An effective theory of quarkonia in QCD matter

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Quarkonia in the QGP

- Quarkonia (e.g. $J/\psi, \Upsilon$), bound states of the heaviest elementary particles, long considered standard candle to characterize QGP properties.
- Most sensitive to the space-time temperature profile.

\[
\left[ -\frac{1}{2\mu_{\text{red}}} \frac{\partial^2}{\partial r^2} + \frac{l(l+1)}{2\mu_{\text{red}}r^2} + V(r) \right] rR_{nl}(r) = (E_{nl} - 2m_Q) rR_{nl}(r) \quad \frac{1}{\langle r \rangle}
\]

\[
\psi(r) = Y_l^m(\hat{r}) R_{nl}(r)
\]

Mocsy et al. (2007)

Bazavov et al. (2013)

Dynamics is more complicated – non-equilibrium transport, recombination ...

\[
\begin{array}{cccccc}
 l & n & E_{nl} & \sqrt{\langle r^2 \rangle} & k^2 & \text{Meson} \\
 0 & 1 & 0.700 & 2.24 & 0.30 & J/\psi \\
 0 & 2 & 0.086 & 5.39 & 0.05 & \psi(2S) \\
 1 & 1 & 0.268 & 3.50 & 0.20 & \chi_c \\
 0 & 1 & 1.122 & 1.23 & 0.99 & \Upsilon(1S) \\
 0 & 2 & 0.578 & 2.60 & 0.22 & \Upsilon(2S) \\
 0 & 3 & 0.214 & 3.89 & 0.10 & \Upsilon(3S) \\
 1 & 1 & 0.710 & 2.07 & 0.58 & \chi_b(1P) \\
 1 & 2 & 0.325 & 3.31 & 0.23 & \chi_b(2P) \\
 1 & 3 & 0.051 & 5.57 & 0.08 & \chi_b(3P)
\end{array}
\]
Challenges and hypothesae

- **Suppression puzzle** - similar dissociation behavior observed in small system, p+A and even in p+p (where QGP is not expected)
- **Recently proposed co-mover dissociation model, energy loss model** – need cross check and microscopic explanation (if applicable)

**E. Ferreiro (2014)**

![Graph showing suppression of 
\( \frac{\Upsilon(2S)}{\Upsilon(1S)} \)](attachment:graph_suppression.png)

**Chatrchyan et al. (2014)**

- **EFT** - capture the interactions without explicitly specifying their nature
Zooming in on the correct theory – excluding energy loss
Production of quarkonia at intermediate and high $p_T$

- Non-Relativistic QCD (NRQCD) - a particular type of effective theory (EFT)

- Explores all regimes of QCD

- Perturbative

- Non-Perturbative

- QCD without the heavy flavor

- Ultra-soft

- $b\bar{b}$: $v^2 \sim 0.1$

- $c\bar{c}$: $v^2 \sim 0.3$

- NRQCD factorization formula. Short distance cross sections (perturbatively calculable) and long distance matrix elements (fit to data, scaling relations)

\[ d\sigma(a + b \rightarrow Q + X) = \sum_n d\sigma(a + b \rightarrow Q\bar{Q}(n) + X)\langle O^Q_n \rangle \]
NRQCD practical examples

- One has to be careful, the simple power counting approximately manifest in the LDMEs can be affected by the partonic cross section – a large number of singlet and octet; S wave and P wave terms enter

\[
d\sigma(J/\psi) = d\sigma(Q\bar{Q}([^3S_1]_1))\langle \mathcal{O}(Q\bar{Q}([^3S_1]_1) \to J/\psi) \rangle + d\sigma(Q\bar{Q}([^1S_0]_8))\langle \mathcal{O}(Q\bar{Q}([^1S_0]_8) \to J/\psi) \rangle \\
+ d\sigma(Q\bar{Q}([^3S_1]_8))\langle \mathcal{O}(Q\bar{Q}([^3S_1]_8) \to J/\psi) \rangle + d\sigma(Q\bar{Q}([^3P_0]_8))\langle \mathcal{O}(Q\bar{Q}([^3P_0]_8) \to J/\psi) \rangle \\
+ d\sigma(Q\bar{Q}([^3P_1]_8))\langle \mathcal{O}(Q\bar{Q}([^3P_1]_8) \to J/\psi) \rangle + d\sigma(Q\bar{Q}([^3P_2]_8))\langle \mathcal{O}(Q\bar{Q}([^3P_2]_8) \to J/\psi) \rangle + \cdots
\]

- The situation is similar for bottomonia
- Excited states have their own expansion

The question is – is there a simplification at high \(p_T\) where the \(p_T\) dependence of the short distance cross section dominates (and expressions simplify)
Leading power factorization

**Singlet contribution**

S. Fleming et al. (2012)  
M. Baumgart et al. (2014)  
Y. Ma et al. (2014)

**Octet contribution**

Only a subset of contributions survive, now interpretable as parton fragmentation in quarkonia.
LP example and applicability

\[
\frac{d\sigma_h}{dp_{\perp}}(p_{\perp}) = \sum_i \int_z^1 \frac{dx}{x} \frac{d\sigma_i}{dp_{\perp}} \left( \frac{p_{\perp}}{x}, \mu \right) D_{i/h}(x, \mu) + \mathcal{O} \left( \frac{m_h^2}{p_{\perp}^2} \right)
\]

\( p_T \gg m_Q \)

\[
\ln \left( \frac{\mu}{p_T} \right) - \ln \left( \frac{\mu}{2m_Q} \right) d_{i/n}(x, \mu) \langle \mathcal{O}_{h/n}^n \rangle
\]

DGLAP Evolution

\[
\mu \frac{d}{d\mu} D_{i/h}(z, \mu) = \sum_i \int_z^1 \frac{dx}{x} P_{i,j}(x) D_{i/h} \left( \frac{z}{x}, \mu \right)
\]

Resummation of

\( \ln(p_T/m_h) \)

Contributions we take in our work

Applicability must be determined phenomenologically
Comparison of energy loss phenomenology to data

- Suppression of $J/\psi$ overestimated by factor of 2 to 3. Constrained by light hadrons
- Included $\chi_c$ and $\psi(2S)$ feeddown.

\[
\psi(2S): \quad \text{Br} \left[ \psi(2S) \to J/\psi + X \right] = 61.4 \pm 0.6\% ,
\]
\[
\chi_{c1}: \quad \text{Br} \left[ \chi_{c1} \to J/\psi + \gamma \right] = 34.3 \pm 1.0\% ,
\]
\[
\chi_{c2}: \quad \text{Br} \left[ \chi_{c2} \to J/\psi + \gamma \right] = 19.0 \pm 0.5\% .
\]

- Discrepancy persists over centralities. Somewhat different $p_T$ dependence
The energy loss picture of quarkonium suppression in the $p_T$ range measured by ATLAS and CMS (up to 40 GeV) is definitively excluded.

In the double suppression ratio $R_{AA}(\psi(2S))/R_{AA}(J/\psi)$, the discrepancy is not simply in magnitude. There is a discrepancy in the sign of the theoretical prediction.
NRQCD with Glauber Gluons & phenomenology

“I think you should be more explicit here in step two.”
NRQCD in a background medium

- Take a closer look at the NRQCD Lagrangian below

**Scales in the problem**

- \( p_s^\mu \sim m_Q v(1,1,1,1) \) soft \( \sim \lambda \)
- \( p_{us}^\mu \sim m_Q v^2(1,1,1,1) \) ultrasoft \( \sim \lambda^2 \)

- Ultrasoft gluons included in covariant derivatives

**Soft gluons are included explicitly**

- Double soft gluon emission
- Heavy quark-antiquark potential
- (can also be interaction with soft particles)

\[
\mathcal{L} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \sum_p |p^\mu A_\mu^p - p^\nu A_\nu^p|^2 + \sum_p \psi_p^\dagger \left\{ i D^0 - \frac{(p - iD)^2}{2m} \right\} \psi_p \\
- \frac{4\pi \alpha_s}{q^2} \sum_{q,p,p'} \left\{ \frac{1}{g^0} \psi_{p'}^\dagger \left[ A_q^0, A_q^0 \right] \psi_p \\
+ g^{\mu 0} (q' - p + p')^\mu - g^{\mu 0} (q - p + p')^\nu + g^{\mu \nu} (q - q')^0 \right\} \psi_{p'}^\dagger \left[ A_q^\mu, A_q^\mu \right] \psi_p \right\} \\
+ \psi \leftrightarrow \chi, \ T \leftrightarrow \bar{T} \\
+ \sum_{p,q} \frac{4\pi \alpha_s}{(p - q)^2} \psi_q^\dagger T^A \psi_p \chi_{-q}^\dagger \bar{T}^A \chi_{-p} + \ldots
\]
Allowed interactions in the medium

- At the level of the Lagrangian

\[ \mathcal{L}_{\text{NRQCD}_G} = \mathcal{L}_{\text{NRQCD}} + \mathcal{L}_{Q-G/C}(\psi, A^\mu_a G/C) \]
+ \mathcal{L}_{g-G/C}(A^\mu_s b, A^\mu_a G/C) + \psi \leftrightarrow \chi

Possible scaling for the virtual gluons interacting with the heavy quarks

| \( q_C \sim (\lambda^2, \lambda^1, \lambda^1, \lambda^2) \sim (\lambda^2, \lambda, \lambda, \lambda) \_n \)
| \( q_C \sim (\lambda^2, \lambda^1, \lambda^1, \lambda^1) \sim (\lambda^1, \lambda^1, \lambda, \lambda) \_n \)

- Energy component must always be suppressed
- Glauber gluons - transverse to the direction of propagation contribution
- Coulomb gluons - isotropic momentum distribution

- Calculated the leading power and next to leading power contributions 3 different ways

<table>
<thead>
<tr>
<th>Source</th>
<th>Collinear</th>
<th>Static</th>
<th>Soft</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A^\mu_C \sim )</td>
<td>n.a.</td>
<td>( (\lambda^1, \lambda^2, \lambda^2, \lambda^2) )</td>
<td>( (\lambda^1, \lambda^1, \lambda^1, \lambda^1) )</td>
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</tbody>
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- Background field method
  Perform a shift in the gluon field in the NRQCD Lagrangian then perform the power-counting

- Hybrid method
  From the full QCD diagrams for single effective Glauber/Coulomb gluon perform the corresponding power-counting, read the Feynman rules

- Matching method
  Full QCD diagrams describing the forward scattering of incoming heavy quark and a light quark or a gluon. We also derive the tree level expressions of the effective fields in terms of the QCD ingredients
Example of the background field method (collinear source)

- Perform the label momentum representation and field substitution (u.s. -> u.s. + Glauber)

\[ iD_t = i\partial_t - g A_U^0 - g A_G^0, \]
\[ \sim \lambda^2 \]
\[ iD = \{ P - (i\partial + gA_U + gn A_G^0) \} + O(\lambda^3), \]
\[ \sim \lambda \]
\[ E = \partial_t (A_U + A_G) + (\partial + iP)(A_U^0 + A_G^0) + gT^c f^{cda}(A_U^0 + A_G^0)_a (A_U + A_G)^a \]
\[ = iP \perp A_G^0 + O(\lambda^4), \]
\[ \sim \lambda^3 \]
\[ B = -(\partial + iP) \times (A_U + A_G) + \frac{g}{2} T^c f^{cda}(A_U + A_G)_a (A_U + A_G)^a \]
\[ = -(iP \perp n) A_G^n + O(\lambda^4). \]
\[ \sim \lambda^3 \]

Example for a collinear source (note results depend on the type of source)

Substitute, expand and collect terms up to order $\lambda^3$

- Results: depend on the type of the source of scattering in the medium

**Leading medium correction**

**Sub-leading medium corrections**

\[ \mathcal{L}^{(0)}_{Q-G/C}(\psi, A_{G/C}^{\mu}) = \sum_{P, q_T} \psi^\dagger_{P+q_T} \left( -g A_G^0 \right) \psi_P \text{ (collinear/static/soft).} \]

\[ \mathcal{L}^{(1)}_{Q-G}(\psi, A_G^{\mu}) = g \sum_{P, q_T} \psi^\dagger_{P+q_T} \left( \frac{2A_G^n (n \cdot P)}{2m} - i (P \perp n A_G^n) \cdot \sigma \right) \psi_P \text{ (collinear)} \]

\[ \mathcal{L}^{(1)}_{Q-G}(\psi, A_G^{\mu}) = 0 \text{ (static)} \]

\[ \mathcal{L}^{(1)}_{Q-G}(\psi, A_C^{\mu}) = g \sum_{P, q_T} \psi^\dagger_{P+q_T} \left( \frac{2A_C \cdot P + [P \cdot A_C] - i [P \times A_C] \cdot \sigma}{2m} \right) \psi_P \text{ (soft)} \]
Looking at t-channel scattering we can also extract the form of the Glauber/Coulomb fields in terms of QCD ingredients (and recover Lagrangian).

\[ t_{\text{coll.}} = \begin{array}{c} p' \\ p_n \\ p \\ p_n' \end{array} \]

Glauber field for collinear source

\[ A_{G}^{\mu,a} = \frac{n_{T}^{\mu}}{q_{T}^{2}} \sum_{\ell} \bar{\xi}_{n,\ell-q_{T}} \frac{\hat{n}}{2}(gT_{a}^{\mu})\xi_{n,\ell} \]

Coulomb field for soft source

\[ \frac{1}{q^{2}} \sum_{\ell} \bar{\phi}_{\ell-q}^{\mu}(gT^{A})\phi_{\ell} \]

Note that for the gluon the last 2 diagrams are necessary for gauge invariance but the first diagram the leading forward scattering contribution.

In the medium the momentum exchange can get dressed \( \sim \) Debye screening.
Possible phenomenology applications

- Phenomenology built so far is connected to the leading term – collisional dissociation, thermal effects put in quarkonium wavefunctions. Also neglected possible transitions between different color states.

- If one takes into account the modification of the heavy quark potential in the medium - distinct suppression of ground and excited states.

Conclusions

- Effective theories of QCD have enabled important conceptual and technical breakthroughs in our understanding of strong interactions and very significant improvement in the accuracy of the theoretical predictions.

- In the leading power factorization (high $p_T$) limit of NRQCD we investigated energy loss phenomenology and showed that it severely overpredicts the J/$\psi$ modification and gives the wrong hierarchy of ground/excited suppression.

- Motivated by this we constructed an effective theory of quarkonia in matter - NRQCD$_G$. Derived the Feynman rules (3 different ways) to leading and subleading power for different sources of interactions in the medium. We showed the connection to existing quarkonium dissociation phenomenology.

- In the future it will be interesting to investigate off-diagonal transitions and have an evolution of a matrix of states.