Quantifying heavy quark transport coefficients with an improved transport model

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1. Extraction of heavy quark transport coefficients

2. Improved treatment of medium-induced radiation

3. Bayesian extraction of heavy quark $\hat{q}$

4. Summary
1 Extraction of heavy quark transport coefficients

2 Improved treatment of medium-induced radiation

3 Bayesian extraction of heavy quark $\hat{q}$

4 Summary
Extraction of heavy quark transport coefficients

- Heavy-quark-medium interactions are quantified by transport coefficients,

\[ \hat{q} = \frac{d\langle (\Delta p_\perp)^2 \rangle}{dt}, \ldots \]

- Hard to compute from first principle. Phenomenological determination is complementary.

- Accuracy of the dynamical modeling (radiation) affects the \( \hat{q} \) extraction.

- Not an easy task to implement the gluon radiation in transport equation.
Transport equations: a class of widely used dynamical model

Semi-classical transport:
- Time-evolution particle’s distribution function: $\frac{df}{dt} = C[f]$.
- Localized interactions: $C[f(t, x, p)]$.

Challenge:
- Medium-induced radiation has finite formation time $\tau_f$ (de-localized): hard to fit in the above framework.
- Need an improved, non-local implementation of medium-induced gluon radiation.
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Improved treatment of medium-induced radiation

- The LPM effect: non-factorizable multiple-collisions, suppressed radiation rate. ($\tau_f \gg \lambda_{el}$).

\[ \sum_m \sum_{l,n} \quad \text{with diagrams} \]
Improved treatment of medium-induced radiation

- The LPM effect: non-factorizable multiple-collisions, suppressed radiation rate. \( (\tau_f \gg \lambda_{el}) \).

- Semi-classical transport: similar kinematics, wrong probability

Looking for multiplicative corrections.
Improved treatment of medium-induced radiation

Let \( N = \frac{\tau_f}{\lambda} \). From analysis\(^1\) of the AMY equation\(^2\) for the single-gluon emission rate:

<table>
<thead>
<tr>
<th>Semi-classical rate ((N &lt; 1))</th>
<th>Leading-ln ( N ) ((N \gg 1))</th>
<th>NLL</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{dR_{\text{incoh}}}{d\omega} )</td>
<td>( \propto \frac{dR_{\text{incoh}}}{d\omega} \frac{1}{N} )</td>
<td>( \propto \frac{dR_{\text{incoh}}}{d\omega} \frac{1}{N'} ) improved ( N' )</td>
</tr>
</tbody>
</table>

- Evolve the system from \( t = 0 \) to \( \tau_f \), and determine \( \tau_f(t) = \frac{2x(1-x)E}{k_{\perp}^2(t)} \). Compute \( N = \frac{\tau_f}{\lambda} \).
- Correct the radiation rate with \( \min\{1, \frac{1}{N}\} \).
- Naturally include the incoherent limit when \( N < 1 \).

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\(^1\) PRD 78 065008, JHEP 07 057

\(^2\) A thermal field theory approach to the radiation rate. It resums multiple scattering in an infinite medium.
Improved treatment of medium-induced radiation

Compare to the numerical solution of the Zakharov formula\(^3\) (labeled “Theory”) which has

- AMY at large \(L\).
- Essential finite size effect at small \(L\).

The improved transport model (labeled “MC”):

- **Quantitatively** reproduce the radiation rate deep-inside the medium.
- **Qualitatively** describe the finite size effects.

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\(^3\)PRC 82 064902, JETP Lett. 65, 615
Improved treatment of medium-induced radiation: mass effect

**Bottom quark,** $M = 4.2$ GeV  
$E = 100$ GeV, $\alpha_s = 0.3$

**Mass effect for gluon radiation from heavy quark**
- Massive kinematics: turn off radiation for $p \lesssim M$.
- A shortened formation time
  $$\tau_f = \frac{2x(1-x)E}{k^2 + x^2M^2}$$
- A dead-cone approximation to the cross-section,
  $$\frac{dR^M}{d\omega dk_\perp} = \frac{dR^{M=0}}{d\omega dk_\perp} \left( \frac{\theta^2}{\theta^2 + \theta_D^2} \right)^2, \theta_D = \frac{M}{E}, \theta = \frac{k_\perp}{\omega}$$
- Agree with theory at large $L$. Deviate at small $L$. 

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A comprehensive simulation framework

Initial Condition
[Geometry, Hard vertex]

Pre-equilibrium → High-Q evolution

Hydrodynamics

Particlize

Hadronic stage

Transport model

Hadronization

Hydro-based model w/ calibrated parameters ($\eta/s$, etc)$^4$.

$^4$arXiv:1804.06469: geometry initial condition, pre-equilibrium dynamics, 2+1D viscous hydrodynamics, Cooper-Frye freezeout, and a hadronic afterburner.

$^5$PRC 88 044907
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Parametrization of probe-medium interaction

A flexible way of parametrization: separate treatments for different regions of momentum transfer ($q$) between probe and medium\textsuperscript{6}

\[
q > Q_{\text{cut}} \gtrsim m_D
\]

Vacuum-like collision. Hard contribution to $\hat{q}$: $\hat{q}^H_{\text{LO}}[Q_{\text{cut}}]

\[
q < Q_{\text{cut}}
\]

Drag & diffusion:

$\hat{q}^S_{\text{LO}}[Q_{\text{cut}}] + \Delta \hat{q}$.

Soft contribution to $\hat{q}$

\textsuperscript{6} JHEP 1603 095 developed such a separation for the perturbative approach.
Parametrization of probe-medium coupling

Transverse momentum broadening: \( \hat{q} = \hat{q}_{LO}^{H+S}(\alpha_s) + \Delta \hat{q} \)

Longitudinal momentum broadening: \( \hat{q}_L = \hat{q}_{LO}^{H+S}(\alpha_s) + \Delta \hat{q}_L \)

\[
\Delta \hat{q} = \frac{KT^3}{1 + \left( a \frac{T}{T_c} \right)^p} \left[ 1 + \left( b \frac{E}{T} \right)^q \right]
\]

\[
\Delta \hat{q}_L = \frac{\Delta \hat{q}}{2} \left( \frac{E}{M} \right)^{\gamma}
\]

\( \alpha_s(\text{max}\{Q, \mu\pi T\}) \)

\( T = 0.3 \text{ GeV} \)

\( \mu = 1 \), \( \mu = 2 \), \( \mu = 4 \)

\( Q \text{ [GeV]} \)

\( T \text{ [GeV]} \)

\( \Delta \hat{q}/T^3 \)

\( E = 5 \text{ GeV} \), \( E = 20 \text{ GeV} \)

Example 1

Example 2
The overall description of the data after global Bayesian analysis

Experimental data taken at RHIC (up) and LHC (left). Bands show the 90% credible region of the model prediction.

LHC: PRL 120, 102301; JHEP 10 (2018) 174; PRL 120, 202301; PLB 782, 474
RHIC: PRL 118, 212301; PRC 99, 034908
Extracted confidence limits of the transport properties

Effective coupling (LHC+RHIC)

\[ \hat{g} = \sqrt{4\pi\alpha_s} \] is still large.

- \( \hat{q}/T^3 \) slowly increases with energy, and decreases with temperature.
- At \( p \approx 10 \text{ GeV} \), the extraction is consistent with earlier estimation for light quark\(^7\).

\(^7\)JET: PRC 90, 014909. Lattice: PRD 85, 014510 and PRD 86, 014509
Extraction of heavy quark transport coefficients

Improved treatment of medium-induced radiation

Bayesian extraction of heavy quark $q$

Summary
Summary

- A transport model with improved treatment of medium-induced gluon radiation. Application to jet observables by Wenkai Fan, Poster JT #9
- The extracted charm $\hat{q}$ shows moderate temperature and energy dependence. Consistent with JET Collaboration light quark $\hat{q}$ extraction at large momentum.
- Comparison between lattice results and phenomenology extraction needs further investigation.
Back-up: more details

\[
\frac{dl^{\text{coh}}}{dx} \propto |\langle \frac{1}{2} \rangle|^2 \times \frac{\lambda_{\text{el}}}{\sqrt{2x(1-x)E/\hat{q}_\text{eff}(\#_1)}}.
\]

1. Suppose "a → b + c" is sampled from $|\langle \frac{1}{2} \rangle|^2$ at $t = t_0$. Compute $\tau_f = \frac{2x(1-x)E}{k_{\perp}^2}$.

2. Keep propagating. Elastic processes $|\langle \frac{1}{2} \rangle|^2$ increase $k_{\perp}^2$ (decrease $\tau_f$) until $t > t_0 + \tau_f(t)$.

3. Now, reject the semi-classically sampled splitting with probability $\text{Prob} = \lambda_{\text{el}}/\tau_f$.

   Why: $\tau_f = \frac{2x(1-x)E}{k_{\perp}^2}$, on average $k_{\perp}^2 \sim \hat{q}_\text{eff}\tau_f$, so $\langle \tau_{f^{-1}} \rangle \sim \sqrt{2x(1-x)E/\hat{q}_\text{eff}(\#_0)}$.

More accurately, mimic the next-to-leading-log(#) effect:

\[
\text{Prob} \rightarrow 0.75\sqrt{\ln \#_1/\ln \#_0} \text{Prob}, \quad \#_1 = 1 + \tau_f/\lambda_{\text{el}} \quad \#_0 = 1 + 6ET/m_D^2
\]

0.75: a fitted numerical constant
Back-up: infinite medium & expanding medium tests

\[ \text{NLL } \omega \gg T \]
\[ \text{Incoherent, } \omega \ll T \]

\[ g \rightarrow g^* + g^* \]
\[ q \rightarrow q + g^* \]

\[ g \rightarrow q^- + q^+ \]

\[ \text{Expand } [\nu]/\text{static } [\nu = 0] \]

\[ \nu = 3/4 \]
\[ \nu = 1 \]
\[ \nu = 3/2 \]
Backup: a $2 \rightarrow 3$ matrix-elements example: $q \rightarrow q + g$

Contributions to the $y_k > 0$ region in the few-body center-of-mass frame.

$$
\begin{align*}
|\mathcal{M}^2|_{ggq \rightarrow ggq} &= g^4 C_F \frac{4 s^2}{d_F q_{\perp}^4} x (1-x) g^2 \frac{1 + (1-x)^2}{x} \left( C_F \vec{A}^2 + C_F \vec{B}^2 - (2 C_F - C_A) \vec{A} \cdot \vec{B} \right) \\
\vec{A} &= \frac{\vec{k}_{\perp} - \vec{q}_{\perp}}{(\vec{k}_{\perp} - \vec{q}_{\perp})^2} - \frac{\vec{k}_{\perp} - x \vec{q}_{\perp}}{(\vec{k}_{\perp} - x \vec{q}_{\perp})^2} \\
\vec{B} &= \frac{\vec{k}_{\perp} - \vec{q}_{\perp}}{(\vec{k}_{\perp} - \vec{q}_{\perp})^2} - \frac{\vec{k}_{\perp}}{k_{\perp}^2}
\end{align*}
$$

The $y_k < 0$ region is obtained by a redefinition of $x$ and $\vec{q}_{\perp}$.
Backup: pure weakly coupled results

- $\mu = \pi T$ (dash-dotted), $2\pi T$ (solid), and $4\pi T$ (dashed).
- $\mu = 2\pi T$ gives good description of the yield (in terms of $R_{AA}$). But underestimates $v_2$ at high-$p_T$.
- The pure weakly coupled approach is often formulated in the limit $E, \omega \gg T, k_\perp, q_\perp$, which may not be true at low-$p_T$. 
DGLAP evolution: high-virtuality parton evolution.

Transport: low-virtuality, up to $\Delta k_{\perp}^2 = \int_0^{\tau_f} \hat{q} dt$, can be determined from the simulation.

Our current prescription: stop the DGLAP evolution when $Q^2 < R_v \Delta k_{\perp}^2$. 