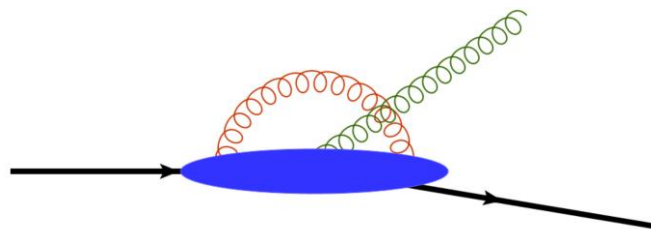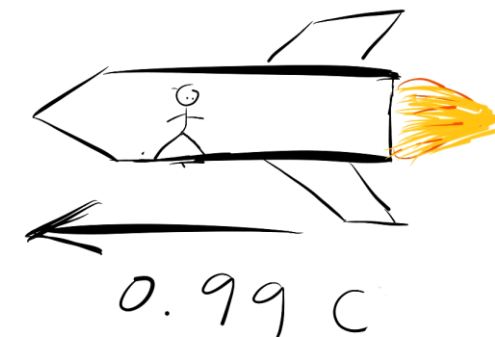# The Problem of Overlapping Formation Times:
In-medium Virtual Corrections for QCD

Shahin Iqbal

CCNU Wuhan

and

NCP Islamabad.

Reporting on work done in collaboration with Peter Arnold, Tyler Gorda Tanner Rase.

Not my real voice!!!

The Problem of Overlapping Formation Times:
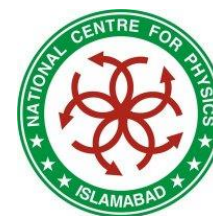# In-medium Virtual Corrections for QCD

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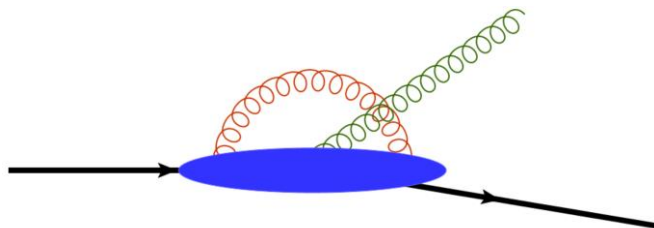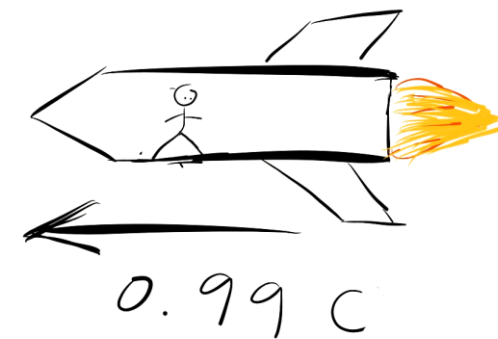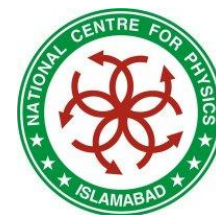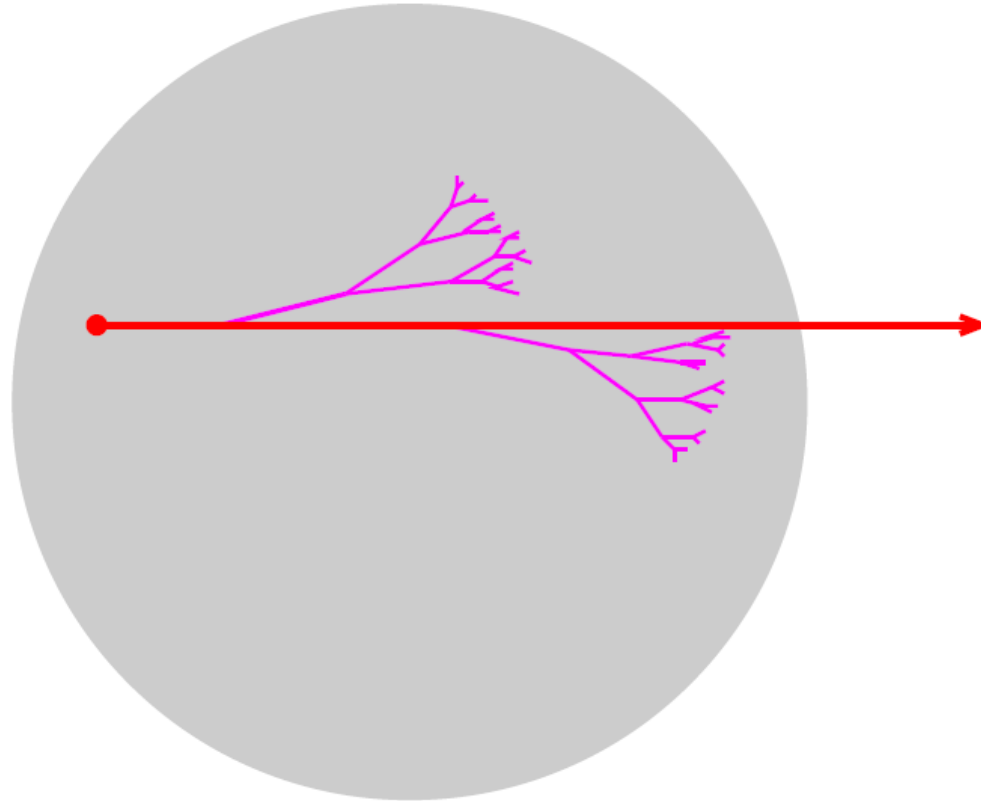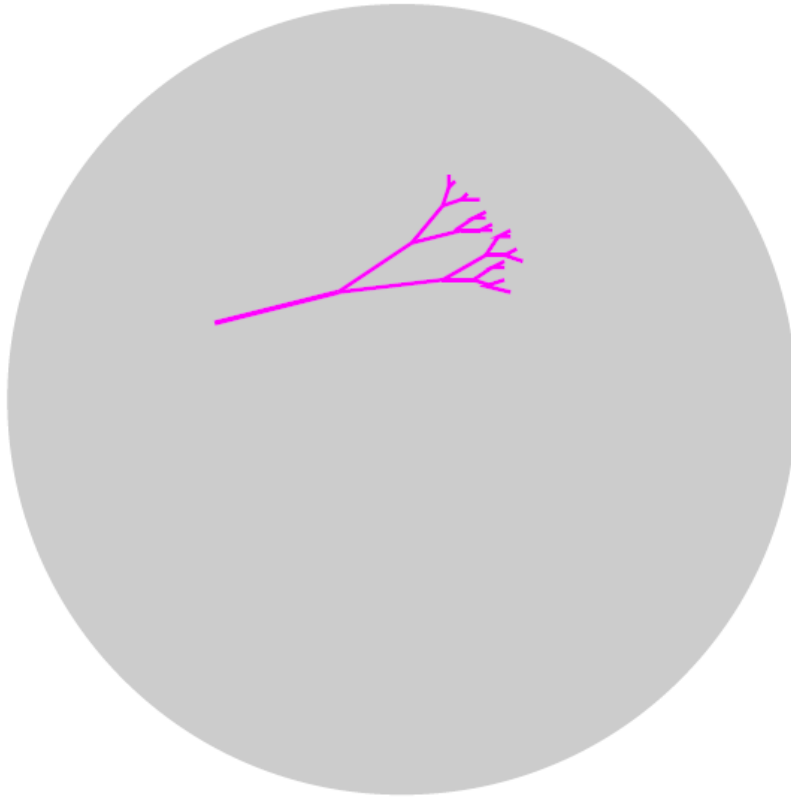
Institute of Particle Physics

1- Background

Consider the energy loss of a high energy parton in a QGP

Consider the energy loss of a high energy parton in a QGP

- Large, homogenous and static medium.
- Imagine a cascade that stops in the medium.
- Complication: LPM effect!

Landau-Pomeranchuk-Migdal effect

Light cannot resolve details smaller
than its wavelength!

0.99 c

Indistinguishable from

Formation Time
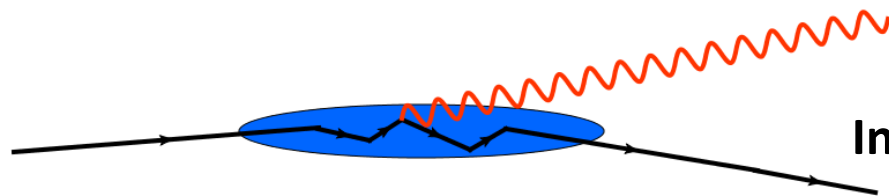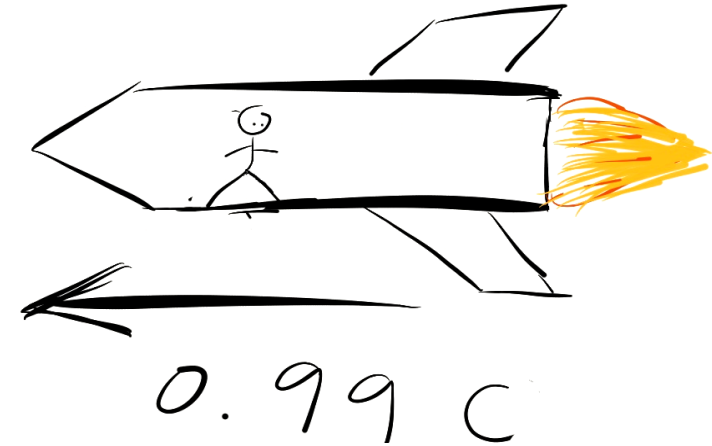
LPM effect: actual rate is smaller than the naive expectation!

LPM effect for QED developed in 1950s.
QCD generalization in 1990s.

2- Beyond LPM?

Idealized Monte Carlo Picture
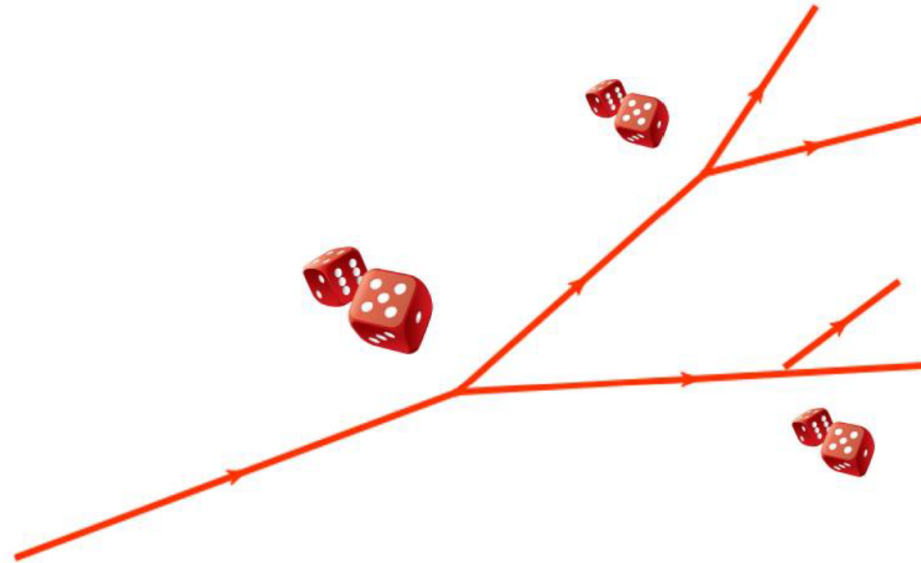
- Rolls a dice for each splitting with probability weighted by the LPM splitting rate.
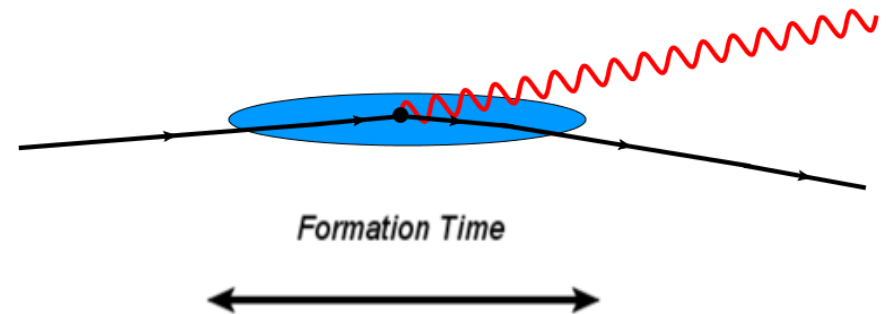- Inherently assumes consecutive splittings quantum *mechanically independent.*

  But are consecutive emissions really independent?

Are consecutive emissions independent?

Note: Time between splittings $t_{rad} \sim \frac{t_{form}}{\alpha}$.

Yes!

No!

$\alpha \ll 1$
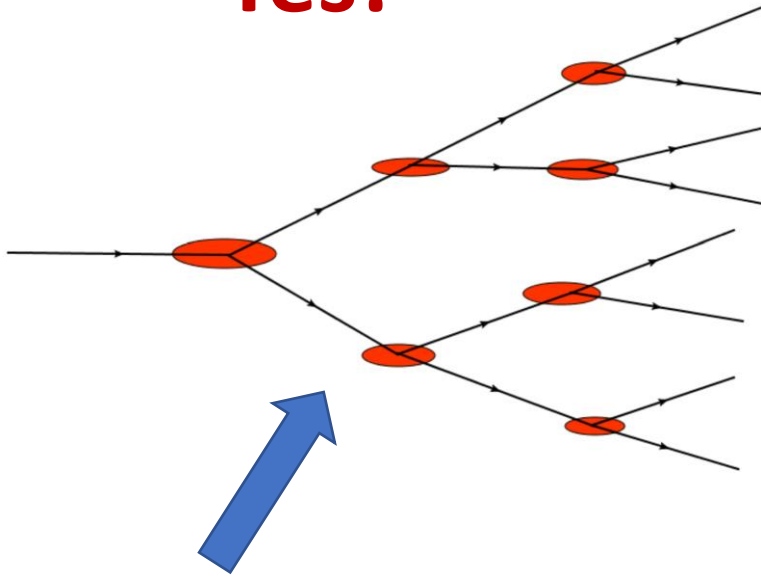
$\alpha \sim not\ too\ small$

Size of overlap corrections
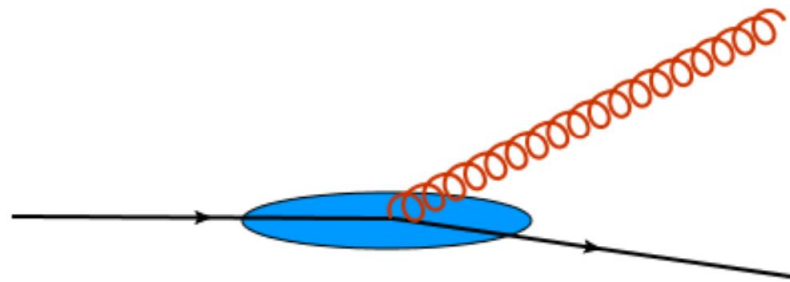
Prob $\sim \alpha$
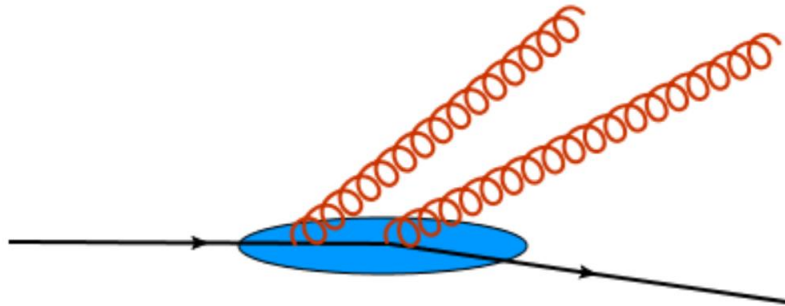
Prob $\sim \alpha^2$

Corrections parametrically of the order of coupling constant $\alpha(Q_\perp)$.

What we want to do?

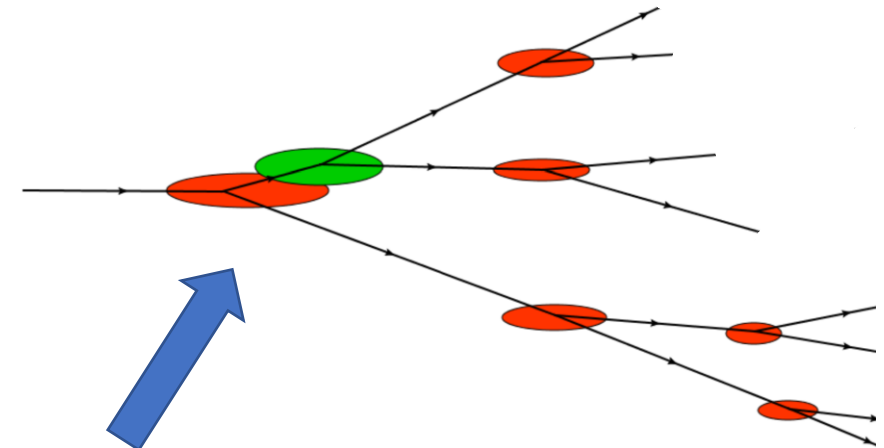Calculate $O(\alpha_s)$ corrections from emissions with overlapping formation times to figure out whether rolling a dice for in-medium showers is good, bad, or ugly.

+
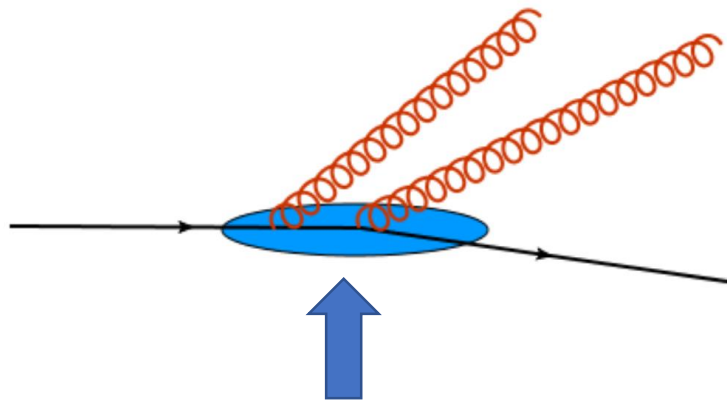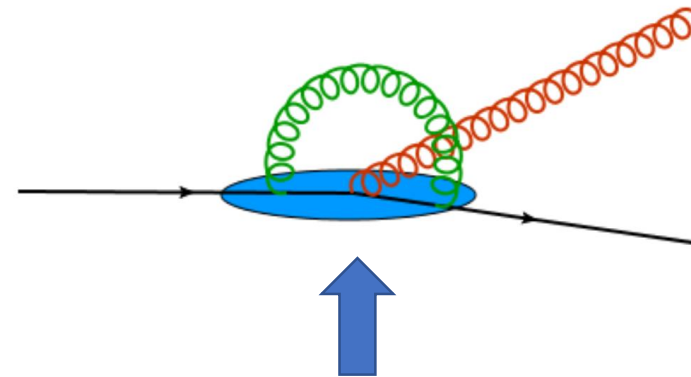
etc.

- Already done (calculated with Han-Chih Chang).
- These give power-law IR divergent contributions to energy loss.

Work in Progress.

3- In-medium Virtual Corrections

Review of Single splitting result.

LPM effect in terms of Feynman diagrams:

Splitting vertices given by QCD Feynman rules.

Medium effects given by non-Hermitian Hamiltonian.

NOT Vacuum!

$$rate \quad \frac{d\Gamma}{dx} = \frac{P(x)Re[i\Omega]}{\pi\, x(1-x)}$$ in the often used Harmonic Oscillator approximation.
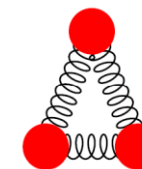
Constraints
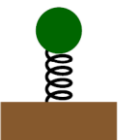
Our formalism:

Same idea, but a lot more complicated!

- Many different time orderings and permutations.
- Non-trivial helicity structure. Splitting matrix elements related to *Helicity dependent* DGLAP splitting functions.
- In-medium evolution between splittings governed by an effective non-Hermitian Hamiltonian.
- Use Harmonic Oscillator (a.k.a. multiple scattering approx. or $\hat{q}$ approx.) and Large-Nc limit to simplify things.
- The final result

$$\frac{d\Gamma}{dx dy} = \int d\Delta t \; (complicated..)$$

$\propto \sqrt{P_{i \to j}(x)}$

Harmonic Oscillator approx.

Various symm. of the problem.
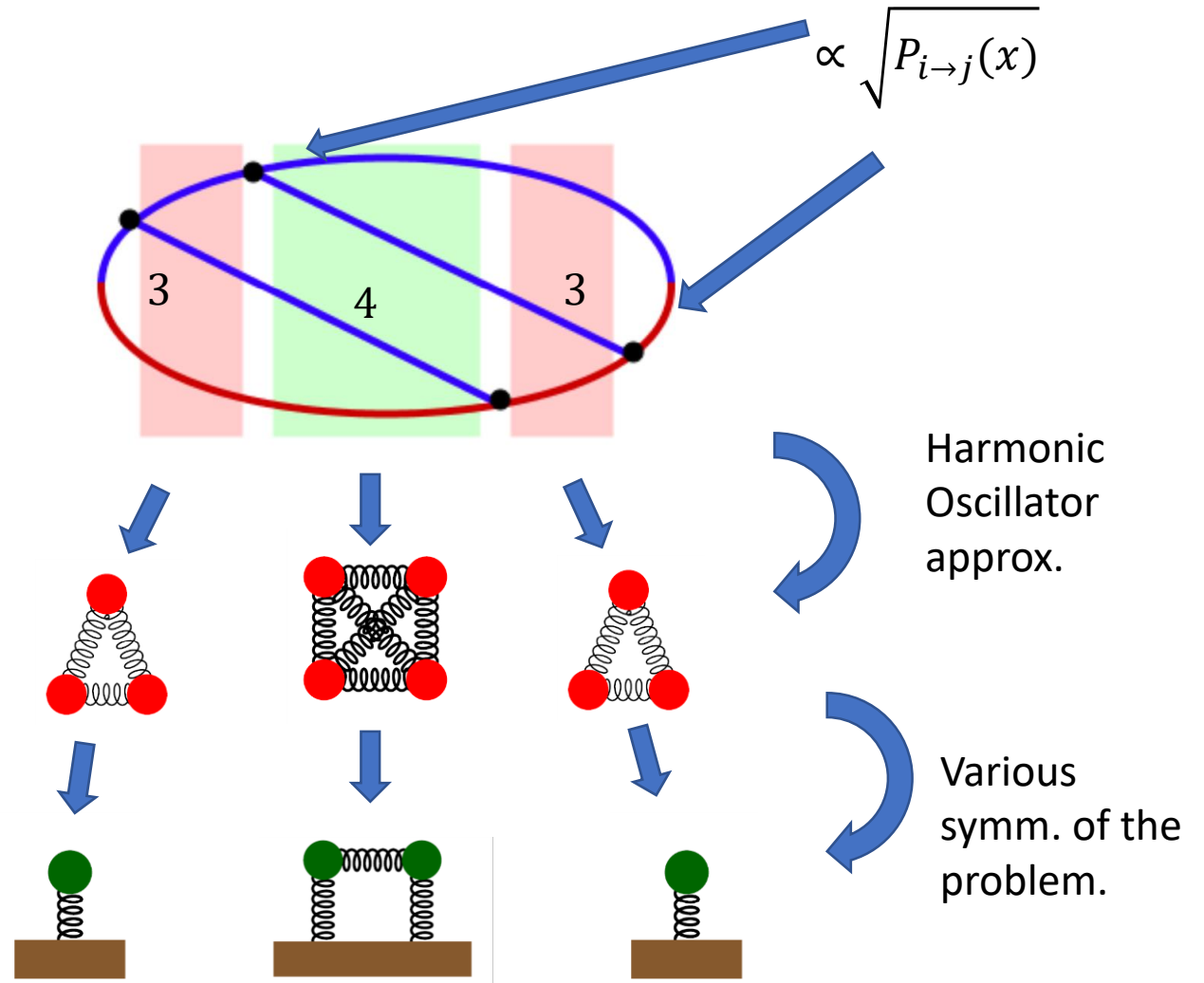
But we ran into a (virtual) dead-end.

Note: All gluons here!
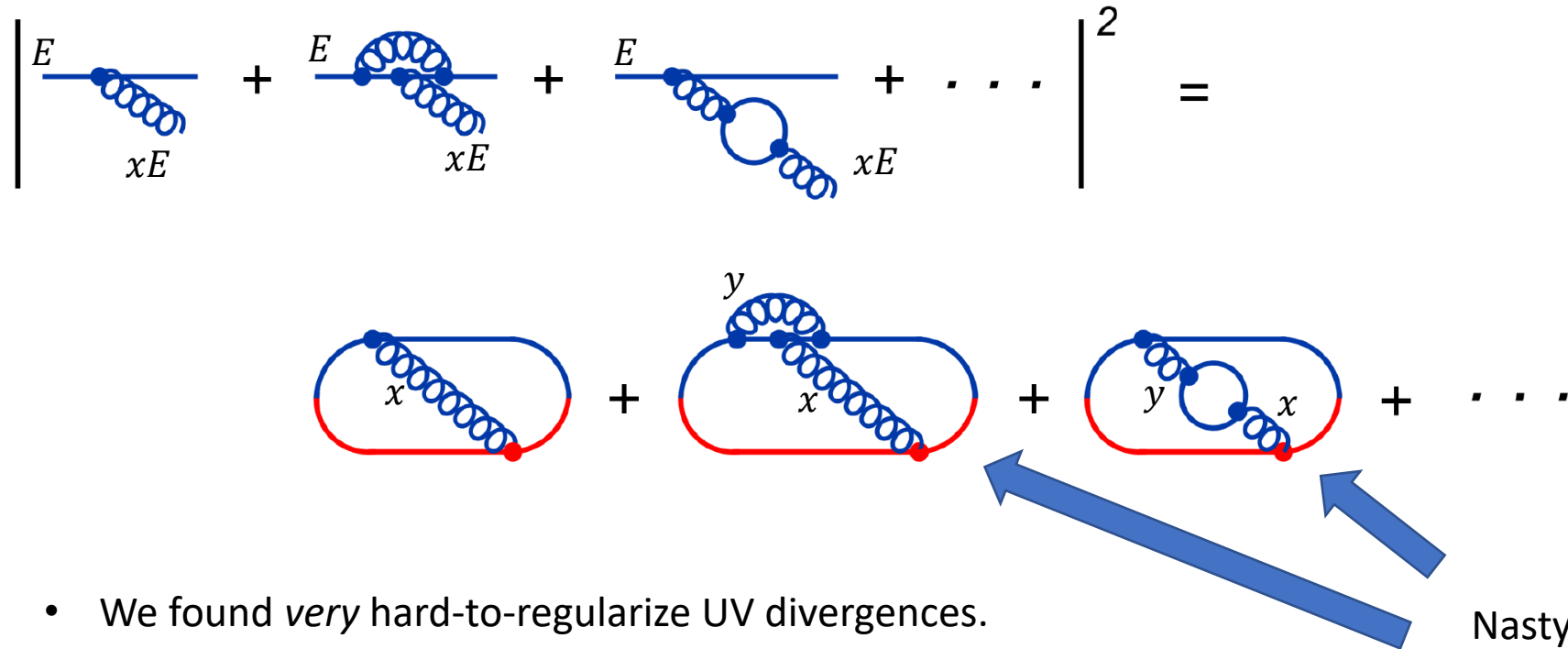
- We found *very* hard-to-regularize UV divergences.
- Switched over to large-Nf QED as a slightly simpler test case…

Nasty UV divergences!

A complete (test?) calculation in Large-Nf QED:
1- Overlapping virtual corrections to bremsstrahlung rate.
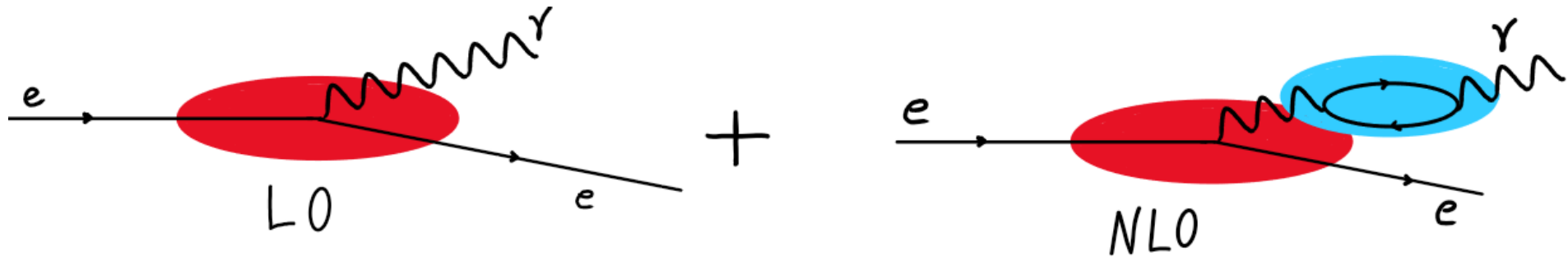
Non-canceling UV divergences regularized and correctly absorbed into renormalization of α_{QED}.

$$\left[\frac{d\Gamma}{dx_e}\right]^{NLO} = -\frac{N_f \alpha_{EM}}{6\pi}\left[\frac{d\Gamma}{dx_e}\right]^{LO} \ln\left(\frac{x_e \mu^4}{(1-x_e)^3 \, \hat{q}E}\right) + Stuff$$

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What we find?
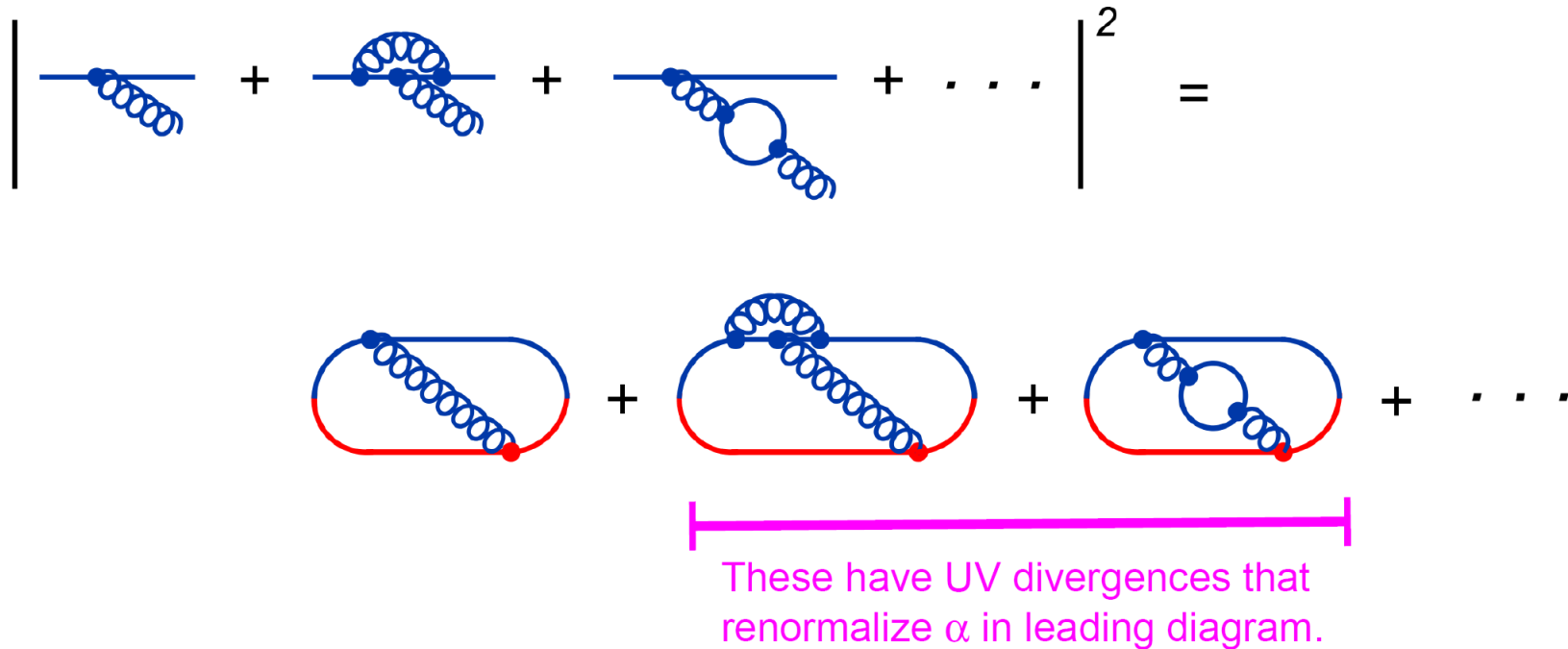
Overlap effects *enhance* energy loss and *reduce* stopping distance.

(a)

(b)
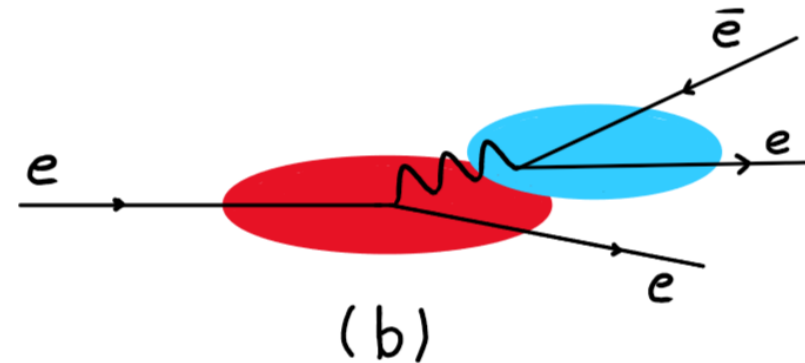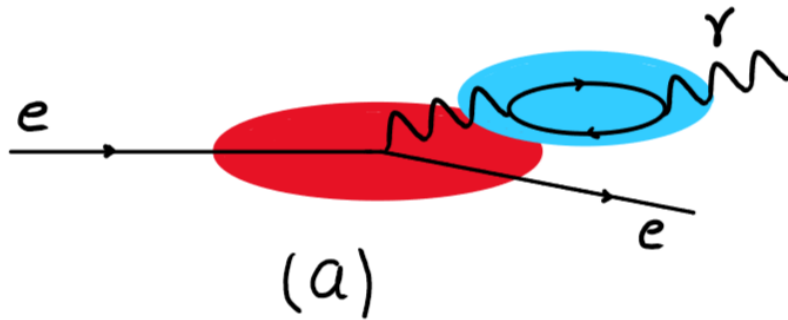
$$\frac{\Delta l}{l_{stop}} = -1.302 \, N_f \, \alpha_{QED}(\mu) \Big|_{\mu = (\hat{q}E)^{\frac{1}{4}}}$$

Calculated with Tanner Rase.

JHEP 1905 (2019) 004

Back to the QCD case: Qualitative results so far….

These have UV divergences that renormalize α in leading diagram.

- All non-canceling UV divergences can be absorbed into a renormalization of α_s.
- Cancel power law IR divergences when calculating IR safe quantities.

Thank You

4- Backup Slides

Previously on the Problem of overlapping formation times….

- Real double gluon bremsstrahlung.
- Avoiding soft emission approximations, we used large-N QCD.

Previously on the Problem of overlapping formation times….

Interesting results: Overlapping emissions are enhanced unless one emission is very soft!

Infrared Issue:
for $y \ll x \ll 1$

$$\frac{\Delta d\Gamma}{dxdy} \sim -\frac{\alpha_s^2}{xy^{\frac{3}{2}}}\sqrt{\frac{\hat{q}}{E}}$$

→Power law IR divergence to energy loss etc.

$$\pi^2 x\, y^{3/2}\, \Delta\, \frac{d\Gamma}{dx\, dy}$$

0.32
0.16
0
−0.16
−0.32
−0.48

Calculated with Han-Chih Chang.
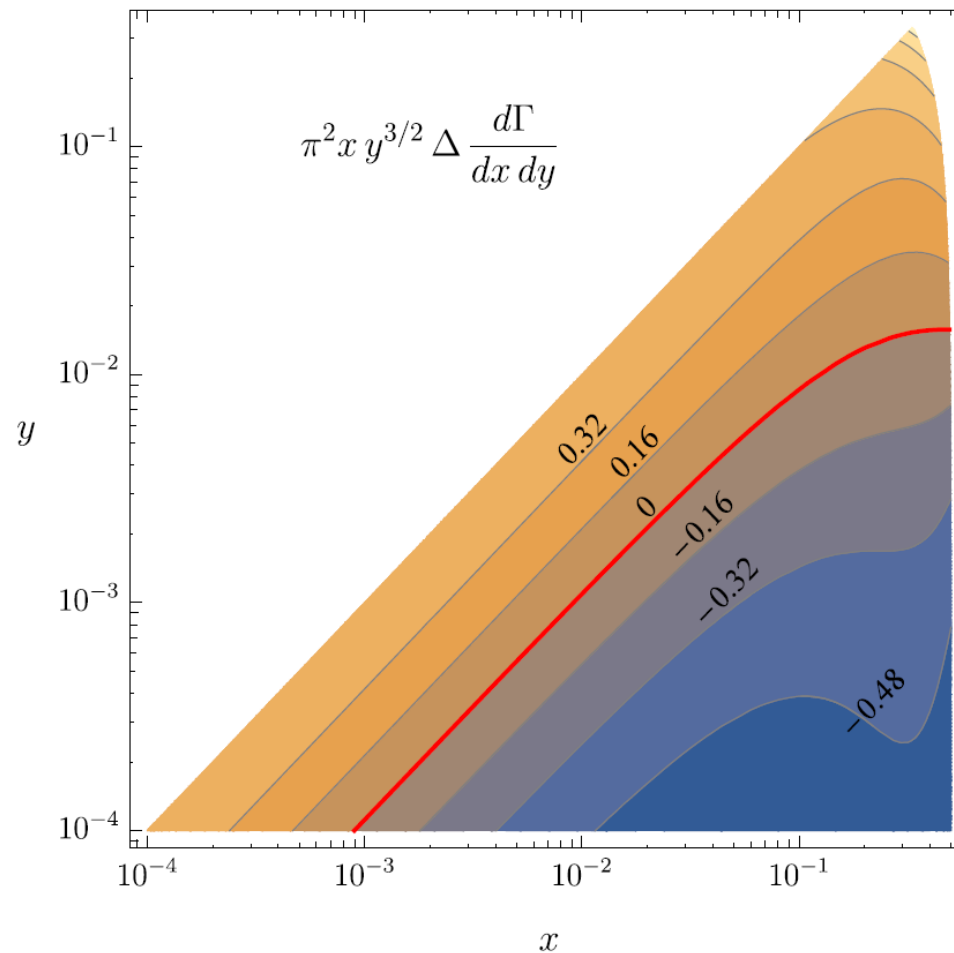
Number of high energy particles remains the same between splittings!
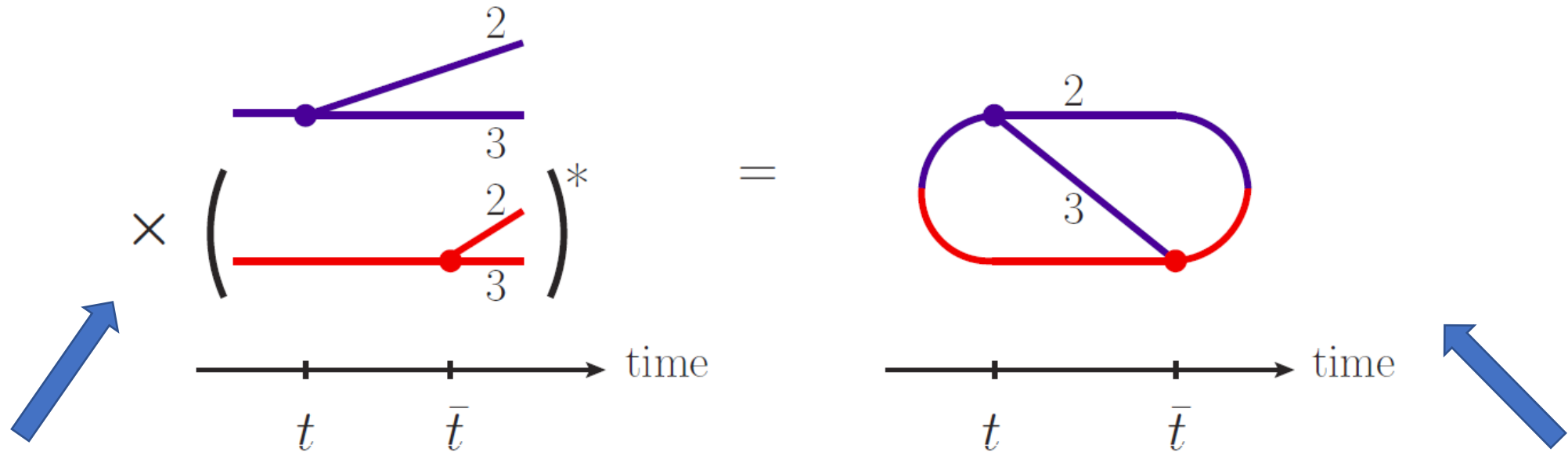
NOT Vacuum!

Medium effects given by non-Hermitian Hamiltonian.

$$rate \ \frac{d\Gamma}{dx} \sim \int (splitting\ vertex\ at\ t) \times (3 - particle\ evolution) \times (splitting\ vertex\ at\ \bar{t})$$

Imagine no medium!

- We need the time evolution of the object $\rho = |p_2 p_3\rangle\langle p_1|$. (Really, an off-diagonal density matrix element!)
- Given by an *effective* Hamiltonian

$$H_{total} = H_{free} + H_{med}$$

In vacuum,

$$H = H_{free} = \epsilon_3 + \epsilon_2 - \epsilon_1$$

- Jets are extremely collinear.

$$p_z \gg p_\perp$$

$$H_{free} \approx \frac{p_{1\perp}^2}{2p_{1z}} + \frac{p_{2\perp}^2}{2p_{2z}} + \frac{p_{3\perp}^2}{2p_{3z}} = \frac{P^2}{2M}$$

Here $M = -x_1 x_2 x_3 E$, $x_i = \frac{p_{iz}}{E}$ and $P = x_1 p_{\perp 2} - x_2 p_{\perp 1}$.

Medium effects

- The medium scatters particles passing through it.

Consider the probability of finding a particle with a certain transverse momentum $\boldsymbol{p}_\perp$,

$$\partial_t n(\boldsymbol{p}_\perp) = \int d^2 \boldsymbol{q}_\perp \, d\Gamma/d^2 \boldsymbol{q}_\perp [n(\boldsymbol{p}_\perp - \boldsymbol{q}_\perp) - n(\boldsymbol{p}_\perp)]$$

Fourier Transform the above,

$$\partial_t n(\boldsymbol{b}_\perp) = -\int d^2 \boldsymbol{q}_\perp \, d\Gamma/d^2 \boldsymbol{q}_\perp \left[1 - e^{i\boldsymbol{b}_\perp \cdot \boldsymbol{q}_\perp}\right] n(\boldsymbol{b}_\perp)$$

$$= \gamma(\boldsymbol{b}_\perp) n(\boldsymbol{b}_\perp)$$

So,

$$n(\boldsymbol{b}_\perp, t) \propto e^{\gamma(\boldsymbol{b}_\perp) t} n(\boldsymbol{b}_\perp, 0)$$

Medium effects

- The off-diagonal density matrix element ρ has the same time evolution:

$$\rho(\boldsymbol{b}_\perp, t) \propto \boldsymbol{e}^{-i\boldsymbol{H}t} \boldsymbol{\rho}(\boldsymbol{b}_\perp, 0)$$

i.e.

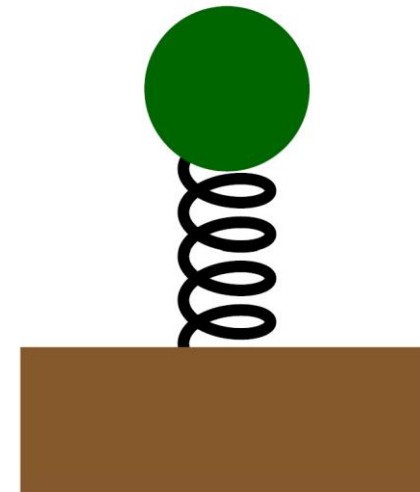$$H_{med} = i\gamma(\boldsymbol{b}_\perp)$$

- Medium introduces an *imaginary potential* into the effective Hamiltonian.
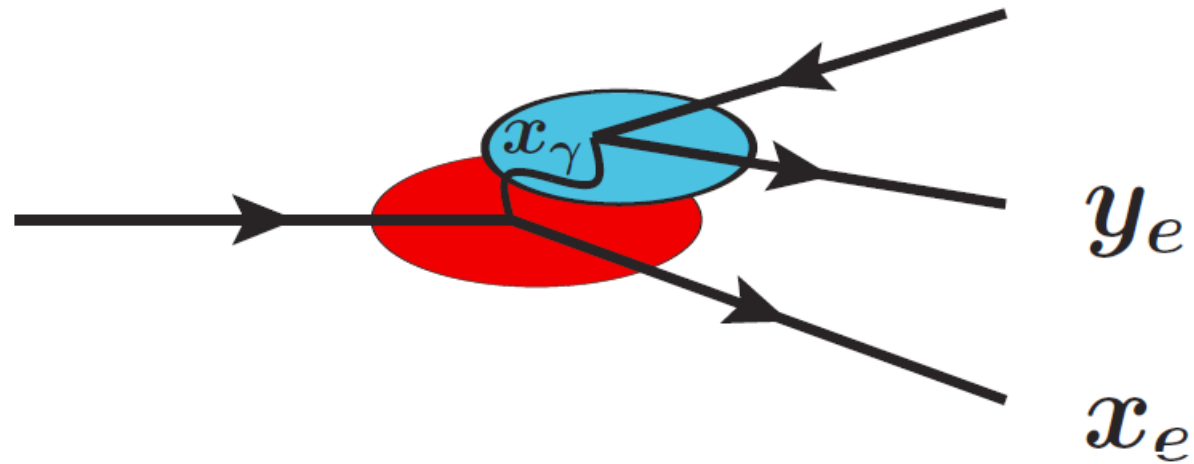
Harmonic approximation

- Particles in the shower stay very close together.
- We can expand H_{med} in a small $\boldsymbol{b}_\perp$ limit.

$$H = \frac{P^2}{2M} + \frac{1}{2} M\Omega^2 B^2$$

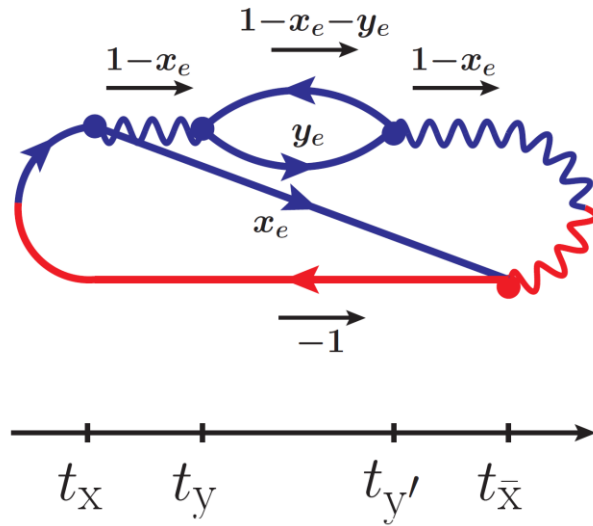Here $\Omega = \sqrt{-i\frac{\hat{q}}{2E}\left(\frac{1}{x_1} + \frac{1}{x_1} + \frac{1}{x_1}\right)}$

Focus on Large-Nf QED for now
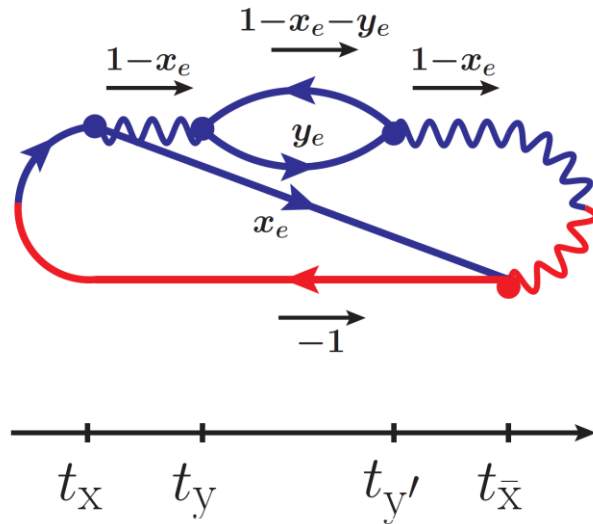

Much smaller set of diagrams to calculate.

We focused on Large-Nf QED for now…

+ *Permuations*

Much smaller set of diagrams to calculate.

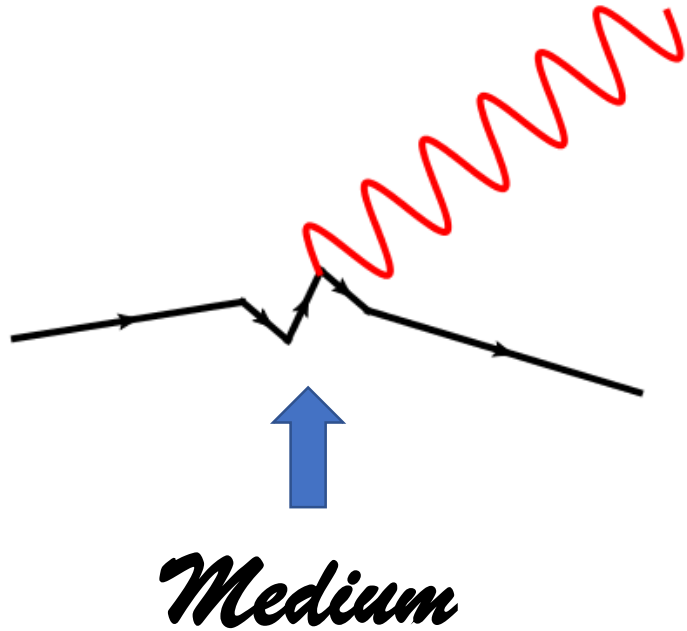# We focused on Large-Nf QED for now...

$+$ *Permuations*

Much smaller set of diagrams to calculate.

We find: Non-canceling UV divergences can be absorbed into the usual renormalization of QED coupling constant.
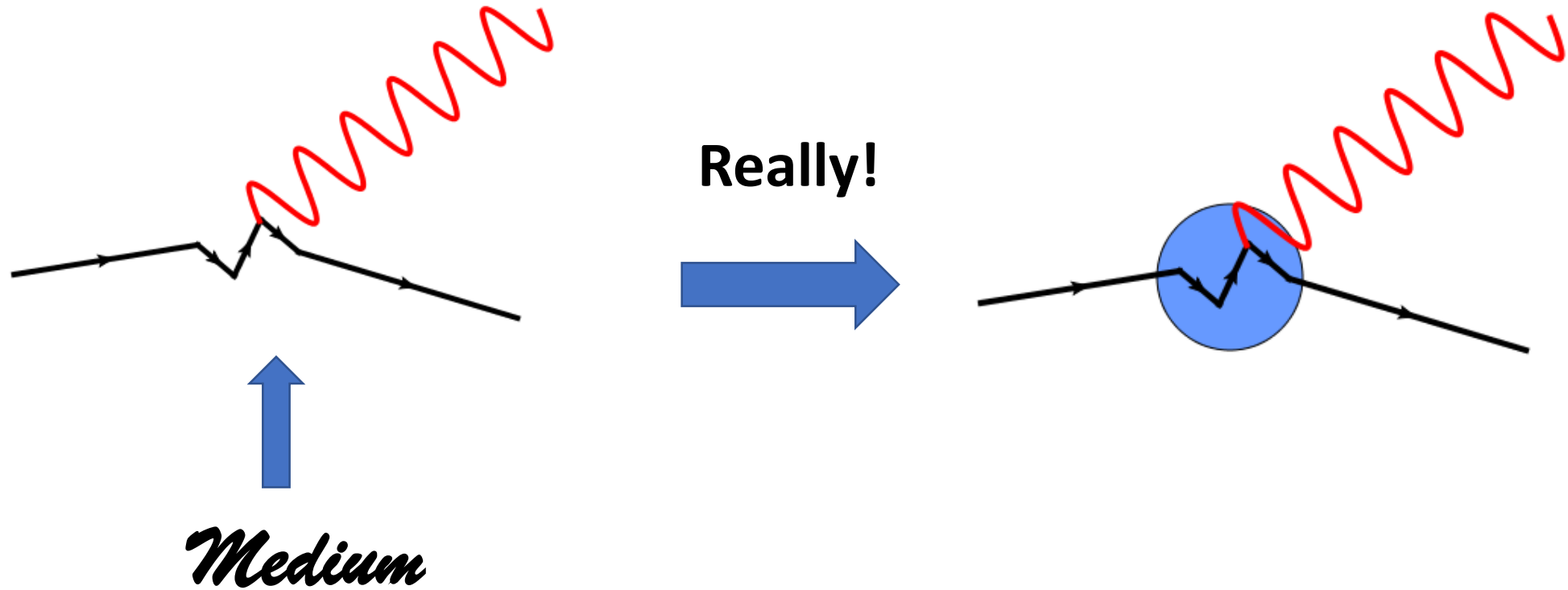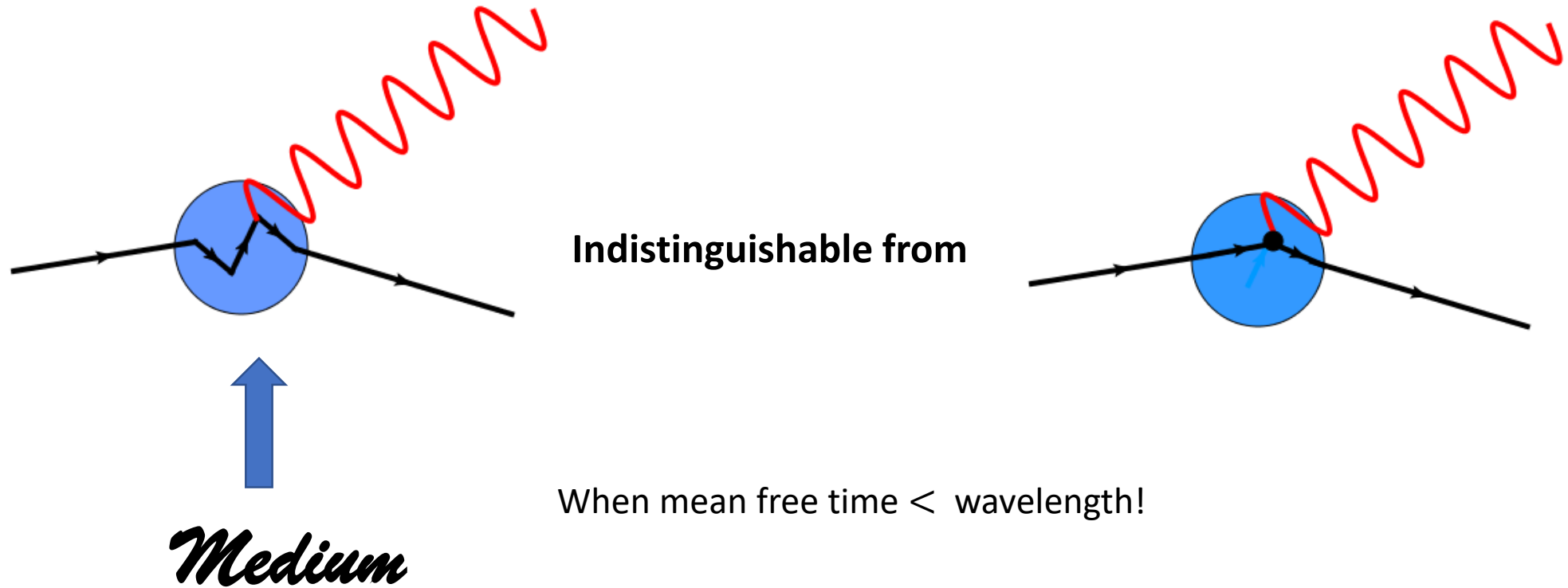
Consider an electron scattering through medium.

Naively, each collision provides an opportunity for radiation.

Probability ~ α per collision.

Medium

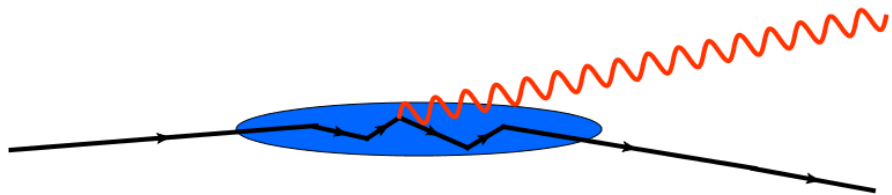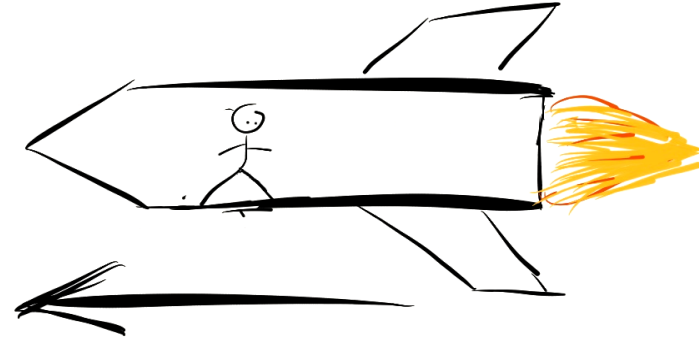# But the photon has a finite wavelength (obviously!)

Really!

Medium

But the photon has a finite wavelength (obviously!)

Indistinguishable from

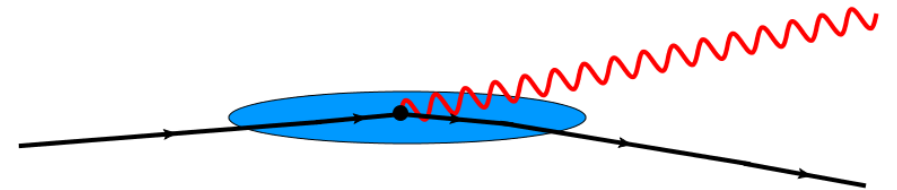When mean free time $<$ wavelength!

Medium

Landau-Pomeranchuk-Migdal effect

At relativistic speeds, the fuzzy region becomes elongated due to relativistic time dilation.

Indistinguishable from

Formation Time

$l_{form} \propto \sqrt{E}$ for fixed x

Emission probability ~ α per formation length

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