Chirality and spin transport from Wigner function approach

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ArXiv:1910.11060 JHG, Z.T. Liang, Q. Wang
ArXiv:1902.06510 JHG, Z.T. Liang PRD100,056021,2019
ArXiv:1802.06216 JHG, Z.T. Liang, Q. Wang, X.N. Wang PRD98,036019,2018

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Outline

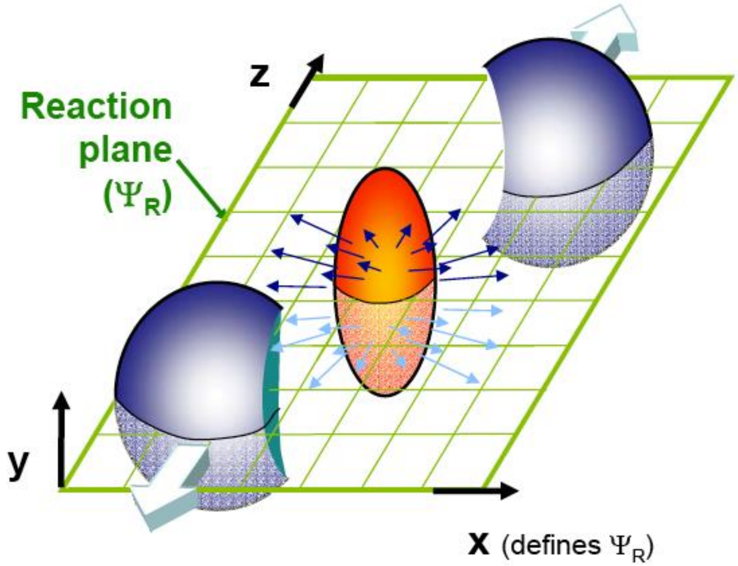
- Introduction
- Chirality transport theory
- Spin transport theory
- Dirac sea and chiral anomaly
- Summary

Chiral and Spin Effects in HIC

Chiral Magnetic Effect

Chiral Vortical Effect

Chiral Magnetic Wave



Chiral Vortical Wave

Chiral Separate Effect

Local Polarization

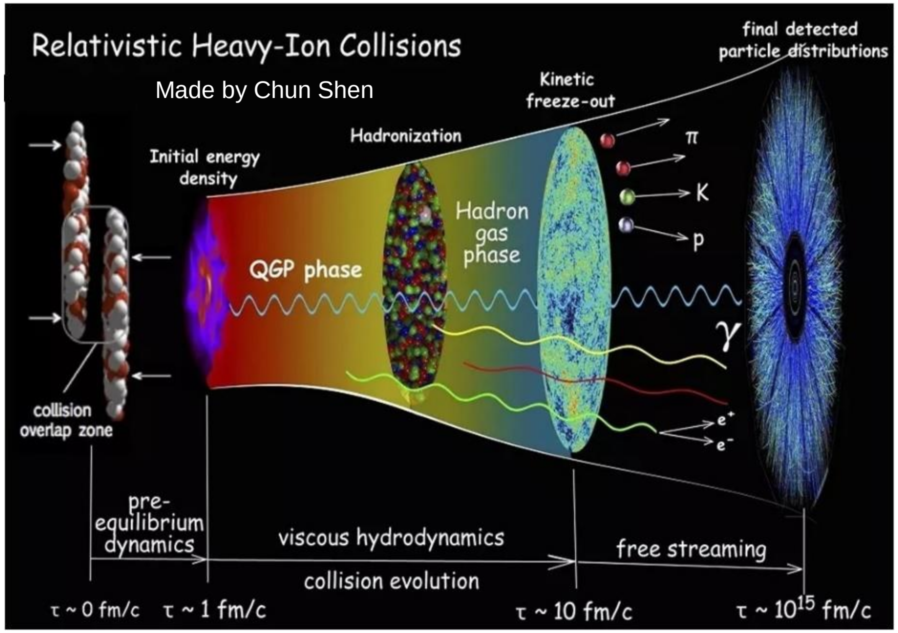
Spin Alignment

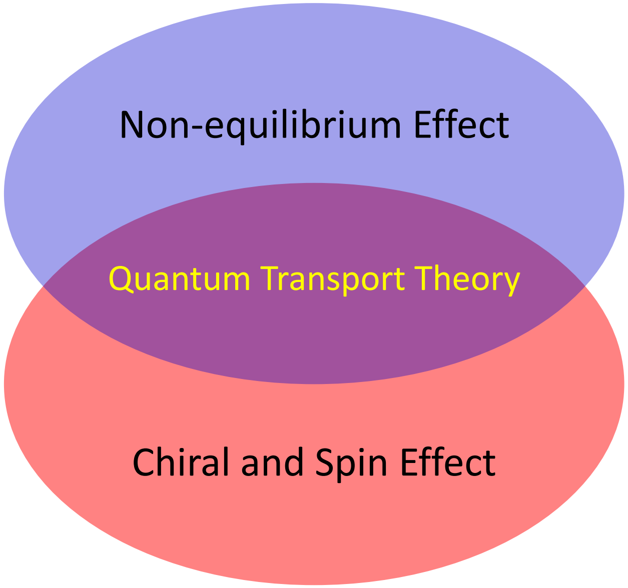
Global Polarization

Parallel Sessions: Chirality I, II, III

Plenary Session V: Huang, Lisa, Liao

Quantum Transport Theory





Parallel talks by: S. Z. Shi, Z. Y. Wang, N. Weickgenannt, S. Y. Li

Wigner Function and Equation

Wigner function for spin-1/2 fermion : Heinz, Phys. Rev. Lett. 1983

$$\boxed{W(x,p) = \left\langle \int \frac{d^4y}{(2\pi)^4} e^{-ip\cdot y} \bar{\psi}(x)\, e^{\frac{1}{2}y\cdot D^\dagger(x)} \otimes e^{-\frac{1}{2}y\cdot D(x)} \psi(x) \right\rangle}$$

16 independent components:

$$\boxed{W = \frac{1}{4}\left[\underbrace{\mathscr{F}}_{\text{scalar}} + i\gamma^5 \underbrace{\mathscr{P}}_{\text{pseudo}} + \gamma^\mu \underbrace{\mathscr{V}_\mu}_{\text{vector}} + \gamma^5\gamma^\mu \underbrace{\mathscr{A}_\mu}_{\text{axial}} + \frac{1}{2}\sigma^{\mu\nu} \underbrace{\mathscr{S}_{\mu\nu}}_{\text{tensor}}\right]}$$

32 Wigner equations in background EM field : Vasak, Gyulassy, Elze, Annals Phys. 1987

$$\boxed{\Pi^\mu \equiv p^\mu - \frac{1}{2} j_1\left(\frac{1}{2}\Delta\right) F^{\mu\nu}\partial_\nu^p}$$

$$\Delta \equiv \partial^p \cdot \partial_x$$

$$\begin{aligned}
\Pi^\mu \mathscr{V}_\mu &= m\mathscr{F}, \\
-G^\mu \mathscr{A}_\mu &= 2m\mathscr{P}, \\
2\Pi_\mu \mathscr{F} + G^\nu \mathscr{S}_{\mu\nu} &= 2m\mathscr{V}_\mu, \\
G_\mu \mathscr{P} - \epsilon_{\mu\nu\rho\sigma}\Pi^\nu \mathscr{S}^{\rho\sigma} &= 2m\mathscr{A}_\mu, \\
(G_\mu \mathscr{V}_\nu - G_\nu \mathscr{V}_\mu) - 2\epsilon_{\mu\nu\rho\sigma}\Pi^\rho \mathscr{A}^\sigma &= 2m\mathscr{S}_{\mu\nu}.
\end{aligned}$$

$$\boxed{G^\mu \equiv \partial_x^\mu - j_0\left(\frac{1}{2}\Delta\right) F^{\mu\nu}\partial_\nu^p}$$

$$\begin{aligned}
G^\mu \mathscr{V}_\mu &= 0, \\
\Pi^\mu \mathscr{A}_\mu &= 0, \\
G_\mu \mathscr{F} - 2\Pi^\nu \mathscr{S}_{\mu\nu} &= 0, \\
4\Pi_\mu \mathscr{P} + \epsilon_{\mu\nu\rho\sigma} G^\nu \mathscr{S}^{\rho\sigma} &= 0, \\
2(\Pi_\mu \mathscr{V}_\nu - \Pi_\nu \mathscr{V}_\mu) + \epsilon_{\mu\nu\rho\sigma} G^\rho \mathscr{A}^\sigma &= 0.
\end{aligned}$$

The Wigner functions in Wigner equations are not normal ordered!

Massless Fermions

Decoupled Wigner equations for massless Fermions:

$$
\begin{aligned}
\Pi^\mu \mathscr{V}_\mu &= 0, \quad \Pi^\mu \mathscr{A}_\mu = 0, \\
G^\mu \mathscr{V}_\mu &= 0, \quad G^\mu \mathscr{A}_\mu = 0, \\
\epsilon_{\mu\nu\rho\sigma} G^\rho \mathscr{A}^\sigma &= -2\left(\Pi_\mu \mathscr{V}_\nu - \Pi_\nu \mathscr{V}_\mu\right), \\
\epsilon_{\mu\nu\rho\sigma} G^\rho \mathscr{V}^\sigma &= -2\left(\Pi_\mu \mathscr{A}_\nu - \Pi_\nu \mathscr{A}_\mu\right).
\end{aligned}
$$

$$
\begin{aligned}
\Pi_\mu \mathscr{F} + \frac{1}{2} G^\nu \mathscr{S}_{\mu\nu} &= 0, \\
\frac{1}{2} G_\mu \mathscr{F} - \Pi^\nu \mathscr{S}_{\mu\nu} &= 0, \\
-G_\mu \mathscr{P} + \epsilon_{\mu\nu\rho\sigma} \Pi^\nu \mathscr{S}^{\rho\sigma} &= 0, \\
\Pi_\mu \mathscr{P} + \frac{1}{4} \epsilon_{\mu\nu\rho\sigma} G^\nu \mathscr{S}^{\rho\sigma} &= 0.
\end{aligned}
$$

8 functions 16 equations

8 functions 16 equations

Right/Left ($s = \pm 1$)

$$\mathscr{J}_s^\mu \equiv \frac{1}{2}\left(\mathscr{V}_\mu + s\mathscr{A}_\mu\right)$$

$$j^\mu = \int d^4p \mathscr{V}^\mu, \; j_5^\mu = \int d^4p \mathscr{A}^\mu, \; T^{\mu\nu} = \int d^4p\, p^\mu \mathscr{V}^\nu$$

$$\Pi_\mu \mathscr{J}_s^\mu = 0, \quad G_\mu \mathscr{J}_s^\mu = 0, \quad \epsilon_{\mu\nu\rho\sigma} G^\rho \mathscr{J}_s^\sigma = -2s\left(\Pi_\mu \mathscr{J}_{s\nu} - \Pi_\nu \mathscr{J}_{s\mu}\right)$$

4 independent functions + 8 coupled equations

Ochs, Heinz AP1998; JHG, Q. Wang PLB 2015; Huang, Shi, Jiang, Liao, Zhuang PRD 2018

Disentanglement Theorem

Component decomposition: $$\boxed{\mathscr{J}_\mu = \mathscr{J}_n n^\mu + \bar{\mathscr{J}}_\mu}$$ $n^2=1 \quad \mathscr{J}_n = n\cdot\mathscr{J}$

Perturbative expansion: $$\boxed{\mathscr{J}_\mu = \mathscr{J}_\mu^{(0)} + \hbar\,\mathscr{J}_\mu^{(1)} + \hbar^2\,\mathscr{J}_\mu^{(2)} + \cdots}$$

The components $\bar{\mathscr{J}}_\mu$ can be expressed as the function of $F \equiv \mathscr{J}_n/p_n$:

$$\boxed{\bar{\mathscr{J}}_\mu^{(0)} = \bar{p}_\mu F^{(0)}, \qquad \bar{\mathscr{J}}_\mu^{(1)} = \bar{p}_\mu F^{(1)} - \frac{s}{2p_n}\epsilon^{\mu\nu\rho\sigma} n_\nu \nabla_\sigma \left(p_\rho F^{(0)}\right), \qquad \ldots\ldots}$$

$$\boxed{F^{(0)} = f^{(0)}\delta(p^2), \qquad F^{(1)} = f^{(1)}\delta(p^2) - \frac{s}{p_n} B\cdot p f^{(0)}\delta'(p^2), \qquad \ldots\ldots}$$

1 distribution function: $\boxed{f}$ 1 transport equation: $\boxed{\nabla_n \mathscr{J}_n + \bar{\nabla}\cdot\bar{\mathscr{J}} = 0}$

This conclusion holds up to any order of $\hbar$!

JHG, Z.T. Liang, Q. Wang, X.N. Wang PRD (2018)

Chiral Transport

Covariant chiral kinetic equation up to $O(\hbar)$: $\nabla^\mu \equiv \partial_x^\mu - F^{\mu\nu}\partial_\nu^p$

$$\boxed{\nabla^\mu \left\{ \left(g_{\mu\nu} + \frac{\hbar s}{2p_n} \epsilon_{\mu\nu\rho\sigma} n^\rho \nabla^\sigma \right) \left[p^\nu f \delta\left(p^2 - \hbar s \frac{B\cdot p}{p_n} \right) \right] \right\} = 0}$$

Chiral kinetic equation for particle by $\int_0^\infty dp_n$:

$$\begin{aligned} (1 + s\hbar B\cdot\Omega)\, n\cdot\partial^x f \\ + \left[v^\mu + s\hbar(\hat{\bar{p}}\cdot\Omega) B^\mu + s\hbar \epsilon^{\mu\nu\rho\sigma} n_\rho E_\sigma \Omega_\nu \right] \bar{\partial}^x_\mu f \\ + \left(\tilde{E}^\mu + \epsilon^{\mu\nu\alpha\beta} v_\nu n_\alpha B_\beta + s\hbar E\cdot B\Omega^\mu \right) \bar{\partial}^p_\mu f \\ + s\hbar E\cdot B \left(\bar{\partial}^\mu_p \Omega_\mu \right) f &= 0 \end{aligned}$$

Stephanov & Yin PRL 109,(2012)162001, Son & Yamamoto PRD 87 (2013) 8, 085016

Berry curvature: $\Omega^\mu = \frac{\bar{p}^\mu}{2|\bar{p}|^3}$ Berry monopole: $\bar{\partial}_p \cdot \Omega_p = 2\pi\delta^3(\bar{p})$

Massive Fermions

32 Wigner equations for **16** Wigner functions at $O(\hbar)$: $\nabla^\mu \equiv \partial_x^\mu - F^{\mu\nu}\partial_\nu^p$

$$
\begin{aligned}
\nabla^\mu \mathscr{V}_\mu = 0, \quad p^\mu \mathscr{A}_\mu &= 0 & m\mathscr{F} &= p^\mu \mathscr{V}_\mu, \quad m\mathscr{P} = -\frac{\hbar}{2}\nabla^\mu \mathscr{A}_\mu \\
\frac{\hbar}{2}\nabla_\mu \mathscr{F} - p^\nu \mathscr{S}_{\mu\nu} &= 0 & m\mathscr{V}_\mu &= p_\mu \mathscr{F} + \frac{\hbar}{2}\nabla^\nu \mathscr{S}_{\mu\nu} \\
p_\mu \mathscr{P} + \frac{\hbar}{4}\epsilon_{\mu\nu\rho\sigma}\nabla^\nu \mathscr{S}^{\rho\sigma} &= 0 & m\mathscr{A}_\mu &= \frac{\hbar}{2}\nabla_\mu \mathscr{P} - \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}p^\nu \mathscr{S}^{\rho\sigma} \\
p_{[\mu}\mathscr{V}_{\nu]} + \frac{\hbar}{2}\epsilon_{\mu\nu\rho\sigma}\nabla^\rho \mathscr{A}^\sigma &= 0 & m\mathscr{S}_{\mu\nu} &= \frac{\hbar}{2}\nabla_{[\mu}\mathscr{V}_{\nu]} - \epsilon_{\mu\nu\rho\sigma}p^\rho \mathscr{A}^\sigma
\end{aligned}
$$

11 of **32** :

$$
\begin{aligned}
\mathscr{P} &= -\frac{\hbar}{2m}\nabla^\mu \mathscr{A}_\mu, \quad \mathscr{V}_\mu = \frac{1}{m}p_\mu \mathscr{F} - \frac{\hbar}{2m^2}\epsilon_{\mu\nu\rho\sigma}\nabla^\nu p^\rho \mathscr{A}^\sigma \\
\mathscr{S}_{\mu\nu} &= -\frac{1}{m}\epsilon_{\mu\nu\rho\sigma}p^\rho \mathscr{A}^\sigma + \frac{\hbar}{2m^2}\left(\nabla_\mu p_\nu - \nabla_\nu p_\mu\right)\mathscr{F}
\end{aligned}
$$

5 of **32** :

$$
\begin{aligned}
\mathscr{F} &= \delta\left(p^2 - m^2\right)\mathcal{F} + \frac{\hbar}{m}\tilde{F}_{\mu\nu}p^\mu \mathcal{A}^\nu \delta'\left(p^2 - m^2\right) \\
\mathscr{A}_\mu &= \delta\left(p^2 - m^2\right)\mathcal{A}_\mu + \frac{\hbar}{m}p^\nu \tilde{F}_{\mu\nu}\mathcal{F}\delta'\left(p^2 - m^2\right)
\end{aligned}
$$

Independent

$\mathcal{F}, \mathcal{A}^\mu$

Fundamental

10 of **32** : **Hold automatically!**

Vasak, Gyulassy, Elze AP 1987; JHG, Liang PRD 2019

Spin Transport

6 of 32 --- Covariant kinetic equations:

$$p\cdot\nabla\left[\mathcal{F}\delta\left(p^2-m^2\right)+\frac{\hbar}{m}\tilde{F}_{\mu\nu}p^\mu\mathcal{A}^\nu\delta'\left(p^2-m^2\right)\right] = \frac{\hbar}{2m}(\partial_\lambda^x\tilde{F}_{\mu\nu})\partial_p^\lambda\left[p^\mu\mathcal{A}^\nu\delta\left(p^2-m^2\right)\right]$$

$$\boxed{p_\mu\mathcal{A}^\mu\,\delta\left(p^2-m^2\right) = 0}$$

$$p\cdot\nabla\left[\mathcal{A}_\mu\delta\left(p^2-m^2\right)+\frac{\hbar}{m}p^\nu\tilde{F}_{\mu\nu}\mathcal{F}\delta'\left(p^2-m^2\right)\right] = F_{\mu\nu}\left[\mathcal{A}^\nu\delta\left(p^2-m^2\right)+\frac{\hbar}{m}p_\lambda\tilde{F}^{\nu\lambda}\mathcal{F}\delta'\left(p^2-m^2\right)\right]+\frac{\hbar}{2m}(\partial_\lambda^x\tilde{F}_{\mu\nu})\partial_p^\lambda\left[p^\nu\mathcal{F}\delta\left(p^2-m^2\right)\right]$$

Manifest Lorentz covariance ! Singular Dirac delta function !

4 independent kinetic equations for particle by $\int_0^\infty dp_0$:

$$\boxed{\mathcal{A}^0=\vec{v}\cdot\vec{\mathcal{A}}}$$

$$\left(\nabla_t+\vec{v}\cdot\vec{\nabla}\right)\mathcal{F} = -\frac{\hbar}{2mE_p}\left[(\vec{B}+\vec{E}\times\vec{v})(\vec{v}\cdot\vec{\nabla}+E_p\overleftarrow{\nabla}_x\cdot\vec{\nabla}_p)-(\vec{B}\cdot\vec{v})(\vec{v}\cdot\vec{\nabla}+E_p\overleftarrow{\nabla}_x\cdot\vec{\nabla}_p)\vec{v}\right]\cdot\vec{\mathcal{A}}$$

$$\left(\nabla_t+\vec{v}\cdot\vec{\nabla}\right)\vec{\mathcal{A}} = \vec{B}\times\vec{\mathcal{A}}-\vec{E}(\vec{v}\cdot\vec{\mathcal{A}})-\frac{\hbar}{2mE_p}(\vec{B}+\vec{E}\times\vec{v})(\vec{v}\cdot\vec{\nabla}+E_p\overleftarrow{\nabla}_x\cdot\vec{\nabla}_p)\mathcal{F}$$

Manifest Lorentz covariance broken! More suitable for simulation!

Chiral Anomaly

Chiral anomaly from QFT:

$$\hbar\partial_\mu j_5^\mu = -2mj_5 - \frac{\hbar^2}{2\pi^2} E\cdot B$$

Wigner equation and function relevant to chiral anomaly:

$$\hbar\nabla^\mu \mathscr{A}_\mu = -2m\mathscr{P} \quad \text{with} \quad \mathscr{A}_\mu = \delta\left(p^2-m^2\right)\mathcal{A}_\mu + \frac{\hbar}{m} p^\nu \tilde{F}_{\mu\nu}\mathcal{F}\delta'\left(p^2-m^2\right)$$

Integrate over momentum

$$j_5^\mu = \int d^4p\,\mathscr{A}^\mu \quad j_5 = \int d^4p\,\mathscr{P}$$

$$\hbar\partial_x^\mu j_\mu^5 = -2mj_5 + \hbar X - \frac{\hbar^2}{2\pi^2} C E\cdot B$$

$$X = F^{\mu\lambda}\int d^4p\,\partial_\lambda^p\left[\mathcal{A}_\mu\delta\left(p^2-m^2\right)\right] \stackrel{?}{=} 0$$

$$C = -\frac{2\pi^2}{m}\int d^4p\,\partial_p^\lambda\left[p_\lambda\mathcal{F}\delta'\left(p^2-m^2\right)\right] \stackrel{?}{=} 1$$

Dirac Sea or Vacuum Contribution

Unnormal ordered Wigner function for free field:

$$W(x,p) \sim \sum_s c_s \langle a_s^\dagger a_s \rangle + \sum_s \bar{c}_s \langle b_s b_s^\dagger \rangle = \sum_s c_s \langle a_s^\dagger a_s \rangle + \sum_s \bar{c}_s \left(\langle b_s^\dagger b_s \rangle + \bar{f}_\mathrm{V} \right) \qquad \bar{f}_\mathrm{V} = -1$$

Normal distribution function:

Vanish at infinite momentum

$$f^s = \langle a_s^\dagger a_s \rangle, \qquad \bar{f}^s = \langle b_s^\dagger b_s \rangle, \qquad s = \pm$$

Sheng, Rischke, Vasak, Wang EPJA 2018, Sheng, Fang, Wang, Rischke PRD 2019

$$\mathcal{A}_\mu = \begin{cases} \frac{m}{4\pi^3} s_\mu \left[f^+ - f^- \right], & p_0 > 0 \\ \frac{m}{4\pi^3} s_\mu \left[(\bar{f}^+ + \bar{f}_\mathrm{V}) - (\bar{f}^- + \bar{f}_\mathrm{V}) \right], & p_0 < 0. \end{cases}$$

$$\mathcal{F} = \begin{cases} \frac{m}{4\pi^3} \left[f^+ + f^- \right], & p_0 > 0 \\ \frac{m}{4\pi^3} \left[(\bar{f}^+ + \bar{f}_\mathrm{V}) + (\bar{f}^- + \bar{f}_\mathrm{V}) \right], & p_0 < 0. \end{cases}$$

Negative energy particle distribution function:

$$\mathcal{F} = \frac{-m}{4\pi^3} \left[(1 - \bar{f}^+) + (1 - \bar{f}^-) \right]$$

Dirac Sea and Chiral Anomaly

Chiral current from Wigner equation: $\boxed{\hbar\partial_x^\mu j_\mu^5 = -2mj_5 + \hbar X - \frac{\hbar^2}{2\pi^2} C E\cdot B}$

$$\boxed{X \;=\; F^{\mu\lambda}\int d^4p\,\partial_\lambda^p\left[\mathcal{A}_\mu\delta\left(p^2-m^2\right)\right] = 0}$$

$$\boxed{C \;=\; -\frac{2\pi^2}{m}\int d^4p\,\partial_p^\lambda\left[p_\lambda\mathcal{F}_\mathrm{v}\delta'\left(p^2-m^2\right)\right]}$$

$$\boxed{C \;=\; -\int\frac{d^4p}{4\pi}\partial_p^\lambda\left[\frac{p_\lambda}{p_0E_p}\bar{f}_\mathrm{v}\delta'(p_0+E_p)\right] = \int\frac{d^3\mathbf{p}}{4\pi}\partial_\mathbf{p}\cdot\left(\frac{\mathbf{p}}{E_p^3}\right) = 1}$$

$$\Rightarrow \quad \boxed{\hbar\partial_\mu j_5^\mu = -2mj_5 - \frac{\hbar^2}{2\pi^2}E\cdot B}$$

Chiral limit: $\boxed{m = 0 \qquad E_p = |\mathbf{p}|}$ 3d Berry monopole: $\boxed{\partial_\mathbf{p}\cdot\left(\frac{\mathbf{p}}{|\mathbf{p}|^3}\right) = 4\pi\delta^3(\mathbf{p})}$

CKE Particle vs Antiparticle

CKE for particle by $\int_0^\infty dp_0$

$$\begin{aligned} & (1+s\hbar B\cdot\Omega)\, n\cdot\partial^x f \\ & +\left[v^\mu + s\hbar(\hat{\bar{p}}\cdot\Omega)B^\mu + s\hbar\epsilon^{\mu\nu\rho\sigma}n_\rho E_\sigma \Omega_\nu\right]\bar{\partial}^x_\mu f \\ & +\left(\tilde{E}^\mu + \epsilon^{\mu\nu\alpha\beta}v_\nu n_\alpha B_\beta + s\hbar E\cdot B\Omega^\mu\right)\bar{\partial}^p_\mu f \\ & + s\hbar E\cdot B\left(\bar{\partial}^p_\mu \Omega^\mu\right) f = 0 \end{aligned}$$

CKE for antiparticle by $\int_{-\infty}^0 dp_0$

$$\begin{aligned} & (1-s\hbar B\cdot\Omega)\, n\cdot\partial^x \bar{f} \\ & +\left[v^\mu - s\hbar(\hat{\bar{p}}\cdot\Omega)B^\mu - s\hbar\epsilon^{\mu\nu\rho\sigma}n_\rho E_\sigma \Omega_\nu\right]\bar{\partial}^x_\mu \bar{f} \\ & -\left(\tilde{E}^\mu + \epsilon^{\mu\nu\alpha\beta}v_\nu n^\alpha B^\beta - s\hbar E\cdot B\Omega^\mu\right)\bar{\partial}^p_\mu \bar{f} \\ & + s\hbar E\cdot B\left(\bar{\partial}^p_\mu \Omega^\mu\right)\left(\bar{f}-1\right) = 0 \end{aligned}$$

Total derivative

$$E\cdot B\Omega^\mu \bar{\partial}^p_\mu f + E\cdot B\left(\bar{\partial}^p_\mu \Omega^\mu\right) f = E\cdot B\bar{\partial}^p_\mu\left(\Omega^\mu f\right)$$

Total derivative

$$E\cdot B\Omega^\mu \bar{\partial}^p_\mu \bar{f} + E\cdot B\left(\bar{\partial}^p_\mu \Omega^\mu\right) \bar{f} = E\cdot B\bar{\partial}^p_\mu\left(\Omega^\mu \bar{f}\right)$$

Chiral anomaly:

$$\partial_\mu j^\mu_s = s\hbar E\cdot B\int\frac{d^3\bar{p}}{(2\pi)^3}\cancel{\bar{\partial}^\mu_p\left[\Omega_\mu(f+\bar{f})\right]} - s\hbar E\cdot B\int\frac{d^3\bar{p}}{(2\pi)^3}\bar{\partial}^\mu_p\Omega_\mu$$

Summary

- Chirality transport for massless fermion can be described sufficiently by one distribution function with one transport equation up to any order of $\hbar$.
- Spin transport for massive fermion can be described sufficiently by one distribution function and one spin vector with coupled transport equations.
- Chiral anomaly in quantum transport theory can be derived from the Dirac sea or the vacuum contribution in the un-normal-ordered Wigner function.
- Dirac sea could modify the chiral kinetic equation for antiparticles.

Thanks for your attention!