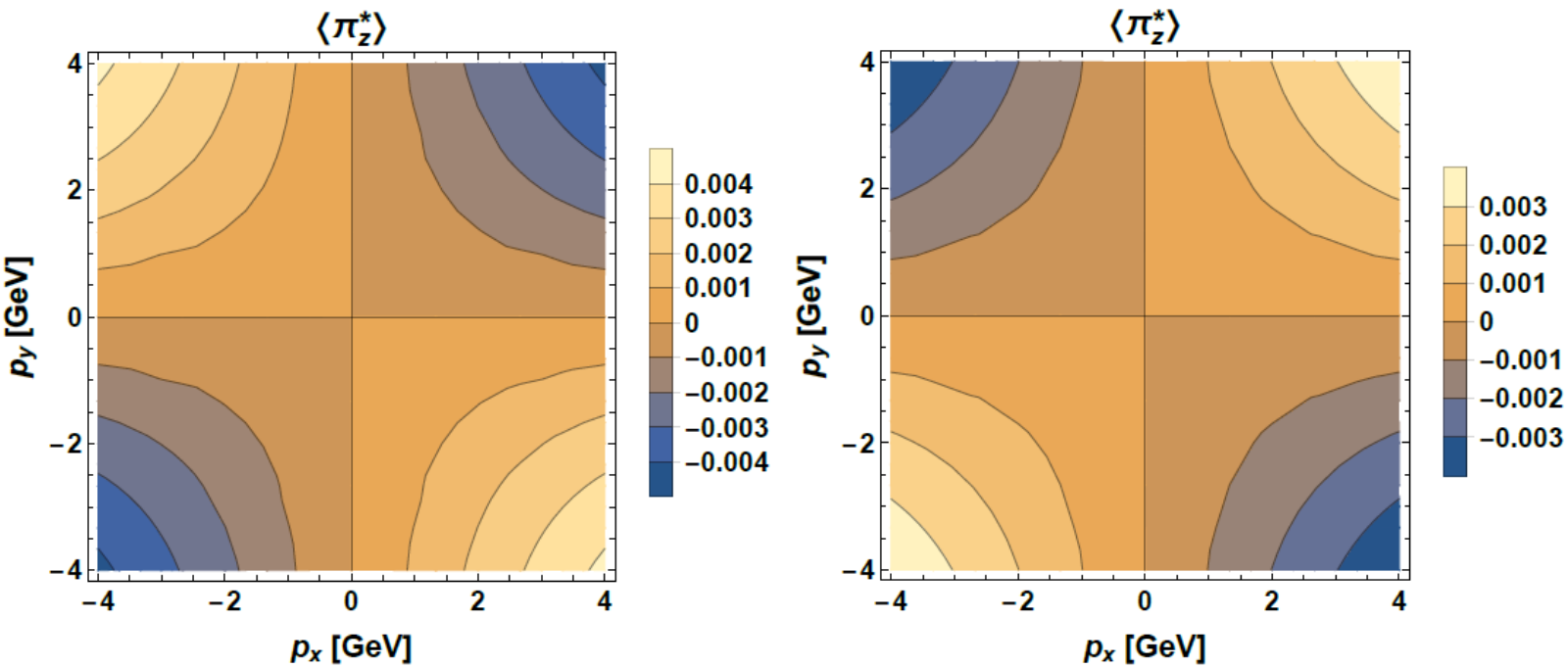
Polarization along the beam direction





- **Sin(2ϕ) structure as expected from the elliptic flow**
- **Opposite sign to the hydrodynamic model and transport model (AMPT)** **[Hydro model: Becattini, Karpenko (2018); Transport model (AMPT): Xie, Li, Tang, Wang (2018)]. Not from resonance decays [Xia,Li,Huang,Huang (2019); Becattini,Cao,Speranza (2019)]**
- **Same sign in chiral kinetic approach [Sun, Ko (2019)]**
- **Same sign in a simple phenomenological model [Voloshin (2017/2018)]**

Polarization along the beam direction



- **Left: the spin polarization is defined by the thermal vorticity. Right: projected spatial thermal vorticity in the Lab frame.**
- **[Florkowski, Kumar, Ryblewski, Mazeliauskas (1904.00002)]**

$$\omega_{\mu\nu} = \varpi_{\alpha\beta}\bar{\Delta}^{\alpha}_{\ \mu}\bar{\Delta}^{\beta}_{\ \nu}$$

$$\bar{\Delta}^{\mu\nu} = g^{\mu\nu} - u^{\mu}_{\rm LAB}u^{\nu}_{\rm LAB}$$

$$u^{\mu}_{\rm LAB} = (1,0,0,0)$$

Spin chemical potential

- **Normal hydrodynamics**
- **-- Energy and momentum conservation: T and u^μ**
- **-- Baryon number conservation: μ_B**

- **Including spin into hydrodynamics**
  -- Angular momentum conservation: $\omega^{\mu\nu}$ (spin chemical potential)
  [Becattini, Florkowski, Speranza (2018); Florkowski, Ryblewski, Kumar (2018)]

- **Ambiguity of localization of energy and spin densities**
  -- pseudo-gauge transformations: $T^{\mu\nu}$ and $S^{\lambda,\mu\nu}$ $\Longrightarrow$ $T'^{\mu\nu}$ and $S'^{\lambda,\mu\nu}$
  -- Belinfante construction: $T^{\mu\nu}_{\rm Bel} = T^{\nu\mu}_{\rm Bel} \qquad S^{\lambda,\mu\nu}_{\rm Bel} = 0$

- **In general, the information about spin cannot be encoded in the form of OAM, since OAM can be eliminated by Lorentz transformation**

- **In general $\omega^{\mu\nu} \neq -\frac{1}{2}(\partial^\mu\beta^\nu - \partial^\nu\beta^\mu)$ (spin chemical potential is not thermal vorticity) unless in global equilibrium**

Different relativistic vorticities

There are different relativistic vorticities: definition is not unique:

- **Kinematic** $\omega^{(K)}_{\mu\nu} = -\frac{1}{2}(\partial_\mu u_\nu - \partial_\nu u_\mu) = \varepsilon_\nu u_\mu - \varepsilon_\mu u_\nu + \boxed{\epsilon_{\nu\mu\rho\eta} u^\rho \omega^\eta}$

- **Non-Relativistic** $\omega^{(\mathrm{NR})}_{\mu\nu} = \epsilon_{\nu\mu\rho\eta} u^\rho \omega^\eta$

- **T-vorticity**
$$\begin{aligned}\omega^{(T)}_{\mu\nu} &= -\frac{1}{2}[\partial_\mu(T u_\nu) - \partial_\nu(T u_\mu)] \\ &= T\omega^{(K)}_{\mu\nu} + \frac{1}{2}(u_\mu \partial_\nu T - u_\nu \partial_\mu T) \\ &\equiv T\omega^{(K)}_{\mu\nu} + \omega^{(T)}_{\mu\nu}(T),\end{aligned}$$

- **Thermal**
$$\begin{aligned}\omega^{(\mathrm{th})}_{\mu\nu} &= -\frac{1}{2}[\partial_\mu(\beta u_\nu) - \partial_\nu(\beta u_\mu)] \\ &= \frac{1}{T}\omega^{(K)}_{\mu\nu} - \frac{1}{2T^2}(u_\mu \partial_\nu T - u_\nu \partial_\mu T) \\ &= \frac{1}{T}\omega^{(K)}_{\mu\nu} + \omega^{(\mathrm{th})}_{\mu\nu}(T),\end{aligned}$$

Becattini, Inghirami, Rolando, et al., Eur. Phys. J. C75,406 (2015) [1501.04468]

Wu, Pang, Huang, QW, PRR 1, 033058(2019) [1906.09385]

A test of different vorticities in (3+1)D hydro

- **Polarization at freezeout**

Becattini, et al., Ann.Phys. 338,32(2013)

$$S^\mu(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int d\Sigma_\lambda p^\lambda \Omega_{\rho\sigma} f_{FD}(1-f_{FD})}{\int d\Sigma_\lambda p^\lambda f_{FD}}$$

- **where we choose different vorticities**

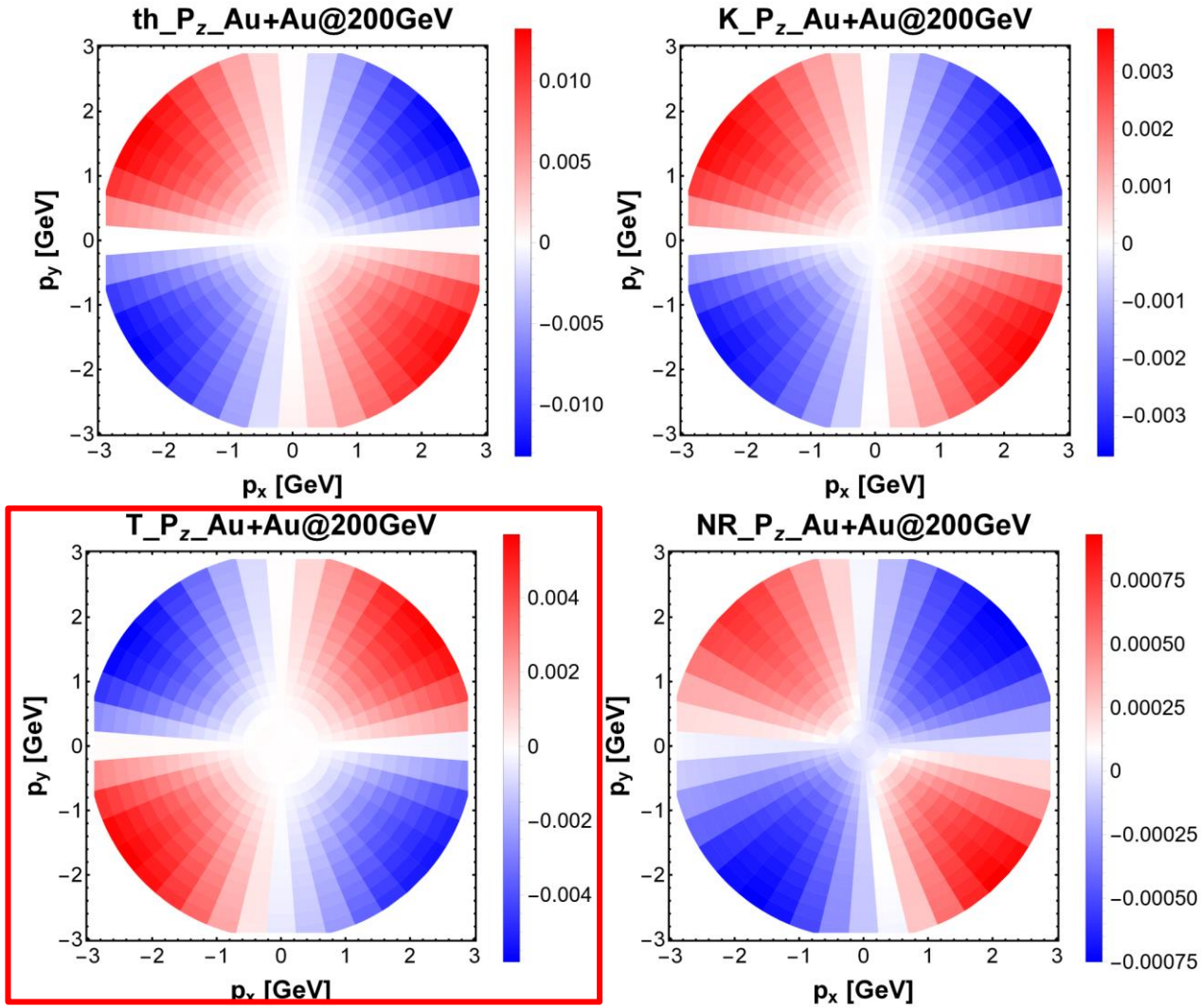
$$\Omega_{\rho\sigma} = \frac{1}{T}\omega^{(K)}_{\rho\sigma}, \frac{1}{T^2}\omega^{(T)}_{\rho\sigma}, \omega^{(\mathrm{th})}_{\rho\sigma}, \frac{1}{T}\omega^{(\mathrm{NR})}_{\rho\sigma}$$

Wu, Pang, Huang, QW, PRR 1, 033058(2019) [1906.09385]

- **(3+1)D Hydro model CLVisc: with AMPT initial condition (OAM encoded)**

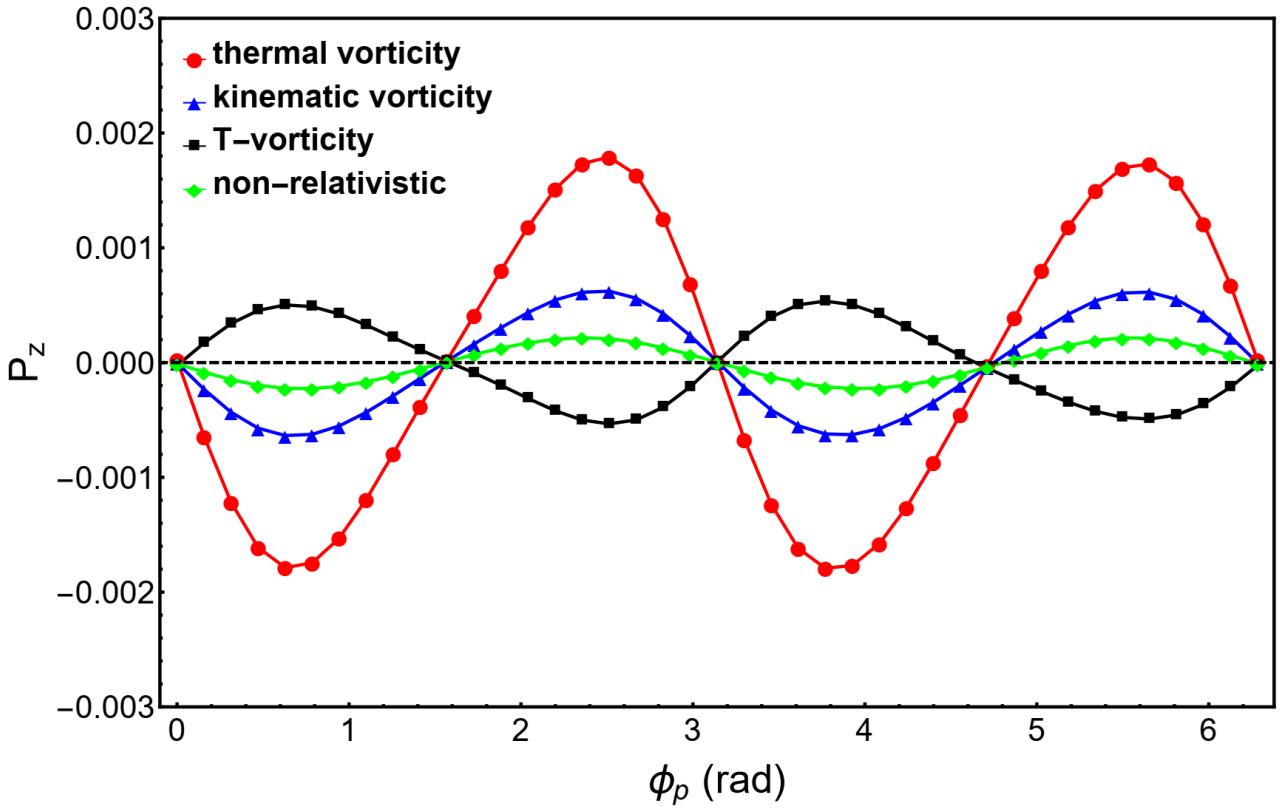
[Pang, QW, Wang (2012); Pang, Petersen, Wang (2018)]

Longitudinal polarization



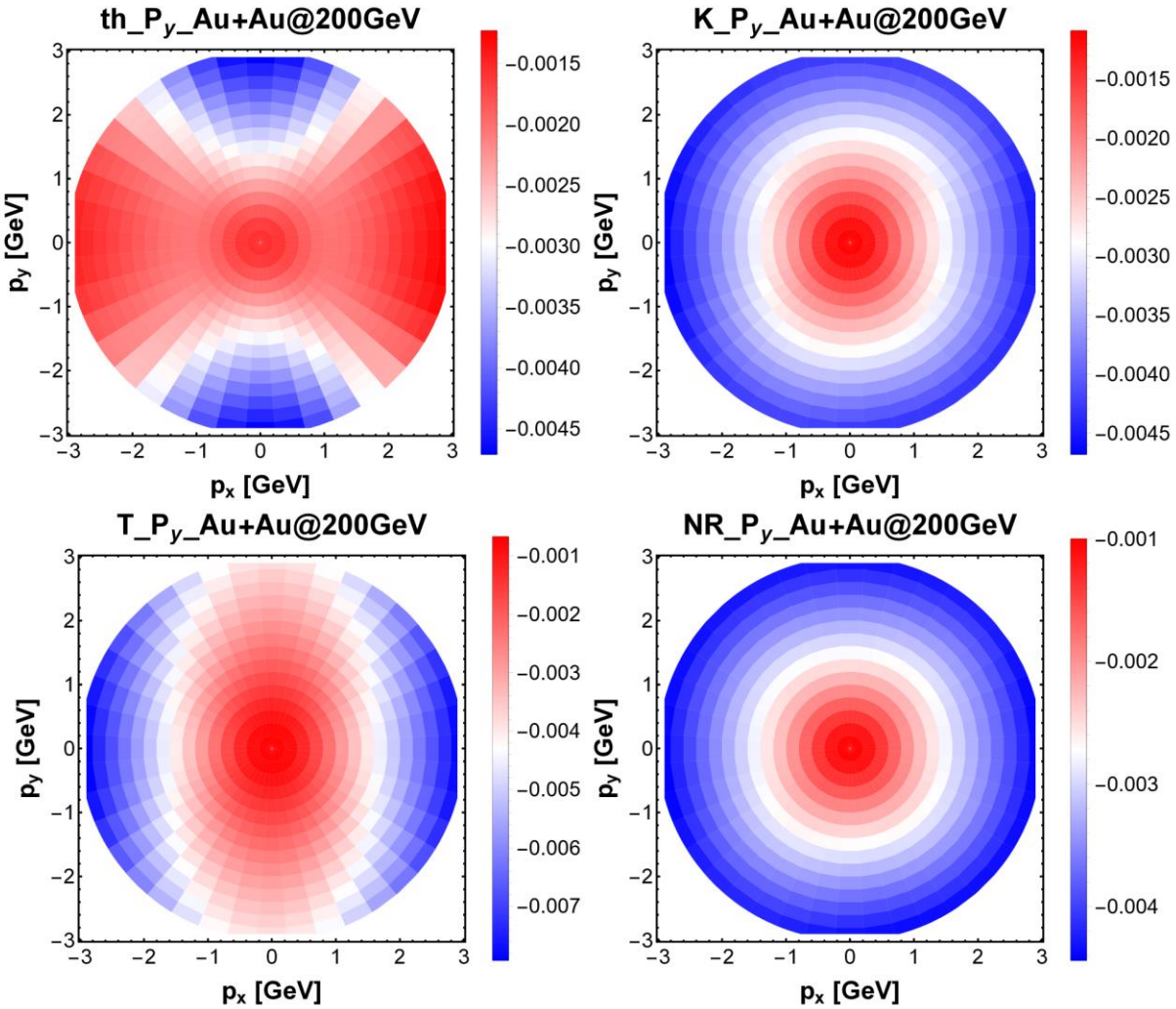
The longitudinal polarization in Au+Au collisions at 200 GeV in the rapidity range Y=[-1; 1] with the AMPT initial condition as functions of (p_x; p_y).

Longitudinal polarization



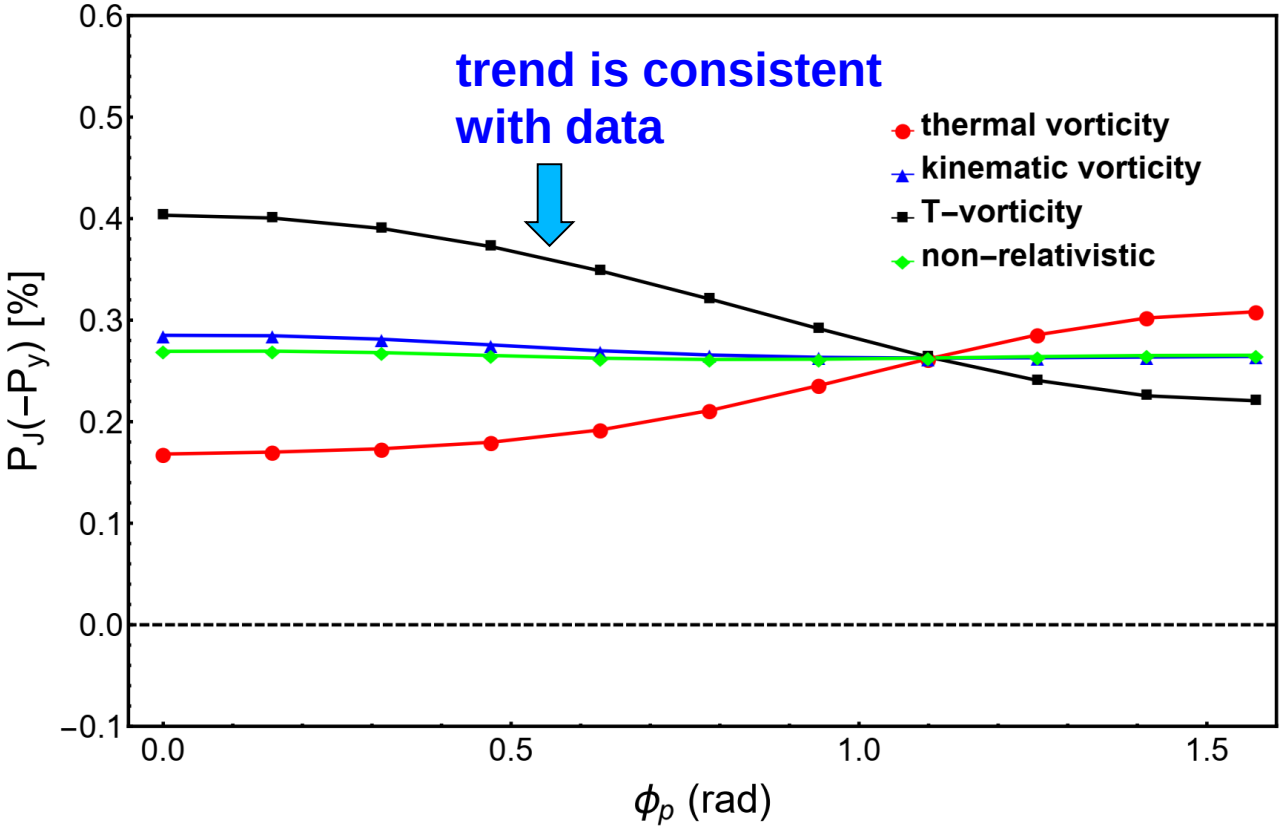
The longitudinal polarization in Au+Au collisions at 200 GeV in the rapidity range Y=[-1; 1] with the AMPT initial condition as functions of (p_x; p_y).

Polarization in direction of OAM



The polarization along –y direction in Au+Au collisions at 200 GeV in the rapidity range Y=[-1; 1] with the AMPT initial condition as functions of (p_x; p_y).

Polarization in direction of OAM



The polarization along –y direction in Au+Au collisions at 200 GeV in the rapidity range Y=[-1; 1] with the AMPT initial condition as functions of (p_x; p_y).

All four vorticities give the same magnitude, but only T-vorticity gives the right trend

Discussions and Messages

Take-home message II:

1. Same message as I: P_y is a significant and robust effect no matter what vortcity you choose -- all four types of vorticity give correct sign and magnitude

2. About P_z: T-vorticity seems to work!

Indication: T-vorticity might be coupled with the spin similar to the way that a magnetic moment is coupled to a magnetic field.
Or
It may be a coincidence from the main assumption that the spin vector is given by the T-vorticity in the same way as the thermal vorticity

A quark coalescence model for polarized vector mesons and baryons

1. **A non-relativistic quark coalescence model is formulated for polarized vector mesons and baryons of spin-1/2 octet and spin-3/2 decuplet.**
2. **With the spin density matrix, one can compute in a uniform way the polarizations of vector mesons and baryons from those of quarks and antiquarks with explicit momentum dependence.**

Yang, Fang, QW, Wang, 1711.06008

A new insight about the global spin alignment of the phi meson

Sheng, Oliva, QW, 1910.13684

A puzzle for spin alignment of ϕ meson

- **The STAR preliminary data imply that $\rho_{00}^{\phi} > 1/3$ and significantly deviate from 1/3**
- **In the coalescence model we have** $P_\Lambda \approx P_s \qquad P_{\overline{\Lambda}} \approx P_{\overline{s}}$

$$\rho_{00}^{\phi} \approx \frac{1}{3} - \frac{4}{9} P_s P_{\overline{s}} \approx \frac{1}{3} - \frac{4}{9} P_\Lambda P_{\overline{\Lambda}} \lesssim \frac{1}{3}$$

- **It seems that there is a discrepancy**
- **The puzzle can be resolved in our coalescence model and can shed light on a new aspect of dense matter!**

Quark spin polarization in vorticity and EM field

- **The spin polarization vector for massive fermions (upper sign) and anti-fermions (lower sign) in the vorticity and electromagnetic field is**

$$P^\mu_\pm(x,p) \approx \frac{1}{2m}\left(\tilde{\omega}^{\mu\nu}_{\rm th} \pm \frac{1}{E_p T} Q\tilde{F}^{\mu\nu}\right) p_\nu$$

- **where** $p^\mu = (E_p, \pm\mathbf{p})$

- **Applying to s and $\bar{s}$ quark, we obtain the polarization along the y-direction**

$$P^y_{s/\bar{s}}(t,\mathbf{x},\mathbf{p}_{s/\bar{s}}) = \frac{1}{2}\omega^y_{\rm th} \pm \frac{1}{2m_s}\hat{\mathbf{y}}\cdot(\boldsymbol{\varepsilon}\times\mathbf{p}_{s/\bar{s}}) \pm \frac{Q_s}{2m_s T}B_y + \frac{Q_s}{2m_s^2 T}\hat{\mathbf{y}}\cdot(\mathbf{E}\times\mathbf{p}_{s/\bar{s}})$$

$$\boldsymbol{\varepsilon} = -\frac{1}{2}[\partial_t(\beta\mathbf{u}) + \nabla(\beta u^0)]$$

electric part of vorticity tensor

Polarization of Λ in coalescence model

- **In the coalescence model we have**

$$
\begin{aligned}
P^y_{\Lambda/\bar{\Lambda}}(t,\mathbf{x}) &= \frac{1}{3}\int \frac{d^3\mathbf{r}}{(2\pi)^3}\frac{d^3\mathbf{q}}{(2\pi)^3}\left|\psi_{\Lambda/\bar{\Lambda}}(\mathbf{q},\mathbf{r})\right|^2 \\
&\quad \times \left[P^y_{s/\bar{s}}(\mathbf{p}_1,\mathbf{x}) + P^y_{s/\bar{s}}(\mathbf{p}_2,\mathbf{x}) + P^y_{s/\bar{s}}(\mathbf{p}_3,\mathbf{x})\right] \\
&= \frac{1}{2}\omega^y_{\rm th} \pm \frac{Q_s}{2m_sT}B_y
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
\mathbf{p}_1 &= \frac{1}{3}\mathbf{p} + \frac{1}{2}\mathbf{r} + \mathbf{q} \\
\mathbf{p}_2 &= \frac{1}{3}\mathbf{p} + \frac{1}{2}\mathbf{r} - \mathbf{q} \\
\mathbf{p}_3 &= \frac{1}{3}\mathbf{p} - \mathbf{r}
\end{aligned}
}
$$

- **We see that only magnetic field appears.**

Spin density matrix element for vector mesons

- **Spin density matrix element for ϕ meson in coalescence model**

$$\rho_{00}^{\phi}(t,\mathbf{x}) \approx \frac{1}{3} - \frac{4}{9}\int \frac{d^3\mathbf{p}}{(2\pi)^3} P_s^y(\mathbf{p}) P_{\bar{s}}^y(-\mathbf{p}) \left|\psi_\phi(\mathbf{p})\right|^2$$

- **where we have applied $\mathbf{p}_s = \mathbf{p}$ and $\mathbf{p}_{\bar{s}} = -\mathbf{p}$**

- **Inserting $P_{s/\bar{s}}^y(t,\mathbf{x},\mathbf{p}_{s/\bar{s}})$ and taking an average of $\rho_{00}^{\phi}(t,\mathbf{x})$ over the fireball volume V and the polarization time t with an effective temperature $T_{\rm eff}$, we obtain**

$$\rho_{00}^{\phi} \approx \frac{1}{3} - \underline{\frac{4}{9}\left\langle P_{\bar{\Lambda}}^y P_{\Lambda}^y \right\rangle} - \underline{\frac{1}{27 m_s^2}\left\langle \mathbf{p}^2 \right\rangle_\phi \left\langle \varepsilon_z^2 + \varepsilon_x^2 \right\rangle} + \underline{\frac{e^2}{243 m_s^4 T_{\rm eff}^2}\left\langle \mathbf{p}^2 \right\rangle_\phi \left\langle E_z^2 + E_x^2 \right\rangle}$$

$c_\Lambda \sim 6\times 10^{-5}$

$c_\varepsilon \sim 10^{-5} - 10^{-6}$

$c_E \sim 10^{-5}$

$\sqrt{s} \leq 200$ GeV

Coherent ϕ field

- So with vorticity and EM field, we have

$$\rho_{00}^{\phi} \approx \frac{1}{3}$$

Coherent vector meson field was proposed to explain $P_{\Lambda} - P_{\bar{\Lambda}}$ Csernai, Kapusta, Welle (2019)

- How to accommodate a large positive deviation for ρ_{00}^{ϕ} ?
- Coherent ϕ field may be the key:
- There may exist non-zero strangeness current [e.g. $s(x) \neq \bar{s}(x)$ in nucleon]:

$$\partial_\mu J_s^\mu = 0$$

$$J_s^\mu = (0, \mathbf{J}_s(t, \mathbf{x})) = (0, j_s^{(x)}(y, z, t), j_s^{(y)}(x, z, t), j_s^{(z)}(x, y, t))$$

- The electric and magnetic part of ϕ field that contribute to the spin alignment along +y direction are given by

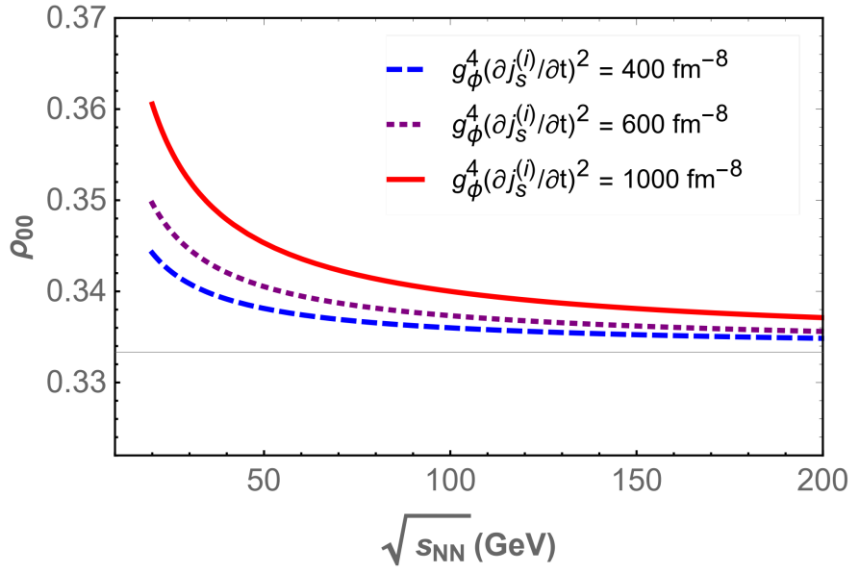
$$\mathbf{E}_\phi = \hat{\mathbf{z}} \frac{g_\phi}{m_\phi^2} \frac{\partial j_s^{(z)}}{\partial t} + \hat{\mathbf{x}} \frac{g_\phi}{m_\phi^2} \frac{\partial j_s^{(x)}}{\partial t} \qquad \mathbf{B}_\phi = -\frac{g_\phi}{m_\phi^2} \nabla \times \mathbf{J}_s = \hat{\mathbf{y}} \frac{g_\phi}{m_\phi^2} \left[\frac{\partial j_s^{(z)}}{\partial x} - \frac{\partial j_s^{(x)}}{\partial z} \right]$$

Coherent ϕ field contribution to ρ_{00}

- **The main contribution is from the electric part of the coherent ϕ field and positive definite!**

$$\rho_{00}^{\phi} \approx \frac{1}{3} + \frac{g_\phi^4}{27 m_s^4 m_\phi^4 T_{\rm eff}^2} \left\langle \mathbf{p}^2 \right\rangle_\phi \left\langle \left(\frac{\partial j_s^{(z)}}{\partial t} \right)^2 + \left(\frac{\partial j_s^{(x)}}{\partial t} \right)^2 \right\rangle$$

- **Our prediction:**



Message

- **Take-home message III:**
- **A significant deviation of ρ_{00} for ϕ from 1/3 may possibly come from the electric part of the coherent ϕ field**

Backup

Spin density matrix

- **We consider an ensemble of particles with spin quantum number S. The normalized spin states are labeled by ψ_i, spin density matrix is defined**

$$\rho = \sum_i P_i |\psi_i\rangle\langle\psi_i|$$

- **The properties of the spin density operator**
  (a) ρ is Hermitian. (b) The trace of ρ is 1. (c) ρ is positive-definite operator for any state ϕ

$$\langle\phi|\rho|\phi\rangle = \sum_i P_i \left|\langle\phi|\psi_i\rangle\right|^2 \geqslant 0$$

- **For a spin-1/2 particle,**

$$\rho = \frac{1}{2}(1 + \vec{\mathscr{P}} \cdot \boldsymbol{\sigma}) \quad \Longrightarrow \quad \vec{\mathscr{P}} = \frac{\mathrm{Tr}(\rho\boldsymbol{\sigma})}{\mathrm{Tr}(\rho)}$$

Spin density matrix for vector mesons in coalescence model

- **For mesons we define a quark-antiquark state with momenta and spins along a fixed direction (the z-direction)**

$$|\mathrm{q}_1, \overline{\mathrm{q}}_2; s_1, s_2; \mathbf{p}_1, \mathbf{p}_2\rangle \equiv |\mathrm{q}_1, \overline{\mathrm{q}}_2; s_1, s_2\rangle \, |\mathrm{q}; \mathbf{p}_1, \mathbf{p}_2\rangle$$

- **The density operator for quarks and antiquarks**

$$\begin{aligned} \rho &= V^2 \sum_{s_1, s_2} \sum_{\mathrm{q}_1, \overline{\mathrm{q}}_2} \int \frac{d^3\mathbf{p}_1}{(2\pi)^3} \frac{d^3\mathbf{p}_2}{(2\pi)^3} w_{\mathrm{q}1, s1}(\mathbf{p}_1) w_{\overline{\mathrm{q}}2, s2}(\mathbf{p}_2) \\ &\quad \times |\mathrm{q}_1, \overline{\mathrm{q}}_2; s_1, s_2; \mathbf{p}_1, \mathbf{p}_2\rangle \langle \mathrm{q}_1, \overline{\mathrm{q}}_2; s_1, s_2; \mathbf{p}_1, \mathbf{p}_2| \end{aligned}$$

- **with probability**

$$\begin{aligned} w_{\mathrm{q}, \pm 1/2}(\mathbf{p}) &= \frac{1}{2}\left[1 \pm \mathscr{P}_{\mathrm{q}}(\mathbf{p})\right] \\ w_{\overline{\mathrm{q}}, \pm 1/2}(\mathbf{p}) &= \frac{1}{2}\left[1 \pm \mathscr{P}_{\overline{\mathrm{q}}}(\mathbf{p})\right] \end{aligned}$$

Meson state in coalescence model

- **Meson spin-momentum state**

$$|\mathrm{M}; S, S_z, \mathbf{p}\rangle \equiv |\mathrm{M}; S, S_z\rangle\, |\mathrm{M}; \mathbf{p}\rangle$$

- **In the quark model, the momentum state of the meson in coordinate representation is given by**

$$\langle \mathbf{x}_1, \mathbf{x}_2 | \mathrm{M}; \mathbf{p}\rangle = \frac{1}{V^{1/2}} \exp(i\mathbf{p}\cdot\mathbf{x})\varphi_{\mathrm{M}}(\mathbf{y})$$

- **with the Gaussian form wave function**

$$\begin{aligned} \varphi_{\mathrm{M}}(\mathbf{q}) &= \int d^3\mathbf{y} e^{-i\mathbf{q}\cdot\mathbf{y}} \varphi_{\mathrm{M}}(\mathbf{y}) \\ &= \left(\frac{2\sqrt{\pi}}{a_{\mathrm{M}}}\right)^{3/2} \exp\left(-\frac{\mathbf{q}^2}{2a_{\mathrm{M}}^2}\right) \end{aligned}$$

Spin density matrix for Vector meson

- **Spin density matrix element for vector meson can be calculated by projection of the vector meson wave function on ρ (in terms of quark state)**

$$\rho^{S=1}_{S_{z1}S_{z2}}(\mathbf{p}) \quad = \quad \langle \mathrm{M}; 1, S_{z1}, \mathbf{p} | \, \rho \, | \mathrm{M}; 1, S_{z2}, \mathbf{p} \rangle$$

- **with the Gaussian form wave function**

$$\begin{aligned}
\rho^{S=0}_{00}(\mathbf{p}) &= \frac{1}{2}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\left[w_{\mathrm{q},+}\left(\frac{\mathbf{p}}{2}+\mathbf{q}\right)w_{\bar{\mathrm{q}},-}\left(\frac{\mathbf{p}}{2}-\mathbf{q}\right)\right. \\
&\quad \left. +w_{\mathrm{q},-}\left(\frac{\mathbf{p}}{2}+\mathbf{q}\right)w_{\bar{\mathrm{q}},+}\left(\frac{\mathbf{p}}{2}-\mathbf{q}\right)\right]|\varphi_{\mathrm{M}}(\mathbf{q})|^2 \\
\boxed{\rho^{S=1}_{00}(\mathbf{p}) &= \frac{1}{2}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\left[w_{\mathrm{q},+}\left(\frac{\mathbf{p}}{2}+\mathbf{q}\right)w_{\bar{\mathrm{q}},-}\left(\frac{\mathbf{p}}{2}-\mathbf{q}\right)\right. \\
&\quad \left. +w_{\mathrm{q},-}\left(\frac{\mathbf{p}}{2}+\mathbf{q}\right)w_{\bar{\mathrm{q}},+}\left(\frac{\mathbf{p}}{2}-\mathbf{q}\right)\right]|\varphi_{\mathrm{M}}(\mathbf{q})|^2} \\
\rho^{S=1}_{11}(\mathbf{p}) &= \int \frac{d^3\mathbf{q}}{(2\pi)^3} w_{\mathrm{q},+}\left(\frac{\mathbf{p}}{2}+\mathbf{q}\right)w_{\bar{\mathrm{q}},+}\left(\frac{\mathbf{p}}{2}-\mathbf{q}\right)|\varphi_{\mathrm{M}}(\mathbf{q})|^2 \\
\rho^{S=1}_{-1,-1}(\mathbf{p}) &= \int \frac{d^3\mathbf{q}}{(2\pi)^3} w_{\mathrm{q},-}\left(\frac{\mathbf{p}}{2}+\mathbf{q}\right)w_{\bar{\mathrm{q}},-}\left(\frac{\mathbf{p}}{2}-\mathbf{q}\right)|\varphi_{\mathrm{M}}(\mathbf{q})|^2
\end{aligned}$$

Can be measured in experiments by strong decays

Spin density matrix for Vector meson

- **The normalized spin density matrix element**

$$\overline{\rho}_{00}^{S=1}(\mathbf{p}) = \frac{\int d^3\mathbf{q}[1-\mathscr{P}_{\mathrm{q}}(\mathbf{p}/2+\mathbf{q})\mathscr{P}_{\overline{\mathrm{q}}}(\mathbf{p}/2-\mathbf{q})]|\varphi_{\mathrm{M}}(\mathbf{q})|^2}{\int d^3\mathbf{q}[3+\mathscr{P}_{\mathrm{q}}(\mathbf{p}/2+\mathbf{q})\mathscr{P}_{\overline{\mathrm{q}}}(\mathbf{p}/2-\mathbf{q})]|\varphi_{\mathrm{M}}(\mathbf{q})|^2}$$

$$\approx \frac{1}{3} - \frac{4}{9}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\mathscr{P}_{\mathrm{q}}\left(\frac{\mathbf{p}}{2}+\mathbf{q}\right)\mathscr{P}_{\overline{\mathrm{q}}}\left(\frac{\mathbf{p}}{2}-\mathbf{q}\right)|\varphi_{\mathrm{M}}(\mathbf{q})|^2$$

if polarization is small

for ϕ meson: s and $\bar{s}$

- **Polarization along the z-direction (for ϕ meson, it cannot be measured)**

$$\mathscr{P}_{S=1} = \frac{\rho_{11}^{S=1} - \rho_{-1-1}^{S=1}}{\rho_{11}^{S=1} + \rho_{00}^{S=1} + \rho_{-1-1}^{S=1}}$$

Density matrix and polarization for baryon in coalescence model

- **We can also define the density matrix for baryons in terms of three quark states and project onto the baryon spin state. The polarization of Λ hyperons is**

$$\mathscr{P}_{\mathrm{B},1/2}(\mathbf{p}) = \frac{\rho^{\mathrm{B}}_{++}(\mathbf{p}) - \rho^{\mathrm{B}}_{--}(\mathbf{p})}{\rho^{\mathrm{B}}_{++}(\mathbf{p}) + \rho^{\mathrm{B}}_{--}(\mathbf{p})}$$

for spin-1/2 baryon

momentum of s-quark is p1, p2 or p3

$$\mathscr{P}_{\Lambda}(\mathbf{p}) \approx \frac{1}{3}\int \frac{d^3\mathbf{r}}{(2\pi)^3}\frac{d^3\mathbf{q}}{(2\pi)^3}\left|\varphi_{\Lambda}(\mathbf{r},\mathbf{q})\right|^2\left[\mathscr{P}_{\mathrm{s}}(\mathbf{p}_1)+\mathscr{P}_{\mathrm{s}}(\mathbf{p}_2)+\mathscr{P}_{\mathrm{s}}(\mathbf{p}_3)\right]$$

- **where relative coordinates**

$$\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3$$

$$\mathbf{r} = \frac{1}{3}(\mathbf{p}_1 + \mathbf{p}_2 - 2\mathbf{p}_3)$$

$$\mathbf{q} = \frac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2)$$

- **We take Gaussian form of baryon wave function**

$$\begin{aligned}\varphi_{\mathrm{B}}(\mathbf{r},\mathbf{q}) &= \int d^3\mathbf{y}d^3\mathbf{z}\exp(-i\mathbf{q}\cdot\mathbf{z} - i\mathbf{r}\cdot\mathbf{y})\varphi_{\mathrm{B}}(\mathbf{y},\mathbf{z}) \\ &= (2\sqrt{\pi})^3\left(\frac{1}{a_{\mathrm{B1}}a_{\mathrm{B2}}}\right)^{3/2}\exp\left(-\frac{\mathbf{r}^2}{2a^2_{\mathrm{B1}}} - \frac{\mathbf{q}^2}{2a^2_{\mathrm{B2}}}\right)\end{aligned}$$