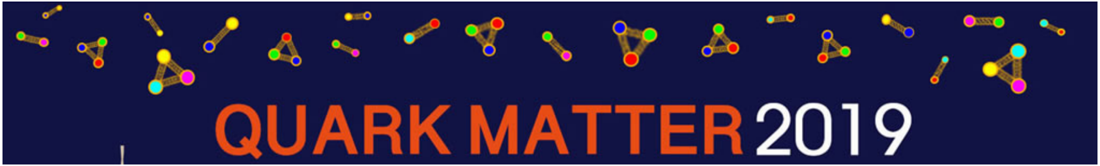

Quantifying heavy quark transport coefficients with an improved transport model

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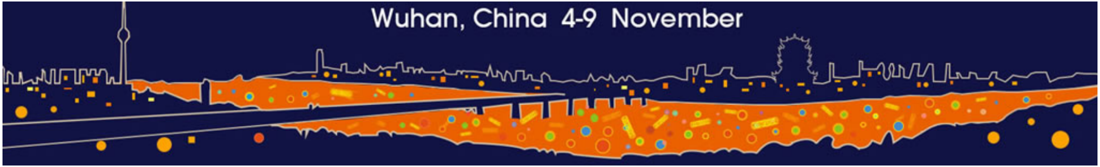
November 5, 2019



This work is supported by US DOE Grant no. DE-FG02-05ER41367 & NSF Grant no. ACI-1550225.

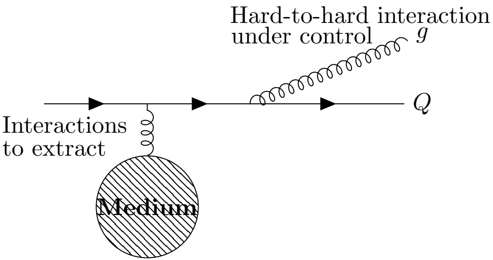
1. Extraction of heavy quark transport coefficients
2. Improved treatment of medium-induced radiation
3. Bayesian extraction of heavy quark $\hat{q}$
4. Summary

Extraction of heavy quark transport coefficients

- Heavy-quark-medium interactions are quantified by transport coefficients,

$$\hat{q} = \frac{d\langle(\Delta p_\perp)^2\rangle}{dt}, \cdots$$

- Hard to compute from first principle. Phenomenological determination is complementary.

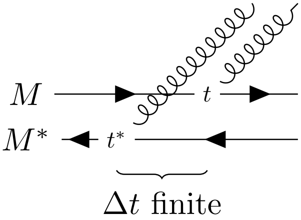


- Accuracy of the dynamical modeling (radiation) affects the $\hat{q}$ extraction.
- Not an easy task to implement the gluon radiation in transport equation.

Transport equations: a class of widely used dynamical model

Semi-classcical transport:

- Time-evolution particle's distribution function: $\frac{df}{dt} = \mathcal{C}[f]$.
- Localized interactions: $\mathcal{C}[f(t, \mathbf{x}, \mathbf{p})]$.

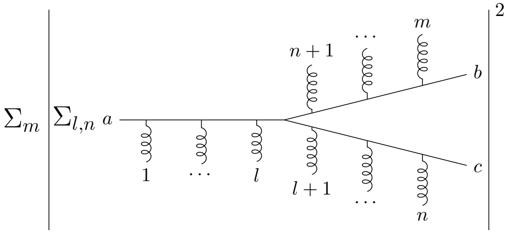


Challenge:

- Medium-induced radiation has finite formation time τ_f (de-localized): hard to fit in the above framework.
- Need an improved, non-local implementation of medium-induced gluon radiation.

Improved treatment of medium-induced radiation

- The LPM effect: non-factorizable multiple-collisions, suppressed radiation rate. ($\tau_f \gg \lambda_{\rm el}$).



Improved treatment of medium-induced radiation

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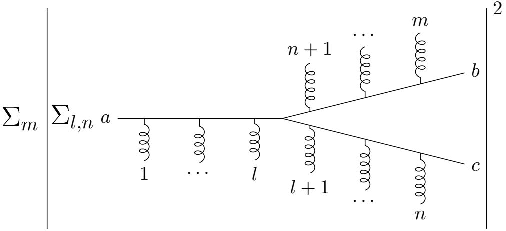
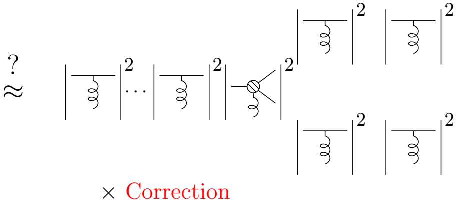


- Semi-classical transport: similar kinematics, wrong probability

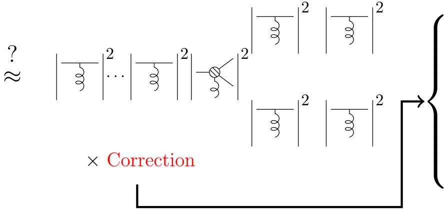


Looking for multiplicative corrections.

Improved treatment of medium-induced radiation

Let $N = \tau_f/\lambda$. From analysis[1] of the AMY equation[2] for the single-gluon emission rate:

Semi-classical rate ($N < 1$)	Leading-ln N ($N \gg 1$)	NLL
$\frac{dR^{\mathrm{incoh}}}{d\omega}$	$\propto \frac{dR^{\mathrm{incoh}}}{d\omega}\frac{1}{N}$	$\propto \frac{dR^{\mathrm{incoh}}}{d\omega}\frac{1}{N'}$ improved N'

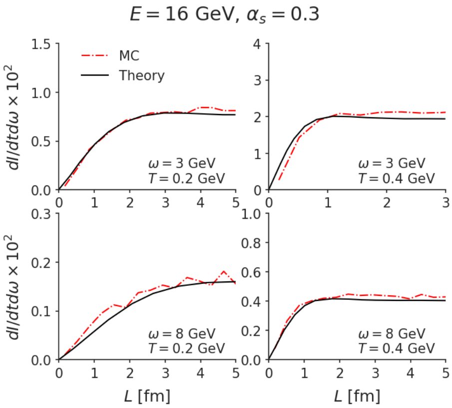


- Evolve the system from $t = 0$ to τ_f, and determine $\tau_f(t) = \frac{2x(1-x)E}{k_\perp^2(t)}$. Compute $N = \tau_f/\lambda$.
- Correct the radiation rate with $\min\{1, \frac{1}{N}\}$.
- Naturally include the incoherent limit when $N < 1$.

[1] PRD 78 065008, JHEP 07 057

[2] A thermal field theory approach to the radiation rate. It resums multiple scattering in an infinite medium.

Improved treatment of medium-induced radiation



Compare to the numerical solution of the Zakharov formula[3] (labeled "Theory") which has

- AMY at large L.
- Essential finite size effect at small L.

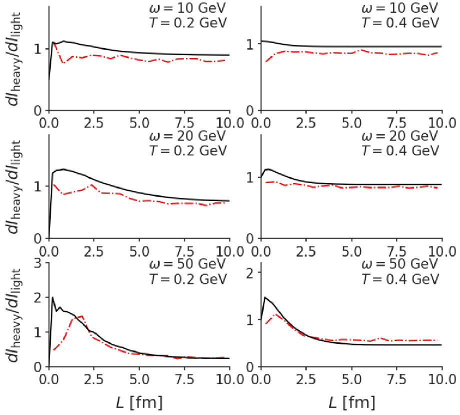
The improved transport model (labeled "MC")):

- **Quantitatively** reproduce the radiation rate deep-inside the medium.
- **Qualitatively** describe the finite size effects.

[3] PRC 82 064902, JETP Lett. 65, 615

Improved treatment of medium-induced radiation: mass effect

Bottom quark, $M = 4.2$ GeV
$E = 100$ GeV, $\alpha_s = 0.3$



Mass effect for gluon radiation from heavy quark

- Massive kinematics: turn off radiation for $p \lesssim M$.
- A shortened formation time

$$\tau_f = \frac{2x(1-x)E}{k_\perp^2 + x^2 M^2}$$

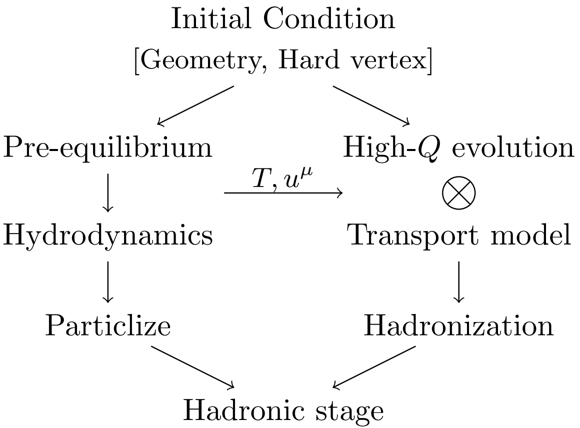
- A dead-cone **approximation** to the cross-section,

$$\frac{dR^M}{d\omega dk_\perp^2} = \frac{dR^{M=0}}{d\omega dk_\perp^2}\left(\frac{\theta^2}{\theta^2+\theta_D^2}\right)^2, \theta_D = \frac{M}{E}, \theta = \frac{k_\perp}{\omega}$$

- Agree with theory at large L. Deviate at small L.

A comprehensive simulation framework

Hydro-based model w/ calibrated parameters (η/s, etc)[4].



Fragmentation& Recombination[5]

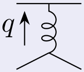
[4] arXiv:1804.06469: geometry initial condition, pre-equilibrium dynamics, 2+1D viscous hydrodynamics, Cooper-Frye freezeout, and a hadronic afterburner.

[5] PRC 88 044907

Parametrization of probe-medium interaction

A flexible way of parametrization: separate treatments for different regions of **momentum transfer** (q) between probe and medium[6]

$q > Q_{\rm cut} \gtrsim m_D$



Vacuum-like collision. Hard contribution to $\hat{q}$: $\hat{q}^H_{\rm LO}[Q_{cut}]$

+

$q < Q_{\rm cut}$

Drag & diffusion: $\hat{q}^S_{LO}[Q_{cut}] + \Delta\hat{q}$. Soft contribution to $\hat{q}$

[6] JHEP 1603 095 developed such a separation for the perturbative approach.

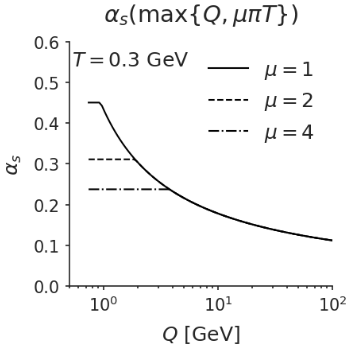
Parametrization of probe-medium coupling

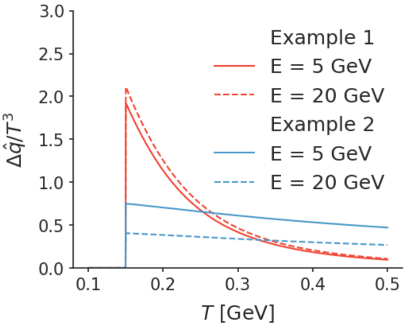
Transverse momentum broadening: $\hat{q} = \hat{q}_{\mathrm{LO}}^{H+S}(\alpha_s) + \Delta\hat{q}$

Longitudinal momentum broadening: $\hat{q}_L = \hat{q}_{L,\mathrm{LO}}^{H+S}(\alpha_s) + \Delta\hat{q}_L$

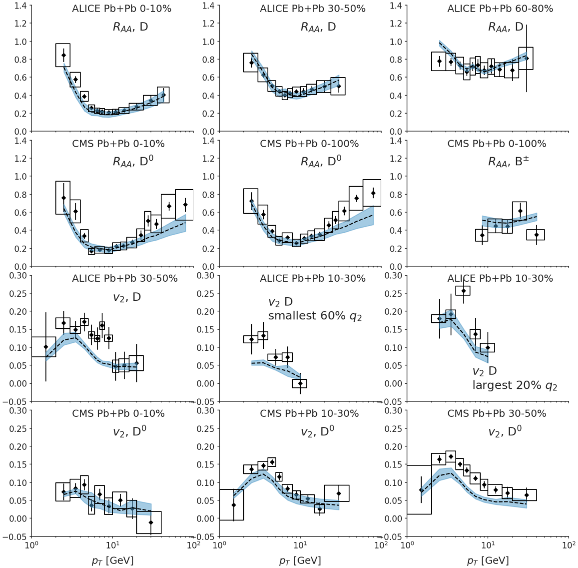
$$\Delta\hat{q} = \frac{KT^3}{\left[1+\left(a\frac{T}{T_c}\right)^p\right]\left[1+\left(b\frac{E}{T}\right)^q\right]}$$

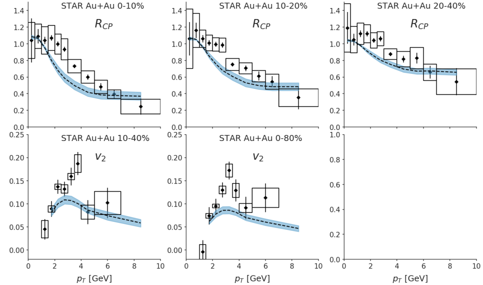
$$\Delta\hat{q}_L = \frac{\Delta\hat{q}}{2}\left(\frac{E}{M}\right)^\gamma$$





The overall description of the data after global Bayesian analysis





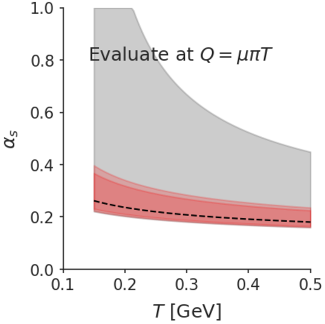
Experimental data taken at RHIC (up) and LHC (left). Bands show the 90% credible region of the model prediction.

LHC: PRL 120, 102301; JHEP 10 (2018) 174; PRL 120, 202301; PLB 782, 474

RHIC: PRL 118, 212301; PRC 99, 034908

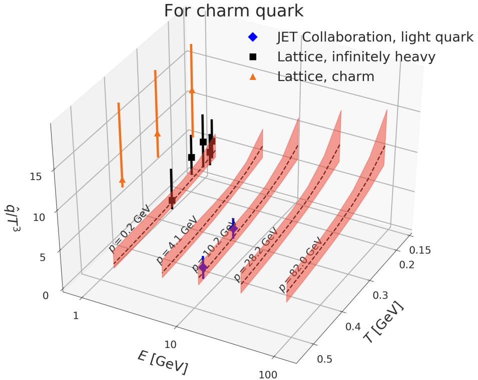
Extracted confidence limits of the transport properties

Effective coupling (LHC+RHIC)



Note $g = \sqrt{4\pi\alpha_s}$ is still large.

$\hat{q}$ as function of E and T (LHC+RHIC)



- $\hat{q}/T^3$ slowly increases with energy, and decreases with temperature.
- At $p \approx 10$ GeV, the extraction is consistent with earlier estimation for light quark[7].

[7] JET: PRC 90, 014909. Lattice: PRD 85, 014510 and PRD 86, 014509

Summary

- A transport model with improved treatment of medium-induced gluon radiation.
  Application to jet observables by Wenkai Fan, Poster JT #9
- Flexible parametrization of probe-medium interaction. Interpolate scattering & diffusion.
- The extracted charm $\hat{q}$ shows moderate temperature and energy dependence. Consistent with JET Collaboration light quark $\hat{q}$ extraction at large momentum.
- Comparison between lattice results and phenomenology extraction needs further investigation.

Back-up: more details

$$\frac{dI^{\mathrm{coh}}}{dx} \propto \left| \quad \right|^2 \times \boxed{\frac{\lambda_{\mathrm{el}}}{\sqrt{2x(1-x)E/\hat{q}_{\mathrm{eff}}(\#_1)}}}.$$

1. Suppose "$a \to b + c$" is sampled from $\left| \quad \right|^2$ at $t = t_0$. Compute $\tau_f = \frac{2x(1-x)E}{k_\perp^2}$.
2. Keep propagating. Elastic processes $\left| \quad \right|^2$ increase $k_\perp^2$ (decrease τ_f) until $t > t_0 + \tau_f(t)$.
3. Now, reject the semi-classically sampled splitting with probability $\mathrm{Prob} = \lambda_{\mathrm{el}}/\tau_f$.

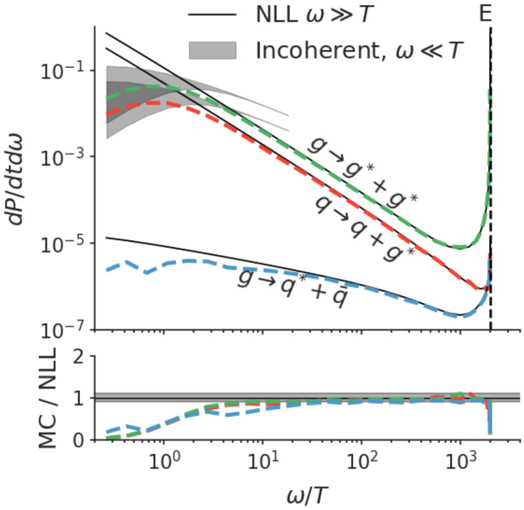
Why: $\tau_f = \frac{2x(1-x)E}{k_\perp^2}$, on average $k_\perp^2 \sim \hat{q}_{\mathrm{eff}}\tau_f$, so $\langle \tau_f^{-1} \rangle \sim \sqrt{2x(1-x)E/\hat{q}_{\mathrm{eff}}(\#_0)}$

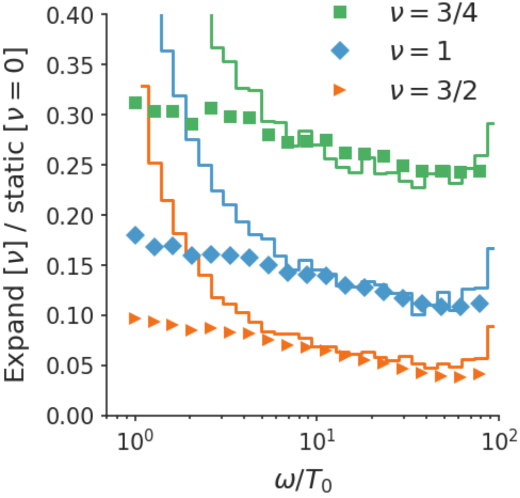
More accurately, mimic the next-to-leading-log($\#$) effect:

$$\mathrm{Prob} \to 0.75\sqrt{\ln \#_1 / \ln \#_0}\,\mathrm{Prob},$$

$\#_1 = 1 + \tau_f/\lambda_{el}$
$\#_0 = 1 + 6ET/m_D^2$
0.75: a fitted numerical constant

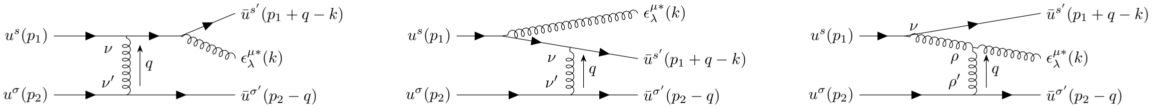
Back-up: infinite medium & expanding medium tests





Backup: a 2 $\rightarrow$ 3 matrix-elements example: $q \rightarrow q + g$

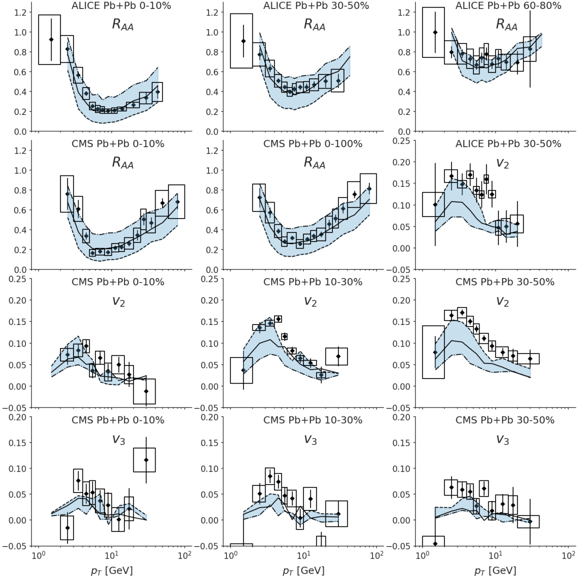
Contributions to the $y_k > 0$ region in the few-body center-of-mass frame.



$$
\begin{aligned}
\overline{|M^2|}_{gq\rightarrow ggq} &= g^4 \frac{C_F}{d_F}\frac{4s^2}{q_\perp^4} x(1-x) g^2 \frac{1+(1-x)^2}{x}\left(C_F \vec{A}^2 + C_F \vec{B}^2 - (2C_F - C_A)\,\vec{A}\cdot\vec{B}\right)\\
\vec{A} &= \frac{\vec{k}_\perp - \vec{q}_\perp}{(\vec{k}_\perp - \vec{q}_\perp)^2} - \frac{\vec{k}_\perp - x\vec{q}_\perp}{(\vec{k}_\perp - x\vec{q}_\perp)^2}\\
\vec{B} &= \frac{\vec{k}_\perp - \vec{q}_\perp}{(\vec{k}_\perp - \vec{q}_\perp)^2} - \frac{\vec{k}_\perp}{\vec{k}_\perp^2}
\end{aligned}
$$

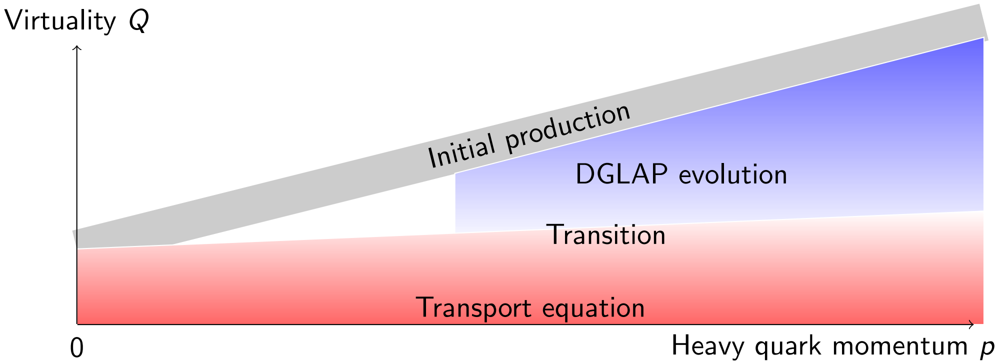
The $y_k < 0$ region is obtained by a redefinition of x and $\vec{q}_\perp$.

Backup: pure weakly coupled results



- $\mu = \pi T$ (dash-dotted), $2\pi T$ (solid), and $4\pi T$(dashed).
- $\mu = 2\pi T$ gives good description of the yield (in terms of R_{AA}). But underestimates v_2 at high-p_T.
- The pure weakly coupled approach is often formulated in the limit $E, \omega \gg T, k_\perp, q_\perp$, which may not be true at low-p_T.

Back-up: Interfacing initial production and in-medium transport



- DGLAP evolution: high-virtuality parton evolution.
- Transport: low-virtuality, up to $\Delta k_\perp^2 = \int_0^{\tau_f} \hat{q} dt$, can be determined from the simulation.
- Our current prescription: stop the DGLAP evolution when $Q^2 < R_v \Delta k_\perp^2$.