Quarkyonic matter and neutron star

arXiv:1908.04799 and some preliminary results

Quark matter - The 28th International Conference on Ultra-relativistic Nucleus-Nucleus Collisions

November 6, 2019, Wuhan

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Two extreme limits

- QCD phase diagram

In $T > T_c$ region, super-colliders have provided wonderful observations

In $\mu >> \Lambda_{QCD}$ region, dense QCD and effective field theory within scale hierarchy

For intermediate density region ($\mu < \Lambda_{QCD}, T < T_c$) was not fully understood because of lack of experimental observations

Figure from RevModPhys.80.1455
Motivation – astrophysical observation

• $2M_\odot$ neutron star measured from a compact binary

![Graphs showing mass and properties of neutron stars](image1)

(Science.340(2010)1233232 J. Antoniadis et al.)


• To reach such a massive state, hard enough EOS is required
  
  ▪ If new degrees of freedom (eg. hyperons) appear in the nuclear matter → matter becomes softer → maximal NS mass is bounded near $1.5M_\odot$
  
  ▪ Intrinsic strong repulsive interaction is required
Motivation – astrophysical observation

• Tidal deformability observed from GW170817 (LIGO-Virgo)

\[
\Lambda = \frac{16 \left( m_1 + 12m_2 \right) m_1^4 \Lambda_1 + \left( m_2 + 12m_1 \right) m_2^4 \Lambda_2}{(m_1 + m_2)^5}
\]

Great observation
(PRL119.161101, PRL121.161101 B.P. Abbott et al.)

\[
400 < \Lambda(1.4M\odot) < 800,
\]

\[
P(2n_0) = 3.5^{+2.7}_{-1.7} \times 10^{34}\text{dyn/cm}^2
\]

\[
P(6n_0) = 9.0^{+7.9}_{-2.6} \times 10^{34}\text{dyn/cm}^2
\]

• \(\tilde{\Lambda} < 800 \ (R_{1.4} < 13.4 \text{ km})\) (Astphys.J857.L23 C. Raithel et al.)

→ stiffness of EOS should be bounded at some point
Possible equation of state

- Based on machinery computations

- Gradual “soft” increment after “stiff” increment (small $v_s^2$ after sudden increment)
- The soft part is constrained by pQCD limit
Speed of sound

- Fast and slow speed

\[ v_s^2 = \frac{\partial p}{\partial \epsilon} = \frac{n}{\mu(\partial n/\partial \mu)} \quad , \quad \frac{\delta \mu}{\mu} \sim v_s^2 \frac{\delta n}{n} \quad , \quad v_s^2 < 1 \quad (\text{causality}) \]

If \( v_s^2 \sim 1 \), the unit increment of chemical potential will be directly proportional to the unit increment of density \( \rightarrow \) requires tremendously large repulsive interaction

At some points, this huge force should be turned off to satisfy the \( \tilde{\Lambda} < 800 \) constraint
Large $N_C$ theory and Quarkyonic matter

- Quark Fermi sea cannot screen gluon

\[
\Pi_{\mu\nu}^{ab}(Q) = g^2 \delta^{ab} \int \frac{d^4K}{(2\pi)^4} \text{Tr} \left[ \gamma_\mu S_F(K)\gamma_\nu S_F(K - Q) \right] \\
= m^2 \delta^{ab} \int \frac{d\Omega}{4\pi} \left( \delta_{\mu4} \delta_{\nu4} + \hat{K}_\mu \hat{K}_\nu \frac{i\omega}{Q \cdot \hat{K}} \right) \\
m^2 = \frac{1}{3} g^2 T^2 \left( C_A + \frac{1}{2} n_f \right) + \frac{1}{2} g^2 \sum_f \frac{\mu_f^2}{\pi^2} \sim \frac{1}{N_C} \\
g_T^2 N_C = \text{const}. \sim \mathcal{O}(1/N_C)
\]

Quarks on the Fermi surface → confined as done in vacuum confinement mechanism

- A lot of rooms for quarks in $\Lambda_{QCD}$ ball

(Figure from PRL122(2019)122701)
Broadly distributed quarks

- Quarks in $\Lambda_{QCD}$ ball can broadly distributed ($k_N^F < \Lambda_{QCD}$)

(Figure from PRL122(2019)122701)
Fast enough nucleon-like shell

- Once quark Fermi sea is formed ($k_N^F \sim \Lambda_{QCD}$)

  - There is no sudden jump of total baryon number density
    \[ n_B^{\text{tot}} = \frac{2N_f}{3\pi^2} k_q^3 \]

  - If one assumes constituent quark model ($m_q \sim \Lambda_{QCD}$), $\mu_N$ grows as
    \[ \mu_N = \sqrt{(N_f k_q^F)^2 + m_N^2} \]

  - Total energy density is kept
    \[ \epsilon_q \sim \Lambda_{QCD} n_q \]
    \[ \sim N_c \Lambda_{QCD}^4 = M_N \Lambda_{QCD}^3 \sim \epsilon_N \]
    \[ \rightarrow \text{ pressure is enhanced (stiff EOS)} \]
    \[ p = -\epsilon + \mu n \]
    (\(\epsilon, n\) are kept in similar order)

Detailed explanations are given in many literatures Nucl.Phys.A796.83, arXiv:1904.05080, etc.
Phenomenological approaches

- $N_c \to \infty$ limit contains intrinsic scale ($\Lambda_{QCD}$) and divergence ($N_c$)

- Like Hagedorn model

  $$\lim_{E \to \infty} \rho(E) dE = \frac{a}{E^{5/2}} e^{E/T_0} dE$$

  $$\lim_{T \to T_0} E \sim \alpha \frac{T_0^2}{T_0 - T}$$

  Density distribution measure for high energy state
  → leads to singular entropy $S \sim |T - T_0|^{-\alpha}$
  Near the critical temperature, the energy diverges
  → system prefer to make new degree of freedom

- If one tries singular $\mu$ near critical density $n_0$ and quark d.o.f.

  $\gamma = 0.7$
  $n_0 \sim 4 \rho_0$

  $\mu_N = M + \kappa \frac{M}{N_c^2} \left\{ (1 - n_N^N/n_0)^{-\gamma} - 1 \right\}$

  $\nu_s^2 = \frac{n_N^N + n_Q^N}{\mu_N (dn_N^N/d\mu_N + dn_Q^N/d\mu_N)}$

  Sudden increment of $\mu$ with onset of quarks and recovery of pQCD limit by large number of quarks

Hard-core repulsion by “effective size”

• To be more realistic

Lattice QCD study for hadron interaction (Front.Phys.13(2018)132105 T. Hatsuda)

• Hard-core nature can be embodied by semi-classical size $v_0$

\[ n_0 = 1/v_0 \quad V_{\text{ex}} = V (1 - n/n_0) \]
\[ n_{\text{ex}} = \frac{n}{1 - n/n_0} = \frac{2}{(2\pi)^3} \int^{k_F} d^3p \]
\[ \epsilon_{\text{ex}} = \frac{2}{(2\pi)^3} \int^{k_F} E_p d^3p \]

Hardcore density and excluded volume
\[ \rightarrow \text{reduced available space leads to accelerated nucleons} \]

Energy density of the total system size can be calculated by multiplying excluded volume factor
Quarkyonic-like shell structure

- Considering Pauli exclusion principle

\[ \tilde{\epsilon} = 4 \left(1 - \frac{n_N}{n_0} \right) \int_{k_F}^{k_F+\Delta} \frac{d^3k}{(2\pi)^3} \left( (N_c m_Q)^2 + k^2 \right)^{\frac{1}{2}} + \frac{2N_c}{\pi^2} \int_{0}^{k_F/N_c} \frac{kk}{\Lambda} \left( \Lambda^2 + k^2 \right)^{\frac{1}{2}} \left( m_Q^2 + k^2 \right)^{\frac{1}{2}} \]

- Quark density is determined by minimization of energy density
- IR cutoff \( \Lambda \) is introduced to regularize singularity (artifact) at onset of quark

\[ \tilde{n}_Q^N = \frac{2}{3\pi^2} \left( (k_Q^2 + \Lambda^2)^{\frac{3}{2}} - \Lambda^3 \right) \]

- Appearance of quarks makes the stiff increment of chemical potential

![Graphs showing the relationship between quark density and quark chemical potential](image)
Quarkyonic-like shell structure

- After tuning hard-core density to match energy density scale

- Hardcore repulsive interaction reproduces the EOS deduced from Quarkyonic picture

- What would be happening if we consider the EM charge and strangeness?
Electrons and strangeness

- EM charge neutrality and $\beta$-equilibrium

\[
\begin{align*}
\mu_n &= \mu_p + \mu_e \\
\mu_d &= \mu_u + 3\mu_e \\
\mu_n &= \mu_\Lambda \text{ (when } n_\Lambda \neq 0, n_\Lambda = 0 \text{ if } \mu_\Lambda < m_\Lambda) \\
\mu_d &= \mu_\bar{s} \text{ (when } n_\bar{s} \neq 0, n_\bar{s} = 0 \text{ if } \mu_\bar{s} < N_cm_s) \\
\end{align*}
\]

$q$ represents unit in baryon number
3 flavor-matter

- As naïve trial, mean-field type mixture of nucleon gases

\[
\begin{align*}
n_{N}^{ex} &= \frac{n_n}{1 - n_B/n_0} + \frac{n_p}{1 - n_B/n_0} + \frac{n_\Lambda}{1 - n_B/n_0} \\
n_{\bar{B}} &= n_p + n_n + (1 + \alpha)n_\Lambda \\
\alpha \text{ determines the mean-field repulsion of } \Lambda \\
\Lambda \text{ exists only when } 0 < \alpha < 0.35 \\
\text{Lattice QCD calculation shows } \alpha \sim 0.2 \\
\end{align*}
\]
3 flavor mean-field mixture of fermions

- if quarks are included (for now we forget about Pauli's principle)

\[
\epsilon = 2 \left(1 - \frac{n_B}{n_0}\right) \sum_i \int_{k_{F_i}}^{k_{F_i}^B} \frac{d^3 k}{(2\pi)^3} \left( m_{B_i}^2 + k^2 \right)^{1/2} + \frac{N_c}{\pi^2} \sum_i \int_0^{k_{F_i}^Q} d k k \left( \Delta_{Q_i}^2 + k^2 \right)^{1/2} \left( m_{Q_i}^2 + k^2 \right)^{1/2}
\]

Too soft EOS (6 flavor mixture)

Shell structure can make stiff EOS!
Further issues

• 3 flavor quarkyonic-like matter

\[ \epsilon = 2 \left(1 - \frac{n_B}{n_0}\right) \sum_i \int_{k_{F_i}}^{[k_{F}+\Delta]_B} \frac{d^3k}{(2\pi)^3} \left( m_{B_i}^2 + k^2 \right)^{\frac{1}{2}} + \frac{N_c}{\pi^2} \sum_i \int_0^{k_{Q_i}} dk k \left( \Lambda_{Q_i}^2 + k^2 \right)^{\frac{1}{2}} \left( m_{Q_i}^2 + k^2 \right)^{\frac{1}{2}} \]

→ in progress

• What is in the shell? quasiparticles or crystalized lattice?
  - Skyrme like crystal structure may exist in large $N_c$ limit
  - If the crystal story is true, what would be the order parameter?
  - How to understand the hardcore repulsion from QCD($N_c \to \infty$)?