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BROOKHAVEN NATIONAL LABORATORY

Computing real-time correlation functions on a hybrid classical/quantum computer

Niklas Mueller
with Andrey Tarasov, Raju Venugopalan
Brookhaven National Laboratory

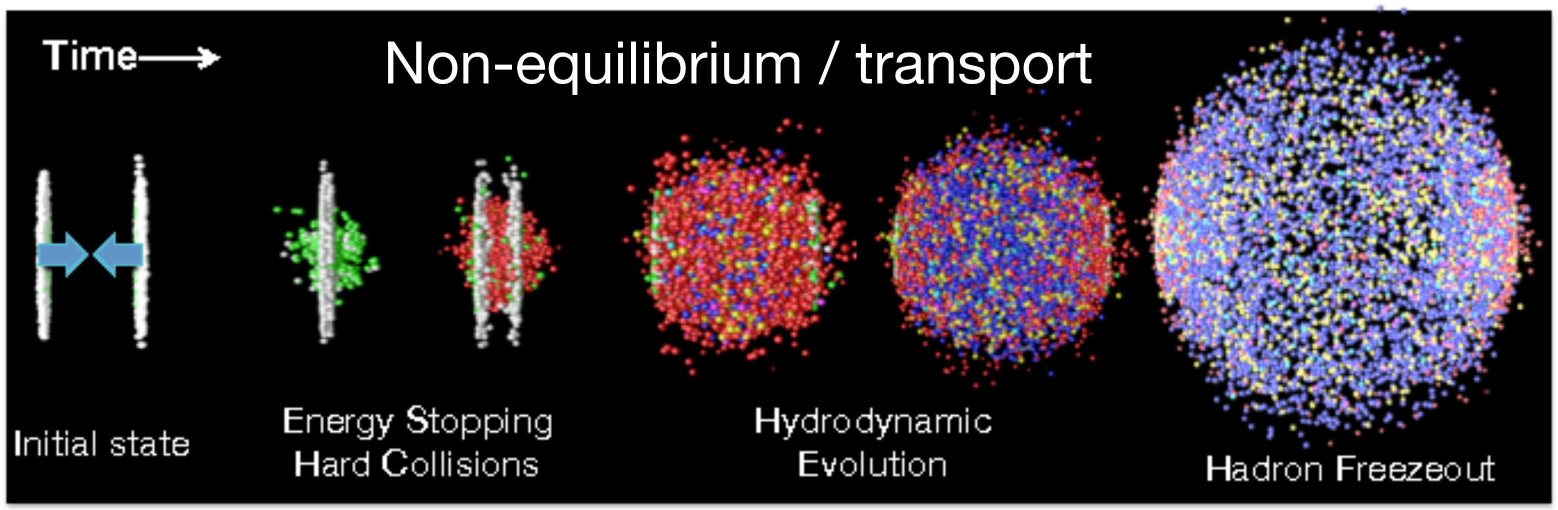
arXiv:1908.07051

Quark Matter 2019
Wuhan, China — Nov 5th, 2019

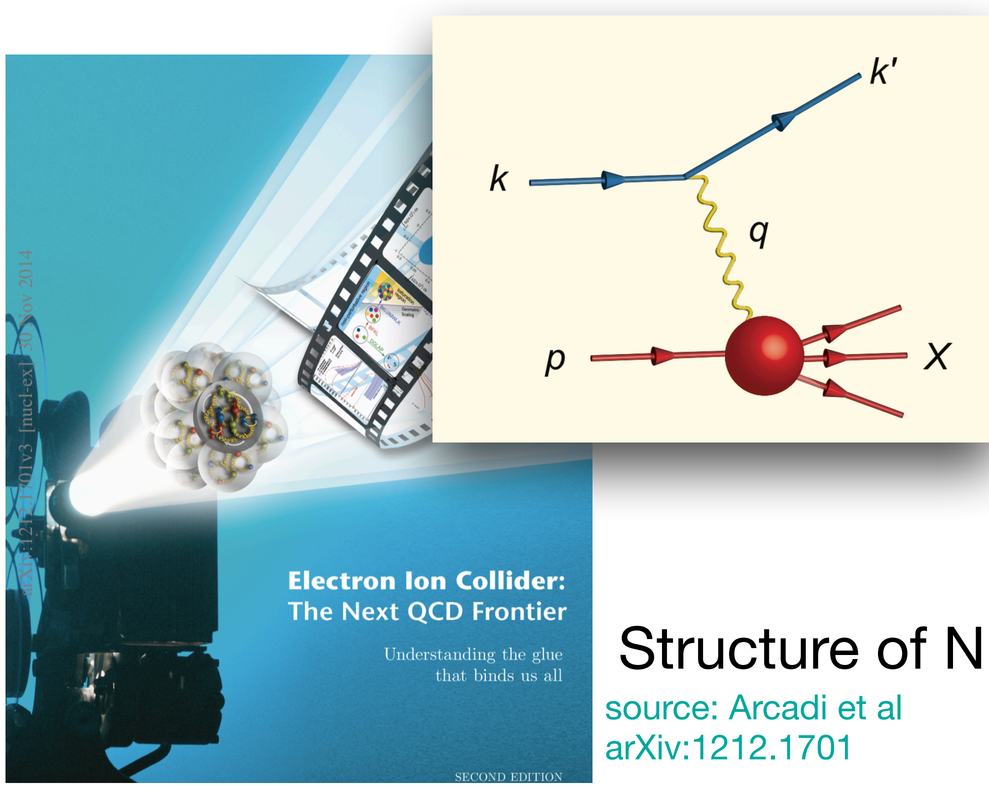
Motivation

Quantum-many-body systems hard to compute classically

Real-time correlation functions



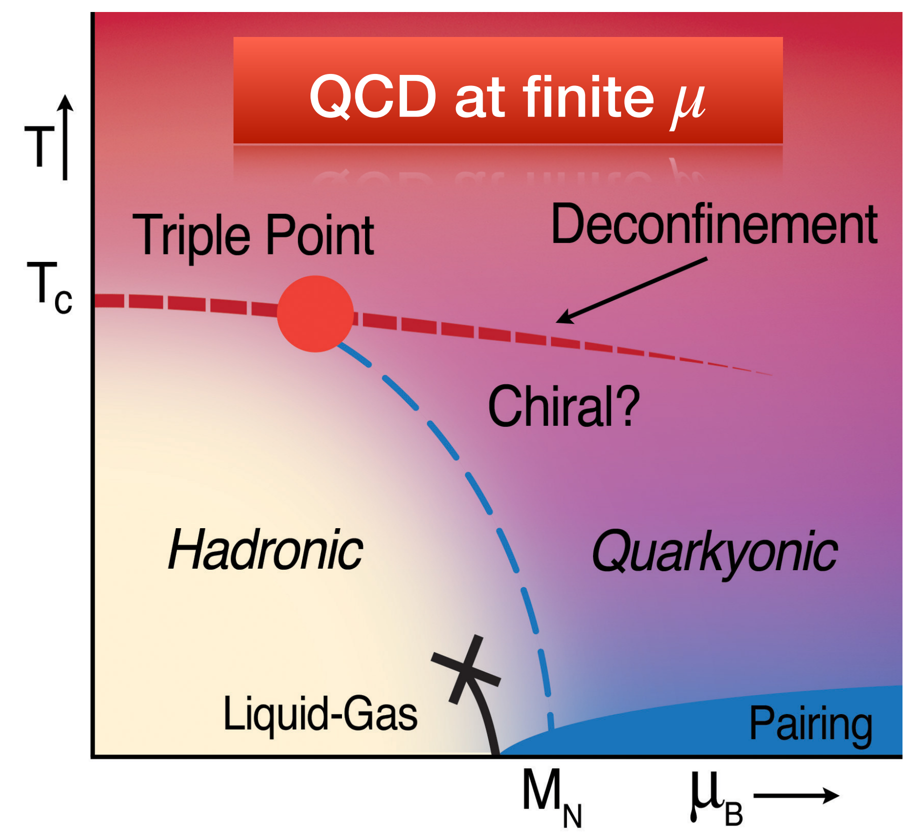
source: Nayak et al. arXiv:1201.4264



Structure of Nuclei

source: Arcadi et al
arXiv:1212.1701

Sign problem(s)



source: https://quark.phy.bnl.gov/

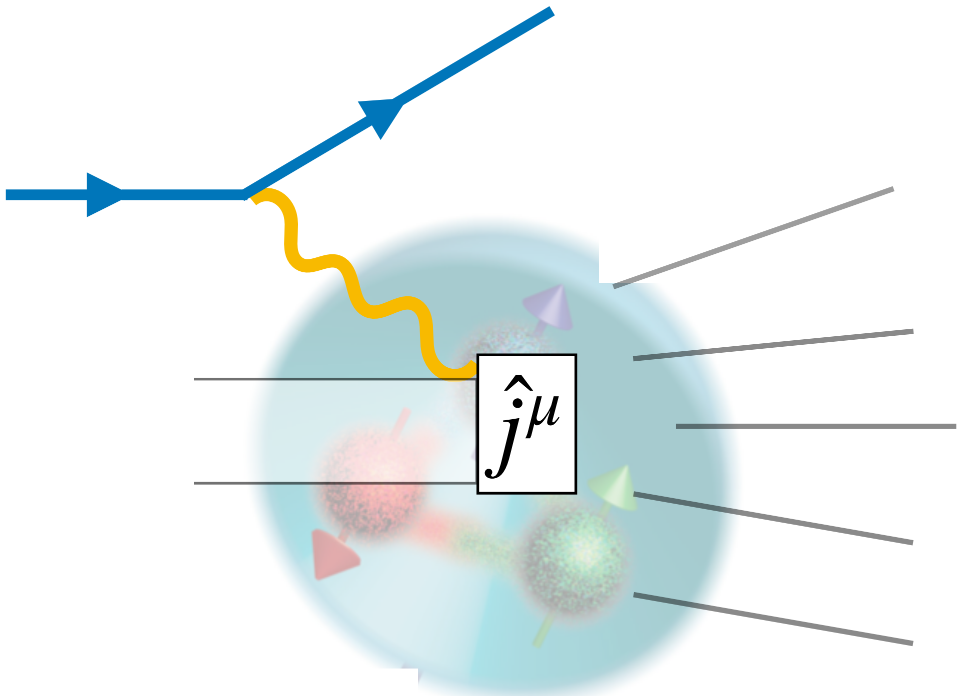
Motivation

Deeply Inelastic Scattering (DIS) structure functions

- **Structure of nuclei via** **<u>real-time correlation functions</u>**

$$W_{\mu\nu}(q,P,S) = \mathrm{Im}\,\frac{i}{\pi}\int d^4\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} \langle P,S|\mathbb{T}\,\hat{j}^{\mu}(\mathbf{x})\hat{j}^{\nu}(0)|P,S\rangle$$

- **proton state** $|P,S\rangle = \hat{\mathcal{U}}_{(0,-\infty)}\,\hat{\Phi}_{P,S}|0\rangle$
- **EM current operators, separated in Minkowski spacetime**



source: Arcadi et al arXiv:1212.1701

- **Lattice Simulations restricted to <u>Euclidean spacetime</u>**

- **quasi-pdf's: Ji, PRL 110, 262002 (2013),**
- **lattice cross sections: Ma, Qiu, PRD98, 074021 (2018)**
- Alexandrou C. et al PRD 92, 014502 (2015)
- Chen, Cohen, Ji, Lin, Zhang, NPB911, 246 (2016)
- Radyushkin, PLB 767, 314 (2017)
- Lin et al, Prog. Part Nucl Phys 100, 107 (2018),
- Detmold et al (USQCD), arXiv1904.09512

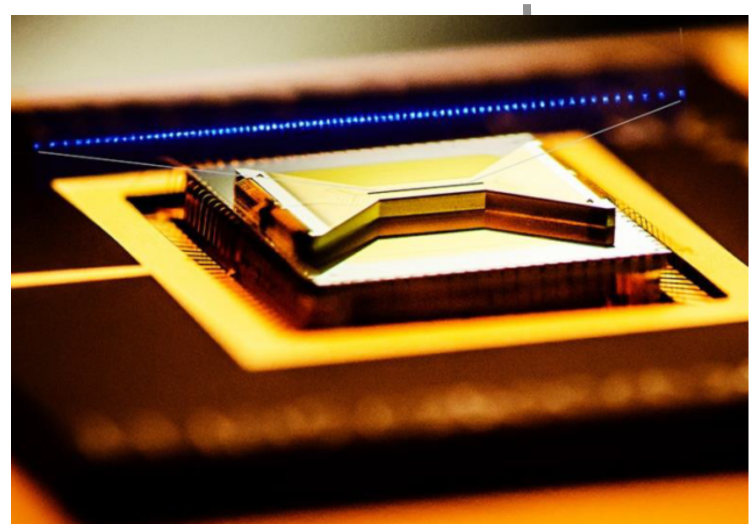
- Lamm, Lawrence, Yamauchi, arXiv:1908.10439

Motivation

Simulating nature with nature

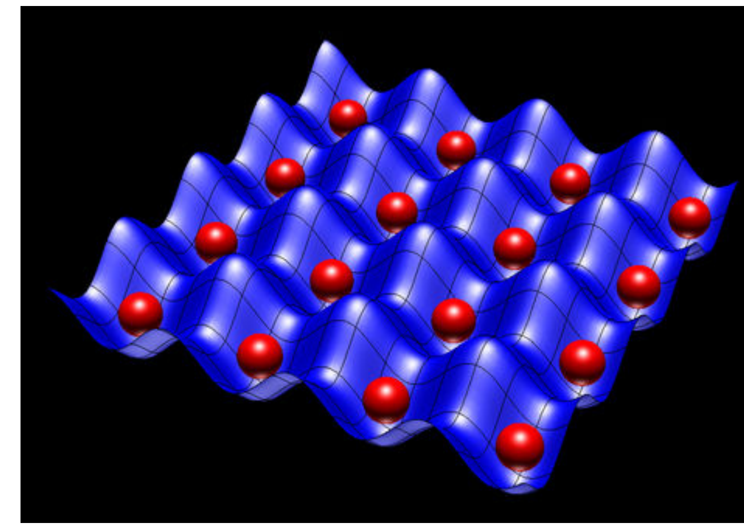
Digital Quantum Computers

source: IBM



source: Christopher Monroe

Analog Quantum Simulators



source: NIST

Quantum Field Theory
infinite dimensional Hilbert space
$\hat{\phi}(\mathbf{x}, t), \hat{\pi}(\mathbf{x}, t)$

"digitization"

Quantum Computer
qubits and gates 'quantum mechanics'
(finite dimensional Hilbert space)

Difficult, because

- **(bosons)** large Hilbert space
- **(gauge theories)** most of it unphysical

Bern, Kosower NPB370 (1992) 451
Strassler, NPB385 (1992), 145
Berezin, Marinov, Ann Phys 104 (1977) 336
D'Hoker Gagne, NPB467 (1996), 272&297

1. Worldline approach to DIS

- **How to map QFT to QM?** worldline approach

$$\phi(\mathbf{x}, t), \pi(\mathbf{x}, t) \to x_\mu(\tau), p_\mu(\tau)$$

"quantum field theory"   "quantum mechanics"

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"quantum field theory" "quantum mechanics"

- **Fermions**, spin via Grassman variables (*)

$$\psi(\mathbf{x}, t), \bar{\psi}(\mathbf{x}, t) \to x_\mu(\tau), p_\mu(\tau), \theta_i(\tau), \theta_i^*(\tau)$$

"spinor fields" "susy quantum mechanics"

(*) similar for color, see NM, Venugopalan PRD99 (2019), 056003

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"spinor fields" "susy quantum mechanics"

- $W^{\mu\nu}$ ~ $\langle P, S \,|\, \hat{j}^\mu(x)\hat{j}^\nu(0) \,|\, P, S\rangle$ **"in-in" correlation function**

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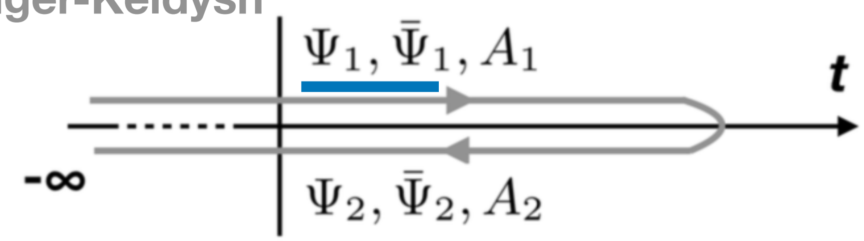
- $W^{\mu\nu} \sim \langle P, S \,|\, \hat{j}^\mu(x)\hat{j}^\nu(0) \,|\, P, S\rangle$ **"in-in" correlation function**

"Schwinger-Keldysh"



$$\hat{\rho}_{\text{quarks}} \otimes \hat{\rho}_{\text{Yang-Mills}}$$

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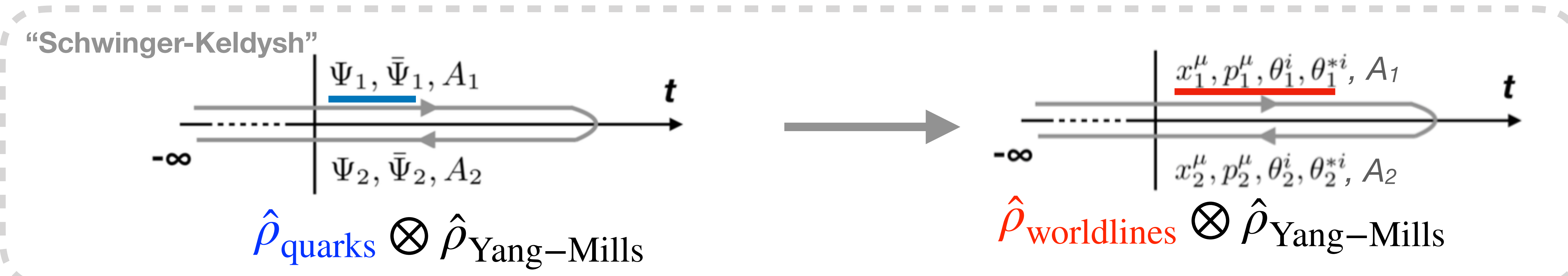

- **Quark part:** worldline Hamiltonian problem

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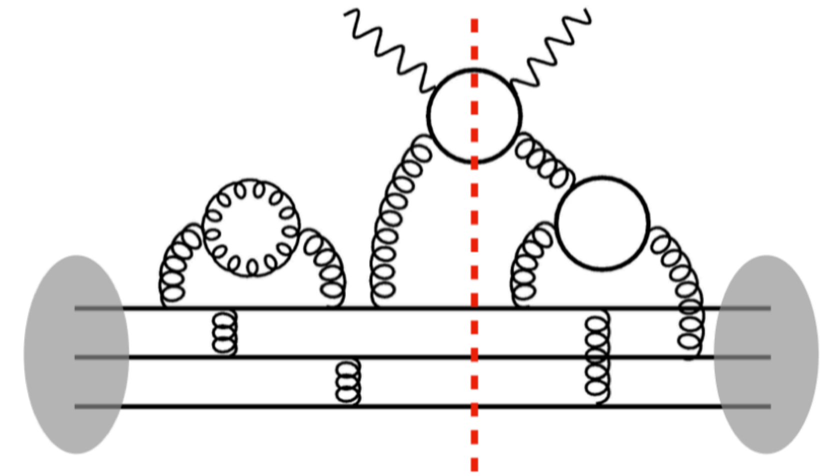
1. Worldline approach to DIS

- **Hadron tensor as worldline SK path integral**

$$W^{\mu\nu} =$$



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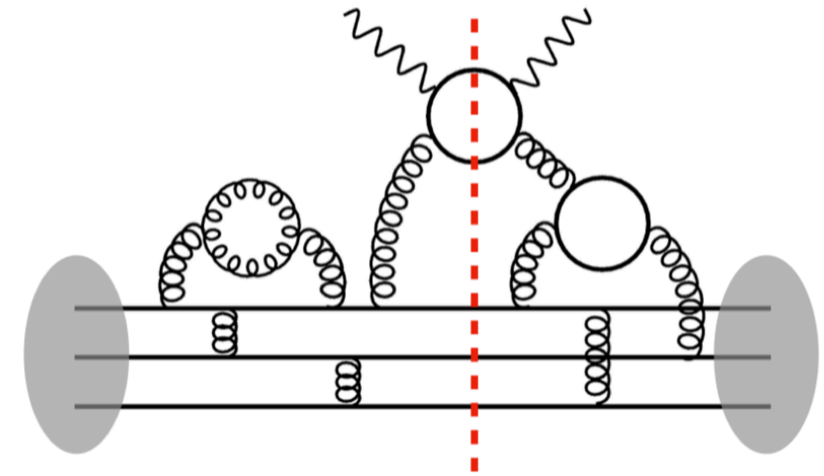
$$W^{\mu\nu} =$$ 

$$W^{\mu\nu} = \frac{1}{\pi e^2}\mathrm{Im}\int d^4z\, e^{iq\cdot z}\sum_{n=0}^{\infty}\frac{i^{n+4}}{n!}\int\Big[\prod_{k=1}^{n+4} d^4x_1^k d^4x_2^k\; d^2\theta_1^k d^2\theta_2^k\Big]\int dA_1 dA_2\;\; \mathrm{tr}_c\,\langle x_1, -\theta_1, A_1|\hat{\rho}_{\text{init}}|x_2, \theta_2, A_2\rangle$$
$$\times\,\langle x_2, \theta_2, A_2|\hat{\mathbb{U}}_{(-\infty,z)}\hat{J}^{\mu}_{(4)}(z)\hat{\mathbb{U}}_{(z,\infty)}\,\hat{\mathbb{U}}_{(\infty,0)}\hat{J}^{\nu}_{(4)}(0)\hat{\mathbb{U}}_{(0,-\infty)}|x_1, \theta_1, A_1\rangle$$

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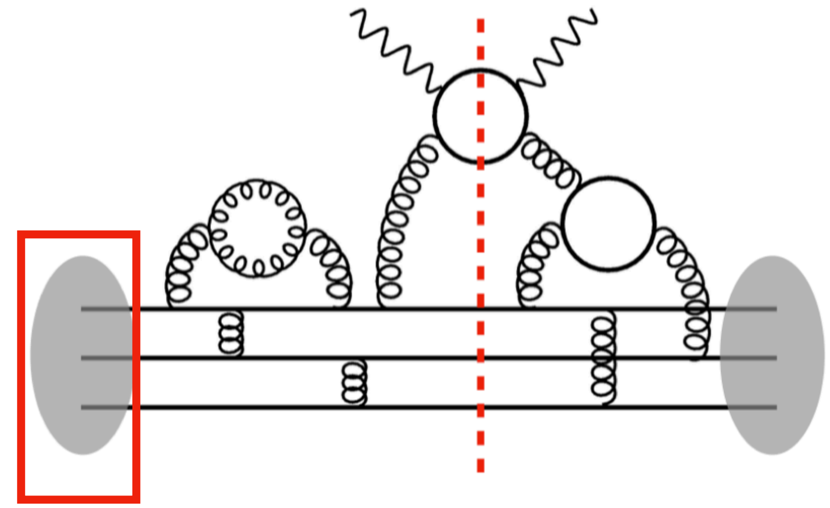
$W^{\mu\nu} =$ 

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initial valence 'worldlines' density matrix
$\hat\rho_{\mathrm{init}} \equiv \hat\Phi_{P,S}|0\rangle(\hat\Phi_{P,S}|0\rangle)^\dagger$

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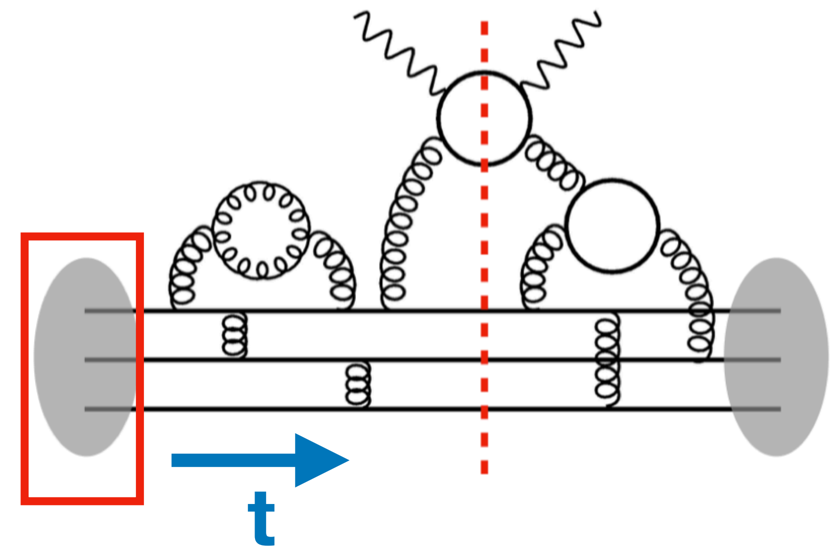

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time evolution

$$\hat{\mathbb{U}}_{(t,t')} \;\equiv\; \exp\{-i\hat{H}(t-t')\}$$
$$\hat{H} \;=\; \hat{H}_{\mathrm{YM}} + \textstyle\sum_{k=1}^{4+n}\hat{H}^k$$

Yang-Mills Hamiltonian

worldline Hamiltonian for quarks

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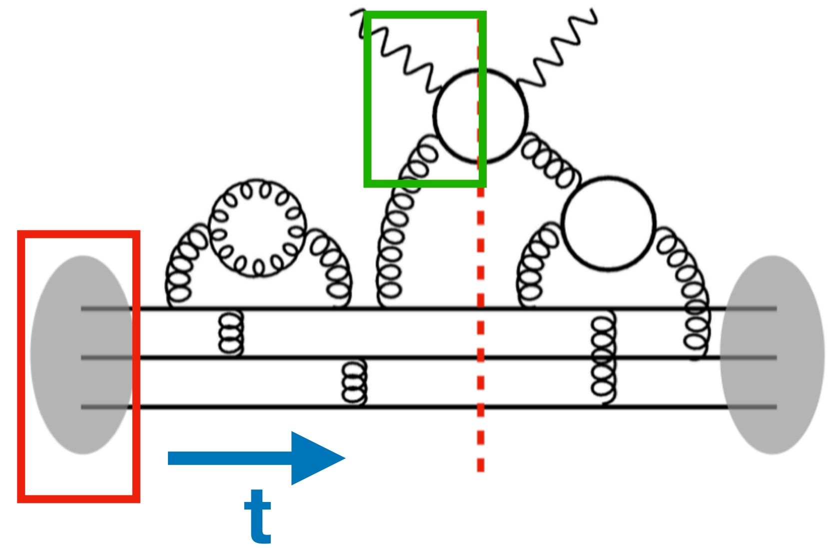
$$W^{\mu\nu} =$$ 

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current measurement

$$\hat{J}^{\mu}_{(4)}(z) \equiv -\textstyle\sum_{k=1}^{4}(\delta\hat{H}^k/\delta a_\mu(z))_{a=0}$$

$$\hat{j}^{\mu}_k(z) \equiv \frac{e}{p_k^0}\Big[\hat{P}^{\mu}_k + i\hat{\psi}^{\nu}_k\hat{\psi}^{\mu}_k q_\nu\Big]\delta^{(3)}(\mathbf{z}-\hat{\mathbf{x}}_k(z^0))$$

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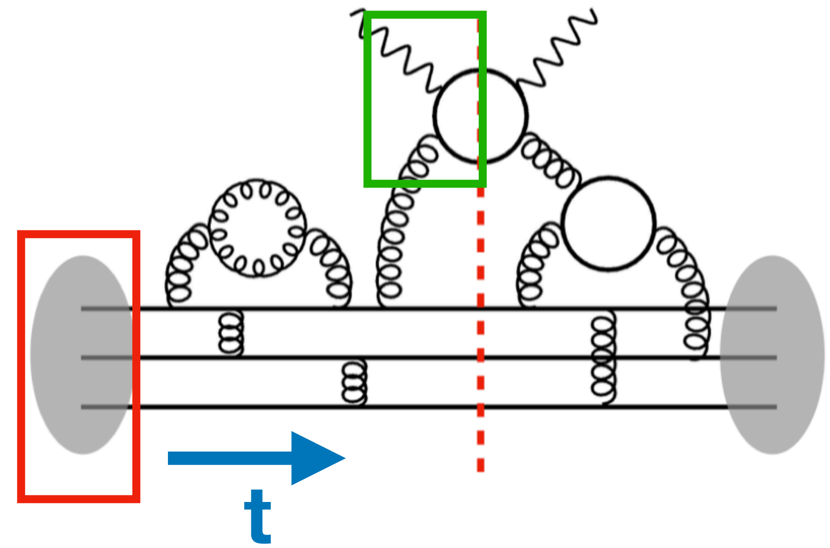
$$W^{\mu\nu} =$$ 

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- **von-Neumann problem** $\partial_t\hat{\rho} = -i[\hat{H},\hat{\rho}]$ $\quad(\in \mathscr{H}_{x,\theta}\otimes\mathscr{H}_{\mathrm{YM}})$

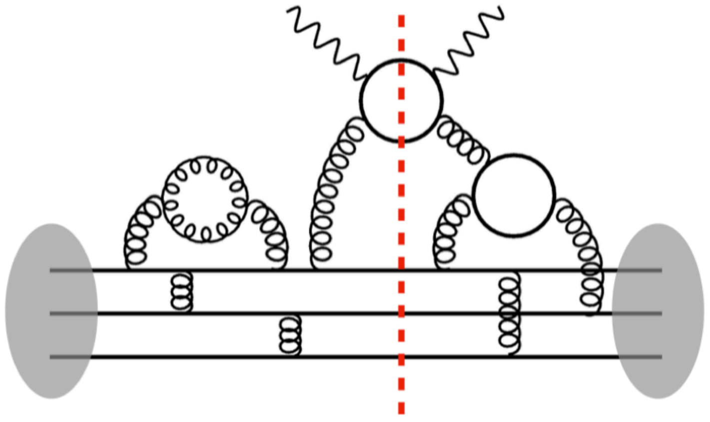
worldlines Yang-Mills

2. Hybrid quantum computation

- **difficult with noisy quantum technology**

$$\hat{\rho}_{\text{worldlines}} \otimes \hat{\rho}_{\text{Yang-Mills}}$$

hard!



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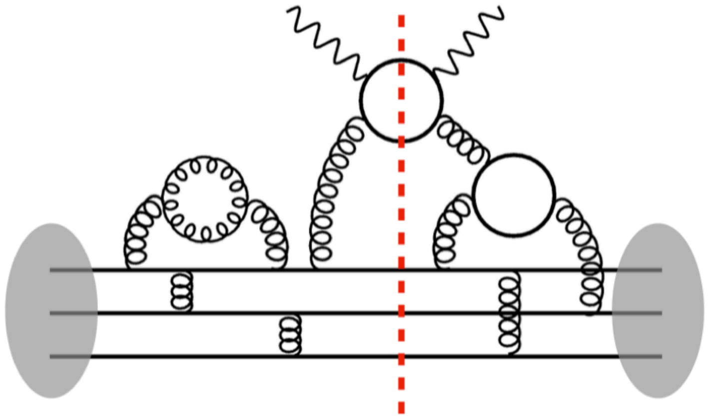

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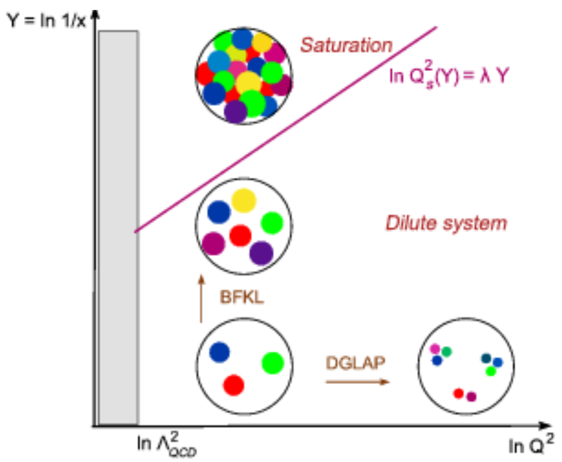
- **High energy 'Regge' limit: gluon saturation**

Color Glass Condensate EFT



Gelis, Iancu, Jalilian-Marian, Venugopalan,
Ann Rev Nucl Part Sci 60, 463 (2010)

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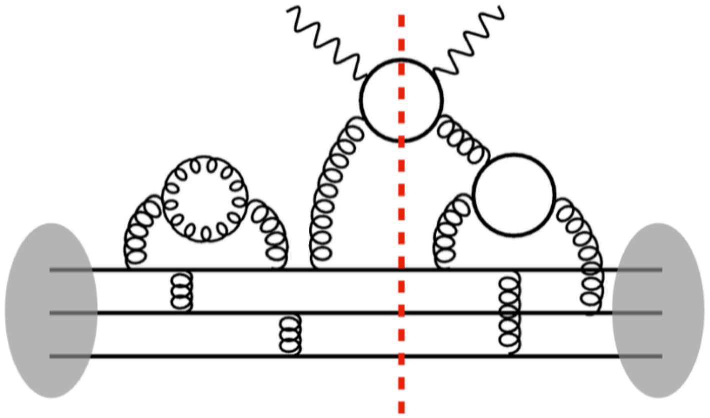

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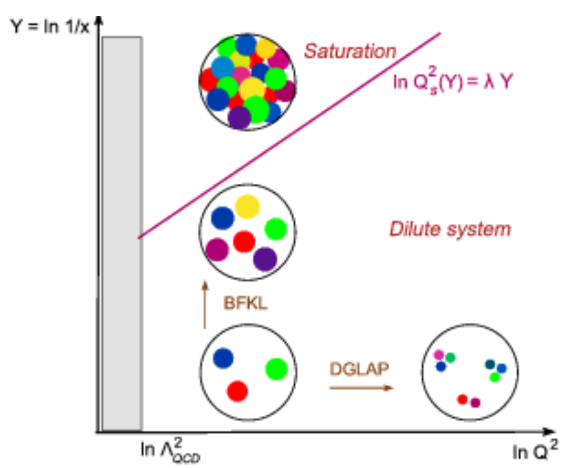

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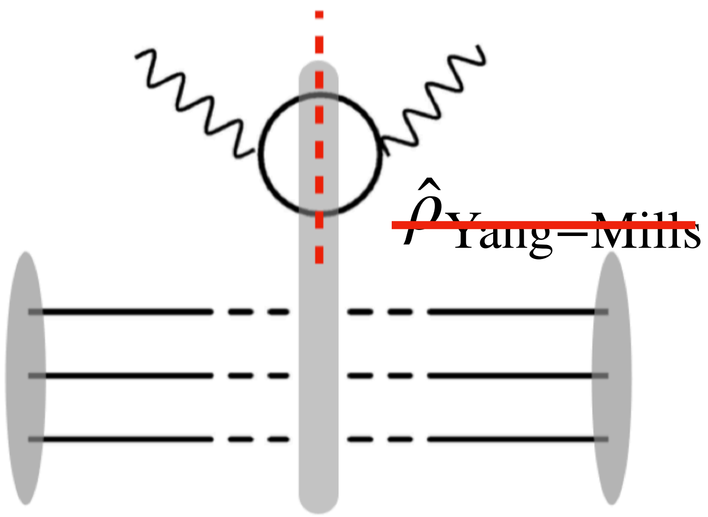

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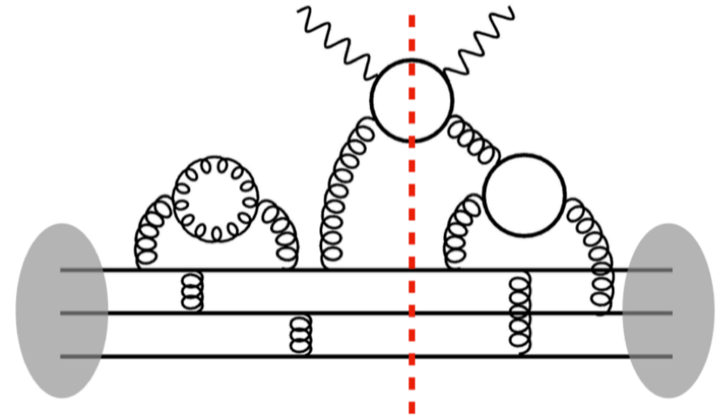

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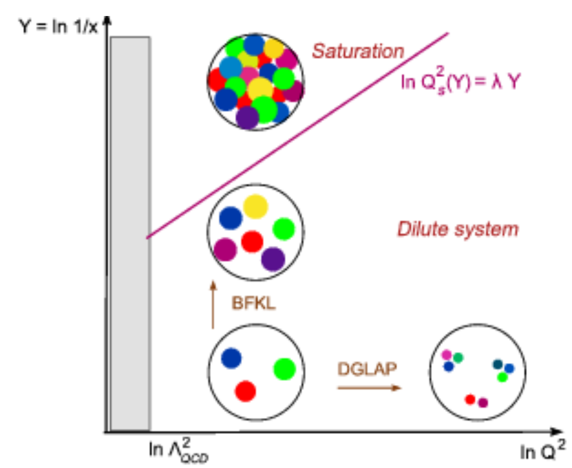

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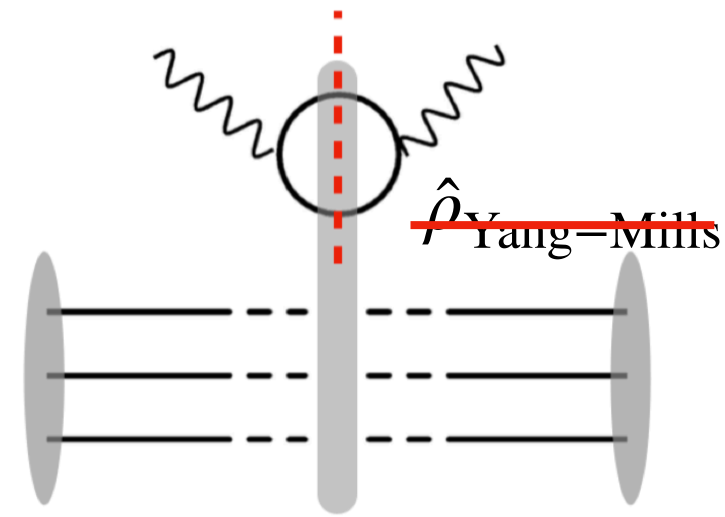

- $$\boxed{W^{\mu\nu} = \ldots \int d^4x d^2\theta \ldots \langle -\theta, x | \ldots e^{-i\hat{H}t} \ldots | \theta, x \rangle}$$

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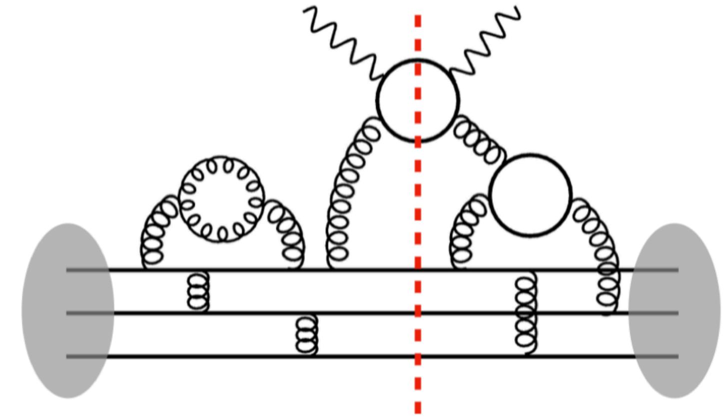

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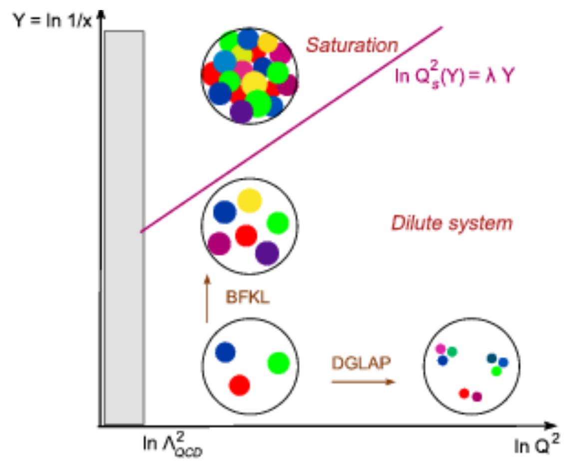

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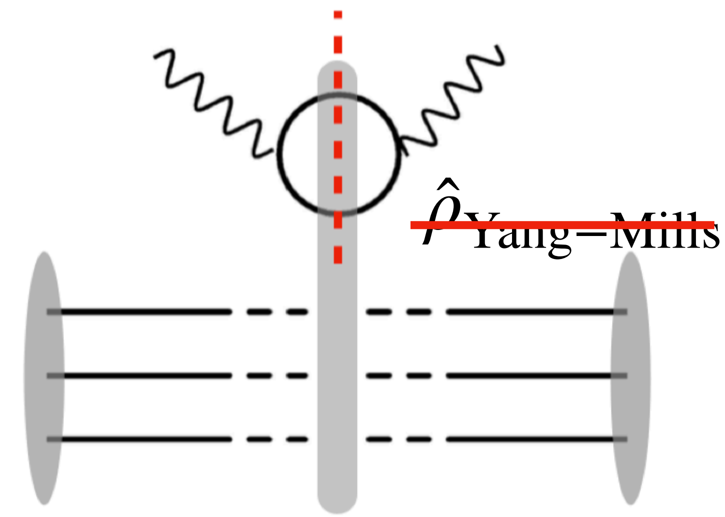

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- **Our strategy**
  - quantum compute fermionic part θ_i, θ_i^*,
  - bosonic part x_μ, p_μ analytically

2. Hybrid quantum computation

arXiv:1908.07051

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- **Quantum computing** $F_2 \equiv \Pi_2^{\mu\nu} W_{\mu\nu}$

$$F_2(q,P) = \frac{\sigma\, Q^2}{2\pi e^2} \int [\mathcal{D}\rho] W[\rho] \int_{x_\perp} \int_z \sum_{L,T;\,f} |\Psi^f_{L,T}(z,x_\perp)|^2 \; D(x_\perp)\, i \int d^2\theta \langle -\theta | \big[\Omega_{L,T}(z,x_\perp)\big] |\theta\rangle$$

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2. Hybrid quantum computation

arXiv:1908.07051

- **Quantum computing** $F_2 \equiv \Pi_2^{\mu\nu} W_{\mu\nu}$

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- **Clifford algebra via 2-fermion operators**

$$[\gamma_\mu, \gamma_\nu]_+ = 2g_{\mu\nu}$$

$$\updownarrow$$

$$[\hat{b}_i, \hat{b}_j^\dagger]_+ = \delta_{ij}$$

$$\begin{aligned}
\gamma_5\gamma^0 &= \hat{b}_1^\dagger - \hat{b}_1 \equiv \sqrt{2}\hat{\psi}_0 \\
\gamma_5\gamma^1 &= \hat{b}_2^\dagger + \hat{b}_2 \equiv \sqrt{2}\hat{\psi}_1 \\
\gamma_5\gamma^2 &= -i(\hat{b}_2^\dagger - \hat{b}_2) \equiv \sqrt{2}\hat{\psi}_2 \\
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\gamma_5 &= -(-1)^{\sum_{i=1}^2 \hat{b}_i^\dagger \hat{b}_i}
\end{aligned}$$

$$\hat{b}_i |\theta\rangle = \theta_i |\theta\rangle$$
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$$\hat{b}_i^\dagger|\theta*\rangle = \theta_i^*|\theta*\rangle$$

- **Trace** $\mathrm{Tr}(\Omega_L) = i\int d^2\theta \langle -\theta | \Omega_L | \theta \rangle$ **on 2 (+1) qubit quantum computer**

$$\Omega_L(z,x_\perp) = \frac{1}{2z(1-z)}\Big\{ -\frac{3}{4}\left[(2z-1) + 2\hat{\psi}^-\hat{\psi}^+\right]$$
$$\times \left[(2z-1) - 2\hat{\psi}^-\hat{\psi}^+\right] - \hat{\psi}^+\hat{\psi}^-\hat{\psi}^+\hat{\psi}^- - \hat{\psi}^j\hat{\psi}^+\hat{\psi}^j\hat{\psi}^-$$
$$- z(1-z) + \frac{3}{4}\Big\},$$

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\gamma_5\gamma^3 &= \hat{b}_1^\dagger + \hat{b}_1 \equiv \sqrt{2}\hat{\psi}_3 \\
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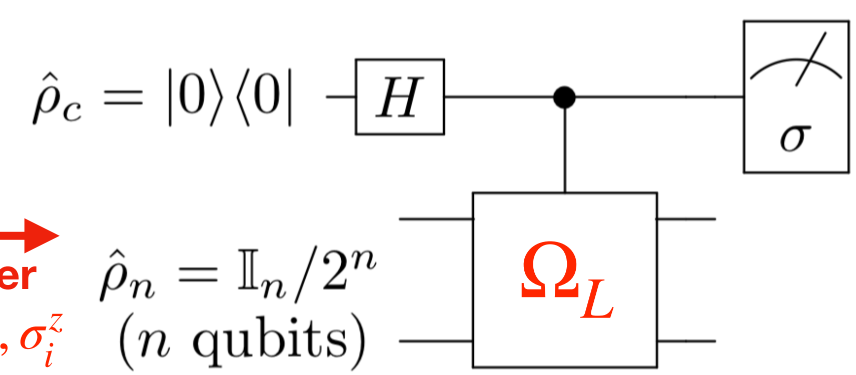
Jordan-Wigner

$\hat{b}_i, \hat{b}_i^\dagger \rightarrow \sigma_i^x, \sigma_i^y, \sigma_i^z$



Knill, Laflamme, PRL 81, 5672 (1998)

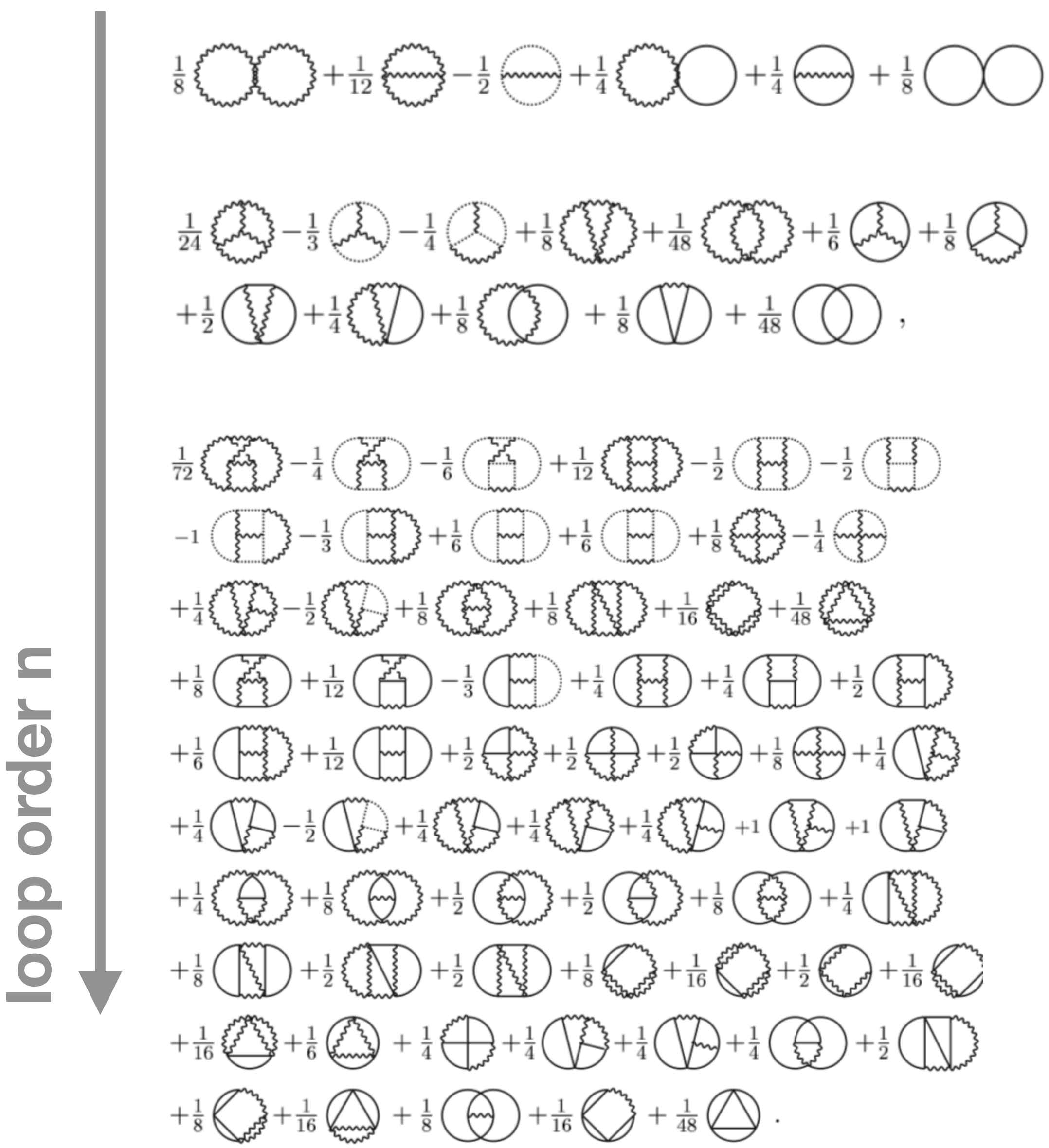
Next steps

Quantum advantage beyond toy problem?

$\mathscr{H}_{x,\theta} \otimes \mathscr{H}_{\mathrm{YM}}$

worldlines Yang-Mills

perturbation theory



A. Vuorinen 2001

- **computational complexity grows ~ n!**
- **S-matrix scattering through quantum algorithms & hamiltonian formulation**
  Jordan, Lee, Preskill Science 336, 1130
- **Worldlines "made for" perturbation theory**
  Bern, Kosower NPB370 (1992) 451
- **number of worldlines grows as n**

for non-perturbative problem $\mathscr{H}_{\mathrm{Yang-Mills}}$ see also:
Quantum link models (Wiese Chandrasekharan,Brower NPB 492, 455; PRD60 094502; NPA 931 (201
prepotentials (Mathur, Raychowdhury, Stryker J Phys A43 (2010) 035403),
discrete groups (Alexandru, Bedaque, Harmalkar, Lamm, Lawrence, Warrington, arXiv:1906.11213)

Conclusions

- **Real-time correlation functions difficult to compute**
  - not for a quantum computer

- **Presented simple proof of principles NISQ study:**
  no quantum advantage yet

- **Digitization of QFT via worldline quantum mechanics,**
  bosonic (coordinate) and fermionic (spin+color) worldlines

- **Quantum Advantage: scale up and extend**

  - **Perturbation theory: scattering of quark and gluon worldlines**
    Hamiltonian formulation vs. Feynman diagrams

  - **non-perturbative Yang-Mills part hard:**
    quantum link models, prepotential formulations etc.

Back-up

Worldline representation: formalism I

Back-up

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- **Bern & Kosower: compute perturbative Feynman diagrams/S-amplitudes via n-point amplitudes of ‘worldlines’**

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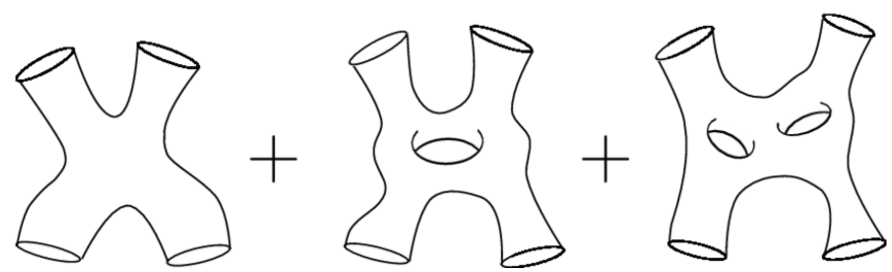

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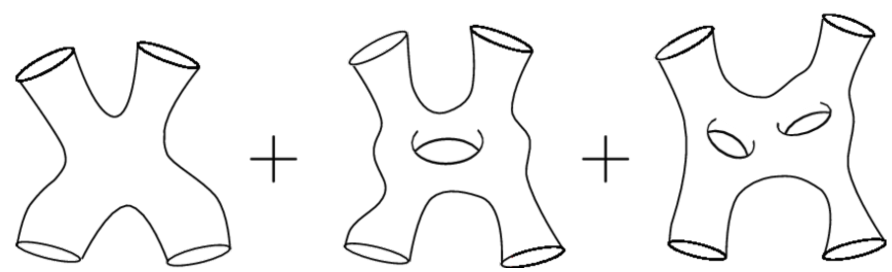

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  Strassler, NPB385 (1992), 145

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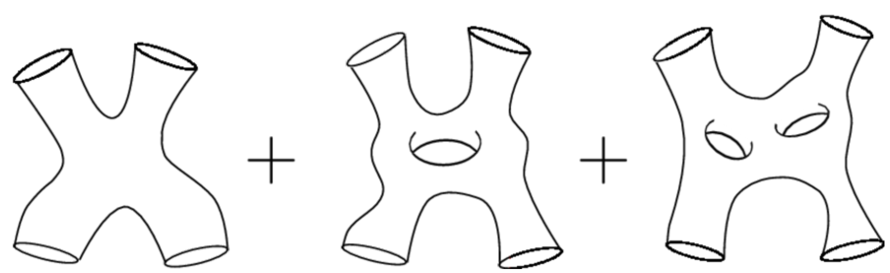

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  Alvarez-Gaume, Witten Nucl.Phys. B234 (1984) 269 theory

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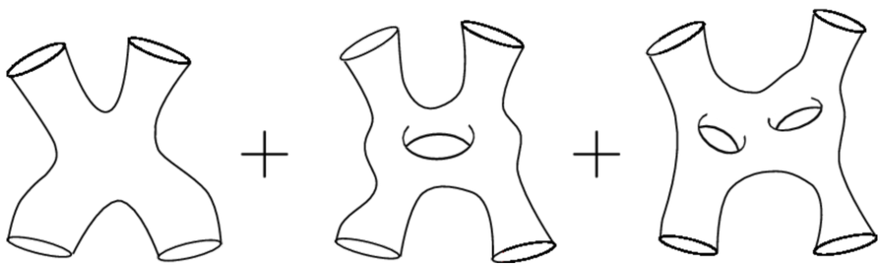

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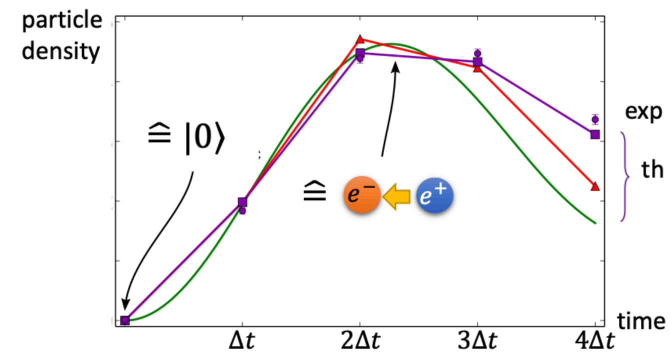
- **Semi-classics: quantum kinetic theory with internal symmetries**

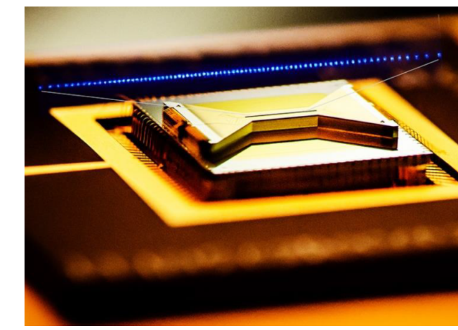
  NM, Venugopalan, PRD97 (2018) no.5, 051901, PRDD96 (2017) no.1, 016023, PRD99 (2019) no.5, 056003

Back-up
State of the Art

Kogut Susskind Hamiltonian LGT

- **Trapped Ion Computer**
  1+1D QED, 4 lattice sites, very very short times





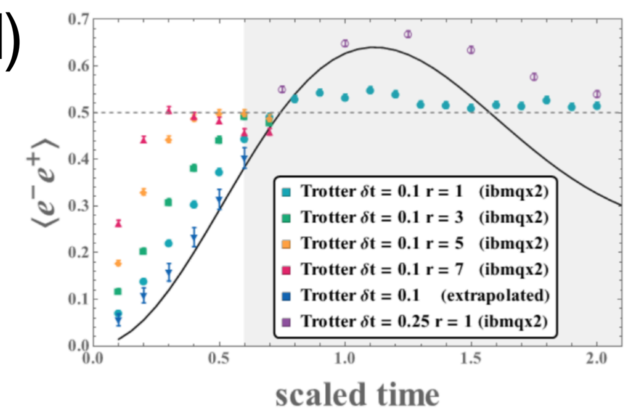
source: Christopher Monroe

Nature 534 (2016), 516

- **IBM-Q,** 1+1D QED, 2 & 4 lattice sites (staggered)

N. Klco, E.F. Dumitrescu, A.J. McCaskey, T.D. Morris, R.C. Pooser, M. Sanz,

E. Solano, P. Lougovski, M.J. Savage, PRA A98 (2018), 032331



source: IBM

- **Cold atom simulators**

T.V. Zache, F. Hebenstreit, F. Jendrejewski, M.K. Oberthaler, J. Berges, P. Hauke, Sci. Techol. 3 (2018), 034010

Z. Davoudi, M. Hafezi, C. Monroe, G. Pagano, A. Seif, A. Shaw arXiv:1908.03210

- **2+1D and more**

E. Zohar, A. Farace, B. Reznik, J.I. Cirac, Phys Rev Lett 118, 070501 (2017) and Phys Rev A 95, 023604 (2017)

Back-up

Worldline representation: formalism II

Strassler, NPB385 (1992), 145

Worldline representation: formalism II

- **Consider vacuum path integral (euclidean)**

$$\int \mathscr{D}\phi \; \exp\{-S[\phi]\} \qquad S[\phi] = \int d^d x \, \phi(D^2[A] - m^2)\phi + \dots$$

Strassler, NPB385 (1992), 145

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- **Gaussian integral yields determinant**

$$\Gamma = \log(\int \mathscr{D}\phi \exp\{-S[\phi]\}) = -\log\det[-D^2 + m^2]$$

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$$\Gamma[A] = -\,\mathbf{Tr}\log[-D^2 + m^2] = \int_0^\infty \frac{dT}{T}\mathscr{N}\int \mathscr{D}x \,\mathbf{tr}\mathscr{P} \ \exp\Big\{-\int_0^T \frac{\dot{x}^2}{2\epsilon} + igA[x]\cdot\dot{x}\Big\}$$

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- **Internal symmetries: Grassmann coordinates** $\quad x_\mu(\tau) \to \; x_\mu(\tau), \psi_\mu(\tau)$

Strassler, NPB385 (1992), 145

Back-up

Worldline representation: formalism III

Back-up

Worldline representation: formalism III

- **Spin: Dirac theory, fermion determinant**

$$S = \int d^4x \; \overline{\chi}(i \, \not{D} - m)\chi$$

Back-up

Worldline representation: formalism III

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Back-up

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- **Worldline representation, coordinate and Grassmann variables**

$$\Gamma_{\text{Dirac}} \propto \int_0^\infty \frac{dT}{T} \int \mathscr{D}x \mathscr{D}\psi \, \mathbf{tr} \mathscr{P} \, \exp\left\{ -\int d\tau \frac{\dot{x}^2}{2\epsilon} + \dot{x} \cdot A[x] + \frac{i}{2}\psi_\mu \dot{\psi}_\mu - \frac{i\epsilon}{2}\psi_\mu F_{\mu\nu}\psi_\nu + \ldots \right\}$$

Back-up

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- **Coherent state representation for spin**

$$\hat{a}_i|\theta\rangle = \theta_i|\theta\rangle\,, \qquad \hat{a}_i^\dagger|\theta^*\rangle = \theta_i^*|\theta^*\rangle$$

e.g. $\psi_1 = \frac{1}{\sqrt{2}}(\theta_1^* + \theta_1)$ (i=1,2)

Back-up

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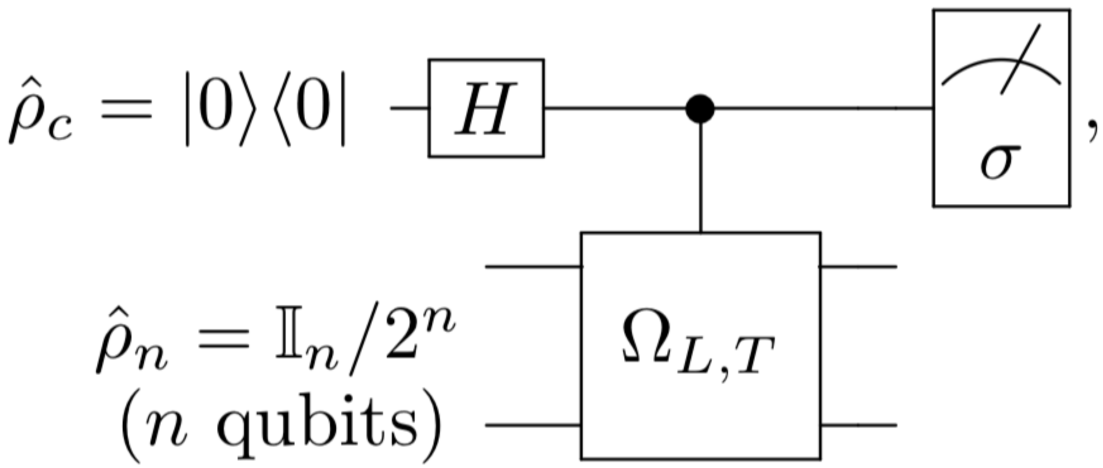
- **similar for color**

Back-up

circuit details



Back-up

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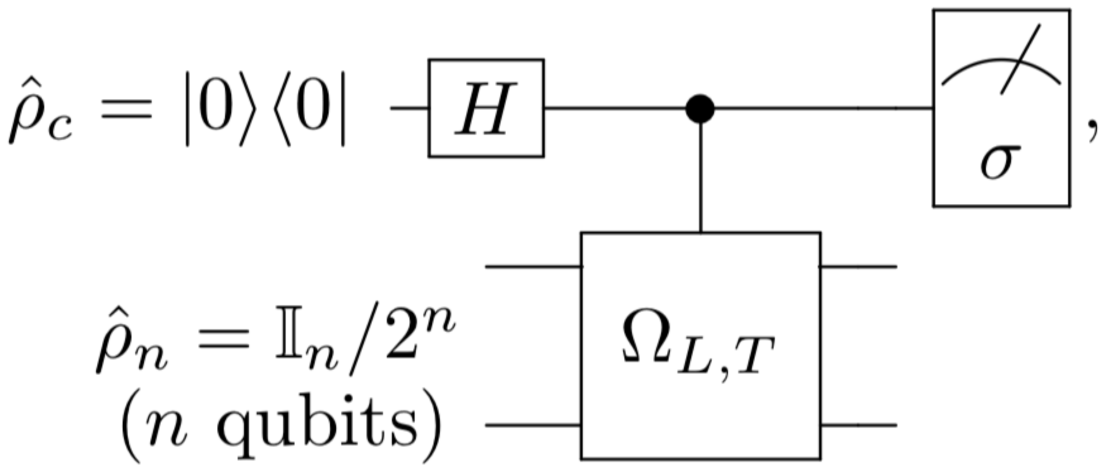

change of measurement basis

$$\langle \Psi|\sigma|\Psi\rangle = \langle \Psi|V^\dagger \mathrm{diag}(-1,1)V|\Psi\rangle$$

$$\boxed{V_{\sigma^x}} = \boxed{\sigma^z}\boxed{H}\boxed{\sigma^z}$$

$$\boxed{V_{\sigma^y}} = \boxed{S}\boxed{H}\boxed{\sigma^y}$$

Back-up

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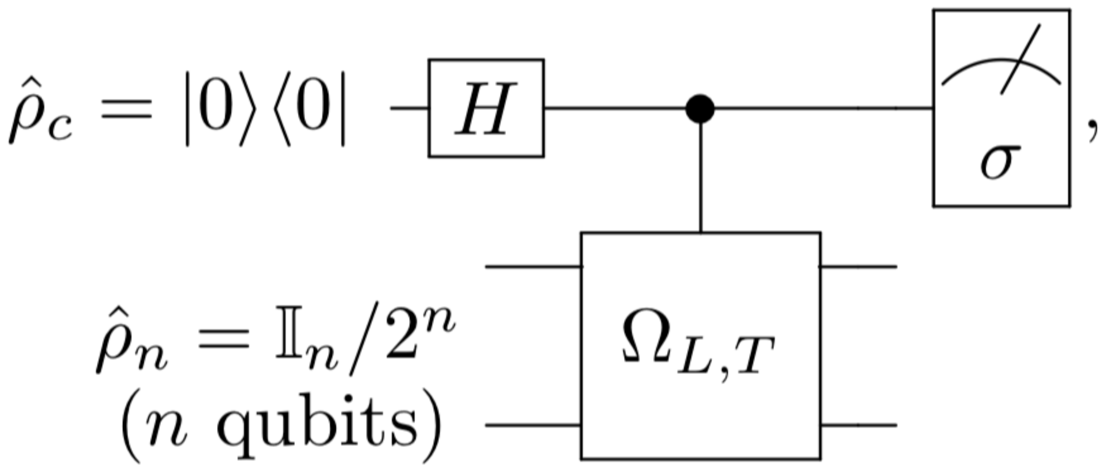

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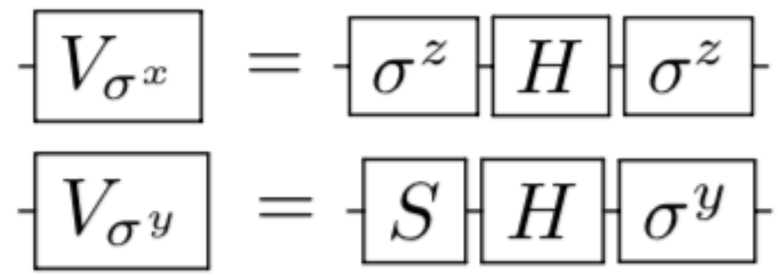

- **Maximally mixed state, poor man's solutions: run 4 times with pure state**

$$\rho_2 = \frac{\mathbb{I}_2}{2^2} = \frac{1}{4}\left(|00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| + |11\rangle\langle 11|\right)$$

Back-up

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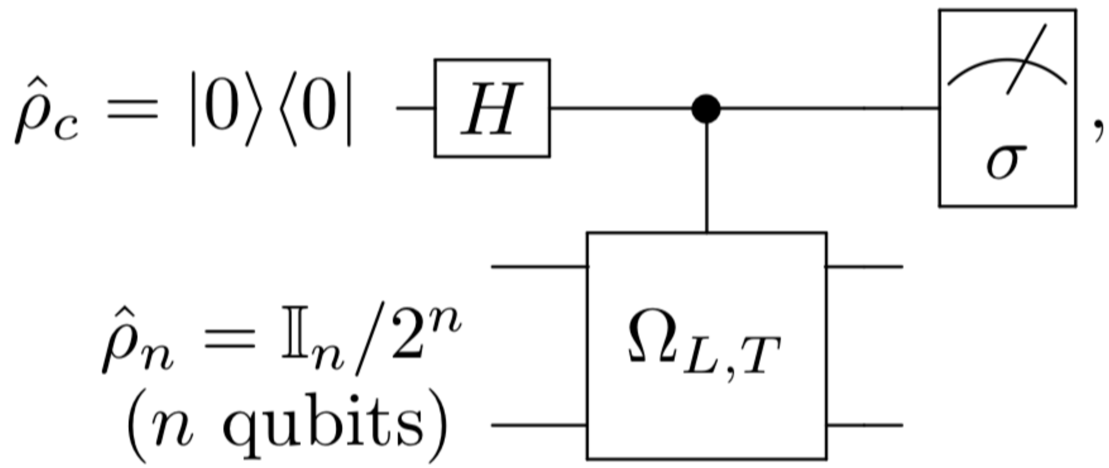

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- **Currently working on pure state algorithm for large traces (phase estimation)**

Back-up

IBM Q: qiskit

- **So far only “ideal” quantum computer (= simulator, no errors)**

```python
import numpy as np
import math
from qiskit import(
                    QuantumCircuit,
                    ClassicalRegister,
                    QuantumRegister,
                    execute,
                    Aer)
from qiskit.visualization import plot_histogram

# Use Aer's qasm_simulator
simulator = Aer.get_backend('qasm_simulator')

trace=0.0

for i in range(4):

    # Initializing a three-qubit quantum state
    if i==0:
        print("# Preparing |000>")
        init_vector = [1,0,0,0,0,0,0,0]
    elif i==1:
        print("# Preparing |010>")
        init_vector = [0,0,1,0,0,0,0,0]
    elif i==2:
        print("# Preparing |110>")
        init_vector = [0,0,0,0,0,0,1,0]
    elif i==3:
        print("# Preparing |100>")
        init_vector = [0,0,0,0,1,0,0,0]

    # initialize state
    q = QuantumRegister(3)
    c = ClassicalRegister(3)

    circuit = QuantumCircuit(q,c)
    circuit.initialize(init_vector, [q[0],q[1],q[2]])


    # build circuit
    circuit.h(0)
```

```python
    # build circuit
    circuit.h(0)

    # C(U) -------------------------------------------------------------------------------------------
    case=0 # 0 for U = 1 \otimes 1, 1 for U = sigma_z \otimes 1, 2 for  U = sigma_z \otimes sigma_z

    if case == 0:
    # 0) EMPTY CIRCUIT, WHEN COMPUTING TRACE OF U = 1 \otimes 1 = 1
        print("# computing trace of U = 1 otimes 1")
    elif case == 1:
    # 1) U = sigma_z \otimes 1
        print("# computing trace of U = sigma_z otimes 1")
        circuit.cz(0,1)
    elif case == 2:
    # 2) U = sigma_z \otimes sigma_z
         print("# computing trace of U = sigma_z otimes sigma_z")
         circuit.cz(0,1)
         circuit.cz(0,2)
    # -----------------------------------------------------------------------------------------------

    # basis transformation in sigma_x measurement basis
    circuit.z(0)
    circuit.h(0)
    circuit.z(0)

    circuit.measure(q[0],c[0])

    shots=1000
    job = execute(circuit, simulator, shots=shots)

    result = job.result()

    counts = result.get_counts(circuit)
    print("# Result is:",counts,"\n")

    controlbit_state=0
    for state in counts.keys():
        controlbit_state=float(state[2])
        if controlbit_state == 0:
            trace += (-1.0)*float(counts[state]/(4*shots))
        elif controlbit_state == 1:
            trace += (+1.0)*float(counts[state]/(4*shots))

rint("\nThe (normalized) trace is ",trace, "(need to multiply by 2^n, where n is no. of qubits)")
```

Back-up

IBM Q: qiskit

- **So far only “ideal” quantum computer (= simulator, no errors)**

```python
import numpy as np
import math
from qiskit import(
                    QuantumCircuit,
                    ClassicalRegister,
                    QuantumRegister,
                    execute,
                    Aer)
from qiskit.visualization import plot_histogram

# Use Aer's qasm_simulator
simulator = Aer.get_backend('qasm_simulator')

trace=0.0

for i in range(4):

    # Initializing a three-qubit quantum state
    if i==0:
        print("# Preparing |000>")
        init_vector = [1,0,0,0,0,0,0,0]
    elif i==1:
        print("# Preparing |010>")
        init_vector = [0,0,1,0,0,0,0,0]
    elif i==2:
        print("# Preparing |110>")
        init_vector = [0,0,0,0,0,0,1,0]
    elif i==3:
        print("# Preparing |100>")
        init_vector = [0,0,0,0,1,0,0,0]

    # initialize state
    q = QuantumRegister(3)
    c = ClassicalRegister(3)

    circuit = QuantumCircuit(q,c)
    circuit.initialize(init_vector, [q[0],q[1],q[2]])


    # build circuit
    circuit.h(0)
```

```python
    # build circuit
    circuit.h(0)

    # C(U) -------------------------------------------------------------------------------------------------
    case=0 # 0 for U = 1 \otimes 1, 1 for U = sigma_z \otimes 1, 2 for  U = sigma_z \otimes sigma_z

    if case == 0:
    # 0) EMPTY CIRCUIT, WHEN COMPUTING TRACE OF U = 1 \otimes 1 = 1
        print("# computing trace of U = 1 otimes 1")
    elif case == 1:
    # 1) U = sigma_z \otimes 1
        print("# computing trace of U = sigma_z otimes 1")
        circuit.cz(0,1)
    elif case == 2:
    # 2) U = sigma_z \otimes sigma_z
         print("# computing trace of U = sigma_z otimes sigma_z")
         circuit.cz(0,1)
         circuit.cz(0,2)
    # -------------------------------------------------------------------------------------------------------

    # basis transformation in sigma_x measurement basis
    circuit.z(0)
    circuit.h(0)
    circuit.z(0)

    circuit.measure(q[0],c[0])

    shots=1000
    job = execute(circuit, simulator, shots=shots)

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    counts = result.get_counts(circuit)
    print("# Result is:",counts,"\n")

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print("\nThe (normalized) trace is ",trace, "(need to multiply by 2^n, where n is no. of qubits)")
```

- **Computed traces of** $\mathbb{I}\otimes\sigma^z$ $\quad\sigma^z\otimes\sigma^z\quad$ $\mathbb{I}\otimes\mathbb{I}$
  (and a few others in our circuit)

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```

- **Computed traces of** $\mathbb{I} \otimes \sigma^z$ $\quad \sigma^z \otimes \sigma^z$ $\quad \mathbb{I} \otimes \mathbb{I}$ **(and a few others in our circuit)**

- **Got correct answer**

Back-up

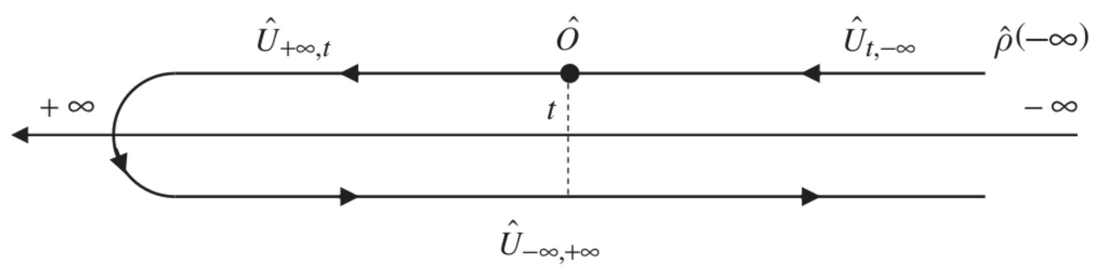
Internal symmetries from SK worldline path integral I

Back-up

Internal symmetries from SK worldline path integral I

- **semi-classical phase space: <u>Truncated Wigner Approximation</u>**

review: Polkovnikov 2009



$$\bar{x} = \frac{x^+ + x^-}{2}$$

$$\tilde{x} = x^+ - x^-$$

Back-up

Internal symmetries from SK worldline path integral I

- **semi-classical phase space: Truncated Wigner Approximation**

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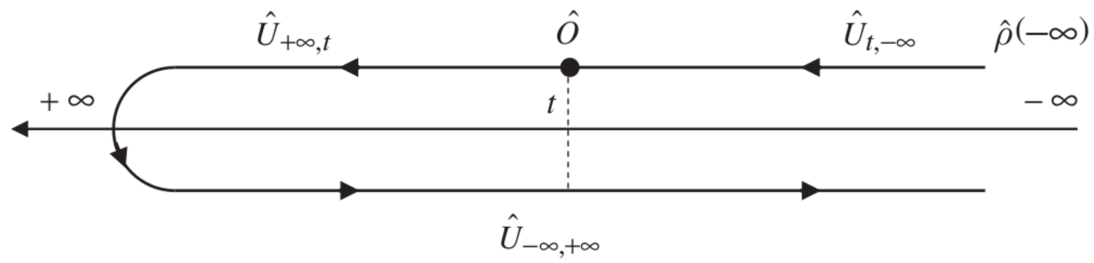

$$\bar{x} = \frac{x^+ + x^-}{2} \quad \text{“classical”}$$

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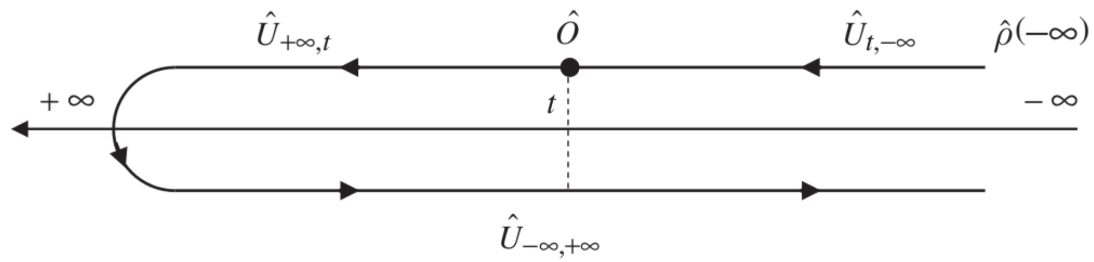

$$\bar{x} = \frac{x^+ + x^-}{2} \quad \text{“classical”}$$
$$\tilde{x} = x^+ - x^- \quad \text{“quantum”}$$

$$\boxed{\begin{aligned}\Gamma_{\mathcal{C}} \approx &\int d^4\bar{x}_i d^4\bar{p}_i d\bar{\lambda}_i d\bar{\lambda}_i^\dagger \; W_A^\chi(\bar{x}_i, \bar{p}_i, \bar{\lambda}_i, \bar{\lambda}_i^\dagger) \\ &\times \int_{\mathcal{C}} \mathcal{D}x\mathcal{D}p\mathcal{D}\lambda\mathcal{D}\lambda^\dagger\mathcal{D}\epsilon D\phi \; \exp\left\{iS_0 + i\int d\tau \left(\left[\dot{\bar{p}} - \frac{\partial H}{\partial \tilde{x}}\right]\tilde{x} \right.\right. \\ &\qquad \left.\left. - \left[\dot{\bar{x}} + \frac{\partial H}{\partial \tilde{p}}\right]\tilde{p} + \left[i\dot{\bar{\lambda}} - \frac{\partial H}{\partial \tilde{\lambda}^\dagger}\right]\tilde{\lambda}^\dagger - \left[i\dot{\bar{\lambda}}^\dagger + \frac{\partial H}{\partial \tilde{\lambda}}\right]\tilde{\lambda}\right)\right\}\end{aligned}}$$

Back-up

Internal symmetries from SK worldline path integral I

- **semi-classical phase space: <u>Truncated Wigner Approximation</u>**

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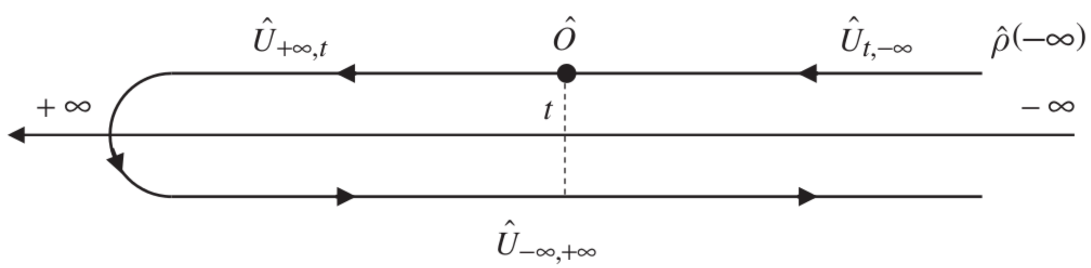

$$\bar{x} = \frac{x^+ + x^-}{2}$$ "classical"

$$\tilde{x} = x^+ - x^-$$ "quantum"

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- **yields (quantum-) Liouville equation**

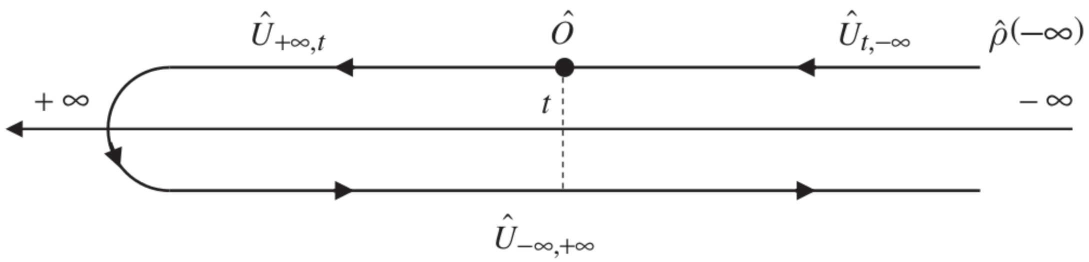
$$\boxed{\frac{d}{d\tau}W_A^\chi = \left(\dot{\bar{x}}_\mu \frac{\partial}{\partial \bar{x}_\mu} + \dot{\bar{P}}_\mu \frac{\partial}{\partial \bar{P}_\mu} + \dot{\bar{\lambda}}_a \frac{\partial}{\partial \bar{\lambda}_a} + \dot{\bar{\lambda}}_a^\dagger \frac{\partial}{\partial \bar{\lambda}_a^\dagger}\right) W_A^\chi(x, P, \lambda, \lambda^\dagger)}$$

Back-up

Internal symmetries from SK worldline path integral I

- **semi-classical phase space: Truncated Wigner Approximation**

review: Polkovnikov 2009



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$$\times \int_{\mathcal{C}} \mathcal{D}x\mathcal{D}p\mathcal{D}\lambda\mathcal{D}\lambda^\dagger\mathcal{D}\epsilon D\phi \; \exp\left\{iS_0 + i\int d\tau \left(\left[\dot{\bar{p}} - \frac{\partial H}{\partial \tilde{x}}\right]\tilde{x} - \left[\dot{\bar{x}} + \frac{\partial H}{\partial \tilde{p}}\right]\tilde{p} + \left[i\dot{\bar{\lambda}} - \frac{\partial H}{\partial \tilde{\lambda}^\dagger}\right]\tilde{\lambda}^\dagger - \left[i\dot{\bar{\lambda}}^\dagger + \frac{\partial H}{\partial \tilde{\lambda}}\right]\tilde{\lambda}\right)\right\}$$

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Grassmann coordinates for color

Back-up

Internal symmetries from SK worldline path integral II

Back-up

Internal symmetries from SK worldline path integral II

- **Spin via anti-commuting variables** (Berezin and Marinov 1976)

$$\boxed{W_A^{\chi}(x, P, \lambda, \lambda^{\dagger}) \longrightarrow W_A^{\chi}(x, P, \lambda, \lambda^{\dagger}, \underline{\psi})}$$

Back-up

Internal symmetries from SK worldline path integral II

- **Spin via anti-commuting variables** (Berezin and Marinov 1976)

$$\boxed{W_A^{\chi}(x, P, \lambda, \lambda^{\dagger}) \longrightarrow W_A^{\chi}(x, P, \lambda, \lambda^{\dagger}, \underline{\psi})}$$

- **bilinear form**

$$S_{\mu\nu} = -i\psi_{\mu}\psi_{\nu}$$

$$Q^a \equiv \lambda_c^{\dagger} t_{cd}^a \lambda_d$$

Back-up

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- **Spin via anti-commuting variables** (Berezin and Marinov 1976)

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- **bilinear form**

$$S_{\mu\nu} = -i\psi_\mu\psi_\nu$$

$$Q^a \equiv \lambda_c^{\dagger} t_{cd}^a \lambda_d$$

$$\begin{aligned}
\dot{x}^\mu &= \epsilon P^\mu \,, \\
\dot{P}^\mu &= \epsilon g F^{a,\mu\nu} Q^a P_\nu - \frac{i\epsilon g}{2}\psi^\alpha (D^\mu F_{\alpha\beta})^a Q^a \psi^\beta \,, \\
\dot{\psi}^\mu &= \epsilon g F^{a,\mu\nu} Q^a \, \psi_\nu \,, \\
\dot{\lambda}_a^{\dagger} &= -igv^\mu t_{ab}^c A_\mu^c \lambda_b^{\dagger} - \frac{\epsilon g}{2}\psi^\mu F_{\mu\nu}^b t_{ac}^b \lambda_c^{\dagger} \psi^\nu \,, \\
\dot{\lambda}_a &= igv^\mu t_{ab}^c A_\mu^c \lambda_b^{\dagger} + \frac{\epsilon g}{2}\psi^\mu F_{\mu\nu}^b t_{ac}^b \lambda_c \psi^\nu \,,
\end{aligned}$$

$$\dot{Q}^a = -igv^\mu f^{abc} A_\mu^b Q^c - \frac{g\epsilon}{2} f^{abc} \psi^\mu F_{\mu\nu}^b \psi^\nu Q^c$$

= Wong's equation (1970)

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\dot{\psi}^\mu &= \epsilon g F^{a,\mu\nu} Q^a\,\psi_\nu\,,\\
\dot{\lambda}_a^\dagger &= -igv^\mu t^c_{ab} A^c_\mu \lambda_b^\dagger - \frac{\epsilon g}{2}\psi^\mu F^b_{\mu\nu} t^b_{ac}\lambda_c^\dagger \psi^\nu\,,\\
\dot{\lambda}_a &= igv^\mu t^c_{ab} A^c_\mu \lambda_b^\dagger + \frac{\epsilon g}{2}\psi^\mu F^b_{\mu\nu} t^b_{ac}\lambda_c \psi^\nu\,,
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= Wong's equation (1970)

exact spin structure

$$\begin{aligned}
&f_A(x,P,Q,S)\\
&= f_A(x,P,Q)\left[\underbrace{i\Sigma_\mu(x,P,Q)S^{\mu\nu}v_\nu}_{\text{polarized}} - \underbrace{\frac{i}{6}\epsilon_{\mu\nu\alpha\beta}v^\mu S^{\nu\alpha}S^{\beta\lambda}v_\lambda}_{\text{unpolarized}}\right]
\end{aligned}$$

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\end{aligned}$$

exact color structure

$$\begin{aligned}
&f(x,P,Q)\\
&= \underbrace{f(x,P)\Big[1+\frac{2}{A_R d^2}d^{abc}Q^aQ^bQ^c\Big]}_{\text{singlet}} + \underbrace{2f^a(x,P)Q^a}_{\text{octet}}
\end{aligned}$$

Back-up

Things you learn on the way

T. Zache, NM, J Schneider, Fred Jendrzejewski, Juergen Berges, Philipp Hauke,
PRL 122 (2019) 050403

- Tried to find simple benchmark for quantum simulator / computer to tell us about CP violation in QCD
- Discovered that Schwinger model with theta term has DQPT

- robust on 8 (!) lattice sites and short times

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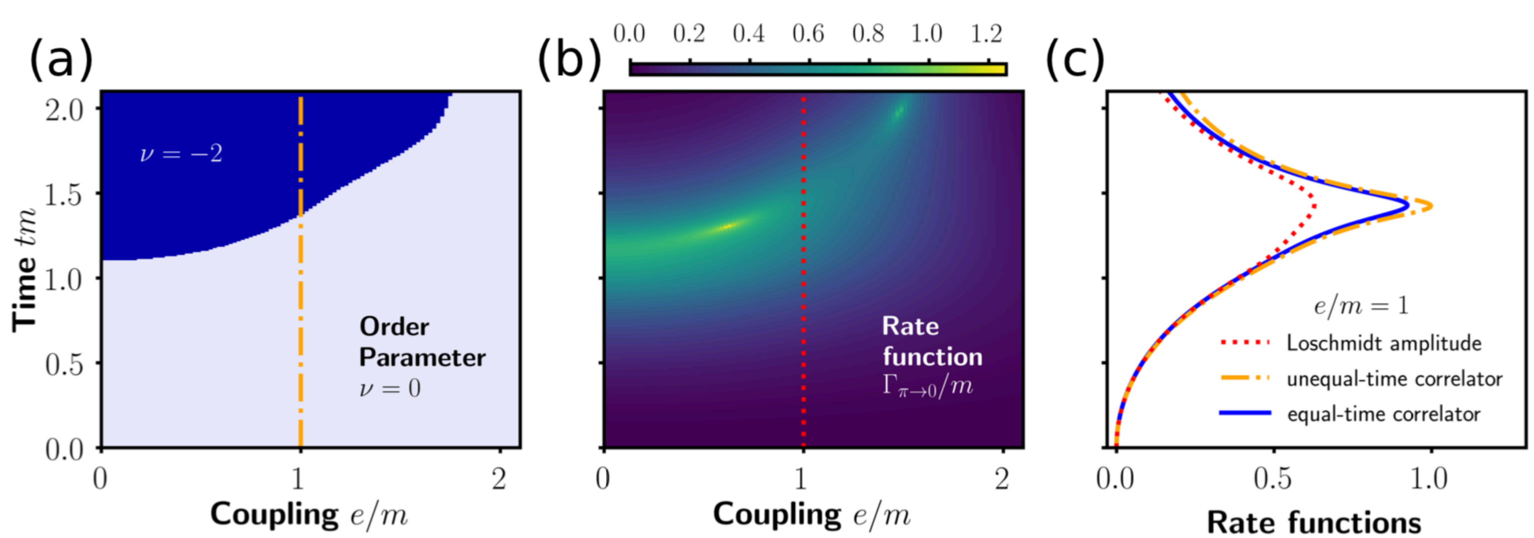

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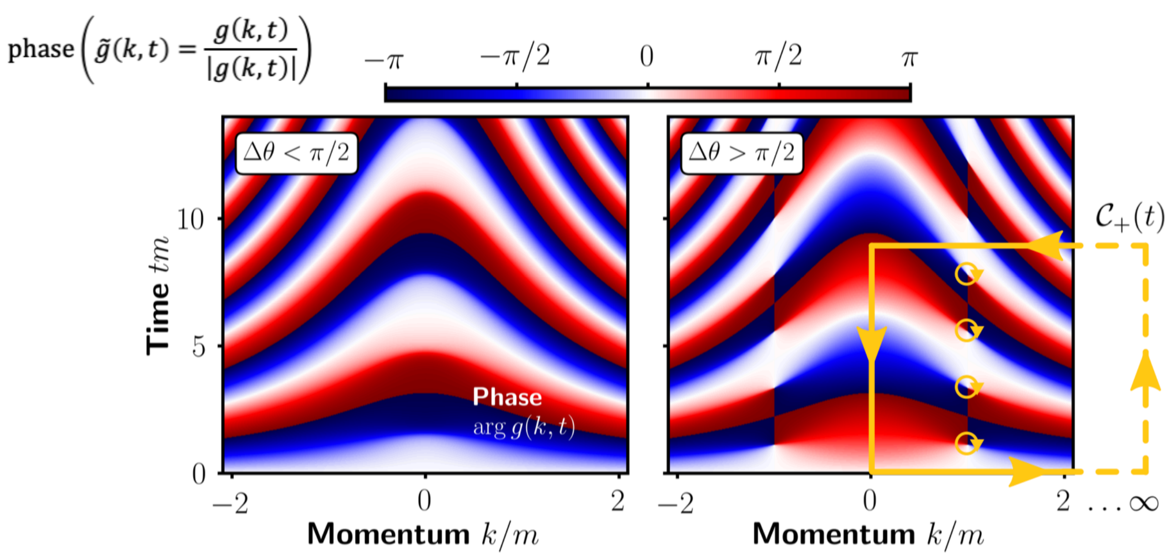

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