Chirality transfer & Chiral turbulence in gauge theories

Sören Schlichting | Universität Bielefeld

Based on:
Mace, Mueller, SS, Sharma arXiv:1910:01654

Mace,Mueller,SS, Sharma, PRD95 (2017) no.3, 036023;
Mueller,SS, Sharma PRL117 (2016) no.14, 142301;
Mace,SS, Venugopalan PRD 93 (2016) no.7, 074036;

UNIVERSITÄT BIELEFELD
Faculty of Physics

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CRC-TR 211
Strong-interaction matter under extreme conditions

Outline

1. Introduction & Motivation

2. Chirality transfer & Chiral turbulence in strongly coupled QED plasmas

3. Conclusions & Outlook

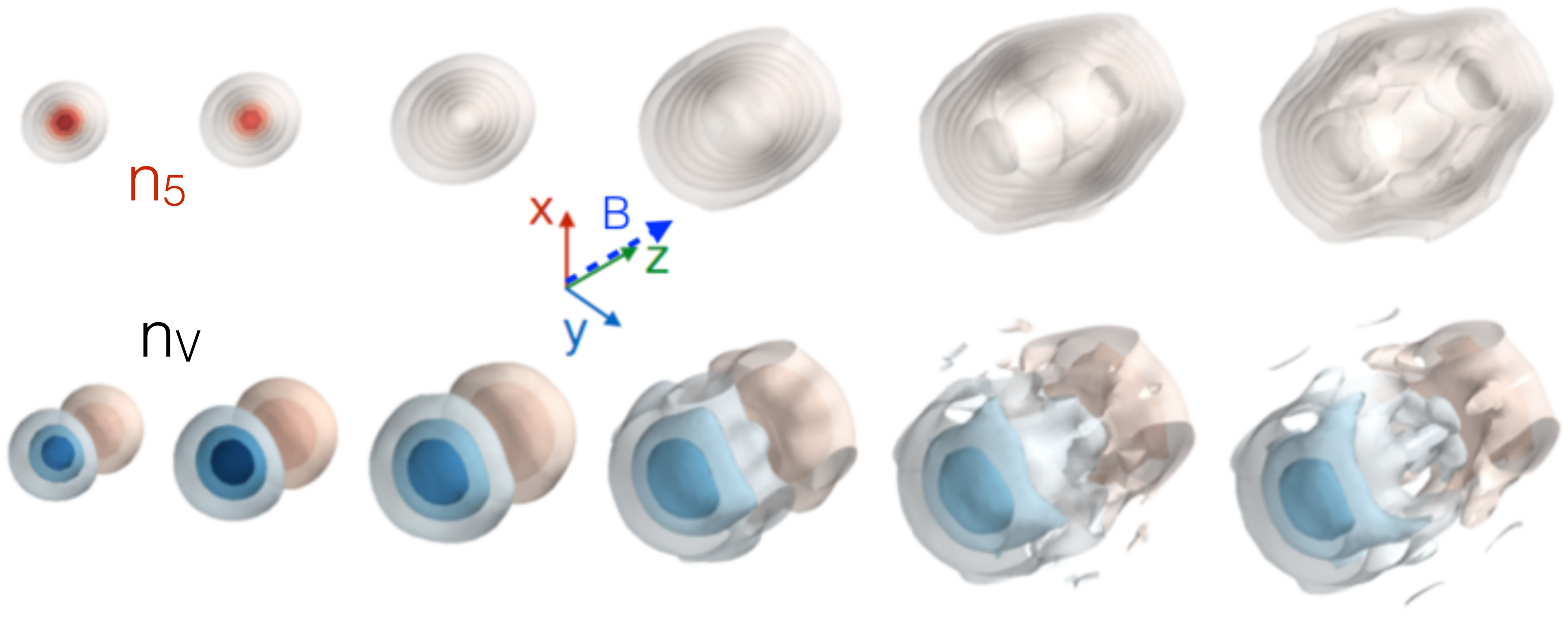
Chiral magnetic effect & anomalous transport

Discovery of new conductivities for systems with chiral fermions and (net-) chirality charge density

Fukushima, Kharzeev, Warringa PRD 78 (2008) 074033

$$\vec{j}_V \propto n_5 \vec{B}$$

j_V: vector current
n_5: chiral charge density
B: magnetic field



Mueller, SS, Sharma PRL 117 (2016) no.14, 142301

Expected manifestations from Heavy-Ion collisions to Dirac/Weyl semi-metals

Difference to ordinary transport phenomena:

Energy-momentum tensor $T^{\mu\nu}$
(Net-)baryon number n_B
conserved

(Net-)chiral charge density n_5
not conserved

-> Chiral transport phenomena are intrinsically non-equilibrium effects

Chiral anomaly in QCD and QED

Chiral charge density of fermions (n_5) not conserved due to the axial anomaly

$$\partial_\mu j^\mu_{5,f} = 2m_f \bar{q}\gamma_5 q - 2\,\partial_\mu K^\mu \qquad \partial_\mu K^\mu = \frac{g^2}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$$

quark mass

Chern-Simons current

(non)-abelian field-strength fluctuations

Gauge field contribution ∝ E.B represents change of chiral charge density of gauge fields

$$N_{CS}(t) = \int d^3x\, K^0(t,x)$$

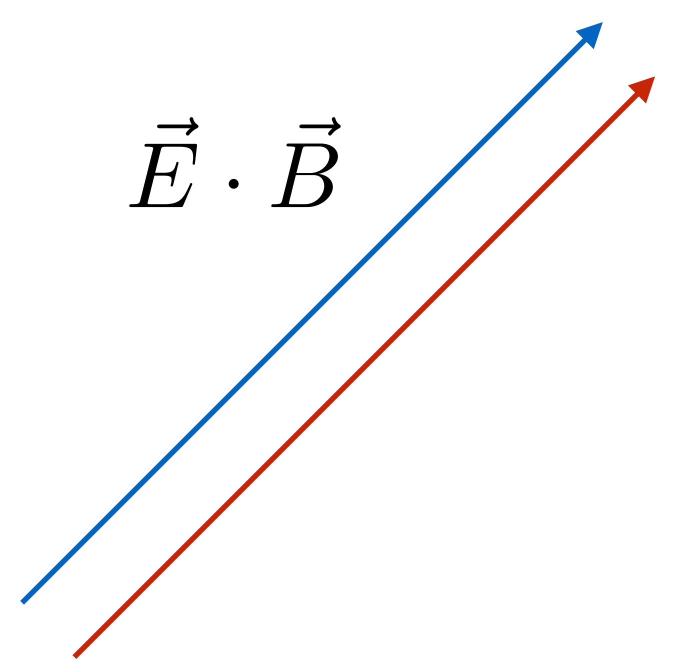
QED: Magnetic Helicity

QCD: Chern-Simons number

Chiral limit (m->0): Net chirality of the system $N_5 + 2N_{CS}$ conserved and thus shared & transferred between fermions and gauge fields

Extremely important to understand transfer/balance of chiral charge between fermions and gauge fields for macroscopic manifestations of anomaly

Chirality transfer in QED and QCD

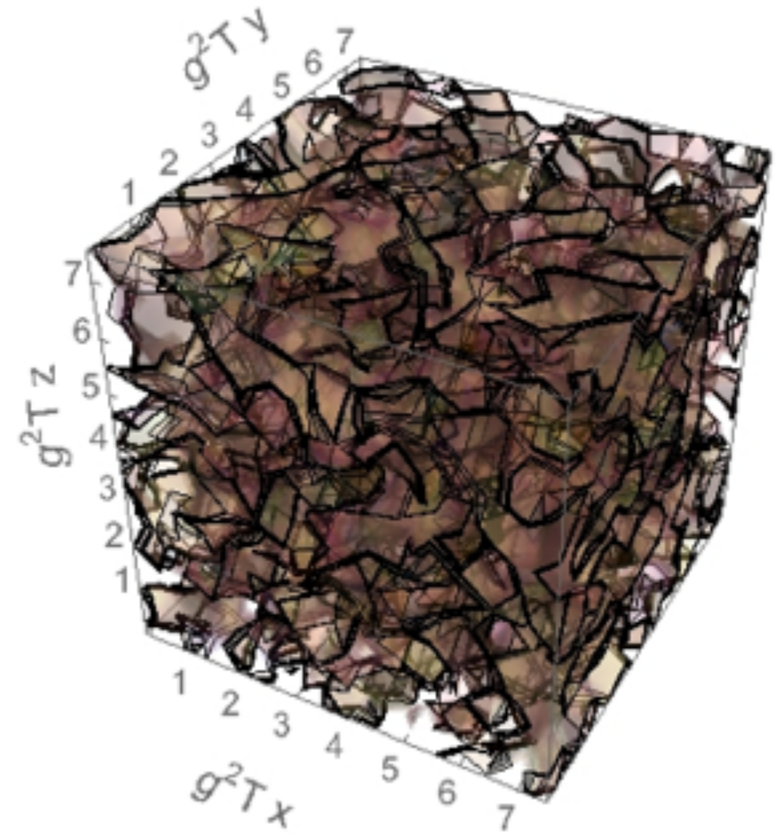


Chirality transfer from gauge fields to fermions

QED: External E,B fields

QED & QCD: Space-time dependent fluctuations of (chromo-) electro-magnetic fields

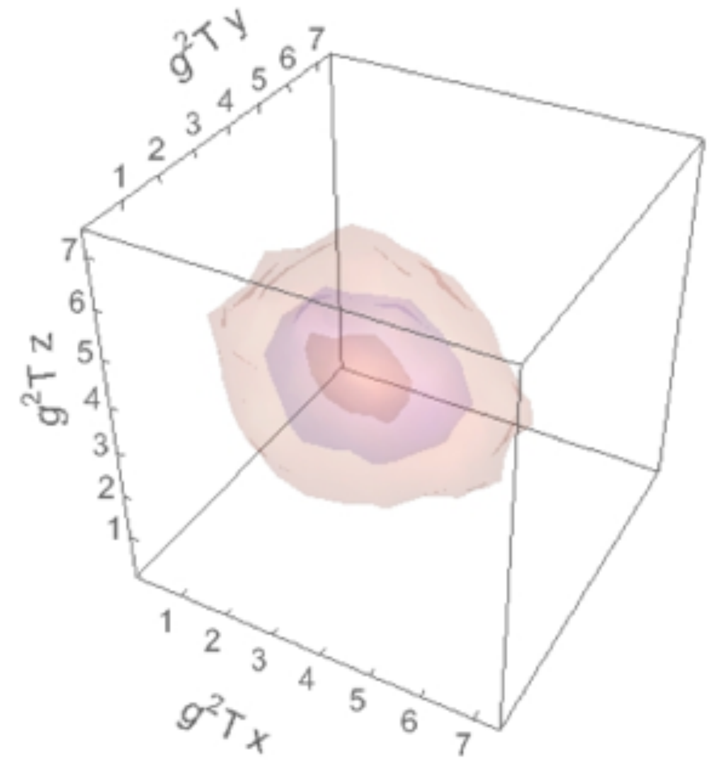
QCD: Sphaleron transitions



Chirality transfer from fermions to gauge fields

QCD: Bias of sphaleron transition rate
McLerran,Mottola,Shaposnikov PRD43 (1991) 2027-2035

QED & QCD: Chiral plasma instabilities
Akamatsu, Yamamoto PRL111 (2013) 052002



Mace, SS, Venugopalan
PRD93 (2016) no.7, 074036

Crucial for understanding of chiral transport in HIC

Interesting applications in Astrophysics & Cosmology
=>Chirality imbalance through weak interaction processes

Masada et al. PRD98 (2018) no.8, 083018; Brandenburg et al. Astrophys.J. 845 (2017) no.2, L21

Microscopic description for QED

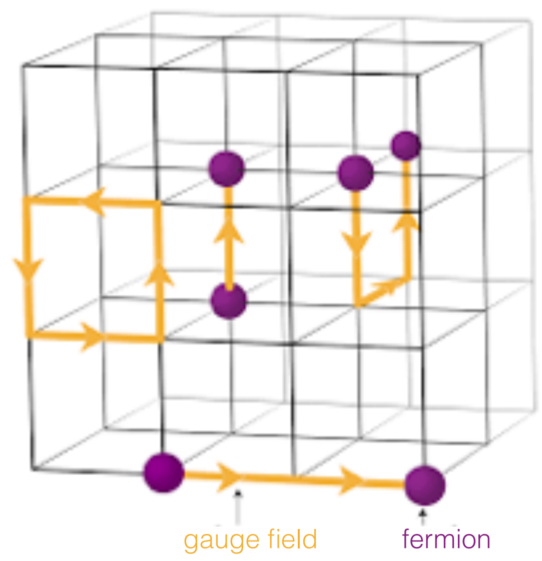
Classical-statistical lattice simulations of dynamics of underlying QFT

Solve operator Dirac equation
with QED gauge fields

$$i\gamma^0\partial_t\hat{\psi} = (-i\not{D}^s_W + m)\hat{\psi}$$

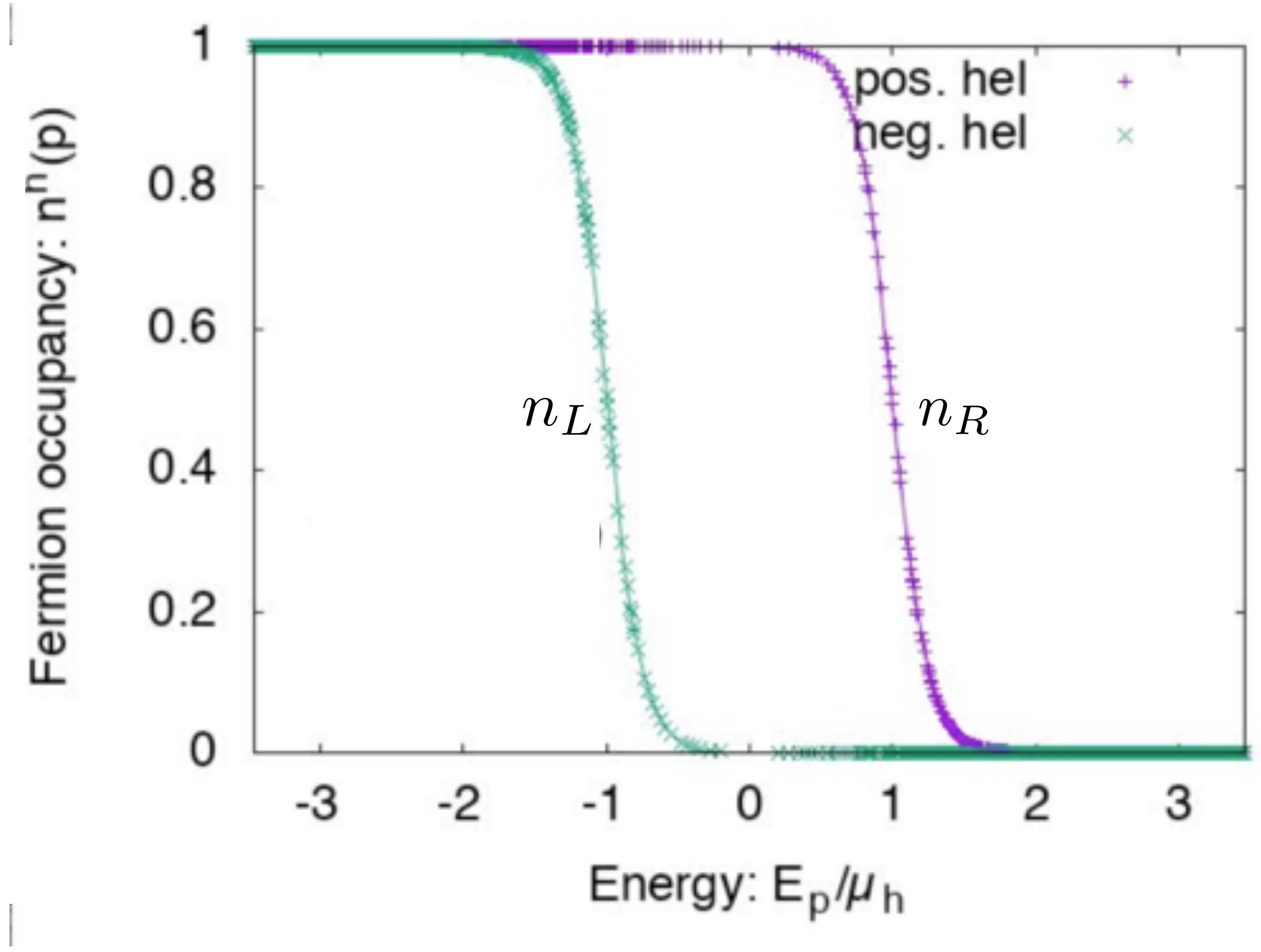
Solve Maxwell's equations
with fermion currents

$$D_\mu F^{\mu\nu} = \langle\hat{\bar{\psi}}(x)\gamma^\mu\hat{\psi}(x)\rangle$$



Start with spin polarization of the fermion sector, characterized by helicity chemical potential $\mu_h >> T$

=> energy density & chiral charge initially concentrated in fermion sector

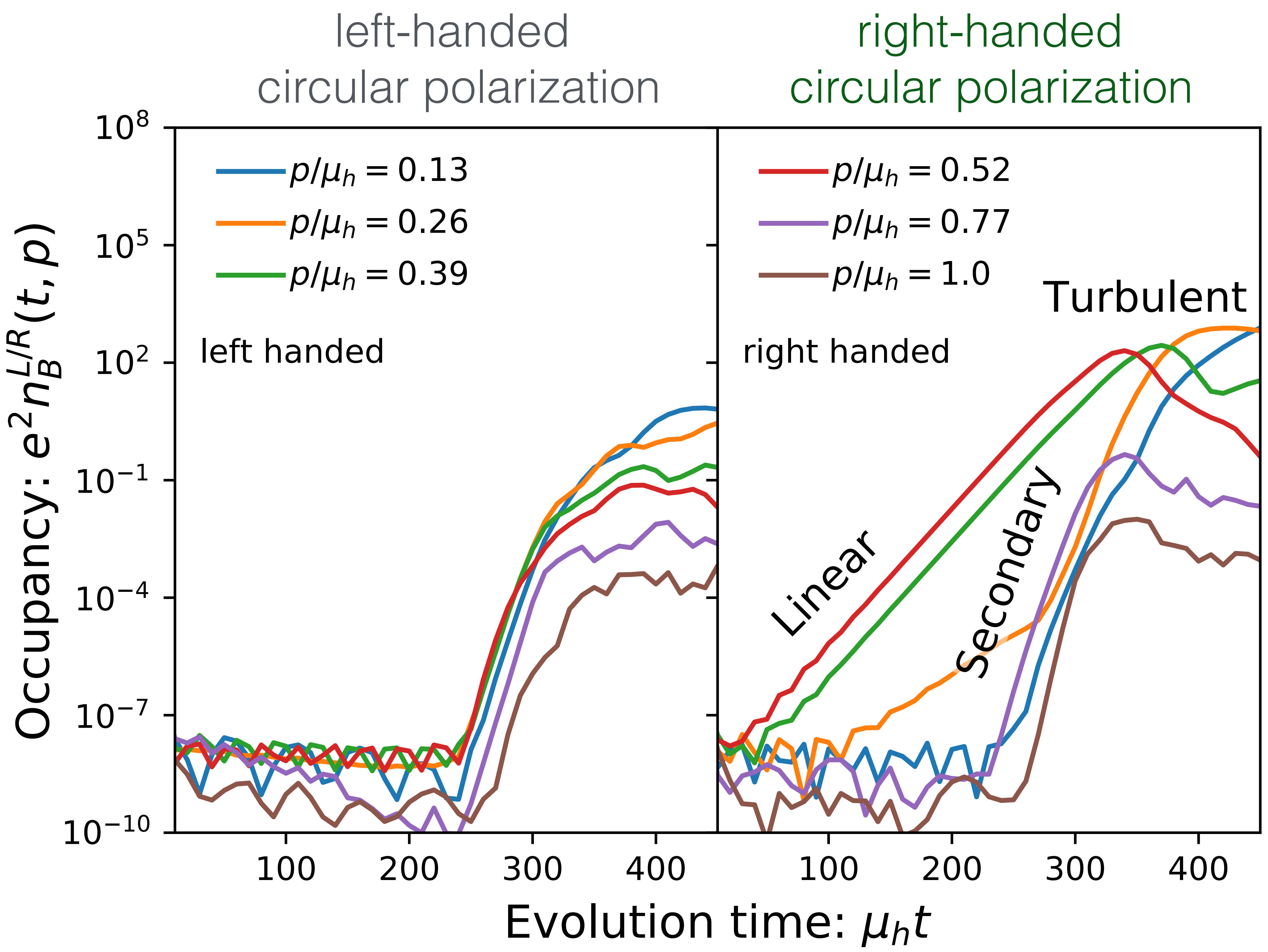


Simulate dynamics in strongly coupled QED $e^2N_f=64$ to properly resolve all dynamical scales on the lattice

Simulations performed close to chiral limit $m << \mu_h$ where net-chirality is effectively conserved
trivial to extend to finite quark mass but not considered so far

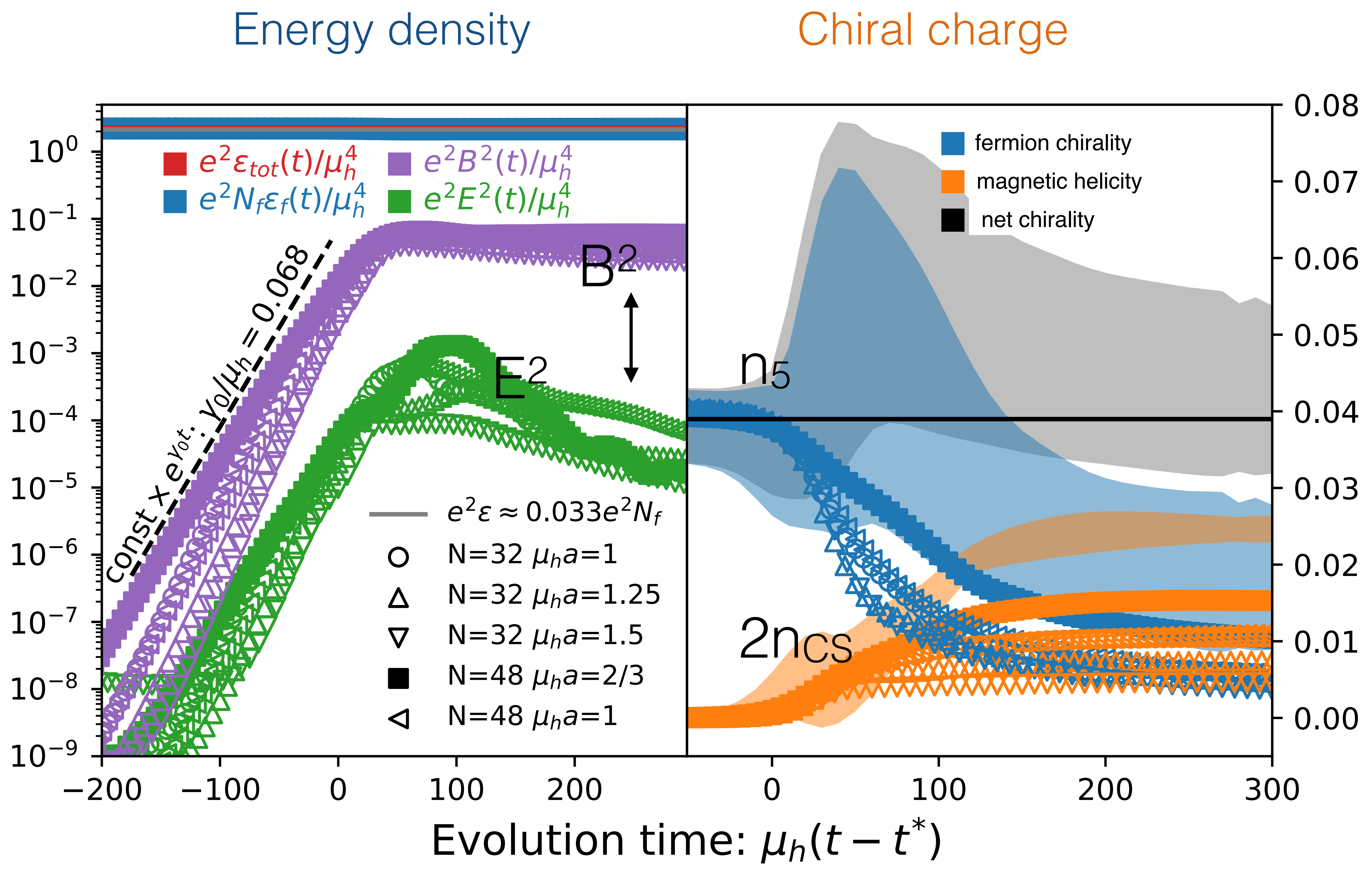
Chirality transfer in large N_f QED

Evolution of left/right-handed magnetic field modes



Chiral plasma instability of leads to a characteristic sequence of dynamics in unstable systems

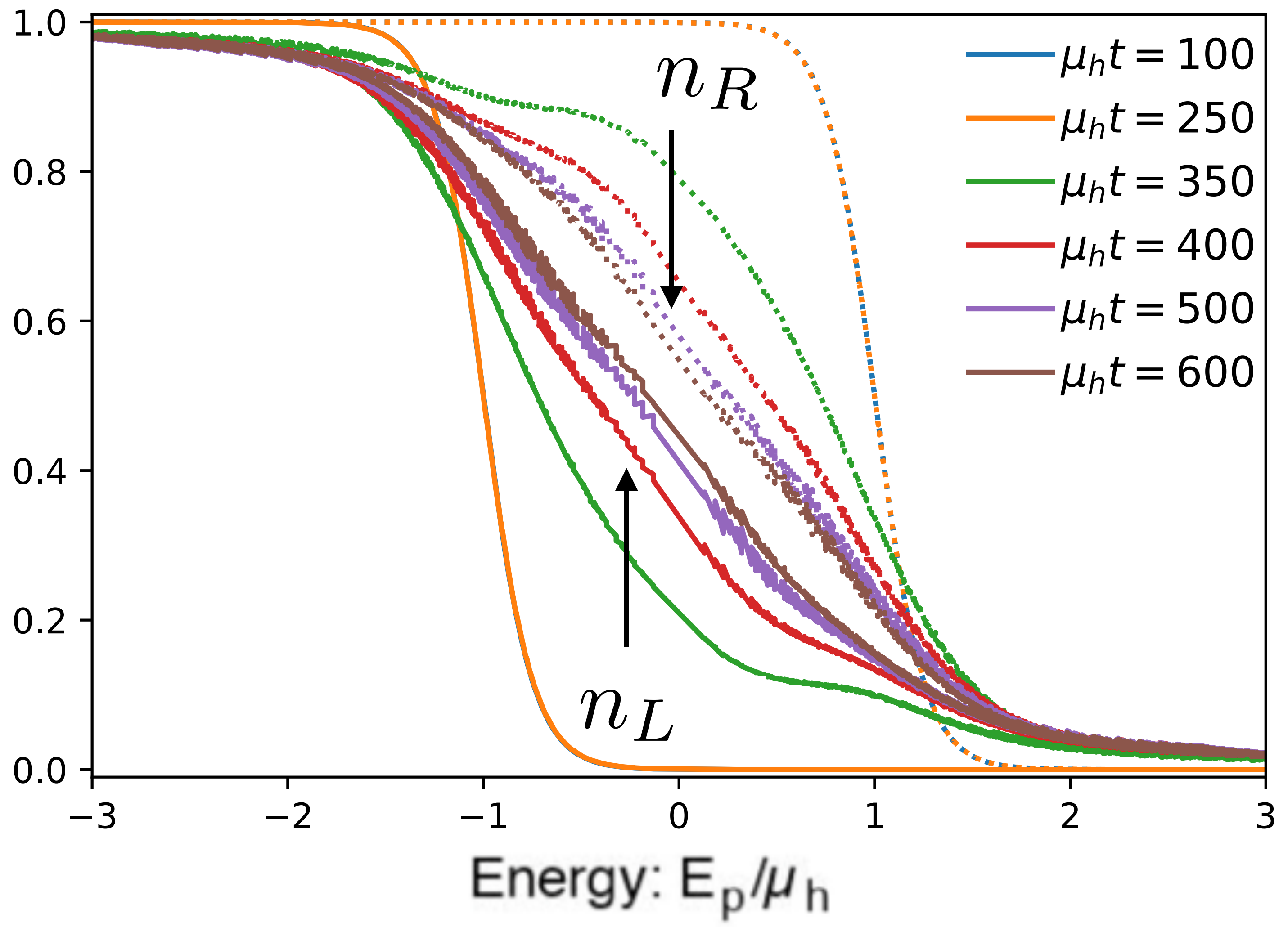
Evolution of conserved quantities



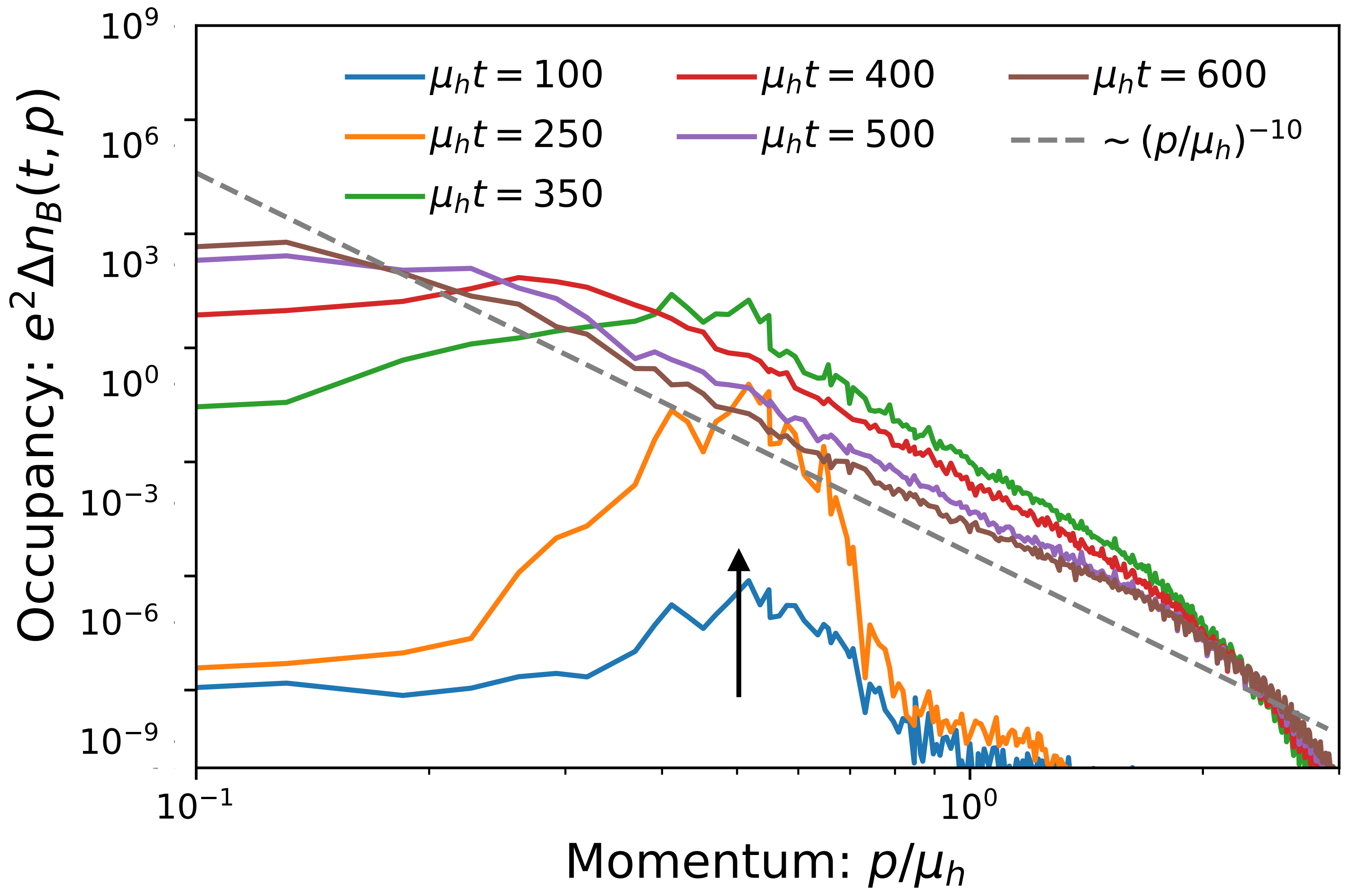
Small fraction of energy but significant fraction of **chiral charge transferred from fermions to gauge fields**

Chirality transfer in large N_f QED

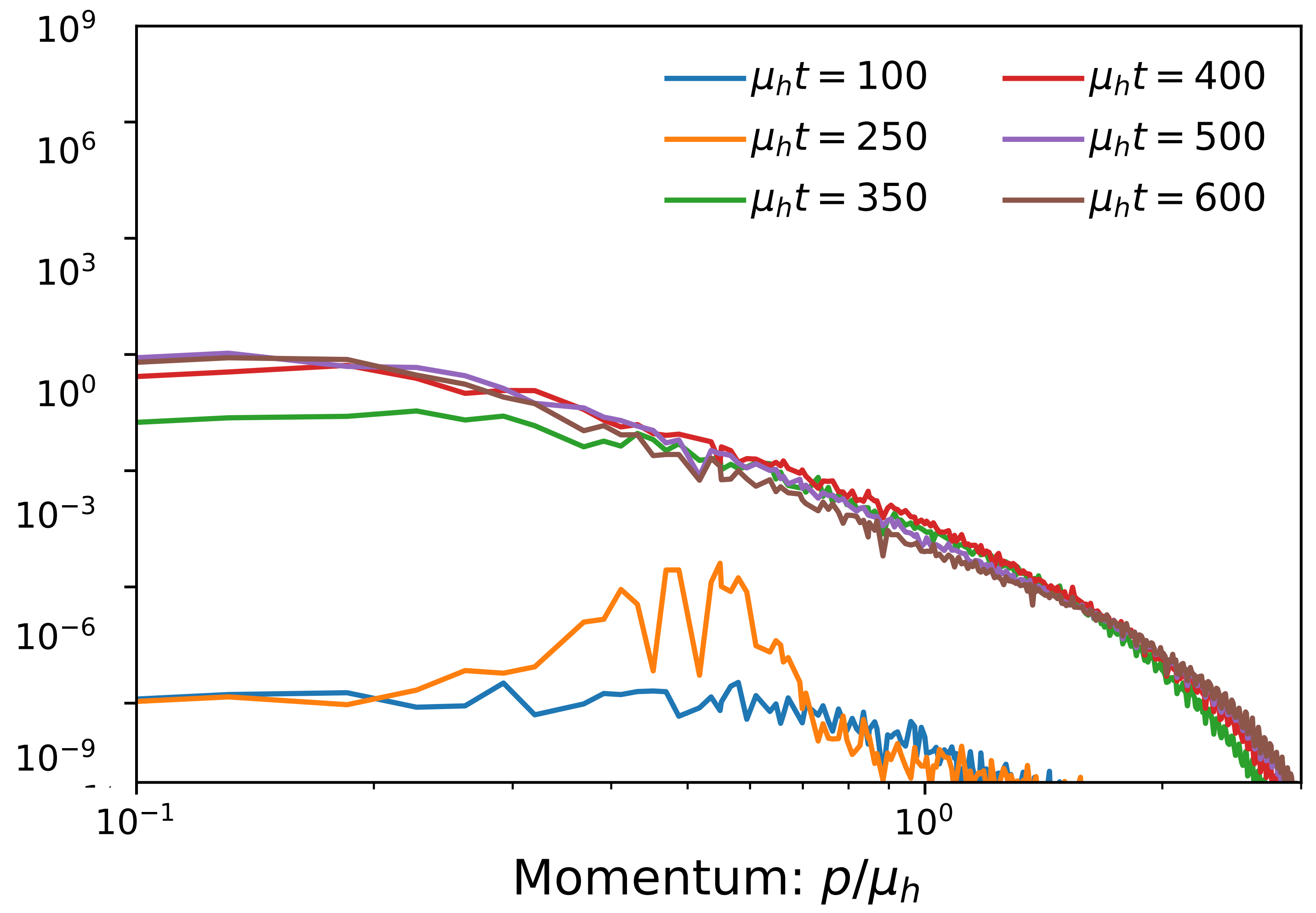
left/right handed fermions



left-handed magnetic fields



right-handed magnetic fields



Non-linearities lead to **depletion of helicity imbalance** (stabilisation)

Strong **heating of the fermion sector** upon saturation of instability

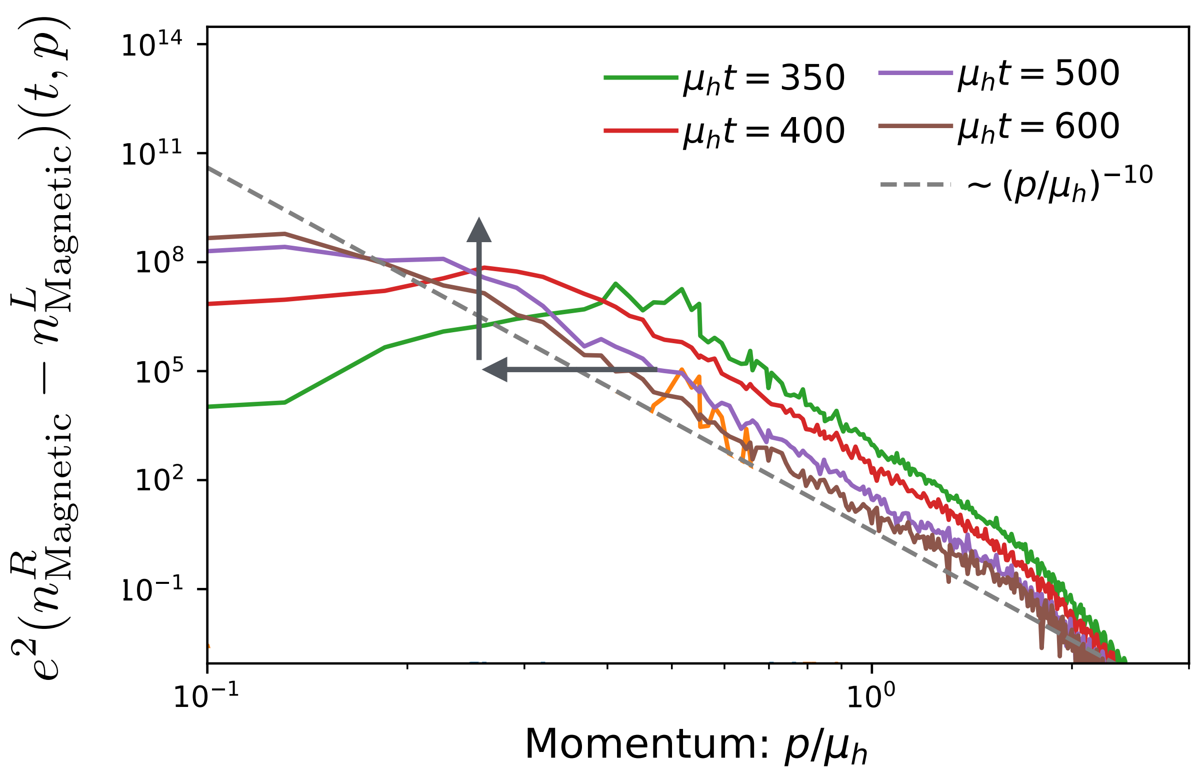
Non-perturbatively large occupations of left & right-handed magnetic fields

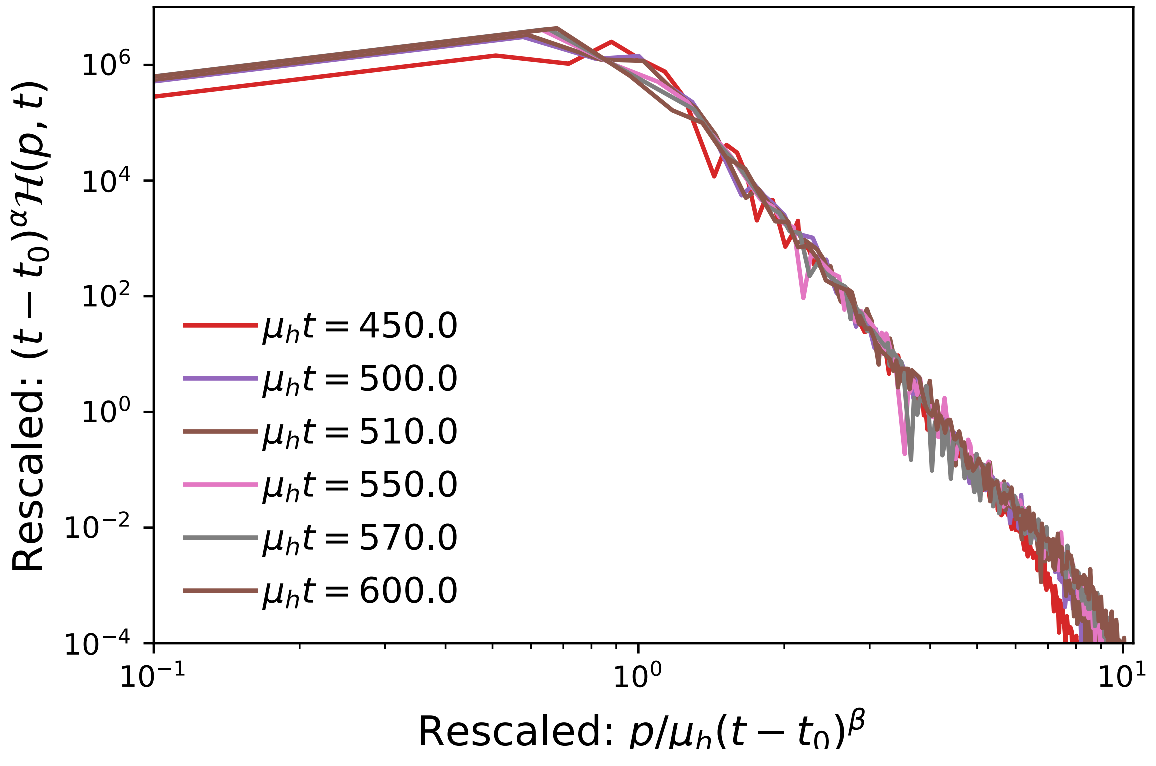
Strong IR excess of right vs. left-handed modes characterized by power law ~$(p/\mu)^{-10}$ with large spectral exponent

-> Chirality imbalance eventually absorbed by large scale helical magnetic fields

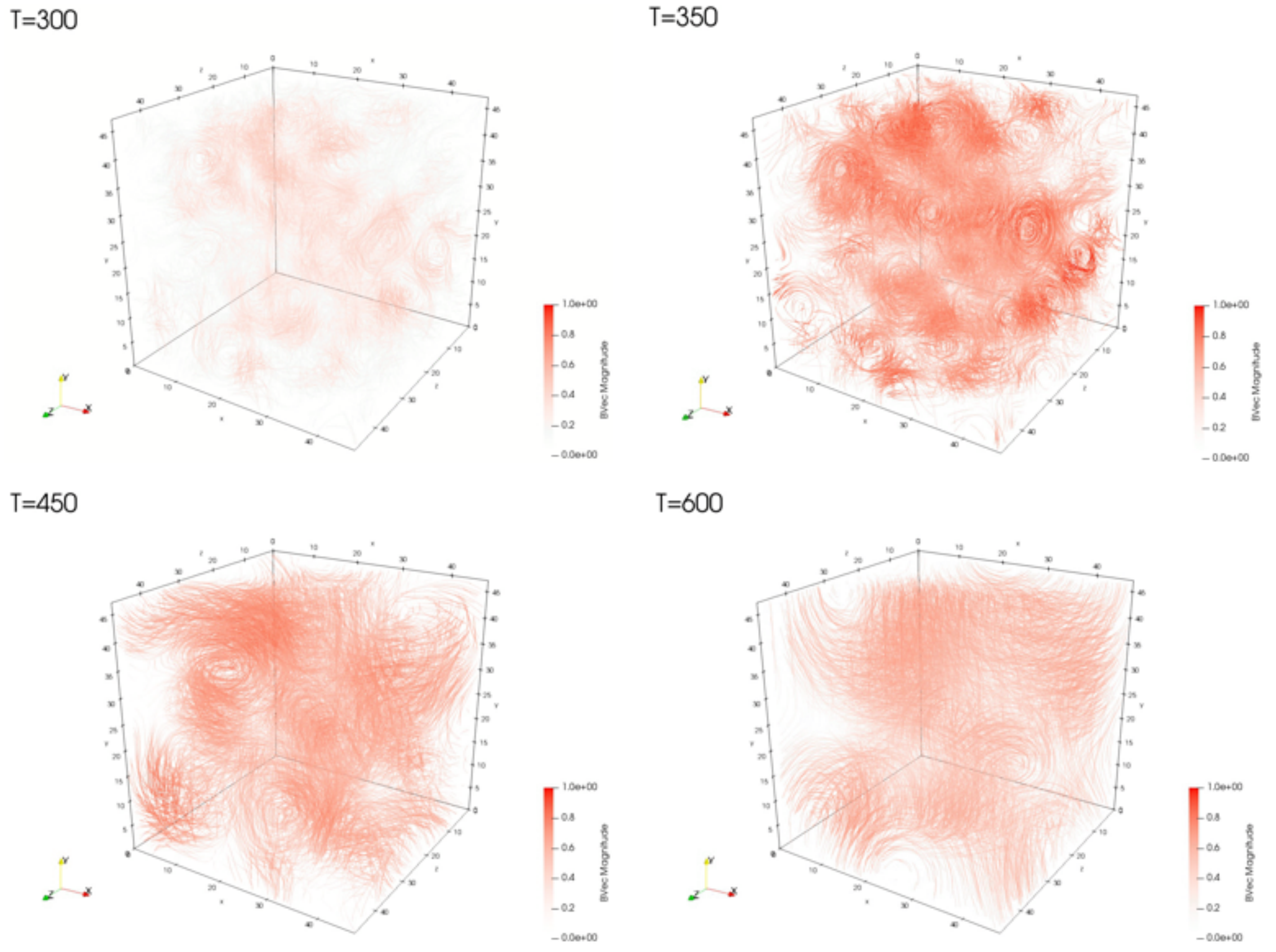
Chirality turbulence

Evolution of net-magnetic helicity spectra





Evolution of magnetic fields in coordinate space



Evolution in the turbulent regime proceeds via **self-similar evolution of net magnetic helicity**

$$\Delta n_B(t,\mathbf{p}) = \tau^{\alpha} f_s(\tau^{\beta}|\mathbf{p}|)$$
$$\tau \equiv \mu_h(t - t^*)$$

Scaling exponents α,β determined from statistical scaling analysis

$$\alpha = 1.14 \pm 0.50$$
$$\beta = 0.37 \pm 0.13$$
$$\alpha \approx 3\beta$$

Generation of large scale magnetic fields via self-similar inverse cascade of magnetic helicity

Conclusions & Outlook

Non-conservation of chiral charge complicates the macroscopic description
-> Important to understand chirality transfer between fermions and gauge fields

QED plasmas: Chiral plasma instability provides efficient mechanism to transfer chiral charge from fermions to gauge fields

-> Eventually chiral charge imbalance leads to generation of large scale helical magnetic fields via self-similar (inverse) cascade of magnetic helicity

Established for the first time from real-time simulations of underlying microscopic theory

QCD plasmas: Sphaleron transitions provide an additional mechanism to absorb chirality imbalance into topology of non-abelian gauge fields

Competition between effectively abelian and non-abelian effects requires further investigation

Need to take into account in macroscopic descriptions of anomalous transport phenomena

$$\partial_\mu j^\mu_a = -4\Gamma_{\rm sph}\frac{\mu_A}{T}$$

SS, deBruin work in progress

Backup

Classical-statistical lattice simulations

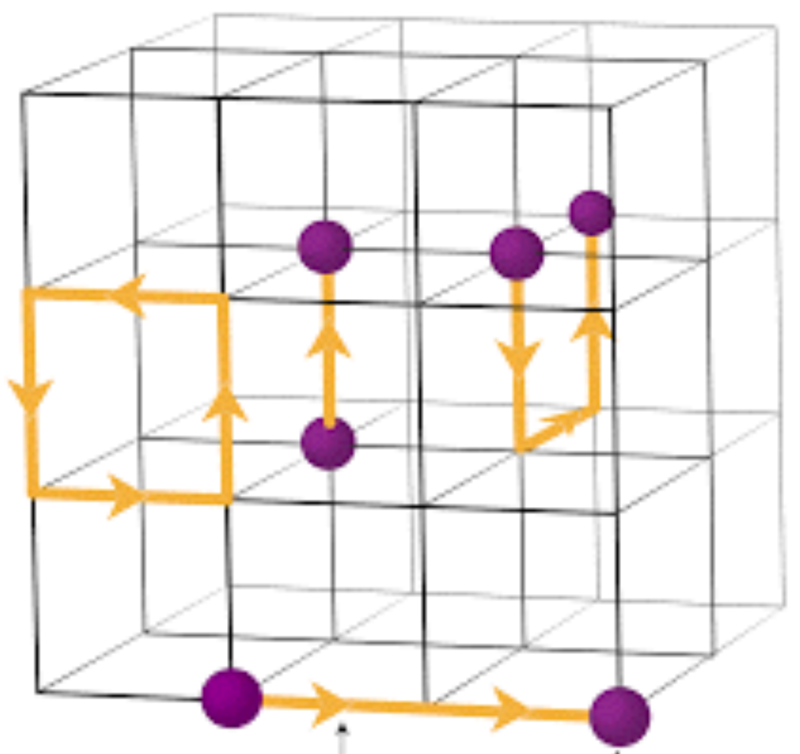
Discretize theory on 3D spatial lattice using the Hamiltonian lattice formalism

$$i\gamma^0\partial_t\hat{\psi} = (-i\not{D}^s_W + m)\hat{\psi}$$

quantum fermions

$$D_\mu F^{\mu\nu} = \langle\hat{\bar{\psi}}(x)\gamma^\mu\hat{\psi}(x)\rangle$$

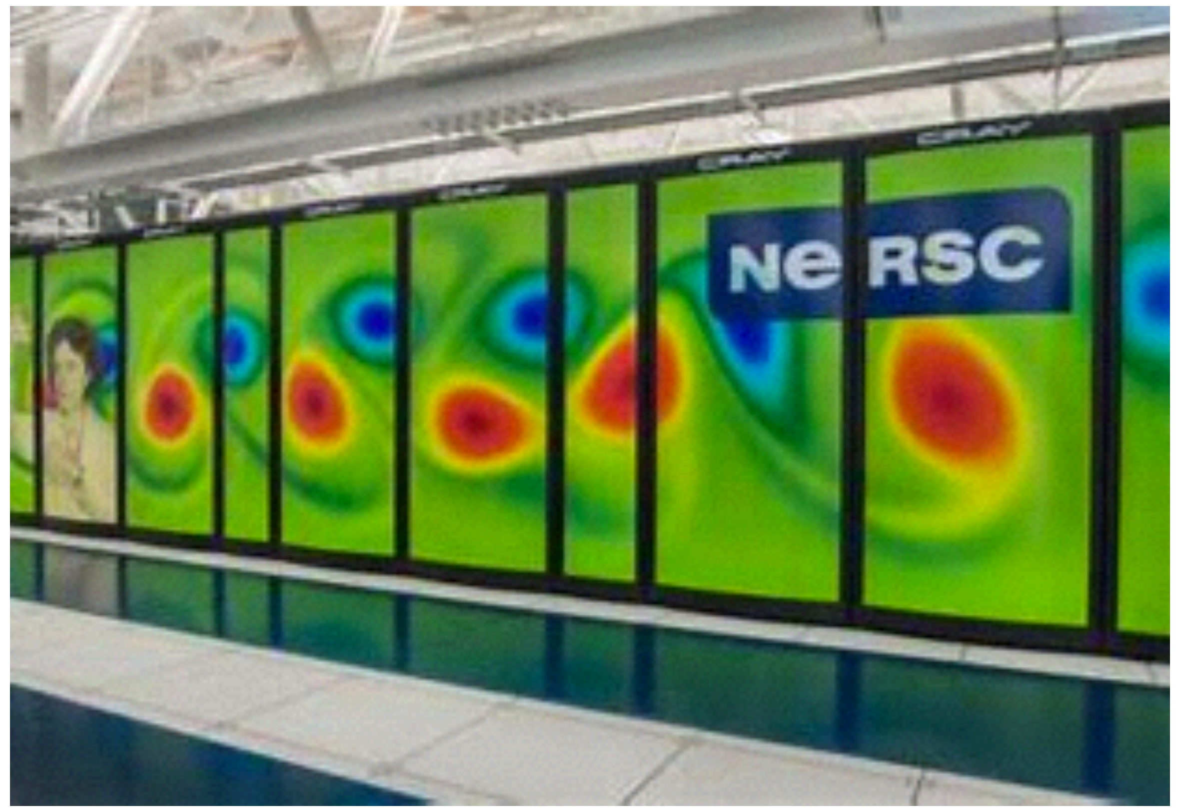
classical gauge fields



Expansion to leading order in e^2 but all orders in e^2N_f (exact at large Nf)

Solution to operator Dirac obtained by expanding the fermion field in operator basis at initial time

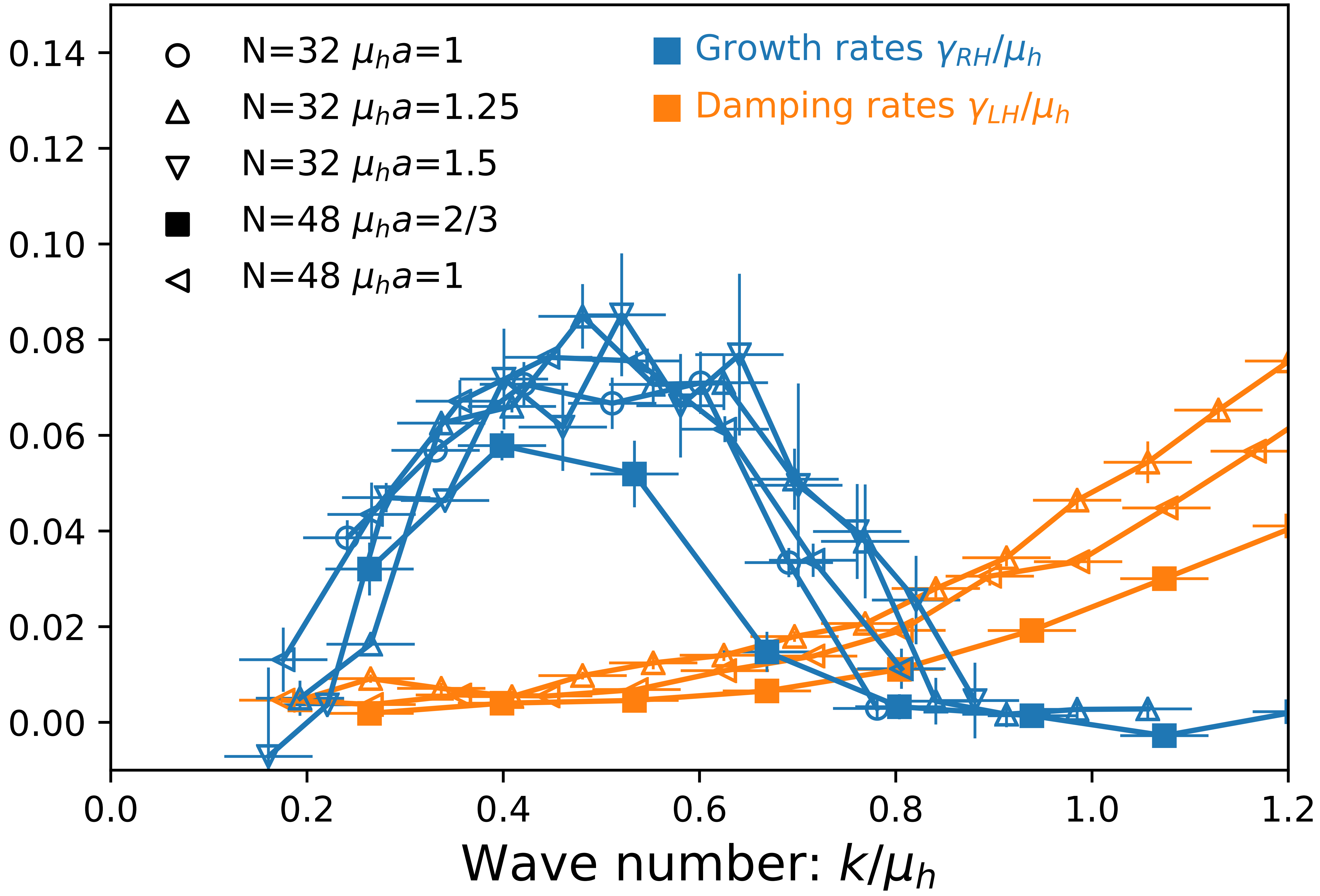
$$\hat{\psi}(x,t) = \sum_{p,\lambda} \hat{b}_{p,\lambda}(t=0)\phi_u^{p,\lambda}(x,t) + \hat{d}^\dagger_{p,\lambda}(t=0)\phi_v^{p,\lambda}(x,t)$$



and solving the Dirac equation for evolution of $4N_cN^3$ wave-functions

Computationally extremely demanding -> lattice sizes up to 48^3
(~5M CPU hours per simulation on Cori@NERSC)

Growth & Damping rates in large N_f QED

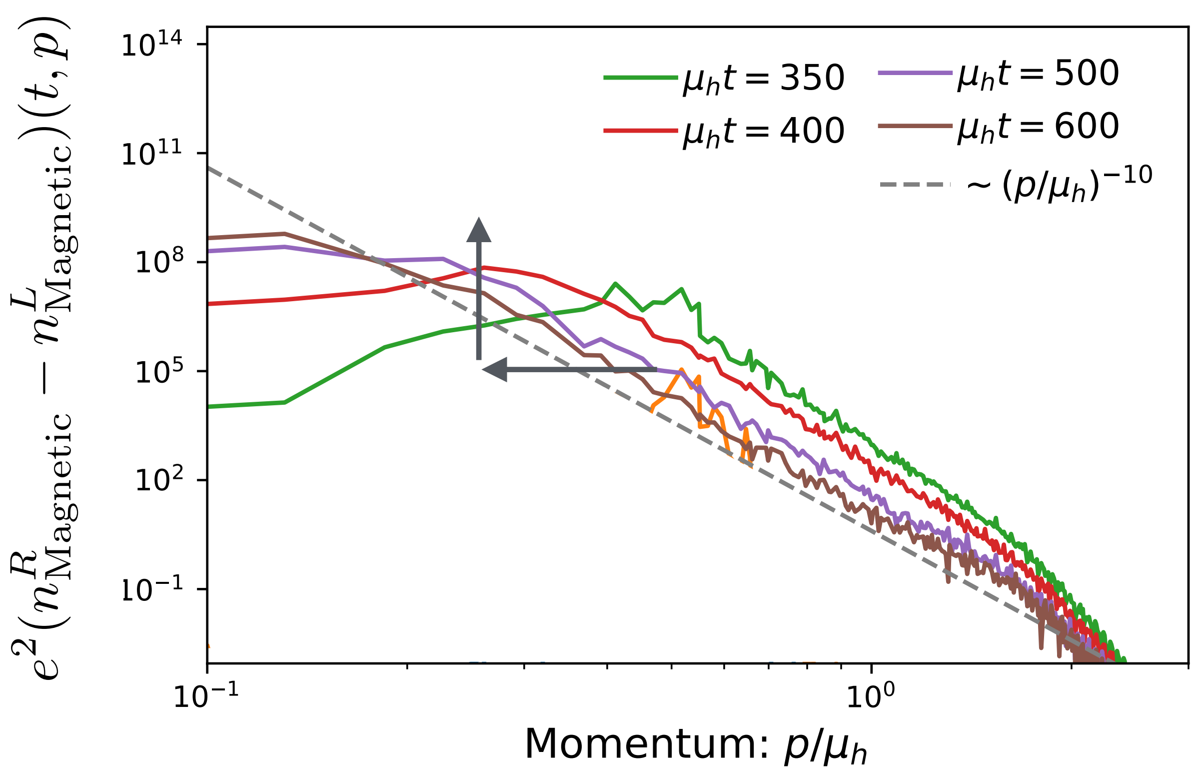


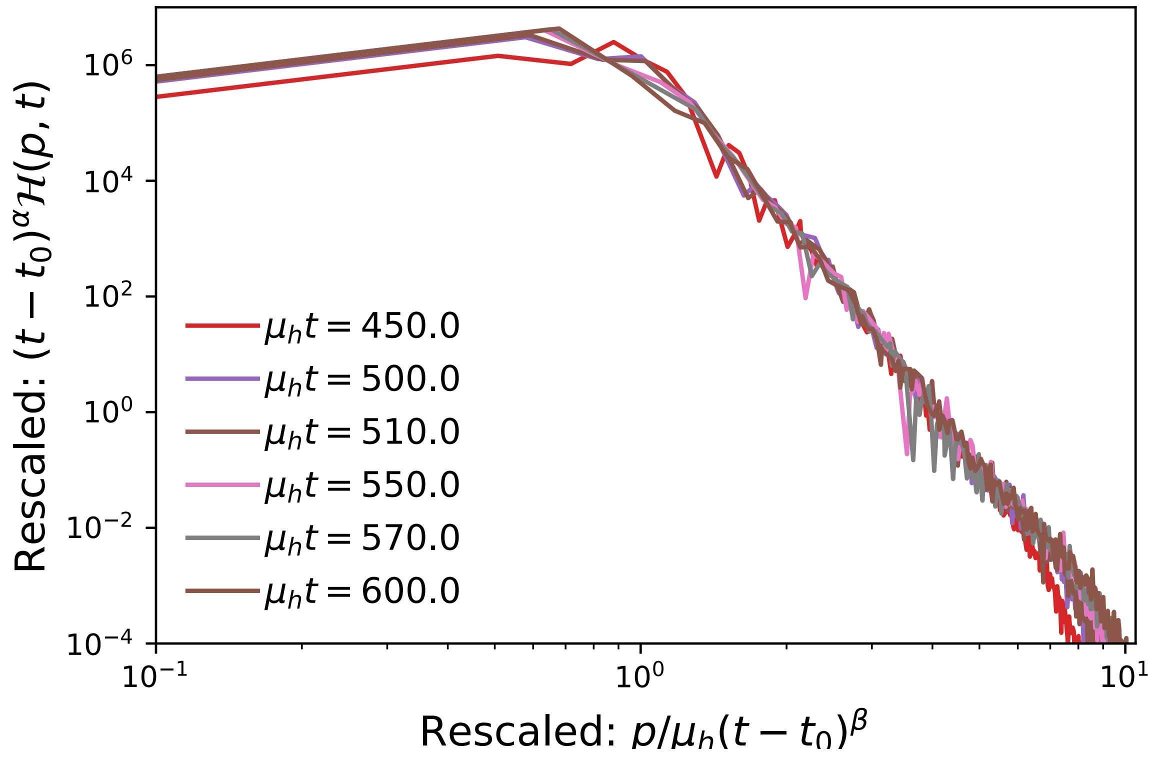
Growth and damping rates in qualitative agreement with expectation from one-loop HTL calculation

By use of improved operator definitions (Wilson NLO) errors due to finite lattice size and lattice spacing appear to be under control

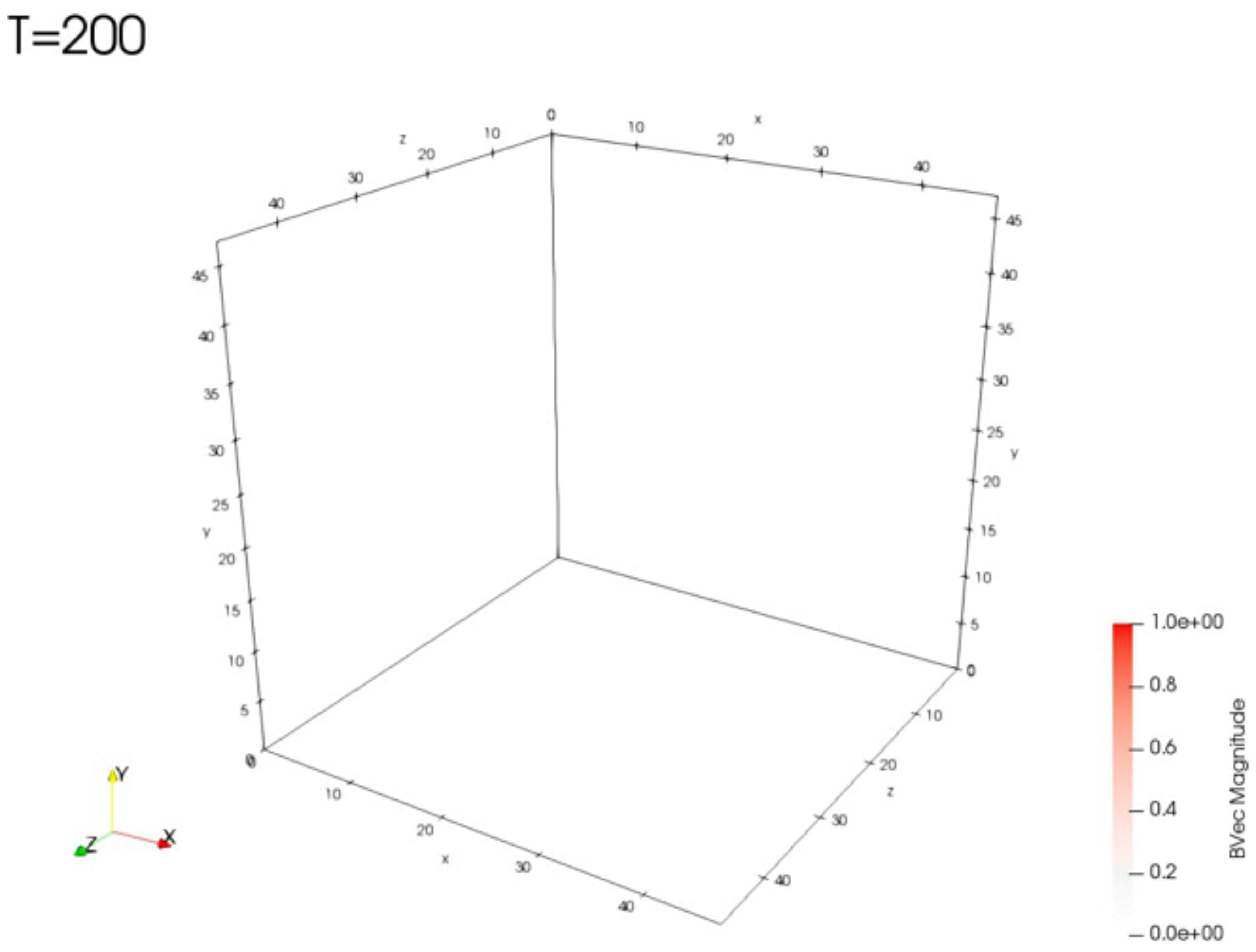
Chirality turbulence

Evolution of net-magnetic helicity spectra





Evolution of magnetic fields in coordinate space



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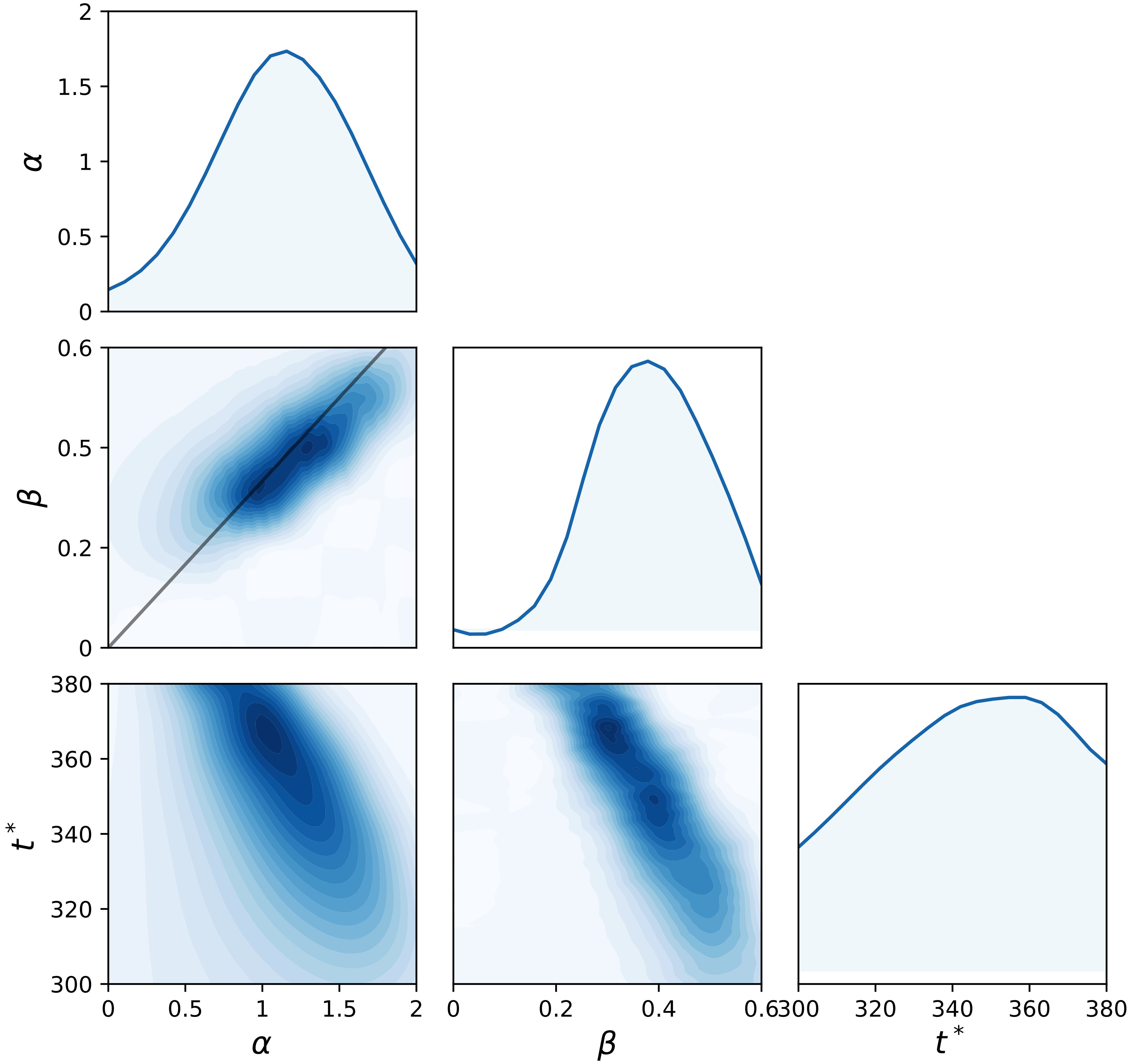
Extraction of scaling exponents



Statistical scaling analysis

$$f_{\mathrm{ref}}(t, \tilde{p} = \tau_{\mathrm{ref}}^{\beta} p) = \log \tau_{\mathrm{ref}}^{-\alpha} \Delta n_B(\tau_{\mathrm{ref}}^{\beta} p)\,,$$

$$\chi^2(\alpha, \beta, t^*) = \frac{1}{N_{\mathrm{test}}} \sum_{\tau_{\mathrm{test}}} \frac{\int \frac{d\tilde{p}}{\tilde{p}} \left(f_{\mathrm{ref}}(\tilde{p}) - f_{\mathrm{test}}(\tilde{p})\right)^2}{\int \frac{d\tilde{p}}{\tilde{p}} \left(f_{\mathrm{ref}}(\tilde{p})\right)^2}\,,$$

determines likelihood

$$W(\alpha, \beta, t^*) = \frac{1}{\mathcal{N}} \exp\left[-\frac{\chi^2(\alpha, \beta, t^*)}{\chi^2_{\mathrm{min}}}\right]$$