Hard probes of non-equilibrium quark-gluon plasma

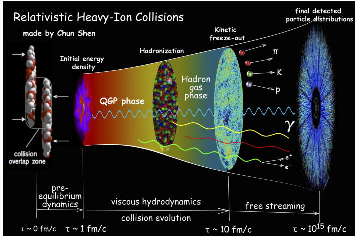
Sigtryggur Hauksson
McGill University

In collaboration with S. Jeon, C. Gale

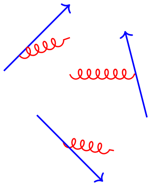
Quark Matter 2019, Wuhan, November 5, 2019

Introduction

- Want to explore non-equilibrium properties of QGP.
  - Initial state physics.
  - Transport coefficients.
- Hard probes like jets and photons promising.
  - Requires detailed theoretical understanding.
    [e.g. Schenke, Strickland, 2007]
    [see also talk by J. Churchill, Wednesday 17:20]
- Weakly coupled non-equilibrium QGP:
  [Blaizot, Iancu, 2001; Arnold, Moore, Yaffe, 2003]
  - Quark and gluon quasiparticles, $E \sim \Lambda$
  - Occasionally interact.
  - Radiate soft gluons, $E \sim g\Lambda$



[Chun Shen, 2014]



Challenges out of equilibrium

- Get coupled equations

$$\mathcal{D}_\mu F^{\mu\nu} = j^\nu$$

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{r}} + \mathbf{F}\cdot\frac{\partial f}{\partial \mathbf{p}} = \mathcal{C}[f, A]$$

- Gives HTL and kinetic theory.

  [Xu, Greiner, 2005; Kurkela et al., 2018]

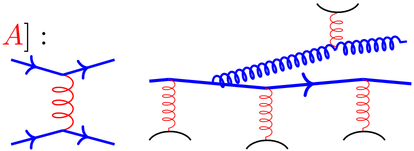
- For any anisotropic $f(\mathbf{p})$ get Weibel instabilities

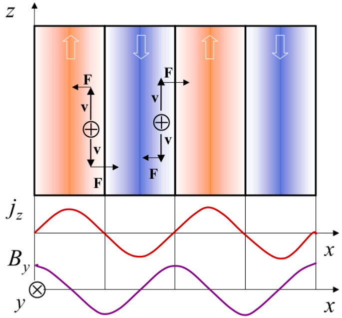
  [e.g. Mrowczynski, Schenke, Strickland 2016]

- A naive calculation gives a divergent rate of bremsstrahlung.
- Role of instabilities in early stages of HIC:

  [Berges, Boguslavski, Schlichting, Venugopalan, 2014]

$\mathcal{C}[f, A]$:

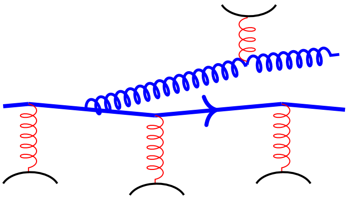




[Mrowczynski 2005]

Overview of talk

- We calculate bremsstrahlung correctly in non-equilibrium systems.

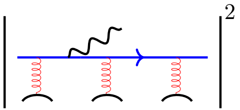


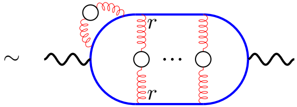
- Completes kinetic theory of quarks and gluons.
- Allows for jet physics and photon emission in a non-equilibrium QGP.
- Understand how to treat divergences.
- Calculate effect of long-wavelength fields on photon emission.
- Applications to phenomenology.

- Momentum broadening in thermal equilibrium

  [Arnold, Moore, Yaffe]

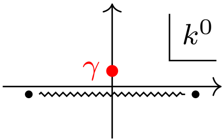
$$\mathbf{p}_\perp = i\delta E\, \mathbf{f}(\mathbf{p}_\perp) + \int_{\mathbf{q}_\perp} \mathcal{C}(\mathbf{q}_\perp)\left[\mathbf{f}(\mathbf{p}_\perp) - \mathbf{f}(\mathbf{p}_\perp + \mathbf{q}_\perp)\right].$$





- $G_{rr}^{\mu\nu} = \frac{1}{2}\langle\{A^\mu, A^\nu\}\rangle$ describes density of soft gluon fields.

$$\mathcal{C}(\mathbf{q}_\perp) \sim \int_{q^0} G_{rr}(Q)\big|_{q^z=q^0} \sim \int_{q^0} G_{\mathrm{ret}}\Pi_{aa}G_{\mathrm{adv}}\big|_{q^z=q^0}$$



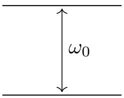
- Out of equilibrium a simple substitution, $f_{\mathrm{eq}}(p) \to f(\mathbf{p})$, in momentum space fails.

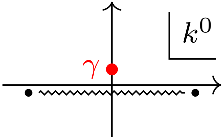
- Get divergent $\mathcal{C}$ because of instability poles, γ.

Analogy from QM

- Transition between two levels in a potential $V(\mathbf{r})e^{\gamma t}$.
  - On since $T_0 = -\infty$: $|c_b(t)|^2 \sim \left|\frac{e^{(i\omega_0+\gamma)t}}{i\omega_0+\gamma}\right|^2$
  - Divergent when $\omega_0, \gamma \to 0$.
- With an intial time $T_0 = 0$: $|c_b(t)|^2 \sim \left|\frac{e^{(i\omega_0+\gamma)t}-1}{i\omega_0+\gamma}\right|^2$
- Lesson: Need time evolution of instability fields starting at $T_0 = 0$.
- Use tools of out-of-equilibrium QFT to get finite $\mathcal{C}$ from

$$G_{rr}(x^0, y^0) = \int_0 dz^0 \int_0 dw^0 \, G_{\mathrm{ret}}(x^0, z^0)\, \Pi_{aa}(z^0, w^0)\, G_{\mathrm{adv}}(w^0, y^0) + \ldots$$





$$
\begin{aligned}
G_{rr}(x,y) = &\int_\alpha \frac{dk_1}{2\pi} \int \frac{dk_2}{2\pi} \int_{\widetilde{\alpha}} \frac{dk_3}{2\pi} \left[e^{-ik_2(x-y)} - e^{-ik_1 x} e^{ik_2 y} - e^{-ik_2 x} e^{ik_3 y} + e^{-ik_1 x} e^{ik_3 y}\right] \\
&\times \frac{1}{8}\Bigg[\left(\frac{1}{k_2 - k_1 + i\epsilon_2 - i\epsilon_1} + \frac{1}{k_2 - k_1 + i\epsilon_2 + i\epsilon_1}\right)\left(\frac{1}{k_2 - k_3 + i\epsilon_2 - i\epsilon_3} + \frac{1}{k_2 - k_3 + i\epsilon_2 + i\epsilon_3}\right) \\
&+ \left(\frac{1}{k_2 - k_1 - i\epsilon_2 - i\epsilon_1} + \frac{1}{k_2 - k_1 - i\epsilon_2 + i\epsilon_1}\right)\left(\frac{1}{k_2 - k_3 - i\epsilon_2 - i\epsilon_3} + \frac{1}{k_2 - k_3 - i\epsilon_2 + i\epsilon_3}\right)\Bigg] \\
&\times \left(\widehat{G}_{\mathrm{ret}}(k_1) + \sum_i \frac{A_i}{k_1 - a_i}\right) \Pi_{aa}(k_2) \left(\widehat{G}_{\mathrm{adv}}(k_3) + \sum_j \frac{A_j^*}{k_3 - a_j^*}\right).
\end{aligned}
$$

Finding G_{rr}

- Initial momentum distribution $f(\mathbf{p})$ at $T_0 = 0$.
- The instability modes $\gamma \sim \xi g \Lambda$ must grow slowly.
- Need anisotropy $\xi \ll g$ to have kinetic theory.
-

$$G_{rr}(t_x, t_y) \approx \int \frac{dk^0}{2\pi}\, e^{-ik^0(t_x - t_y)}\, \widehat{G}_{\mathrm{ret}}(k^0)\, \Pi_{aa}(k^0)\, \widehat{G}_{\mathrm{adv}}(k^0) + \frac{B\,\Pi_{aa}(0)B^\dagger}{2\gamma}\left[e^{\gamma t_x} e^{\gamma t_y} - 1\right]$$

- Two scales:
  - Fluctuating fields at $g\Lambda$
  - Long wavelength instabilities at $\xi g \Lambda$.

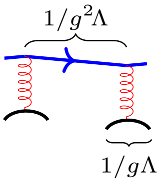
$$G_{\mathrm{ret}}(k^0) = \widehat{G}_{\mathrm{ret}}(k^0) + \sum_i \frac{B}{k^0 - i\gamma}$$

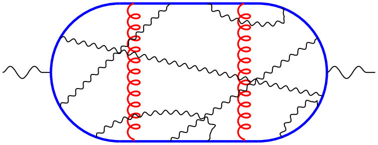
Photon emission in long wavelength background

- What is effect of long wavelength field on photon emission and jet splitting?
- Calculation valid for any

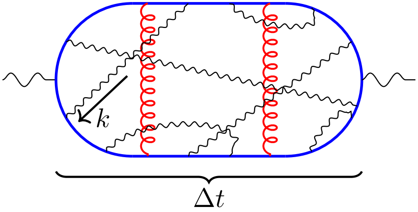
$$G_{rr} = [g\Lambda] + [\xi g\Lambda]$$

- $g\Lambda$ kicks are time ordered.
- Long wavelength background field give a new time scale, $1/\xi g\Lambda \gg 1/g^2\Lambda$.
- Interaction with them not time ordered.
- Complicated color factors: Can consider Abelian and large N_c limit.





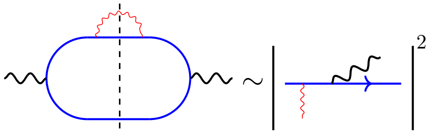
Photon emission in long wavelength background



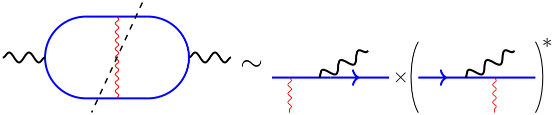
- Sum up all Abelian $\xi g \Lambda$ kicks.
- Expand in long wavelength:
  - $k\Delta t \sim \xi/g \ll 1$
- LO: $\sim e^{-\# \int_k G_{rr}\,(\Delta t)^2}$ gives phase rotation
- NLO: $\sim e^{-\# \int_k G_{rr}\,k^2(\Delta t)^4}$ gives EM change to dispersion relations.
- Higher order corrections: Spin precession, rotation of momentum distribution, ...

Photon emission in long wavelength background

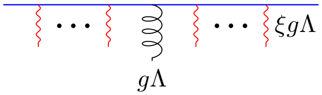
- Result: Effect of $\xi g\Lambda$ field cancels out at LO and NLO in $k\Delta t$.
- Cancellation between



and



- Similar to cancellation of IR divergences in vacuum jets.
- Effect of $\xi g\Lambda$ field does not vanish for $N_c \to \infty$.



Phenomenology

- Non-equilibrium effects all in

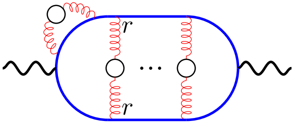
$$\mathcal{C} \sim \int_{q^0} \widehat{G}_{\mathrm{ret}} \Pi_{aa} \widehat{G}_{\mathrm{adv}} |_{q^z = q^0}$$

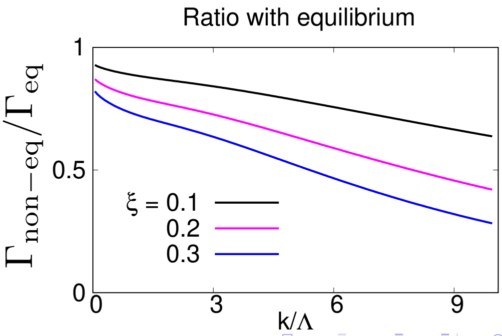
- Have solved integral equation numerically.

  [S. Hauksson et al. (2018)]

- Can evaluate jet-medium interaction in a non-thermal medium.

- Combining with hydrodynamic simulations gives access to transport coefficients.





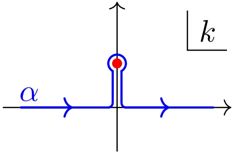
Conclusions

- Complete kinetic theory by including non-equilibrium effects interaction rates.
- Allows for study of jets and photons in non-equilibrium QGP.
- Apparent divergence in bremsstrahlung disappears when taking time evolution into account.
- Effect of long wavelength field disappears in Abelian case.
- Can use jets and photons to extract transport coefficients of QGP.

Calculation of G_{rr}

- $$G_{rr}(x^0, y^0) = \int_0 dz^0 \int_0 dw^0 \, G_{\text{ret}}(x^0, z^0) \, \Pi_{aa}(z^0, w^0) \, G_{\text{adv}}(w^0, y^0) + \dots$$
- In HTL approximation can write

$$G_{rr}(x, y) = \int_\alpha \frac{dk_1}{2\pi} \int \frac{dk_2}{2\pi} \int_{\tilde{\alpha}} \frac{dk_3}{2\pi} \int_0^{x^0} dz^0 \int_0^{y^0} dw^0 e^{-ik_1(x^0 - z^0)} e^{-ik_2(z^0 - w^0)} e^{-ik_3(w^0 - y^0)}$$
$$\times \; G_{\text{ret}}(k_1) \, \Pi_{aa}(k_2) \, G_{\text{adv}}(k_3)$$



- Evaluate integrals using principal value prescriptions.

Calculation of G_{rr}

- Can ignore rapid decay e^{-at}, with $a \sim g\Lambda$ and $t \geq 1/g^2\Lambda$.
- Can ignore rapid oscillations e^{iat} with $a \sim g\Lambda$ and t varying over $1/g^2\Lambda$.
- $$\begin{aligned} G_{rr}(x,y) \approx & \int \frac{dk}{2\pi}\, \widehat{G}_{\mathrm{ret}}(k)\, \Pi_{aa}(k)\, \widehat{G}_{\mathrm{adv}}(k) \;\; e^{-ik(x-y)} \\ & + \sum_i \int \frac{dk}{2\pi}\, \frac{A_i}{k-a_i}\, \Pi_{aa}(k)\, \widehat{G}_{\mathrm{adv}}(k) \;\left(e^{-ikx} - e^{-ia_i x}\right) e^{iky} \\ & + \sum_j \int \frac{dk}{2\pi}\, \widehat{G}_{\mathrm{ret}}(k)\, \Pi_{aa}(k)\, \frac{A_j^*}{k-a_j^*}\; e^{-ikx} \left(e^{iky} - e^{ia_j^* y}\right) \\ & + \sum_{i,j} \int \frac{dk}{2\pi}\, \frac{A_i}{k-a_i}\, \Pi_{aa}(k)\, \frac{A_j^*}{k-a_j^*} \;\left(e^{-ikx} - e^{-ia_i x}\right)\left(e^{iky} - e^{ia_j^* y}\right). \end{aligned}$$

Final result:

$$G_{rr}(t_x, t_y) \approx \int \frac{dk^0}{2\pi}\, e^{-ik^0(t_x - t_y)}\, \widehat{G}_{\mathrm{ret}}(k^0)\, \Pi_{aa}(k^0)\, \widehat{G}_{\mathrm{adv}}(k^0) + \frac{B\,\Pi_{aa}(0) B^\dagger}{2\gamma} \left[e^{\gamma t_x} e^{\gamma t_y} - 1\right]$$

$$G_{\mathrm{ret}}(k^0) = \widehat{G}_{\mathrm{ret}}(k^0) + \sum_i \frac{B}{k^0 - i\gamma}$$

Effect of long wavelength fields

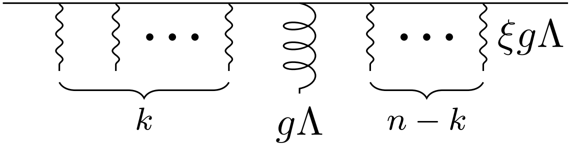
- Assume that $k\Delta t \ll 1$
- Also assume $k \gg 1\ \Lambda(\Delta t)^2$
  - Two subsequent gluon emission not correlated by background field.
- Summing over all ways of attaching background field insertions to quark propagator:

$$S^{(n)}_{\mathrm{ret}}(t_x, t_y; \mathbf{p}; \{\mathbf{l}_1, \ldots, \mathbf{l}_n\}) \approx \frac{1}{2}\theta(\Delta t)\Big[e^{-ip\Delta t}\widehat{\not{P}}(igA_\mu \widehat{P}^\mu)^n \prod_{i=1}^{n} \frac{e^{-i\, l_i^{||}\Delta t} - 1}{-i\, l_i^{||}} - (-1)^n e^{ip\Delta t}\widetilde{\not{P}}(igA_\mu \widetilde{P}^\mu)^n \prod_{i=1}^{n} \frac{e^{i\, l_i^{||}\Delta t} - 1}{i\, l_i^{||}}\Big]$$

- Effect of background field factors out.

Effect of long wavelength fields

- Same thing happens when have HTL kicks as well.



- Can sum up effect of background fields taking into account combinatorical factors.
- Get cancellation in Abelian case.