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Improved opacity expansion for in-medium parton splitting

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In collaboration with Konrad Tywoniuk

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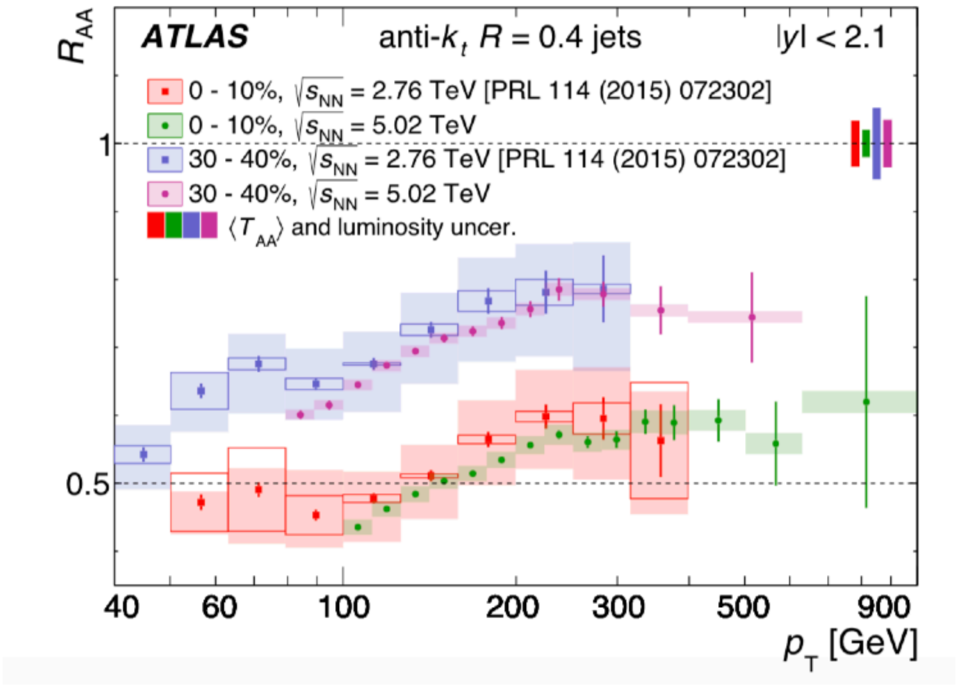
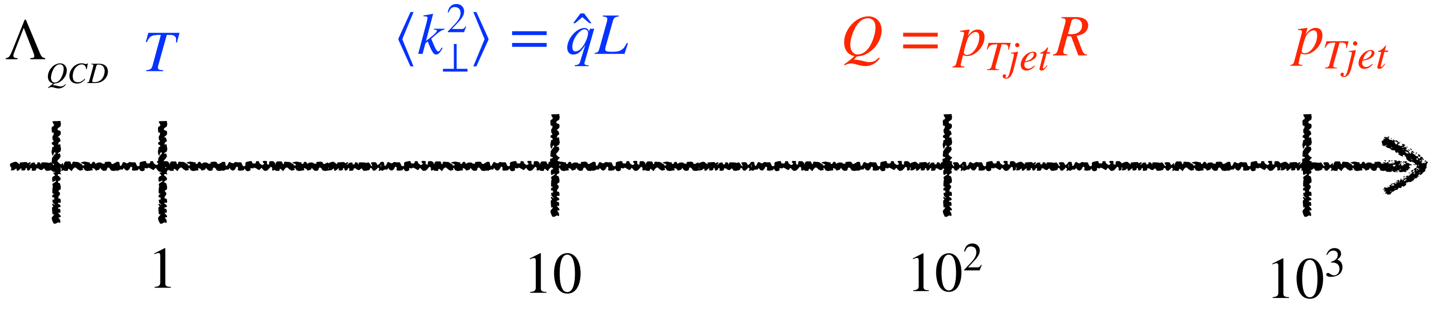
Motivation

- Use jets as test particles to learn about the transport properties and non-equilibrium dynamics of the QGP
- Jets are complex multi-scale quantum systems. Many physics processes at play: QCD collinear cascade, multiple scattering, medium-induced radiation, medium response, interferences, etc
- For quantitative comparison to data we need better control on theoretical uncertainties

Inclusive jets



PLB 790 (2019)



Weak coupling picture of jet quenching in a nutshell

- Momentum broadening in the QGP: diffusion in transverse momentum space

$$\hat{q} \equiv \frac{\mathrm{d}\langle k_T^2 \rangle_{typ}}{\mathrm{d}t} \sim \alpha_s^2\, C_R\, n\, \ln \frac{Q^2}{m_D^2} \sim \alpha_s^2 T^3$$

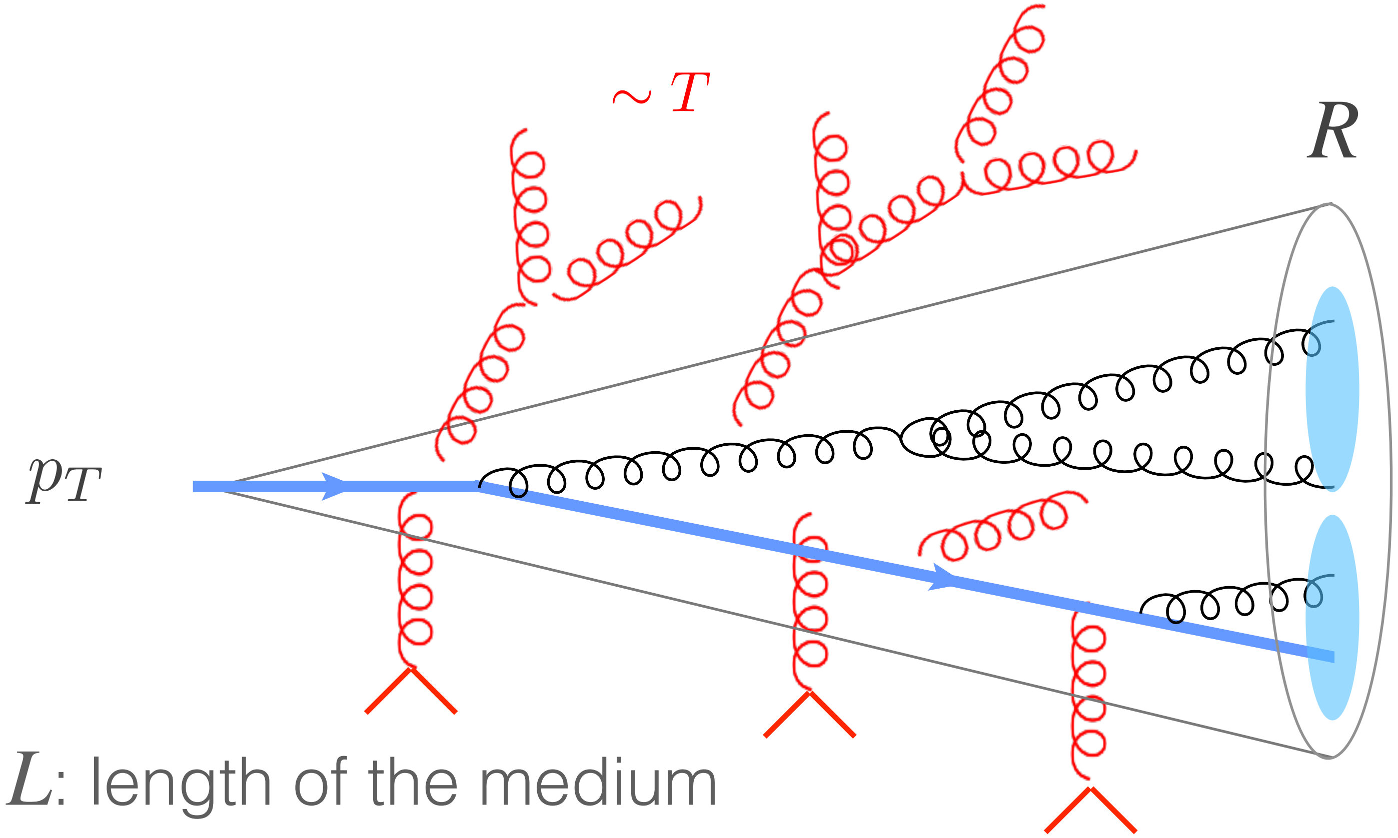
[Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000) Zakharov (1996) Wiedemann (2001) Gyulassy, Levai, Vitev (2001) Arnold, Moore, Yaffe (2002)

- Multiple scattering trigger abundant soft gluon radiation (requires resummation)

- Large angle turbulent cascade
  (constant flow of energy from p_T to T

→ minijet thermalization

[Blaizot, Iancu, MT (2013), Iancu, Wu (2015)]



L: length of the medium

Typical energy loss

$$\langle E \rangle_{typ} \sim \bar{\alpha}^2\, \hat{q}\, L^2$$

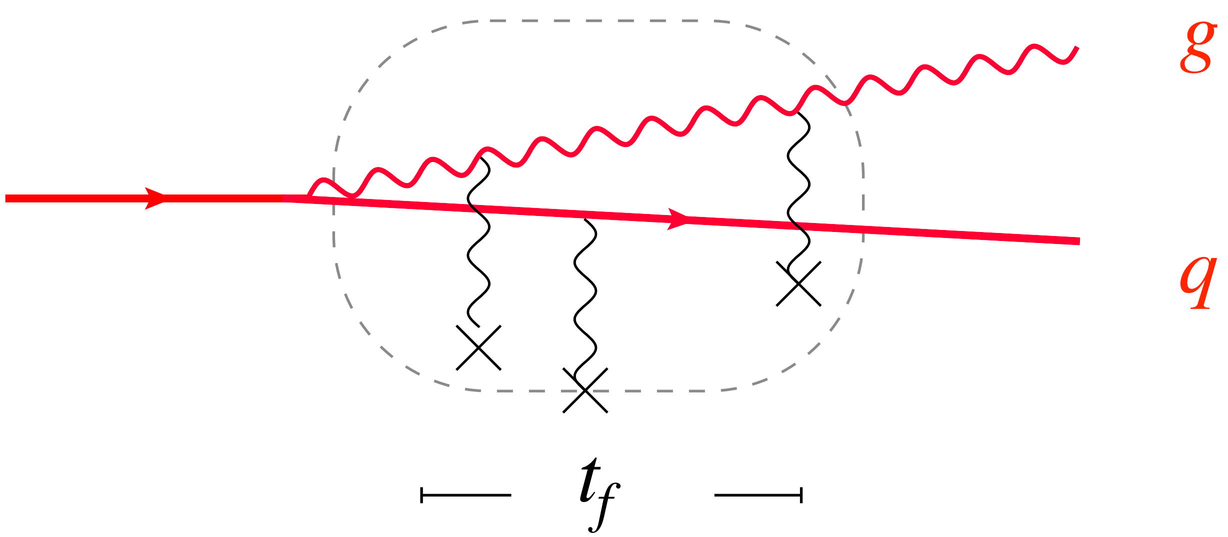
Unresolved subjects at small angles

$$\theta < \theta_c = (\hat{q}L^3)^{-1/2}$$

[Salgado, MT, Tywoniuk (2010-11), Iancu, Casalderrey-Solana (2011)]

The LPM effect on the back of the envelope

- The energy spectrum of photons caused by the propagation of a relativistic charge in a medium is suppressed due to coherence effects (**Landau-Pomeranchuk-Migdal (1953)**).
- Analog effect in QCD except the gluon interacts with the plasma and suffers "brownian kicks"



formation time

$$t_f(\omega) = \frac{\omega}{k_\perp^2}$$

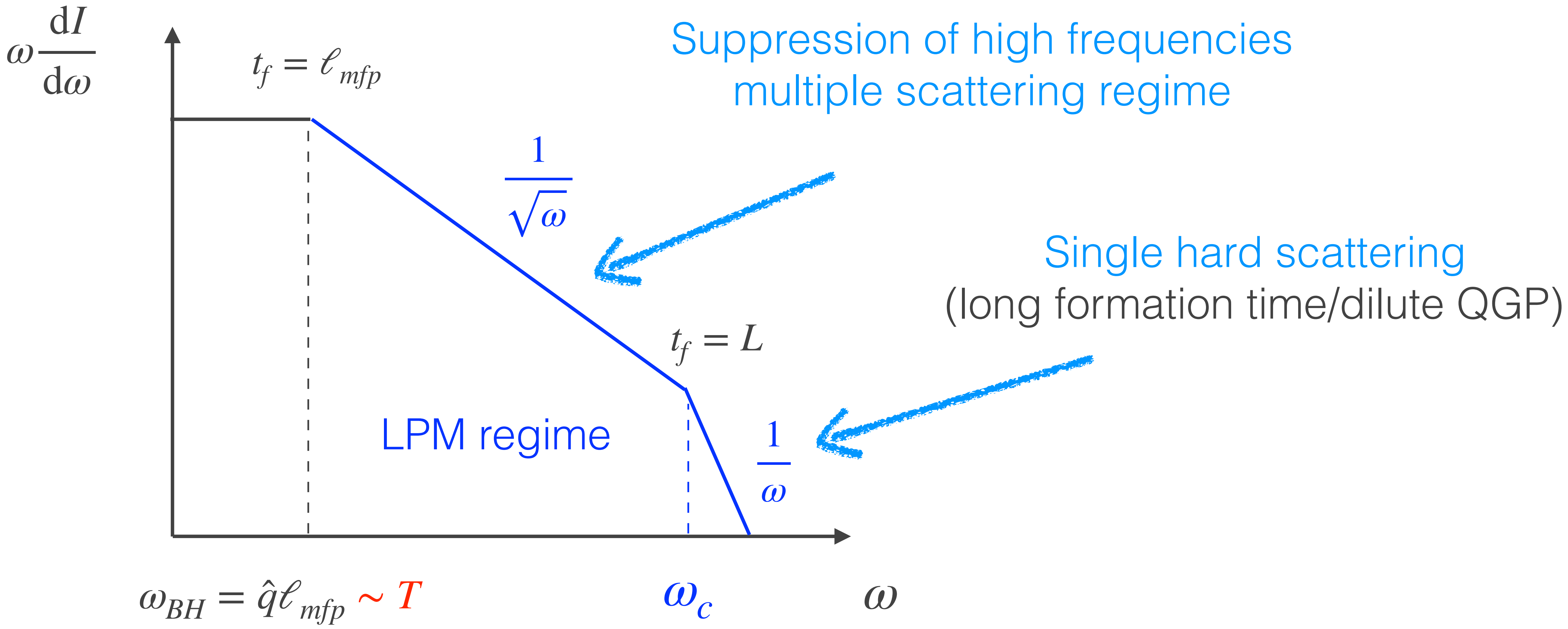
$$k_\perp^2 \sim \hat{q}\, t_f$$

- In QCD the spectrum is suppressed in the UV

$$t_f(\omega) = \sqrt{\frac{\omega}{\hat{q}}} \quad \text{and} \quad \omega \frac{\mathrm{d}I^{LPM}}{\mathrm{d}\omega} \sim \alpha_s \sqrt{\frac{\omega}{\hat{q}}}\, L \propto \frac{1}{\sqrt{\omega}}$$

Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000) Zakharov (1996)

The LPM effect on the back of the envelope



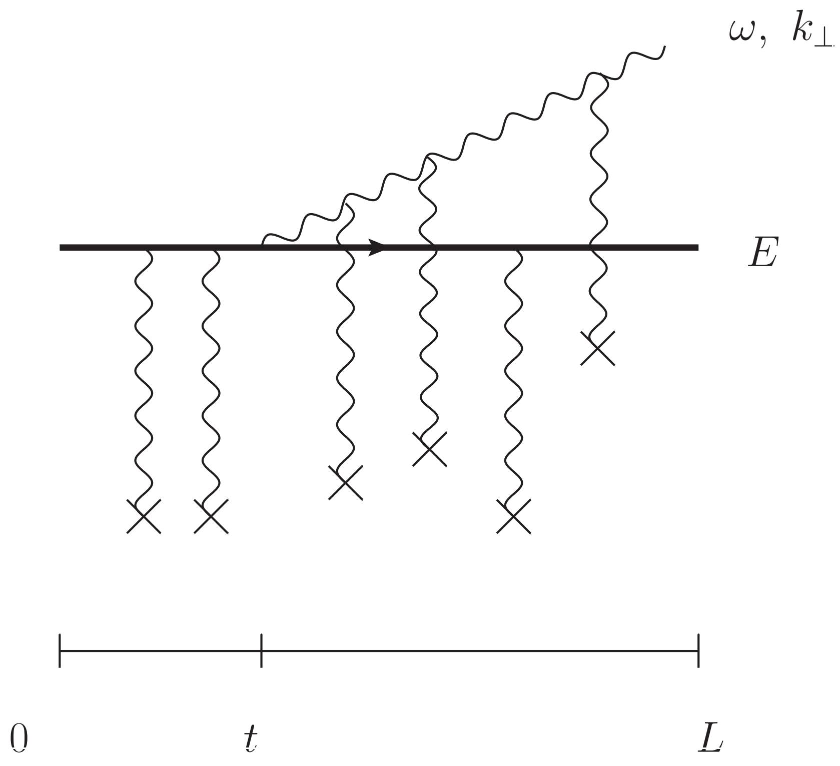
Suppression of high frequencies
multiple scattering regime

Single hard scattering
(long formation time/dilute QGP)

- Maximum radiation frequency: $\omega_c = \hat{q}L^2$
- Minimum radiation angle (no mass singularity): $\theta_c = \frac{1}{\sqrt{\hat{q}L^3}}$

Medium-induced gluon radiation spectrum

Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000) Zakharov (1996)



- Thermal medium (HTL)

$$\frac{d\sigma_{\rm el}}{d^2q_\perp} \equiv \frac{g^2 m_D^2 T}{q_\perp^2(q_\perp^2+\mu^2)}$$

Aurenche-Gelis-Zakaret (2000)

- Static scattering centers

$$\frac{d\sigma_{\rm el}}{d^2q_\perp} \equiv \frac{g^4 n}{(q_\perp^2+\mu^2)^2}$$

Gyulassy-Wang (1992)
Gyulassy-Levai-Vitev (2000)

- The two-body green's function obeys a 2+1 d Schrödinger equation

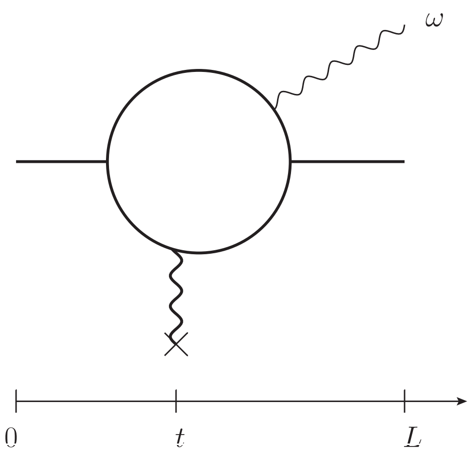
$$\left[i\frac{\partial}{\partial t}+\frac{\boldsymbol{\partial}^2}{2\omega}+i\sigma(\boldsymbol{x})\right]\mathcal{K}(\boldsymbol{x},t|\boldsymbol{y},t_1)=i\delta(\boldsymbol{x}-\boldsymbol{y})\delta(t-t_1)$$

- The imaginary potential is related to the dipole amplitude

$$\sigma(\boldsymbol{x},t)=N_c\int\frac{d^2\boldsymbol{q}}{(2\pi)^2}\frac{d\sigma_{\rm el}}{d^2\boldsymbol{q}}\left(1-e^{i\boldsymbol{q}\cdot\boldsymbol{x}}\right)\quad \sim x_\perp^2\ln\frac{1}{\mu^2x_\perp^2}$$

Medium-induced gluon radiation spectrum

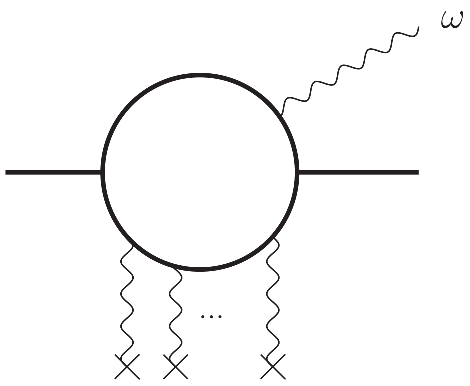
- Two analytic approximations in the literature (implemented in various MC event generators)



1. Dilute medium: single-hard scattering approximation (Opacity expansion, Higher-Twist)

Gyulassy-Levai-Vitev (2000) Guo, Wang (2000) Sievert, Vitev, Yoon, Boram (2019)

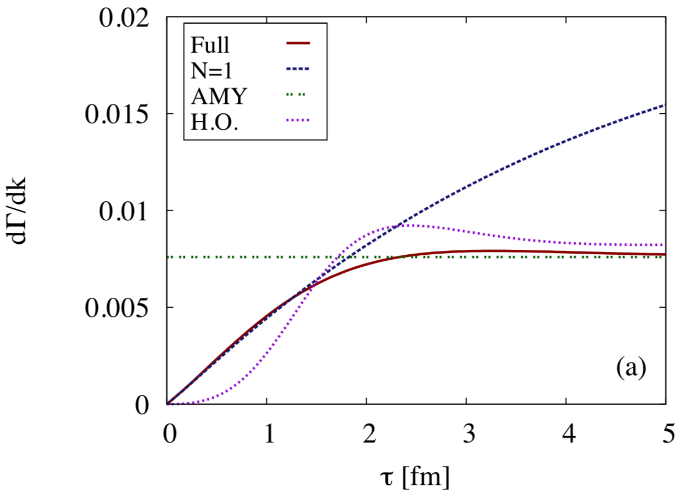
2. Dense medium: multiple-soft scattering. All order resummation by neglecting the Coulomb logarithm: Harmonic oscillator



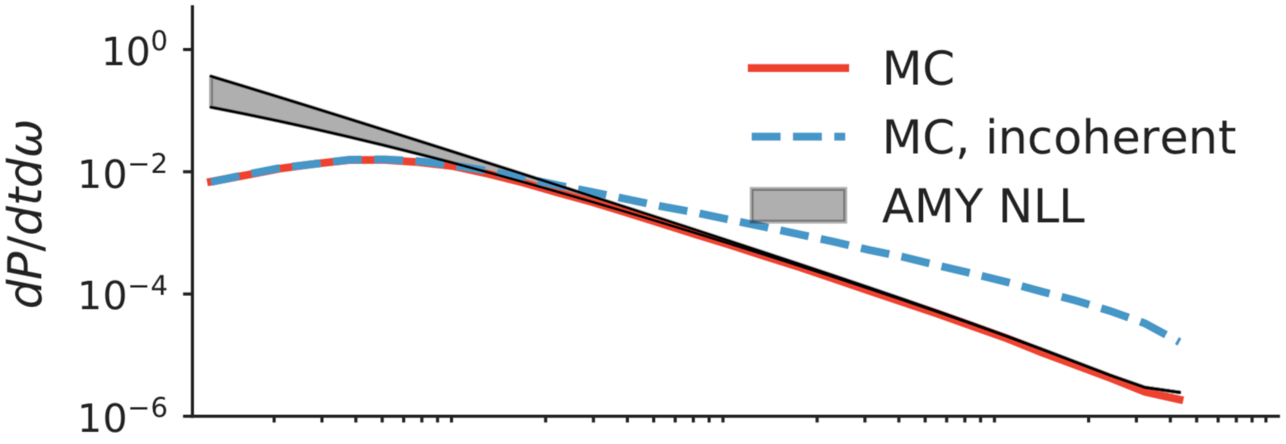
Baier, Dokshitzer, Mueller, Peigné, Schiff (1996)
Zakharov (1997) Wiedemann, Salgado (2003)

$$\sigma(x_\perp) \sim x_\perp^2$$

- Exact numerical solutions:



Caron-Huot and Gale (2010)



Ke , Xu, Bass (2018)

Improved opacity expansion (IOE)

- Extract the leading logarithm from the dipole cross-section

$$\sigma(\boldsymbol{x}) = N_c\,\boldsymbol{x}^2\left(\ln\frac{Q^2}{\mu^2}+\ln\frac{1}{\boldsymbol{x}^2Q^2}\right) \qquad\qquad Q^2 \equiv \langle x_\perp^2\rangle_{\rm HO}$$

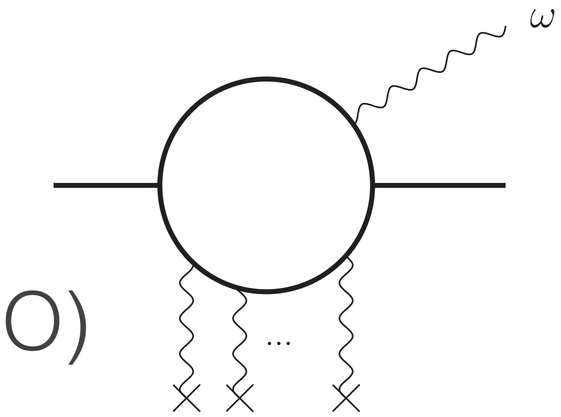
Application to momentum broadening: Molière (1948) Iancu, Itakura Triantafyllopoulos (2004)

- Perturbation around the harmonic oscillator

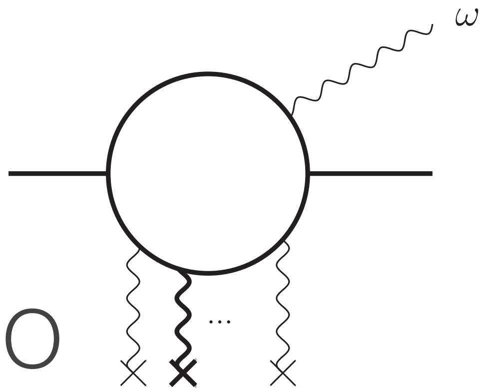
$$H \to H_{\rm HO} + H_I'$$

$$H_{\rm HO} \equiv H_0 + N_c\,\boldsymbol{x}^2\,\ln\frac{Q^2}{\mu^2} \quad \gg \quad H_I' \equiv N_c\,\boldsymbol{x}^2\,\ln\frac{1}{\boldsymbol{x}Q^2}$$

LO (HO)



NLO



- Expansion parameter $$\left(\ln\frac{Q^2}{\mu^2}\right)^{-1}$$

Improved opacity expansion (IOE)

- The radiative spectrum to NLO in the expansion around the Harmonic oscillator (LO)

$$\boxed{\omega\frac{\mathrm{d}I}{\mathrm{d}\omega} \simeq 2\,\bar{\alpha}\,\ln|\cos(\Omega L)| + \frac{1}{2}\,\bar{\alpha}\,\hat{q}_0\,\mathrm{Re}\int_0^L \mathrm{d}s\,\frac{1}{k^2(s)}\left[\ln\frac{k^2(s)}{Q^2}+\gamma\right]}$$

where

$$Q^2 \simeq \sqrt{\omega\hat{q}(Q^2)} \equiv \sqrt{\omega\hat{q}_0\ln(Q^2/\mu^2)} \simeq \sqrt{\omega\hat{q}_0\ln(\sqrt{\omega\hat{q}_0}/\mu^2)}$$

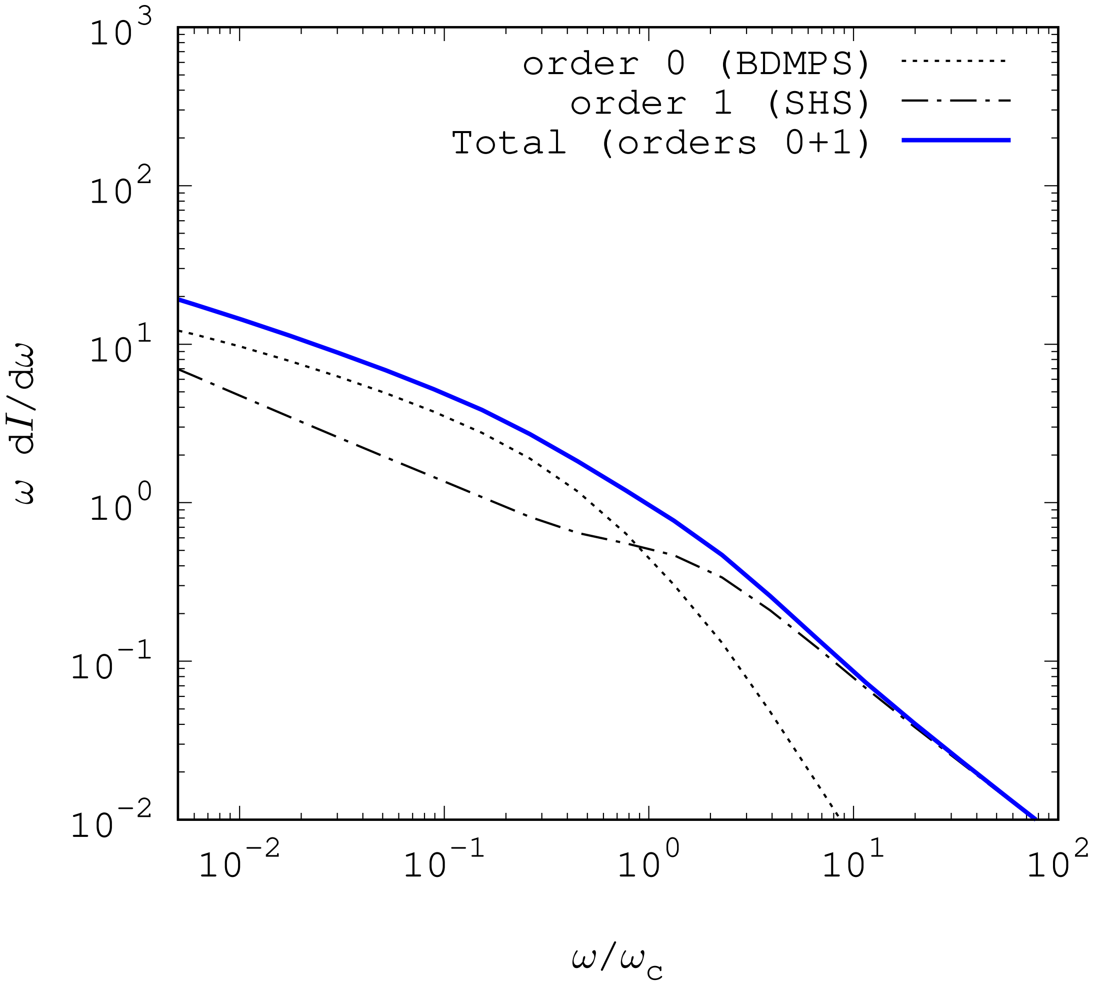
$$k^2(s) = i\frac{\omega\Omega}{2}(\cot(\Omega s) - \tan(\Omega(L-s))) \qquad \Omega \equiv \frac{1-i}{2}\sqrt{\frac{\hat{q}}{\omega}}$$

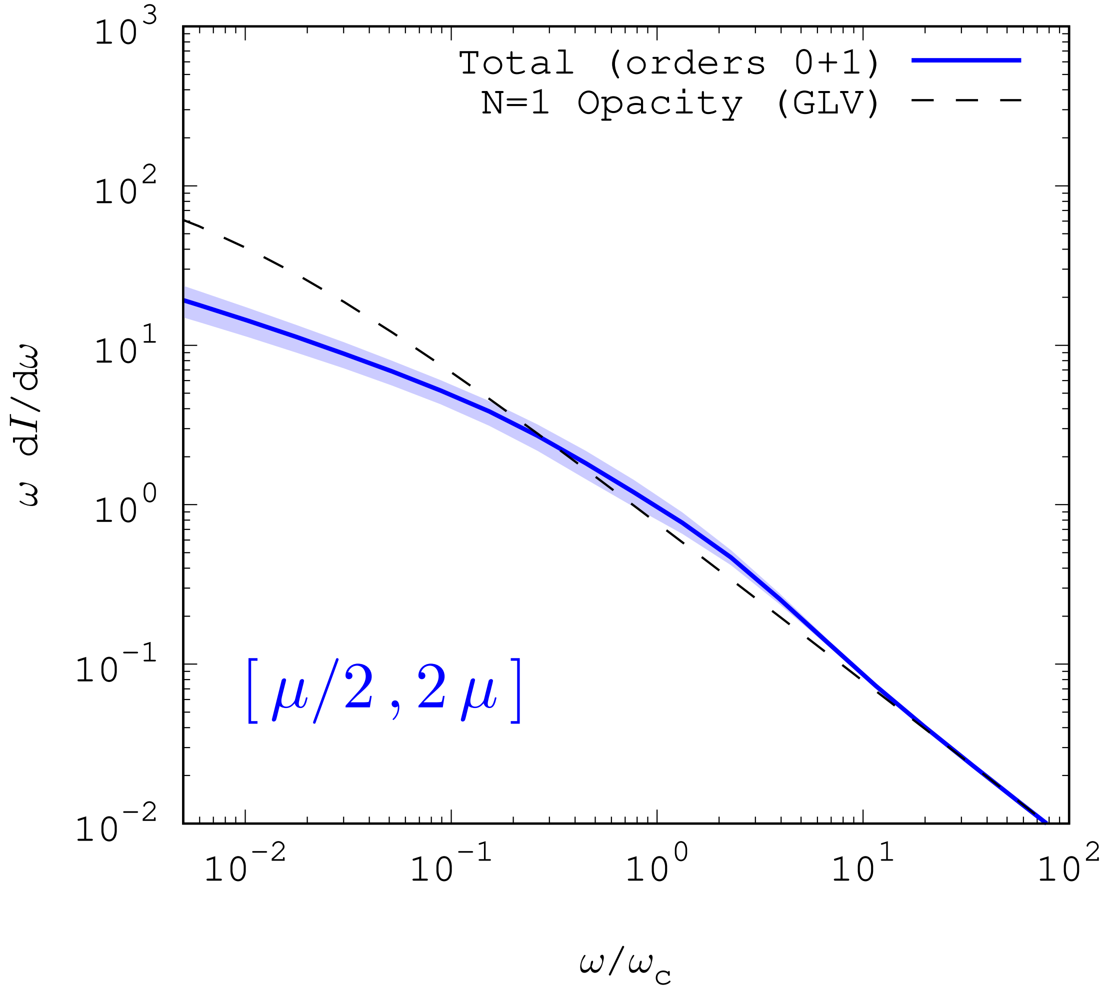
→ Encompasses the large frequency limit of N=1 opacity (GLV spectrum)

Numerical results

Medium-induced gluon spectrum for $L = 3\,fm$ and $\hat{q} \simeq 1.4\,GeV^2/fm$ ($\omega_c = nL^2 = 22\,GeV$)

arXiv: 1903.00506 [hep-ph]





- First two orders. Order zero corresponds to the BDMPS spectrum.

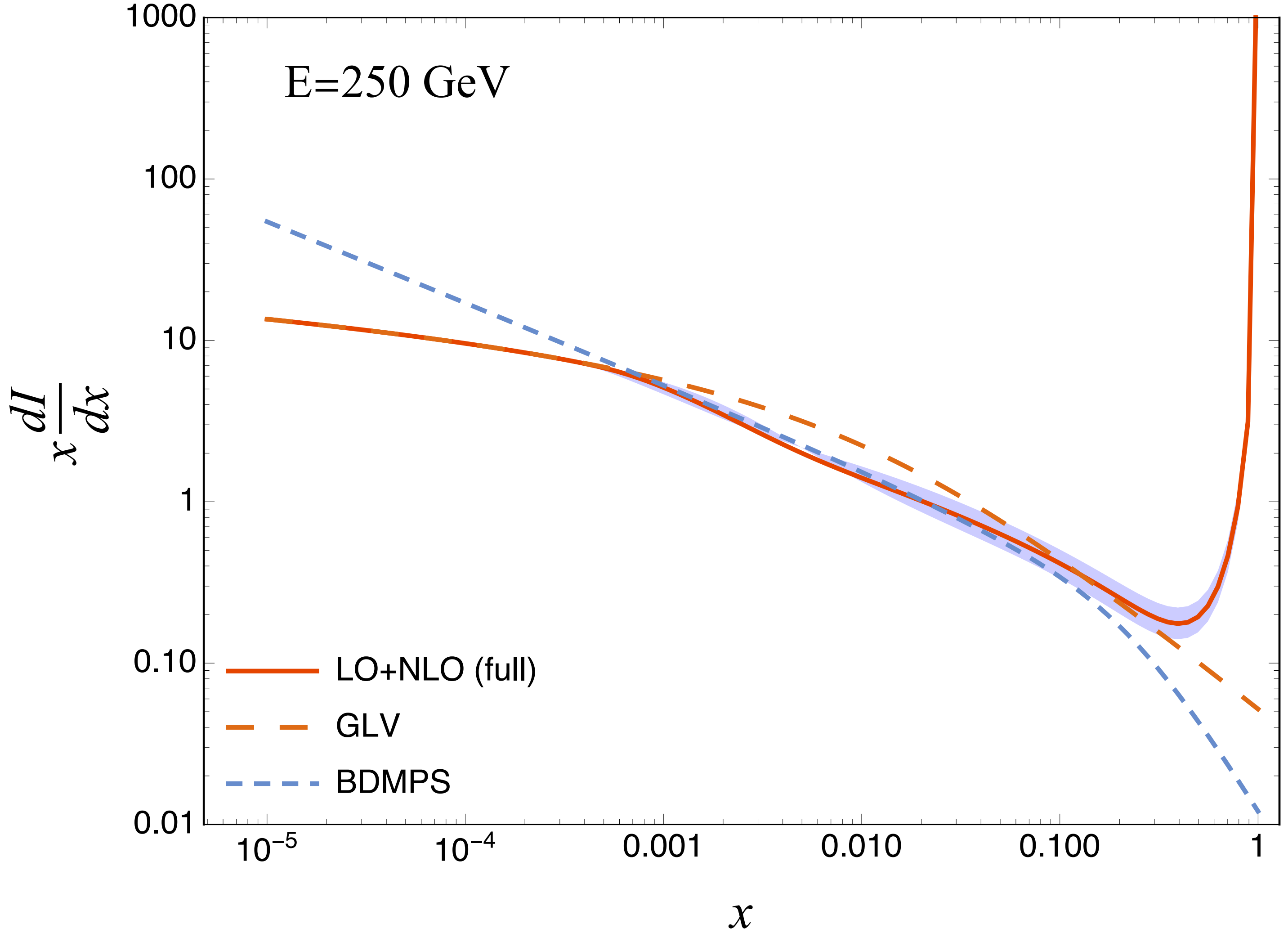
- Improved opacity expansion compared to N=1 Opacity (GLV spectrum)

Numerical results

Generalization:

i) complete splitting function at moderate x

ii) Extension to the Bethe-Heitler regime

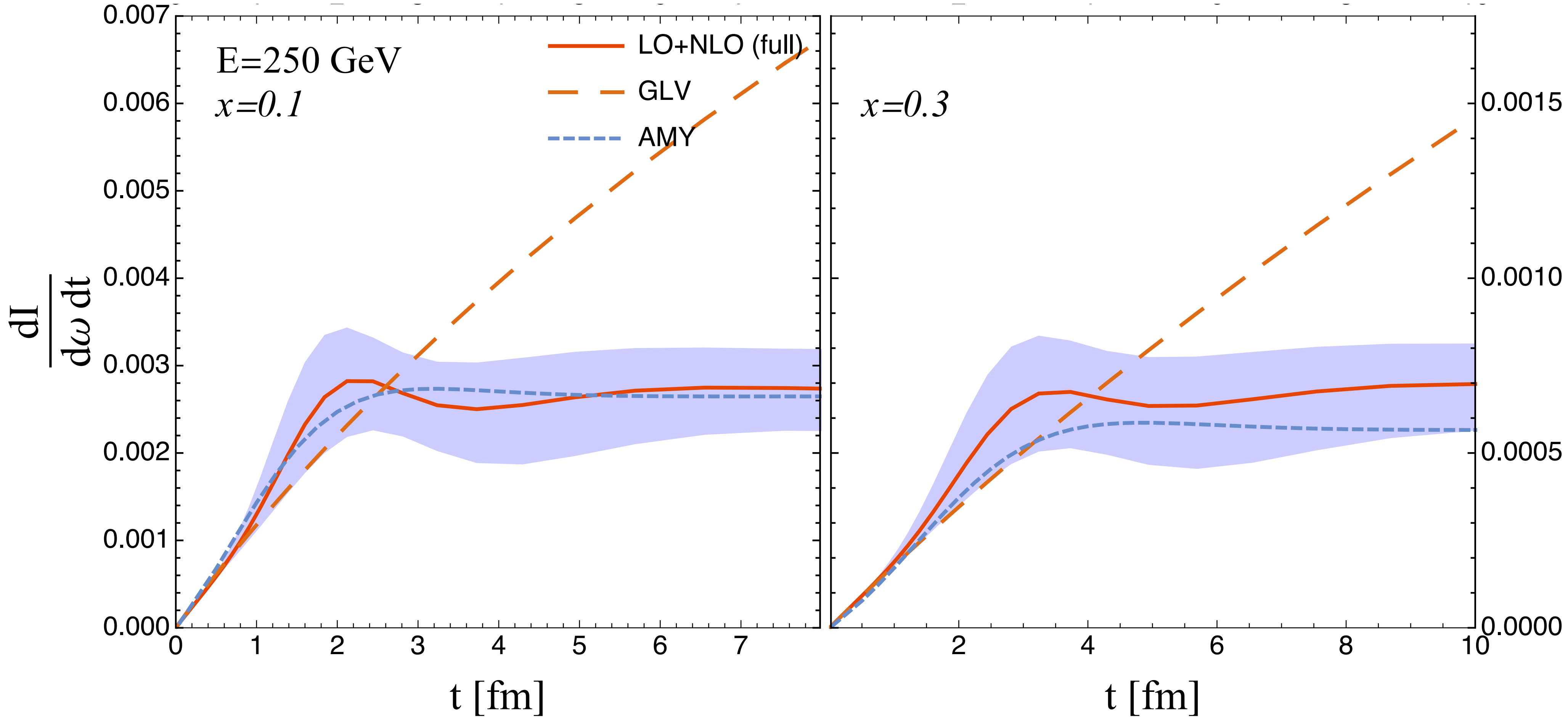


arXiv: 1910.02032 [hep-ph]

Splitting rate: IOE vs exact numerical solution

Splitting rate in the plasma comparison to the exact numerical solution by S. Caron-Huot and C. Gale, PRC 82 (2010)

(blue dashed curve, courtesy of S. Caron-Huot, C. Gale, S. Jeon and C. Park)



Medium parameter: $\hat{q} = 1.34\,\mathrm{GeV}^2/\mathrm{fm}$ and $\mu = 0.9\,\mathrm{GeV}$ corresponding to $g = 1.94$ and $T = 0.4\,\mathrm{GeV}$

Summary and outlook

- Jet quenching involves multiple parton interactions: need to develop new resummation techniques to account for high density effects as well as matching to NLO calculation for quantitative understanding of jet medium modification

- We revisited medium-induced gluon splitting by proposing a novel opacity expansion scheme based expanding around the harmonic oscillator. We have calculated the first two orders that encompass multiple-soft (IR) and single hard (UV) scattering regimes

- Under perturbative control for large media (hard scale much larger than the Debye mass)

- Outlook: generalize to transverse momentum dependent distribution. MC implementation. Phenomenology.