Beyond Color Glass Condensate:
particle production at both low and high transverse momenta

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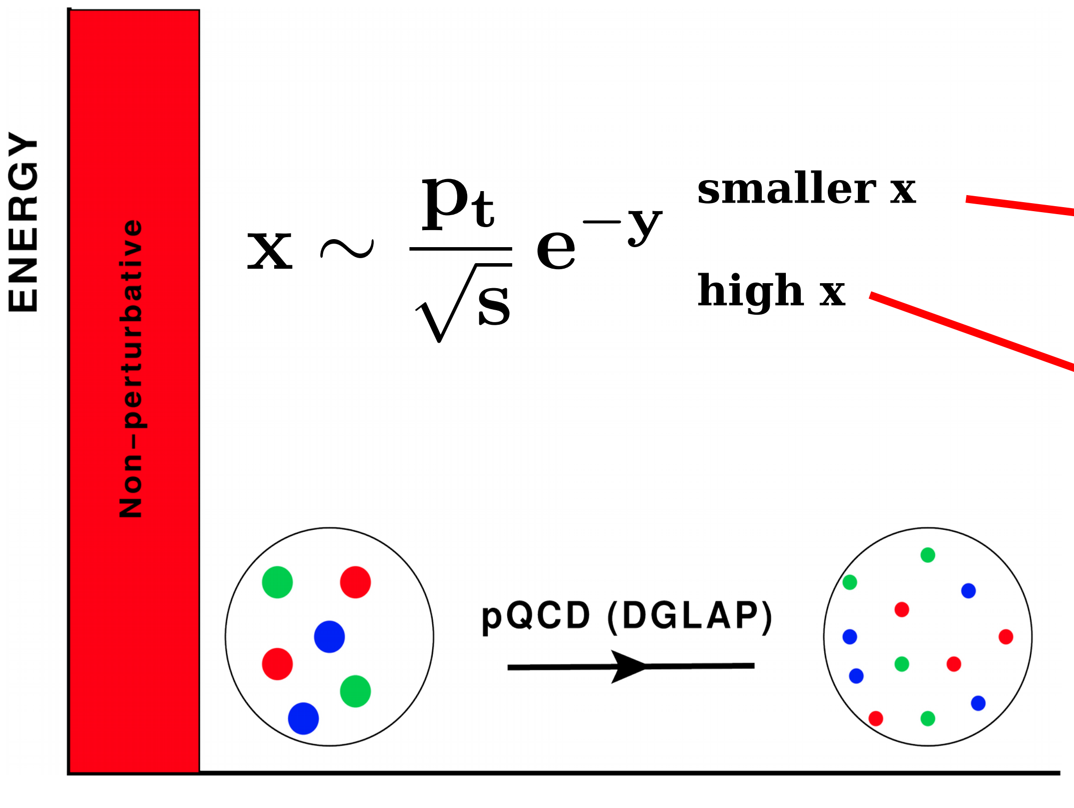
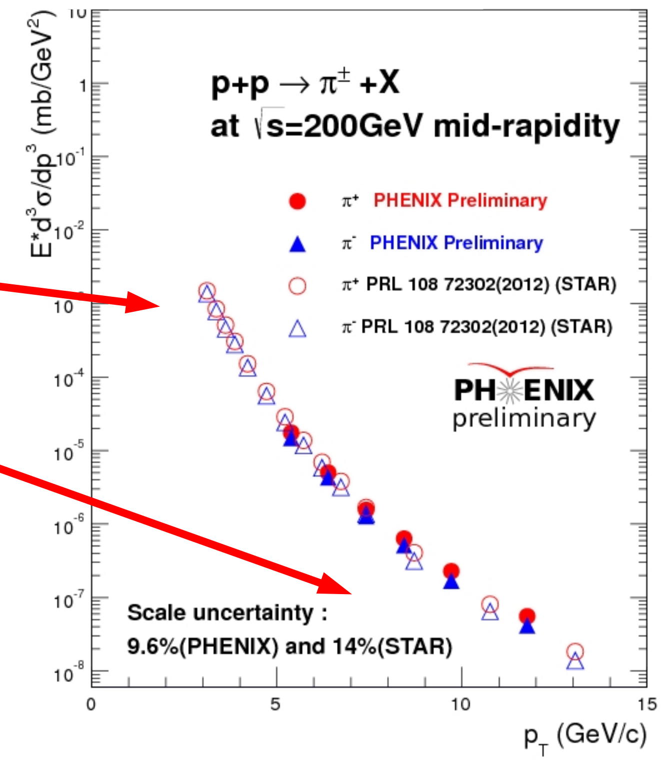
pQCD: the standard paradigm

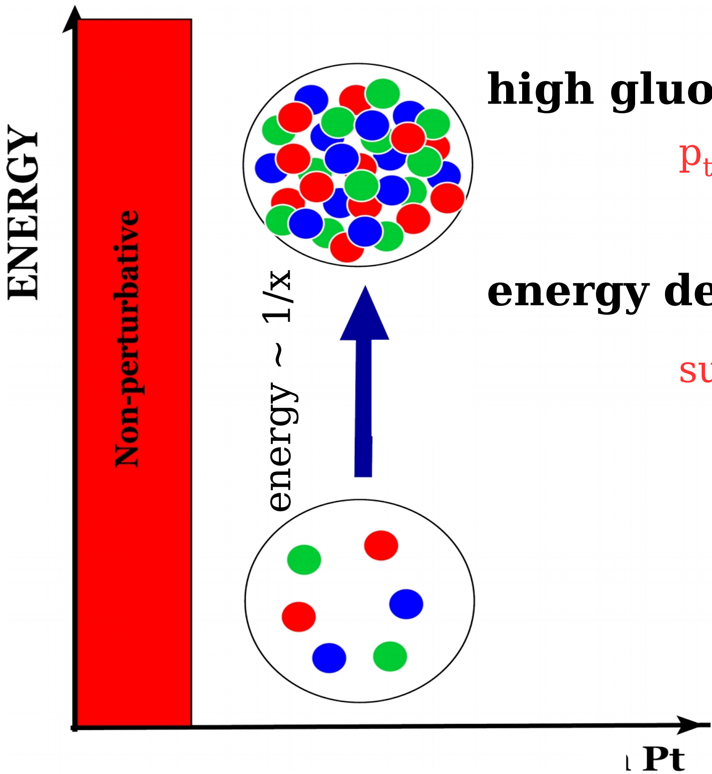
$$E\frac{d\sigma}{d^3p} \sim f_1(x, p_t^2) \otimes f_2(x, p_t^2) \otimes \frac{d\sigma}{dt} \otimes D(z, p_t^2) + \cdots\cdots$$





bulk of QCD phenomena happens at low p_t (small x)

QCD at high energy: gluon saturation



high gluon density: Eikonal multiple scattering

p_t broadening

energy dependence: x-evolution via JIMWLK/BK

suppression of spectra/away side peaks

$$\mathbf{Q_s^2(x, b_t, A) \sim A^{1/3}\,(\frac{1}{x})^{0.3}}$$

$$Q_s^2(x = 3 \times 10^{-4}) \sim 1\, GeV^2$$

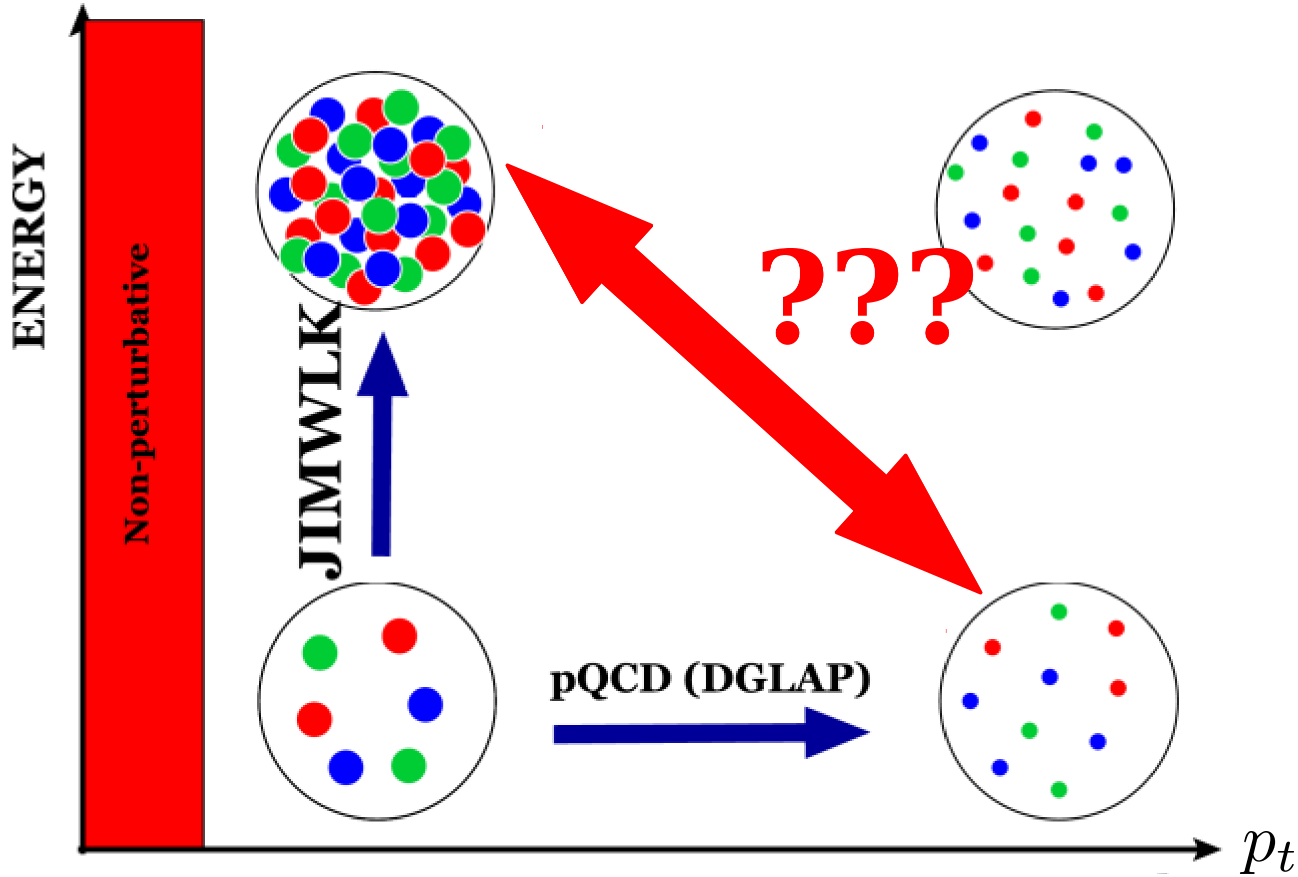
for a proton target (quarks)

a framework for multi-particle production in QCD at small x/low p_t

Shadowing/Nuclear modification factor
Azimuthal angular correlations (di-jets,...)
Long range rapidity correlations (ridge,...)
Initial conditions for hydro
Thermalization (?)

$$\mathbf{x \leq 0.01}$$

QCD kinematic phase space



unifying saturation with high p_t (large x) physics?

<u>kinematics of saturation: *where is saturation applicable?*</u>

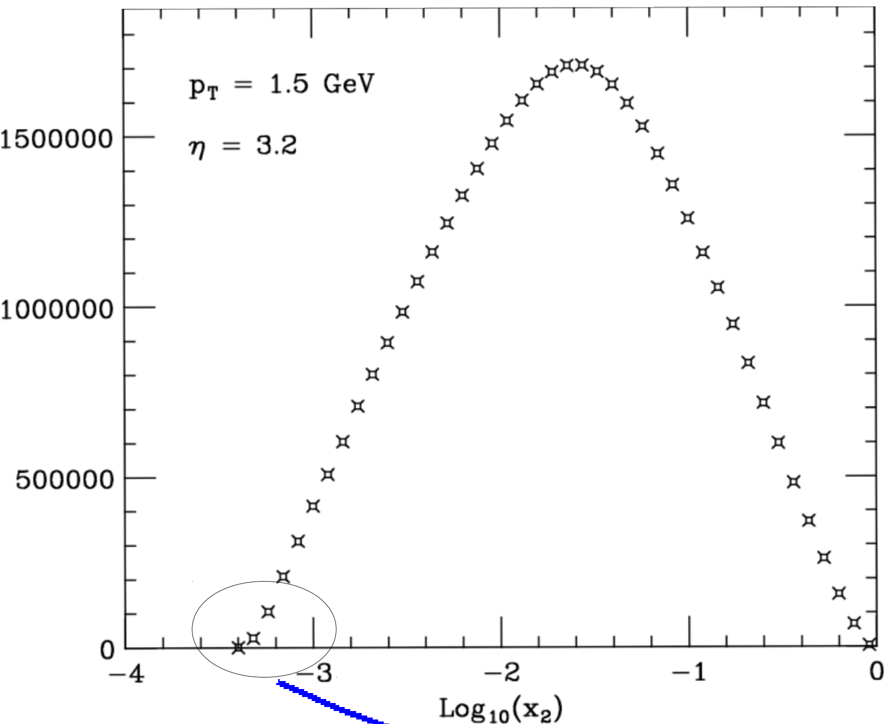
jet physics, high p_t (polar and azimuthal) angular correlations

cold matter energy loss, spin physics, ...

Pion production at RHIC: <u>kinematics</u>

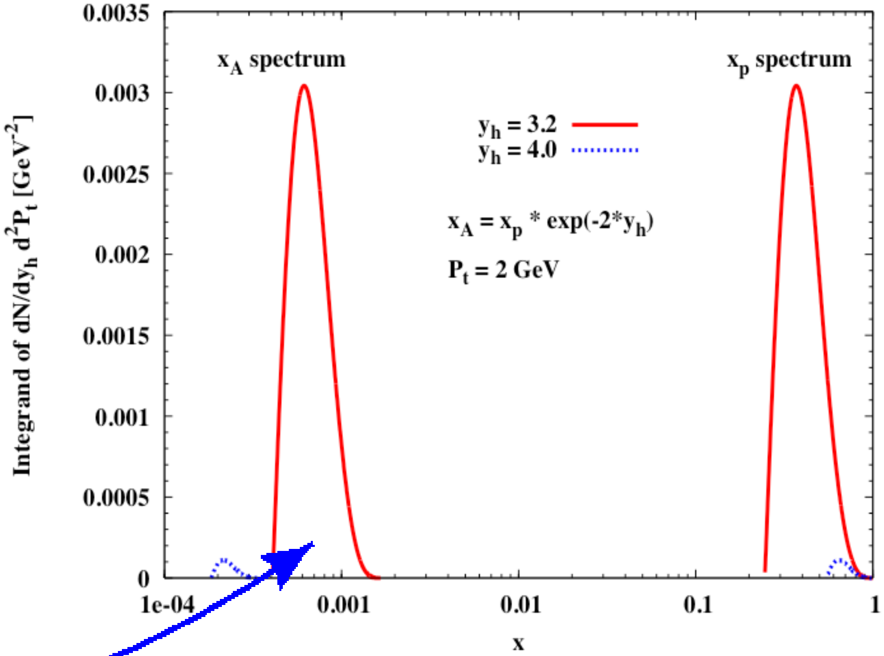
collinear factorization

GSV, PLB603 (2004) 173-183



CGC

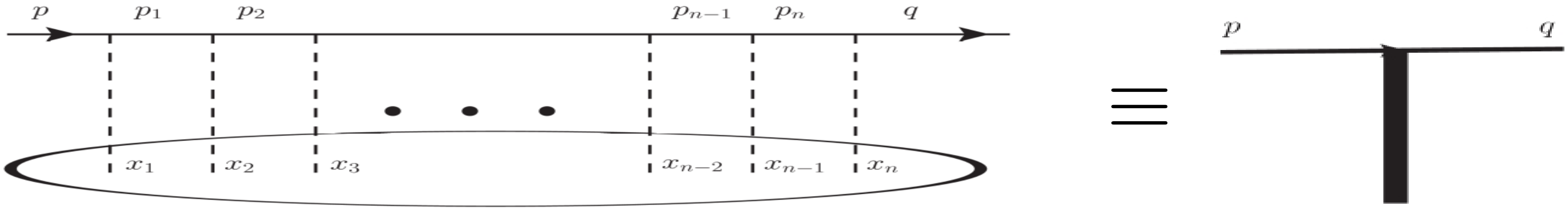
DHJ, NPA765 (2006) 57-70



$$\int_{x_{min}}^{1} dx\, xG(x,Q^2) \cdots\cdots \longrightarrow x_{min} G(x_{min},Q^2) \cdots$$

this is an extreme approximation with severe consequences!

CGC: tree level (eikonal approximation)



$$i\mathcal{M}(p,q) = 2\pi\delta(p^+ - q^+)\bar{u}(q)\not{n}\int d^2x_t\, e^{-i(q_t-p_t)\cdot x_t}\,[V(x_t) - 1]\, u(p)$$

with $V(x_t) \equiv \hat{P}\exp\left\{ig\int_{-\infty}^{+\infty} dx^+ S_a^-(x^+, x_t)t_a\right\}$

Dipole: DIS, proton-nucleus collisions

$$< Tr\, V(x_\perp)\, V^\dagger(y_\perp) >$$

scattering from small x gluons of the target can cause only a *small angle deflection*

$$S_a^-(k^+ \sim 0, k^-/\sqrt{s} \ll 1, k_t \sim Q_s)$$

toward precision: NLO evolution

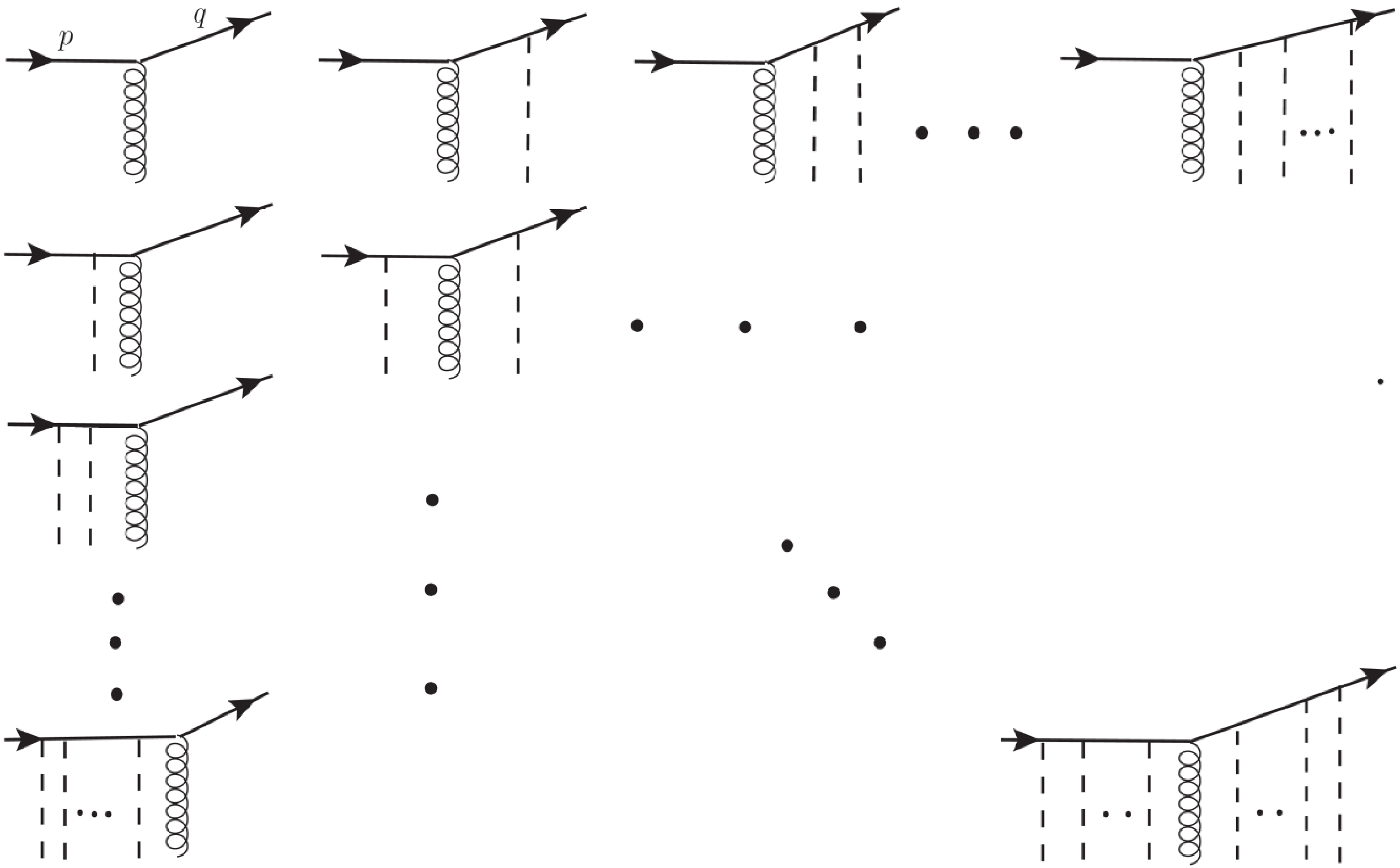
beyond eikonal approximation: tree level

large x partons of target: can cause a large-angle deflection of the quark (high q_t)

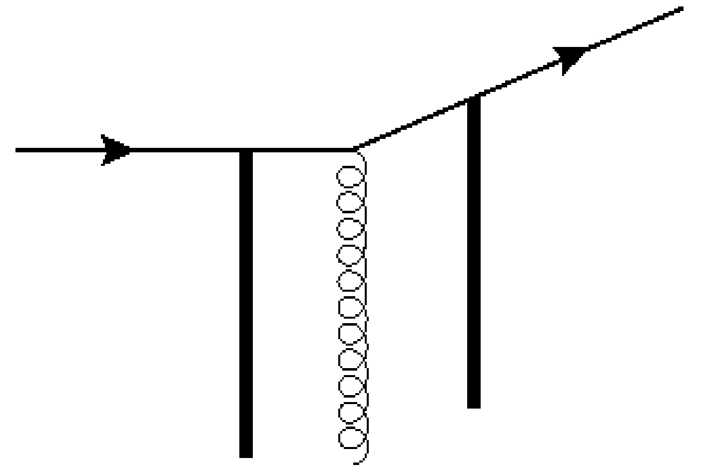
$$A_a^\mu(x^+, x^-, x_\perp)$$

include eikonal multiple scatterings before and after the hard scattering

$$A_a^\mu = S_a^\mu + (A_a^\mu - S_a^\mu) \qquad S_a^\mu = n^\mu \, S_a(x^+, x_t)$$



summing all the terms gives:



$$i\mathcal{M}_1 \quad = \quad \int d^4x\, d^2z_t\, d^2\bar{z}_t \int \frac{d^2k_t}{(2\pi)^2}\,\frac{d^2\bar{k}_t}{(2\pi)^2}\, e^{i(\bar{k}-k)x}\, e^{-i(\bar{q}_t-\bar{k}_t)\cdot\bar{z}_t}\, e^{-i(k_t-p_t)\cdot z_t}$$

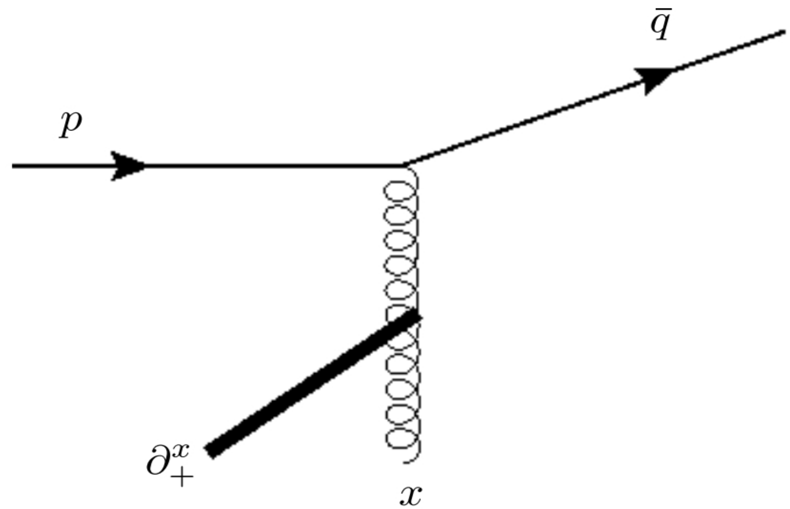
$$\bar{u}(\bar{q})\,\left[\overline{V}_{AP}(x^+,\bar{z}_t)\,\not{\bar{n}}\,\frac{\not{\bar{k}}}{2\bar{k}^+}\,\left[ig\not{A}(x)\right]\,\frac{\not{k}}{2k^+}\,\not{n}\,V_{AP}(z_t,x^+)\right]\,u(p)$$

with

$$\overline{V}_{AP}(x^+,\bar{z}_t) \equiv \hat{P}\,\exp\left\{ig\int_{x^+}^{+\infty} d\bar{z}^+\,\bar{S}_a^-(\bar{z}_t,\bar{z}^+)\,t_a\right\}$$

$$V_{AP}(z_t,x^+) \equiv \hat{P}\,\exp\left\{ig\int_{-\infty}^{x^+} dz^+\,S_a^-(z_t,z^+)\,t_a\right\}$$

all re-scatterings of hard gluon can be re-summed

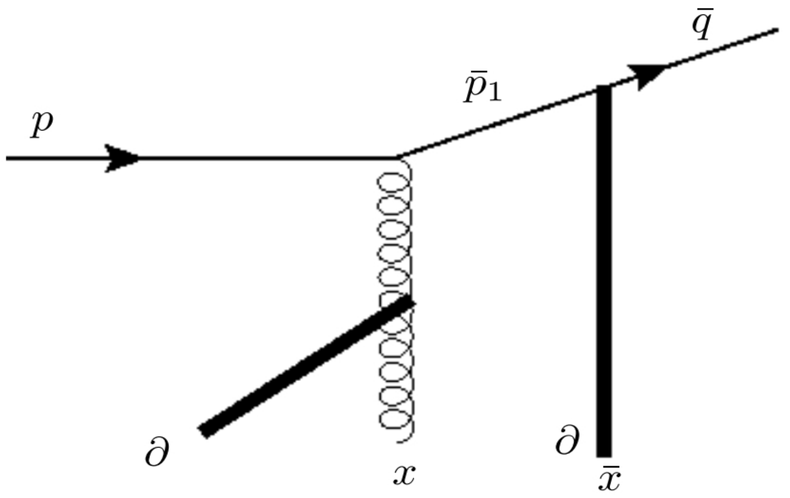


$$i\mathcal{M}_2 \quad = \quad \frac{2\,i}{(p-\bar{q})^2}\int d^4x\, e^{i(\bar{q}-p)x}\,\bar{u}(\bar{q})\Big[(ig\,t^a)\,\left[\partial_{x^+}U^{\dagger}_{AP}(x_t,x^+)\right]^{ab}$$

$$\left[n\cdot(p-\bar{q})\not{A}_b(x) - (p-\bar{q})\cdot A_b(x)\,\not{n}\right]\Big]\,u(p)$$

with $$U_{AP}(x_t,x^+) \equiv \hat{P}\,exp\left\{ig\int_{-\infty}^{x^+} dz^+\,S_a^-(z^+,x_t)\,T_a\right\}$$

Re-scatterings of hard gluon and final state quark re-sum to



$$
\begin{aligned}
i\mathcal{M}_3 \;&=\; -2\,i\int d^4x\,d^2\bar{x}_t\,d\bar{x}^+\,\frac{d^2\bar{p}_{1t}}{(2\pi)^2}\,e^{i(\bar{q}^+-p^+)x^-}\;e^{-i(\bar{p}_{1t}-p_t)\cdot x_t}\;e^{-i(\bar{q}_t-\bar{p}_{1t})\cdot\bar{x}_t} \\
&\bar{u}(\bar{q})\Bigg[\left[\partial_{\bar{x}^+}\,\overline{V}_{AP}(\bar{x}^+,\bar{x}_t)\right]\,\not{n}\,\not{\bar{p}}_1\,(igt^a)\,\left[\partial_{x^+}\,U^\dagger_{AP}(x_t,x^+)\right]^{ab} \\
&\frac{\left[n\cdot(p-\bar{q})\not{A}^b(x)-(p-\bar{p}_1)\cdot A^b(x)\,\not{n}\right]}{\left[2n\cdot\bar{q}\,2n\cdot(p-\bar{q})\,p^- - 2n\cdot(p-\bar{q})\,\bar{p}_{1t}^2-2n\cdot\bar{q}\,(\bar{p}_{1t}-p_t)^2\right]}\Bigg]\,u(p)
\end{aligned}
$$

cross section: $|\mathbf{i}\mathcal{M}|^{\mathbf{2}} = |\mathbf{i}\mathcal{M}_{\mathbf{eik}} + \mathbf{i}\mathcal{M}_{\mathbf{1}} + \mathbf{i}\mathcal{M}_{\mathbf{2}} + \mathbf{i}\mathcal{M}_{\mathbf{3}}|^{\mathbf{2}}$

soft (eikonal) limit: $i\mathcal{M} \longrightarrow i\mathcal{M}_{eik}$

spinor helicity formalism: light-front spinors

spin asymmetries

$$|i\mathcal{M}_2^+|^2 \sim g^2 \frac{q^+}{p^+} \frac{1}{q_\perp^4} \int d^4x \, d^4y \, e^{i(q^+-p^+)(x^- - y^-)} \, e^{-i(q_t - p_t)\cdot(x_t - y_t)}$$
$$\Big\{ \Big[(p^+ - q^+)^2 \, q_\perp^2 \, A_\perp^b(x) \cdot A_\perp^c(y) + 4\, p^+ q^+ \, q_\perp \cdot A_\perp^b(x) \, q_\perp \cdot A_\perp^c(y) \Big]$$
$$+ \, \mathbf{i} \, \epsilon^{ij} \, [(p^+)^2 - (q^+)^2] \Big[q_i \, A_j^b(x) \, q_\perp \cdot A_\perp^c(y) - q_i \, A_j^c(y) \, q_\perp \cdot A_\perp^b(x) \Big] \Big\}$$
$$\big[\partial_{y^+} U_{AP}\big]^{ca} \big[\partial_{x^+} U_{AP}^\dagger\big]^{ab}$$

$$|i\mathcal{M}_2^-|^2 = \left(|i\mathcal{M}_2^+|^2\right)^\star \longrightarrow \mathbf{d}\sigma^{++} - \mathbf{d}\sigma^{--} \neq \mathbf{0}$$

this is zero in CGC

azimuthal asymmetries

rapidity loss

SUMMARY

CGC is a systematic approach to high energy collisions

- ***strong hints from RHIC, LHC,...***
- ***connections to TMD,...***
- ***toward precision: NLO,...***

CGC breaks down at large x (high p_t)

- ***a significant portion of EIC phase space is at large x***
- ***transition from DGLAP physics to CGC***

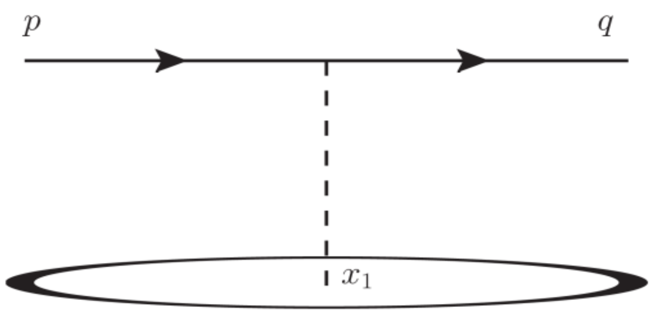
Toward a unified formalism:

- ***particle production in both small and large x (p_t) kinematics***
- ***spin, azimuthal asymmetries in intermediate p_t region***
- ***one-loop correction to cross section: from JIMWLK to DGLAP ?***

$$i\mathcal{M}_1 = (ig)\int d^4x_1\, e^{i(q-p)x_1}\, \bar{u}(q)\, \left[\not{n}\, S(x_1)\right]\, u(p)$$

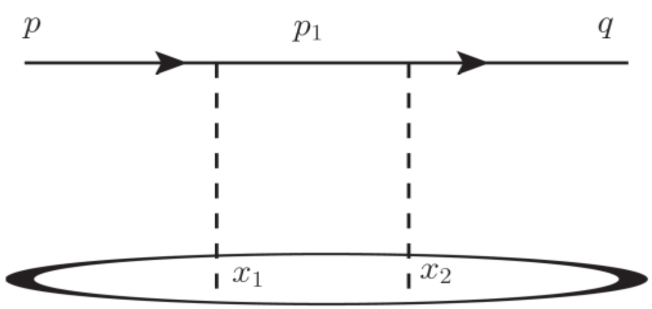
$$= (ig)(2\pi)\delta(p^+ - q^+)\int d^2x_{1t}\, dx_1^+\, e^{i(q^- - p^-)x_1^+}\, e^{-i(q_t - p_t)x_{1t}}$$

$$\bar{u}(q)\left[\not{n}\, S(x_1^+, x_{1t})\right] u(p)$$



$$i\mathcal{M}_2 = (ig)^2\int d^4x_1\, d^4x_2\int \frac{d^4p_1}{(2\pi)^4}\, e^{i(p_1-p)x_1}\, e^{i(q-p_1)x_2}$$

$$\bar{u}(q)\left[\not{n}\, S(x_2)\,\frac{i\not{p}_1}{p_1^2 + i\epsilon}\,\not{n}\, S(x_1)\right] u(p)$$



$$\int \frac{dp_1^-}{(2\pi)}\frac{e^{ip_1^-(x_1^+ - x_2^+)}}{2p^+\left[p_1^- - \frac{p_{1t}^2 - i\epsilon}{2p^+}\right]} = \frac{-i}{2p^+}\,\theta(x_2^+ - x_1^+)\, e^{i\frac{p_{1t}^2}{2p^+}(x_1^+ - x_2^+)}$$

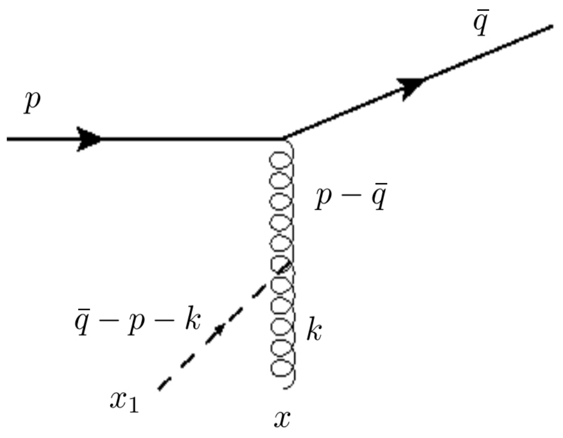
contour integration over the pole leads
to path ordering of scattering

ignore all terms: $O(\frac{p_t}{p^+}, \frac{q_t}{q^+})$ and use $\not{n}\,\frac{\not{p}_1}{2n\cdot p}\,\not{n} = \not{n}$

$$i\mathcal{M}_2 = (ig)^2\,(-i)(i)\,2\pi\delta(p^+ - q^+)\int dx_1^+\, dx_2^+\,\theta(x_2^+ - x_1^+)\int d^2x_{1t}\, e^{-i(q_t - p_t)\cdot x_{1t}}$$

$$\bar{u}(q)\,\left[S(x_2^+, x_{1t})\,\not{n}\, S(x_1^+, x_{1t})\right]\, u(p)$$

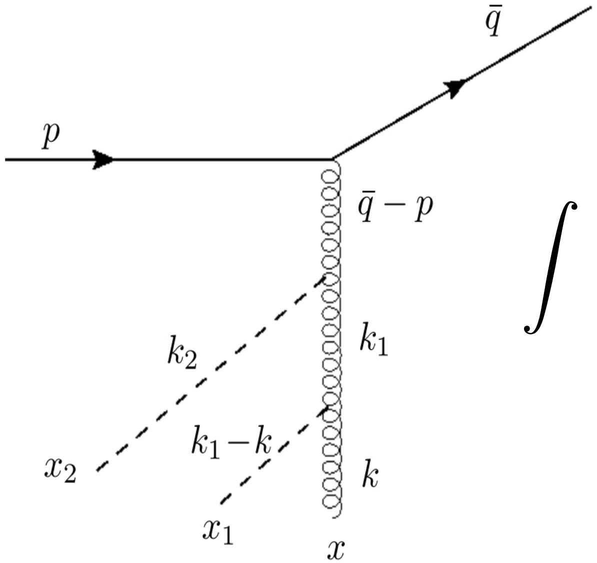
interactions of large and small x modes



$$\begin{aligned} i\mathcal{M} &= f_{acd} \int \frac{d^4k}{(2\pi)^4}\, d^4x\, d^4x_1\, e^{i(\bar{q}-p-k)x_1}\, e^{ikx} \\ &\quad \bar{u}(\bar{q})\,(ig\,\gamma^\mu\, t^a)\, u(p)\, A^c_\lambda(x)\ \left[i\, g\, S^d(x_1)\right] \\ &\quad \frac{1}{(p-\bar{q})^2 + i\epsilon} \Bigg[-g^\mu_\lambda\, n\cdot(p-\bar{q}-k) \\ &\quad + n^\mu \left[p_\lambda - \bar{q}_\lambda\, (1 - \frac{n\cdot k}{n\cdot(p-\bar{q})}) \right] \Bigg] \end{aligned}$$

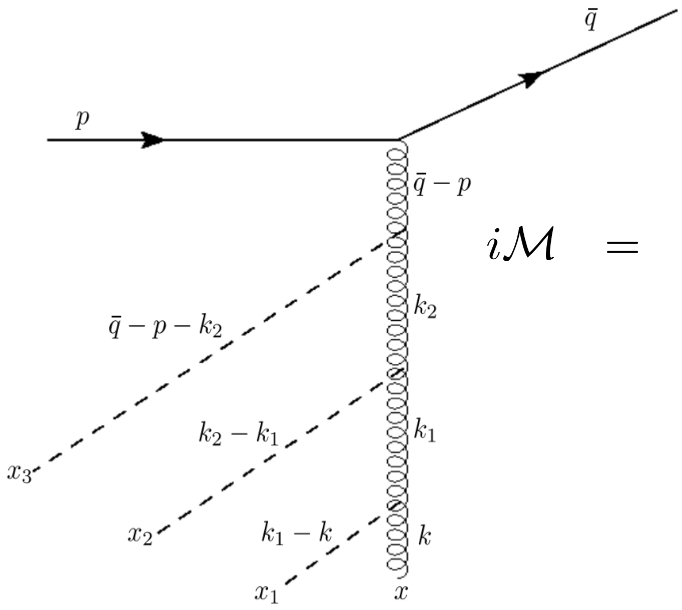
performing k^- integration sets $x_1^+ = x^+$

$$\begin{aligned} i\mathcal{M} &= 2 f_{acd} \int d^4x\, e^{i(\bar{q}-p)x} \\ &\quad \bar{u}(\bar{q})\, \frac{\left[\not{n}\,(p-\bar{q})\cdot A_c(x) - \not{A}_c(x)\, n\cdot(p-\bar{q})\right]}{(p-\bar{q})^2}\, (ig\, t^a)\, u(p)\ \left[i\, g\, S^d(x^+, x_t)\right] \end{aligned}$$



$$\int \frac{dk_1^-}{(2\pi)} \frac{e^{ik_1^-(x^+-x_2^+)}}{2(\bar{q}^+-p^+)\left[k_1^- - \frac{k_{1t}^2-i\epsilon}{2(\bar{q}^+-p^+)}\right]} \sim \theta(x^+-x_2^+)$$

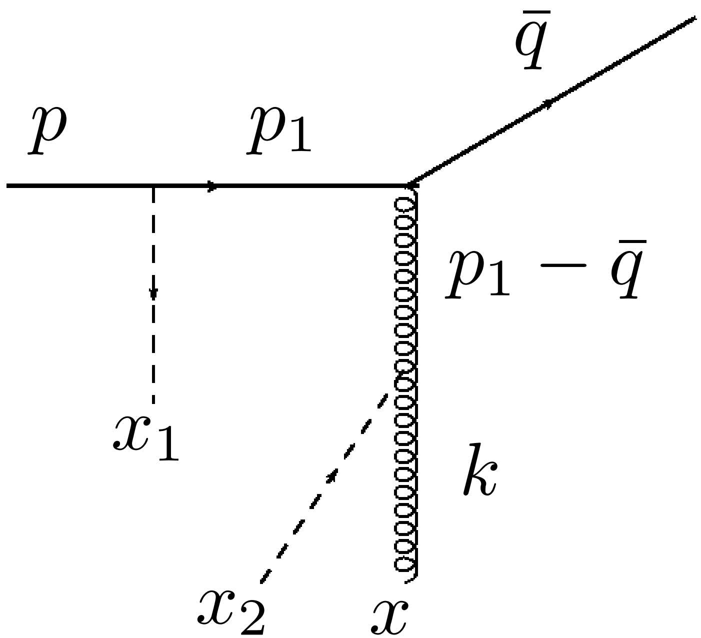
$$\begin{aligned} i\mathcal{M} &= 2\, f_{abc}\, f_{cde} \int d^4x\, dx_2^+\, \theta(x^+-x_2^+)\, e^{i(\bar{q}^+-p^+)x^- - i(\bar{q}_t-p_t)\cdot x_t} \\ & \bar{u}(\bar{q})\, \frac{\left[\not{n}\,(p-\bar{q})\cdot A_e(x) - \not{A}_c(x)\, n\cdot(p-\bar{q})\right]}{(p-\bar{q})^2}\, (ig\, t^a)\, u(p) \\ & \left[i\, g\, S_d(x^+, x_t)\right]\ \left[i\, g\, S_b(x_2^+, x_t)\right] \end{aligned}$$



$$
\begin{aligned}
i\mathcal{M} \quad = \quad & \frac{2(i)^2}{(\bar{q}-p)^2}\, f^{abc} f^{cde} f^{egf} \int d^4x\, dx_2^+ dx_3^+\, \theta(x^+ - x_2^+)\,\theta(x_2^+ - x_3^+) \\
& \bar{u}(\bar{q})\,(ig\,t^a)\left[n\cdot(p-\bar{q})\not{A}_f(x) - (p-\bar{q})\cdot A_f(x)\not{n}\right] u(p) \\
& \left[i\,g\,S_g(x^+, x_t)\right]\,\left[i\,g\,S_d(x_2^+, x_t)\right]\,\left[i\,g\,S_b(x_3^+, x_t)\right] \\
& e^{i(\bar{q}^+ - p^+)x^- - i(\bar{q}_t - p_t)\cdot x_t}
\end{aligned}
$$

recall

$$
\begin{aligned}
\partial_{x^+}\left[U_{AP}^\dagger(x_t, x^+)\right]^{ab} \quad = \quad & (if^{bca})\,[igS_c(x^+, x_t)] \\
+ \quad & (if^{bce})\,(if^{eda}) \int dx_1^+ \theta(x^+ - x_1^+)[[igS_c(x^+, x_t)]\,[igS_d(x_1^+, x_t)] \\
+ \quad & (if^{bch})\,(if^{gdf})\,(if^{fea}) \int dx_1^+\, dx_2^+\, \theta(x^+ - x_1^+)\,\theta(x_1^+ - x_2^+) \\
& [[igS_c(x^+, x_t)]\,[igS_d(x_1^+, x_t)]\,[[igS_c(x_2^+, x_t)] + \cdots\cdots
\end{aligned}
$$



both initial state quark and hard gluon interacting:

integration over p_1^-

$$\int \frac{dp_1^-}{2\pi} \frac{e^{ip_1^-(x_1^+ - x^+)}}{[p_1^2 + i\epsilon]\,[(p_1-\bar{q})^2 + i\epsilon]}$$

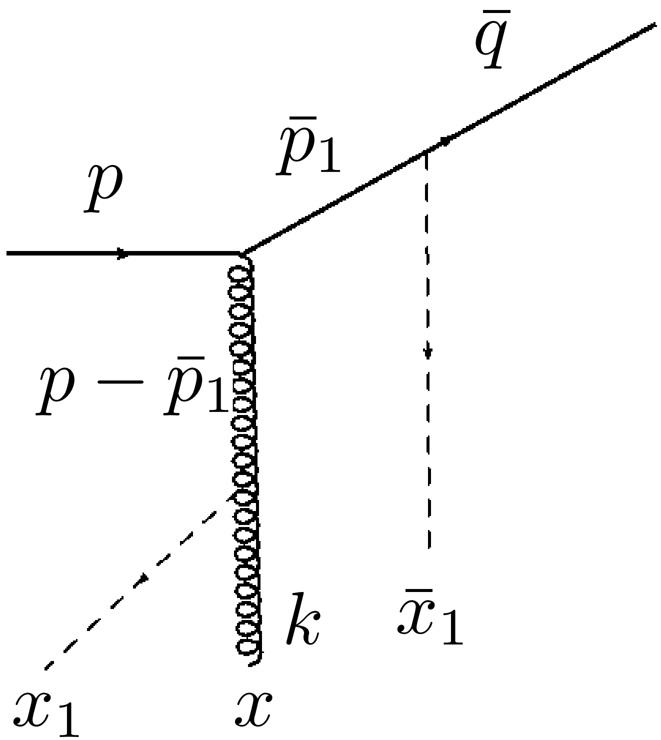
both poles are below the real axis, we get

$$\frac{e^{i\frac{p_{1t}^2}{2p^+}(x_1^+ - x^+)}}{\left[\frac{p_{1t}^2}{2p^+} - \bar{q}^- - \frac{(p_{1t}-\bar{q}_t)^2}{2(p^+ - \bar{q}^+)}\right]} + \frac{e^{i\left[\bar{q}^- + \frac{(p_{1t}-\bar{q}_t)^2}{2(p^+ - \bar{q}^+)}\right](x_1^+ - x^+)}}{\left[\bar{q}^- + \frac{(p_{1t}-\bar{q}_t)^2}{2(p^+ - \bar{q}^+)} - \frac{p_{1t}^2}{2p^+}\right]}$$

ignoring phases we get a cancellation!

this can be shown to hold to all orders whenever both initial state quark and hard gluon scatter from the soft fields!

how about the final state quark interactions?



integration over $\bar{p}_1^-$

$$\int \frac{d\bar{p}_1^-}{2\pi} \frac{e^{i\bar{p}_1^-(\bar{x}_1^+ - x^+)}}{[\bar{p}_1^2 + i\epsilon]\ [(p_1 - \bar{p}_1)^2 + i\epsilon]}$$

now the poles are on the opposite side of the real axis, we get both ordering

$$\theta(x^+ - \bar{x}_1^+) \text{ and } \theta(\bar{x}_1^+ - x^+)$$

ignoring the phases the contribution of the two poles add!

path ordering is lost!

however further re-scatterings are still path-ordered before/after $\mathbf{x_1^+}, \bar{\mathbf{x}}_1^+$