Electrical Cross-Conductivity in the Hadron Gas

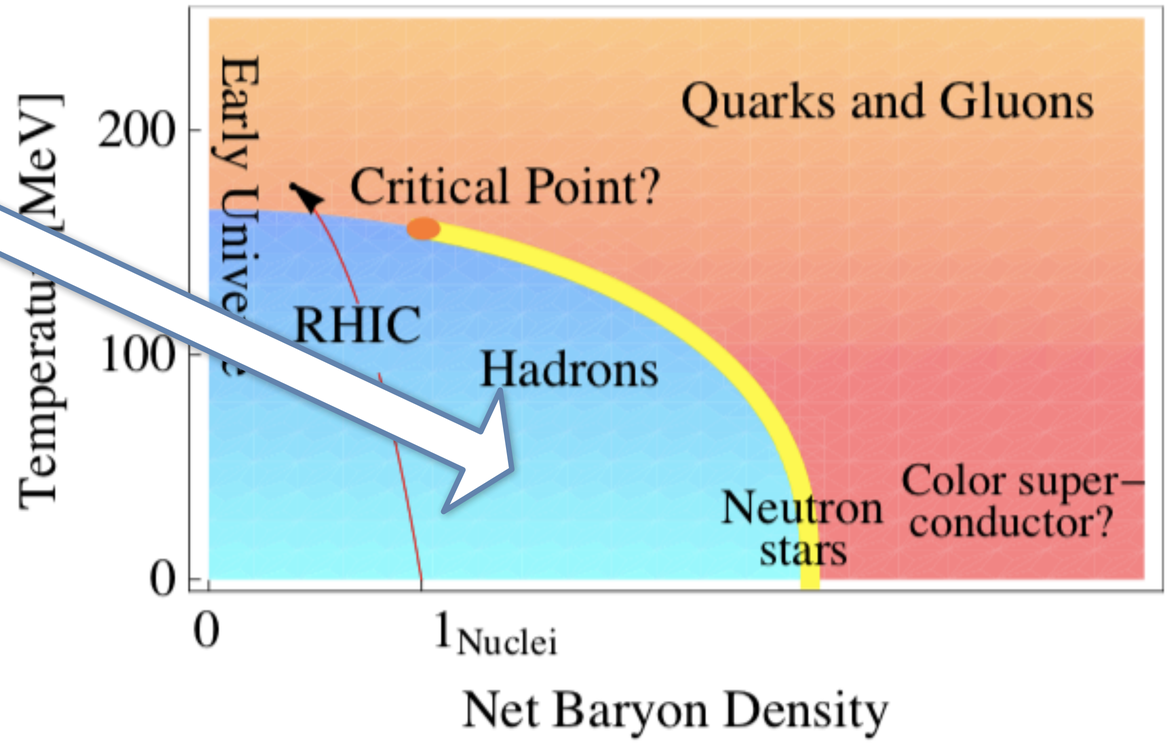
Jean-Bernard Rose

with G. Denicol, M. Greif, J. Hammelmann,
J. Fotakis, H. Elfner, C. Greiner

Quark Matter 2019, Wuhan, China, November 5th, 2019

Constraining hadronic active degrees of freedom

- Composed of hadrons
- Which ones are active degrees of freedom, and do we know them all?
- How do we constrain this?
- **Additional ways of constraining these properties are needed: this talk aims to provide one such new path**



Edelstein et al., AIP Conf.Proc. 1116 (2009) no.1, 265-284

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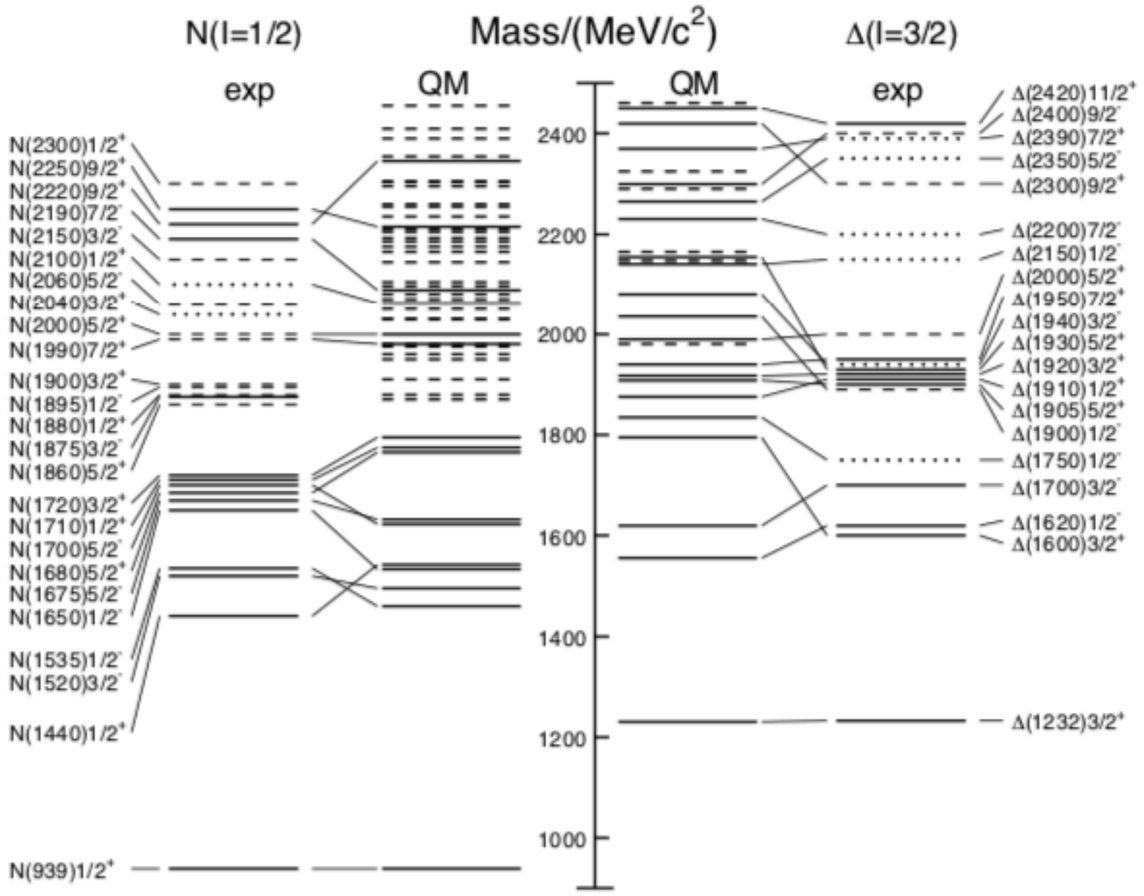

M. Tanabashi et al., Phys. Rev., vol. D98, no. 3, p. 030001, 2018

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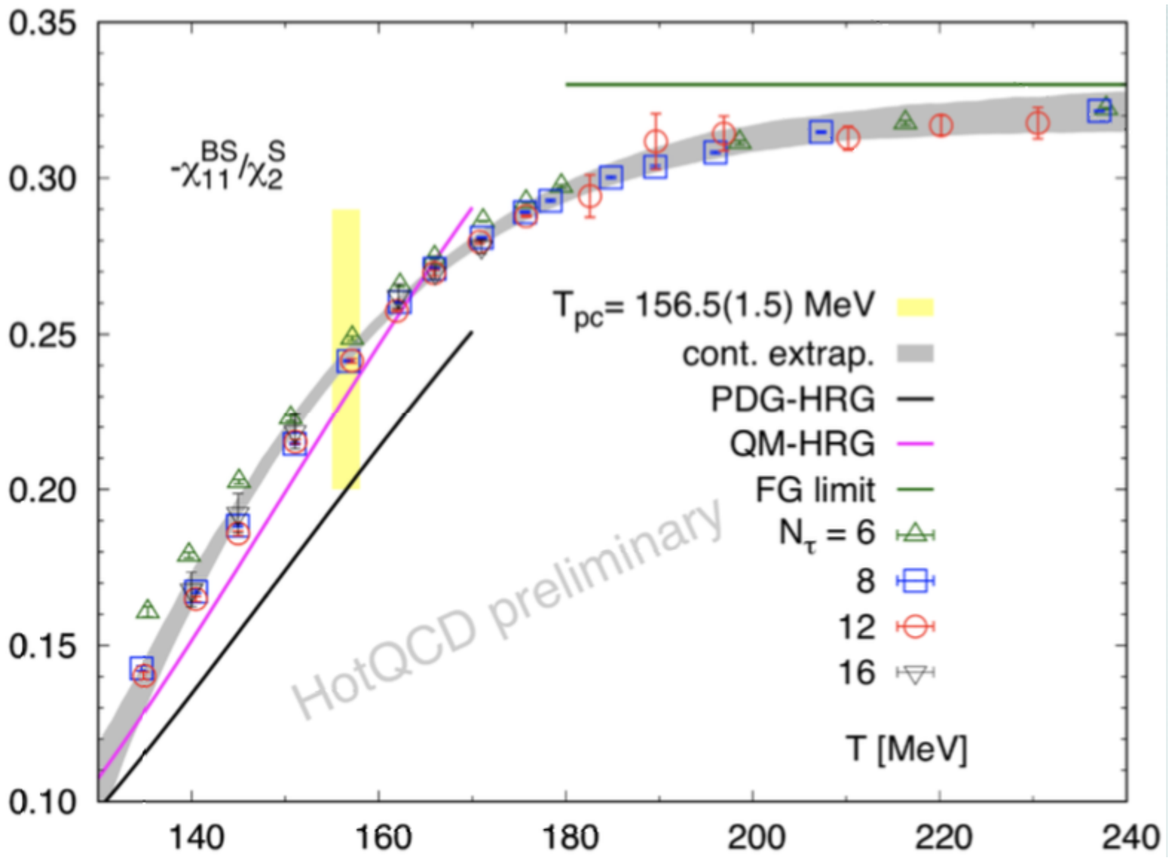

Bellwied et al., 1910.14592

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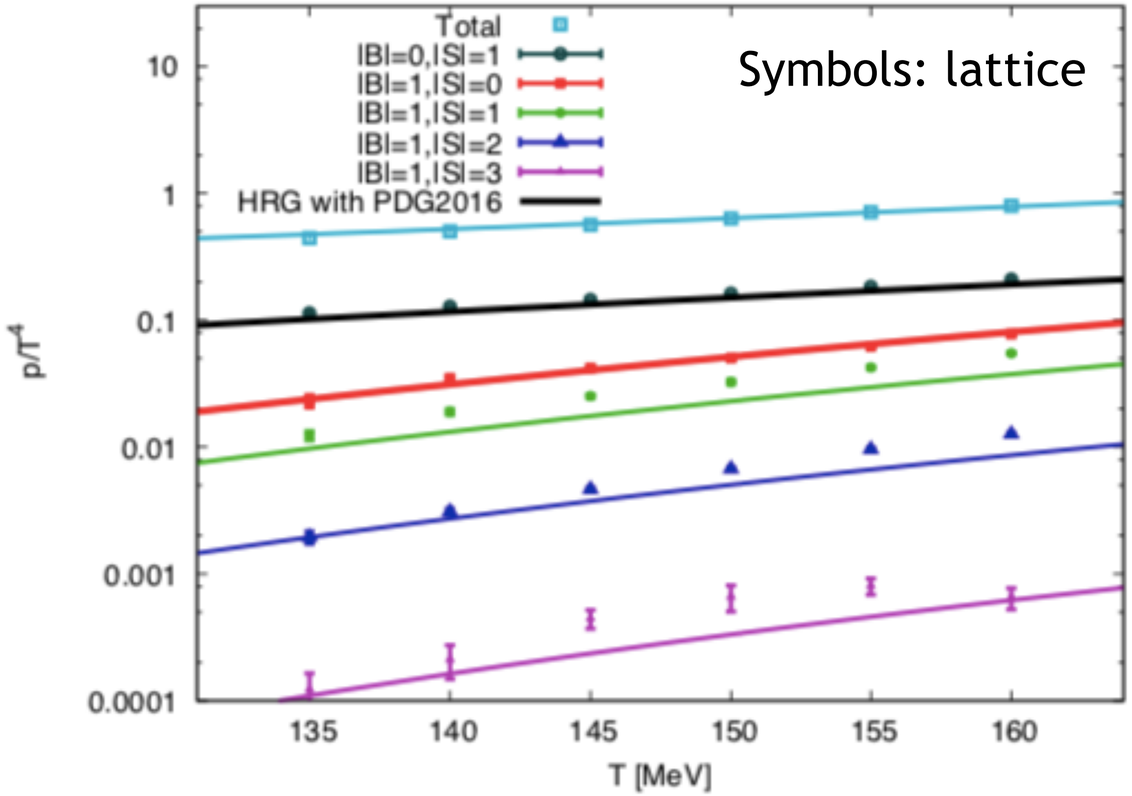
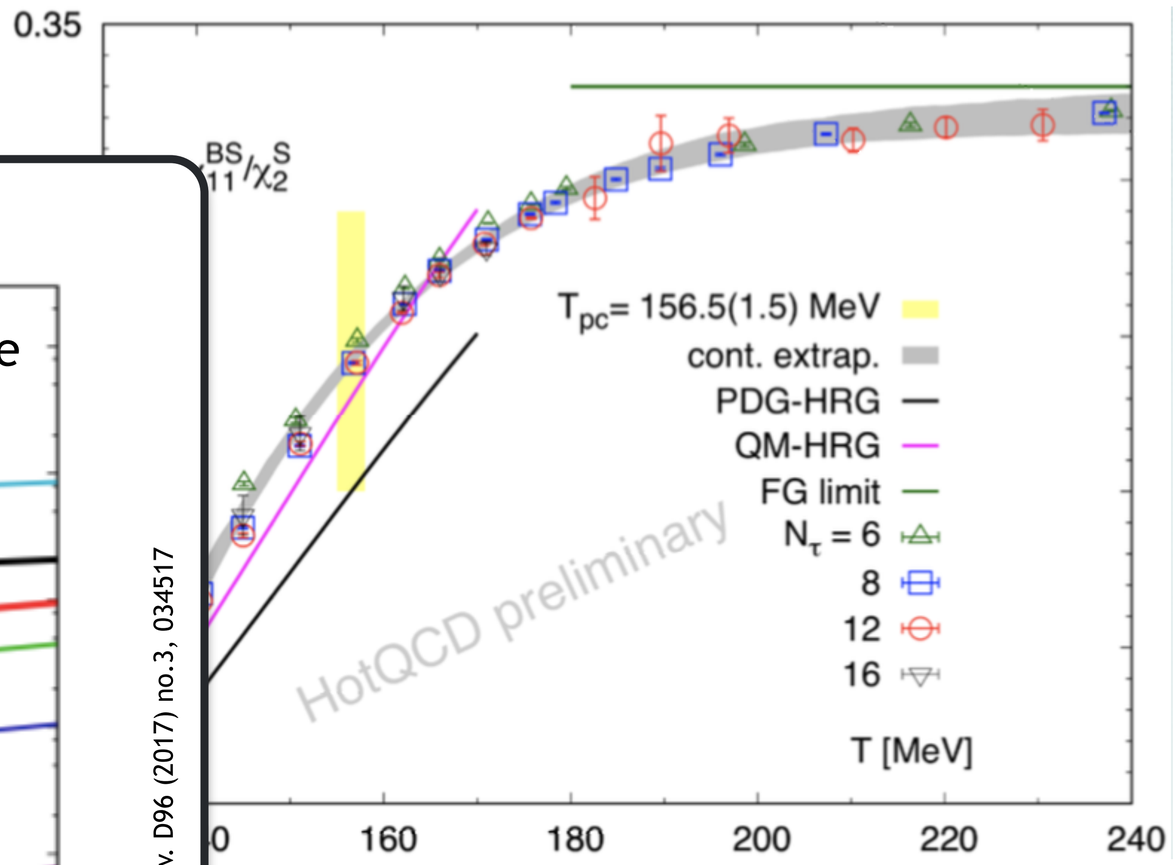


Alba et al., Phys.Rev. D96 (2017) no.3, 034517



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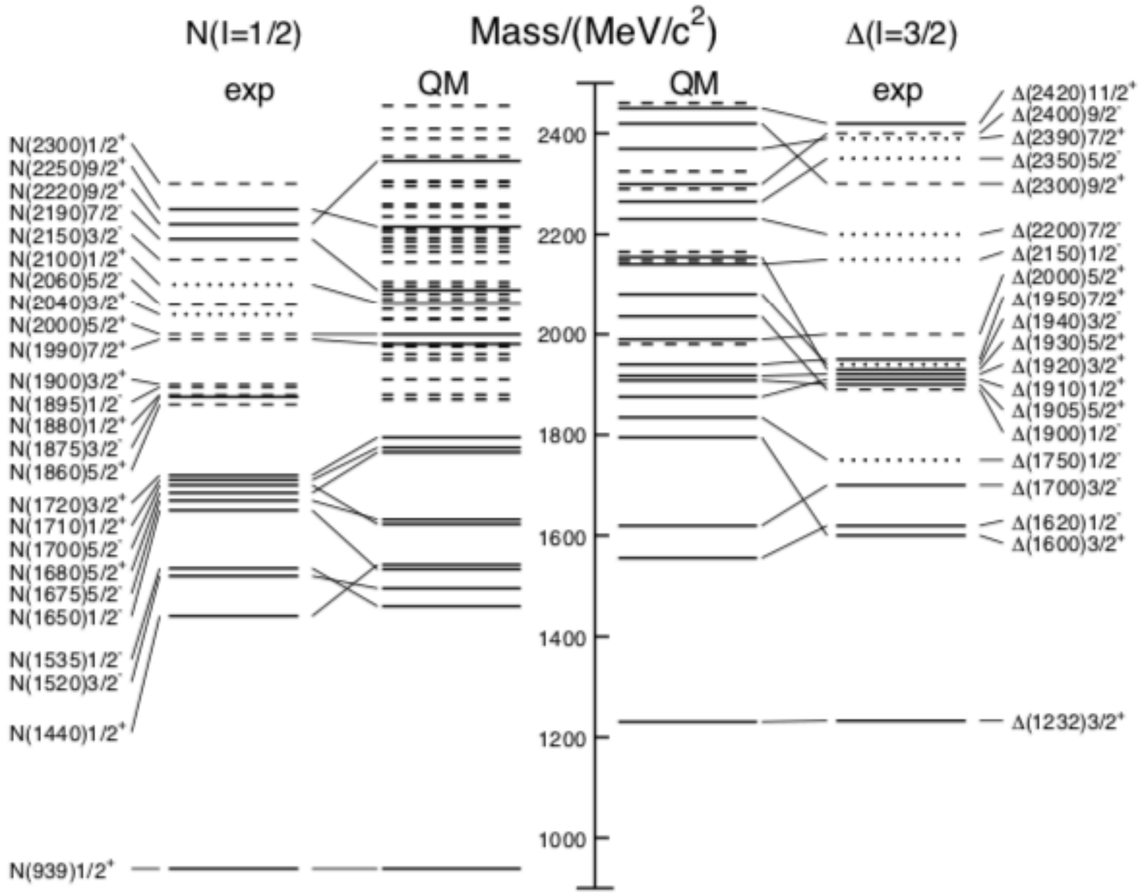

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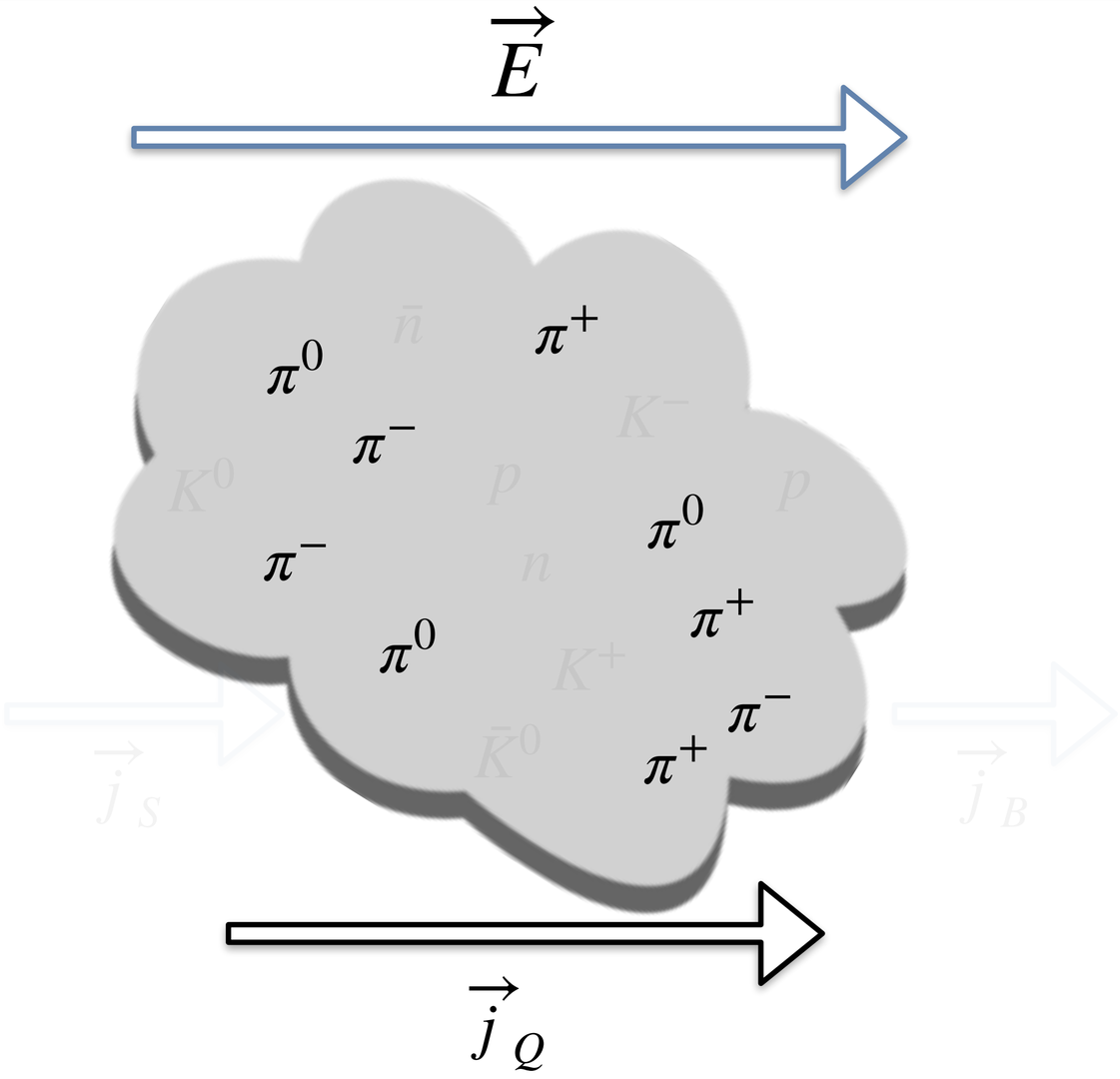
Electric cross-conductivity



- An electric field introduces an electric current:

$$\vec{j}_Q = \sigma_{QQ}\,\vec{E}_Q$$

- Hadrons can have multiple charges: Q, B, S
- So the electric field also introduces other currents:

$$\vec{j}_B = \sigma_{QB}\,\vec{E}_Q$$

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- **Can be calculated both in effective models and on the lattice!**

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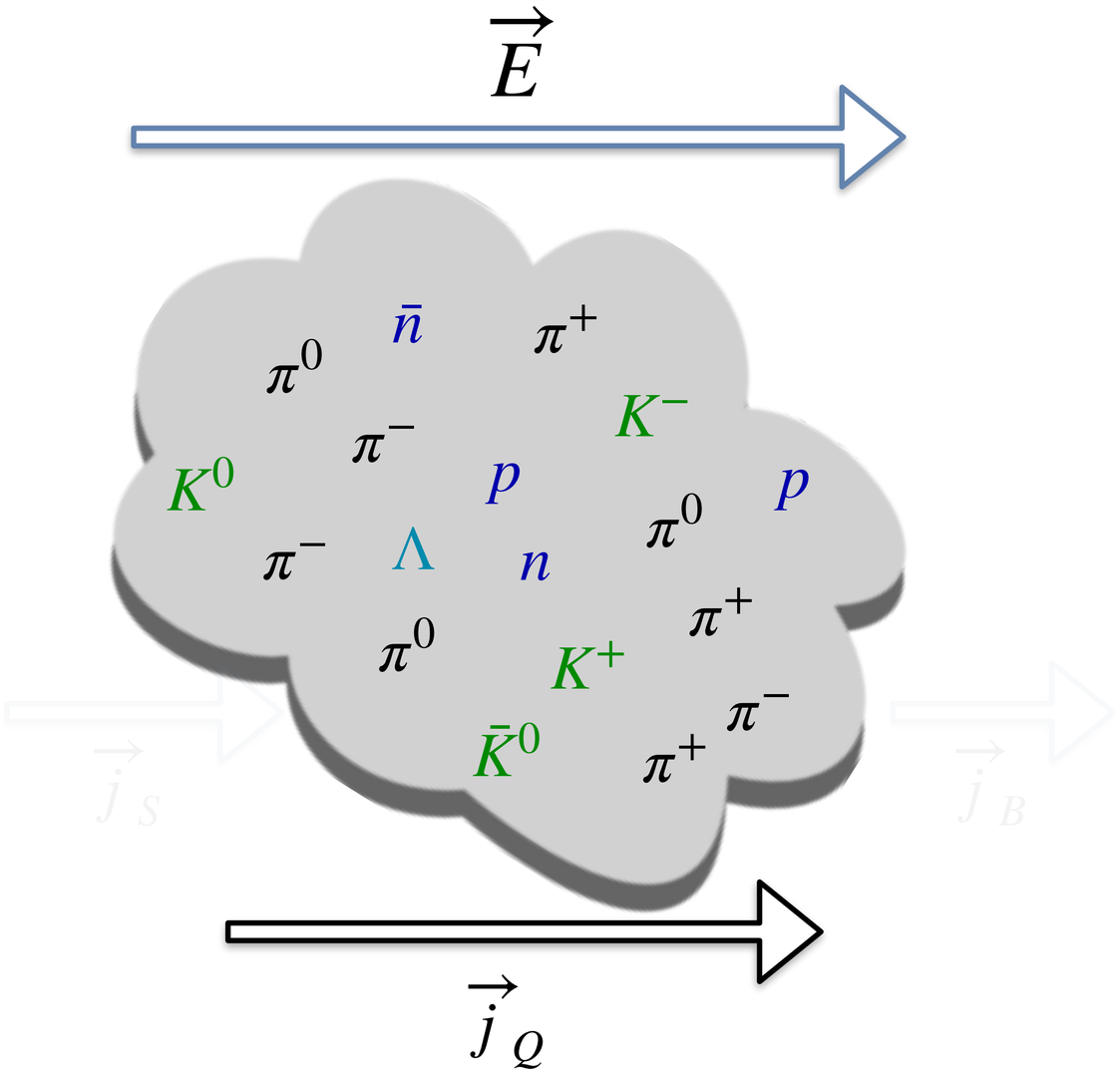

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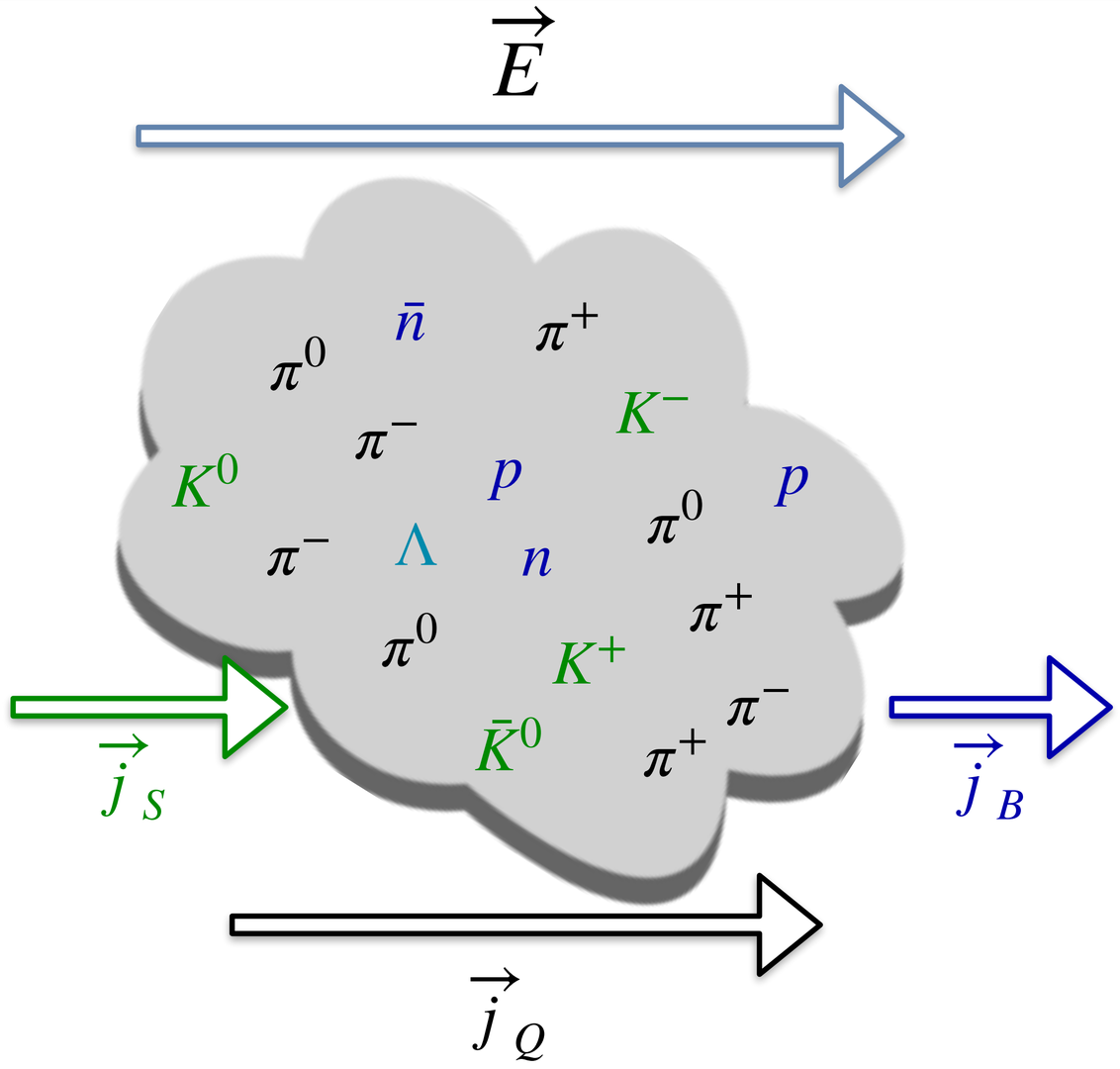

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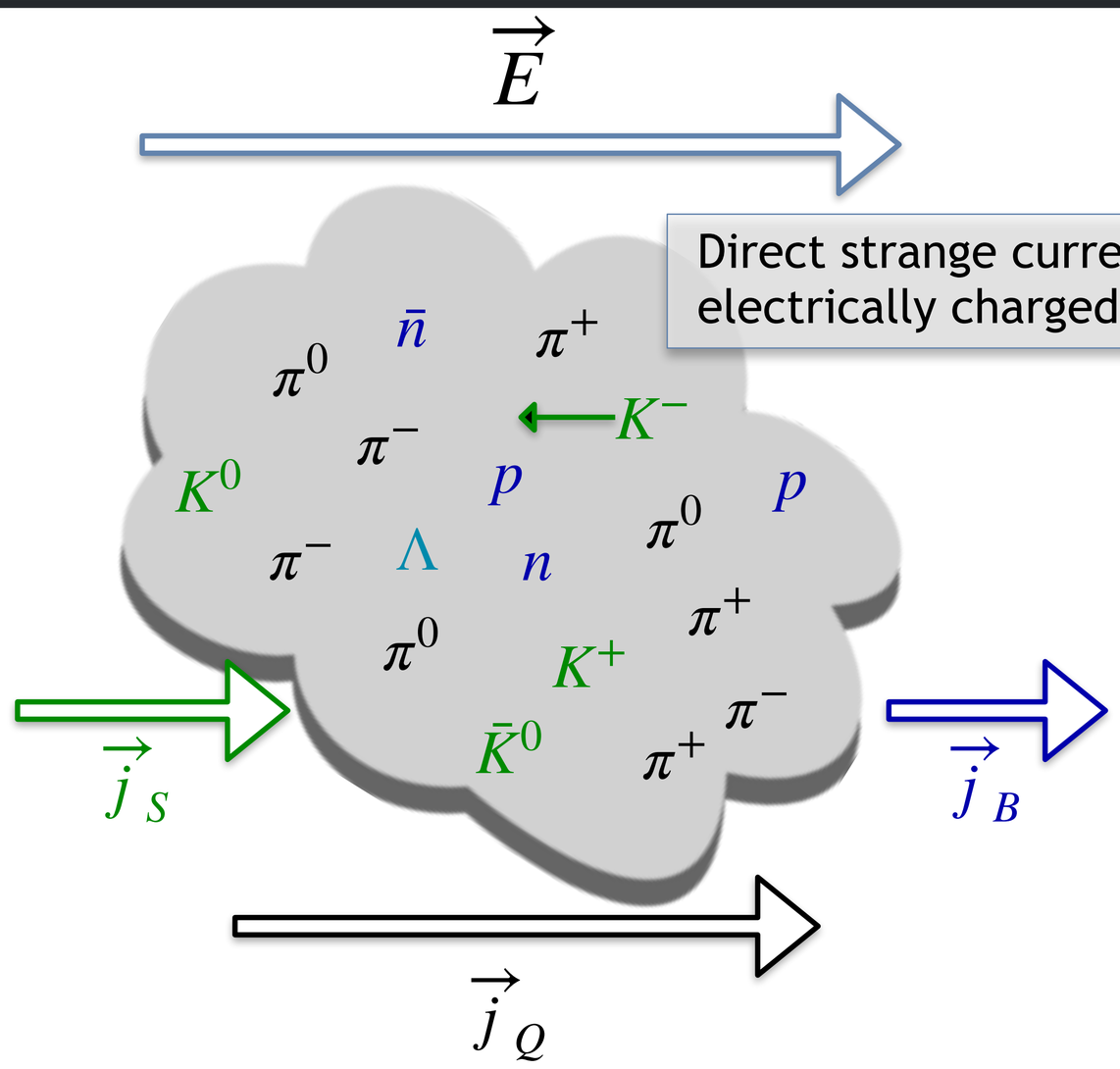

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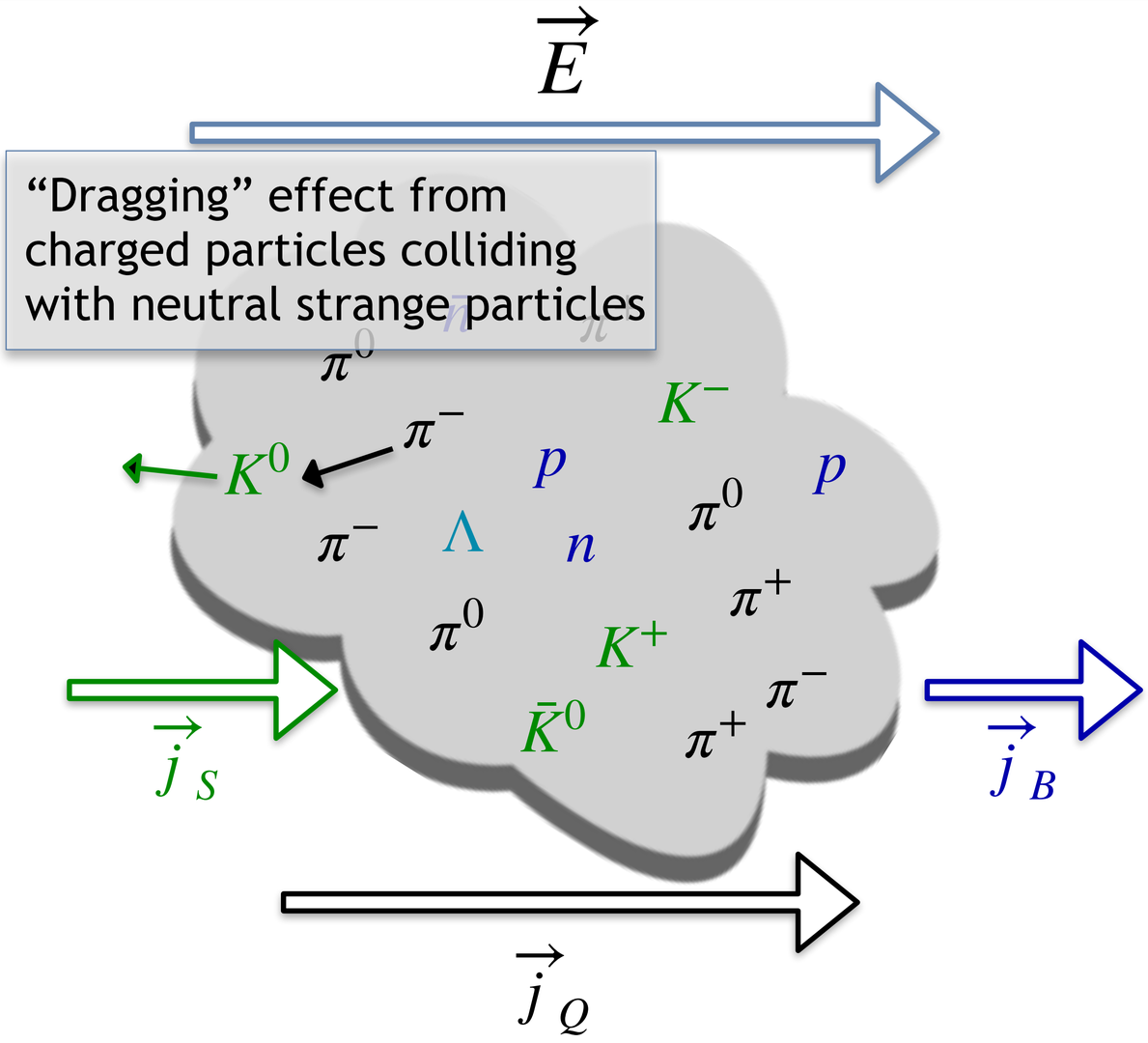

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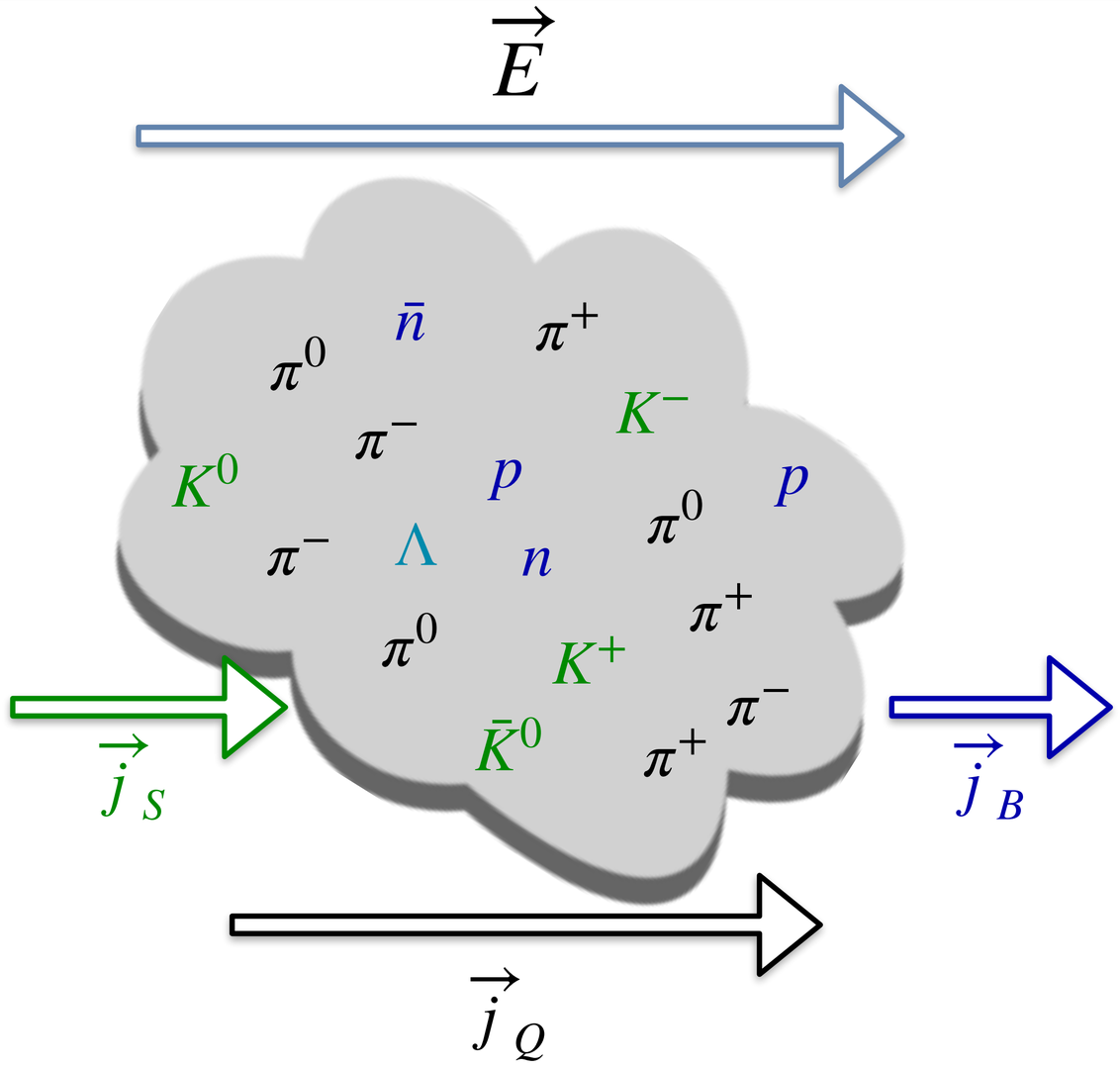

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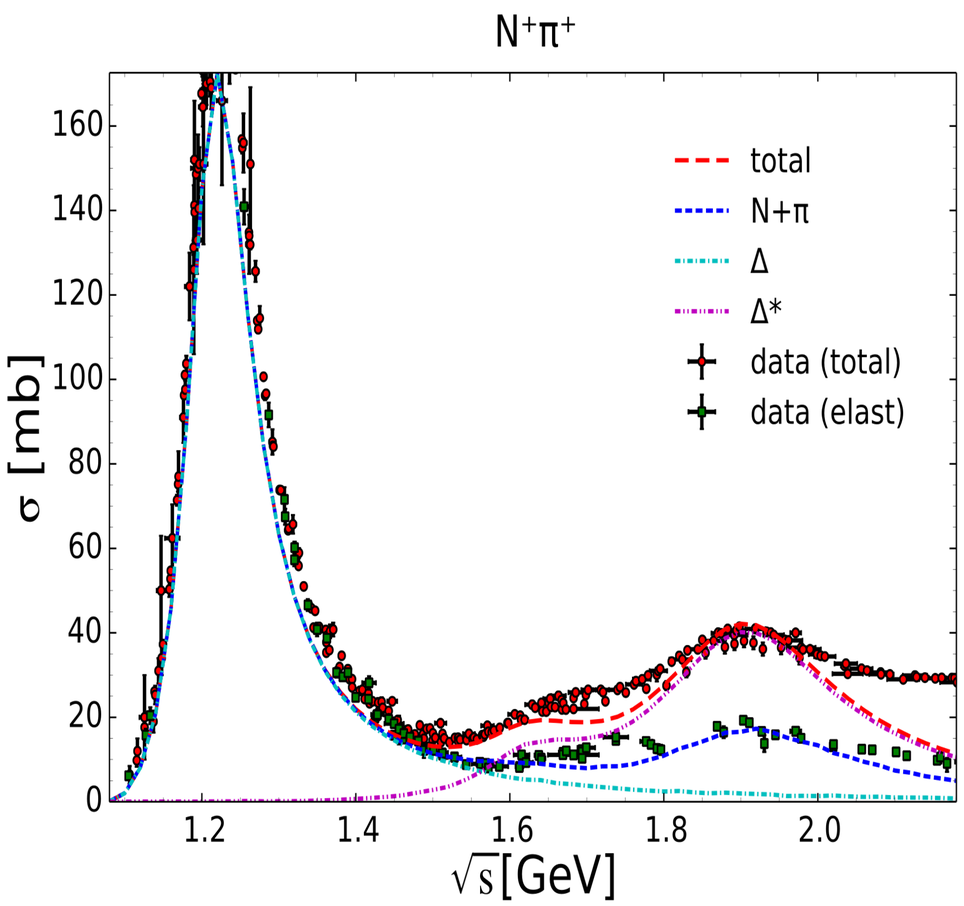
Modelling the hadron gas: SMASH



SMASH v1.6, smash-transport.github.io, DOI:10.5281/zenodo.3485108

See also talk from A. Schäfer at 10:00 earlier in EM I

- **SMASH is a semi-classical transport approach for the hadron gas**
- Geometric collision criterion:

$$d_{trans} < d_{int} = \sqrt{\frac{\sigma_{tot}}{\pi}}$$

- Spectral functions of resonances are described by relativistic Breit-Wigner functions, with resonance lifetime

$$\tau_{\mathrm{res}} = \frac{1}{\Gamma(m)}$$

- Elastic scatterings parameterized for NN; many other elastic scatterings assumed to go through resonances
- All other elastic scatterings go through Additive Quark Model
- Inelastic scatterings, currently include
  - NN↔NR, NN↔ ΔR
  - KN↔KN, KN↔πH
  - +antiparticles
- Strings (turned off for detailed balance)

Weil et al., Phys. Rev. C94 (2016) no.5, 054905

Green-Kubo formalism

The cross-conductivity is calculated from

$$\sigma_{Qi} = \frac{V}{T}\int_0^\infty C_{Qi}\, dt', \quad i = Q, B, S$$

where

$$C_{Qi}(t) \equiv \langle (j_Q^x(t) - \langle j_Q^x \rangle_{eq}) \cdot (j_i^x(t') - \langle j_i^x \rangle_{eq}) \rangle_{eq}$$

In the dilute case, exponential *ansatz*

$$C_{Qi}(t) = C_{Qi}(0)\, e^{-\frac{t}{\tau_{Qi}}}$$

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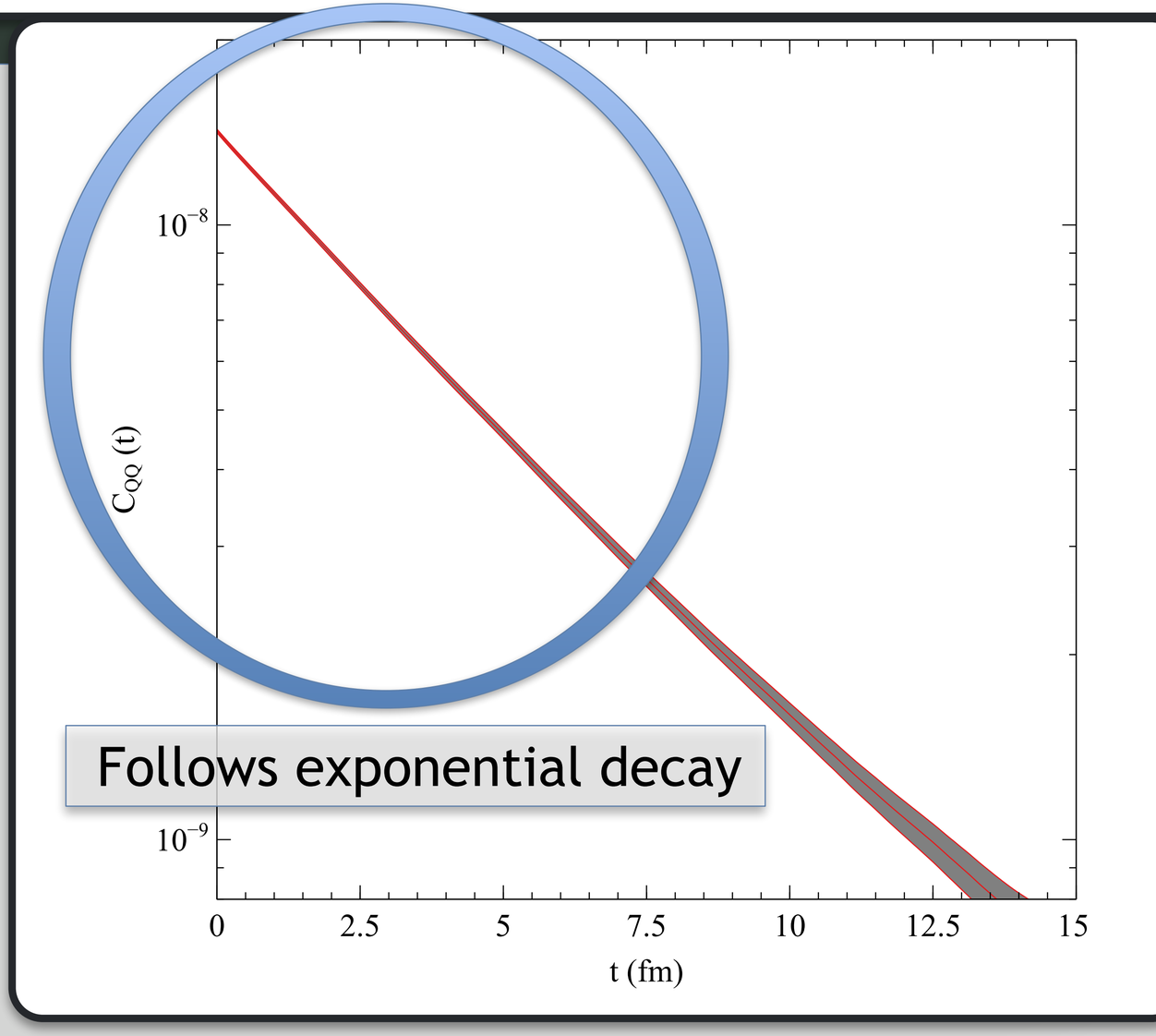
where τ_{Qi} is the relaxation time



Green-Kubo test case: π-K-N with 30 mb

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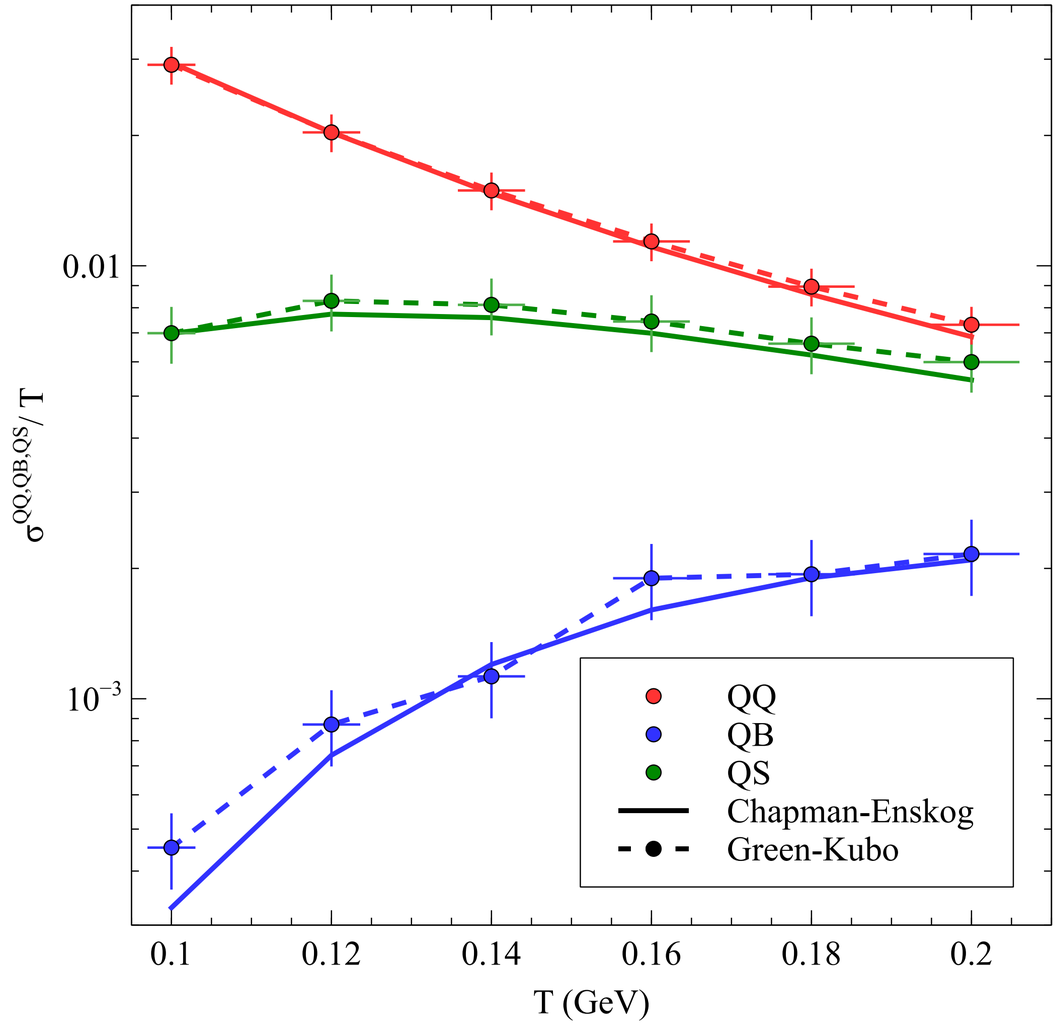

Greif et al., Phys.Rev. D93 (2016) no.9, 096012, Erratum: Phys.Rev. D96 (2017) no.5, 059902

π-K-N resonant gas: Degrees of freedom

N	Δ	Λ	Σ	Ξ	Ω	Unflavored				Strange
N_{938}	Δ_{1232}	Λ_{1116}	Σ_{1189}	Ξ_{1321}	Ω^-_{1672}	π_{138}	$f_{0\ 980}$	$f_{2\ 1275}$	$\pi_{2\ 1670}$	K_{494}
N_{1440}	Δ_{1620}	Λ_{1405}	Σ_{1385}	Ξ_{1530}	Ω^-_{2250}	π_{1300}	$f_{0\ 1370}$	$f_{2\ 1525}'$		K^*_{892}
N_{1520}	Δ_{1700}	Λ_{1520}	Σ_{1660}	Ξ_{1690}		π_{1800}	$f_{0\ 1500}$	$f_{2\ 1950}$	$\rho_{3\ 1690}$	$K_{1\ 1270}$
N_{1535}	Δ_{1905}	Λ_{1600}	Σ_{1670}	Ξ_{1820}			$f_{0\ 1710}$	$f_{2\ 2010}$		$K_{1\ 1400}$
N_{1650}	Δ_{1910}	Λ_{1670}	Σ_{1750}	Ξ_{1950}		η_{548}		$f_{2\ 2300}$	$\varphi_{3\ 1850}$	K^*_{1410}
N_{1675}	Δ_{1920}	Λ_{1690}	Σ_{1775}	Ξ_{2030}		η'_{958}	$a_{0\ 980}$	$f_{2\ 2340}$		$K_0^*{}_{1430}$
N_{1680}	Δ_{1930}	Λ_{1800}	Σ_{1915}			η_{1295}	$a_{0\ 1450}$		$a_{4\ 2040}$	$K_2^*{}_{1430}$
N_{1700}	Δ_{1950}	Λ_{1810}	Σ_{1940}			η_{1405}		$f_{1\ 1285}$		K^*_{1680}
N_{1710}		Λ_{1820}	Σ_{2030}			η_{1475}	φ_{1019}	$f_{1\ 1420}$	$f_{4\ 2050}$	$K_{2\ 1770}$
N_{1720}		Λ_{1830}	Σ_{2250}				φ_{1680}			$K_3^*{}_{1780}$
N_{1875}		Λ_{1890}				σ_{800}		$a_{2\ 1320}$		$K_{2\ 1820}$
N_{1900}		Λ_{2100}					$h_{1\ 1170}$			$K_4^*{}_{2045}$
N_{1990}		Λ_{2110}				ρ_{776}		$\pi_{1\ 1400}$		
N_{2080}		Λ_{2350}				ρ_{1450}	$b_{1\ 1235}$	$\pi_{1\ 1600}$		
N_{2190}						ρ_{1700}				
N_{2220}							$a_{1\ 1260}$	$\eta_{2\ 1645}$		
N_{2250}						ω_{783}				
						ω_{1420}		$\omega_{3\ 1670}$		
						ω_{1650}				

- + anti-particles
- Isospin symmetry

SMASH v1.6, smash-transport.github.io, DOI:10.5281/zenodo.3485108

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Full hadron gas: Degrees of freedom

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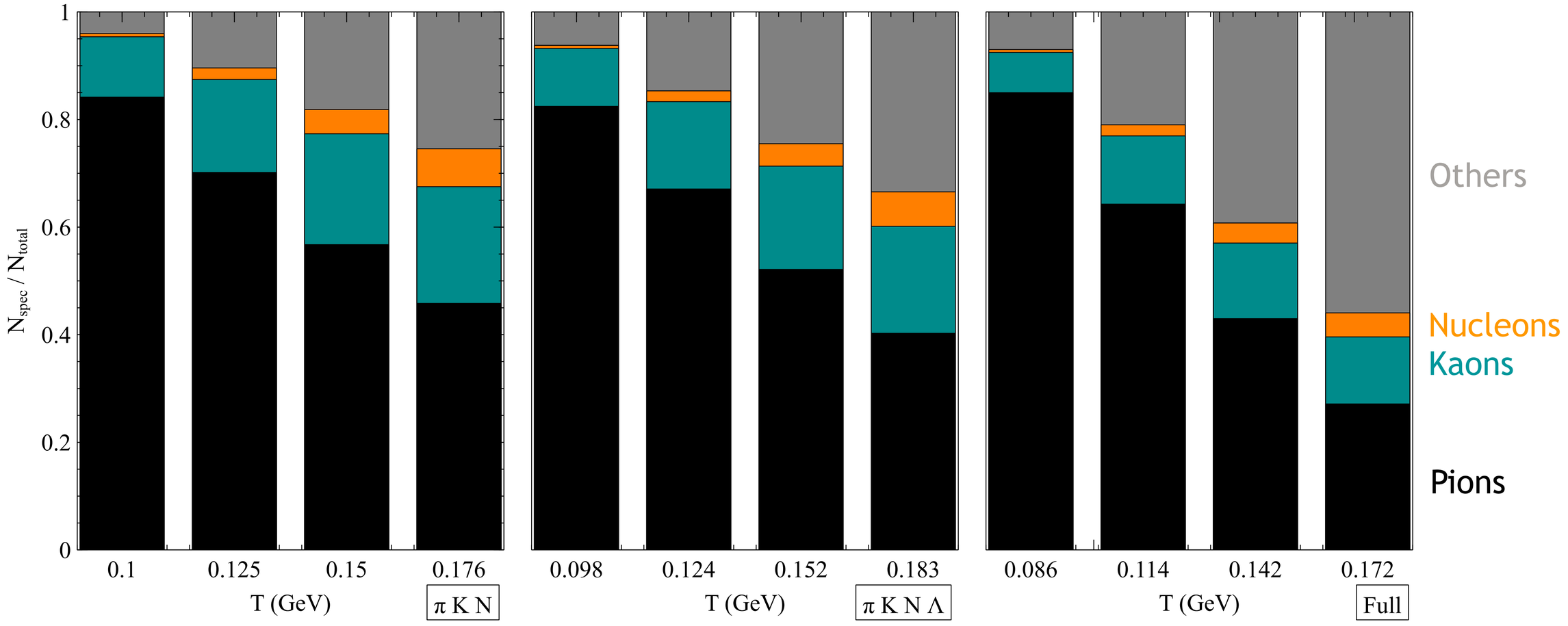
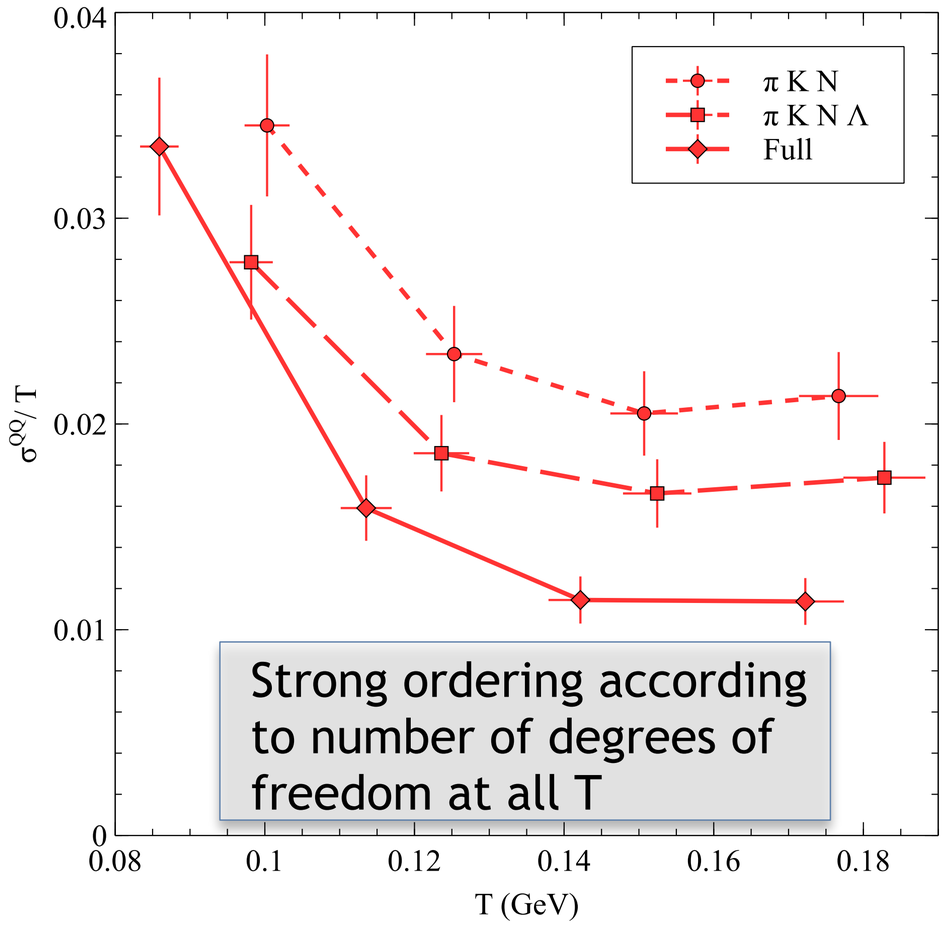
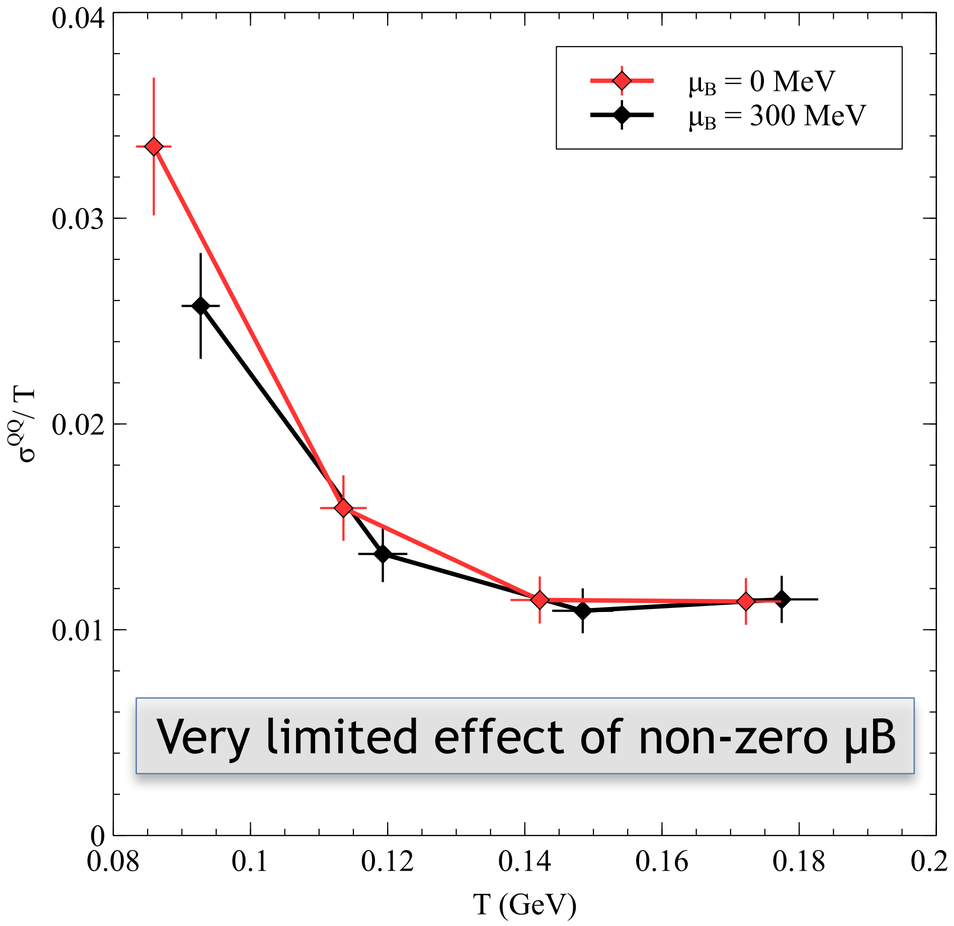
Chemical composition



Electric conductivity

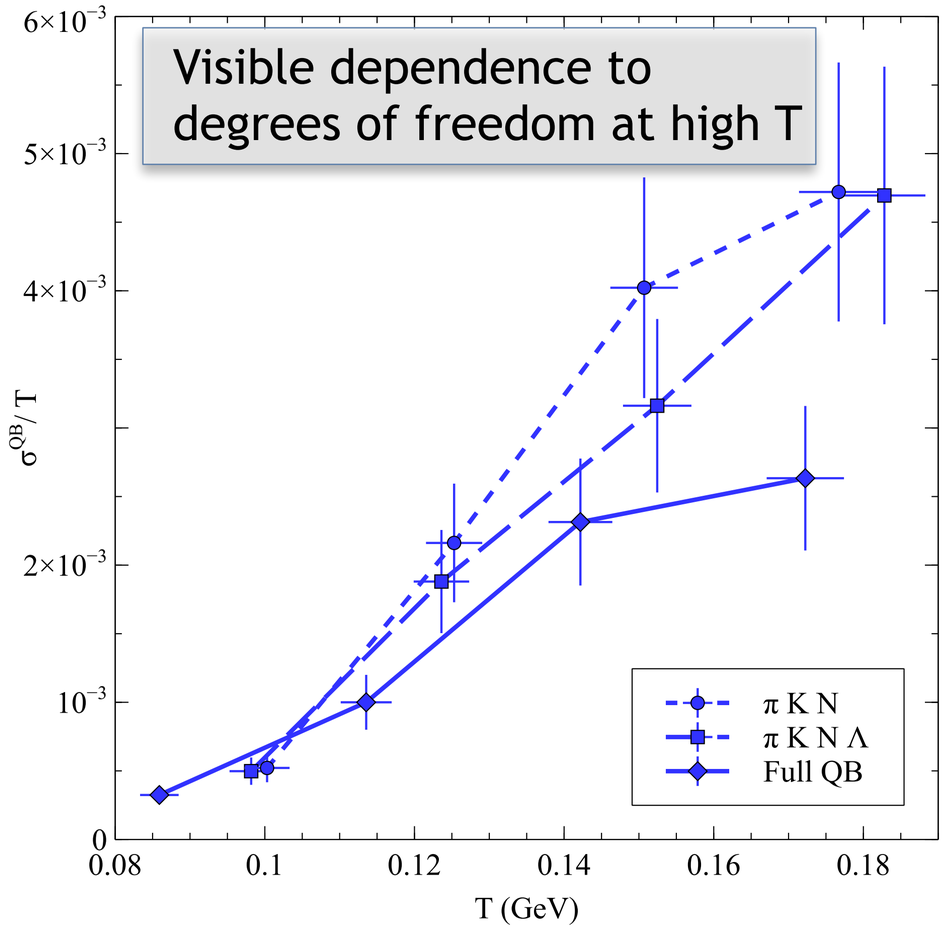


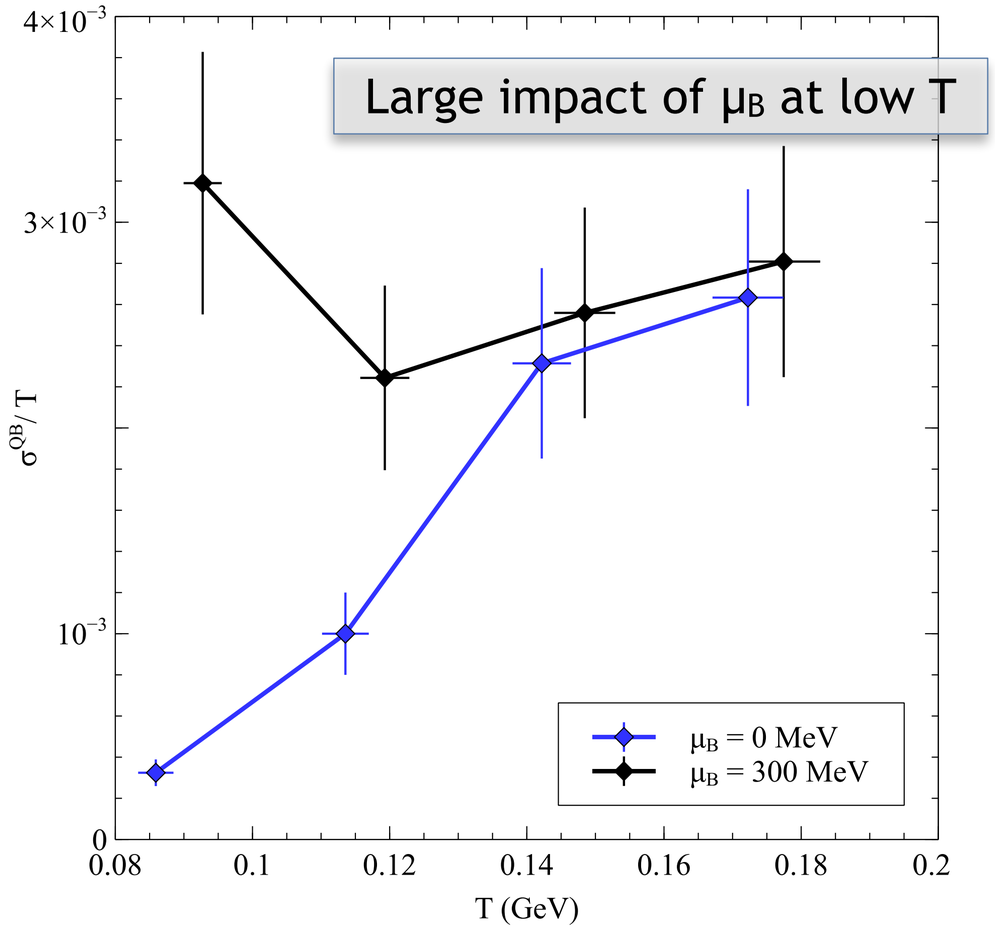
Strong ordering according to number of degrees of freedom at all T



Very limited effect of non-zero μB

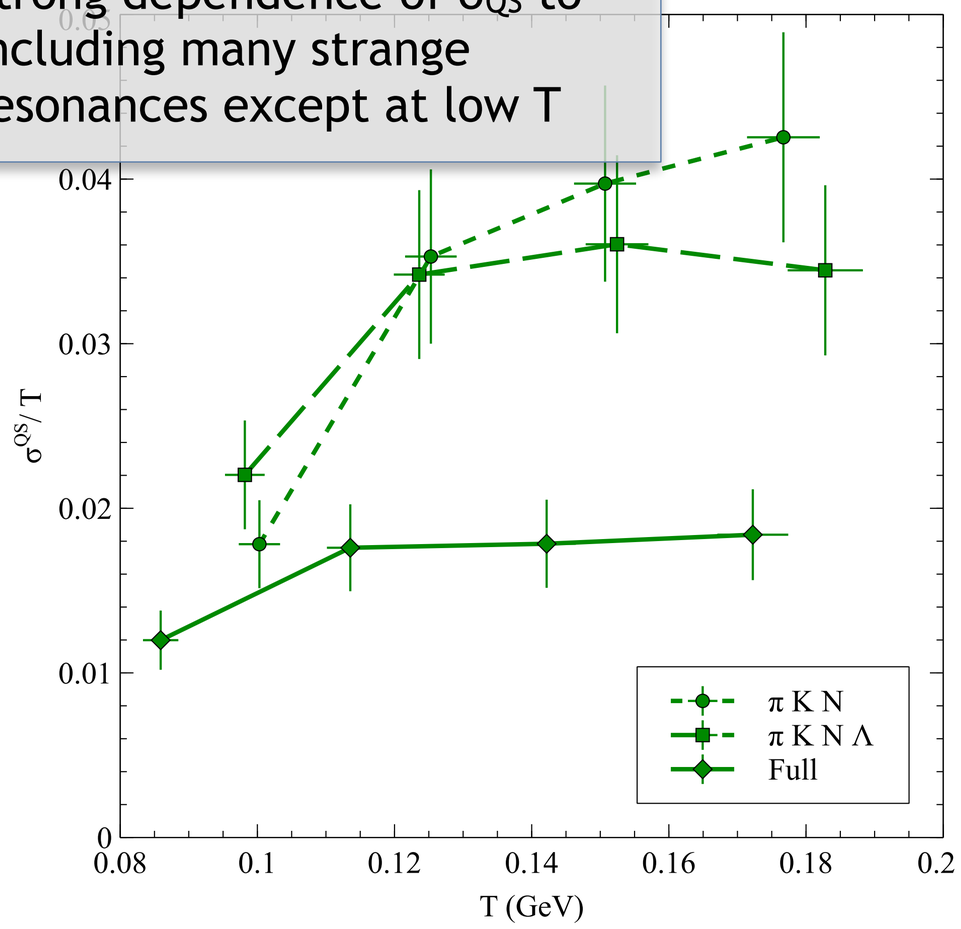
Baryonic-electric conductivity

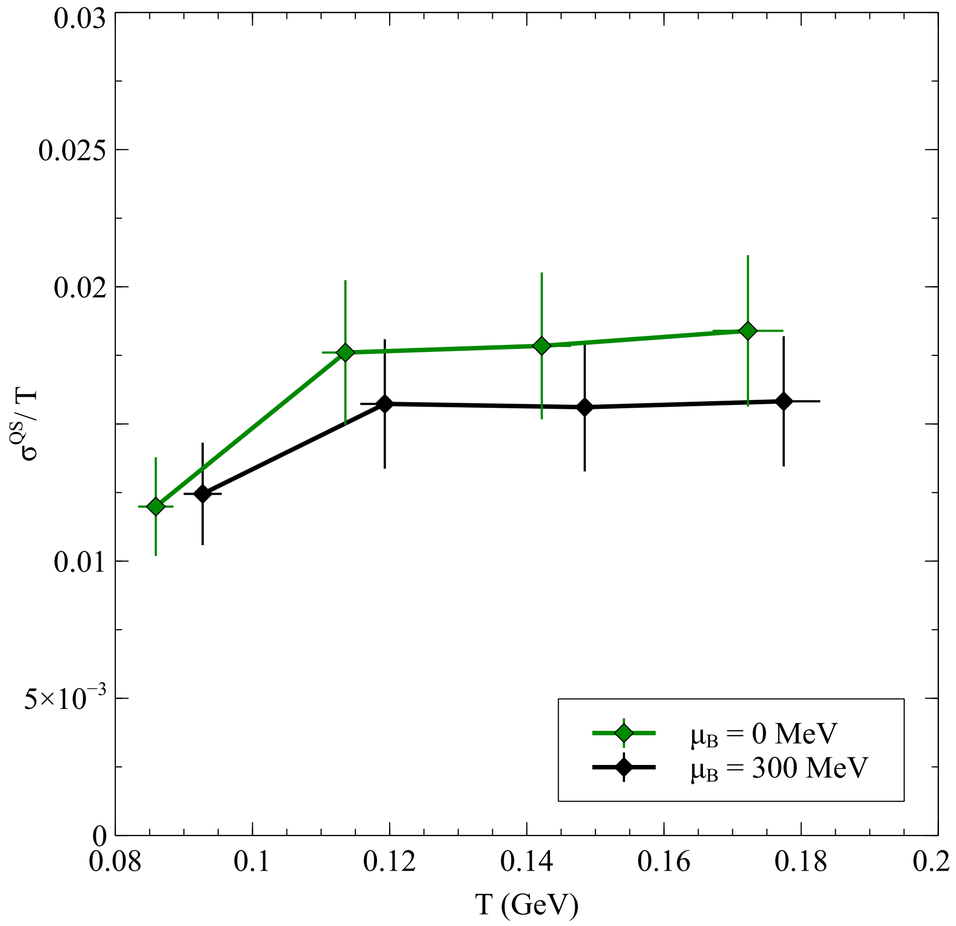




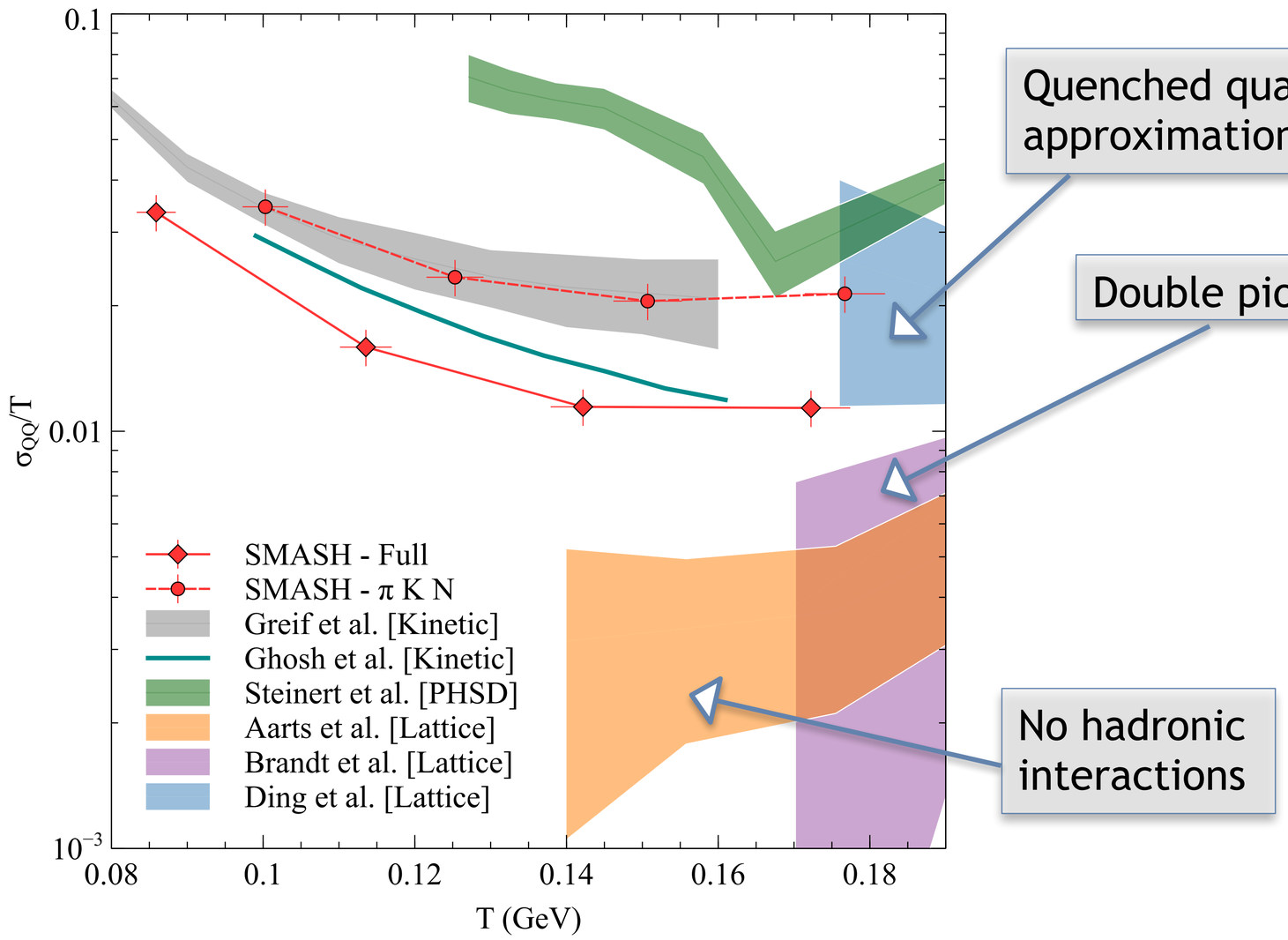
Strange-electric conductivity

Strong dependence of σ_{QS} to including many strange resonances except at low T





Electric conductivity comparison



Quenched quarks approximation

Double pion mass

No hadronic interactions

-Greif et al., Phys. Rev., vol. D93, no. 9, p. 096012, 2016
-Ghosh et al., Nucl. Phys., vol. A969, pp. 237-253, 2018
-Steinert et al., Phys. Rev., vol. C89, no. 3, p. 035203, 2014
-Aarts et al., JHEP, vol. 02, p. 186, 2015
-Brandt et al., Phys. Rev., vol. D93, no. 5, p. 054510, 2016
-Ding et al., Phys. Rev., vol. D94, no. 3, p. 034504, 2016

Multiple lattice calculations exist, but they have large errors and systematic uncertainties

Need consistent results in a larger T range

Summary & Outlook

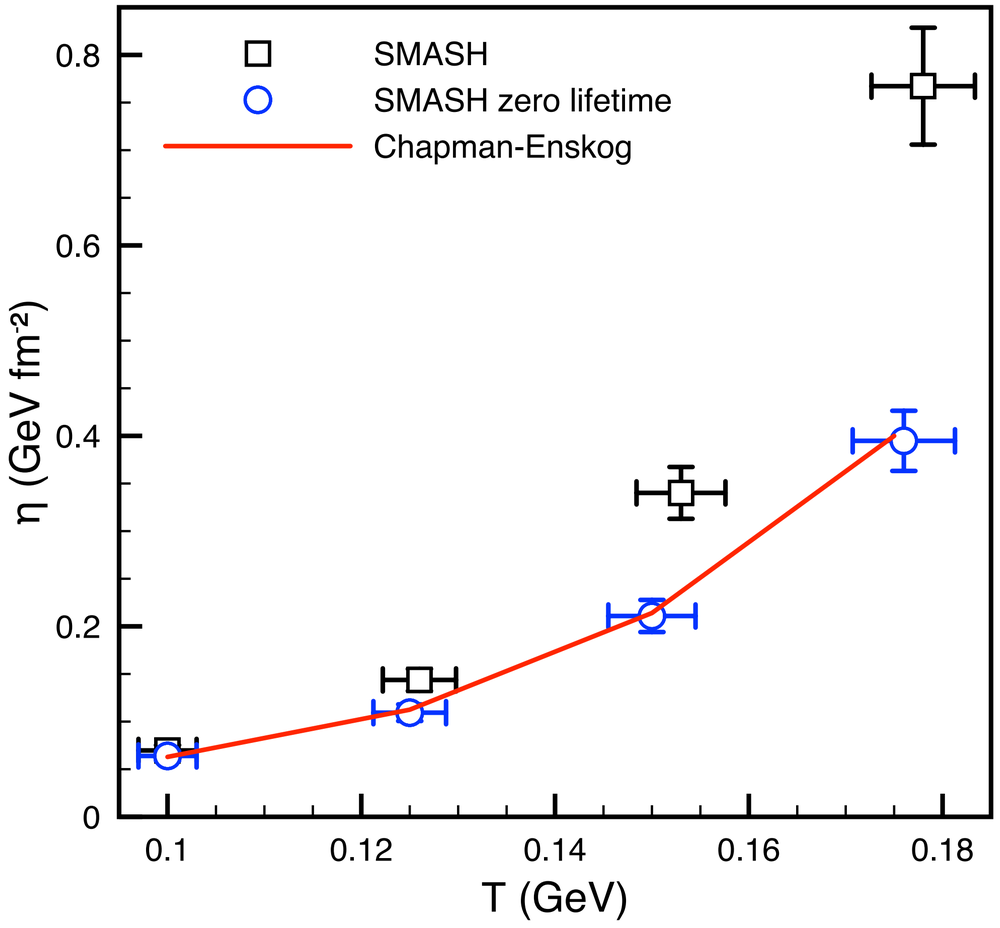
- **Investigated temperature and μ_B dependence of cross-conductivity in systems of increasing complexity**
  - Very good agreement with Chapman-Enskog approximation
  - Cross-conductivity sensitive to increasing number of degrees of freedom
  - Comparison with current lattice data inconclusive
  - Could be used to constrain active degrees of freedom in the hadron gas

- **Outlook:**
  - Investigation of angular dependent interactions
  - Inclusion of multi-particle interaction will play a role at phase transition
  - Precise low temperature conductivity calculations on the lattice are needed
  - Diffusion of multiple charges

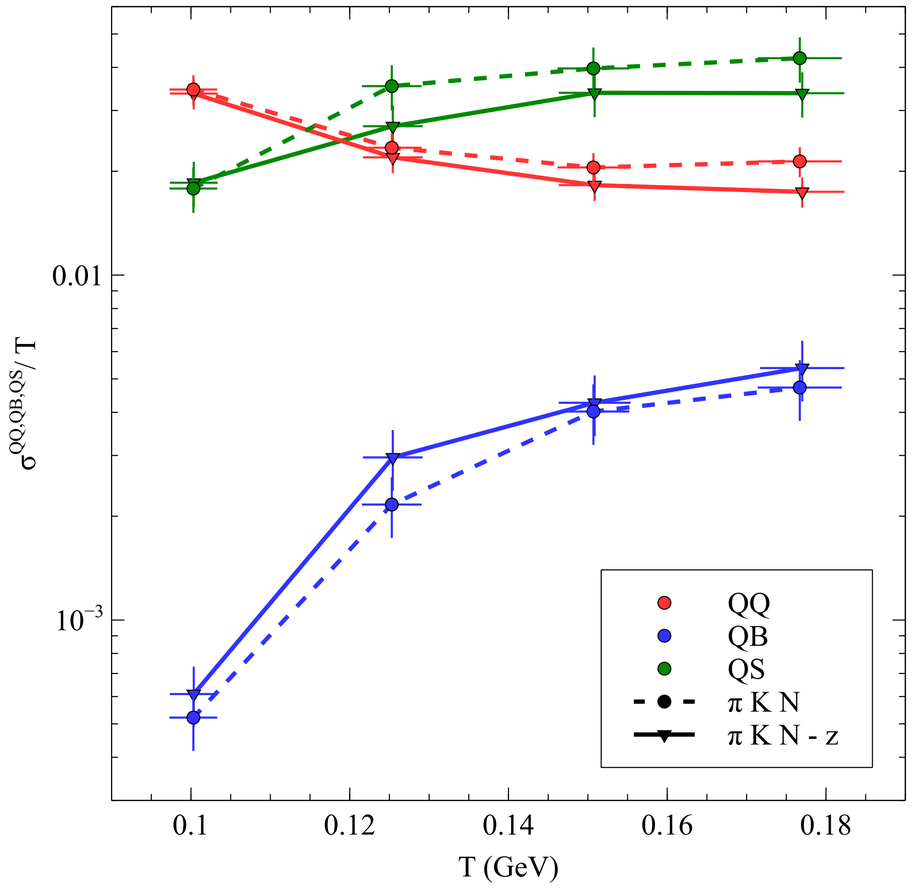
See talk by J. Fotakis at 10:20 today in COL I

Backup slides

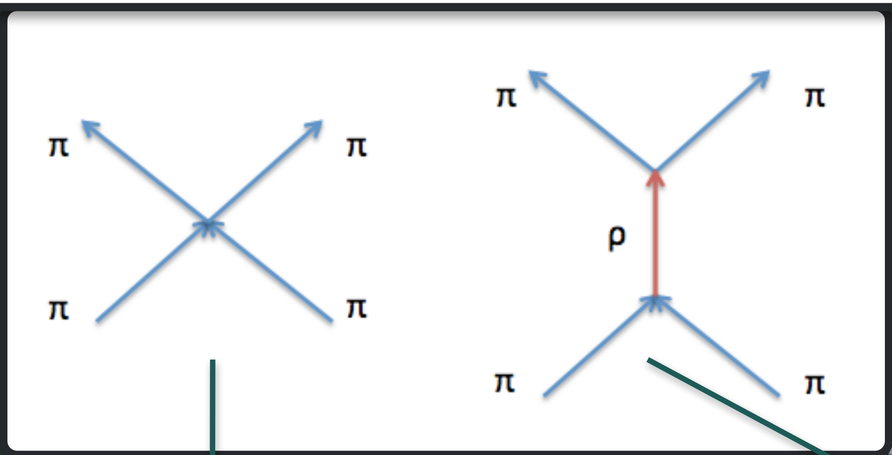
Resonance lifetimes: Shear vs conductivity



-Rose et al., Phys. Rev. C97 (2018) no.5, 055204



High temperature η/s : Resonance lifetimes

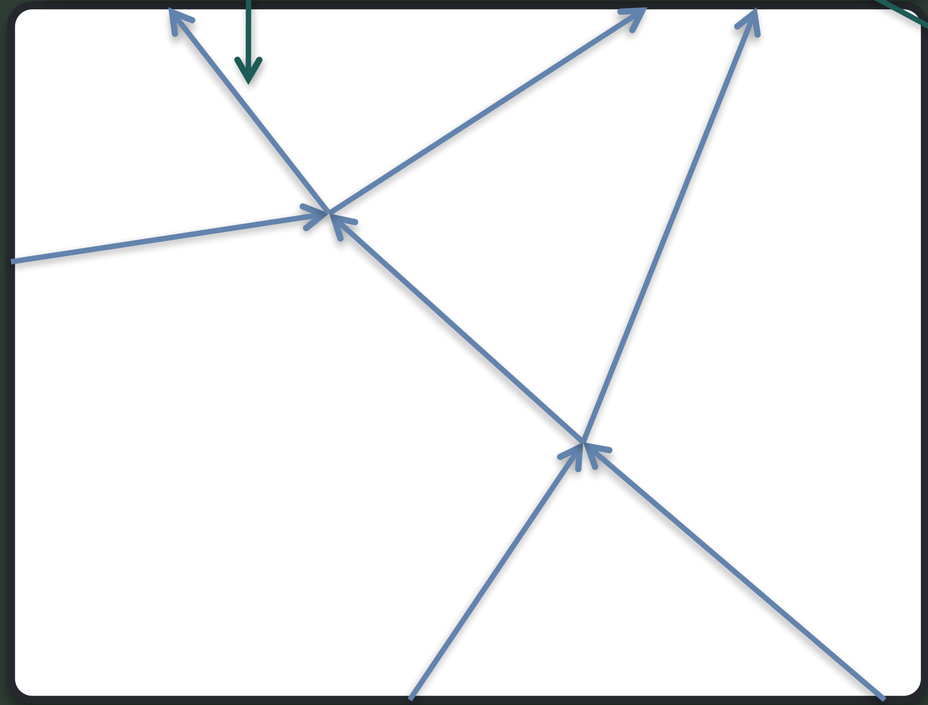


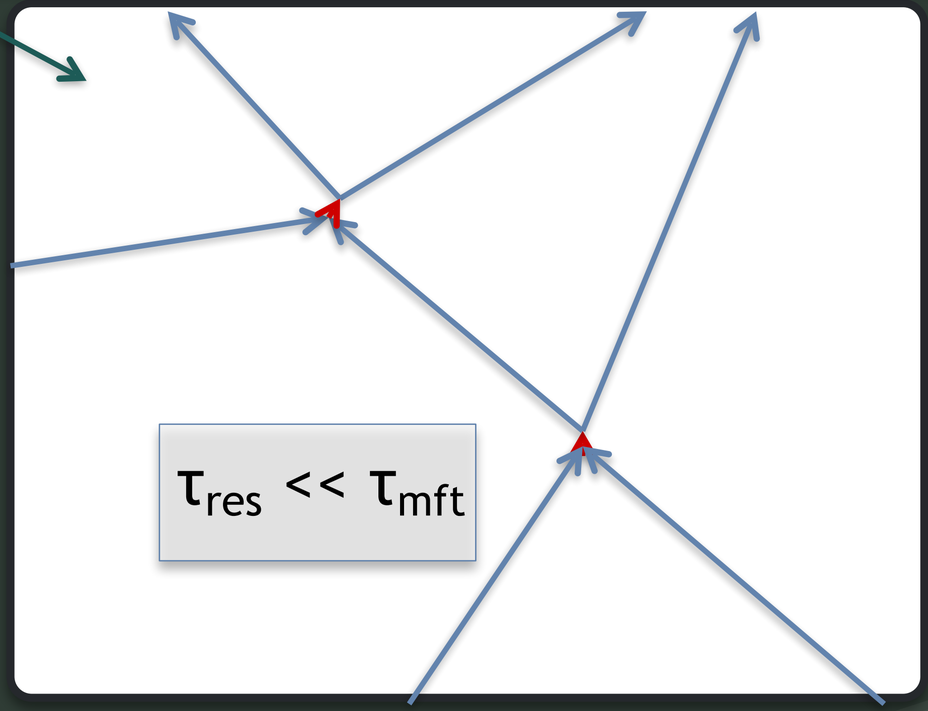
Must look at the microscopic picture from different descriptions

τ_{res} = resonance lifetime

τ_{mft} = mean free time

At low T and density:





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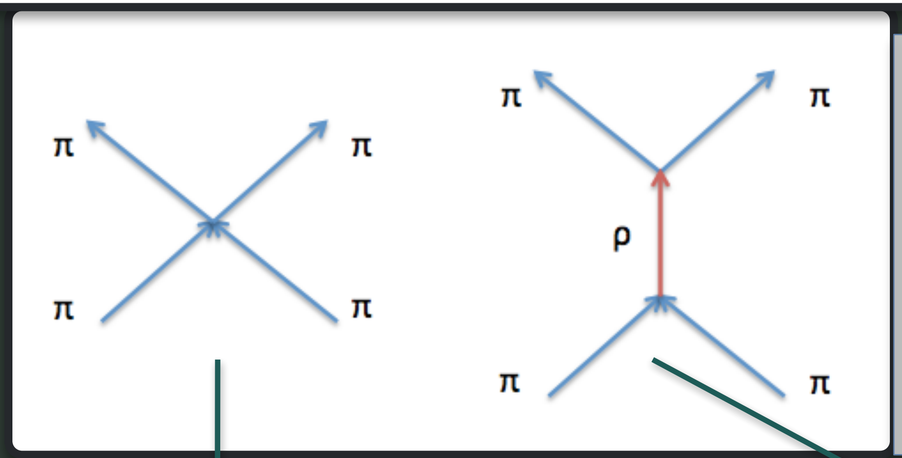

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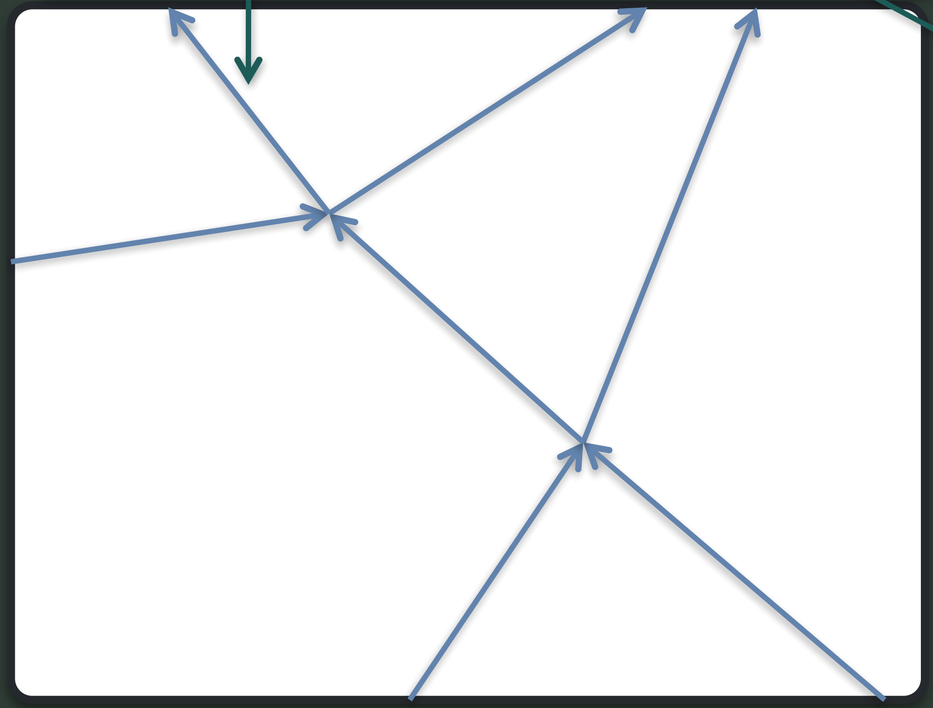

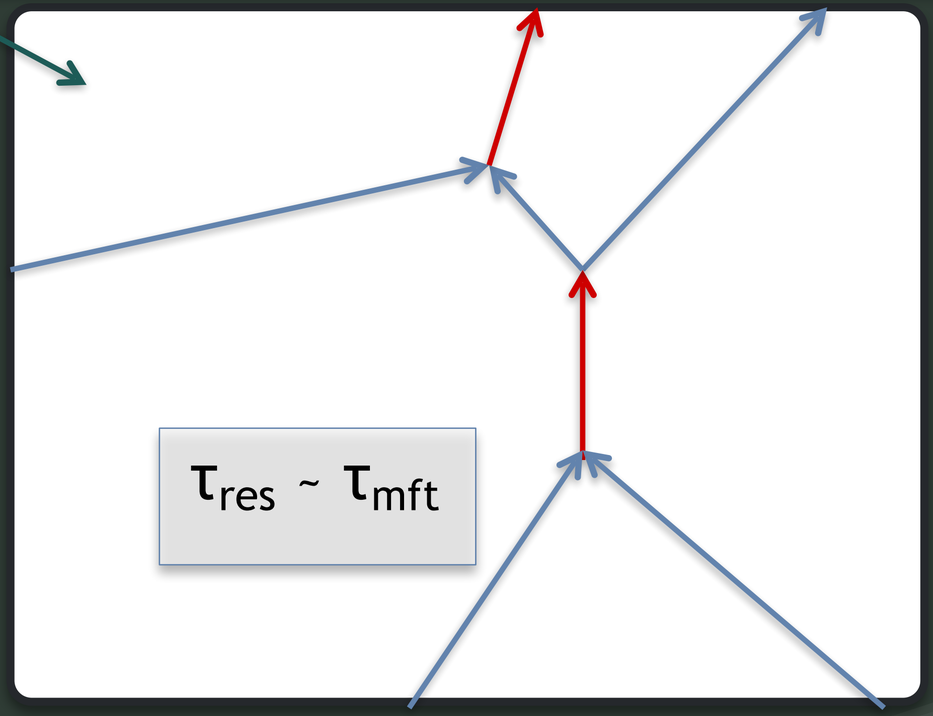

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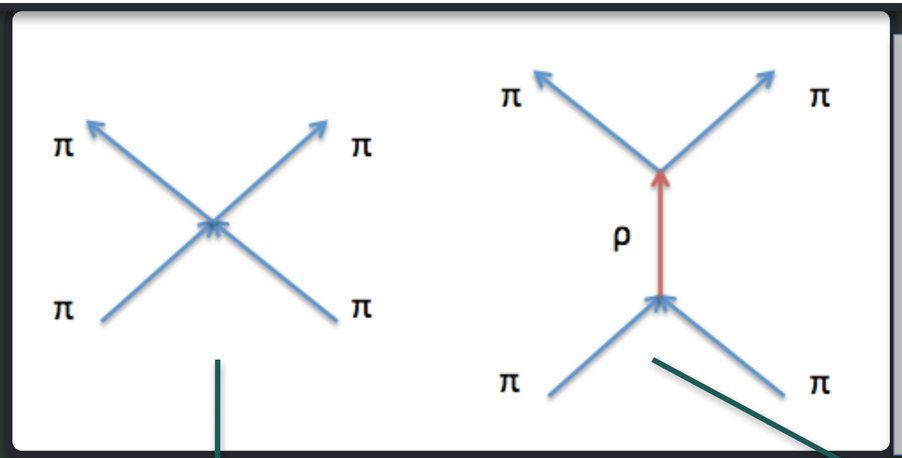

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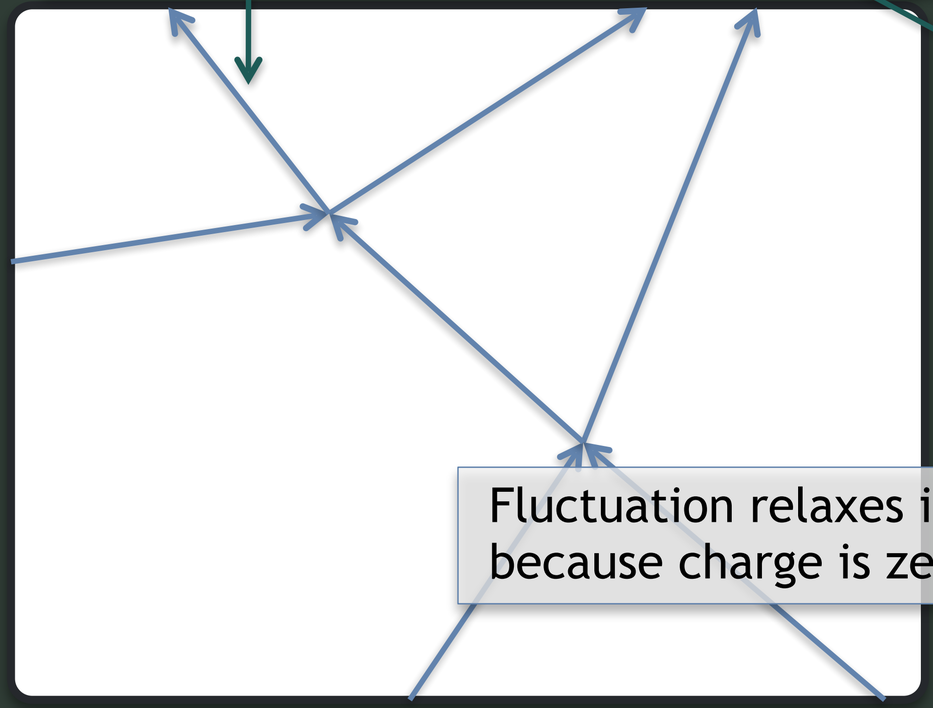
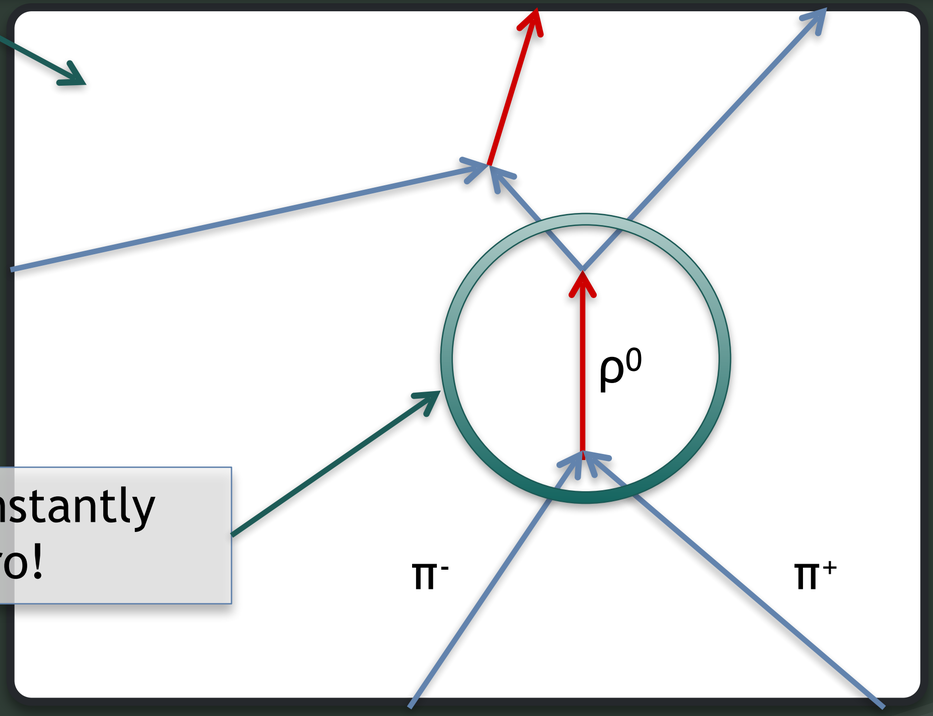


Fluctuation relaxes instantly because charge is zero!



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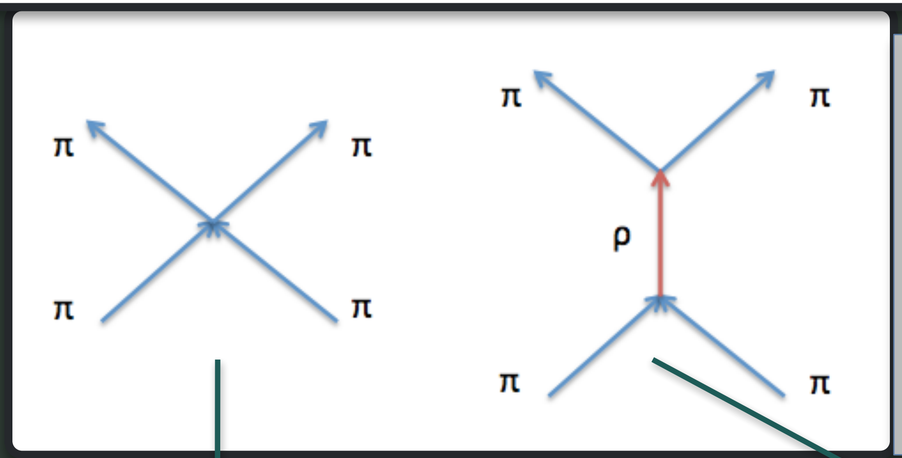

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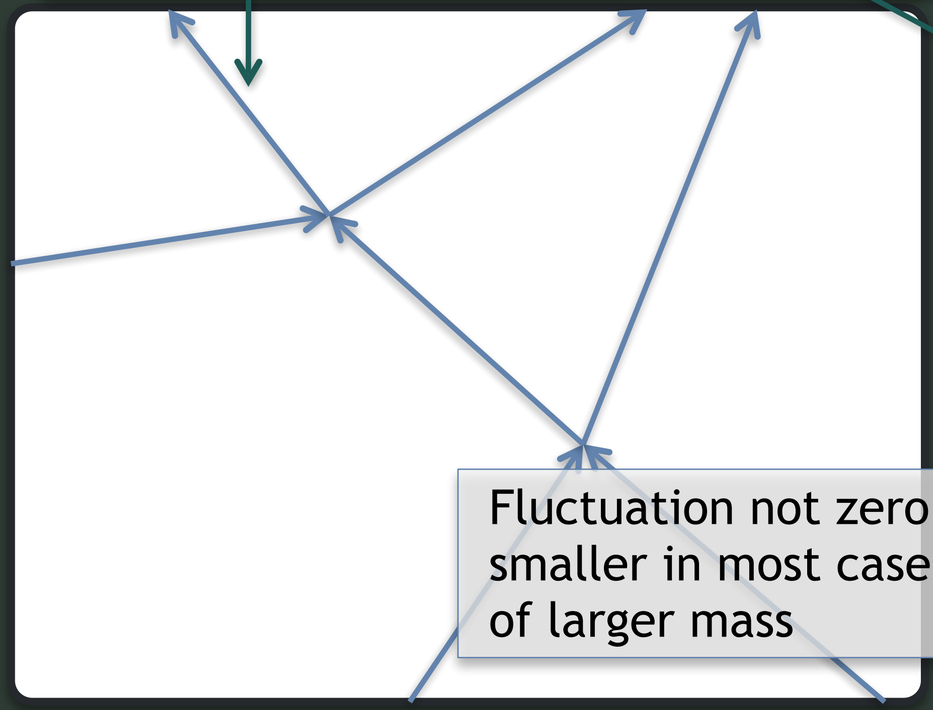
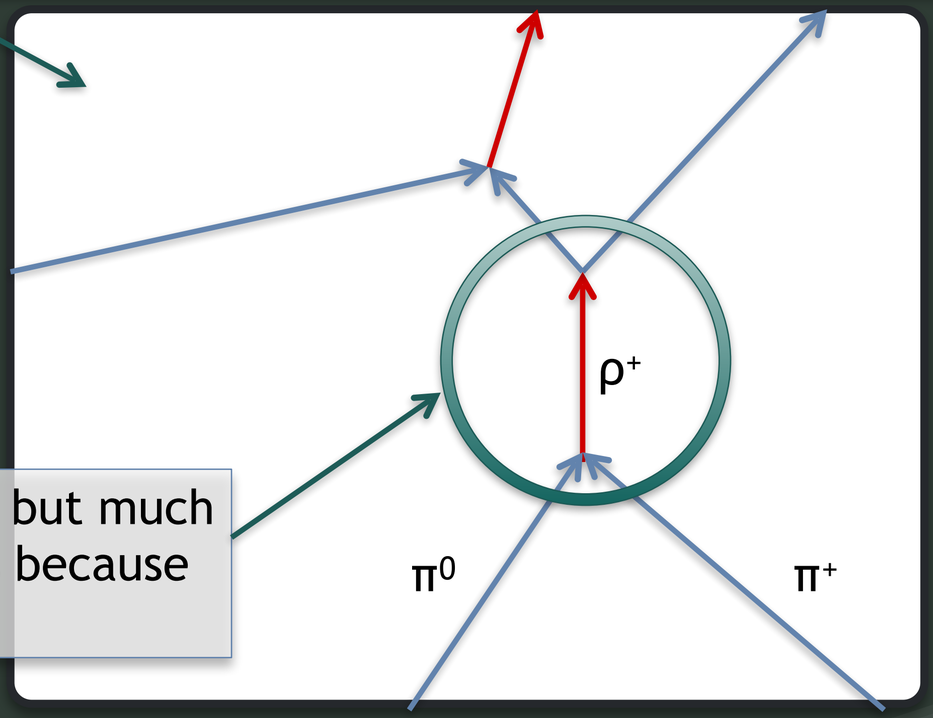
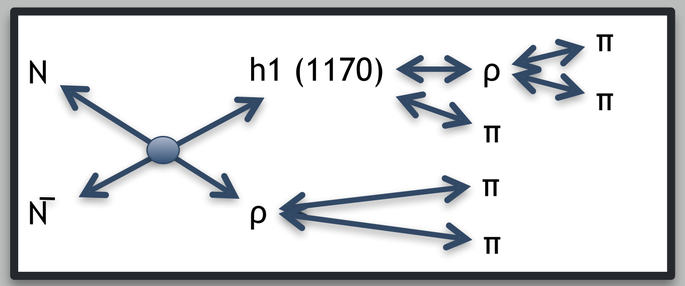


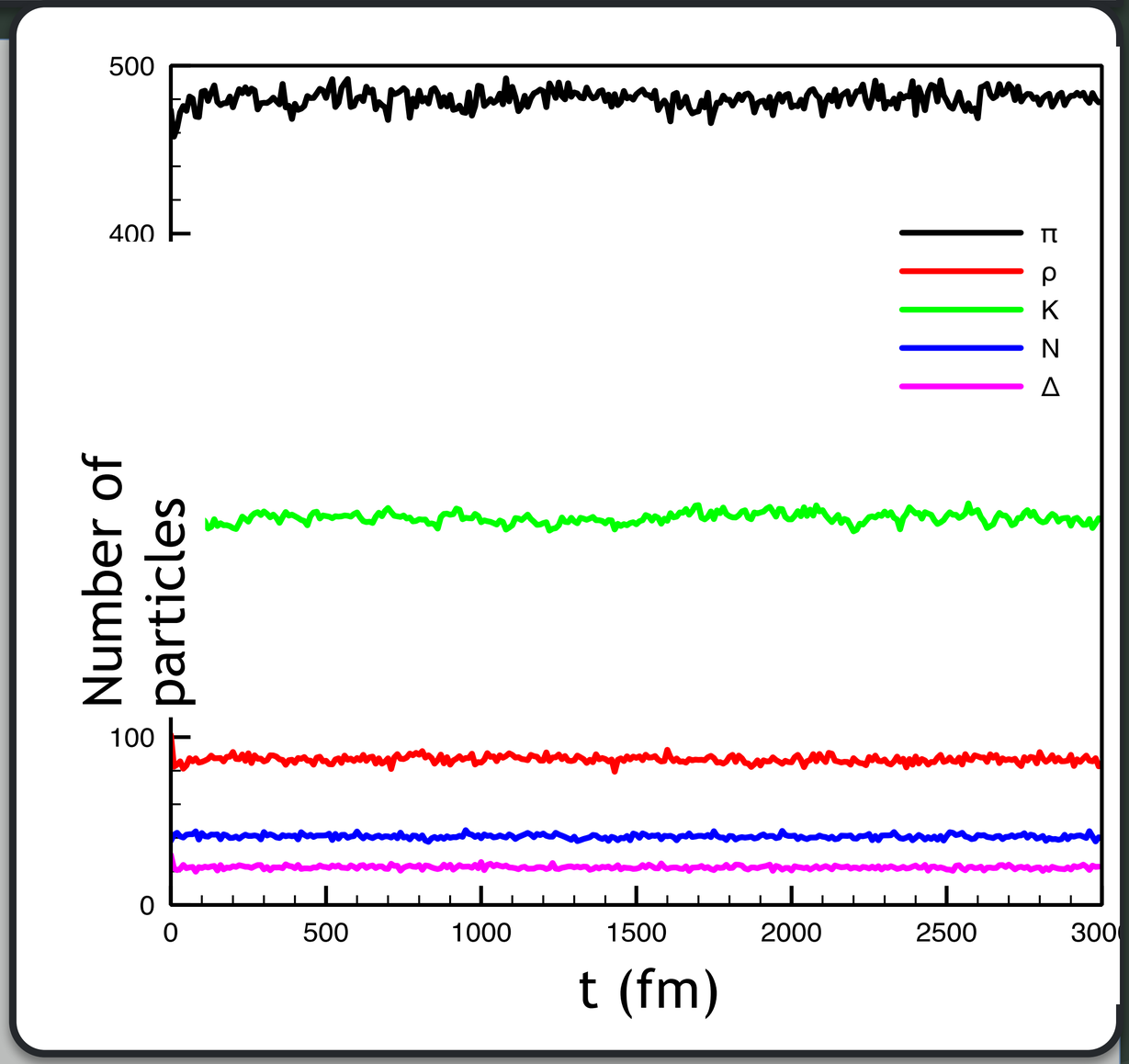


Fluctuation not zero, but much smaller in most cases because of larger mass

Equilibrium in SMASH

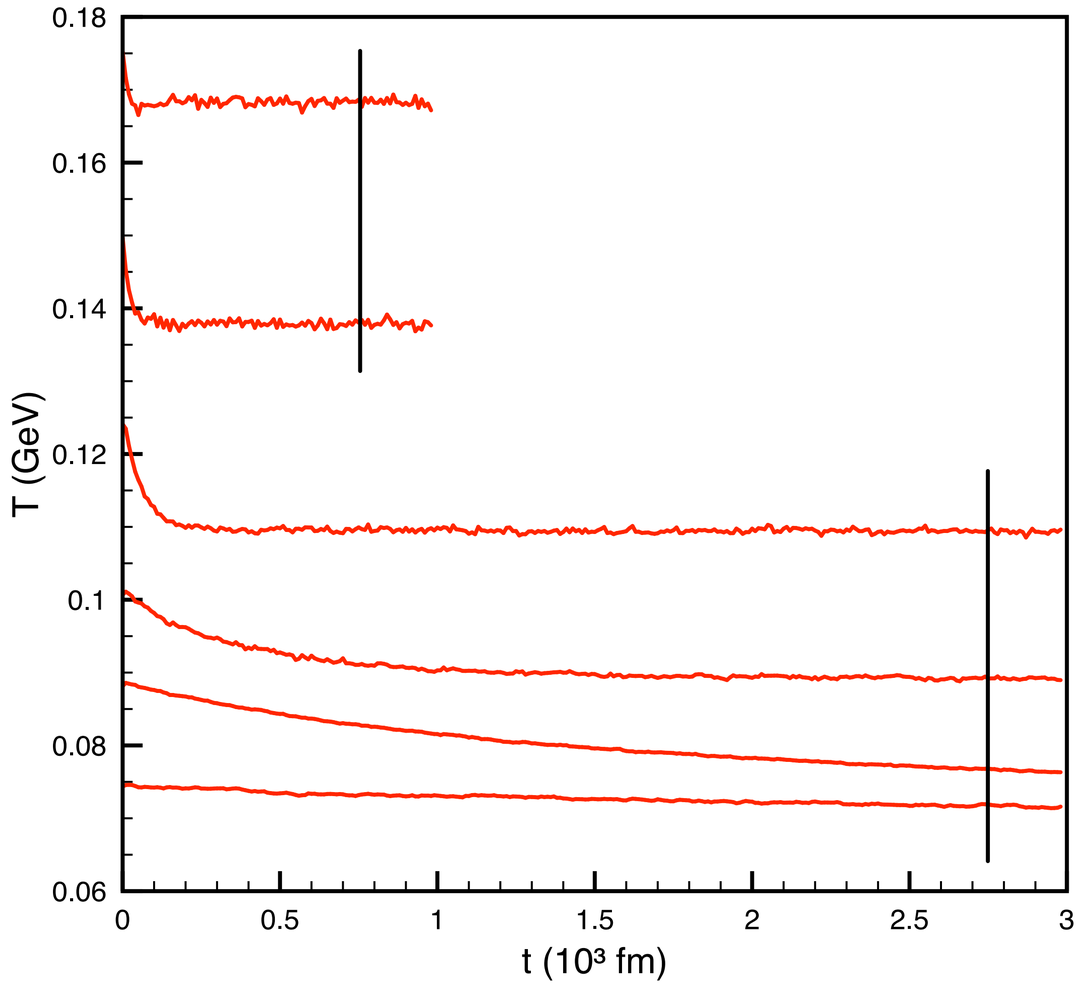
- Box calculations simulating infinite matter to apply the Green-Kubo procedure
- MUST have thermal & chemical equilibrium
- Baryon/antibaryon annihilation implemented to conserve detailed balance via an average decay to 5π





Equilibrium in SMASH

- All particles and resonances initialized to thermal multiplicities (at the pole mass)
- Must wait for equilibration and compute *T*, *μ* once in equilibrium from most abundant particles
  - T fitted from weighted momentum spectra of *π*, *K* & *N*
  - μ_B obtained from *N* / anti-*N* ratio



What about entropy?

The entropy density can be calculated from the Gibbs formula:

$$s = \frac{e + p - \mu n}{T} = \frac{w - \mu n}{T}$$

where the energy density and pressure can be taken from the average shear-stress tensor according to:

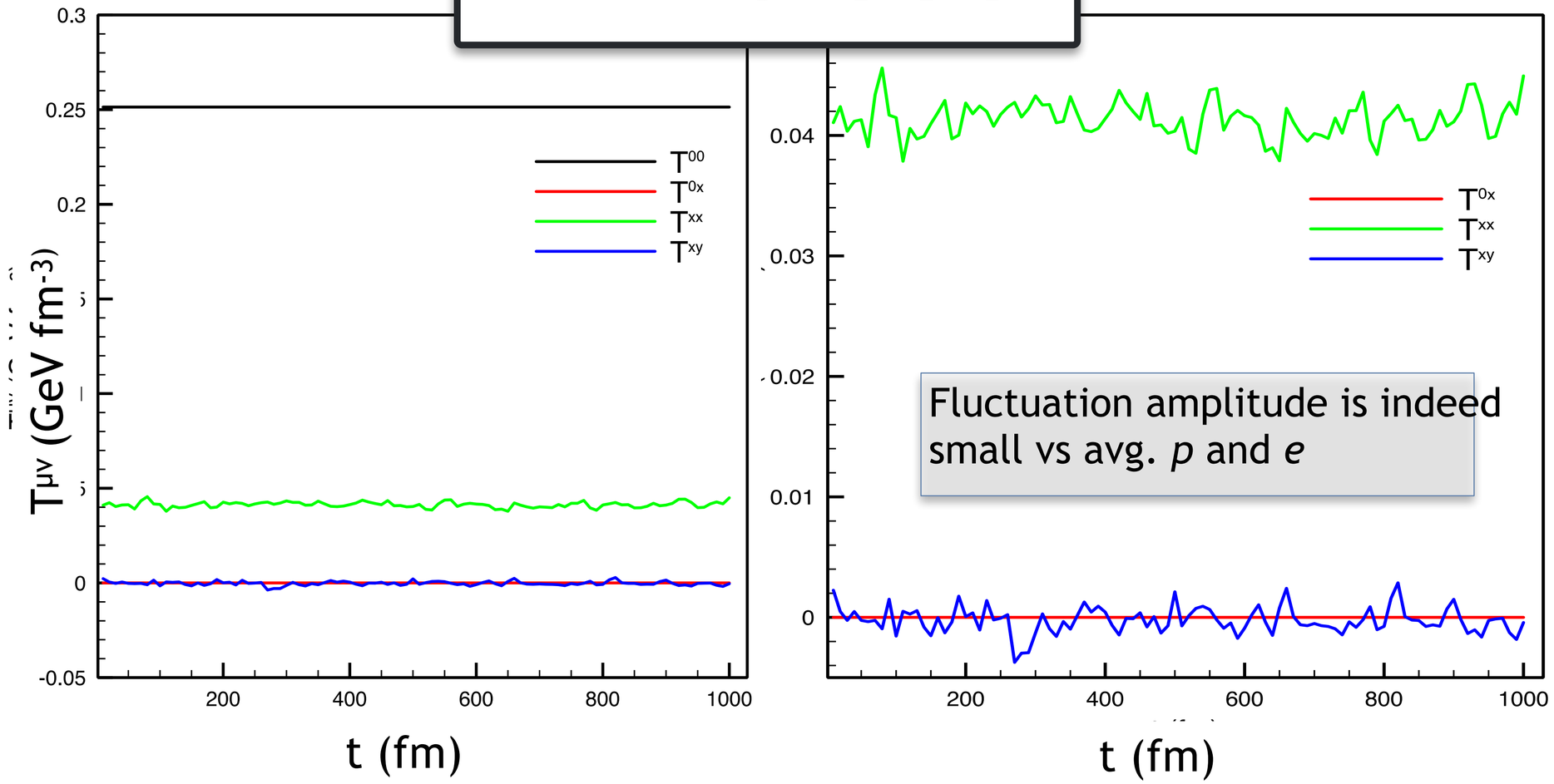
$$T^{\mu\nu} = diag(e, p, p, p)$$

Assuming a nearly ideal gas, one can fit the temperature and chemical potential with momentum distributions:

$$\frac{dN}{dp} = \frac{g}{2\pi^2} V p^2 \exp\left(-\frac{\sqrt{p^2 + m^2} - \mu}{T}\right)$$

Energy density and pressure

$$T^{\mu\nu} = diag(e, p, p, p)$$



Fluctuation amplitude is indeed small vs avg. *p* and *e*